Beyond Δm^2: Absolute Mass Sensitivity in Neutrino Oscillations

(with G. Alves, J. Kaler, S.W. Li, P. Machado, 2608.13736 [hep-ph])

André de Gouvêa – Northwestern University

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$$P_{\rm osc} \propto \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \sin^2\left(\frac{L}{L_{\rm osc}}\right)$$

This expression for $L_{\rm osc}^{-1}$ is approximate. It is the leading order result in a m^2/E^2 expansion.

It has consequences. One is part of the generic neutrino physics lore:

$P_{\rm osc}$ measures neutrino mass-squared differences

This means we don't really measure the neutrino masses, only their differences. In particular oscillation experiments can't tell whether the lightest neutrino is massless or whether the neutrino masses are quasi-degenerate.

What is the next-to-leading order correction to $L_{\rm osc}^{-1}$ (i.e., the term that goes like m^4/E^4)?

The Propagation of Massive Neutrinos

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-iE_i t}|\nu_i\rangle, \qquad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of ν_i's, say:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

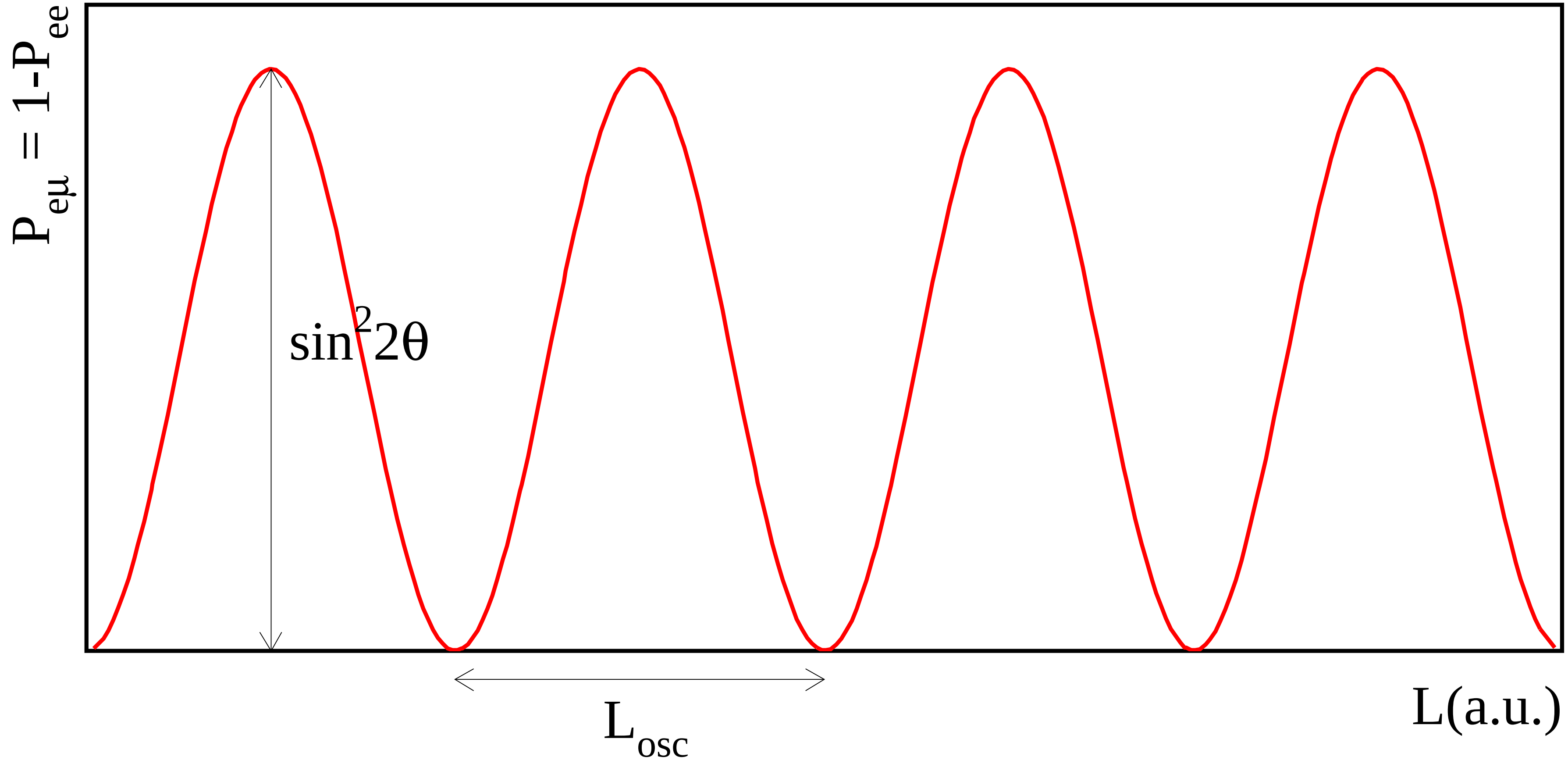
If this is the case, a state produced as a ν_e evolves in vacuum into

$$|\nu(t,\vec{x})\rangle = \cos\theta e^{-ip_1 x}|\nu_1\rangle + \sin\theta e^{-ip_2 x}|\nu_2\rangle.$$

It is trivial to compute $P_{e\mu}(L) \equiv |\langle\nu_\mu|\nu(t,z=L)\rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$\boxed{P_{e\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)}$$

oscillation parameters: $\begin{cases} \pi\frac{L}{L_{\rm osc}} \equiv \frac{\Delta m^2 L}{4E} = 1.267\left(\frac{L}{\rm km}\right)\left(\frac{\Delta m^2}{\rm eV^2}\right)\left(\frac{\rm GeV}{E}\right) \\ \text{amplitude } \sin^2 2\theta \end{cases}$



Neutrino oscillations at the Next-to-Leading Order

The standard hand-wavy arguments don't work at NLO. We need to treat the neutrino states more carefully. We treat them as wave-packets.

Assume a ν_α is produced at the origin at time zero and we work in one space dimension. Its wave-function as a function of the position x and the time t is

$$|\nu_\alpha(x,t)\rangle = \sum_i \frac{U^*_{\alpha i}}{\sqrt{2\pi}} \int \frac{dp}{(2\pi\sigma_p^2)^{1/4}} \exp\left(-\frac{(p-P_i)^2}{4\sigma_p^2} + ipx - iE_i(p)t\right) |\nu_i\rangle$$

P_i is the mean momentum for each ν_i and $E_i(p) = \sqrt{p^2+m_i^2}$.

Narrow-Width Approximation

Expand each energy $E_i(p)$ around its mean momentum P_i,

$$E_i(p) = E_i(P_i) + v_i(p - P_i) + \mathcal{O}\left(\frac{m_i^2(p-P_i)^2}{E_i(P_i)^3}\right)$$

$v_i = P_i/E_i(P_i)$, and keep terms through linear order in $p - P_i$.

This is a narrow-width expansion in σ_p/P_i and is distinct from the relativistic expansion in m_i^2/E^2.

With this approximation,

$$|\nu_\alpha(x,t)\rangle = \sum_i \frac{U_{\alpha i}^*}{(2\pi\sigma_x^2)^{1/4}} \exp\left[-\frac{(x-v_i t)^2}{4\sigma_x^2} + iP_i x - iE_i(P_i)t\right]|\nu_i\rangle$$

The Probability Density in Space and Time

The amplitude to detect the neutrino as flavor β is $A_{\alpha\beta} = \langle\nu_\beta|\nu_\alpha(x,t)\rangle$ and the probability density is

$|A_{\alpha\beta}|^2 =$

$$\sum_{i,j} U^*_{\alpha i}\, U_{\beta i}\, U_{\alpha j}\, U^*_{\beta j} \frac{1}{(2\pi\sigma_x^2)^{1/2}} \exp\left[-\frac{(x-v_i t)^2 + (x-v_j t)^2}{4\sigma_x^2} + i\Delta P_{ij}\, x - i\Delta E_{ij} t\right]$$

with $\Delta P_{ij} = P_i - P_j$ and $\Delta E_{ij} = E_i(P_i) - E_j(P_j)$.

From Probability Density to Probability

We want the probability that the neutrino is detected inside a finite region within the detector, located around the baseline position L.

To keep the integrals and the answers analytic, we approximate the detection region by a Gaussian centered at L with width δL, and extend the integral range to infinity. We define the ancillary quantity

$$\widetilde{P}_{\alpha\beta}(L,t) = \int_{-\infty}^{\infty} dx\; e^{-\frac{(x-L)^2}{2\delta L^2}}\; |A_{\alpha\beta}(x,t)|^2$$

We will return to the normalization of this later.

The Probability Experiments Measure

Experiments measure the flux of ν_β at the detector, $\phi^D_\beta(t)$. This measurement is related to the inferred time-independent probability via

$$\phi^D_\beta(t) \equiv \sum_\alpha P_{\alpha\beta}(L)\phi^S_\alpha(t-L),$$

where $\phi^S_\alpha(t)$ is the flux of ν_α at the source.

On the other hand, given a (time-dependent) flux $\phi^S_\alpha(t)$ of ν_α emitted at the source, the flux of ν_β at the detector is

$$\phi^D_\beta(t) = C_\alpha \int_{-\infty}^{+\infty} dt' \; P_{\alpha\beta}(L,t,t')\phi^S_\alpha(t'),$$

where C_α is a proportionality constant.

Aside: On the Normalization Constant C_α

C_α is chosen such that the total number $N^D_{\rm tot}$ of neutrinos at the detector is the same as the total number $N^S_{\rm tot}$ of neutrinos emitted at the source.

$$\begin{aligned} N^D_{\rm tot} &= \sum_\beta \int_{-\infty}^{+\infty} dt\ \Phi^D_\beta(t), \\ &= \sum_\beta \int_{-\infty}^{+\infty} dt \left(C_\alpha \int_{-\infty}^{+\infty} dt'\ P_{\alpha\beta}(L,t,t') \phi^S_\alpha(t') \right), \\ &= \int_{-\infty}^{+\infty} dt'\ \phi^S_\alpha(t') \left(C_\alpha \sum_\beta \int_{-\infty}^{+\infty} dt\ P_{\alpha\beta}(L,t,t') \right). \end{aligned}$$

$P_{\alpha\beta}(L,t,t')$ depends only on $t''=t-t'$. We take advantage of this to write

$$N^D_{\rm tot} = \left(C_\alpha \sum_\beta \int_{-\infty}^{+\infty} dt''\ P_{\alpha\beta}(L,t'') \right) N^S_{\rm tot}$$

$$C_\alpha = \frac{1}{\sum_\beta \int_{-\infty}^{+\infty} dt\ P_{\alpha\beta}(L,t,t')}$$

If different flavors are produced at the source, each with flux $\phi_\alpha^S(t)$,

$$\phi_\beta^D(t) = \sum_\alpha C_\alpha \int_{-\infty}^{+\infty} dt'\ P_{\alpha\beta}(L,t,t')\phi_\alpha^S(t'),$$

where C_α is given above.

Not only is the total number of neutrinos emitted at the source is the same as the number of neutrinos that go through the detector, but it also implies that the number of neutrinos is conserved for each individual initial flavor: If a total N_α^S of ν_α is produced at the source, the total number of neutrinos of all flavors at the detector associated with the initial-state flavor ν_α is also N_α^S.

The neutrino wave packets are sharply peaked at $t - t' = L/v_i$ and the different v_i are very close to one another and to the speed of light. Assuming $\Phi^S_\alpha(t)$ is relatively smooth,

$$\int_{-\infty}^{\infty} dt' \widetilde{P}_{\alpha\beta}(L, t - t')\Phi^S_\alpha(t') \simeq \Phi^S_\alpha(t - L) \int_{-\infty}^{\infty} dt'' \widetilde{P}_{\alpha\beta}(L, t''),$$

where, in the second expression, we changed the integration variable to $t'' = t - t'$. Comparing this with the definition of $P_{\alpha\beta}(L)$

$$P_{\alpha\beta}(L) \propto \int_{-\infty}^{\infty} dt''\, \widetilde{P}_{\alpha\beta}(L, t'')$$

That is, $P_{\alpha\beta}(L)$ is proportional to the time-averaged value of $\widetilde{P}_{\alpha\beta}$.

$$\mathcal{P}_{\alpha\beta}(L) = \frac{1}{\mathcal{N}_T} \int_{-\infty}^{\infty} dt\, \widetilde{P}_{\alpha\beta}(L, t) = \frac{1}{\mathcal{N}_T} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx\, e^{-\frac{(x-L)^2}{2\delta L^2}} \left|A_{\alpha\beta}(x, t)\right|^2,$$

with $1/\mathcal{N}_T$ a dimensionful normalization constant.

Properly Normalizing the Probability

The quantity $\mathcal{P}_{\alpha\beta}(L)$ is the (time-averaged) probability that the neutrino is detected in the detection region is identified as flavor β. What an oscillation experiment reports, however, is conditioned on a neutrino of any flavor having been detected. Given that a neutrino produced as ν_α is observed in the detection region, the probability that it is observed as ν_β is

$$P_{\alpha\beta}(L) = \frac{\mathcal{P}_{\alpha\beta}(L)}{\sum_{\beta'} \mathcal{P}_{\alpha\beta'}(L)}$$

The denominator is the total probability of detecting a neutrino, independent of its flavor.

$\sum_\beta P_{\alpha\beta}(L) = 1$ for all α and L and that $P_{\alpha\beta}(L)$ does not depend on $\mathcal{N}_T$ and other flavor-independent prefactors.

When the Dust Settles

$$\mathcal{P}_{\alpha\beta}(L) = \frac{1}{\mathcal{N}_T}\sum_{i,j} U^*_{\alpha i}\, U_{\beta i}\, U_{\alpha j}\, U^*_{\beta j}\, \mathcal{N}_{ij} \times$$
$$\exp\left(-\frac{L^2}{2\delta L^2 + (L^{ij}_{\rm coh})^2} - \frac{2\pi i\, L}{L^{ij}_{\rm osc}}\frac{(L^{ij}_{\rm coh})^2}{2\delta L^2 + (L^{ij}_{\rm coh})^2} - c_{ij}\right),$$

where

$$L^{ij}_{\rm coh} = 2\sqrt{\frac{v_i^2+v_j^2}{(v_i-v_j)^2}}\,\sigma_x, \qquad L^{ij}_{\rm osc} = \frac{2\pi}{\Delta E_{ij}}\left(\frac{v_i+v_j}{v_i^2+v_j^2} - \frac{\Delta P_{ij}}{\Delta E_{ij}}\right)^{-1},$$

along with the normalization factor

$$\mathcal{N}_{ij} = \frac{2\sqrt{2}}{|v_i-v_j|}\frac{\sigma_x}{\sqrt{2\delta L^2 + (L^{ij}_{\rm coh})^2}},$$

and a baseline-independent constant

$$c_{ij} = \frac{2\left[(\Delta E_{ij} - v_i \Delta P_{ij})^2 + (\Delta E_{ij} - v_j \Delta P_{ij})^2\right] \delta L^2 \sigma_x^2 + 4\Delta E_{ij}^2\, \sigma_x^4}{(v_i - v_j)^2 [2\delta L^2 + (L_{\rm coh}^{ij})^2]}\,.$$

In the regime relevant to oscillation experiments, $\delta L / L_{\rm coh}^{ij}$ is negligible. We will therefore safely take $\delta L / L_{\rm coh}^{ij} \to 0$ henceforth. In this limit, the first and second terms reduce to the standard wave-packet decoherence damping and oscillation phase, respectively. With all this in mind,

$$\mathcal{P}_{\alpha\beta}(L) = \frac{1}{\mathcal{N}_T} \sum_{i,j} \mathcal{N}_{ij}\, U_{\alpha i}^*\, U_{\beta i}\, U_{\alpha j}\, U_{\beta j}^* \;\exp\left(-\frac{L^2}{(L_{\rm coh}^{ij})^2} - \frac{2\pi i\, L}{L_{\rm osc}^{ij}} - c_{ij} \right).$$

Neutrino Production (Reactor Neutrinos)

$$A \to A' + e + \bar{\nu}_i$$

We describe the $A' + e$ system via an effecive mass $m_{\rm eff}$, which we assume does not depend on the mass-eigenstate.

$$E_i(P_i) = \frac{m_A^2 + m_i^2 - m_{\rm eff}^2}{2m_A}, \qquad P_i = \frac{\sqrt{(m_A^2 + m_i^2 - m_{\rm eff}^2)^2 - 4m_A^2 m_i^2}}{2m_A},$$

where m_A is the mass of the parent nucleus.

Reference energy E_0: the energy the neutrino would have if it were massless.

$$E_0 = \frac{m_A^2 - m_{\rm eff}^2}{2m_A}$$

E_0 is the quantity most directly related to the neutrino energy reconstructed in a reactor experiment.

Expanding to NLO...

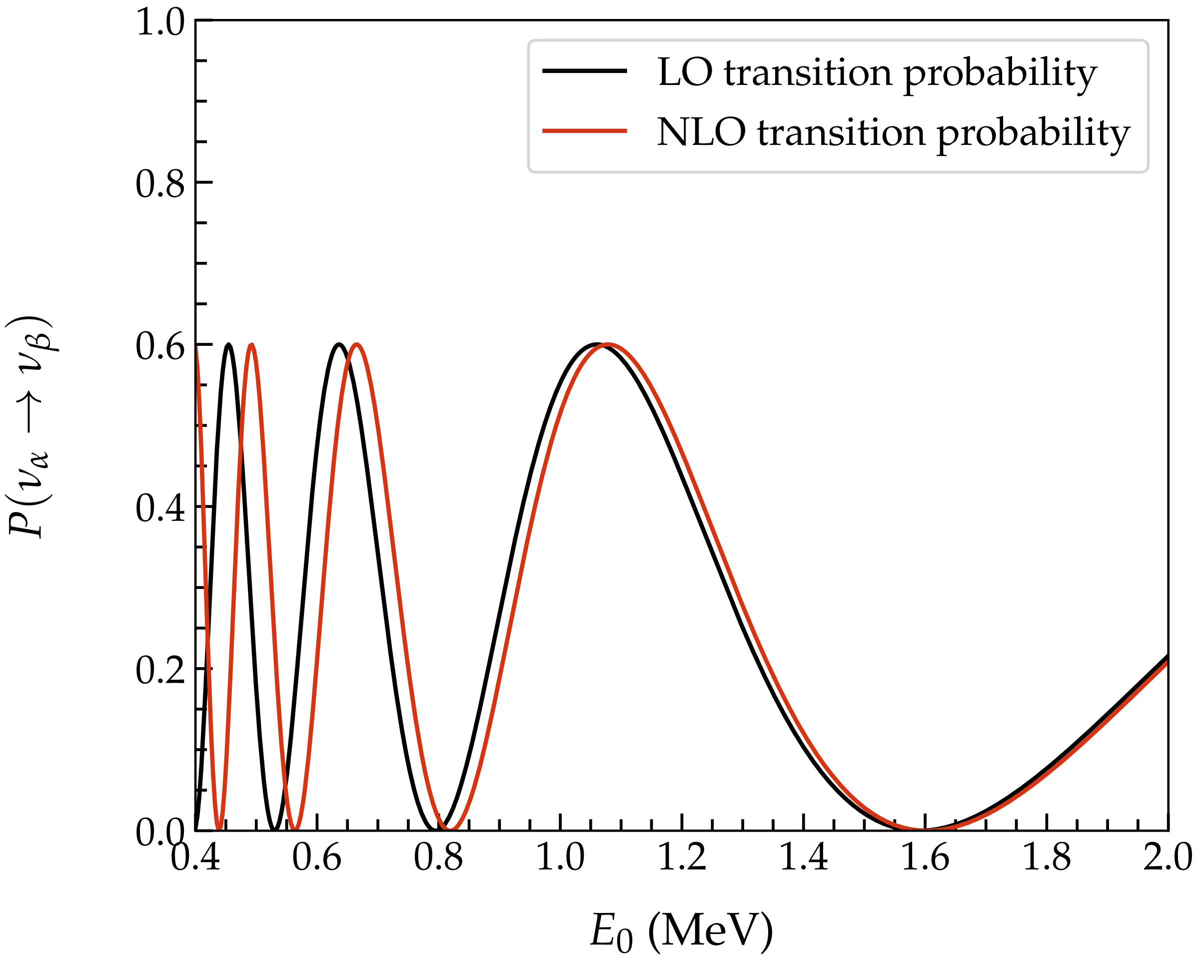
$$\frac{2\pi}{E_0 L^{ij}_{\rm osc}} = \frac{\Delta m^2_{ij}}{2E_0} + \frac{m_i^4 - m_j^4}{8\,E_0^3}\left(1 - \frac{E_0}{m_A}\right) + \mathcal{O}\left(\frac{m_i^6}{E_0^5}\right)$$

The oscillation length depends on the mass of the decaying nucleus and hence on the production process! Fortunately, $E_0 \ll m_A$, so we get, ignoring decoherence effects

$$P_{\alpha\beta}(L) = \sum_{i,j} U^*_{\alpha i}\, U_{\beta i}\, U_{\alpha j}\, U^*_{\beta j}\;\exp\left[-i\left(\frac{\Delta m^2_{ij} L}{2E_0} + \frac{(m_i^4 - m_j^4)L}{8\,E_0^3}\right)\right]$$

This only depends on the reference energy, which we extract from experiment independent from the neutrino mass.

Two Flavor Oscillations



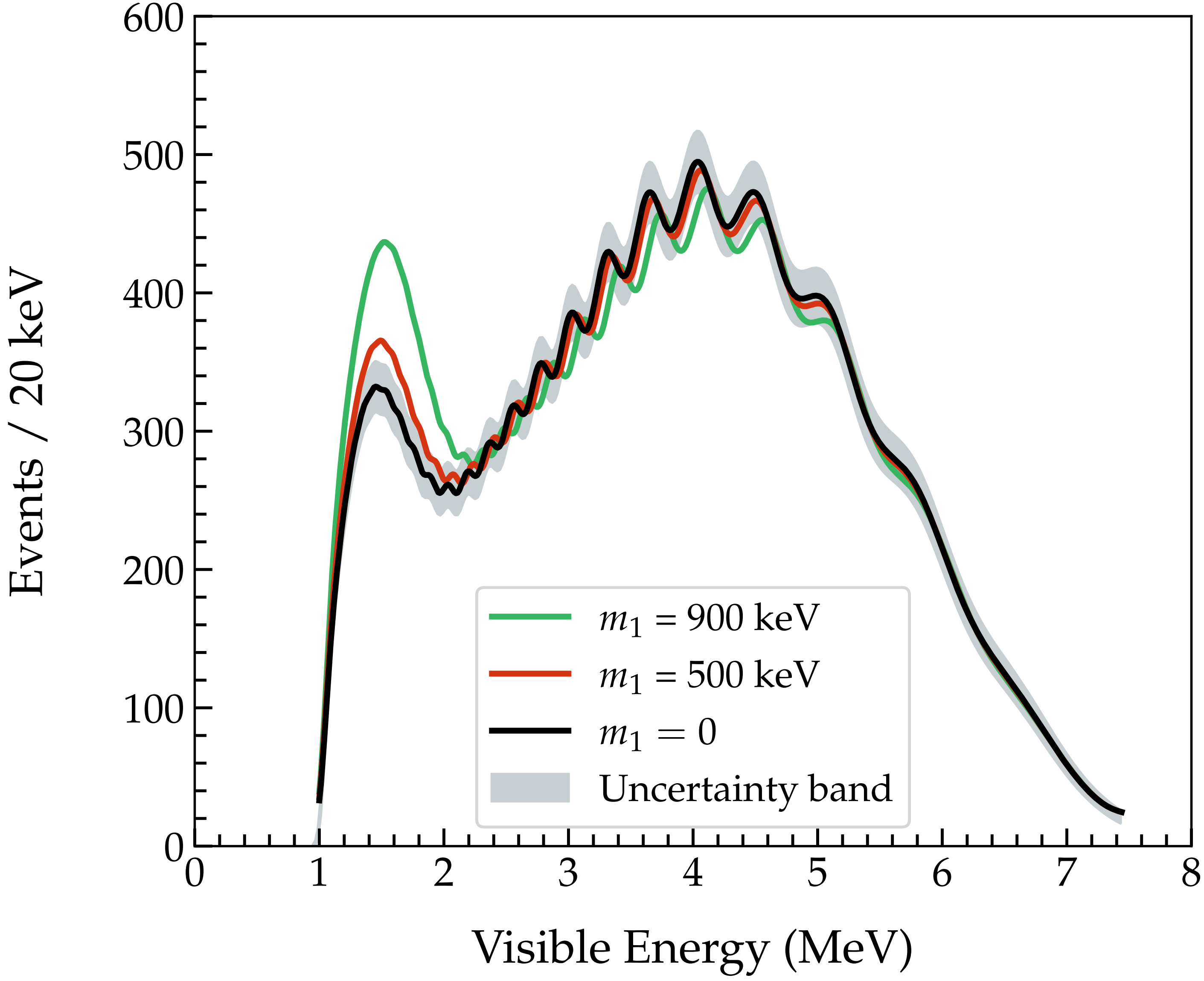
$$P_{\alpha\beta} = \sin^2 2\theta \sin^2\left[\frac{\Delta m^2 L}{4E_0}\left(1 + \frac{m_1^2 + m_2^2}{4E_0^2}\right)\right]$$

Reactor Neutrinos – JUNO's Sensitivity

$$P_{\bar{\nu}_e\to\bar{\nu}_e} = 1-\sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\left[\frac{\Delta m^2_{21}\,L}{4E_0}\left(1+\frac{m_1^2+m_2^2}{4E_0^2}\right)\right]$$
$$-\sin^2 2\theta_{13}\left\{\cos^2\theta_{12}\sin^2\left[\frac{\Delta m^2_{31}\,L}{4E_0}\left(1+\frac{m_1^2+m_3^2}{4E_0^2}\right)\right]\right.$$
$$\left.+\sin^2\theta_{12}\sin^2\left[\frac{\Delta m^2_{32}\,L}{4E_0}\left(1+\frac{m_2^2+m_3^2}{4E_0^2}\right)\right]\right\}$$

Can we see the NLO effects? Depends on $m_1\sim m_2\sim m_3$ (our only hope lies deep inside the quasi-degenerate regime...)

Reactor Neutrinos – JUNO's Sensitivity



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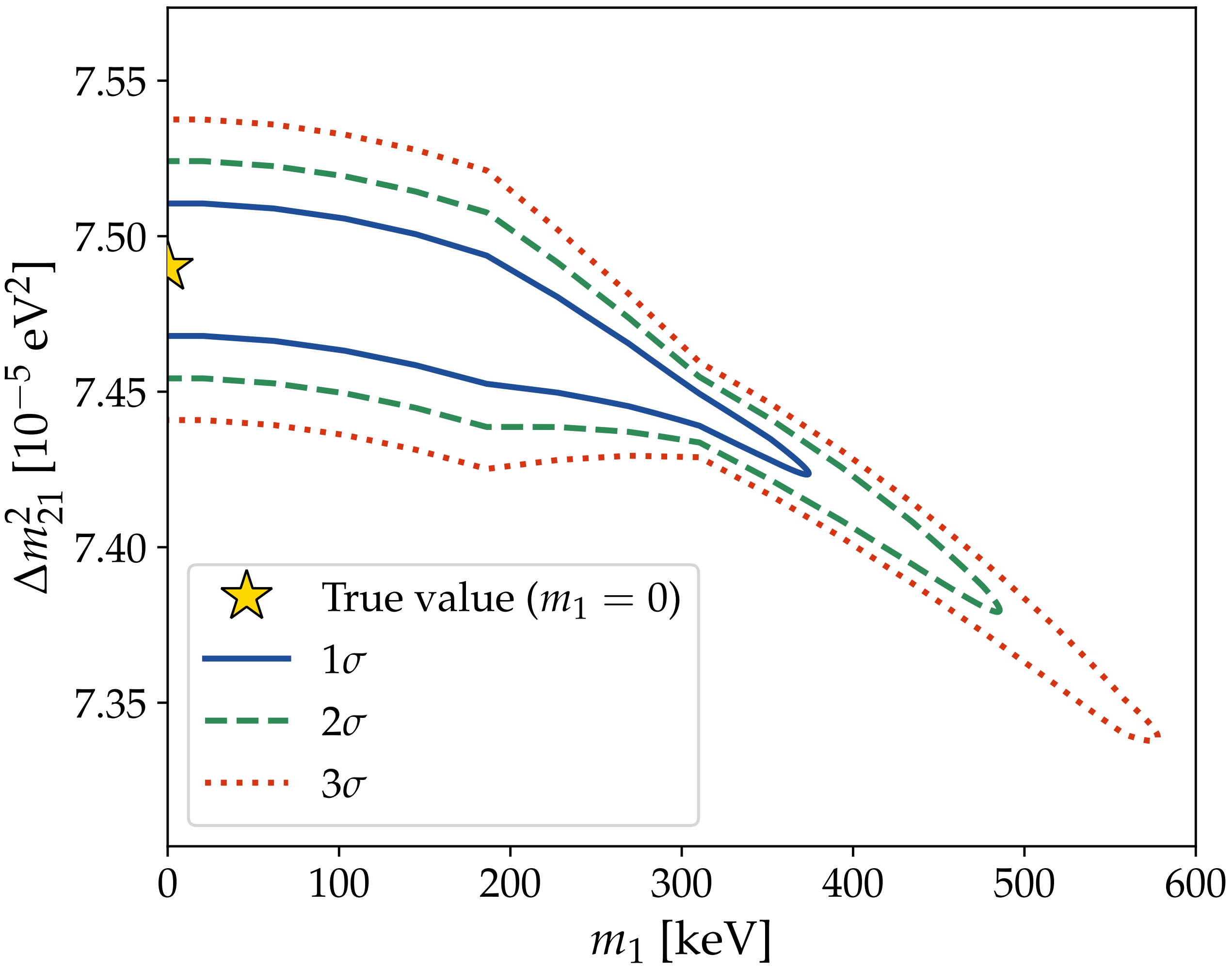
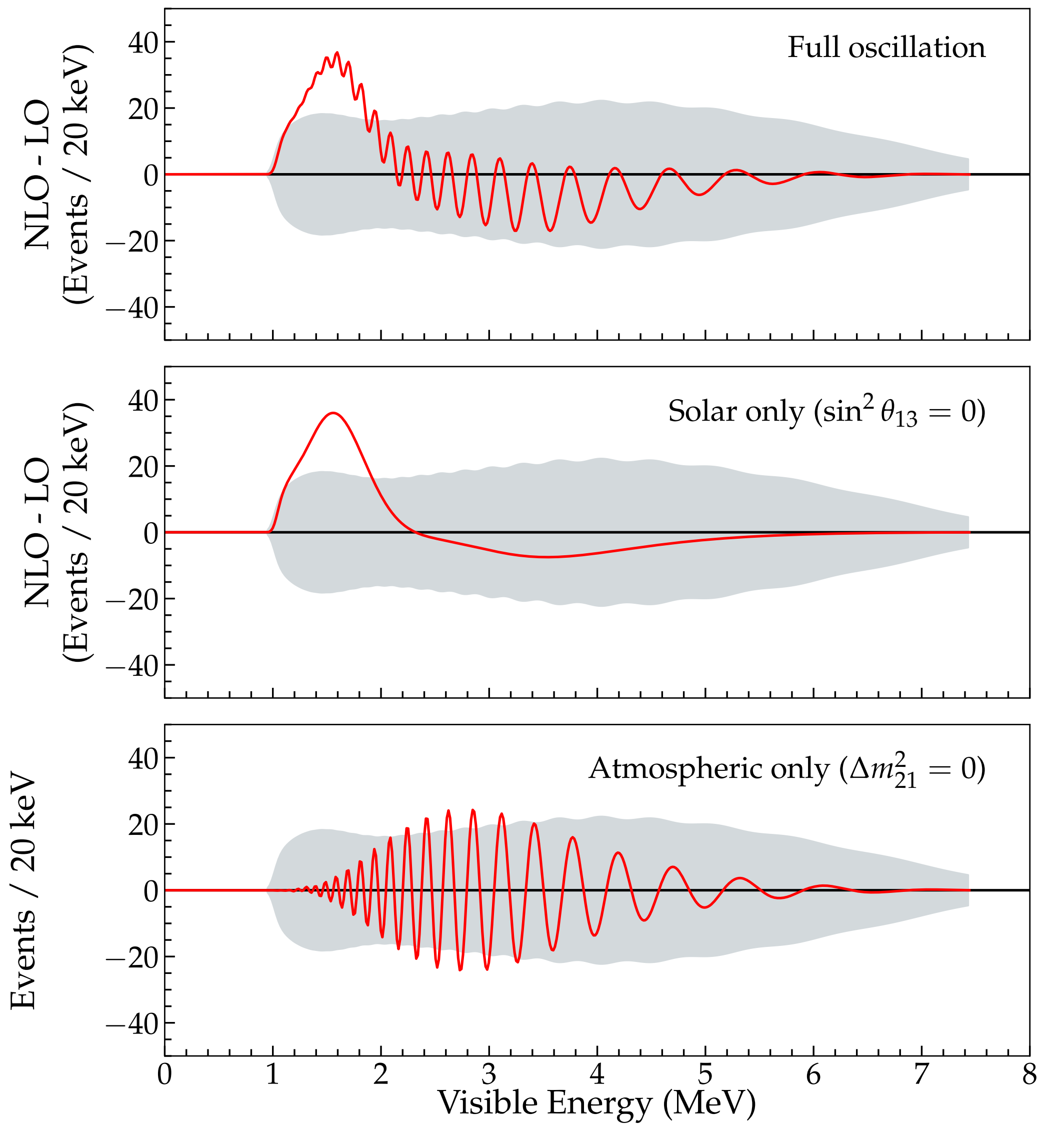


Bottom line: $m_1 < 294$ keV at 1σ and $m_1 < 419$ keV at 2σ.

Reactor Neutrinos – JUNO's Sensitivity



Reality Check – (Some) Existing Bounds

NLO oscillations constrain $m_i^2 + m_j^2$, $i, j = 1, 2, 3$, assuming the LO effect, proportional to $m_i^2 - m_j^2$, is know. There are plenty of other bounds on different combinations of neutrino masses.

- Kinematical bounds constrain $m^2_{\nu_\alpha} = \sum_i |U_{\alpha i}|^2 m_i^2$, $\alpha = e, \mu, \tau$.
  - $m_{\nu_e} < 0.45$ eV at 90% C.L. (KATRIN), $m_{\nu_e} < 15$ eV at 90% C.L., (ECHo), $m_{\nu_e} < 27$ eV at 90% C.L. (HOLMES);
  - $m_{\nu_\mu} < 0.17$ MeV at 90% C.L. (pion decay);
  - $m_{\nu_\tau} < 18.2$ MeV (ALEPH) $m_{\nu_\tau} < 48$ MeV (DELPHI collaboration), both at 95% C.L [τ decays to many pions];
- $m_i < 5.8$ eV (SN1987A);
- $\sum m_i < 0.053$ eV at 95% C.L. ($Lambda$CDM, Cosmic Surveys), model dependent. ($w_0 w_a$CDM model $\Rightarrow$ $\sum m_i < 0.163$ eV (95% C.L.));
- Double-beta decay experiments constrain $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$. $m_{\beta\beta} < 0.028$–0.122 eV (KamLAND-Zen), $m_{\beta\beta} < 0.075$–0.200 eV (LEGEND).

Quick Summary

- $P \propto \sin^2(\frac{\Delta m^2 L}{4E})$ is the first term in an m^2/E^2 expansion. What comes next?
- The second term in the m^2/E^2 expansion is proportional to $(m_i^4 - m_j^4/E^4)$. The coefficient of this next-order must be computed with care. Hand-wavy arguments don't work!
- $(m_i^4 - m_j^4)/E^4 = \Delta m_{ji}^2(m_j^2 + m_i^2)$. Neutrino oscillations are proportional to the mass-squared difference and mass-squared sum and hence the absolute values of the neutrino masses!
- Not surprisingly, effects are really tiny. Nonetheless, we estimate that JUNO is sensitive to neutrino masses of order the electron mass.
- Other applications? Very low energy neutrinos, HNLs, etc.