Can pre-inflationary axions form miniclusters?

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Work in progress, in collaboration with
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- Motivation
- Temperature-sourced fluctuations
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Axion dark matter substructure

Post-inflationary	Pre-inflationary
PQ symmetry breaks after inflation.	PQ symmetry breaks before inflation.
$\mathcal{O}(1)$ misalignment variations on horizon scales at PQ breaking.	θ_{init} uniform across the observable universe.
String + Domain Wall source large $\delta\rho/\rho$	No strings or domain wall
⟶ Miniclusters & Minivoids well-studied	⟶ Naïve expectation: smooth axion DM

But: adiabatic $\delta T/T$ from inflation is still there.

Does it source enough fluctuation to seed substructure?

Eggemeier et al. 2212.00560

Pierobon et al. 2311.17367

Why this matters

Direct detection

Local DM density is set by substructure

Lensing & pulsar timing (?)

Pre-inflationary is the cleaner window

Single θ_{init} removes string/wall uncertainties

Predictions follow from inflation + QCD

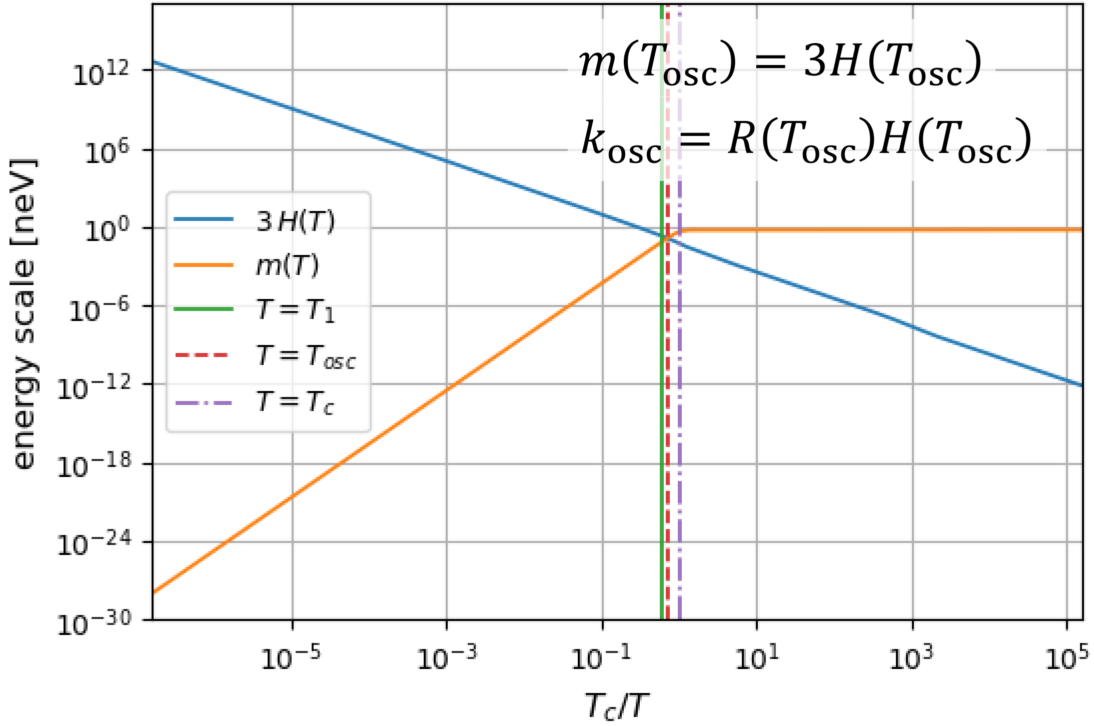
The QCD axion: field and mass

Real scalar $a(x) = f_a\,\theta(x)$, with cosine potential $$V = f_a^2 m^2(T)(1 - \cos\theta)$$

Temperature-dependent topological susceptibility $\chi(T)$ (Lattice QCD)

$$m^2(T) = \frac{\chi(T)}{f_a^2} = \frac{\chi_0/f_a^2}{1 + \left(\frac{T}{T_c}\right)^b}$$

$$\chi_0 = (75.5\ \mathrm{MeV})^4, \qquad T_c = 0.1565\ \mathrm{GeV}, \qquad b = 8.16$$



Linearised axion perturbation

Decompose $\theta(t,x) = \bar{\theta}(t) + \delta\theta(t,x)$, $T(t,x) = \bar{T}(t) + \delta T(t,x)$.

Linearise the cosine potential around the homogeneous mode.

Homogenous part

$$\frac{d^2\bar{\theta}}{dt^2} + 3H\frac{d\bar{\theta}}{dt} + m^2(\bar{T})\sin\bar{\theta} = 0$$

Fluctuation mode

$$\frac{\partial^2\widehat{\delta\theta}}{\partial t^2} + 3H\frac{\partial\widehat{\delta\theta}}{\partial t} + \left(\frac{k^2}{R^2} + m^2(\bar{T})\cos\bar{\theta}\right)\widehat{\delta\theta} = \hat{J}$$

Source term

$$\hat{J} = -\bar{T}\frac{dm^2}{dT}(\bar{T})\frac{\widehat{\delta T}}{\bar{T}}\sin\bar{\theta} - 2m^2(\bar{T})\hat{\Psi}\sin\bar{\theta} + \frac{d\bar{\theta}}{dt}\frac{\partial\left(\hat{\Psi} + 3\hat{\Phi}\right)}{\partial t}$$

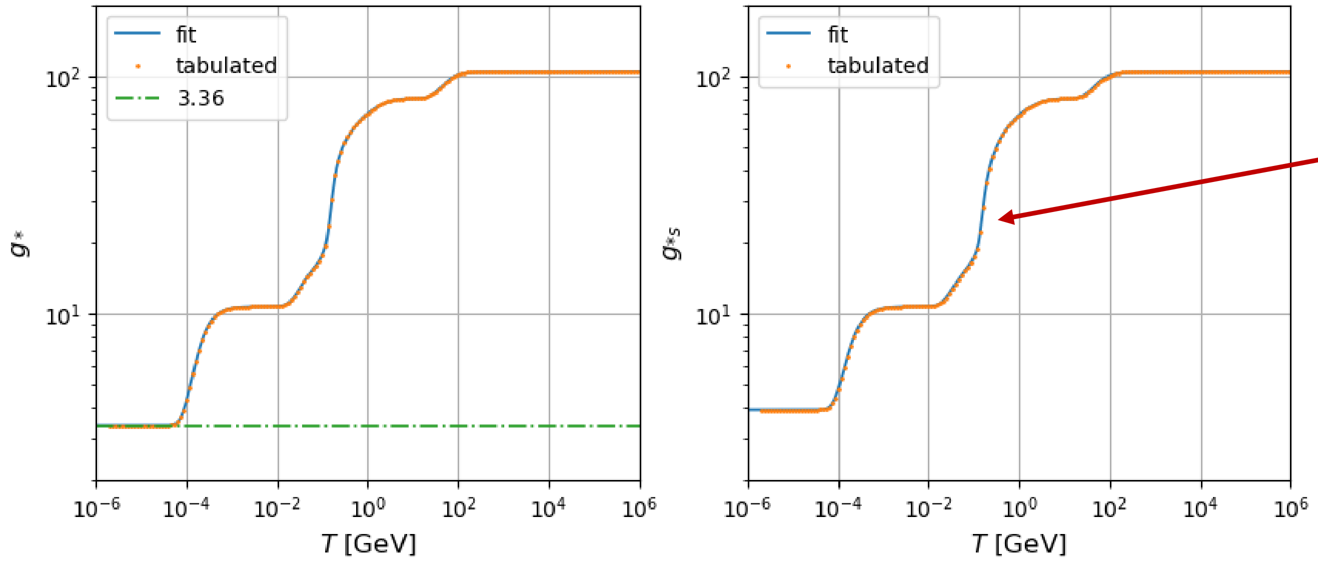
Thermal drive Grav. drive Kinetic

How does $\delta T/T$ excite $\delta\theta$

Source term $\hat{J} = -\bar{T}\frac{dm^2}{dT}(\bar{T})\frac{\widehat{\delta T}}{\bar{T}}\sin\bar{\theta} - 2m^2(\bar{T})\hat{\Psi}\sin\bar{\theta} + \frac{d\bar{\theta}}{dt}\frac{\partial\left(\hat{\Psi}+3\hat{\Phi}\right)}{\partial t}$

	Thermal drive	Grav. drive	Kinetic
Temperature contrast	$\frac{\widehat{\delta T}}{\bar{T}}$	Sikivie & Wei Xue \`22 Dominik & Ahmed \`25	
Metric potentials		Φ and Ψ $ds^2 = (1+2\Psi)dt^2 - R^2(1-2\Phi)d\vec{x}^2$ Newtonian gauge	

Inputs: $g_*(T)$, $g_{*s}(T)$, and curvature spectrum



$g_*(T)$ and $g_{*s}(T)$ from Saikawa & Shirai `18

Dynamical region of interest

We are looking at the edge

where the source is the largest

$$\bar{T}\frac{dm^2}{dT}(\bar{T})$$

Initial condition from inflation

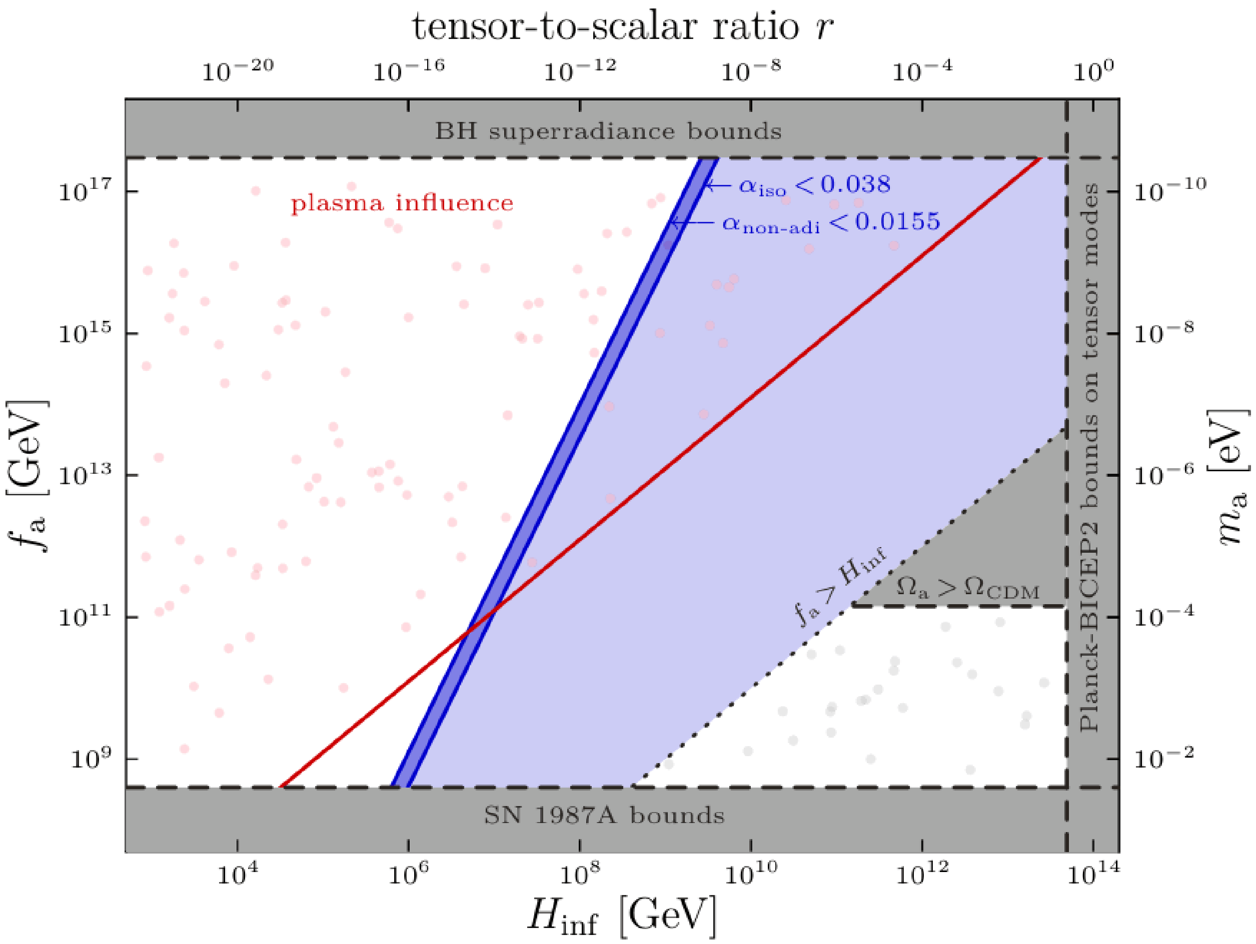
on super-horizon scales

$$\langle \widehat{\Theta}(\vec{p})^* \, \widehat{\Theta}(\vec{q}) \rangle = (2\pi)^3 \delta(\vec{p}-\vec{q}) \, \mathcal{P}_\Theta(p)$$

$$\mathcal{P}_\Theta(k) \to \frac{1}{9}\frac{2\pi^2}{k^3} A_s \qquad A_s = 2.1 \times 10^{-9}$$

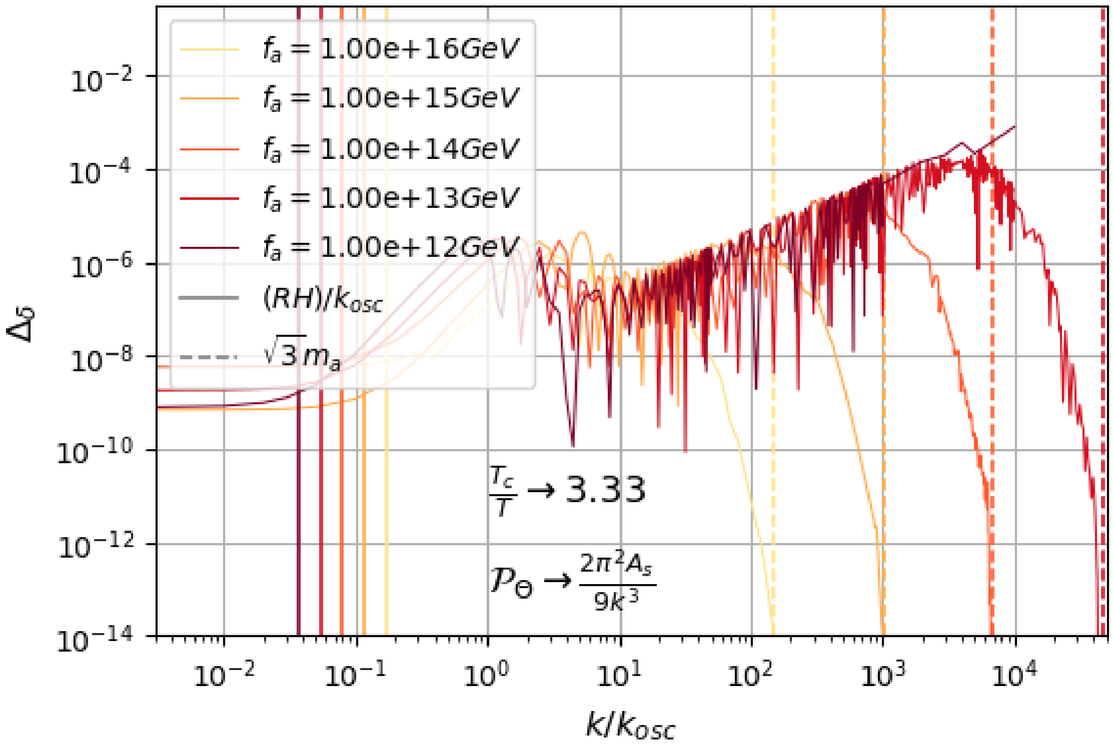
$$\widehat{\Theta} = -\frac{1}{2}\widehat{\Phi} = -\frac{1}{2}\widehat{\Psi}$$

Density contrast power spectrum



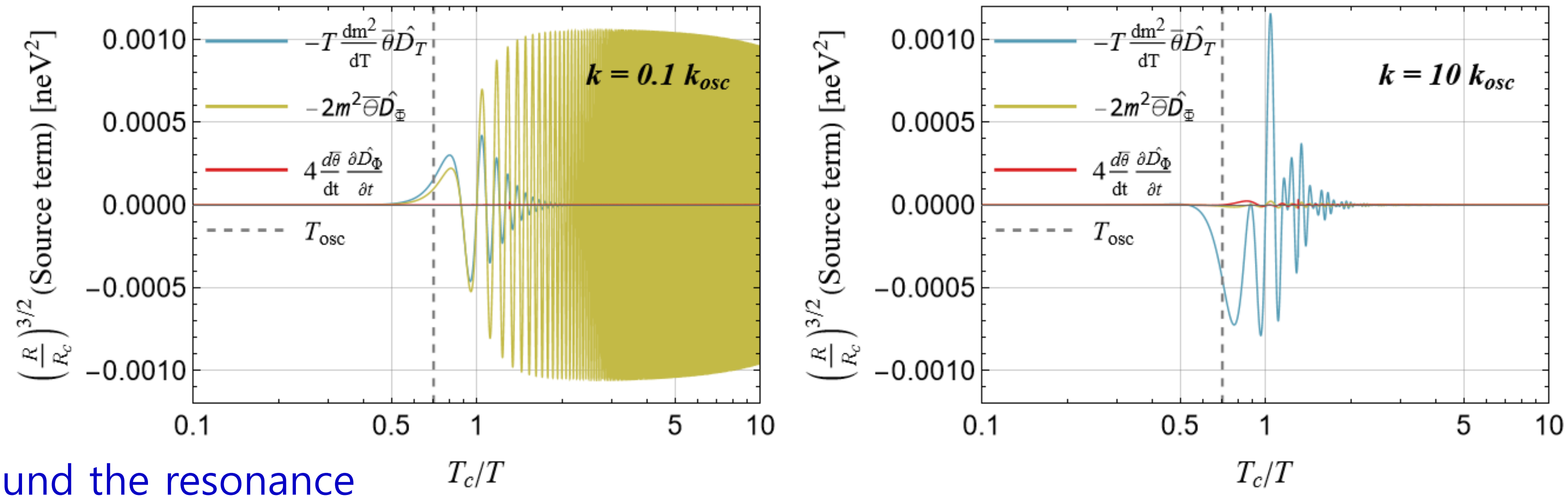
Dominik & Ahmed `25

High f_a, no fine-tunning at hill-top



Density contrast power spectrum

Source terms for $\widehat{\delta\theta}$



Dominant around the resonance

$$\hat{J} = -\bar{T}\frac{dm^2}{dT}(\bar{T})\frac{\widehat{\delta T}}{\bar{T}}\sin\bar{\theta} - 2m^2(\bar{T})\hat{\Psi}\sin\bar{\theta} + \frac{d\bar{\theta}}{dt}\frac{\partial\left(\hat{\Psi}+3\hat{\Phi}\right)}{\partial t}$$

Latest resonance

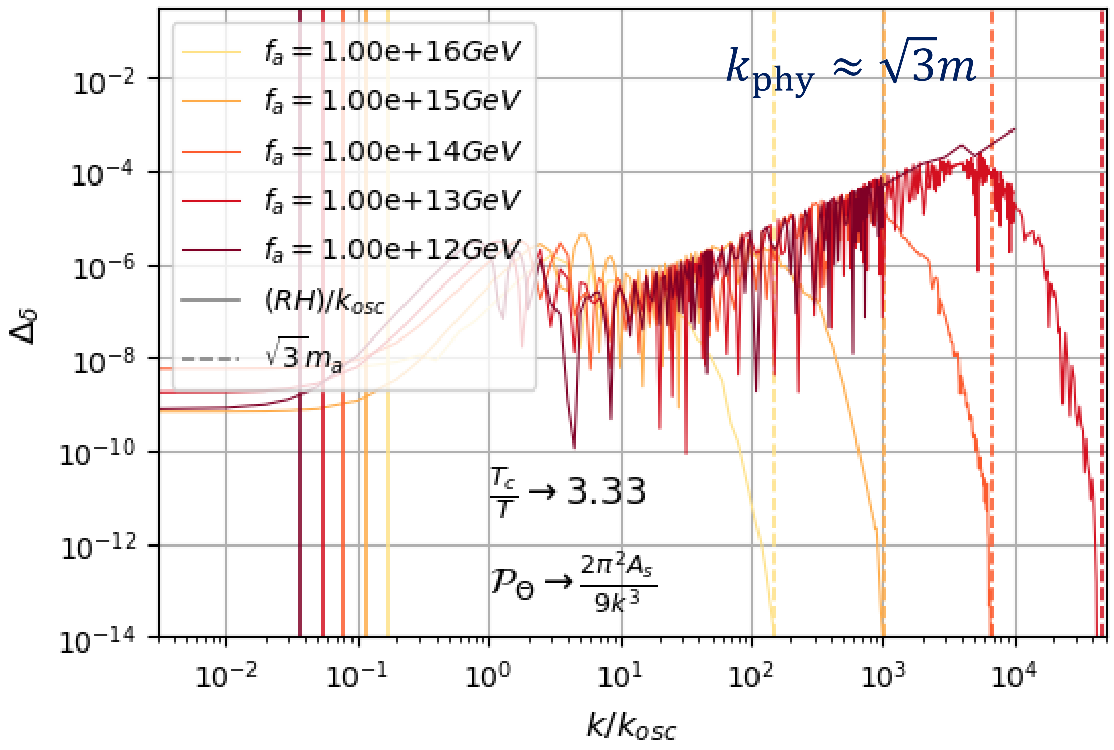
after horizon entry around QCD

$$\hat{\Theta} \sim \cos\left(\frac{k}{\sqrt{3}}\int^t \frac{dt'}{R(t')}\right) \sim \cos\left(\frac{k_{\mathrm{phy}}}{\sqrt{3}}t + \delta'\right)$$

resonance condition

$$\sqrt{m^2 + k_{\mathrm{phy}}^2} \approx m + \frac{k_{\mathrm{phy}}}{\sqrt{3}} \longrightarrow k_{\mathrm{phy}} \approx \sqrt{3}m$$

Density contrast power spectrum



Would this fluctuation grow to collapse?

Growth conditions

$\mathcal{O}(1)$ density contrast

$k < k_{QJ}$ (Quantum Jeans' scale)

Non-relativistic wave description

$$i\partial_t\psi + \frac{\nabla^2}{2mR^2}\psi - m\Phi\psi = 0$$

Two scales compete in the dispersion relation

Gradient pressure

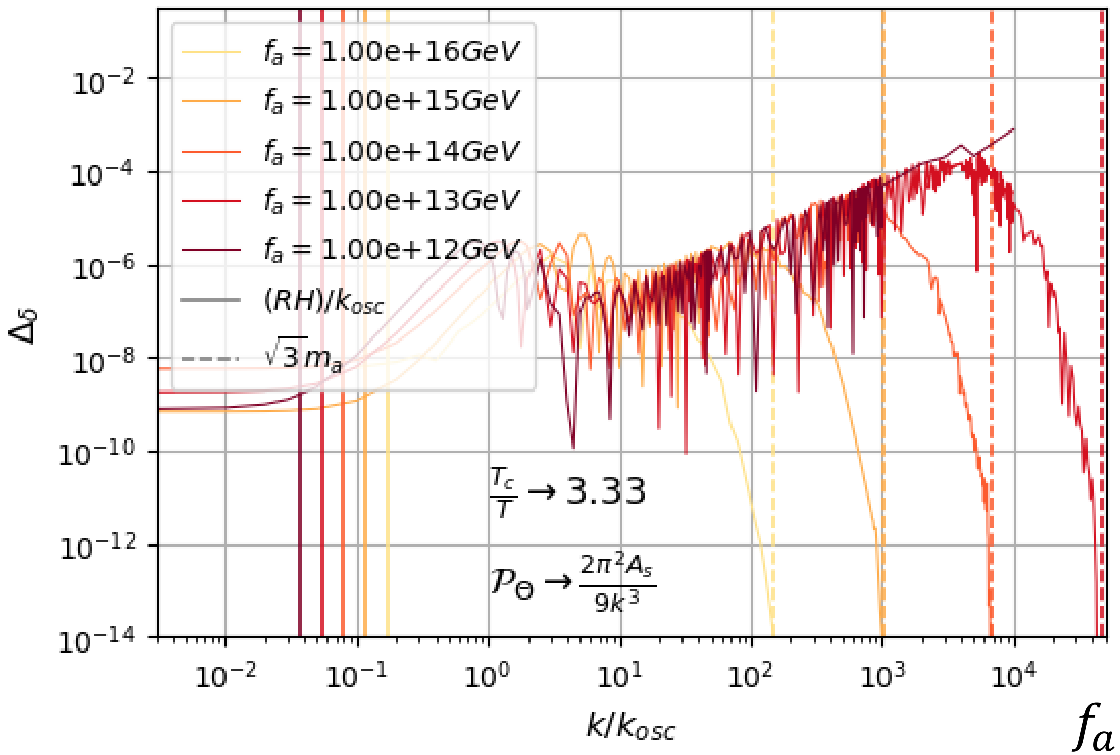
$$\omega_{\rm grad} \sim \frac{k^2}{2mR^2}$$

self-gravity

$$\omega_{\rm grav} \sim \sqrt{4\pi G\bar{\rho}}$$

$$\omega_{\rm grad} = \omega_{\rm grav} \longrightarrow k_{\rm QJ} = R(16\pi G\bar{\rho}m^2)^{1/4}$$

Density contrast power spectrum



In the scanned region, $\frac{k_{\rm peak}}{R(T_c)} \approx m$.

Growth conditions

$\mathcal{O}(1)$ density contrast

$k < k_{QJ}$ (Quantum Jeans' scale)

$$k_{QJ} = R(16\pi G\bar{\rho}m^2)^{1/4}$$

$$\frac{k_{QJ}}{k_{\rm peak}} \approx 0.15 \times \left(\frac{R_0/R}{3400}\right)^{-\frac{1}{4}} \left(\frac{m}{0.57\ \rm neV}\right)^{-\frac{1}{2}} \left(\frac{\Omega_{DM}h^2}{0.12}\right)^{\frac{1}{4}}$$

$f_a \to 10^{16}$ GeV ⟶ $m \approx 0.57$ neV, smaller f_a only worsen.

⟶ Requires the redshift $z < 0.7$ for $k < k_{QJ}$.

For such large f_a, density should grow by $\times 10^3$.

⟶ Requires the redshift $z < 2.4$

Smaller f_a scenario

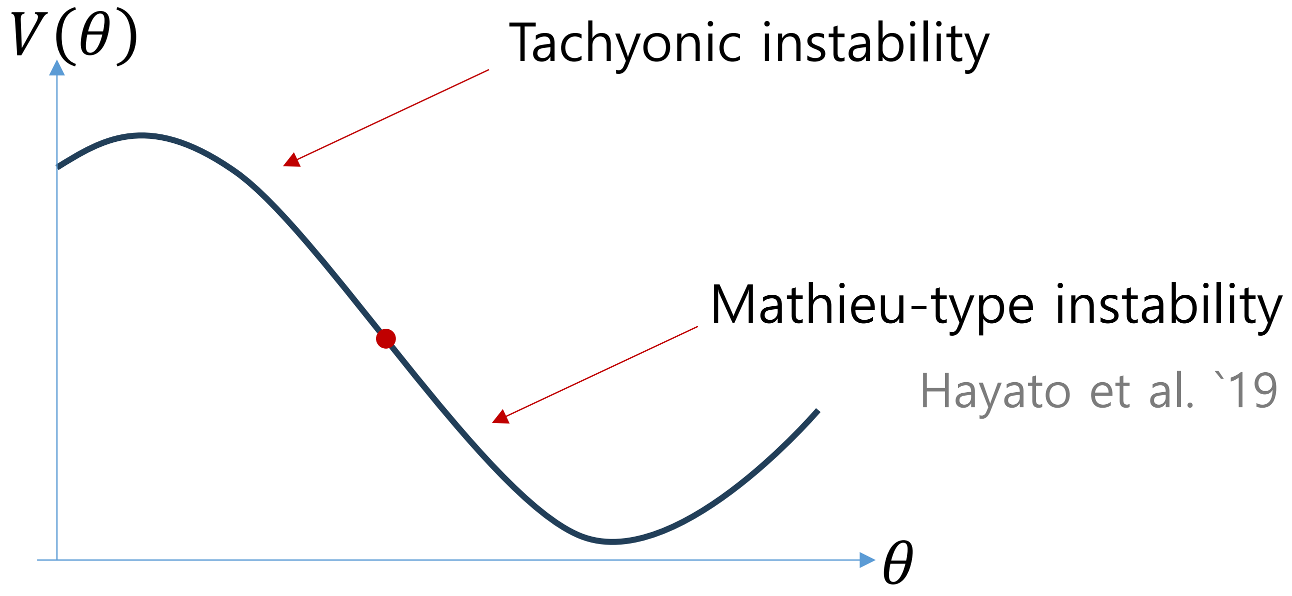
For $f_a \leq 10^{11}\,\mathrm{GeV}$, $\bar{\theta}_{\mathrm{init}}$ must be tuned to π.

$\bar{\theta}_{\mathrm{init}} \approx 0.65$ for $f_a \approx 10^{12}\,\mathrm{GeV}$

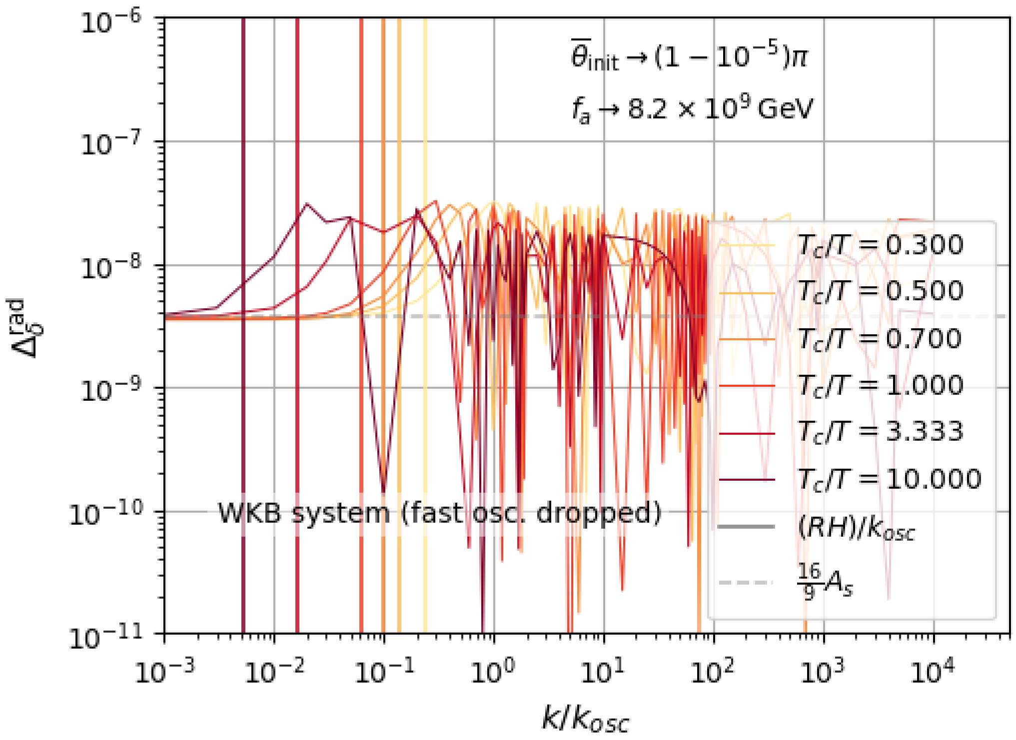
Oscillation is delayed, and Hubble drag becomes smaller at oscillation.

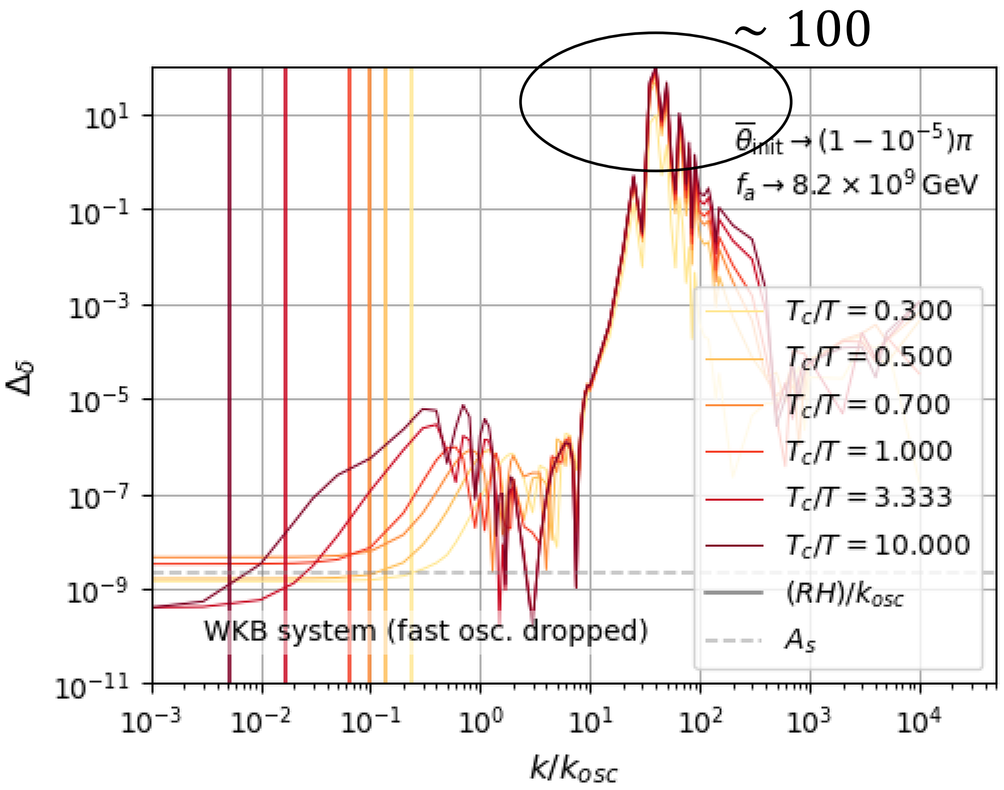
The anharmonicity of the potential becomes important even after the oscillation.

Naoya et al. `22



Density contrast power spectrum: $f_a = 8.2 \times 10^9$ GeV $\quad \bar{\theta}_{\rm init} = (1-10^{-5})\pi$



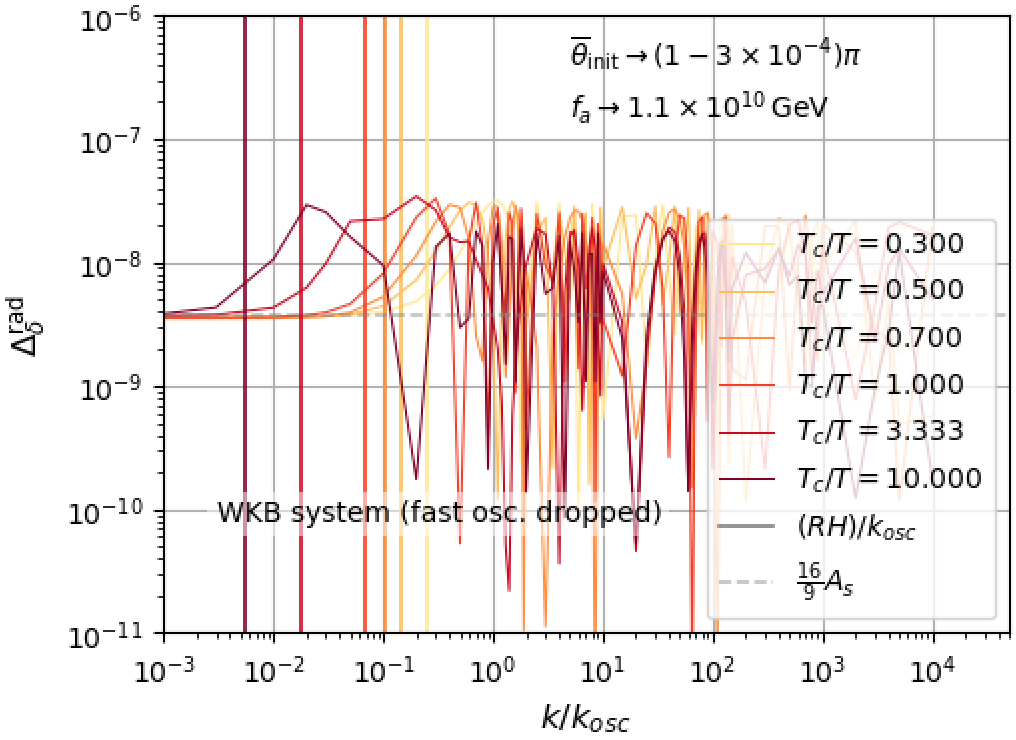


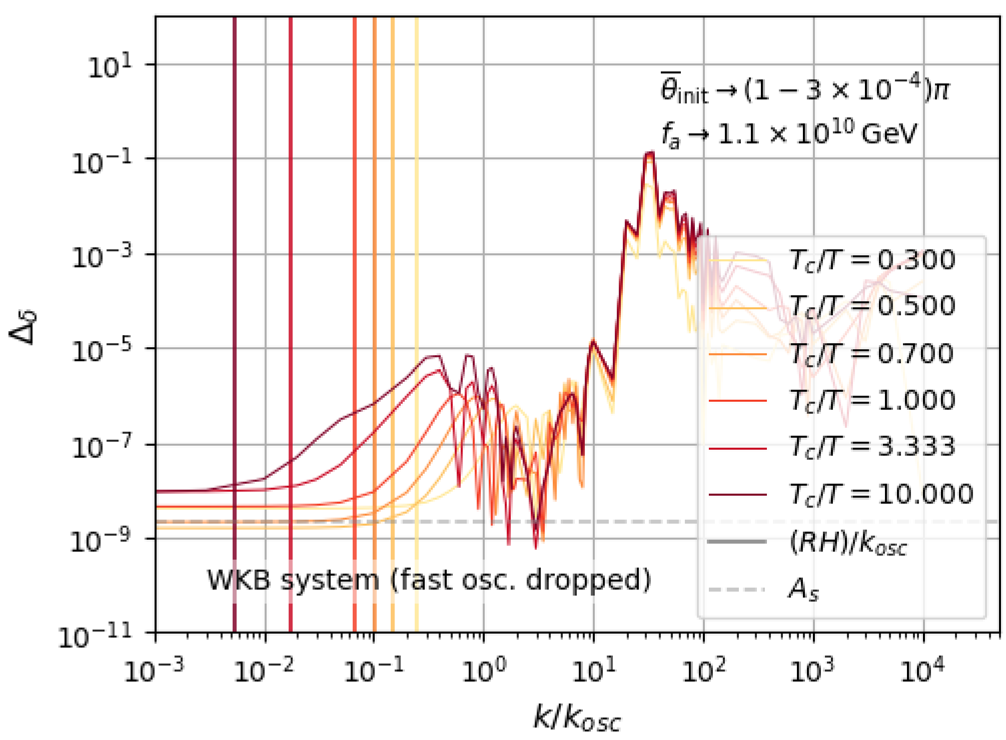
$k_{QJ}(T_{\rm MRE}) = 23000\ {\rm pc}^{-1}$ $\quad$ $k_{\rm peak} = 3500\ {\rm pc}^{-1}$

$\times\, 10$

Density contrast power spectrum: $f_a = 1.1 \times 10^{10}$ GeV $\bar{\theta}_{\rm init} = (1 - 3 \times 10^{-4})\pi$

The peak height is not very sensitive to f_a, but to $\bar{\theta}_{\rm init}$.





$k_{QJ}(T_{\rm MRE}) = 20000\ {\rm pc}^{-1}$ $k_{\rm peak} = 2300\ {\rm pc}^{-1}$

$\times 10$

Smaller f_a scenario

Large scale hierarchy

Completion of QCD cross-over takes $\sim H(T_c)^{-1} \approx (0.018\,\mathrm{neV})^{-1}$

$\times\, 10^7$

Axion oscillation time scale $\sim m^{-1} \approx (0.57\,\mathrm{meV})^{-1}$

Smaller f_a scenario

Slow-envelope formulation

Absorbing the fast phase

$$\varphi(t) = \int^t \omega(t')\, dt' \qquad \omega(t) \to m(t)$$

$$\widehat{\delta\theta} = \frac{1}{\sqrt{2\omega R^3}} \left(A(t, \vec{k})\, e^{-i\varphi} + A^*(t, -\vec{k})\, e^{i\varphi} \right)$$

$$\bar{\theta} = \frac{1}{\sqrt{2\omega R^3}} \left(B(t)\, e^{-i\varphi} + B^*(t)\, e^{i\varphi} \right)$$

Laurent expand the equation of motion in $e^{-i\varphi}$

$$\frac{d^2\bar{\theta}}{dt^2} + 3H\frac{d\bar{\theta}}{dt} + m^2(\bar{T}) \sin\bar{\theta} = 0$$

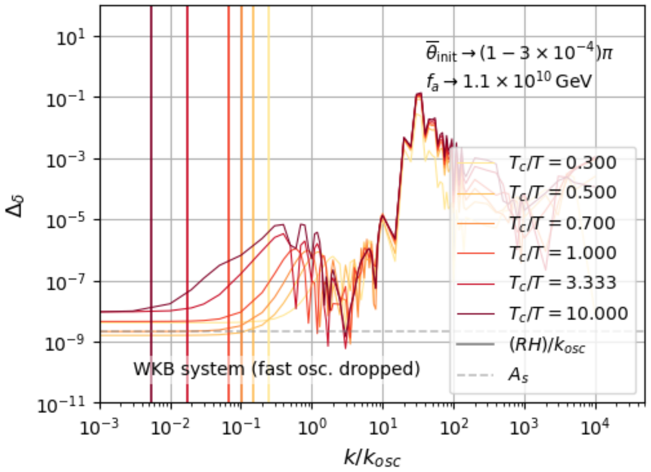
$$\frac{\partial^2 \widehat{\delta\theta}}{\partial t^2} + 3H\frac{\partial \widehat{\delta\theta}}{\partial t} + \left(\frac{k^2}{R^2} + m^2(\bar{T}) \cos\bar{\theta}\right) \widehat{\delta\theta} = \hat{J}$$

$\longrightarrow$ $e^{-in\varphi}$ terms on both sides.

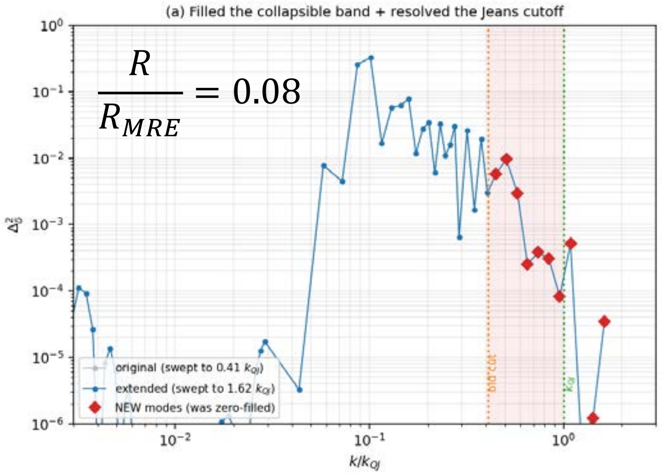
Take only the $e^{-0 \cdot i \cdot \varphi}$ term.

Tracking the evolution

Inflation

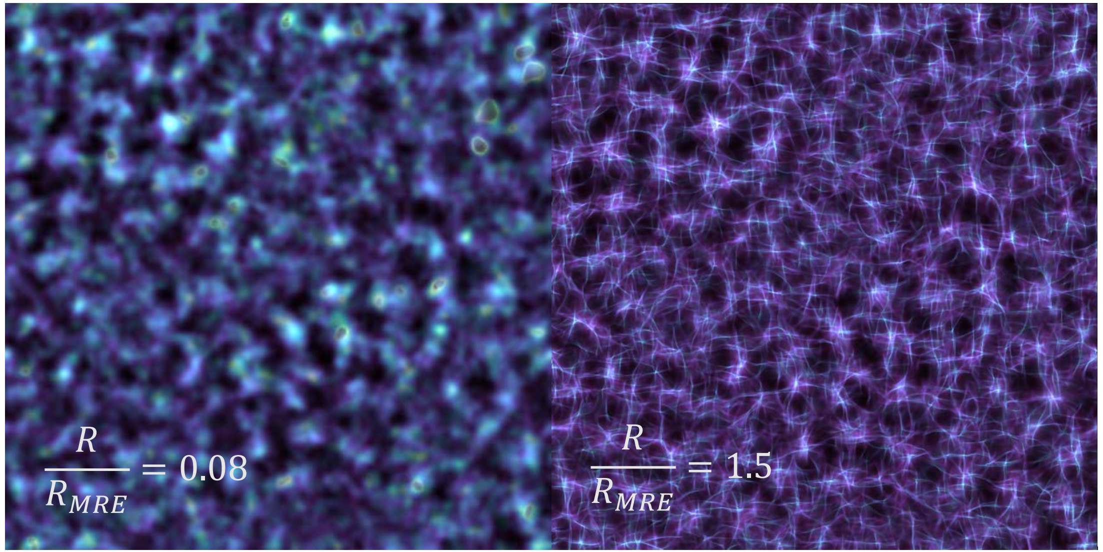


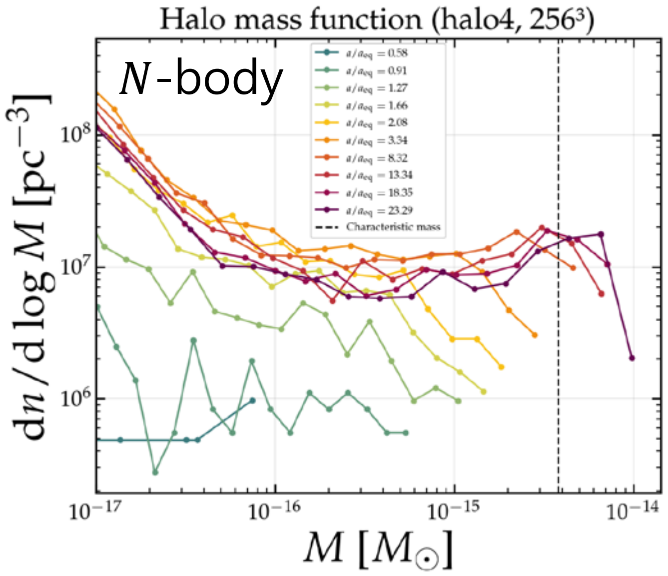
linear pert.



Schrodinger-Poisson

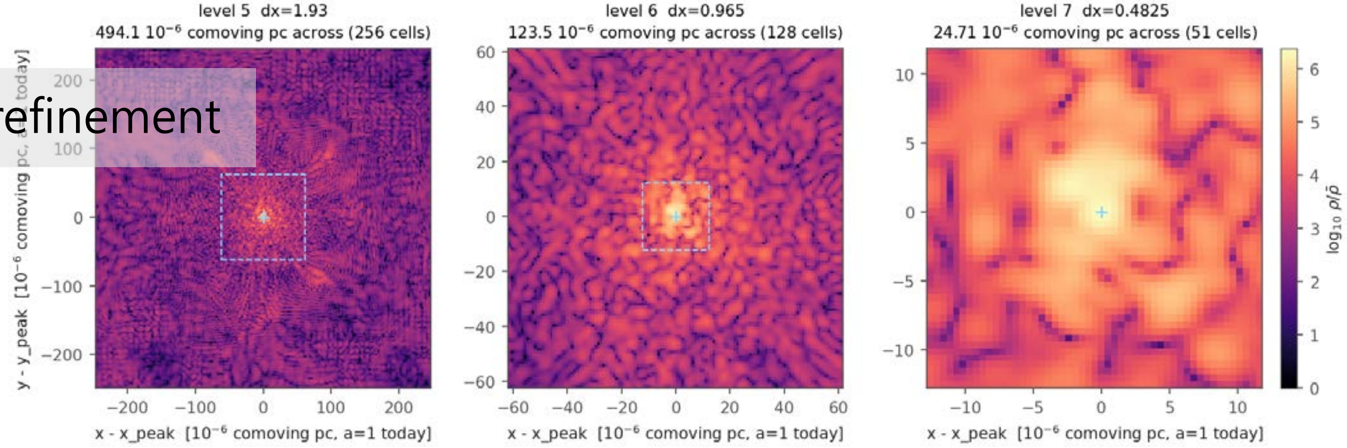
Nonlinear dynamics

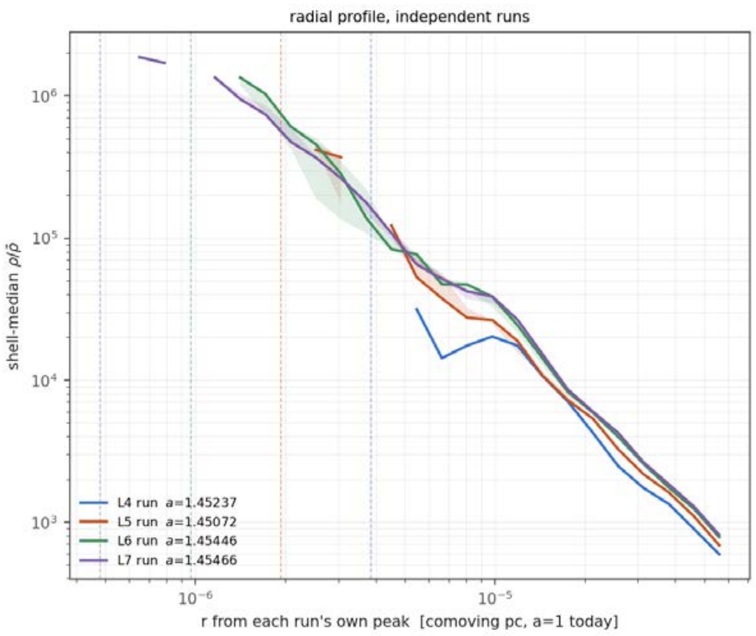




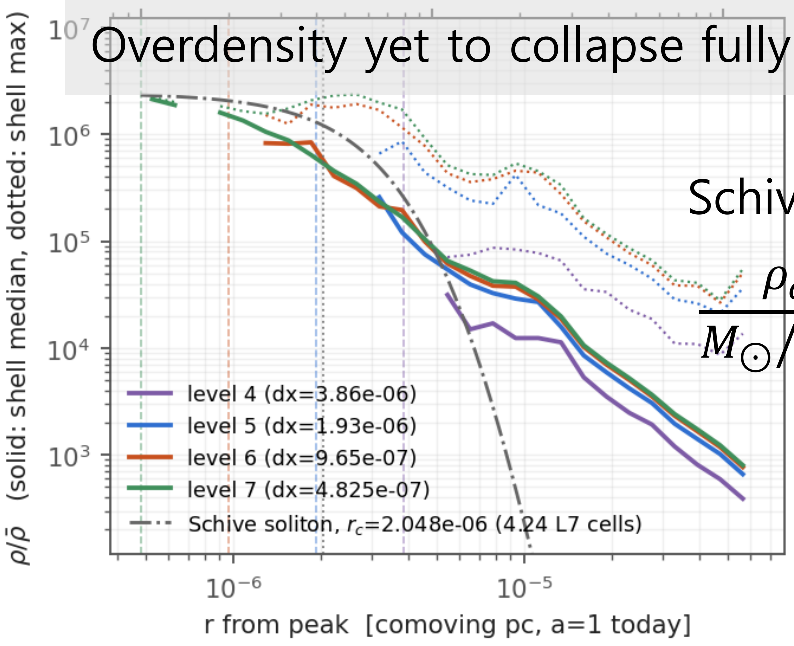
Tracking the evolution

Mesh refinement





Overdensity yet to collapse fully



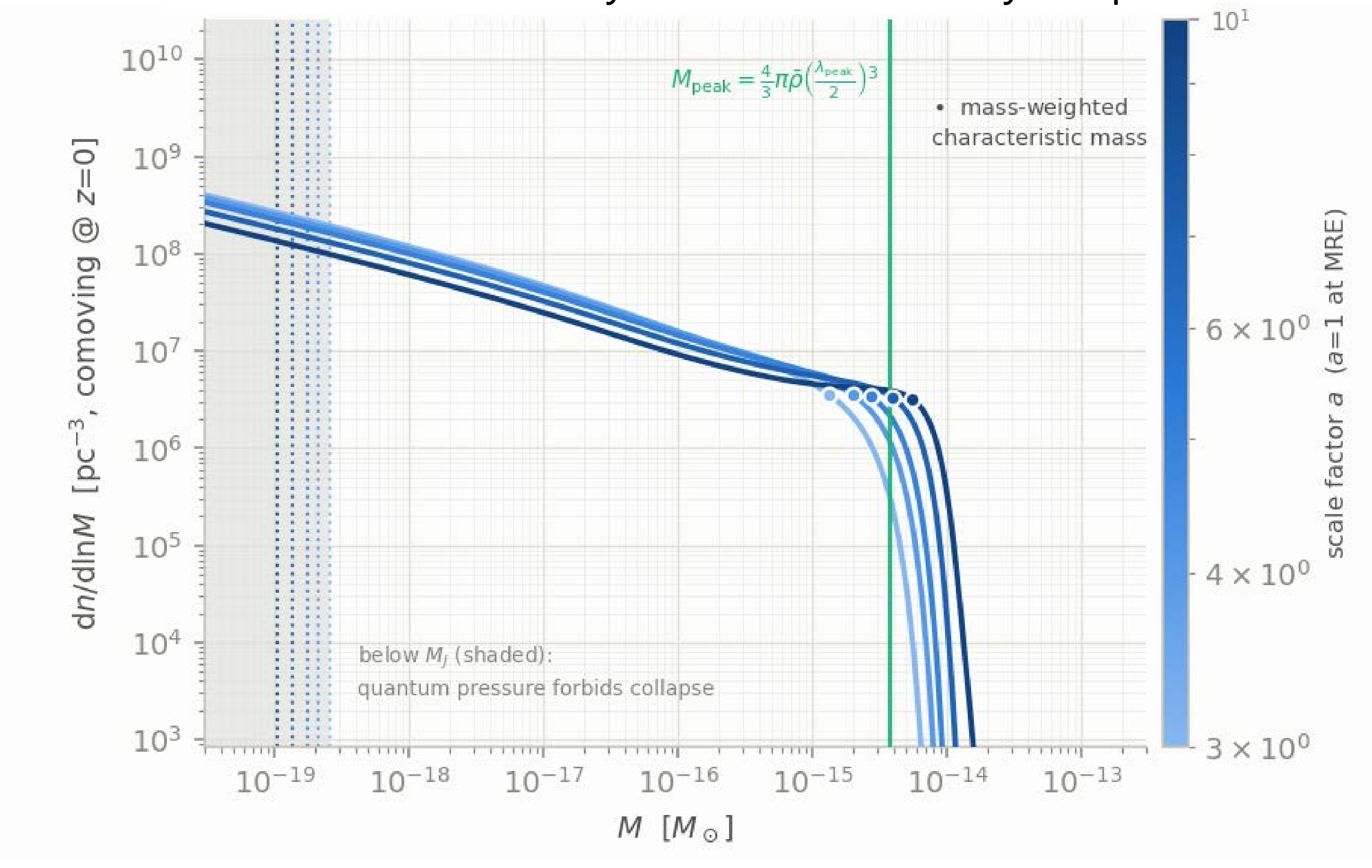
Schive soliton fit with

$$\frac{\rho_c}{M_\odot/\mathrm{pc}^3}\left(\frac{r_c}{10^{-3}\mathrm{pc}}\right)^4 = \frac{1.9\times 10^{-6}}{a}\left(\frac{10^{-8}\mathrm{eV}}{m}\right)^2$$

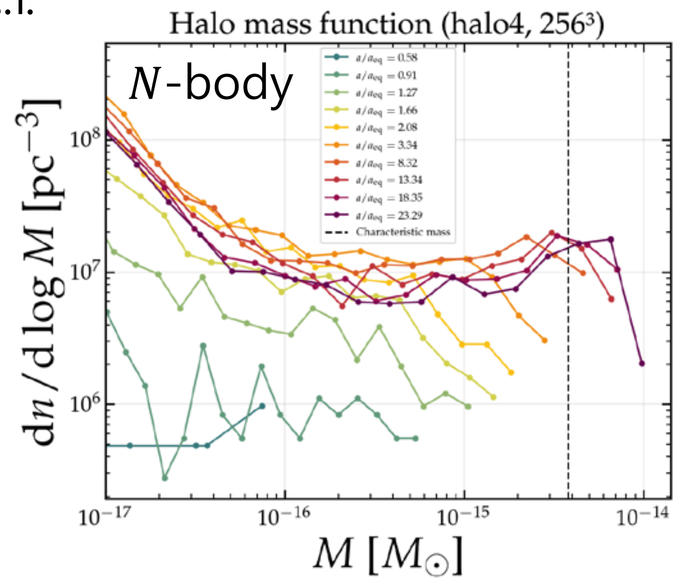
Eggemeier & Niemeyer `19

Tracking the evolution

Press-Schechter analysis on linear theory output



c.f.



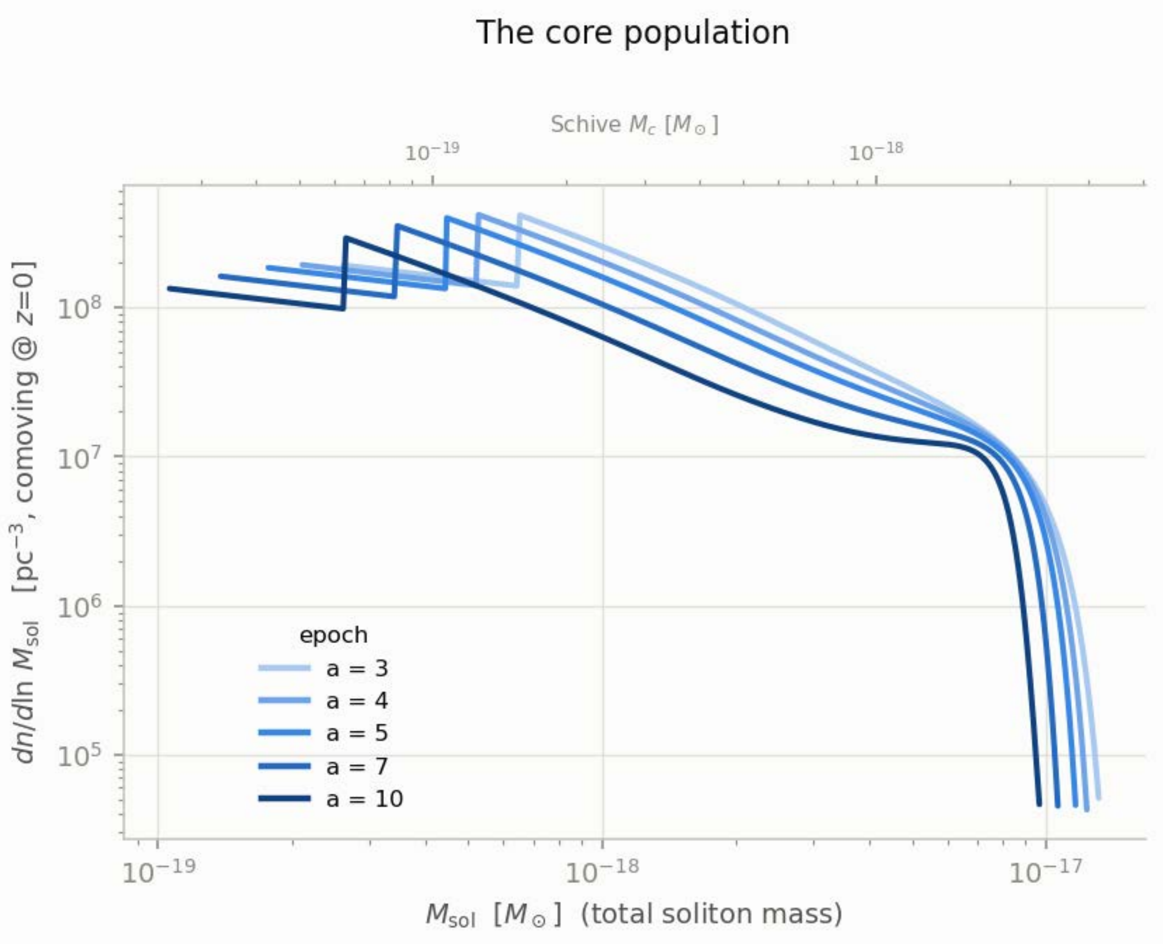
Tracking the evolution

Schive et al. `14

$$M_c = \frac{1}{4} a^{-\frac{1}{2}} \left(\frac{\zeta(z)}{\zeta(0)}\right)^{\frac{1}{6}} \left(\frac{M_h}{M_{\min,0}}\right)^{1/3} M_{\min,0}$$

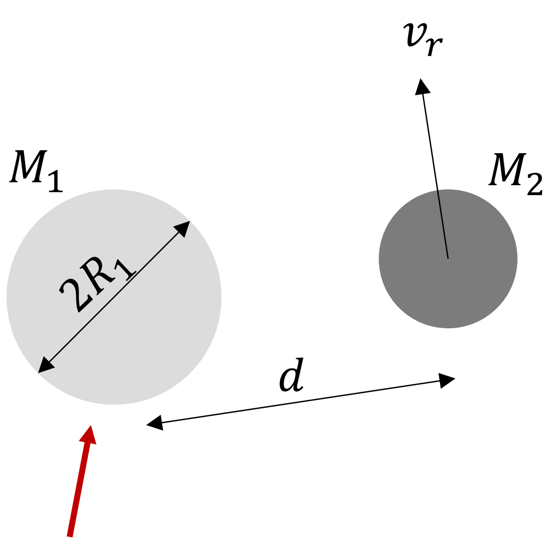
$$r_c = 1.6\mathrm{kpc} \left(\frac{m}{10^{-22}\mathrm{eV}}\right)^{-1} a^{\frac{1}{2}} \left(\frac{\zeta(z)}{\zeta(0)}\right)^{-\frac{1}{6}} \left(\frac{M_h}{10^9 M_\odot}\right)^{-1/3}$$

$$M_{\min,0} \sim 4.4 \times 10^7 M_\odot \left(\frac{m}{10^{-22}\mathrm{eV}}\right)^{-3/2}$$



Total soliton mass $M_{\mathrm{sol}} \sim 4.15\, M_c$

Tidal disruption



Is this object going to be disrupted?

Gorghetto et al. `24

The tidal acceleration $a_{\rm tidal} = \frac{2GM_2R_1}{d^3}$

Duration of encounter $\Delta t \sim \frac{d}{v_r}$

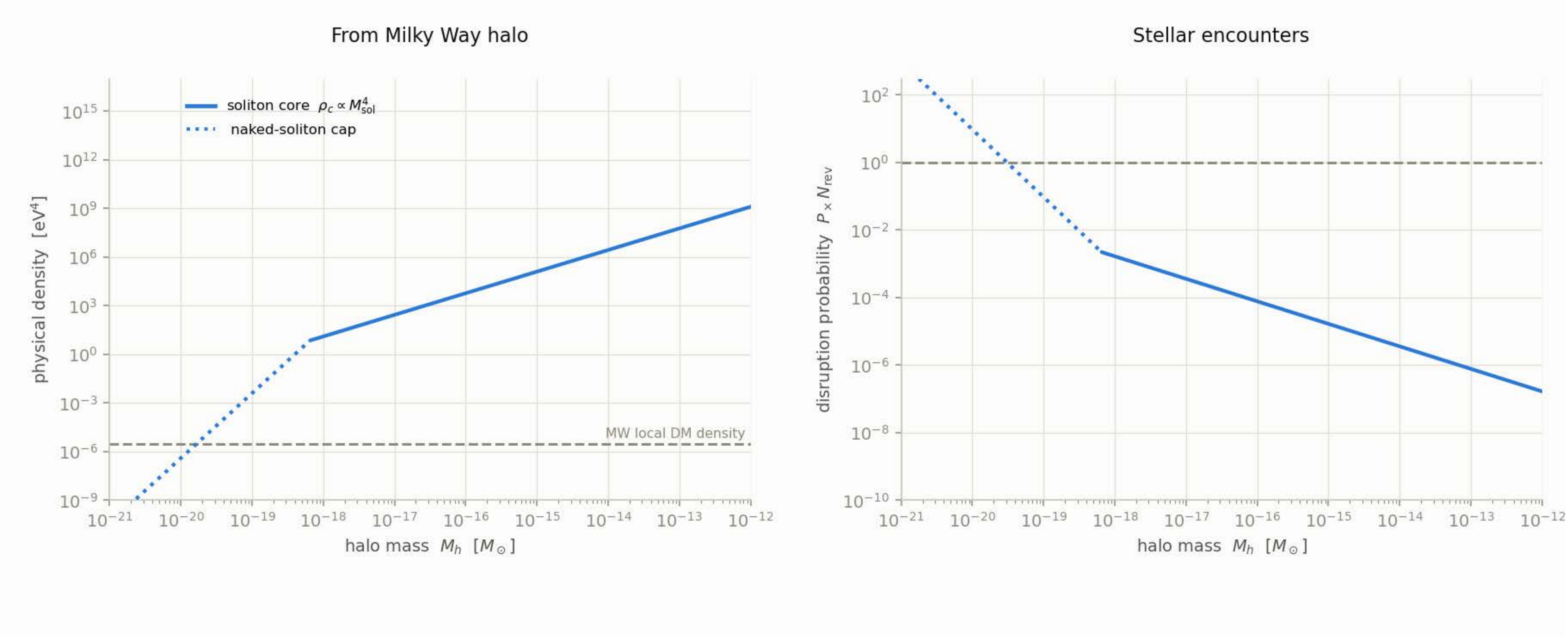
Induced velocity $\Delta v \sim a_{\rm tidal}\,\Delta t$

The object is tidally disrupted if $\Delta v \geq \sqrt{\frac{2GM_1}{R_1}}$

The probability of disruption while passing the Milky Way disc

$$P_{\rm tidal} = \pi d_{\rm crit}^2 n_{\rm star} = 6\times 10^{-4}\left(\frac{\rho_{\rm sol}}{\rm eV^4}\right)^{\frac{1}{2}}\left(\frac{\Sigma}{50M_\odot/{\rm pc}^2}\right)$$

Tidal disruption



The story in one picture

10^9 10^{11} 10^{13} 10^{15} 10^{16} GeV

Temperature fluctuation sources $\delta T/T \to \delta\theta$

anharmonicity ($\bar{\theta}_{\text{init}} \to \delta\theta$)

Low f_a

Potential collapse
at early time

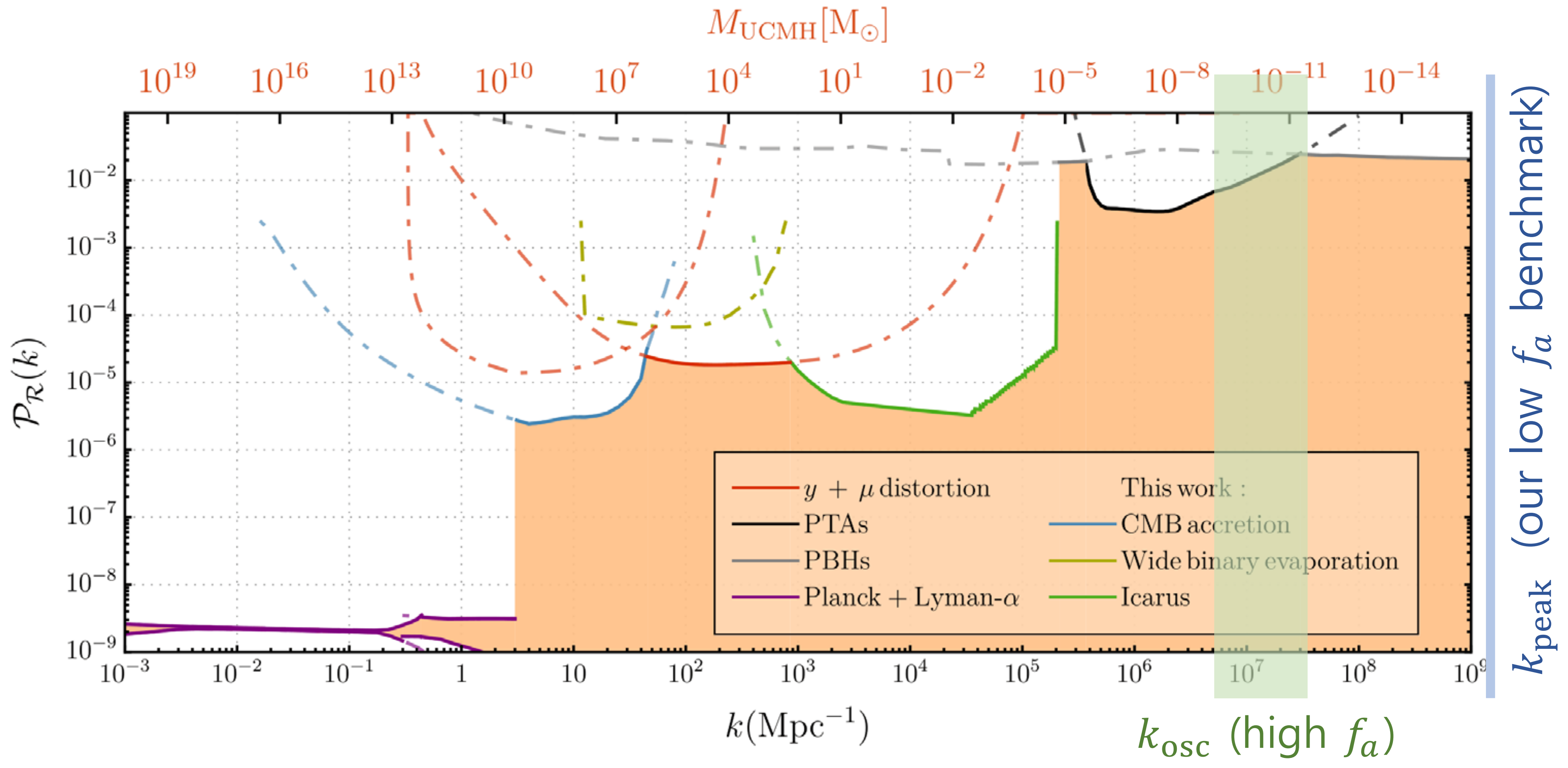
Intermediate f_a

High f_a

Potential collapse
only at very late time

$z < 2.4$

How wildly can pre-inflationary scenario vary



Torsten Bringmann et al. `25

Summary

- **Possibility of having pre-inflationary axion Minicluster formation is explored at linear level.**
- **Effect of gravitational potential and realistic QCD cross-over era are considered.**
- **The cluster formation window for high $10^{12}\ \mathrm{GeV} \le f_a \le 10^{16}\ \mathrm{GeV}$ is very narrow.**
- **Low axion decay constant (e.g. $1.1 \times 10^{10}\ \mathrm{GeV}$) scenario makes a perfect target for S-P.**