TOHOKU UNIVERSITY

COSMIC WISPERS CA21106

Minicharged Particles from Quantum Consistency and Number Theory

August 23-27, 2026
@The 5th workshop on Symmetry and Structure of the Universe

Fumi Takahashi (Tohoku University)

Junseok Lee, FT, and Yu-Dai Tsai, 2603.12320

What I won’t talk about today

- Trapped misalignment

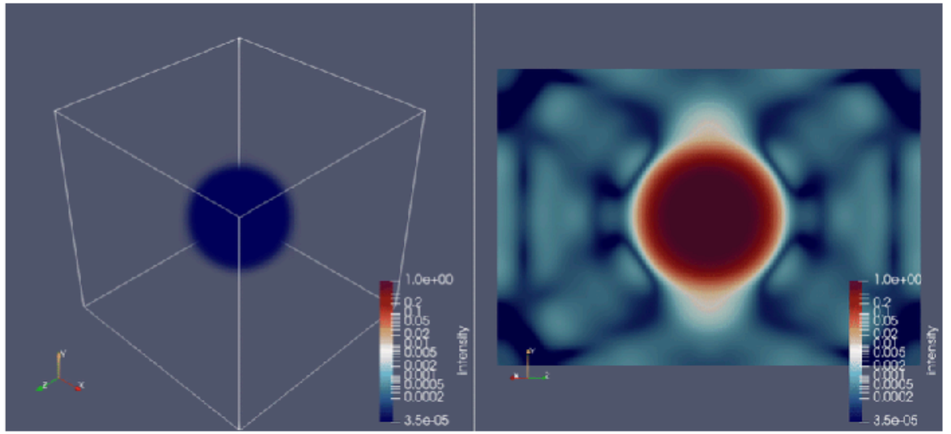
  Higaki, Jeong, Kitajima, FT, 1603.02090,
  Jeong, Matsukawa, Nakagawa FT, 2201.00681
- Bubble misalignment

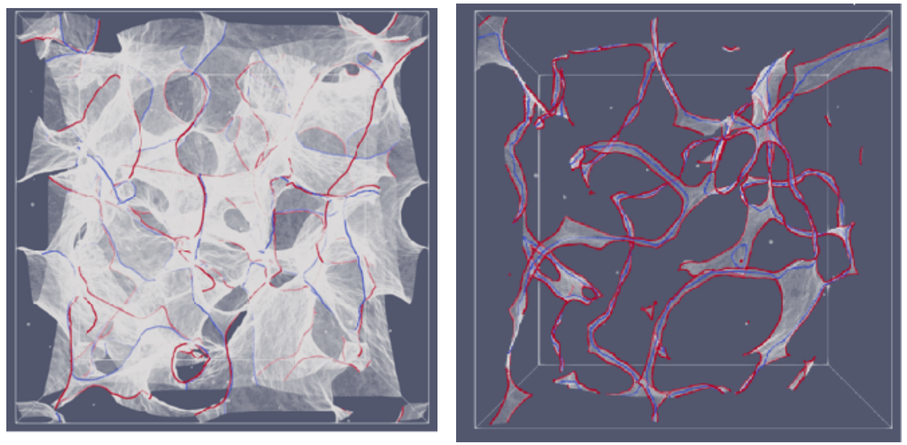
  Kitajima, Lee, Murai, FT and Yin, 2402.09501
- Axion topological defects

  Lee, Murai, FT and Yin 2409.09749, 2407.09478
- Axion DW and isocurvature problem

  Kitajima, Lee, Murai, FT and Yin 2608.13650
- A new solution to the axion DW problem

  Kitajima, Lee, Murai, FT and Yin 2507.07075





Vector portal

The vector portal connects the SM photon or hypercharge field to a hidden U(1) gauge boson through kinetic mixing.

SM sector

$SU(3)_C \times SU(2)_L \times U(1)_Y$

Kinetic mixing

$$\frac{\chi}{2} F'_{\mu\nu} F^{\mu\nu}$$

Hidden sector

$U(1)_H$

A'^{μ}: Dark photon

ψ_i: Hidden matter

Dark photon (γ') : the gauge boson of $U(1)_H$

Kinetic mixing with $U(1)_{EM}$ (or $U(1)_Y$ at high energies)

$$\frac{\chi}{2} F'_{\mu\nu} F^{\mu\nu}$$

We assume **massless dark photons**: $m_{\gamma'} = 0$

Dark photon (γ') : the gauge boson of $U(1)_H$

Kinetic mixing with $U(1)_{EM}$ (or $U(1)_Y$ at high energies)

$$\frac{\chi}{2} F'_{\mu\nu} F^{\mu\nu}$$

We assume **massless dark photons**: $m_{\gamma'} = 0$

In the absence of hidden matter, kinetic mixing can be removed by a field redefinition,

$$A' \to A' - \chi A \qquad\qquad |\chi| \ll 1$$

and the dark photon remains decoupled from the SM.

Introducing hidden matter

$$\mathcal{L} \supset \bar{\Psi}\gamma^\mu \left(i\partial_\mu - g_H q_H A'_\mu\right)\Psi - m\bar{\Psi}\Psi$$

$$\xrightarrow[A' \to A' - \chi A]{} \bar{\Psi}\gamma^\mu \left(i\partial_\mu - g_H q_H \left(A'_\mu - \chi A_\mu\right)\right)\Psi - m\bar{\Psi}\Psi$$

Thus, kinetic mixing induces a small electric charge for the hidden fermion,

$$q_{\text{eff}} = -\chi g_H q_H$$

making it a mini-charged particle (mCP).

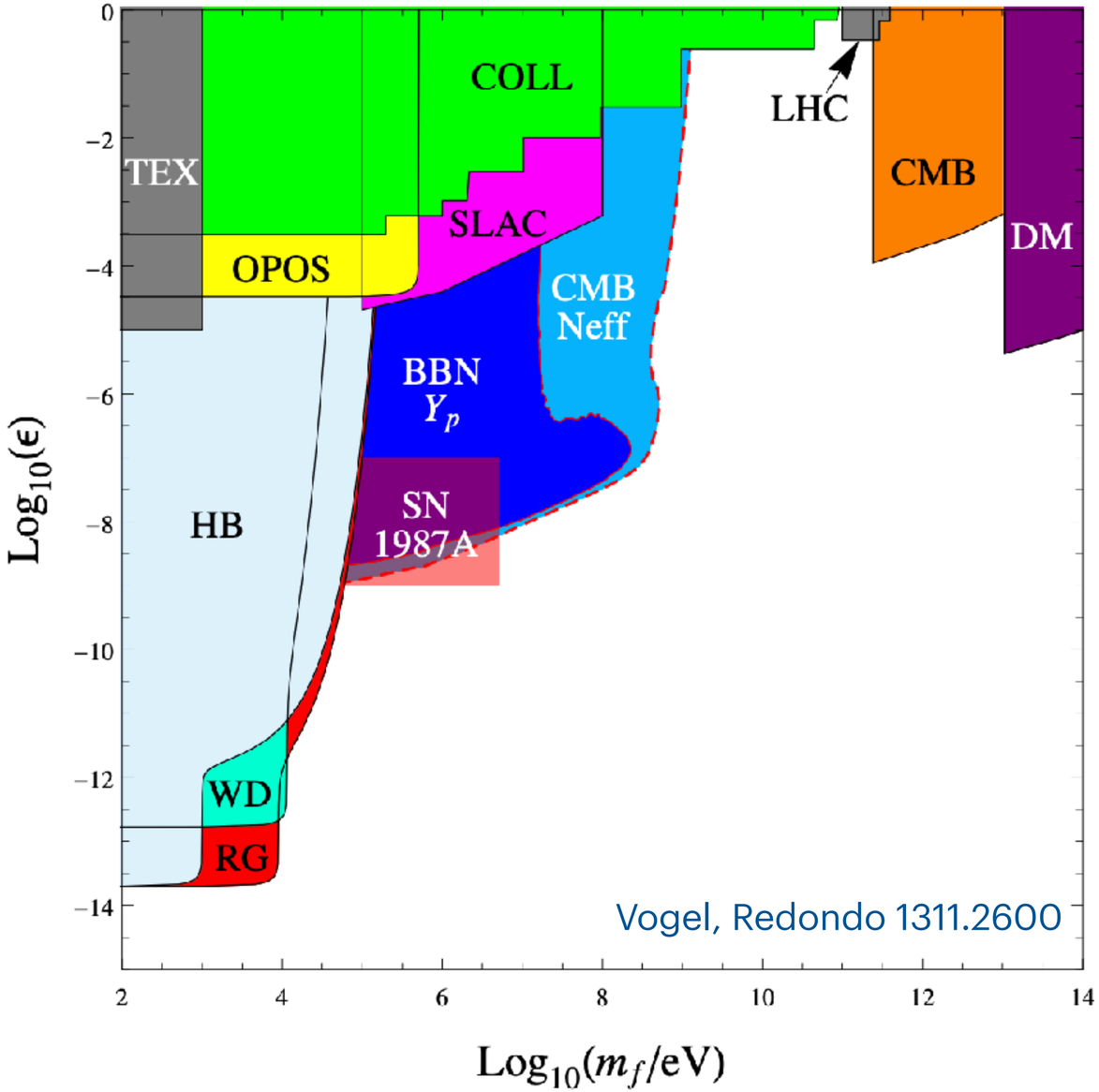
Current bounds on mCPs

mCP searches span from laboratory experiments to cosmology.

Stellar Cooling: Strong constraints from HB, RG, WD, and SN 1987A at low masses.

Cosmological Limits: Constraints from BBN, CMB (N_{eff}), and DM abundance.

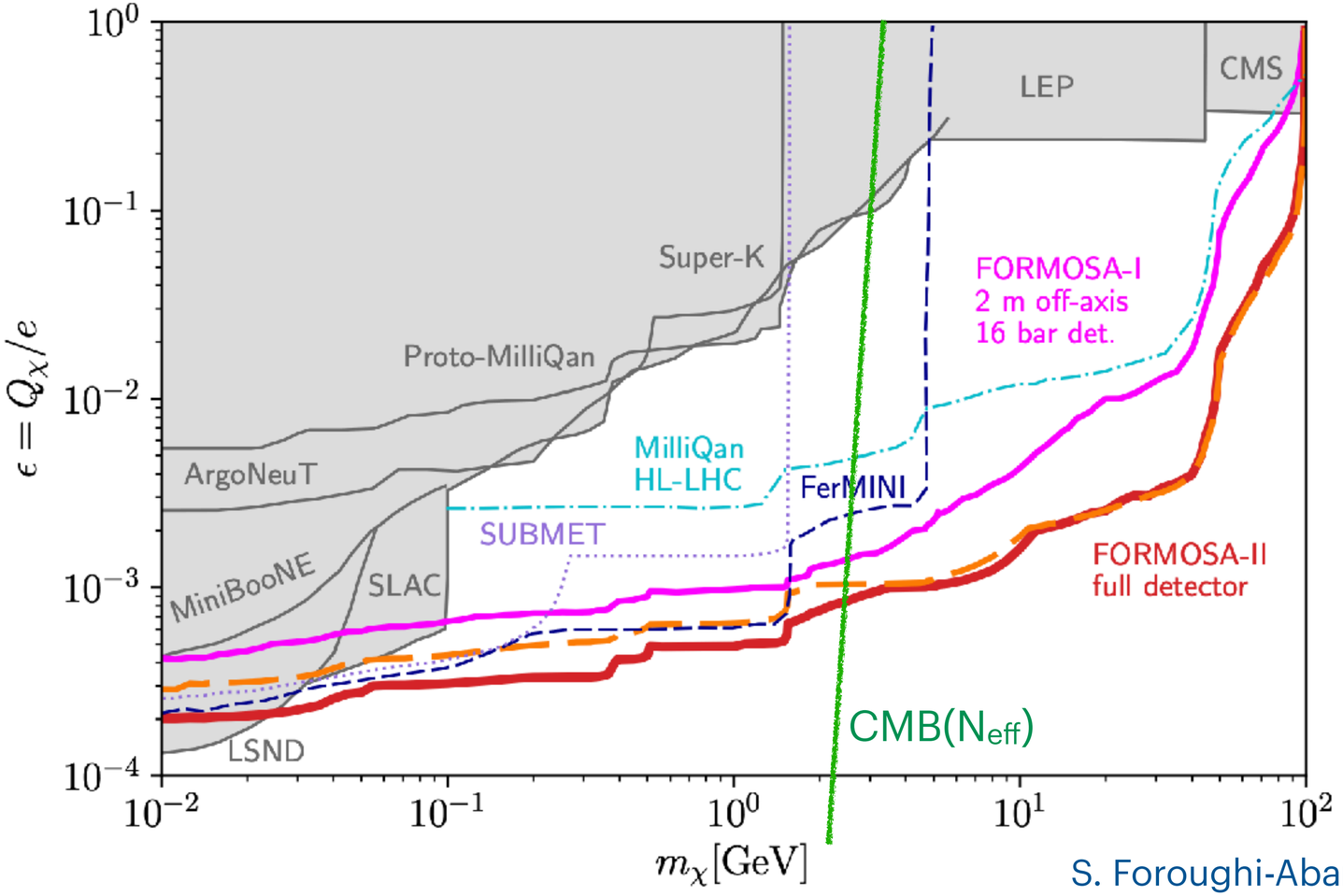
Laboratory Searches: Collider and beam dump experiments probe higher masses and relatively large minicharges.

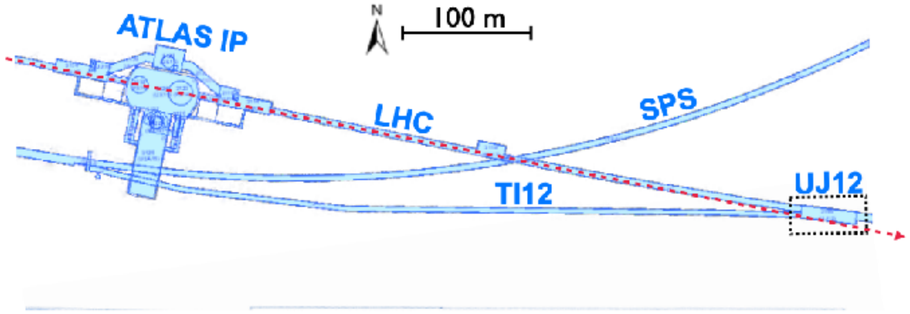


Vogel, Redondo 1311.2600

Future mCP searches: FORMOSA as an example

MilliQan, MAPP, FORMOSA, SUBMET, FerMINI, etc. probe mCPs through tiny ionization signals in scintillators.





S. Foroughi-Abari, F. Kling, and Y-D Tsai, 2010.07941

Why is mCP light?

In most phenomenological studies, the mCP mass is simply treated as a free parameter.

But if a mCP is light enough to be probed by collider, beam-dump experiments or astrophysical observations, we should ask:

Why is mCP light? What protects the mCP mass from the UV scale?

Chiral U(1) model for light mCPs

J. Lee, FT, and Y-D Tsai, 2603.12320

We introduce a chiral $U(1)_{\rm X}$ gauge symmetry, in addition to the $U(1)_{\rm H}$ gauge symmetry, to forbid the mass term.

$$\bar{\Psi}\Psi = \psi_1\psi_2 + \text{h.c.}$$

Dirac spinor Weyl spinor

$$\psi_1\psi_2 = \psi_1^T(i\sigma_2)\psi_2$$

	ψ_1	ψ_2
Vector-like $U(1)_{\rm H}$	$+1$	-1
Chiral $U(1)_{\rm X}$	a	$-b$

If $a \neq b$, the mass term is forbidden by the chiral $U(1)_{\rm X}$

Chiral U(1) model for light mCPs

J. Lee, FT, and Y-D Tsai, 2603.12320

We introduce a chiral $U(1)_{\rm X}$ gauge symmetry, in addition to the $U(1)_{\rm H}$ gauge symmetry, to forbid the mass term.

$$\bar{\Psi}\Psi = \psi_1\psi_2 + \text{h.c.}$$

Dirac spinor Weyl spinor

$$\psi_1\psi_2 = \psi_1^T(i\sigma_2)\psi_2$$

Dark Higgs

	ψ_1	ψ_2	ϕ
Vector-like $U(1)_{\rm H}$	$+1$	-1	0
Chiral $U(1)_{\rm X}$	a	$-b$	1

When $U(1)_{\rm X}$ is spontaneously broken, a small fermion mass is induced:

$$yM_c\left(\frac{\phi^{(*)}}{M_c}\right)^{|a-b|}\psi_1\psi_2 + \text{h.c.}$$

$y = \mathcal{O}(1)$ complex const.

M_c : UV cutoff scale

In general, we may introduce $2n$ left-handed Weyl fermions ψ_i with the following charges

	ψ_i	ψ_j	ϕ
Vector-like $U(1)_{\rm H}$	$+1$	-1	0
Chiral $U(1)_{\rm X}$	a_i	$-b_j$	1

$$i = 1, \ldots, n$$
$$j = n+1, \ldots, 2n$$

with $a_i \neq b_j$

mCP masses are induced after SSB of $U(1)_{\rm X}$:

$$\epsilon = \frac{\langle \phi \rangle}{M_c}$$

$$y_{ij} M_c \left(\frac{\phi^{(*)}}{M_c} \right)^{|a_i - b_j|} \psi_i \psi_j + \text{h.c.}$$

$$y_{ij} = \mathcal{O}(1)$$

$n \times n$ mass matrix:

$$M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}$$

Hierarchical masses can be naturally realized.

Anomaly cancellation conditions

Impose strong constraints on the charge assignments

- $U(1)_{\rm X}$ - graviton-graviton $\quad \sum_{i=1}^{n} a_i = \sum_{j=1}^{n} b_j$
- $\left(U(1)_{\rm X}\right)^2 - U(1)_{\rm H}$ $\quad \sum_{i=1}^{n} a_i^2 = \sum_{j=1}^{n} b_j^2$
- $\left(U(1)_{\rm X}\right)^3$ $\quad \sum_{i=1}^{n} a_i^3 = \sum_{j=1}^{n} b_j^3$

Note that $\left(U(1)_{\rm H}\right)^2 - U(1)_{\rm X}$ is the same as $U(1)_{\rm X}$ - graviton-graviton for our charge assignment and $\left(U(1)_{\rm H}\right)^3$ is trivial due to the vector-like nature of $U(1)_{\rm H}$.

J. Lee, FT, and Y-D Tsai, 2603.12320,

See also P. Batra, B. A. Dobrescu, and D. Spivak, hep-ph/0510181, K. Nakayama, FT, and T. Yanagida, 1102.4688, André de Gouvea, Daniel Hernandez 1507.00916, Generic U(1)xU(1) studied by D.B.Costa, B.A. Dobrescu, P.J. Fox in 1905.13729, 2001.11991

Prouhet-Tarry-Escott problem

Goldbach and Euler (1750-51), E. Prouhet (1851), E. B. Escott (1910), G. Tarry (1895). See P. Borwein 2002) for a review.

The PTE problem asks for two distinct multisets of integers

$$A = \{a_1, \cdots, a_n\}, \quad B = \{b_1, \cdots, b_n\}$$

such that their power sums agree up to the degree k, i.e.

$$\sum_{i=1}^{n} a_i^{\ell} = \sum_{i=1}^{n} b_i^{\ell} \qquad (\ell = 1, 2, \cdots, k)$$

- The case $n = k + 1$ is called an *ideal* solution; for given k, this is the minimal possible size.

e.g.) $n = 4$ is the size of the ideal solution for $k = 3$.

Prouhet-Tarry-Escott problem

Goldbach and Euler (1750-51), E. Prouhet (1851), E. B. Escott (1910), G. Tarry (1895). See P. Borwein 2002) for a review.

No.	A	B
1	$\{0, 4, 7, 11\}$	$\{1, 2, 9, 10\}$
2	$\{0, 6, 7, 13\}$	$\{1, 3, 10, 12\}$
3	$\{0, 5, 10, 15\}$	$\{1, 3, 12, 14\}$
4	$\{0, 7, 9, 16\}$	$\{1, 4, 12, 15\}$
5	$\{0, 8, 9, 17\}$	$\{2, 3, 14, 15\}$
6	$\{0, 7, 11, 18\}$	$\{2, 3, 15, 16\}$
7	$\{0, 6, 13, 19\}$	$\{1, 4, 15, 18\}$
8	$\{0, 8, 11, 19\}$	$\{1, 5, 14, 18\}$
9	$\{0, 10, 11, 21\}$	$\{1, 6, 15, 20\}$
10	$\{0, 11, 13, 22\}$	$\{1, 7, 18, 20\}$

TABLE I. Examples of degree $k = 3$ PTE solutions with $n = 4$.

e.g.

$$4 + 7 + 11 = 1 + 2 + 9 + 10$$

$$4^2 + 7^2 + 11^2 = 1^2 + 2^2 + 9^2 + 10^2$$

$$4^3 + 7^3 + 11^3 = 1^3 + 2^3 + 9^3 + 10^3$$

Anomaly Cancellation for Light mCPs

$\Leftrightarrow$ **A degree** $k = 3$ **PTE problem**

Anomaly Cancellation for Light mCPs

$\Leftrightarrow$ A degree $k = 3$ PTE problem

- The minimal ideal-solution setup contains **eight** left-handed Weyl fermions, forming **four mCP mass eigenstates**.

$$M_{ij}\psi_i\psi_j + \text{h.c.}$$

	ψ_i	ψ_j
$U(1)_\text{H}$	$+1$	-1
$U(1)_\text{X}$	a_i	$-b_j$

$$i = 1, \ldots, n$$
$$j = n+1, \ldots, 2n$$

Anomaly Cancellation for Light mCPs

$\Leftrightarrow$ A degree $k = 3$ PTE problem

- The minimal ideal-solution setup contains **eight** left-handed Weyl fermions, forming **four mCP mass eigenstates**.

There must be at least four mCPs!

What else can we say about mCPs from this correspondence?

A bit more about PTE problem...

- An affine transformation, $A' = \{ca_i + d\}$ and $B' = \{cb_i + d\}$, maps a solution to another solution, where c and d are rational numbers with $c \neq 0$.

- Centralized form: by a common shift, we set

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i = 0$$

A bit more about PTE problem...

Symmetric solutions

A centralized PTE solution is called **symmetric** if it is invariant under the sign flip:

$$\begin{cases} A = -A, \quad B = -B, & n \text{ even}, \\ B = -A, & n \text{ odd}. \end{cases}$$

e.g.) $A = \{-8, -1, 1, 8\}, \quad B = \{-7, -4, 4, 7\}$

often written as $A = \{\pm 8, \pm 1\}, \quad B = \{\pm 7, \pm 4\}$

Note that the linear and cubic sum conditions are trivial for symmetric solutions.

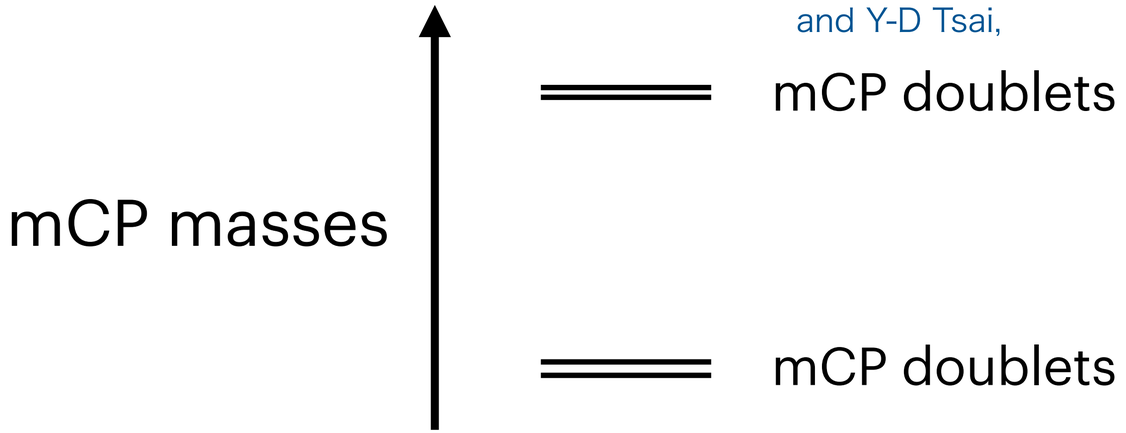
Symmetric solutions predict mCP doublets!

The mCP mass spectrum is determined by the PTE solutions:

$$M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}$$

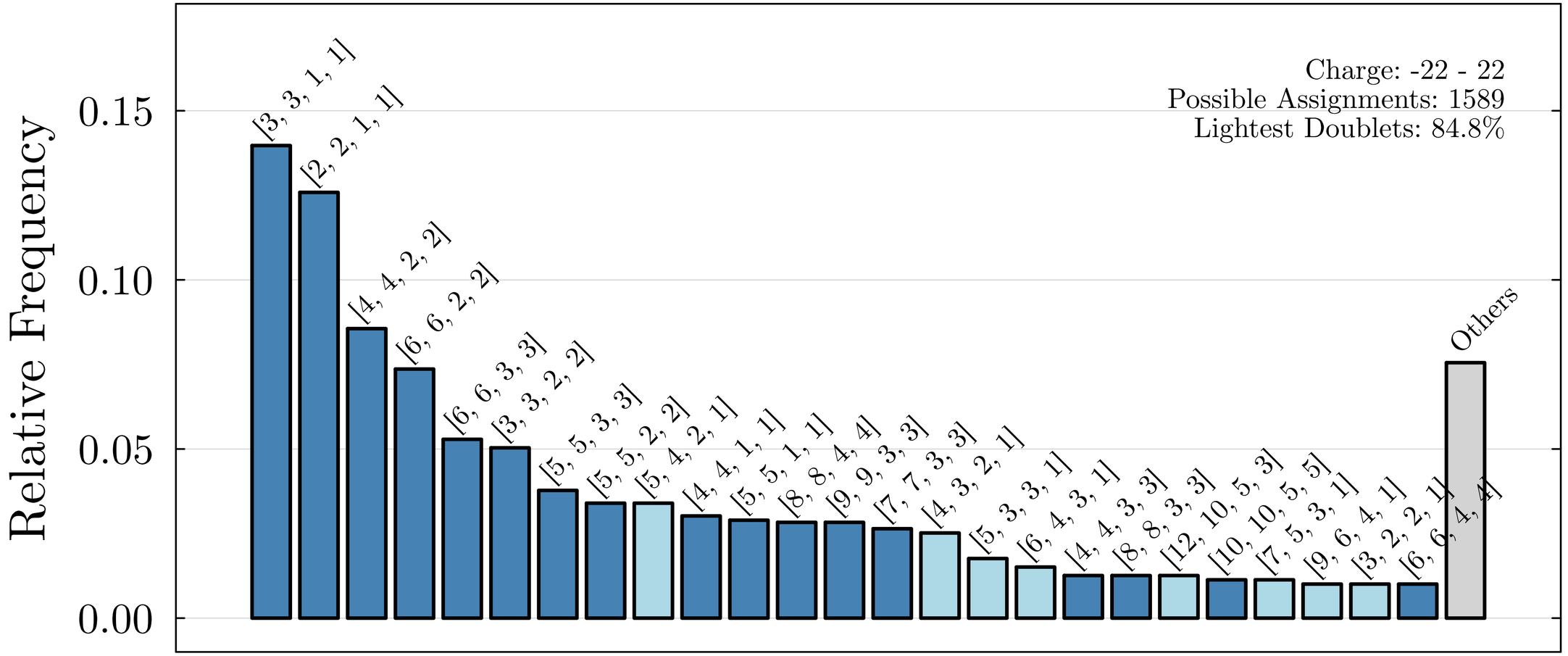
In fact, **symmetric PTE solutions always yield mCP doublets.**

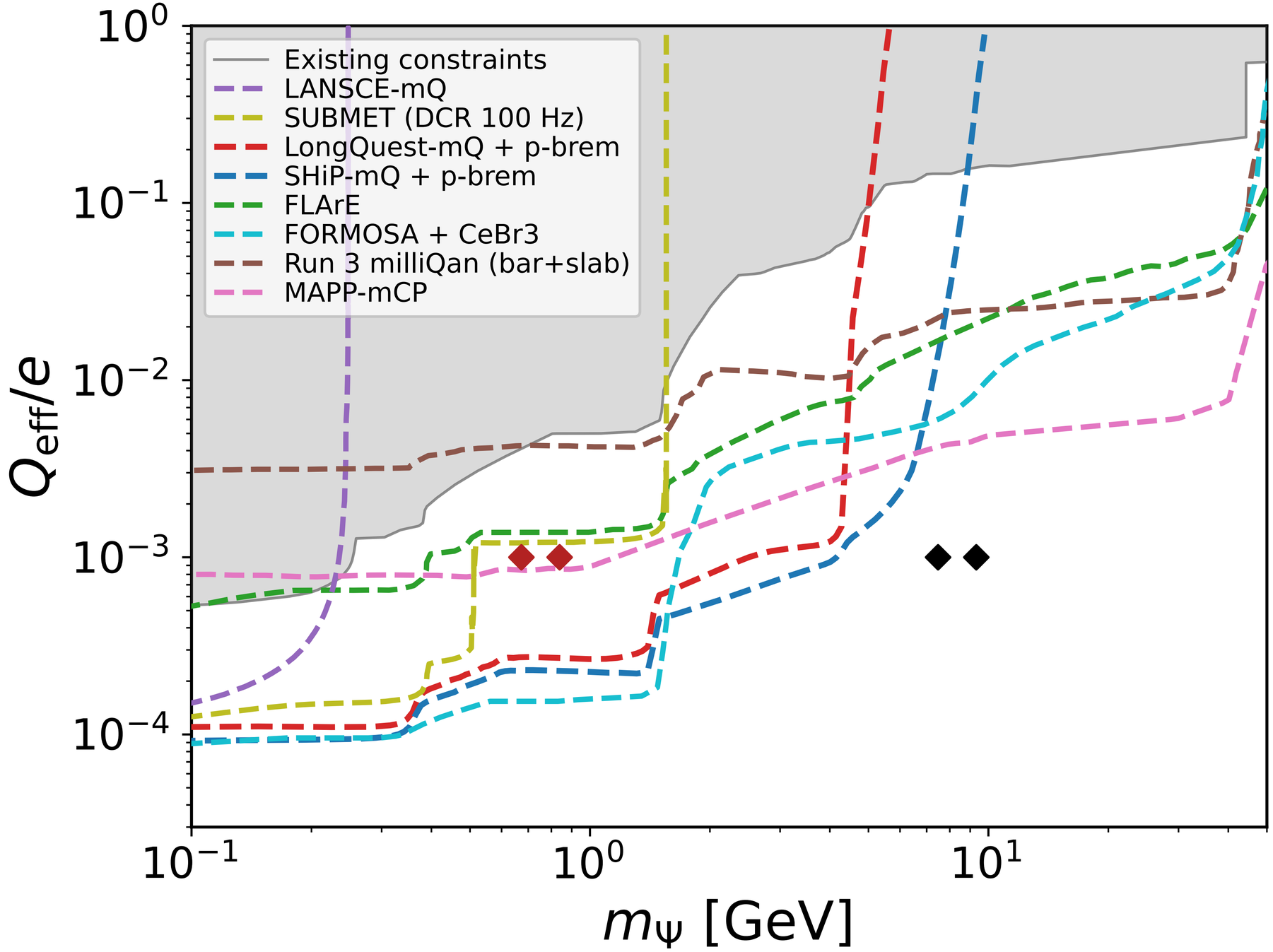
Proof given in the appendix of 2603.12320 by J. Lee, FT, and Y-D Tsai,



Symmetric solutions dominate

More than 80% of ideal solutions with $H = \max[\,|a_i|, |b_j|\,] \leq 22$ are symmetric solutions ➡ **mCPs are likely to appear in doublets.**



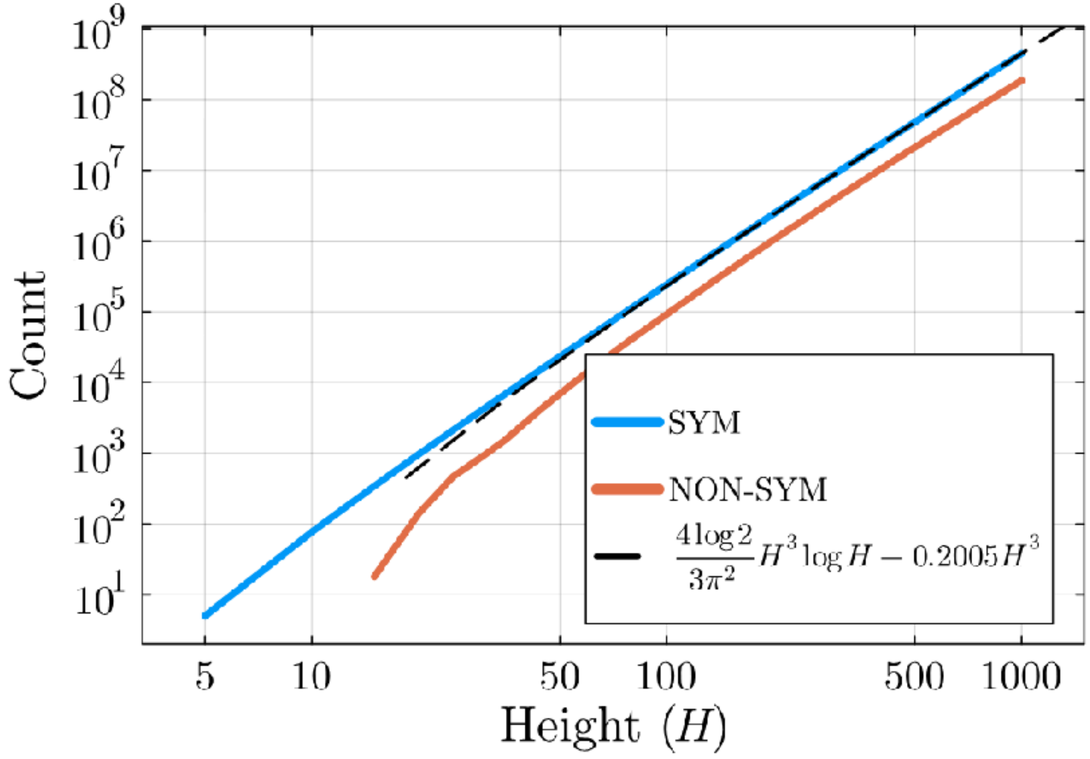

Existing constraints
LANSCE-mQ
SUBMET (DCR 100 Hz)
LongQuest-mQ + p-brem
SHiP-mQ + p-brem
FLArE
FORMOSA + CeBr3
Run 3 milliQan (bar+slab)
MAPP-mCP
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Q_{eff}/e
10^{-1}
10^0
10^1
m_ψ [GeV]

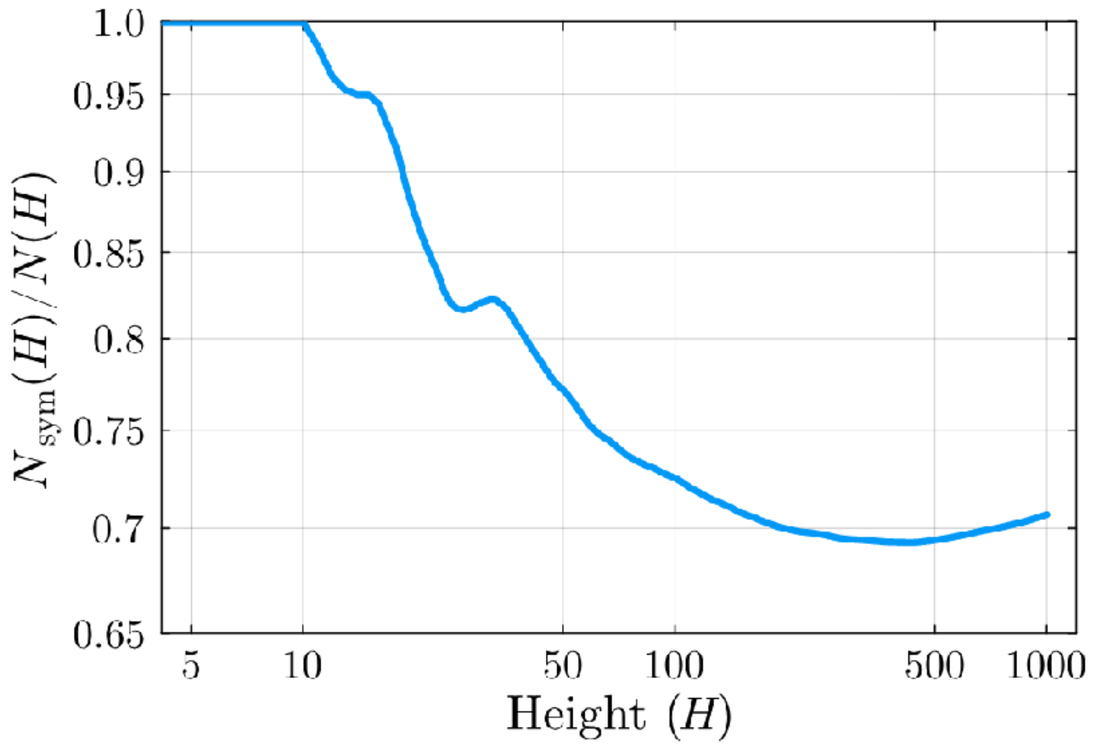
Counting symmetric solutions

J. Lee, FT, and Y-D Tsai, 2606.07735 [math.NT]

We count the number of symmetric solutions as a function of height $H = \max[\,|a_i|, |b_j|\,]$:

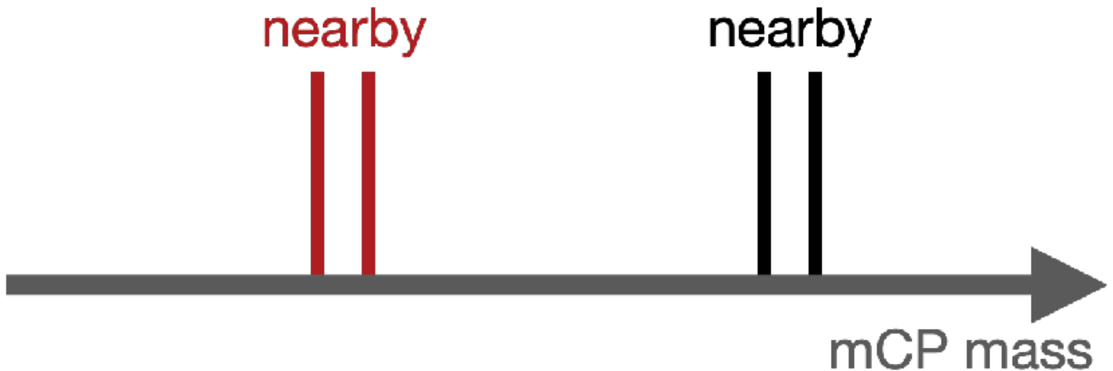
$$N_{\mathrm{sym}}(H) = \frac{4\log 2}{3\pi^2} H^3 \log H + O(H^3).$$





Summary

- Vector portal is a promising candidate for BSM.
- Light mCPs can be naturally realized with a chiral U(1).
- Anomaly cancellation connects the model to **the Prouhet-Tarry-Escott problem** in number theory, implying **at least four mCPs** in the minimal setup.
- Symmetric PTE solutions suggest **a doublet structure.**



Back-up slides

What if the vector-like charge assignment is not universal?

We assumed that the vector-like $U(1)_H$ is flavor blind:

	ψ_i	ψ_j	ϕ
Vector-like $U(1)_{\mathrm{H}}$	$+1$	-1	0
Chiral $U(1)_{\mathrm{X}}$	a_i	$-b_j$	1

$$i = 1, \ldots, n$$
$$j = n+1, \ldots, 2n$$

with $a_i \neq b_j$

so that all the nontrivial flavor structures arise from the chiral $U(1)_X$.

If we instead consider a more general vector-like charge assignment,

	ψ_i	ψ_j	ϕ
Vector-like $U(1)_{\rm H}$	q_i	$-q_{j-n}$	0
Chiral $U(1)_{\rm X}$	a_i	$-b_j$	1

$$i = 1, \ldots, n$$
$$j = n+1, \ldots, 2n$$

the mixed-anomaly cancellation conditions now become

$$\sum_i q_i^2 a_i = \sum_i q_i^2 b_i \qquad \sum_i q_i a_i^2 = \sum_i q_i b_i^2$$

a weighted-version of PTE problem

and the minimum number of mCPs, minicharges, and mass hierarchy will be modified.

Geberic U(1)xU(1) theories were studied by D.B.Costa, B.A. Dobrescu, P.J. Fox, 1905.13729, 2001.11991

Dark photon (γ') : the gauge boson of $U(1)_H$

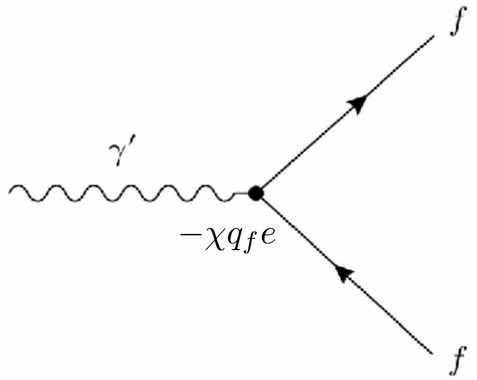
Kinetic mixing with $U(1)_{EM}$ (or $U(1)_Y$ at high energies)

$$\frac{\chi}{2} F'_{\mu\nu} F^{\mu\nu}$$

- Massive dark photons: $m_{\gamma'} \neq 0$

  Coupled to SM with interaction strength suppressed by χ.

  ∵ The kinetic mixing can be removed by $A \to A - \chi A'$ (mass basis)



Dim-6 electric&magnetic dipole couplings to dark-photon

$$\bar{\psi}_L^i \sigma_{\mu\nu} \left(D_M^{ij} + i\gamma_5 D_E^{ij} \right) H \psi_R^j F'^{\mu\nu}$$

Mass ratio of two mCPs in the same doublet

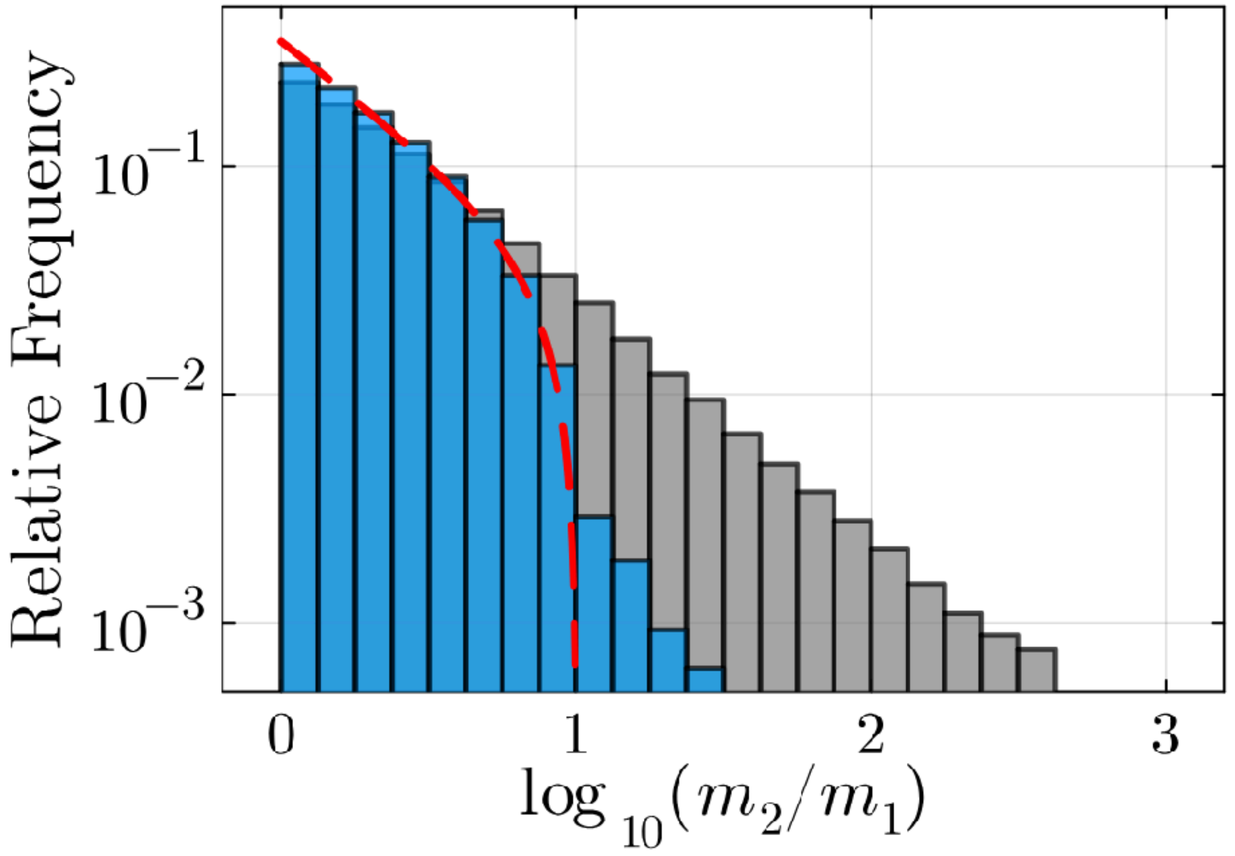


FIG. 2. Mass splitting between the two lightest eigenstates for degree $k = 3$ PTE solutions with $n = 4$. The grey and blue histograms show the distribution of m_2/m_1 with $m_2 > m_1$ for coefficient magnitudes sampled in $[0, 1]$ and $[0.1, 1]$, respectively. The red dashed line shows the estimate based on the diagonal approximation for coefficients with magnitudes in $[0.1, 1]$.