On the Origins of Varying Gauge Couplings

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Motivation

- Several constructions utilize the following effective Lagrangian:

$$\mathscr{L} \supset -\frac{1}{2}\left(\frac{1}{g^2} - \frac{\phi}{\Lambda}\right) \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Varying fine-structure (Lee, et. al., '22, Chluba & Hart, '23, Ghosh, et. al., '25), electroweak confinement (Berger, et. al., '19, Howard, et. al., '21, Bhalla-Ladd, et. al., '25), ultralight scalars (Brzeminski, et. al., '20, Banerjee, et. al., '22, Becker, et. al., '25), early QCD confinement (Ipek & Tait, '18, Lu, et. al., '22)…

- Before BBN/EWPT little is known, dynamics may have differed drastically from standard cosmological evolution

- Gauge couplings drive early-universe dynamics and since $g \to g(\mu)$, different cosmological epochs map to different energy scales

- RGE-driven evolution depends only on heavy thresholds, only slow (logarithmic!) variations in energy are possible

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Is this interpretation theoretically consistent? Can vacuum shifts induce large variations? How fast can gauge couplings change at low energies?

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* RGE-drive... depends only on heavy thresholds, only slow (logarithmic!) variations in energy are possible

* Drastic changes of couplings implies $\langle\phi\rangle \sim \Lambda$... e.g. confined EW requires $\frac{1}{g^2} \approx \frac{\langle\phi\rangle}{\Lambda} \sim 2.37$

Is this interpretation theoretically consistent? Can vacuum shifts induce large variations? How fast can gauge couplings change at low energies?

Varying Gauge Coupling Induced by Singlet Scalar…

- The EFT description breaks down in this regime!

$$\mathscr{L} \supset -\frac{1}{2}\left[\frac{1}{g^2} - \sum_n c_n \left(\frac{\phi}{\Lambda}\right)^n\right] \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Require: Low energy containing only singlet ϕ, described by 4D QFT, including the approach to the UV
- Generically can classify possible UV completions into Higgs-like or Extra dimensions

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- As ϕ shifts, $m(\phi)$ moves the RG threshold where ψ enters the $\beta(\mu)$, inducing a field-dependent low-energy coupling

Outline

1. The Higgs-like (4D) charged matter UV-completion
2. The restriction of gauge invariance
3. A caveat in 4D...
4. A "natural" alternative
5. Phenomenological implications
6. The bottom line

The Charged Matter UV-Completion

The Charged Matter UV-Completion

* Leading term in the one-loop effective action (Shifman, et. al., '79): $\mathscr{L} \supset -\frac{1}{2g^2(\mu)}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$

* $g(\mu)$ determined by RGE, well known solution: $\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}\ln\left(\frac{\mu}{\Lambda}\right)$

* The slope is fixed by all light charged degrees of freedom present at that scale:

$$\tilde{b} = \frac{1}{4\pi^2}\left[\frac{11}{3}C_2(G) - \frac{2}{3}\sum_f S_2(r_f) - \frac{1}{3}\sum_s S_2(r_s)\right]$$

* Distinct coefficients $\tilde{b}_>$ $(m(\phi) < \mu)$ and $\tilde{b}_<$ $(m(\phi) > \mu)$, $g(\mu)$ continuous across thresholds at leading order

* ϕ dependence of the gauge coupling arises when $\mu < m(\phi)$... Running and matching gives

$$\frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}_>\ln\left(\frac{\mu}{\Lambda}\right) + 2\tilde{b}_\Delta\ln\left[\frac{\mu}{m(\phi)}\right]\Theta\left[m(\phi)-\mu\right]$$

Logarithmic, Not Linear

- Re-expressed via the coupling measured today, $g(\mu_0, \phi_0)$:

$$\frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\mu_0,\phi_0)} + 2\tilde{b}_> \ln\left(\frac{\mu}{\mu_0}\right) + 2\tilde{b}_\Delta \left\{ \ln\left[\frac{\mu}{m(\phi)}\right] \Theta\left[m(\phi)-\mu\right] - \ln\left[\frac{\mu_0}{m(\phi_0)}\right] \Theta\left[m(\phi_0)-\mu_0\right] \right\}$$

$$\text{assuming } \mu_0 < m(\phi_0) \implies \mathscr{L}_{\text{eff}} = -\frac{1}{2}\left[\frac{1}{g^2(\mu_0,\phi_0)} + \tilde{b}_< \ln\left(\frac{\mu}{\mu_0}\right) - 2\tilde{b}_\Delta \ln\left(\frac{|M+y\phi|}{m(\phi_0)}\right)\right] \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

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- The dependence on the scalar is logarithmic, not linear, re-summing the EFT tower keeps it valid for $|\Delta m|/m \gtrsim 1$
- Applies for any charged matter whose mass depends on ϕ; multiple fields summed!
- Small excursions recover the dimension-5 form, but the variation is far slower than linear

$$\mathscr{L} \supset -\frac{1}{2}\left[\frac{1}{g^2(\mu,\phi_0)} - 2\tilde{b}_\Delta \frac{1}{m}\frac{dm}{d\phi}\bigg|_0 (\phi-\phi_0) + \ldots\right] \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

Couplings in the Early Universe: Higgs-Like

- Toy abelian model: N_f vector-like fermions + ϕ; a second-order transition profile

$$V(\phi, T) = \frac{\lambda}{2}\left(\phi^2 - \langle\phi\rangle^2\right)^2 + f(\lambda, y, \ldots)\phi^2 T^2$$

$$\Longrightarrow \langle\phi(T)\rangle = \Theta(T_c - T)\langle\phi\rangle\sqrt{1 - \left(\frac{T}{T_c}\right)^2} \qquad m(\phi) = |M \pm y\langle\phi(T)\rangle|$$

- The probed energy scale is the temperature ($\mu \to T$) of the bath; as the universe cools, thresholds shift

- With $N_f = 100$, many distinct UV theories flow to the same IR coupling

- The naïve linear EFT drastically overestimates the achievable variation.

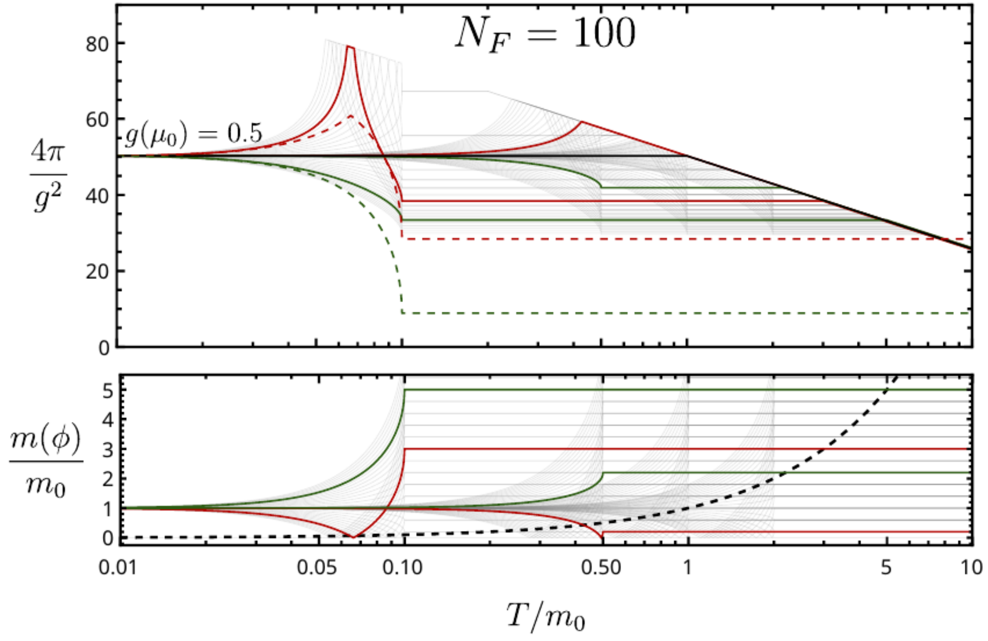


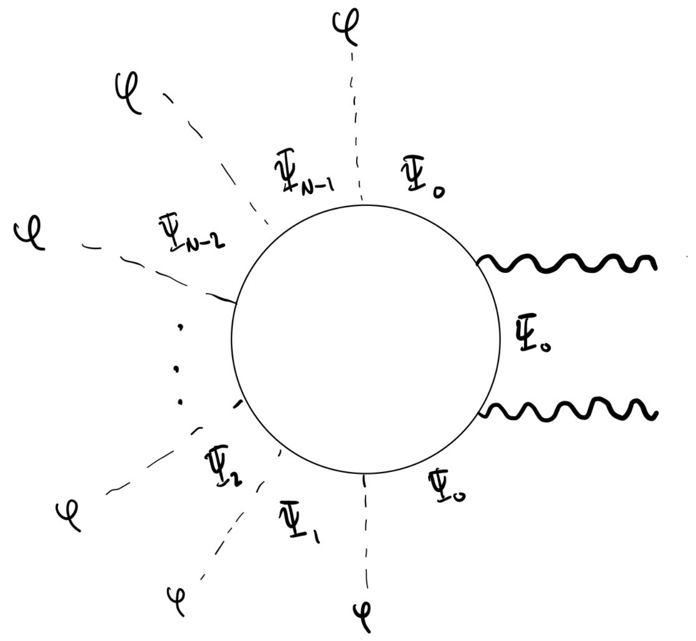
Fig. 1 Coupling fixed in the IR; dashed = naïve linear

Symmetries on the Scalar

- When ϕ transforms non-trivially under a symmetry, dependence of the effective coupling can be modified
- Consider theory with ϕ and N_f fermions ψ_n, $n = 0,1,2,\ldots, N_f - 1$ and invariant under $\mathbb{Z}_{N_f}$ symmetry
  $\phi \to e^{2i\pi/N_f}\phi$, $\psi_n \to e^{-2i\pi n/N_f}\psi_n$

$$\mathscr{L} = -\sum_{k=0}^{N-1}\left[M_k\bar{\psi}_k\psi_k + y_k\phi\bar{\psi}_k\psi_{k+1} + Y\bar{\psi}_0\phi\psi_{N_f} + \mathrm{h.c.}\right]$$

$$\Longrightarrow \frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}_{>}\ln\left(\frac{\mu}{\Lambda}\right) + 2\tilde{b}_{\Delta}\ln\left[\frac{\mu^{N_f}}{|\det\, m(\phi)|}\right]$$



- The symmetry is encoded in $|\det\, m(\phi)|$... e.g. $N_f = 3$:

$$\det\, m(\phi) = M_0M_1M_2 - M_0|y_1\phi|^2 - M_1|y_2\phi|^2 - M_2|y_0\phi|^2 + (y_0y_1Y\phi^3 + \mathrm{h.c.})$$

- ϕ^3 term $\propto Y\ldots$ $Y \to 0$ restores a larger $U(1)$ symmetry.

Enhanced Symmetry → Higher Operators

- Generalizing... $\det\, m(\phi) \supset \prod_i M_i |y_{i+1}\phi|^2 + Y\phi^{N_f}$
- Expand $\phi = \phi_0 + \Delta\phi$...

$$\ln|\det\, m(\phi)| = \ln|\det\, m(\phi_0)| + \mathrm{Re}\left[\mathrm{Tr}\left(m^{-1}\frac{dm}{d\phi}\right)_{\phi_0}\right]\Delta\phi + \frac{1}{2}\mathrm{Re}\left[\mathrm{Tr}\left(m^{-1}\frac{d^2m}{d\phi^2}\right)_{\phi_0} - \mathrm{Tr}\left(m^{-1}\frac{dm}{d\phi}\right)^2_{\phi_0}\right](\Delta\phi)^2$$

- Generic point: the trace is non-zero → leading dependence is linear (a dimension-5 operator).
- Enhanced-symmetry point (e.g. $\phi_0 = 0$): leading dependence is quadratic or higher (dimension-6+).
- Qualitatively different low-energy EFT... significant phenomenological implications (Brzeminski, et. al., '20, Banerjee, et. al., '22, Becker, et. al., '25, Ghosh, et. al., '25)

The Restriction of Gauge Invariance

Gauge Invariance in 4D ensures logarithmic variation

- Gauge invariance forbids mass/kinetic mixing of gauge bosons with neutral fields (exception: Abelian kinetic mixing)
- Up to dimension four, gauge bosons couple only to charged matter, which feeds back at one loop
- To change the coupling we must change charged matter… only masses and non-gauge couplings may vary
- At one loop, only masses reach the gauge running
- How does a moving mass threshold enter the coupling?

The Logarithm Is Generic

- $G_> \supseteq G_<$ matched at massive threshold m
- Spontaneous breaking of scale invariance by $\phi \to e^{\lambda}\phi$
- Above m have renormalizable theory, no ϕ dependence; scale anomaly coefficient at one-loop: $\partial_\mu D^\mu = \tilde{b}_> \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$
- Below $m(\phi)$ expand in powers of g:

$$\mathscr{L}_{\text{eff}} \supset -\frac{1}{2}\left[\frac{1}{g^2(\mu)} + f(\phi) + \mathcal{O}(g^2)\right] \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Scale anomaly coefficient below threshold:

$$\partial_\mu D^\mu = \left(\tilde{b}_< - \frac{1}{2}\frac{df}{d\ln\phi}\right) \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Matching the light-field anomaly across the threshold fixes $f(\phi)$:

$$\frac{df}{d(\ln\phi)} = -2(\tilde{b}_< - \tilde{b}_>) = -2\tilde{b}_\Delta \implies f(\phi) = \text{constant} - 2\tilde{b}_\Delta \ln\phi$$

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There are some caveats!

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A Caveat in 4D

Yukawas Vary at Tree Level

- Yukawas are not protected by gauge invariance

- Froggatt–Nielsen-style UV completion (Froggatt & Nielsen, '79, Leurer, et. al., '92, Leurer, et. al., '93):

$$-\mathscr{L} \supset \xi^c \Phi (y\eta + Y\psi) + (m + f\phi)\psi^c \eta + (M + F\phi)\psi^c \psi$$

	η	ξ^c	ψ	ψ^c	Φ	ϕ
$SU(N)$	$\mathbb{1}$	$\bar{\boldsymbol{N}}$	$\mathbb{1}$	$\mathbb{1}$	$\boldsymbol{N}$	$\mathbb{1}$
$U(1)$	1	-1	1	-1	0	0

- Integrate out ψ and re-sum ϕ insertions:

$$-\mathscr{L}_{\text{eff}} \supset \xi^c \Phi \eta \left(y + \frac{Yf\phi}{M + F\phi} \right) \equiv y(\phi) \xi^c \Phi \eta \qquad y(\phi) = y + \frac{fY\phi}{M + F\phi}$$

- A genuine linear dependence (EFT restructuring once $F\phi \gtrsim M$)

Faster-than-Linear Yukawas

- Add a second heavy pair protected by a $\mathbb{Z}_2$; the leading insertion is now ϕ^2:

$$-\mathscr{L}_{\text{eff}} \supset \xi^c \Phi \eta \left(y + \frac{Y f F_{12} \phi^2}{M_1 M_2 - F_{12} F_{21} \phi^2} \right) \equiv y(\phi) \xi^c \Phi \eta \qquad y(\phi) = y + \frac{Y f F_{12} \phi^2}{M_1 M_2 - F_{12} F_{21} \phi^2}$$

- Larger discrete symmetries push the leading power higher, giving

$$y(\phi) \simeq Y \left[\frac{\phi}{\Lambda(\phi)} \right]^p$$

- Exponent controlled by symmetry... power-law sensitivity, not from any logarithm!

Two-Loop Caveat: Yukawa Couplings Augment the Running

- At two loops the Yukawa enters the gauge β-function:

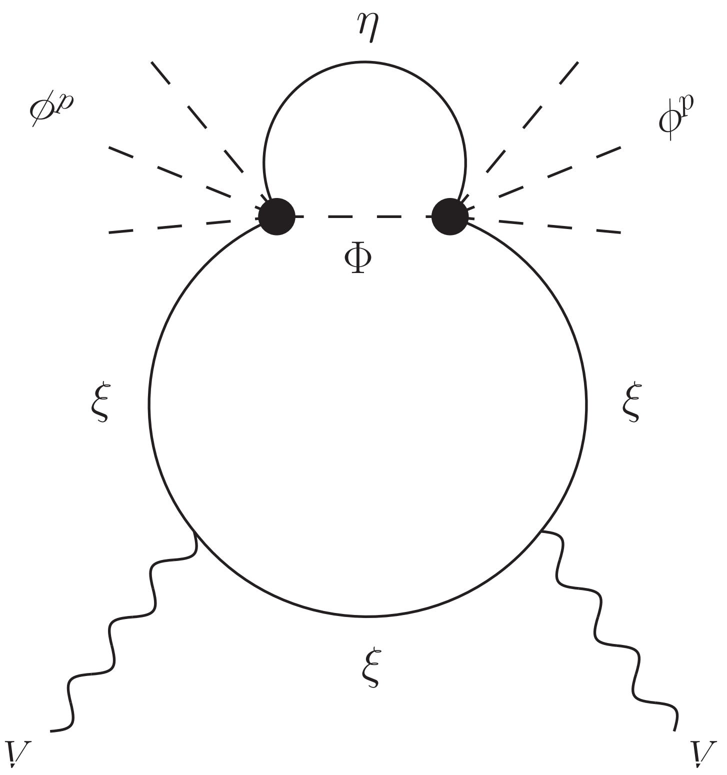
$$\frac{dg}{dt} \simeq -\frac{b_1}{(4\pi)^2}\left[1+\frac{b_2^{(y)}}{b_1(4\pi)^2}y^2\right]g^3$$

- Integrating (with Yukawa as effective β coefficient):

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + \frac{2b_1}{(4\pi)^2}\left[1+\frac{b_2^{(y)}}{b_1(4\pi)^2}y^2\right]\ln\left(\frac{\mu}{\Lambda}\right)$$

- With $y \to y(\phi) \simeq Y\left[\frac{\phi}{\Lambda(\phi)}\right]^p$, the gauge coupling inherits a power-law piece:

$$\left(\frac{1}{g^2}\right)_{2-\text{loop}} \propto y^2(\phi)\ln\left(\frac{\mu}{\Lambda}\right) \sim Y^2\left[\frac{\phi}{\Lambda(\phi)}\right]^{2p}\ln\left(\frac{\mu}{\Lambda}\right)$$



A “Natural” Alternative

A "Natural" Alternative

- In 5D $[g] = M^{-1}$: possible to have stronger variation
- Kaluza-Klein (KK) decomposition provides natural setting for large multiplicity of charged states
- Consider the 5D theory with $z \in \{0, R\}$: $\mathscr{L}_5 = -\frac{1}{4}F_{MN}F^{MN} + \frac{1}{2}\bar{\Psi}i\Gamma^M D_M\Psi - M\bar{\Psi}\Psi - \frac{y}{\sqrt{R}}\Phi\bar{\Psi}\Psi + \frac{1}{2}|\partial\Phi|^2 - V(\Phi)$
- Potential induces $\Phi = \sqrt{R}\phi$; take $\Psi_L|_0 = 0,\ \Psi_R|_R = 0$
- Zero-mode forbidden: $m_n^2(\phi) = (M + y\phi)^2 + \left(\frac{\omega_n}{R}\right)^2,\ \tan(\omega_n) = -\frac{\omega_n}{|M + y\phi|R}$
- For $\mu_0 < m_1$: $\frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\mu_0,\phi_0)} + \tilde{b}_\Psi \sum_n^\infty \ln\left(\frac{\mu^2}{m_n^2}\right)\Theta(\mu - m_n),\ \tilde{b}_\Psi = -\frac{4Q^2}{3(4\pi)^2}$
- Running scale $\mu < \Lambda \simeq \frac{4\pi^2}{g^2 R} \implies n \lesssim \frac{4\pi}{g^2}$

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- $\mu \gg \frac{\pi}{R}, |M + y\phi| \implies N \simeq \frac{\mu R}{\pi}$:

$$\frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\mu_0,\phi)} + 2b_\Psi\left[\ln\left(\frac{\mu R}{N\pi}\right) + \ln\left(\frac{N^N}{N!}\right) - \sum_{n=1}^{N}\ln\left(\frac{m_n R}{n\pi}\right)\right]$$

$$\simeq \frac{1}{g^2(\mu_0,\phi)} - 2b_\Psi\left[\left(\frac{\mu R}{\pi}\right) + \sum_{n=1}^{N}\ln\left(\frac{m_n R}{n\pi}\right)\right]$$

- ϕ VEV does not affect UV gauge coupling, fractional changes in KK masses decrease for increasing KK levels
- Matching gives IR relation: $\frac{1}{g^2(\mu,\phi)} \simeq \frac{1}{g^2(\mu_0,\phi_0)} - b_\Psi\sum_{n=1}^{\infty}\ln\left(\frac{m_n^2(\phi)}{m_n^2(\phi_0)}\right)$

Couplings in the Early Universe: Flat Extra Dimension

- Toy bulk abelian model generates $\Phi \to \sqrt{R}\phi$ with same second-order transition profile:

$$\langle \phi(T) \rangle = \Theta(T_c - T)\langle \phi \rangle \sqrt{1 - \left(\frac{T}{T_c}\right)^2}$$

$$m_n^2(\phi) = \left(M \pm y\langle \phi(T) \rangle\right)^2 + \left(\frac{\omega_n}{R}\right)^2$$

- Again take temperature ($\mu \to T$) of the bath; as the universe cools, thresholds shift
- Sensitive to number of states present at low-energy, key difference: if more modes stack together and allow for faster changes
- Increasing R allows higher multiplicity of modes… edge of reliability of KK description

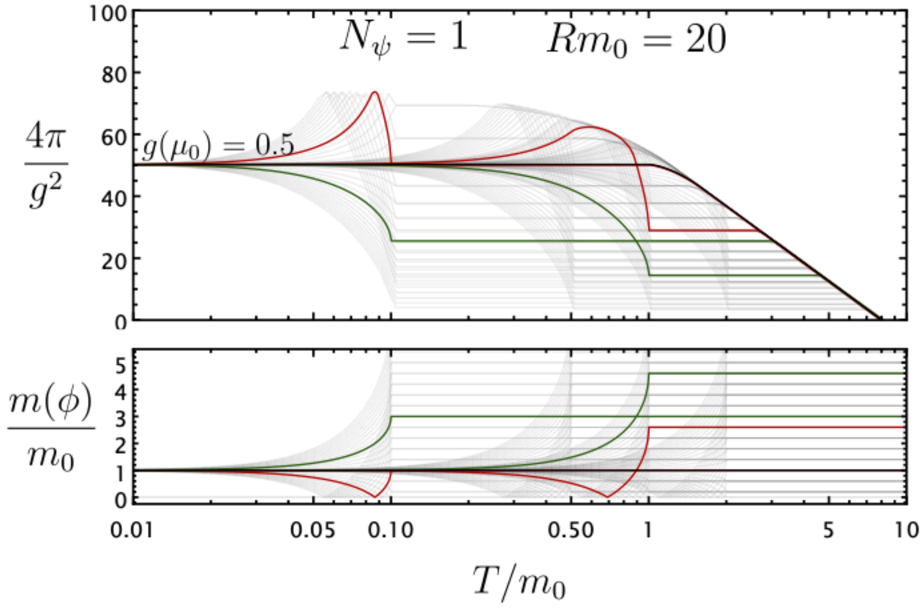

Fig. 1 Coupling fixed in the IR

Phenomenological Implications (mostly) in 4D

Phenomenology of New Charged Matter

- New, stable charged matter very constrained, cosmologically
- Can decay directly or indirectly to SM
  - Requires non-trivial electroweak charge $\Longrightarrow$ changing one gauge coupling changes others
  - Cascade decays: e.g. electroweak multiplet dark matter (Cirelli et. al. Nucl. Phys. B 753 (2006) 178)
- Additional collider constraints on electroweak and color charged matter
  - $\tau \gtrsim 1\mu s$ requires $m \gtrsim 2$ TeV (color) and $m \gtrsim 400 - 1200$ GeV (electrically charged) (CMS Collaboration, JHEP 04 (2025) 109, ATLAS Collaboration, JHEP 7 (2025) 140)
  - More freedom with cascade decays, $m \gtrsim 140$ GeV (Higgsino-like) and $m \gtrsim 1000$ GeV (color) (ATLAS Collaboration JHEP 06 (2026) 094, CMS Collaboration, arXiv:hep-ph/2604.25604)

Dynamical Gauge Variations via Singlet (Thermalized)

- 4D UV completion of EFT with $\phi \to \phi_0$ coupling to new matter charged under $SU(3)_C$ or $SU(2)_L$
- For $\mu < m(\phi)$, change in the coupling relative to the running SM value $\alpha_i(\mu, \phi_0) = g_i^2/4\pi$

$$\Delta\alpha_i(\mu,\phi) \simeq \frac{b_\Delta}{2\pi}\alpha_i^2(\mu,\phi_0)\ln\left(\frac{m(\phi)}{m(\phi_0)}\right),\ b_\Delta = (4\pi)^2\tilde{b}_\Delta = \frac{2N_\psi}{3}$$

- Baryogenesis example (S.A.R Ellis, et. al., JHEP8 (2019) 002): $\Delta\alpha_2 > 0$ or $\Delta\alpha_3 < 0$ at $T \sim 100$ GeV could enable EWBG
- Generating Y_B with $\Delta\alpha_3 = 0$ requires $\Delta\alpha_2 \gtrsim 0.01 \implies b_\Delta \ln\left(m(\phi)/m(\phi_0)\right) \gtrsim 55$ or
  $N_\psi \gtrsim 83$ Dirac fundamentals for $\ln(m/m_0) = 1$
- Generating Y_B with $\Delta\alpha_2 = 0$ requires $\Delta\alpha_3 \lesssim -0.03 \implies b_\Delta \ln\left(m(\phi)/m(\phi_0)\right) \lesssim -16$ or
  $N_\psi \gtrsim 24$ Dirac fundamentals for $\ln(m/m_0) = -1$

Dynamical Gauge Variations via Singlet (Thermalized)

- 4D UV completion of EFT with $\phi \to \phi_0$ coupling to new matter charged under $SU(3)_C$ or $SU(2)_L$
- For $\mu < m(\phi)$, change in the coupling relative to the running SM value $\alpha_i(\mu, \phi_0) = g_i^2/4\pi$

$$\Delta\alpha_i(\mu, \phi) \simeq \frac{b_\Delta}{2\pi}\alpha_i^2(\mu, \phi_0)\ln\left(\frac{m(\phi)}{m(\phi_0)}\right),\ b_\Delta = (4\pi)^2\tilde{b}_\Delta = \frac{2N_\psi}{3}$$

- Early QCD phase transition (D. Berger, et. al. JHEP 07 (2020) 192, S. Ipek, T.M.P Tait PRL 22 (2019) 112001,...) :

$$\Lambda(\phi) \simeq \Lambda_{\rm QCD}\left(\frac{m(\phi)}{m(\phi_0)}\right)^{b_\Delta/b_{\rm QCD}},\ \Lambda_{\rm QCD} \simeq 50\ {\rm MeV},\ b_{\rm QCD} = 7,\ b_\Delta = \frac{2}{3}N_\psi$$

- Envision $\Lambda(\phi) \gtrsim T_{\rm EWPT} \sim 100$ GeV, requires $\mathcal{O}\left(10^3\right)$ enhancement

$$\Longrightarrow \frac{m(\phi)}{m(\phi_0)} \gtrsim 1400,\ \text{for } N_\psi = 10,\ \frac{m(\phi)}{m(\phi_0)} \gtrsim 10,\ \text{for } N_\psi = 30,\ \frac{m(\phi)}{m(\phi_0)} \gtrsim 2,\ \text{for } N_\psi = 100$$

Dynamical Gauge Variations via Singlet (Non-Thermalized)

- Dynamical variations can also be generated by an non-thermalized singlet scalar
- After an initial displacement, oscillate freely until $H < m_\phi$
- Parameterizing the physical excitation as $\eta(x) = \phi(x) - \phi$, the coupling to photons, gluons and light quarks is parameterized as (D. Antypas, et. al. 2203.14915, D.B. Kaplan and M.B. Wise, JHEP 08 (2000) 037,...)

$$\mathscr{L} \supset \sum_{n\geq 1} \frac{1}{n} \left(\frac{\eta}{\sqrt{2} M_{\rm Pl}} \right)^n \left[\frac{d_e^{(n)}}{4e^2} F_{\mu\nu}F^{\mu\nu} - \frac{\beta_3 d_g^{(n)}}{2g_3^2} G^a_{\mu\nu} G^{a\mu\nu} - \sum_q \left(d_{m_q}^{(n)} + \gamma_m d_g^{(n)} \right) m_q \bar{q} q \right]$$

- Constrained by equivalence principle violation (D. Antypas, et. al. 2203.14915), nuclear mass changes (A. Hees, et. al. PRL 117(2016) 061301), from BBN/CMB (S. Ghosh, et. al. PRD 114 (2026) 023525)
  - Especially strong constraints on variations arising on $d^{(1)} = \frac{\alpha b_\Delta}{2\pi} \frac{\sqrt{2} M_{\rm Pl}}{m} \frac{dm(\phi)}{d\phi} \Big|_{\phi=\phi_0}$

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- With $\phi \to -\phi$ and $\phi_0 = 0$ can forbid $n = 1$ with $m^2(\phi) = M^2 \mp y^2\phi^2$
    - For $n = 2$ the variation is $\dfrac{\Delta\alpha}{\alpha} \simeq \mp \dfrac{d^{(2)}M^2}{4y^2M_{\mathrm{Pl}}^2} \ln\left(1 \mp \dfrac{y^2\phi^2}{M^2}\right)$
    - Greatly relaxes constraints in this case (T. Bouley, et. al. JHEP 03 (2023) 104, S. Ghosh, et. al. PRD 114 (2026) 023525)

Gauge Couplings vs. Yukawa Couplings in 4D

GAUGE COUPLINGS

$$\mathscr{L}_{\text{eff}} \supset -\frac{1}{2}\left[\frac{1}{g^2(\mu)} - c\ln\left(1+\frac{\Delta m(\phi)}{M}\right)\right]\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Protected by gauge invariance
- Enter only at one loop, only through mass thresholds
- Sensitivity is logarithmic in ϕ

YUKAWA COUPLINGS

$$y(\phi) \simeq Y\left[\frac{\phi}{\Lambda(\phi)}\right]^p$$

- Unprotected can be modified at tree-level
- Symmetry and mass hierarchy set power p
- Sensitivity can be power-law in ϕ

Bottom line: *in UV-complete 4D theories, gauge couplings must vary logarithmically; reaching power-law and/or drastic variation costs an extra loop (via power-law Yukawas) or a high multiplicity of charged matter (via extra dimensions).*

Thank You!

Back-up Stuff on Dynamics of Singlet Scalar

Dynamics of the Singlet Scalar

- $\psi + \bar{\psi} \to \phi + A_\mu$ populates the scalar drives it toward equilibrium
- Charged matter coupling $\Longrightarrow$corrections to $V_{\text{eff}}(\phi)$
  - Quantum corrections at $T = 0$: $\Delta m_\phi^2 \sim \dfrac{d(r_\psi) N_\psi y^2}{(4\pi)^2} M^2,\ \Delta\lambda \sim \dfrac{d(r_\psi) N_\psi y^4}{(4\pi)^2}$
  - Quantum corrections at $T \neq 0$: $\Delta V(\phi) \propto \begin{cases} m_\psi^2(\phi) T^2,\ T \gg m(\phi) \\ g^2(\mu \sim T, \phi) T^4,\ T \lesssim m(\phi) \end{cases}$

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- When thermalized, finite T corrections $\Longrightarrow$ phase transition, changing ϕ and $g(\mu, \phi)$
  - Significant changes need large $\Delta m(\phi)$ driven by ϕ couplings to *lighter* fields
  - Thermalized ϕ produced abundantly, decays to $\psi + \bar{\psi}$ $(m_\phi > 2m_\psi)$ or to vector bosons $(m_\phi < 2m_\psi)$

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- Non-thermalized scalar coherently oscillates at late times
  - In $y \to 0$ limit with initial displacement ϕ_i and $\phi = 0$ today, trapped until $H \sim m_\phi$
  - ϕ rolls to minimum, oscillates, redshifts as matter

$$f_{\text{DM}} \simeq \left(\frac{m_\phi}{4.5 \times 10^{-17}\ \text{eV}}\right)^2 \left(\frac{\bar{\phi}(t_0)}{100\ \text{GeV}}\right)^2$$