

Quantum Entanglement and Bell Nonlocality for two-qubit system at Colliders

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Quantum Entanglement @ LHC

> 5 σ deviations

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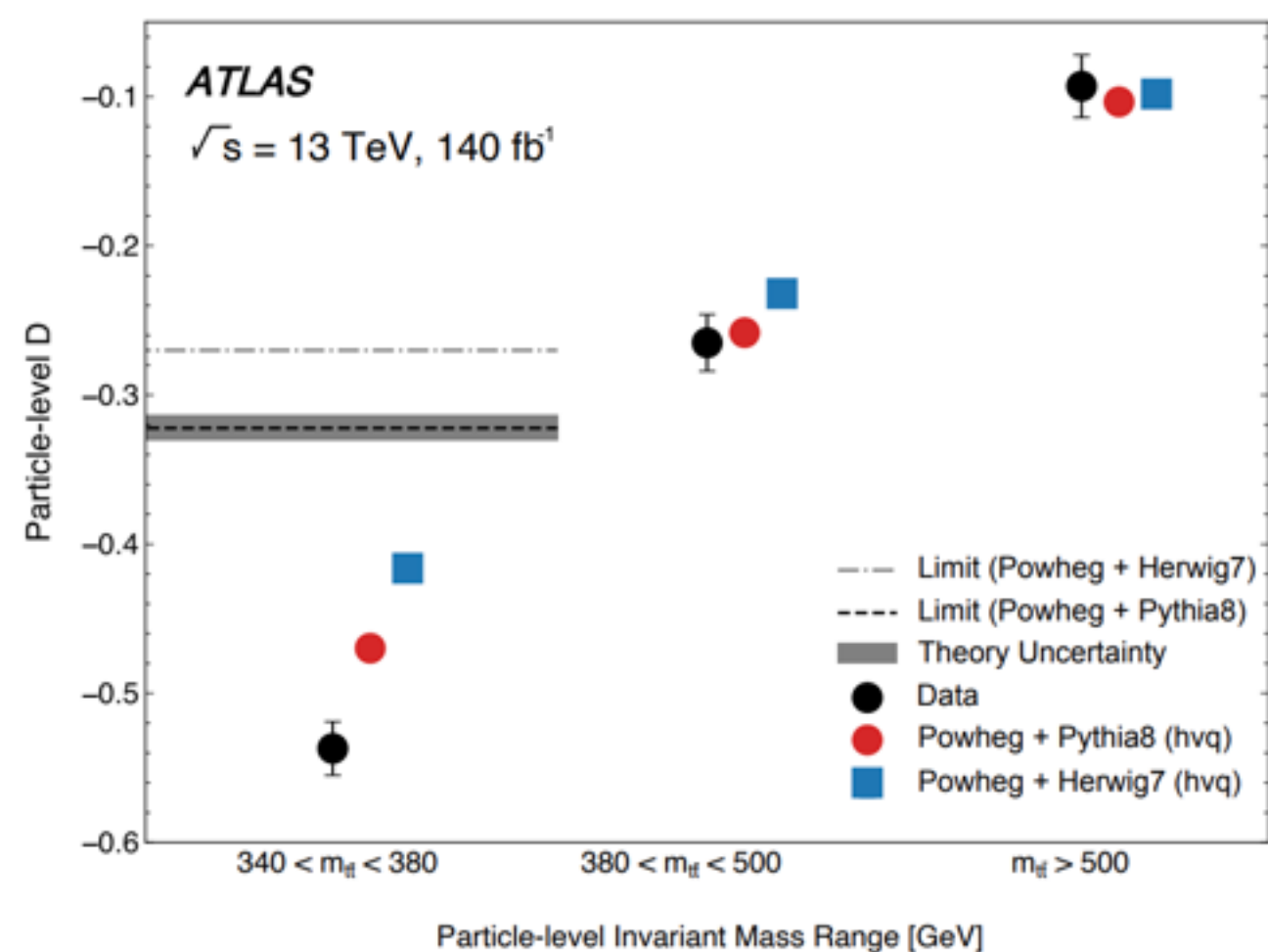
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CERN-EP-2023-230
October 3, 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration



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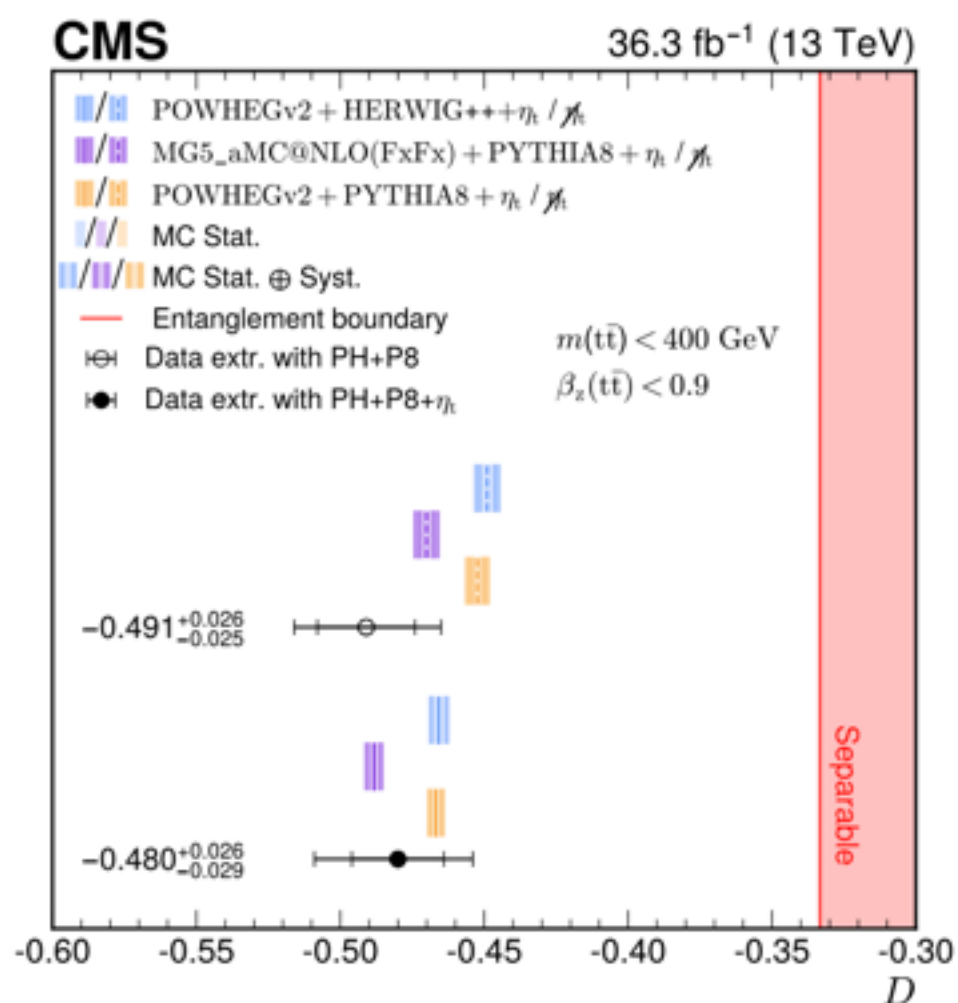
CMS-TOP-23-001



CERN-EP-2024-137
2024/10/30

Observation of quantum entanglement in top quark pair production in proton-proton collisions at $\sqrt{s} = 13 \text{ TeV}$

The CMS Collaboration*



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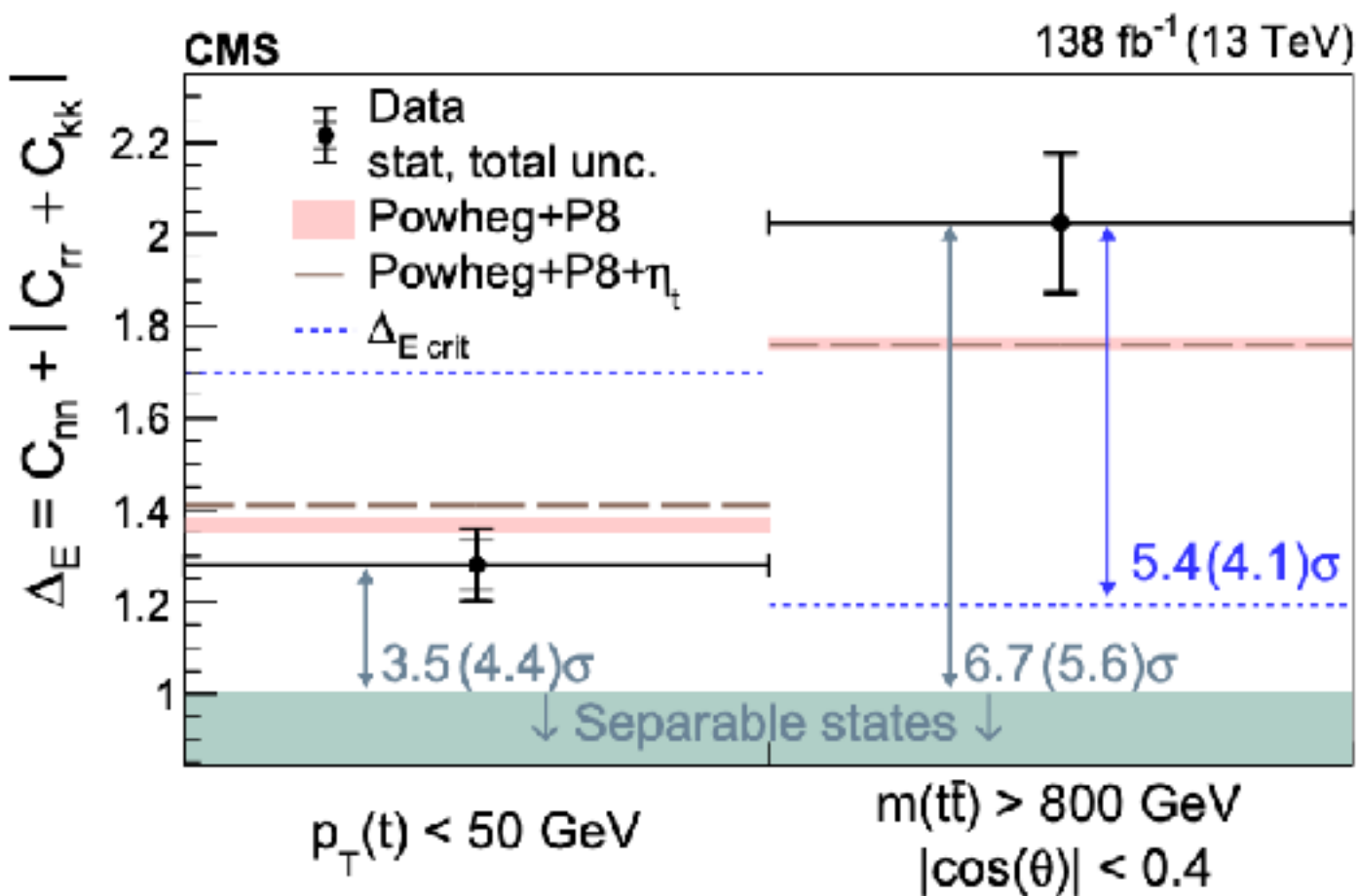
CMS-TOP-23-007



CERN-EP-2024-231
2025/01/09

Measurements of polarization and spin correlation and observation of entanglement in top quark pairs using lepton+jets events from proton-proton collisions at $\sqrt{s} = 13 \text{ TeV}$

The CMS Collaboration*



Quantum Entanglement @PH

- We use **spin correlations** as a diagnostic tool to reveal underlying **quantum correlations**, notably entanglement, in high-energy processes involving heavy-particle production and decay.

- Top quark pair production

[Afik, Nova '20] [Fabbrichesi, Floreanini, Panizzo '21] [Severi, Boschi, Maltoni, Sioli '22]
[Aguilar-Saavedra, Casas '22] [Han Low, Barr '23] [Aguilar-Saavedra '23, '24]
[Dong, Gonçalves, Kong, Navarro '23] [Maltoni, Severi, Tentori, Vryonidou '24]

- Tau pair production

[Ehatäht, Fabbrichesi, Marzola and Veelken, '23]
[Altakach, Lamba, Maltoni, Mawatari, and Sakurai '22]
[Fabbrichesi, Marzola '24] [Zhang, Zhou, Liu, Li, Hsu, Han'25]

- Higgs decays

[Barr '21] [Saavedra, Bernal, Casas, Moreno '23] [Bernal, Caban, Rembielinski '23],
[Ma, Li '23] [Aguilar-Saavedra '24] [Morales '24] [Sullivan '24] [Del Gratta et al '24]
[Gonçalves, Kaladharan, Krauss, Navarro '25] [Gonçalves, Kaladharan, Navarro '25].

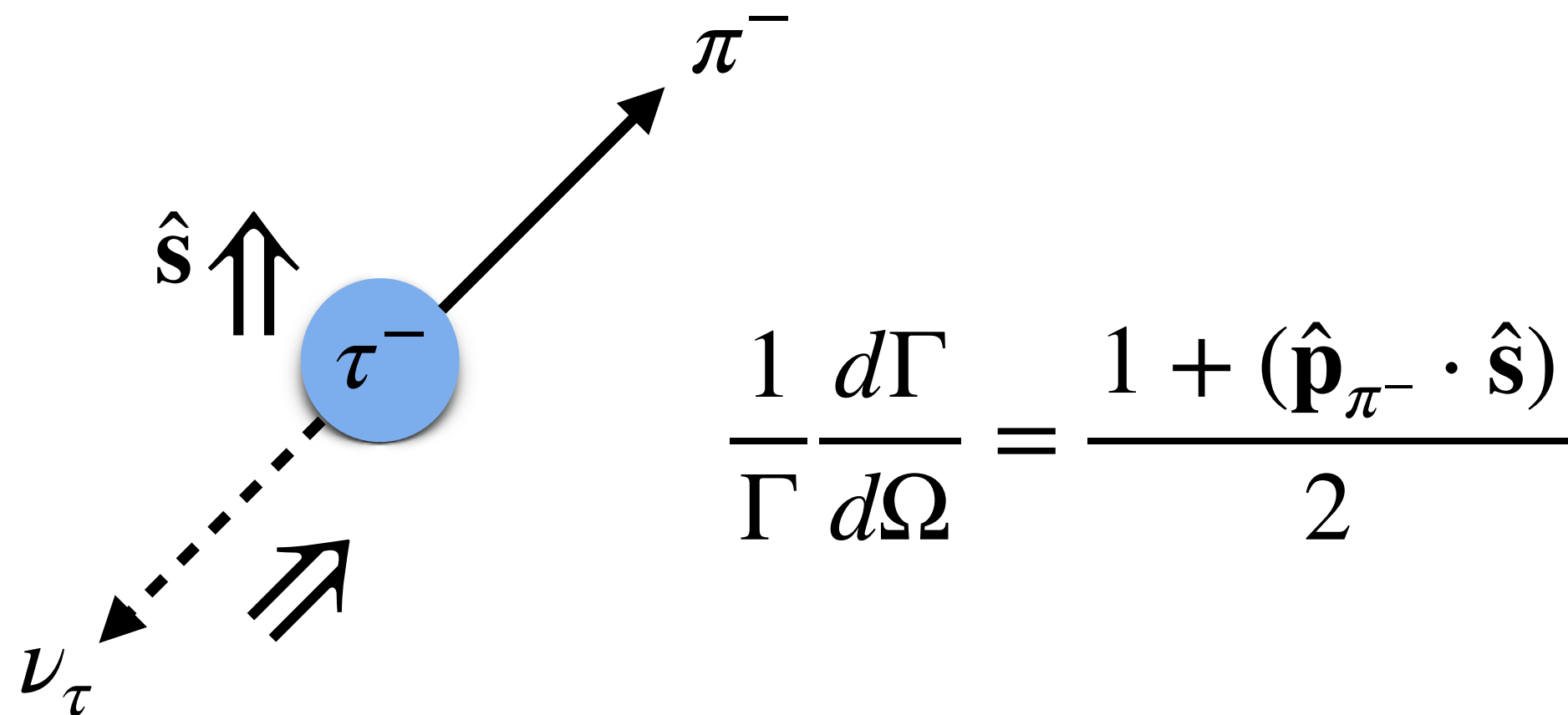
- Vector boson production

[Aoude, Madge, Maltoni, Mantani '22] [Fabbrichesi, Floreanini, Gabrielli, Marzola '22]
[Barr, Caban, Rembielinski '23] [Morales '23] [Ashby-Pickering, Barr, Wierzchucka '23]
[Grossi, Pelliccioli, Vicini '24]

Quantum Tomography

- Polarimeter at collider = particle decay

Ex) $\tau^- \rightarrow \pi^- + \nu$ (at τ^- rest frame)



Spin analyzing power

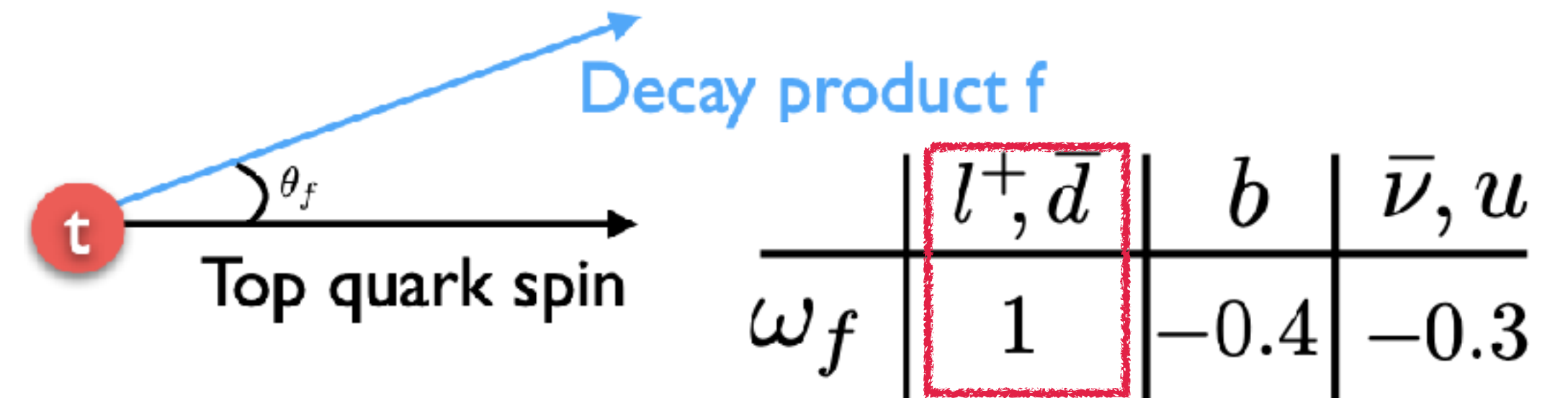


$$(\omega_f \in [-1, 1])$$

In general,

$$\frac{1}{\Gamma_f} \frac{d\Gamma_f}{d\Omega} = \frac{1 + \omega_f(\hat{\mathbf{p}}_f \cdot \hat{\mathbf{s}})}{2}$$

For top quark, $\frac{1}{\Gamma_f} \frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \omega_f \cos\theta_f)$



Quantum Tomography

- One qubit characterized by 3 parameters

$$\rho = \frac{I_2 + B_i \sigma_i}{2} \quad B_i = \langle \sigma_i \rangle = \text{tr}(\sigma_i \rho)$$

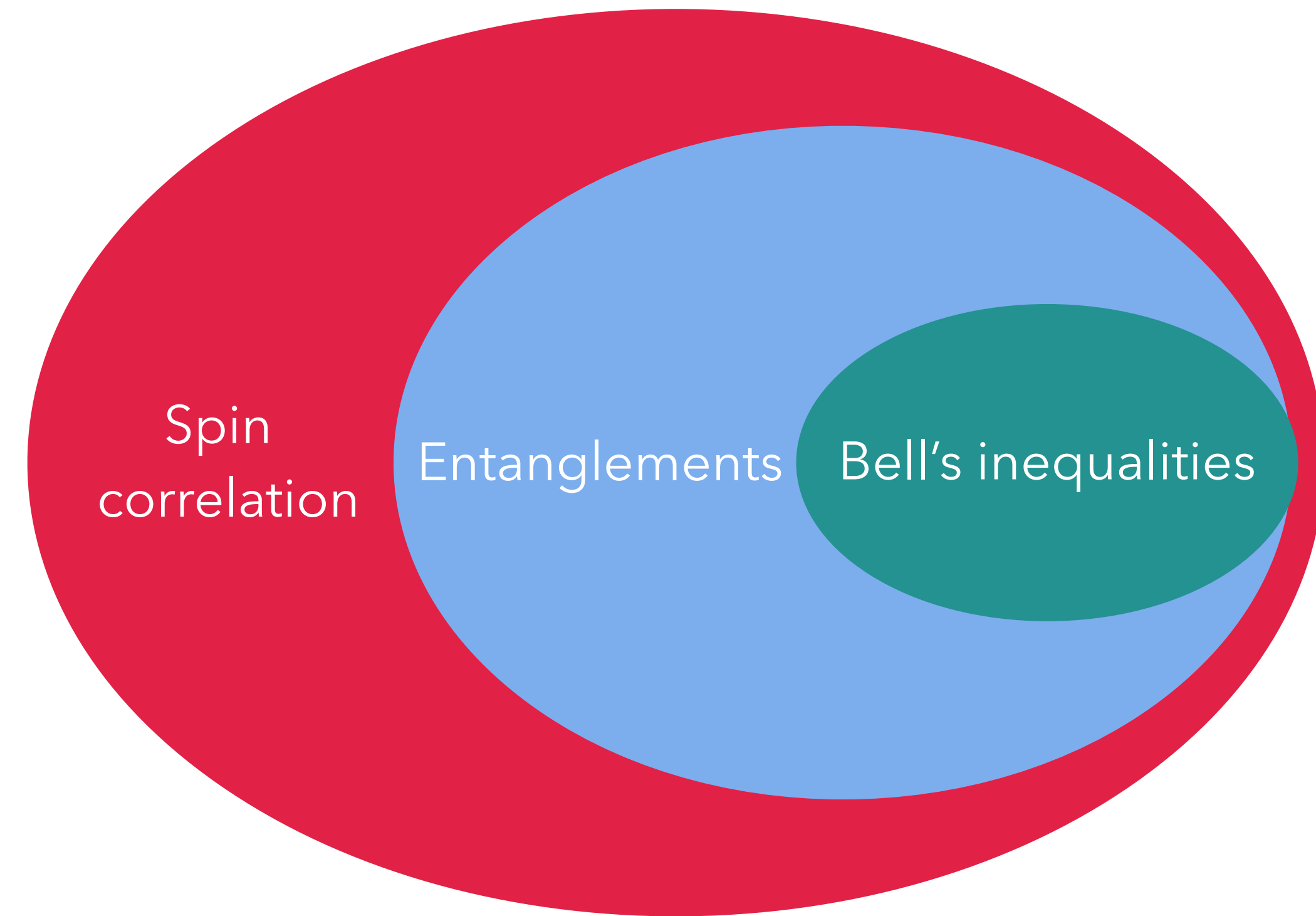
- Two qubit characterized by 15 parameters

$$\rho = \frac{I_2 \otimes I_2 + B_i \sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

$$B_i = \langle \sigma_i \otimes I_2 \rangle \quad \text{Polarizations}$$

$$\bar{B}_i = \langle I_2 \otimes \sigma_i \rangle$$

$$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle \quad \text{Spin correlations}$$



- Symmetry of theory is very useful.
- Ex) P and CP invariance of QCD
 $\rightarrow B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$,
in , $t\bar{t}$ production.

Density Matrix Formalism

$$\rho = |\psi\rangle\langle\psi| \text{ (pure state),} \quad \rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \text{ (mixed state)} \quad \left(p_i \geq 0, \quad \sum_i p_i = 1 \right)$$

- ρ are **Hermitian**, $Tr = 1$, and **positive semidefinite**.

$$\rho = \rho^\dagger, \quad Tr[\rho] = 1, \quad \rho \geq 0$$

- **Purity**, $Tr[\rho^2]$ is 1 for **pure state** and $\frac{1}{D}$ for **completely mixed state**,

$$\frac{1}{D} \leq Tr[\rho^2] \leq 1, \quad (D \text{ is dimension of the Hilbert space})$$

- We can compute expectation value of $\hat{\mathcal{O}}$ as

$$\langle \hat{\mathcal{O}} \rangle = Tr[\hat{\mathcal{O}}\rho]$$

Quantum Entanglement

- A quantum state of two subsystems A and B is **separable** when its density matrix ρ can be expressed as a convex sum

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i \quad \left(p_i \geq 0, \quad \sum_i p_i = 1 \right)$$

- If the state is not separable, it is named **entangled**
 - **Peres-Horodecki criterion**
 - **Concurrence**
 - $D = \frac{1}{3} \text{Tr}[C]$
 - **Clauser-Horne-Shimony-Holt (CHSH) Inequality**

Peres Horodecki Criterion

[Peres '96; Horodecki '97]

- The **Peres-Horodecki criterion** provides a **necessary and sufficient** condition for entanglement in two-qubit systems and qubit-qutrit systems:

Take the **partial transpose** (transpose indices associated only to Bob (or Alice))

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

For a separable system, ρ^{T2} is **non-negative**

→ If ρ^{T2} has **at least one negative eigenvalue**, the system is entangled.

Peres Horodecki Criterion

[Peres '96; Horodecki '97]

- The **Peres-Horodecki criterion** provides a **necessary and sufficient** condition for entanglement in two-qubit systems and qubit-qutrit systems:

Take the **partial transpose** (transpose indices associated only to Bob (or Alice))

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

We consider the entanglement quantifier of **negativity** $\mathcal{N}(\rho)$ as

$$\mathcal{N}(\rho) = \sum_k \frac{|\lambda_k| - \lambda_k}{2} = \sum_{\lambda_k < 0} |\lambda_k|$$

with λ_k being the eigenvalues of the ρ^{T2} .

Concurrence Criterion

- Concurrence

[Wooters '98]

For a separable state, the concurrence $C(\rho) = 0$ and for an entangled state, the concurrence is $C(\rho) > 0$ and $C(\rho) = 1$ is the maximally entangled state.

$$C(\rho) = \inf_{\{p_n, |\psi_n\rangle\}} \sum_n p_n C(|\psi_n\rangle)$$

For a two qubit system, explicit formula is given

$$C(\rho) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

where λ_i are the eigenvalues of the matrix $\sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}$, here $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$.

Entanglement marker D

[Afik, Nova '21]

- Note that this is only a **sufficient condition** for quantum entanglement adopted by the ATLAS and CMS Collaborations near the $t\bar{t}$ threshold at the LHC

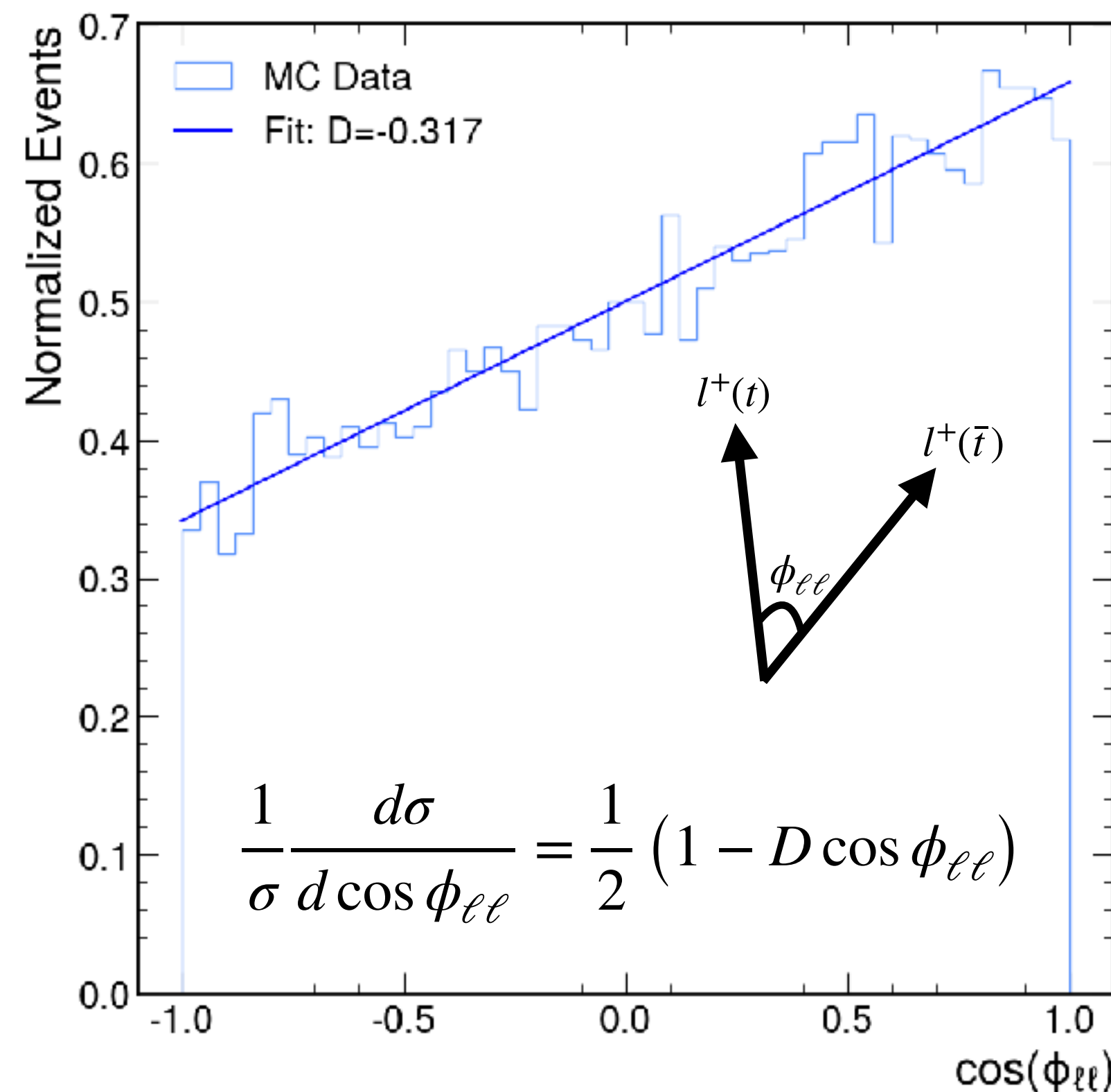
$$D \equiv \frac{1}{3} \text{Tr}[C] < -\frac{1}{3}$$

We can measure it by leptons from the top

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2} \left(1 + \alpha_1 \vec{B}_1 \cdot \hat{\ell}_1 + \alpha_2 \vec{B}_2 \cdot \hat{\ell}_2 + \alpha_1 \alpha_2 C_{ij} \hat{\ell}_{1i} \hat{\ell}_{2i} \right)$$

The spin density matrix of a two-qubit system which describes production of a pair of top-antitop quarks:

$$\rho = \frac{I_2 \otimes I_2 + B_i \sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$



Bell's inequality violation

- Clauser-Horne-Shimony-Holt (CHSH) inequality

[Clauser, Horne, Shimony, Holt '69]

$$\left| \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2) \right| > 2$$

- Augmented by the Horodecki condition

[Horodecki, Horodecki, Horodecki '95]

$$m_{12} \equiv m_1 + m_2 > 1$$

where $m_1 \geq m_2 \geq m_3$ are the three eigenvalues of the following matrix

$$M = CC^T$$

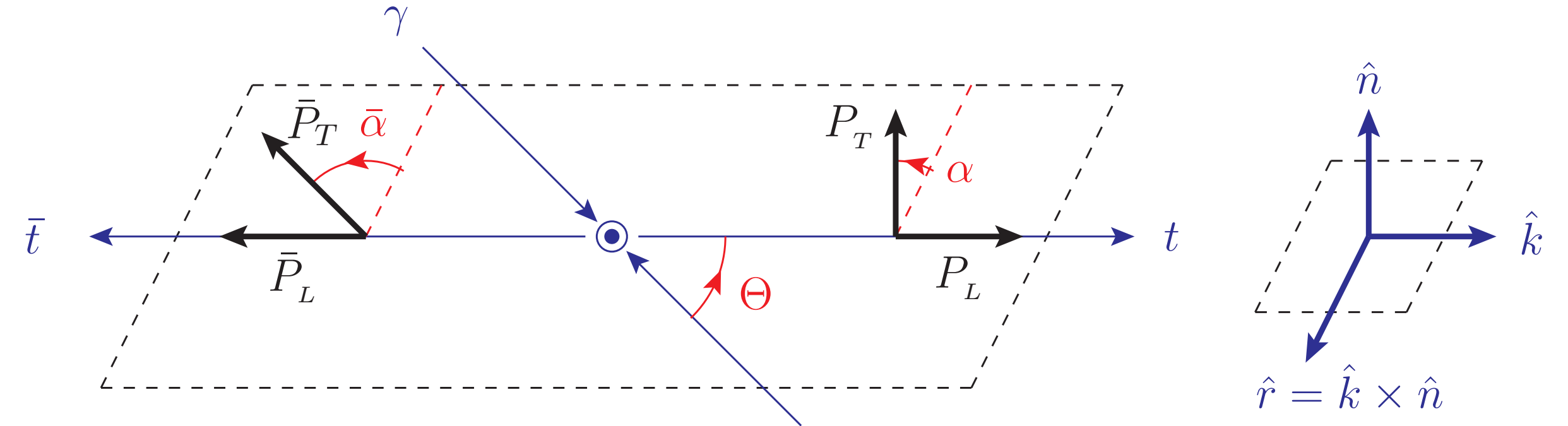
with C being the spin correlation matrix denoted by C_{ij}

Spin density matrix of a two-qubit system

- Let us consider a process

$$\gamma(k_1, \lambda_1) + \gamma(k_2, \lambda_2) \rightarrow t(p, \sigma) + \bar{t}(\bar{p}, \bar{\sigma})$$

- The helicity amplitude is given by



$$\mathcal{M}_{\lambda_1 \lambda_2; \sigma \bar{\sigma}} \equiv \mathcal{A}_C \langle \sigma \bar{\sigma}; \lambda_1 \lambda_2 \rangle = \frac{8\pi\alpha Q_t^2}{1 - \beta^2 \cos^2 \Theta} \langle \sigma \bar{\sigma}; \lambda_1 \lambda_2 \rangle$$

where $\beta^2 = 1 - 4M_t^2/\hat{s}$.

- The reduced amplitude transform under P, CP, and $CP\tilde{T}$:

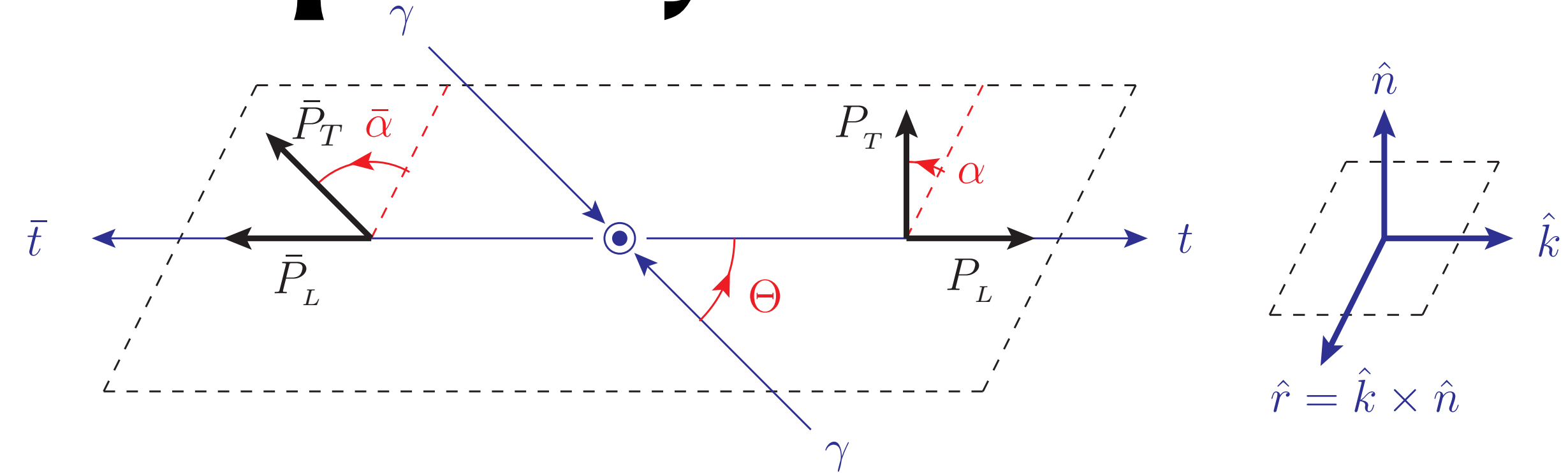
$$\langle \sigma \bar{\sigma}; \lambda_1, \lambda_2 \rangle \xleftrightarrow{P} -(-1)^{(\sigma - \bar{\sigma})/2} \langle -\sigma, -\bar{\sigma}; -\lambda_1, -\lambda_2 \rangle ,$$

$$\langle \sigma \bar{\sigma}; \lambda_1, \lambda_2 \rangle \xleftrightarrow{CP} -(-1)^{(\sigma - \bar{\sigma})/2} \langle -\bar{\sigma}, -\sigma; -\lambda_2, -\lambda_1 \rangle ,$$

$$\langle \sigma \bar{\sigma}; \lambda_1, \lambda_2 \rangle \xleftrightarrow{CP\tilde{T}} -(-1)^{(\sigma - \bar{\sigma})/2} \langle -\bar{\sigma}, -\sigma; -\lambda_2, -\lambda_1 \rangle^* ,$$

Spin density matrix of a two-qubit system

- The helicity amplitude is given by



$$\mathcal{M}_{\lambda_1 \lambda_2; \sigma \bar{\sigma}} \equiv \mathcal{A}_C \langle \sigma \bar{\sigma}; \lambda_1 \lambda_2 \rangle = \frac{8\pi\alpha Q_t^2}{1 - \beta^2 \cos^2 \Theta} \langle \sigma \bar{\sigma}; \lambda_1 \lambda_2 \rangle$$

where $\beta^2 = 1 - 4M_t^2/\hat{s}$.

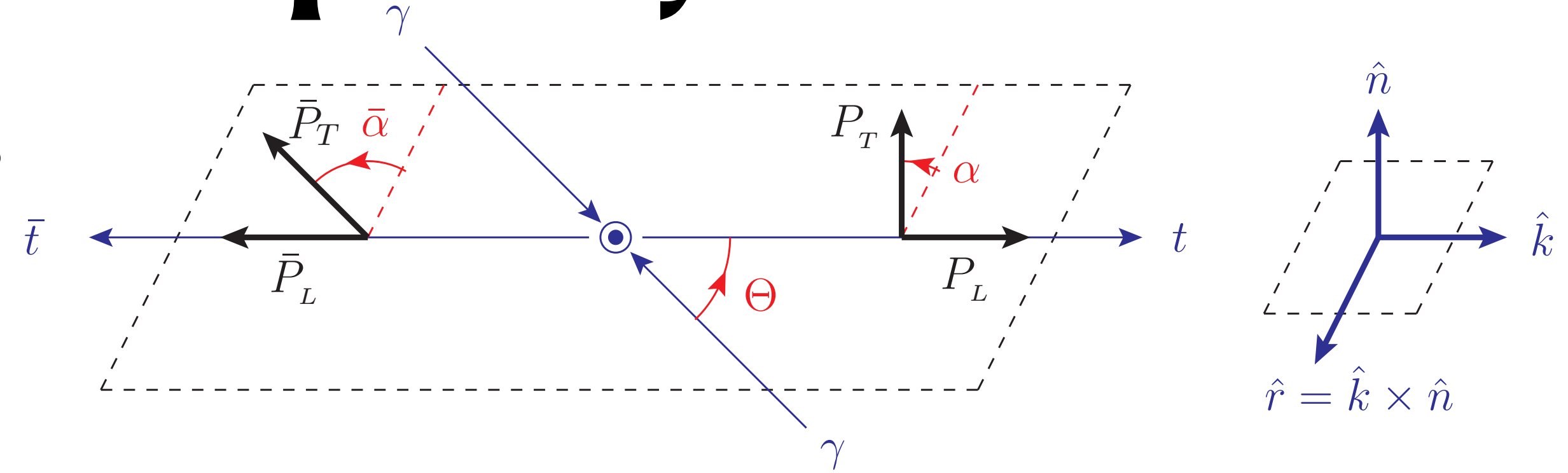
- The reduced amplitude :

$(\lambda_1 \lambda_2) \downarrow (\sigma \bar{\sigma}) \rightarrow$	$\langle ++ \rangle$	$\langle -- \rangle$	$\langle -+ \rangle$	$\langle +- \rangle$
$\langle ++ \rangle$	$\frac{2M_t}{\sqrt{\hat{s}}} [1 + \beta]$	$\frac{2M_t}{\sqrt{\hat{s}}} [1 - \beta]$	0	0
$\langle -- \rangle$	$\frac{2M_t}{\sqrt{\hat{s}}} [-1 + \beta]$	$\frac{2M_t}{\sqrt{\hat{s}}} [-1 - \beta]$	0	0
$\langle -+ \rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$-\beta \sin \Theta (\cos \Theta + 1)$	$-\beta \sin \Theta (\cos \Theta - 1)$
$\langle +- \rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$-\beta \sin \Theta (\cos \Theta - 1)$	$-\beta \sin \Theta (\cos \Theta + 1)$

Spin density matrix of a two-qubit system

- The helicity amplitude can be rewritten as

$$\mathcal{M} = \mathcal{A}_C \begin{pmatrix} \langle ++; \lambda_1 \lambda_2 \rangle & \langle +-; \lambda_1 \lambda_2 \rangle \\ \langle -+; \lambda_1 \lambda_2 \rangle & \langle --; \lambda_1 \lambda_2 \rangle \end{pmatrix} \equiv \mathcal{A}_C \mathbf{M}$$



- The polarization-weighted amplitude squared can be obtained by

[Bernreuther, Heisler and Si '15]

$$\overline{|\mathcal{M}|^2} = \frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm} \text{Tr} [\mathcal{M} \bar{\rho}_t^T \mathcal{M}^\dagger \rho_t] = \frac{1}{4} |\mathcal{A}_C|^2 \sum_{\lambda_1, \lambda_2 = \pm} \text{Tr} [\mathbf{M} \bar{\rho}_t^T \mathbf{M}^\dagger \rho_t] \equiv \frac{1}{4} |\mathcal{A}_C|^2 \hat{\Sigma}$$

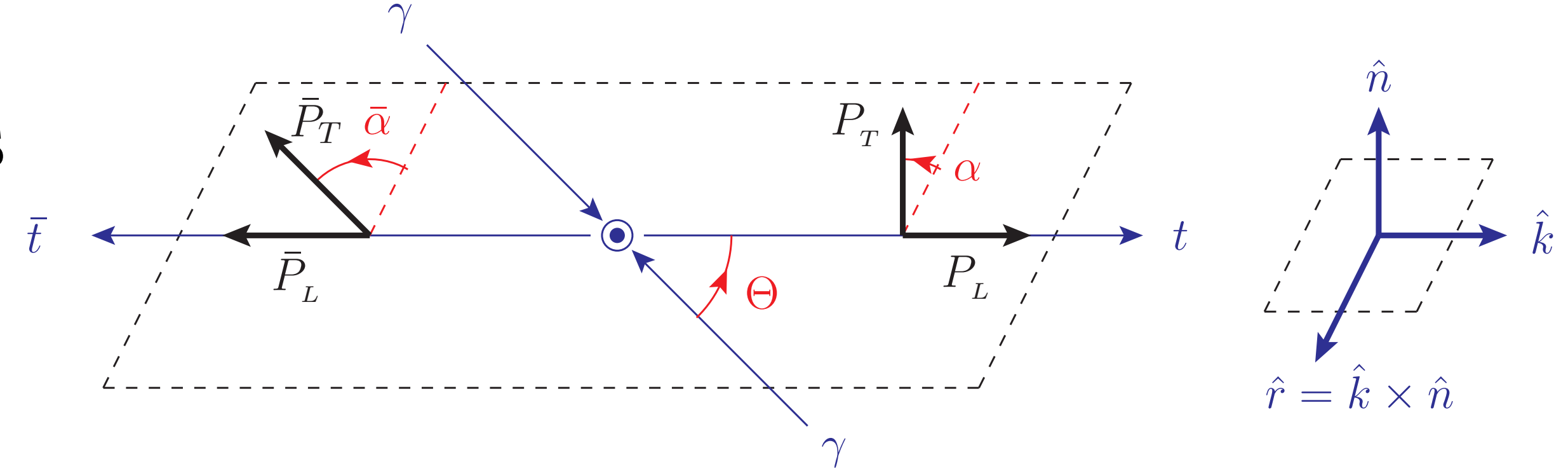
with the polarization density matrices for top and anti-top.

$$\rho_t = \frac{1}{2} \begin{pmatrix} 1 + P_L & P_T e^{-i\alpha} \\ P_T e^{i\alpha} & 1 - P_L \end{pmatrix}, \quad \bar{\rho}_t = \frac{1}{2} \begin{pmatrix} 1 + \bar{P}_L & -\bar{P}_T e^{i\bar{\alpha}} \\ -\bar{P}_T e^{-i\bar{\alpha}} & 1 - \bar{P}_L \end{pmatrix}$$

Spin density matrix of a two-qubit system

- The helicity amplitude can be rewritten as

$$\mathcal{M} = \mathcal{A}_C \begin{pmatrix} \langle ++; \lambda_1 \lambda_2 \rangle & \langle +-; \lambda_1 \lambda_2 \rangle \\ \langle -+; \lambda_1 \lambda_2 \rangle & \langle --; \lambda_1 \lambda_2 \rangle \end{pmatrix} \equiv \mathcal{A}_C \mathbf{M}$$



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with the

$$\begin{aligned} \hat{\Sigma} = & (\hat{C}_1[+++]+ \hat{C}_3[+++]) \\ & + P_n(\hat{C}_6[+-] + \hat{C}_8[+-]) + P_r(-\hat{C}_5[-+] - \hat{C}_7[-]) + P_k(\hat{C}_2[-] + \hat{C}_4[-]) \\ & + \bar{P}_n(\hat{C}_6[+-] - \hat{C}_8[+-]) + \bar{P}_r(-\hat{C}_5[-+] + \hat{C}_7[-]) + \bar{P}_k(-\hat{C}_2[-] + \hat{C}_4[-]) \\ & + P_n \bar{P}_n(\hat{C}_{13}[+++]- \hat{C}_{15}[+++]) + P_n \bar{P}_r(-\hat{C}_{14}[-] - \hat{C}_{16}[-]) + P_n \bar{P}_k(-\hat{C}_{10}[-] - \hat{C}_{12}[-]) \\ & + P_r \bar{P}_n(\hat{C}_{14}[-] - \hat{C}_{16}[-]) + P_r \bar{P}_r(\hat{C}_{13}[+++]+ \hat{C}_{15}[+++]) + P_r \bar{P}_k(\hat{C}_9[+-] + \hat{C}_{11}[+++]) \\ & + P_k \bar{P}_n(\hat{C}_{10}[-] - \hat{C}_{12}[-]) + P_k \bar{P}_r(-\hat{C}_9[+-] + \hat{C}_{11}[+++]) + P_k \bar{P}_k(-\hat{C}_1[+++]+ \hat{C}_3[+++]) , \end{aligned}$$

Spin density matrix of a two-qubit system

• The

$$\hat{C}_1[+++]=\frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}[|++;\lambda_1\lambda_2|^2+|--;\lambda_1\lambda_2|^2],$$

$$\hat{C}_2[- - -]=\frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}[|++;\lambda_1\lambda_2|^2-|--;\lambda_1\lambda_2|^2],$$

$$\hat{C}_3[+++]=\frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}[|-+;\lambda_1\lambda_2|^2|+-;\lambda_1\lambda_2|^2],$$

$$\hat{C}_4[-++]=-\frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}[|-+;\lambda_1\lambda_2|^2|+-;\lambda_1\lambda_2|^2],$$

• The p

$$\hat{C}_5[-++]=\frac{1}{4}\Re\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle-\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle-\langle+-;\lambda_1\lambda_2\rangle)^*,$$

$$\hat{C}_6[+-+]=-\frac{1}{4}\Im\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle-\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle+\langle+-;\lambda_1\lambda_2\rangle)^*,$$

$$\hat{C}_7[- - -]=\frac{1}{4}\Re\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle+\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle+\langle+-;\lambda_1\lambda_2\rangle)^*,$$

$$\hat{C}_8[+-+]=-\frac{1}{4}\Im\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle+\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle-\langle+-;\lambda_1\lambda_2\rangle)^*,$$

$$\hat{C}_9[+- -]=\frac{1}{4}\Re\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle+\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle-\langle+-;\lambda_1\lambda_2\rangle)^*,$$

$$\hat{C}_{10}[- - +]=-\frac{1}{4}\Im\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle+\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle+\langle+-;\lambda_1\lambda_2\rangle)^*,$$

$$\hat{C}_{11}[+++]=\frac{1}{4}\Re\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle-\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle+\langle+-;\lambda_1\lambda_2\rangle)^*,$$

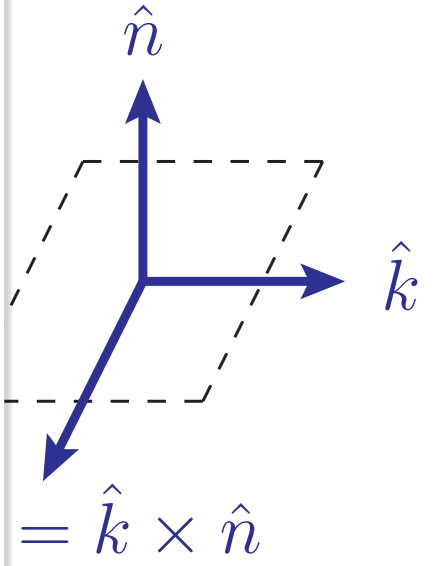
$$\hat{C}_{12}[-+-]=-\frac{1}{4}\Im\sum_{\lambda_1,\lambda_2=\pm}(\langle++;\lambda_1\lambda_2\rangle-\langle--;\lambda_1\lambda_2\rangle)(\langle-+;\lambda_1\lambda_2\rangle-\langle+-;\lambda_1\lambda_2\rangle)^*,$$

$$\hat{C}_{13}[+++]=-\frac{1}{2}\Re\sum_{\lambda_1,\lambda_2=\pm}[\langle++;\lambda_1\lambda_2\rangle\langle--;\lambda_1\lambda_2\rangle^*],$$

$$\hat{C}_{14}[- - +]=\frac{1}{2}\Im\sum_{\lambda_1,\lambda_2=\pm}[\langle++;\lambda_1\lambda_2\rangle\langle--;\lambda_1\lambda_2\rangle^*],$$

$$\hat{C}_{15}[+++]=-\frac{1}{2}\Re\sum_{\lambda_1,\lambda_2=\pm}[\langle-+;\lambda_1\lambda_2\rangle\langle+-;\lambda_1\lambda_2\rangle^*],$$

$$\hat{C}_{16}[-+-]=-\frac{1}{2}\Im\sum_{\lambda_1,\lambda_2=\pm}[\langle-+;\lambda_1\lambda_2\rangle\langle+-;\lambda_1\lambda_2\rangle^*].$$



isler and Si '15]

with th

$$\hat{\Sigma}=(\hat{C}_1[+++]+\hat{C}_3[+++])$$

$$+P_n(\hat{C}_6[+-+]+\hat{C}_8[+-+])+P_r(-\hat{C}_5[-++]-\hat{C}_7[- - -])+P_k(\hat{C}_2[- - -]+\hat{C}_4[-++])$$

$$+\bar{P}_n(\hat{C}_6[+-+]-\hat{C}_8[+-+])+ \bar{P}_r(-\hat{C}_5[-++]+\hat{C}_7[- - -])+ \bar{P}_k(-\hat{C}_2[- - -]+\hat{C}_4[-++])$$

$$+P_n\bar{P}_n(\hat{C}_{13}[+++]-\hat{C}_{15}[+++])+P_n\bar{P}_r(-\hat{C}_{14}[- - +]-\hat{C}_{16}[-+-])+P_n\bar{P}_k(-\hat{C}_{10}[- - +]-\hat{C}_{12}[-+-])$$

$$+P_r\bar{P}_n(\hat{C}_{14}[- - +]-\hat{C}_{16}[-+-])+P_r\bar{P}_r(\hat{C}_{13}[+++]+\hat{C}_{15}[+++])+P_r\bar{P}_k(\hat{C}_9[+- -]+\hat{C}_{11}[+++])$$

$$+P_k\bar{P}_n(\hat{C}_{10}[- - +]-\hat{C}_{12}[-+-])+P_k\bar{P}_r(-\hat{C}_9[+- -]+\hat{C}_{11}[+++])+P_k\bar{P}_k(-\hat{C}_1[+++]+\hat{C}_3[+++]),$$

Spin density matrix of a two-qubit system

- The spin density matrix in $\{n, r, k\}$ basis,

$$\rho = \frac{1}{4} \left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{\hat{A}} \sum_{i=n,r,k} \hat{B}_i^+ (\sigma_i \otimes \mathbf{1}_2) + \frac{1}{\hat{A}} \sum_{j=n,r,k} \hat{B}_j^- (\mathbf{1}_2 \otimes \sigma_j) + \frac{1}{\hat{A}} \sum_{i,j=n,r,k} \hat{C}_{ij} (\sigma_i \otimes \sigma_j) \right]$$

- $\hat{B}_i^+$ and $\hat{B}_i^-$ are polarization of top and anti-top

$$\hat{B}_i^+ = (\hat{C}_6[+-] + \hat{C}_8[+-], -\hat{C}_5[+-] - \hat{C}_7[+-], \hat{C}_2[+-] + \hat{C}_4[+-]) ,$$

$$\hat{B}_j^- = (\hat{C}_6[+-] - \hat{C}_8[+-], -\hat{C}_5[+-] + \hat{C}_7[+-], -\hat{C}_2[+-] + \hat{C}_4[+-]) ,$$

- $\hat{C}_{ij}$ are spin correlation matrix

$$\hat{C}_{ij} = \begin{pmatrix} \hat{C}_{13}[++] - \hat{C}_{15}[++] & -\hat{C}_{14}[-+] - \hat{C}_{16}[-+] & -\hat{C}_{10}[-+] - \hat{C}_{12}[-+] \\ \hat{C}_{14}[-+] - \hat{C}_{16}[-+] & \hat{C}_{13}[++] + \hat{C}_{15}[++] & \hat{C}_9[+-] + \hat{C}_{11}[++] \\ \hat{C}_{10}[-+] - \hat{C}_{12}[-+] & -\hat{C}_9[+-] + \hat{C}_{11}[++] & -\hat{C}_1[++] + \hat{C}_3[++] \end{pmatrix}$$

- $\hat{A}$ is normalization factor

$$\hat{A} = \hat{C}_1[++] + \hat{C}_3[++] \quad \longrightarrow \quad \frac{d\sigma}{d\Omega} = \frac{\alpha^2 \hat{\beta}}{32\pi \hat{s}} \hat{A}$$

Spin density matrix of a two-qubit system

- The spin density matrix in $\{n, r, k\}$ basis,

$$\rho = \frac{1}{4} \left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{\widehat{A}} \sum_{i=n,r,k} \widehat{B}_i^+ (\sigma_i \otimes \mathbf{1}_2) + \frac{1}{\widehat{A}} \sum_{j=n,r,k} \widehat{B}_j^- (\mathbf{1}_2 \otimes \sigma_j) + \frac{1}{\widehat{A}} \sum_{i,j=n,r,k} \widehat{C}_{ij} (\sigma_i \otimes \sigma_j) \right]$$

Polarization coefficient	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$	$\widehat{C}_6$	$\widehat{C}_7$	$\widehat{C}_8$
[P, CP, CPT̃] Parities	[+ + +]	[− − −]	[+ + +]	[− + +]	[− + +]	[+ + −]	[− − −]	[+ − +]
Physical quantity	$\frac{\widehat{A} - \widehat{C}_{kk}}{2}$	$\frac{\widehat{B}_k^+ - \widehat{B}_k^-}{2}$	$\frac{\widehat{A} + \widehat{C}_{kk}}{2}$	$\frac{\widehat{B}_k^+ + \widehat{B}_k^-}{2}$	$-\frac{\widehat{B}_r^+ + \widehat{B}_r^-}{2}$	$\frac{\widehat{B}_n^+ + \widehat{B}_n^-}{2}$	$-\frac{\widehat{B}_r^+ - \widehat{B}_r^-}{2}$	$\frac{\widehat{B}_n^+ - \widehat{B}_n^-}{2}$
Polarization coefficient	$\widehat{C}_9$	$\widehat{C}_{10}$	$\widehat{C}_{11}$	$\widehat{C}_{12}$	$\widehat{C}_{13}$	$\widehat{C}_{14}$	$\widehat{C}_{15}$	$\widehat{C}_{16}$
[P, CP, CPT̃] Parities	[+ − −]	[− − +]	[+ + +]	[− + −]	[+ + +]	[− − +]	[+ + +]	[− + −]
Physical quantity	$\frac{\widehat{C}_{rk} - \widehat{C}_{kr}}{2}$	$\frac{\widehat{C}_{kn} - \widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{rk} + \widehat{C}_{kr}}{2}$	$-\frac{\widehat{C}_{kn} + \widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{nn} + \widehat{C}_{rr}}{2}$	$\frac{\widehat{C}_{rn} - \widehat{C}_{nr}}{2}$	$\frac{\widehat{C}_{rr} - \widehat{C}_{nn}}{2}$	$-\frac{\widehat{C}_{rn} + \widehat{C}_{nr}}{2}$

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A} \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2} = \mathcal{A} \begin{pmatrix} \langle ++; \lambda_1\lambda_2 \rangle & 0 \\ 0 & \langle --; \lambda_1\lambda_2 \rangle \end{pmatrix}$$

Polarization coefficient	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$	$\hat{C}_6$	$\hat{C}_7$	$\hat{C}_8$
[P, CP, CPT̃] Parities	[+++]	[- - -]	[+++]	[- ++]	[- ++]	[+ + -]	[- - -]	[+ - +]
Physical quantity	$\frac{\hat{A} - \hat{C}_{kk}}{2}$	$\frac{\hat{B}_k^+ - \hat{B}_k^-}{2}$	$\frac{\hat{A} + \hat{C}_{kk}}{2}$	$\frac{\hat{B}_k^+ + \hat{B}_k^-}{2}$	$-\frac{\hat{B}_r^+ + \hat{B}_r^-}{2}$	$\frac{\hat{B}_n^+ + \hat{B}_n^-}{2}$	$-\frac{\hat{B}_r^+ - \hat{B}_r^-}{2}$	$\frac{\hat{B}_n^+ - \hat{B}_n^-}{2}$
Polarization coefficient	$\hat{C}_9$	$\hat{C}_{10}$	$\hat{C}_{11}$	$\hat{C}_{12}$	$\hat{C}_{13}$	$\hat{C}_{14}$	$\hat{C}_{15}$	$\hat{C}_{16}$
[P, CP, CPT̃] Parities	[+ - -]	[- - +]	[+++]	[- + -]	[+++]	[- - +]	[+++]	[- + -]
Physical quantity	$\frac{\hat{C}_{rk} - \hat{C}_{kr}}{2}$	$\frac{\hat{C}_{kn} - \hat{C}_{nk}}{2}$	$\frac{\hat{C}_{rk} + \hat{C}_{kr}}{2}$	$-\frac{\hat{C}_{kn} + \hat{C}_{nk}}{2}$	$\frac{\hat{C}_{nn} + \hat{C}_{rr}}{2}$	$\frac{\hat{C}_{rn} - \hat{C}_{nr}}{2}$	$\frac{\hat{C}_{rr} - \hat{C}_{nn}}{2}$	$-\frac{\hat{C}_{rn} + \hat{C}_{nr}}{2}$

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A} \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2} = \mathcal{A} \begin{pmatrix} \langle ++; \lambda_1\lambda_2 \rangle & 0 \\ 0 & \langle --; \lambda_1\lambda_2 \rangle \end{pmatrix}$$

$$\hat{B}_i^+ = (\hat{C}_6[+-] + \hat{C}_8[+-], -\hat{C}_5[-+] - \hat{C}_7[-], \hat{C}_2[-] + \hat{C}_4[-]) ,$$

$$\hat{B}_j^- = (\hat{C}_6[+-] - \hat{C}_8[+-], -\hat{C}_5[-+] + \hat{C}_7[-], -\hat{C}_2[-] + \hat{C}_4[-]) ,$$

$$\hat{C}_{ij} = \begin{pmatrix} \hat{C}_{13}[++] - \hat{C}_{15}[++] - \hat{C}_{14}[-+] - \hat{C}_{16}[-+] & -\hat{C}_{10}[-+] - \hat{C}_{12}[-+] \\ \hat{C}_{14}[-+] - \hat{C}_{16}[-+] & \hat{C}_{13}[++] + \hat{C}_{15}[++] & \hat{C}_9[+-] + \hat{C}_{11}[++] \\ \hat{C}_{10}[-+] - \hat{C}_{12}[-+] & -\hat{C}_9[+-] + \hat{C}_{11}[++] & -\hat{C}_1[++] + \hat{C}_3[++] \end{pmatrix}$$

$$\begin{aligned} B_k^+ &= -B_k^- = C_2/C_1 \equiv B \\ C_n n &= C_r r = C_{13}/C_1 \equiv C \\ C_{nr} &= -C_{rn} = -C_{14}/C_1 \equiv D \\ C_{kk} &= -1 \end{aligned}$$



$$\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A} \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2} = \mathcal{A} \begin{pmatrix} \langle ++; \lambda_1\lambda_2 \rangle & 0 \\ 0 & \langle --; \lambda_1\lambda_2 \rangle \end{pmatrix}$$

- Let us compute quantum observables for $\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$$\text{Tr}[\rho^2] = \frac{1 + B^2 + C^2 + D^2}{2} \leq 1$$

$$\mathcal{C}[\rho] = \sqrt{C^2 + D^2}$$

$$m_{12} = 1 + C^2 + D^2 = 1 + \mathcal{C}^2$$

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A} \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2} = \mathcal{A} \begin{pmatrix} \langle ++; \lambda_1\lambda_2 \rangle & 0 \\ 0 & \langle --; \lambda_1\lambda_2 \rangle \end{pmatrix}$$

- Let us compute quantum observables for $\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$$\text{Tr}[\rho^2] = \frac{1 + B^2 + C^2 + D^2}{2} \leq 1$$

$$\mathcal{C}[\rho] = \sqrt{C^2 + D^2}$$

$$m_{12} = 1 + C^2 + D^2 = 1 + \mathcal{C}^2$$

- For a single scalar exchange, $B = 0$, $C^2 + D^2 = 1$

Maximal Entanglement

Violation of CHSH inequality

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A} \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2} = \mathcal{A} \begin{pmatrix} \langle ++; \lambda_1\lambda_2 \rangle & 0 \\ 0 & \langle --; \lambda_1\lambda_2 \rangle \end{pmatrix}$$

- Let us compute quantum observables for $\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$$\text{Tr}[\rho^2] = \frac{1 + B^2 + C^2 + D^2}{2} \leq 1$$

$$\mathcal{C}[\rho] = \sqrt{C^2 + D^2}$$

$$m_{12} = 1 + C^2 + D^2 = 1 + \mathcal{C}^2$$

- For a single scalar exchange, $B = 0$, $C^2 + D^2 = 1$

Maximal Entanglement

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Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A} \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle \delta_{\sigma\bar{\sigma}} \delta_{\lambda_1\lambda_2} = \mathcal{A} \begin{pmatrix} \langle ++; \lambda_1\lambda_2 \rangle & 0 \\ 0 & \langle --; \lambda_1\lambda_2 \rangle \end{pmatrix}$$

- Let us compute quantum observables for $\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

- For a single scalar exchange, $B = 0, C^2 + D^2 = 1$

$$C_{ij}^{\text{CP-even}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad C_{ij}^{\text{CP-odd}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad C_{ij}^{\text{mixed}} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ \sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

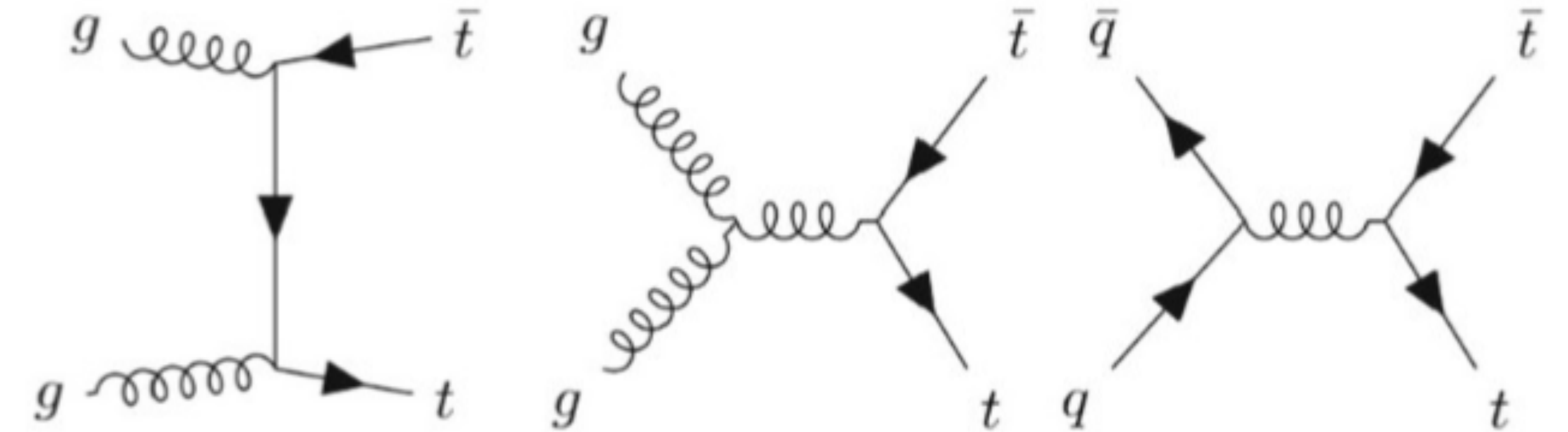
Spin density matrix of a two-qubit system

- Let us consider $t\bar{t}$ process,
- Leading order QCD amplitude is P, CP, and CPT invariant.

Only $[+++]$ coefficient survive!

Polarization coefficient	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$	$\hat{C}_6$	$\hat{C}_7$	$\hat{C}_8$
[P, CP, CPT] Parities	$[+++]$	$[- - -]$	$[+++]$	$[- ++]$	$[- ++]$	$[+ + -]$	$[- - -]$	$[+ - +]$
Physical quantity	$\frac{\hat{A} - \hat{C}_{kk}}{2}$	$\frac{\hat{B}_k^+ - \hat{B}_k^-}{2}$	$\frac{\hat{A} + \hat{C}_{kk}}{2}$	$\frac{\hat{B}_k^+ + \hat{B}_k^-}{2}$	$-\frac{\hat{B}_r^+ + \hat{B}_r^-}{2}$	$\frac{\hat{B}_n^+ + \hat{B}_n^-}{2}$	$-\frac{\hat{B}_r^+ - \hat{B}_r^-}{2}$	$\frac{\hat{B}_n^+ - \hat{B}_n^-}{2}$
Polarization coefficient	$\hat{C}_9$	$\hat{C}_{10}$	$\hat{C}_{11}$	$\hat{C}_{12}$	$\hat{C}_{13}$	$\hat{C}_{14}$	$\hat{C}_{15}$	$\hat{C}_{16}$
[P, CP, CPT] Parities	$[+ - -]$	$[- - +]$	$[+++]$	$[- + -]$	$[+++]$	$[- - +]$	$[+++]$	$[- + -]$
Physical quantity	$\frac{\hat{C}_{rk} - \hat{C}_{kr}}{2}$	$\frac{\hat{C}_{kn} - \hat{C}_{nk}}{2}$	$\frac{\hat{C}_{rk} + \hat{C}_{kr}}{2}$	$-\frac{\hat{C}_{kn} + \hat{C}_{nk}}{2}$	$\frac{\hat{C}_{nn} + \hat{C}_{rr}}{2}$	$\frac{\hat{C}_{rn} - \hat{C}_{nr}}{2}$	$\frac{\hat{C}_{rr} - \hat{C}_{nn}}{2}$	$-\frac{\hat{C}_{rn} + \hat{C}_{nr}}{2}$

Top quark pairs @ LHC



- Spin-density matrix

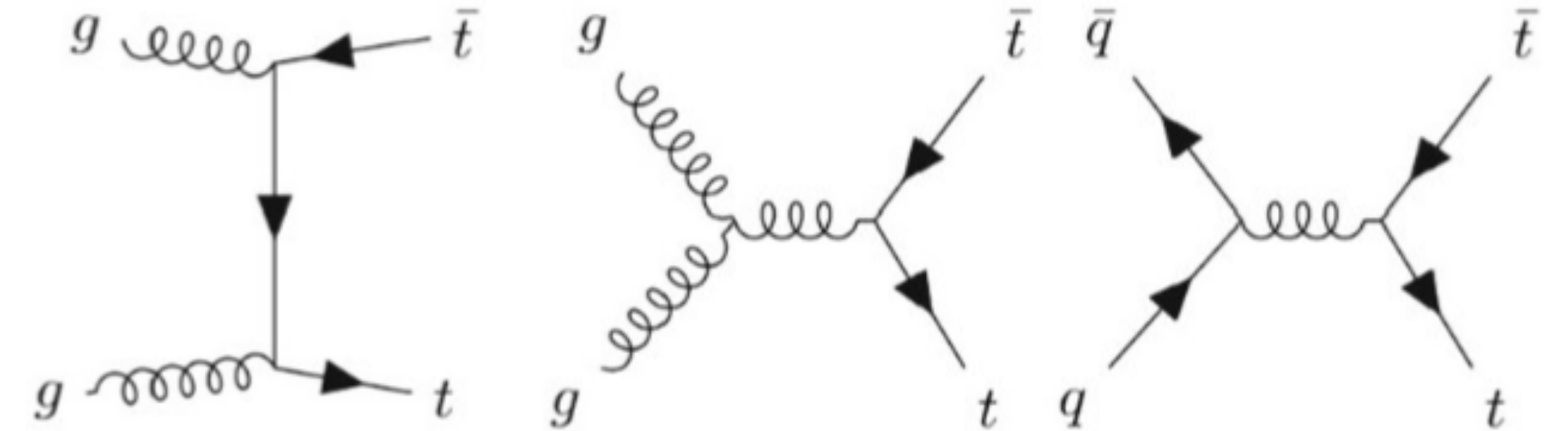
$$\mathcal{R}_{\sigma\sigma'\bar{\sigma}\bar{\sigma}'}(\hat{\beta}, \theta) = \frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm 1} \mathcal{M}_{\lambda_1 \lambda_2; \sigma \bar{\sigma}}(\theta) \mathcal{M}_{\lambda_1 \lambda_2; \sigma' \bar{\sigma}'}^*(\theta)$$

- Fano decomposition

$$\mathcal{R}(\hat{\beta}, \theta) = \tilde{A} I_2 \otimes I_2 + \tilde{B}_i^+ \sigma_i \otimes I_2 + \tilde{B}_i^- I_2 \otimes \sigma_i + \tilde{C}_{ij} \sigma_i \otimes \sigma_j$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \hat{\beta}}{32\pi \hat{s}} \tilde{A}$$

Top quark pairs @ LHC



- Spin-density matrix

$$\mathcal{R}_{\sigma\sigma'\bar{\sigma}\bar{\sigma}'}(\hat{\beta}, \theta) = \frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm 1} \mathcal{M}_{\lambda_1 \lambda_2; \sigma \bar{\sigma}}(\theta) \mathcal{M}_{\lambda_1 \lambda_2; \sigma' \bar{\sigma}'}^*(\theta)$$

- Fano decomposition

$$\mathcal{R}(\hat{\beta}, \theta) = \tilde{A} I_2 \otimes I_2 + \tilde{B}_i^+ \sigma_i \otimes I_2 + \tilde{B}_i^- I_2 \otimes \sigma_i + \tilde{C}_{ij} \sigma_i \otimes \sigma_j$$

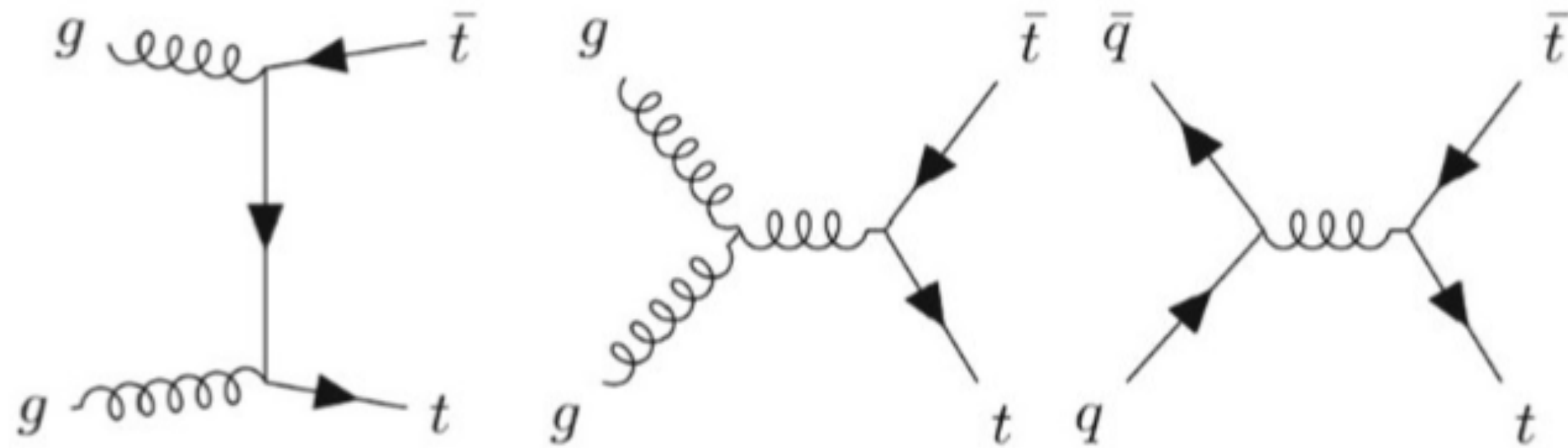
$$\rho = \frac{\mathcal{R}}{\text{Tr}(\mathcal{R})} = \frac{I_2 \otimes I_2 + B_i^+ \sigma_i \otimes I_2 + \bar{B}_i^- I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

Top quark pairs @ LHC

[Afik, Nova '20]

[Severi, Boschi, Maltoni, Sioli '21]

[Saavedra, Casas '22]



Only [+++] coefficient survive!

$C_{rk} = C_{kr}$ also noted!

$$\hat{C}_{ij} = \begin{pmatrix} \hat{C}_{13}[+++]-\hat{C}_{15}[+++] & -\hat{C}_{14}[- - +]-\hat{C}_{16}[- + -] & -\hat{C}_{10}[- - +]-\hat{C}_{12}[- + -] \\ \hat{C}_{14}[- - +]-\hat{C}_{16}[- + -] & \hat{C}_{13}[+++]+\hat{C}_{15}[+++] & \hat{C}_9[+ - -]+\hat{C}_{11}[+++] \\ \hat{C}_{10}[- - +]-\hat{C}_{12}[- + -] & -\hat{C}_9[+ - -]+\hat{C}_{11}[+++] & -\hat{C}_1[+++]+\hat{C}_3[+++] \end{pmatrix}$$

Top quark pairs @ LHC

$$\hat{C}_{ij} = \begin{pmatrix} \hat{C}_{13}[+++]-\hat{C}_{15}[+++] & -\hat{C}_{14}[- - +] - \hat{C}_{16}[- + -] & -\hat{C}_{10}[- - +] - \hat{C}_{12}[- + -] \\ \hat{C}_{14}[- - +] - \hat{C}_{16}[- + -] & \hat{C}_{13}[+++]+\hat{C}_{15}[+++] & \hat{C}_9[+ - -] + \hat{C}_{11}[+++] \\ \hat{C}_{10}[- - +] - \hat{C}_{12}[- + -] & -\hat{C}_9[+ - -] + \hat{C}_{11}[+++] & -\hat{C}_1[+++]+\hat{C}_3[+++] \end{pmatrix}$$

• $gg \rightarrow t\bar{t}$

$$F_g(z) = \frac{7 + 9\beta^2 z^2}{192(1 - \beta^2 z^2)^2},$$

$$\tilde{A}^{gg} = F_g(z)(1 + 2\beta^2(1 - z^2) - \beta^4(2 - 2z^2 + z^4)),$$

$$\tilde{C}_{nn}^{gg} = -F_g(z)(1 - 2\beta^2 + \beta^4(2 - 2z^2 + z^4)),$$

$$\tilde{C}_{kk}^{gg} = -F_g(z)(1 - 2z^2(1 - z^2)\beta^2 - \beta^4(2 - 2z^2 + z^4)),$$

$$\tilde{C}_{rr}^{gg} = -F_g(z)(1 - \beta^2(2 - \beta^2)(2 - 2z^2 + z^4)),$$

$$\tilde{C}_{rk}^{gg} = \tilde{C}_{kr}^{gg} = F_g(z)2z(1 - z^2)^{3/2}\beta^2\sqrt{1 - \beta^2},$$

• $q\bar{q} \rightarrow t\bar{t}$

$$F_q = \frac{1}{18},$$

$$\tilde{A}^{q\bar{q}} = F_q(2 - \beta^2(1 - z^2)),$$

$$\tilde{C}_{nn}^{q\bar{q}} = -F_q\beta^2(1 - z^2),$$

$$\tilde{C}_{kk}^{q\bar{q}} = F_q(2z^2 + \beta^2(1 - z^2)),$$

$$\tilde{C}_{rr}^{q\bar{q}} = F_q(2 - \beta^2)(1 - z^2),$$

$$\tilde{C}_{rk}^{q\bar{q}} = \tilde{C}_{kr}^{q\bar{q}} = F_q 2z\sqrt{(1 - \beta^2)(1 - z^2)},$$

Only $[+++]$ coefficient survive!

$C_{rk} = C_{kr}$ also noted!

$$z = \cos \theta$$

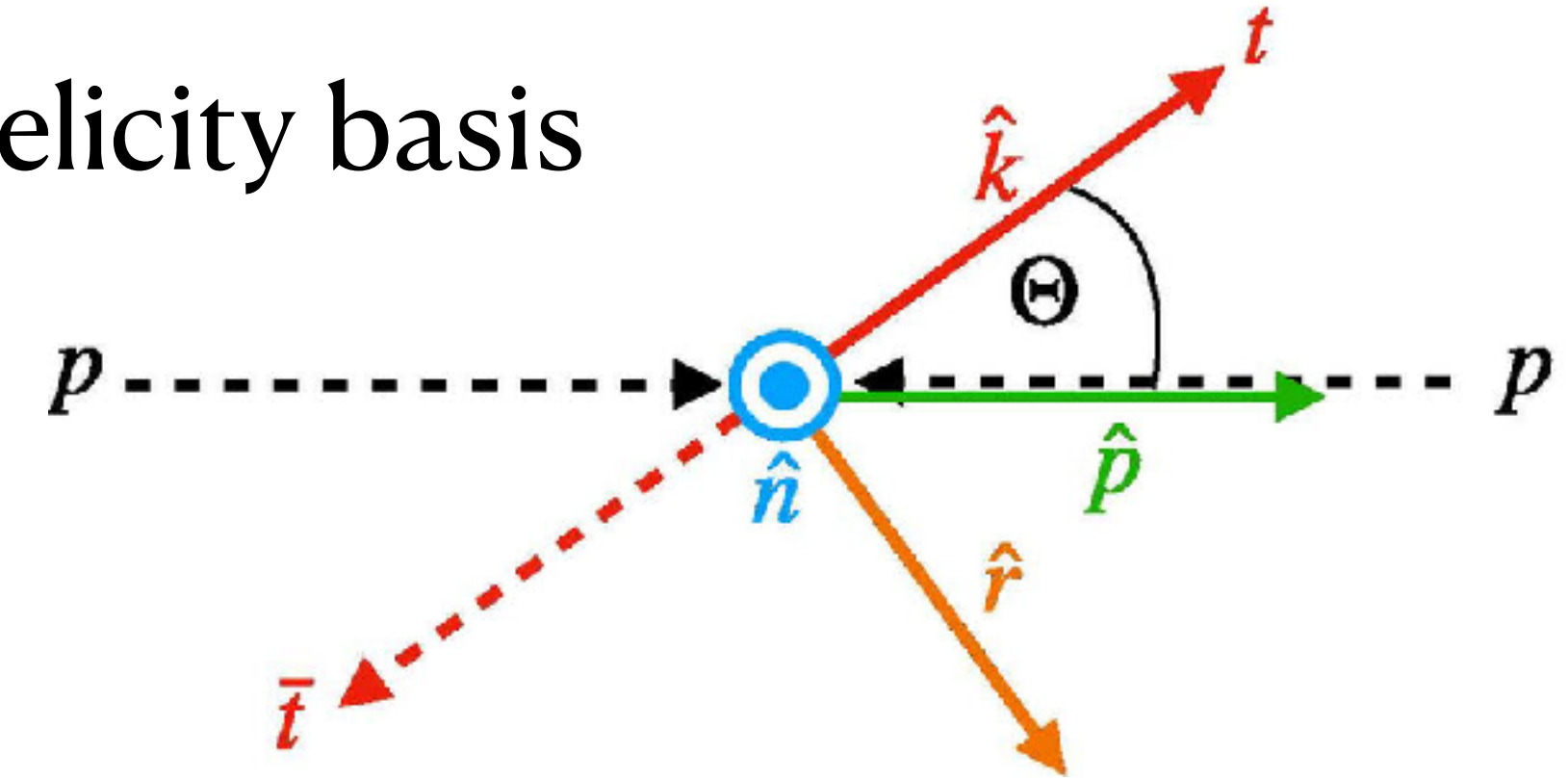
Top quark pairs @ LHC

- Angular averaged spin-density matrix

$$R = \frac{1}{4\pi} \int d\Omega \mathcal{R}(\hat{\beta}, z), \quad \rho = \frac{R}{\text{Tr}(R)}$$

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2} \left(1 + \alpha_1 \vec{B}_1 \cdot \hat{\ell}_1 + \alpha_2 \vec{B}_2 \cdot \hat{\ell}_2 + \alpha_1 \alpha_2 C_{ij} \hat{\ell}_{1i} \hat{\ell}_{2i} \right)$$

- Helicity basis



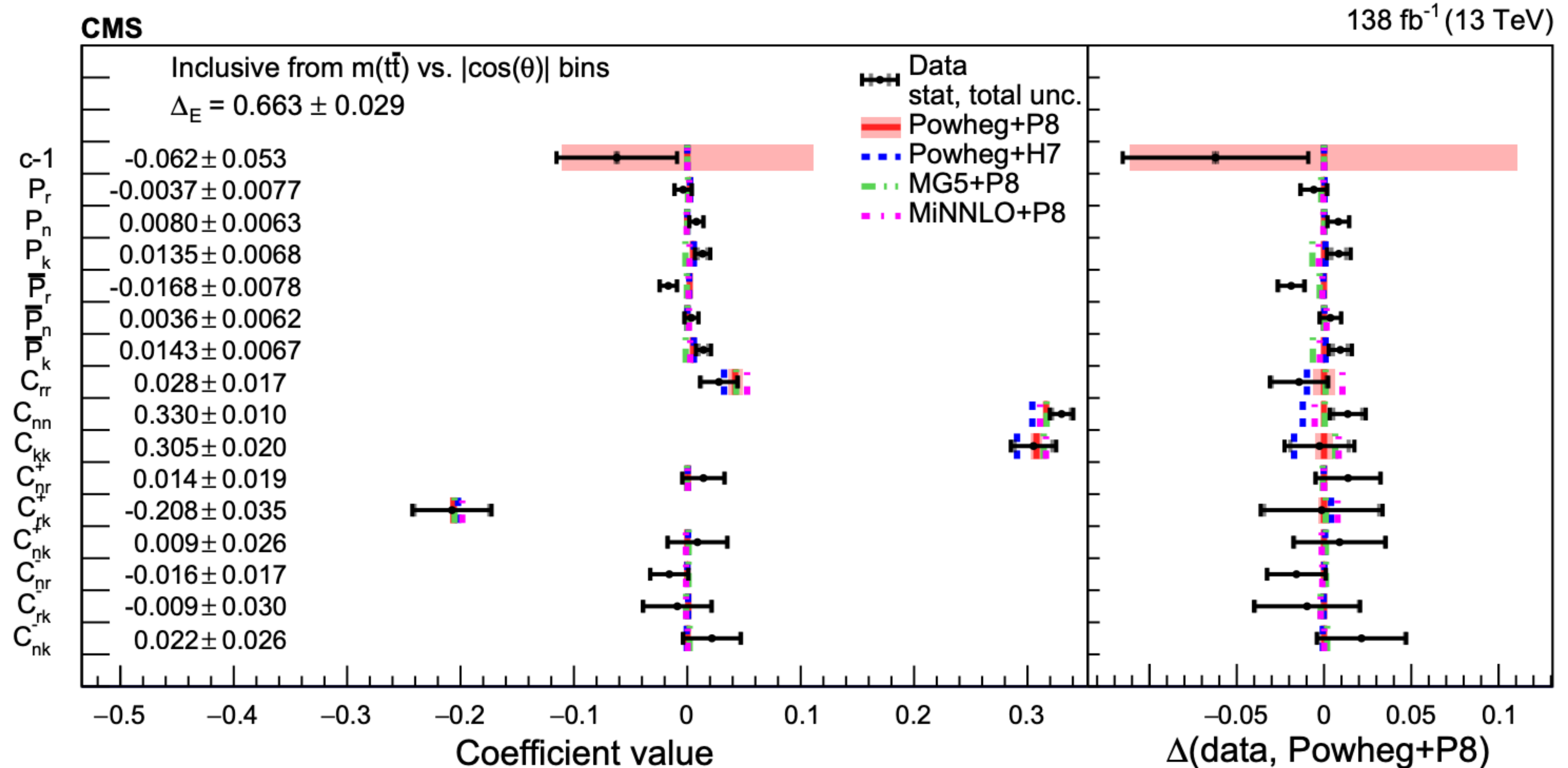
$$\beta = \sqrt{1 - \frac{4m_t^2}{s}} \quad z = \cos \theta$$

[Afik, Nova '20]

[Severi, Boschi, Maltoni, Sioli '21]

[Saavedra, Casas '22]

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Peres-Horodecki criterion

$$\rho^{\text{T}_2} = \frac{1}{4} \begin{bmatrix} 1 + B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} + i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ B_1^- + C_{31} - i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} - C_{22} - i(C_{12} + C_{21}) & B_1^+ - C_{13} - i(B_2^+ + C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} - C_{22} + i(C_{21} + C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} + i(B_2^- - C_{32}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} - i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix} < 0$$

Peres-Horodecki criterion

$$\rho^{\text{T}_2} = \frac{1}{4} \begin{bmatrix} 1 + B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} + i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ B_1^- + C_{31} - i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} - C_{22} - i(C_{12} + C_{21}) & B_1^+ - C_{13} - i(B_2^+ + C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} - C_{22} + i(C_{21} + C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} + i(B_2^- - C_{32}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} - i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix} < 0$$

$$D^{(1)} = 1/3(+C_{kk} + C_{rr} + C_{nn}),$$

$$D^{(k)} = 1/3(+C_{kk} - C_{rr} - C_{nn}),$$

$$D^{(r)} = 1/3(-C_{kk} + C_{rr} - C_{nn}),$$

$$D^{(n)} = 1/3(-C_{kk} - C_{rr} + C_{nn}).$$

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$D_{\min} < -1/3 \quad (\text{entanglement})$$

Peres-Horodecki criterion

$$\rho^{\text{T}_2} = \frac{1}{4} \begin{bmatrix} 1 + B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} + i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ B_1^- + C_{31} - i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} - C_{22} - i(C_{12} + C_{21}) & B_1^+ - C_{13} - i(B_2^+ + C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} - C_{22} + i(C_{21} + C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} + i(B_2^- - C_{32}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} - i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix} < 0$$

$$D^{(1)} = 1/3(+C_{kk} + C_{rr} + C_{nn}),$$

$$D = \frac{\text{tr}[C]}{3}.$$

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$D^{(k)} = 1/3(+C_{kk} - C_{rr} - C_{nn}),$$

$$D^{(r)} = 1/3(-C_{kk} + C_{rr} - C_{nn}),$$

$$D^{(n)} = 1/3(-C_{kk} - C_{rr} + C_{nn}).$$

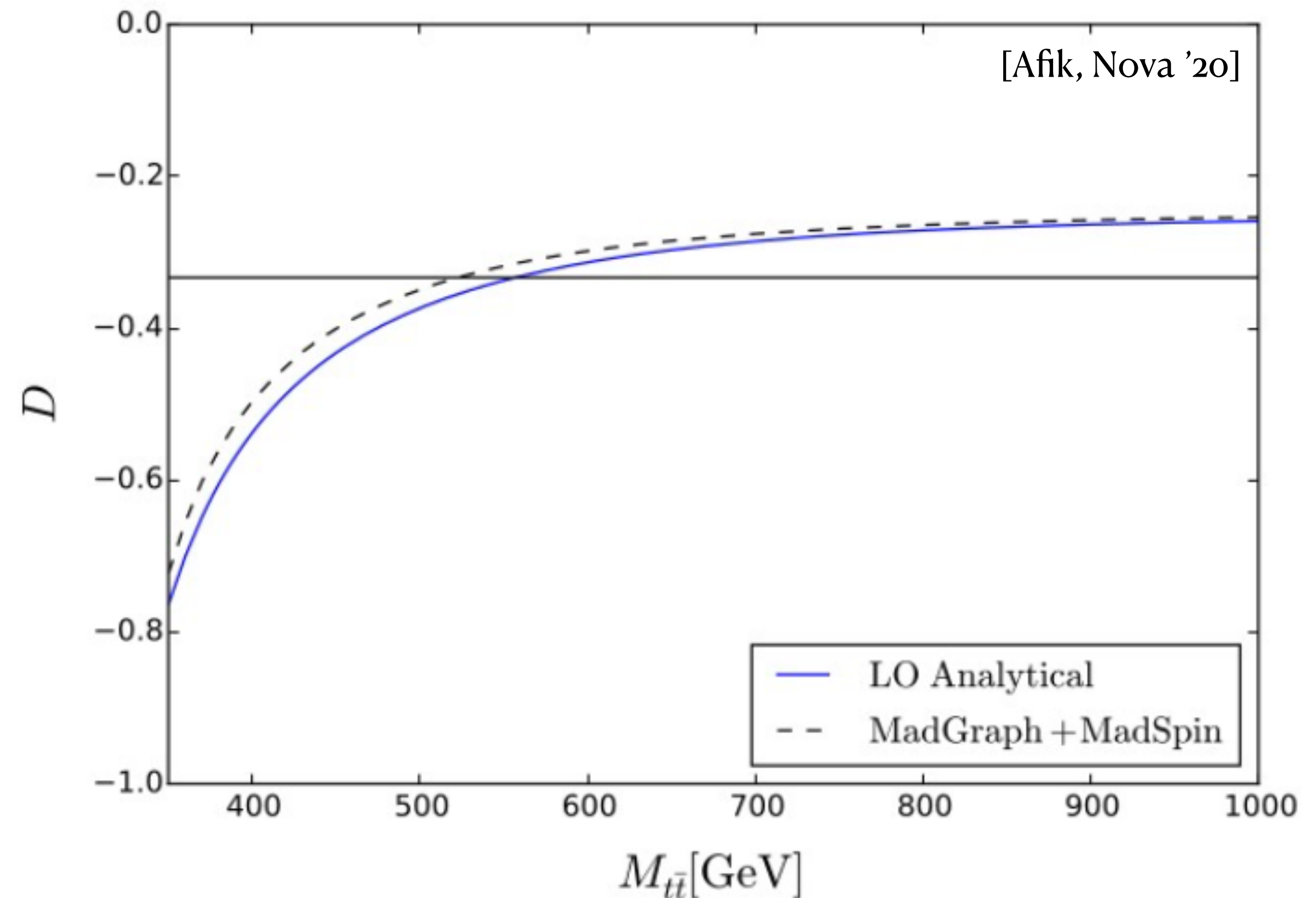
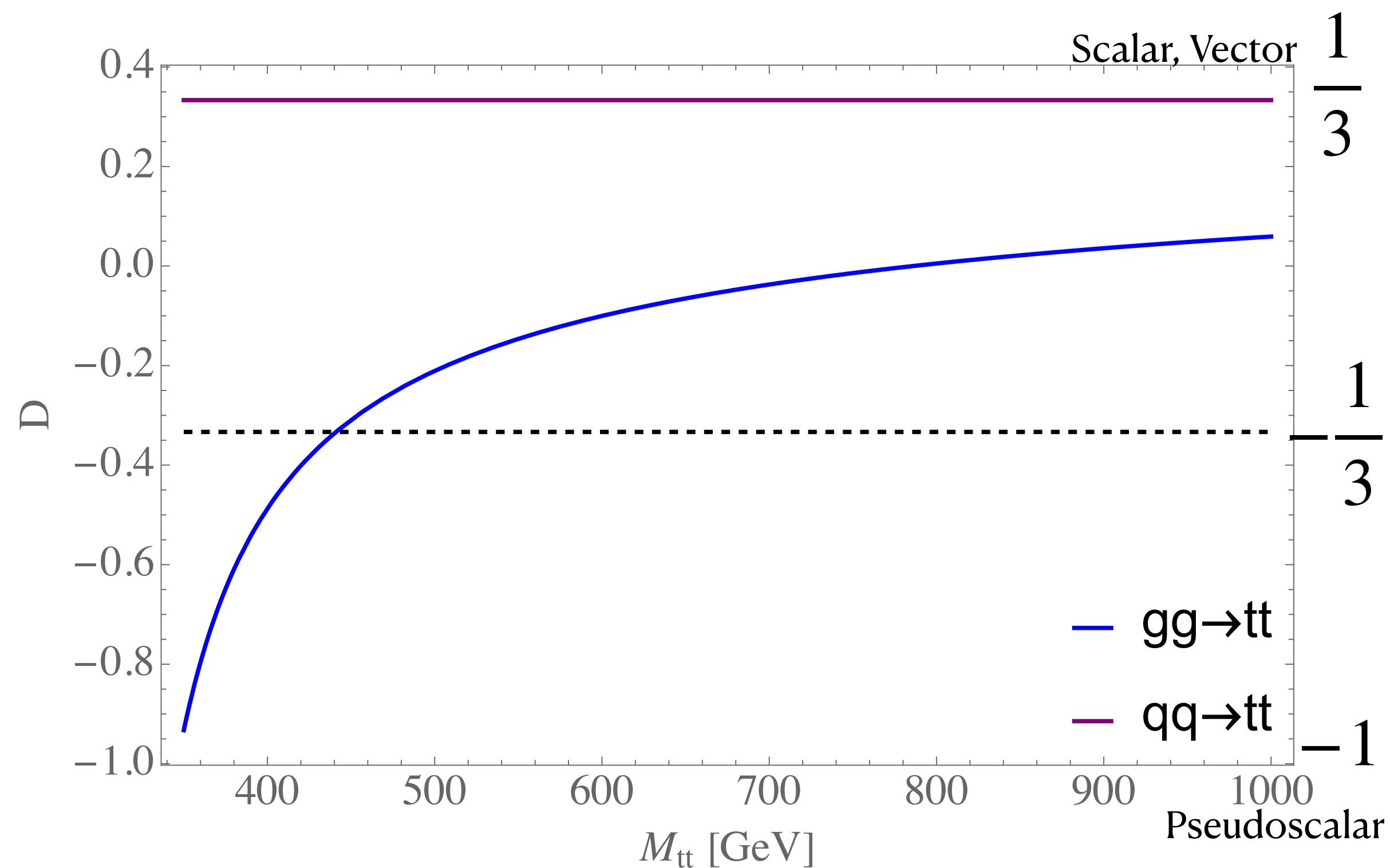
$$D_{\min} < -1/3 \quad (\text{entanglement})$$

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2} \left(1 + \alpha_1 \vec{B}_1 \cdot \hat{\ell}_1 + \alpha_2 \vec{B}_2 \cdot \hat{\ell}_2 + \alpha_1 \alpha_2 C_{ij} \hat{\ell}_{1i} \hat{\ell}_{2i} \right) \longrightarrow \frac{d\sigma}{d\cos\phi} = \frac{1}{2} (1 - D \cos\phi),$$

Entanglement marker D $D < -1/3$

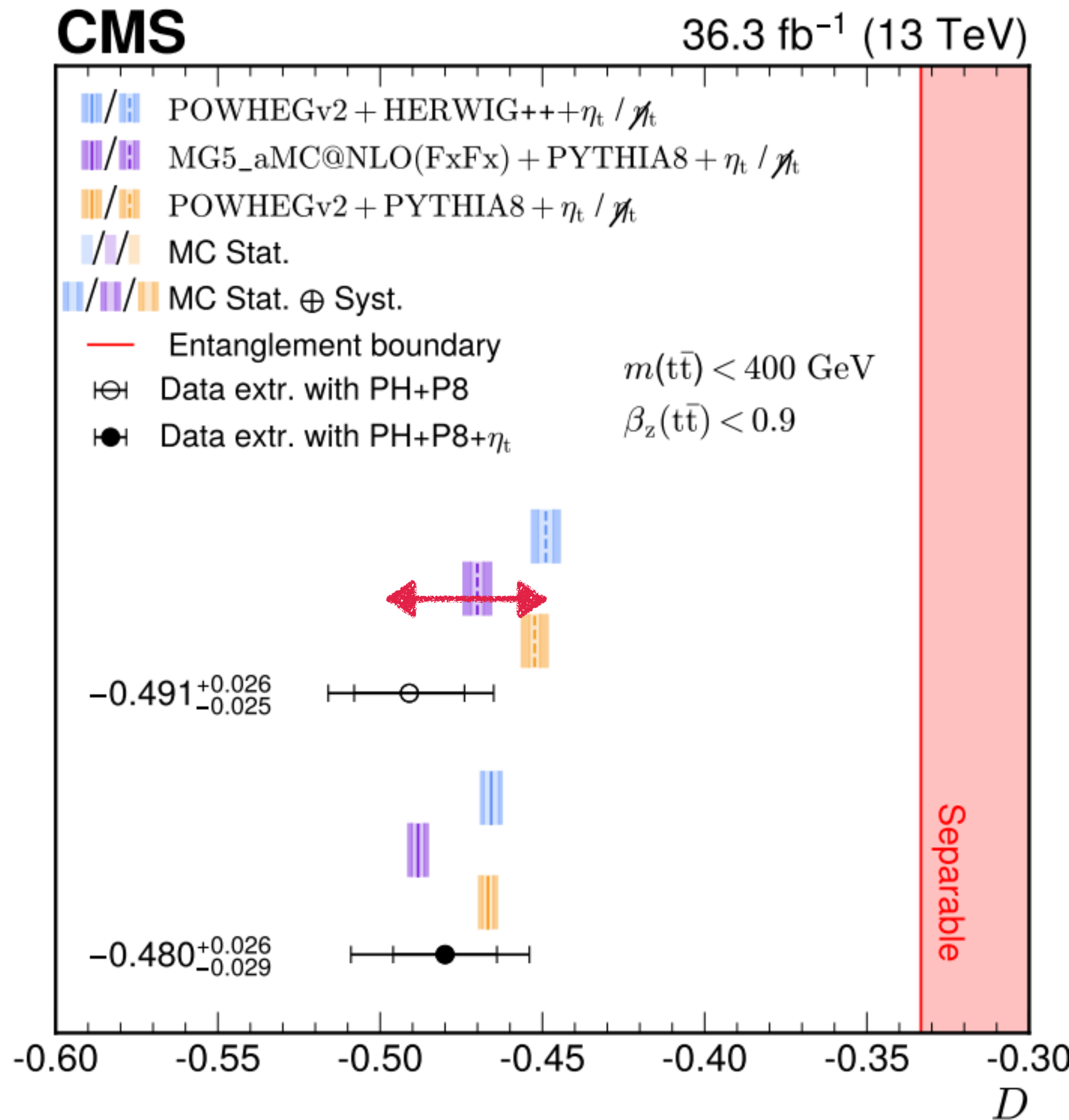
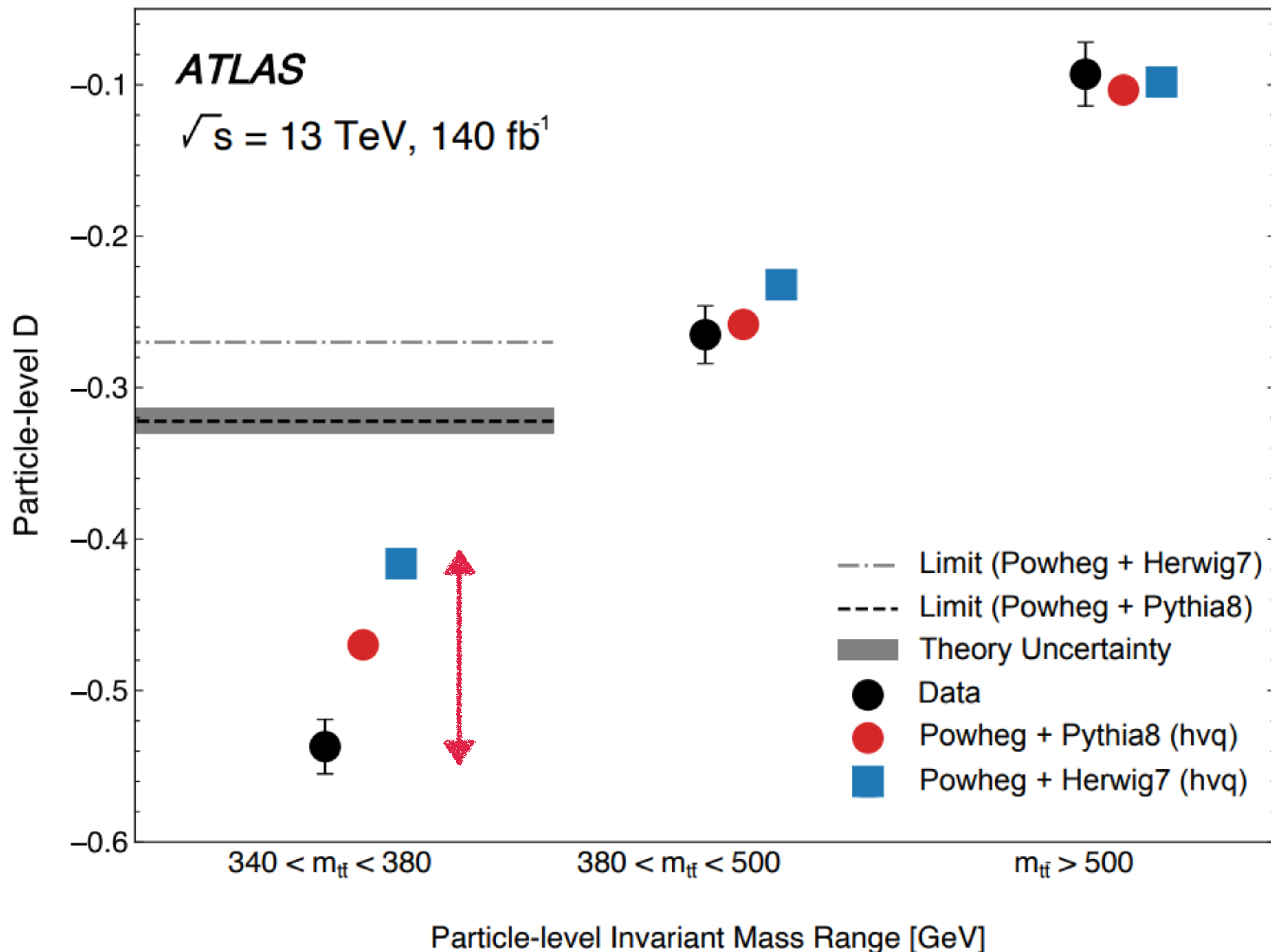
$$\frac{d\sigma}{d\cos\phi} = \frac{1}{2} (1 - D \cos\phi), \quad D = \frac{\text{tr}[C]}{3}.$$

- ϕ is the angle between the leptons measured in the parent top/antitop frames

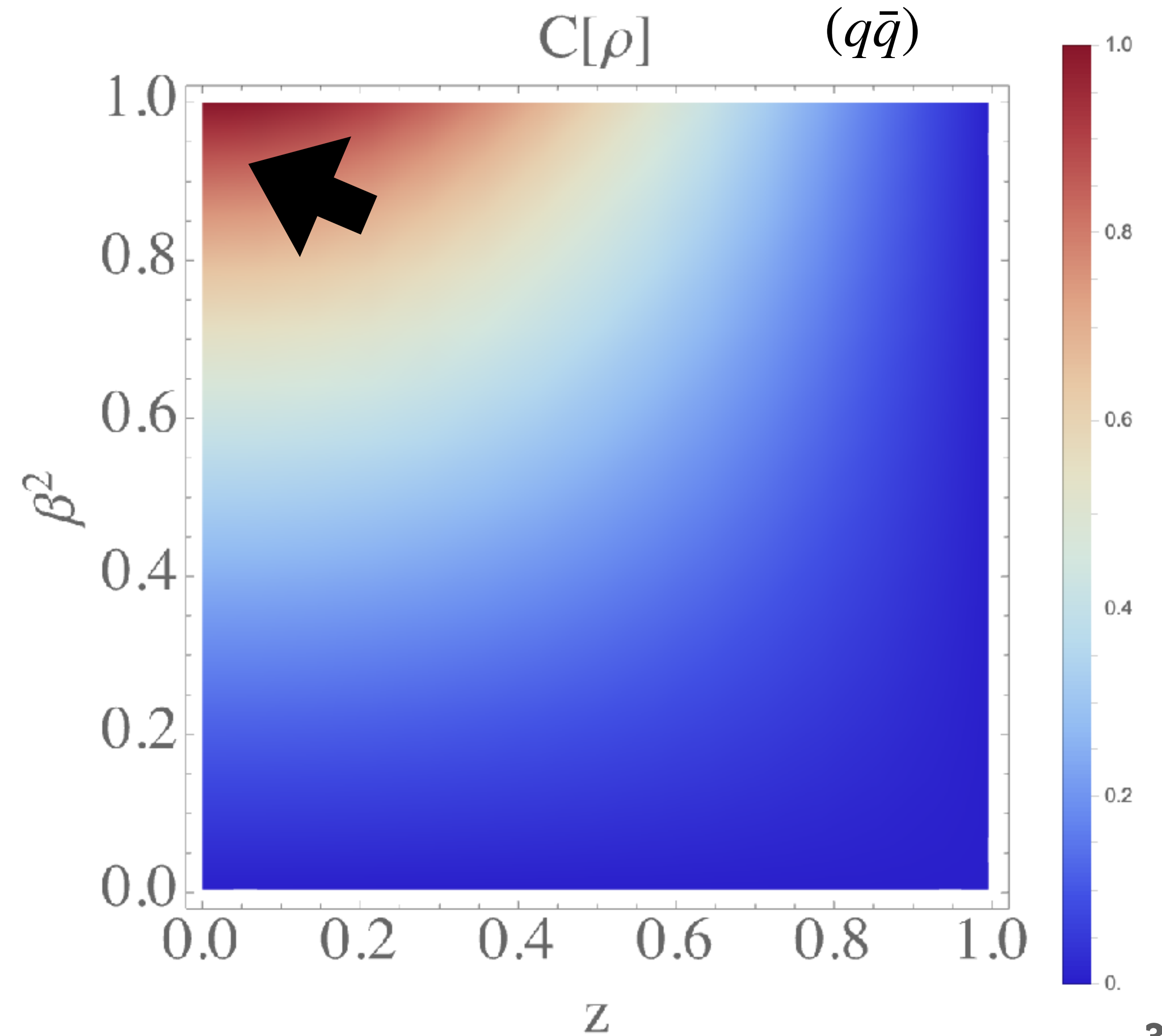
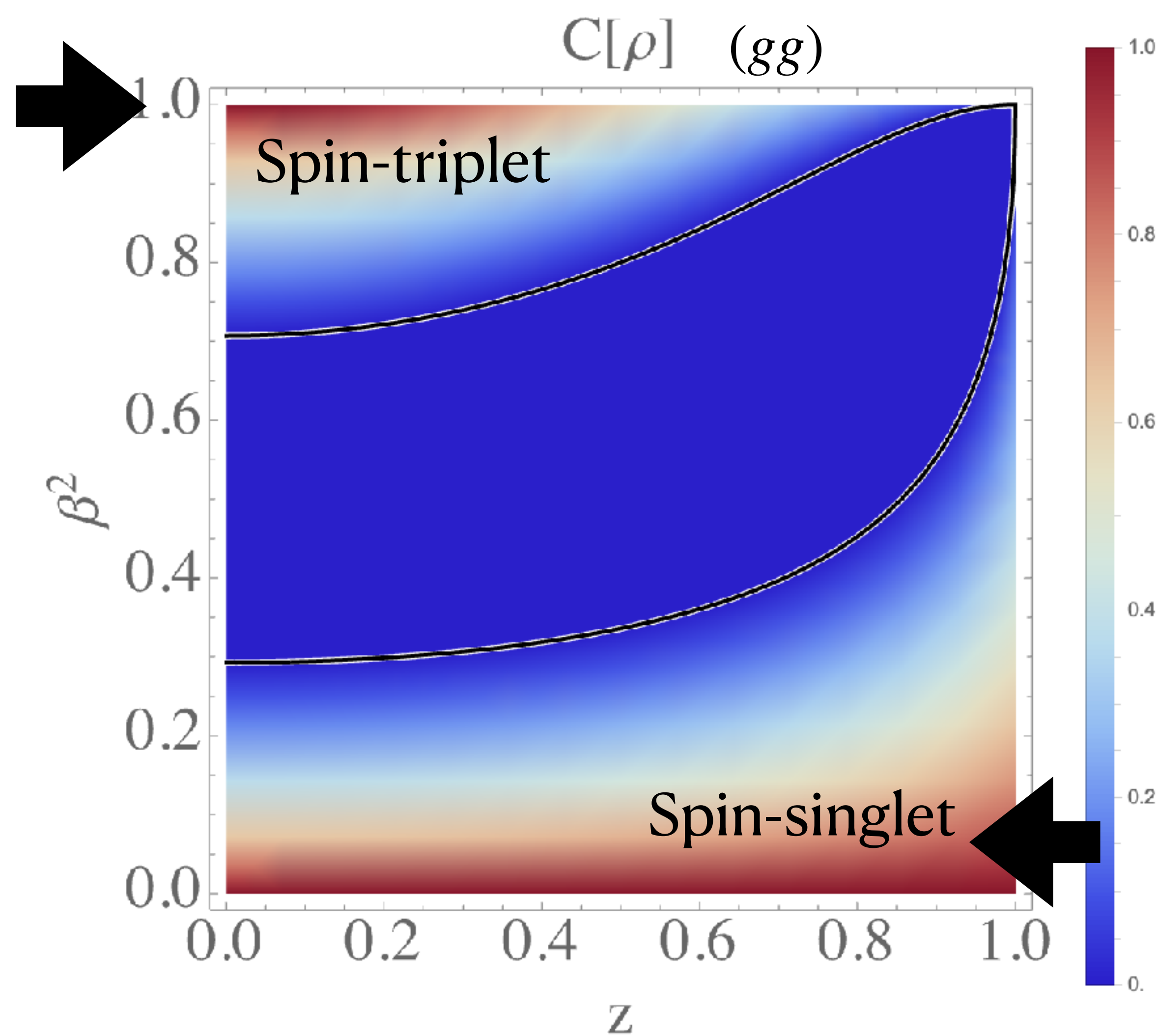


Entanglement marker D

$$D < -1/3$$



Concurrence



Bell's inequality violation

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\left| \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2) \right| > 2.$$

$$D^{(1)} = 1/3(+C_{kk} + C_{rr} + C_{nn}),$$

$$D^{(k)} = 1/3(+C_{kk} - C_{rr} - C_{nn}),$$

$$D^{(r)} = 1/3(-C_{kk} + C_{rr} - C_{nn}),$$

$$D^{(n)} = 1/3(-C_{kk} - C_{rr} + C_{nn}).$$

$$D_{\min} \equiv \min\{D^{(1)}, D^{(k)}, D^{(r)}, D^{(n)}\},$$

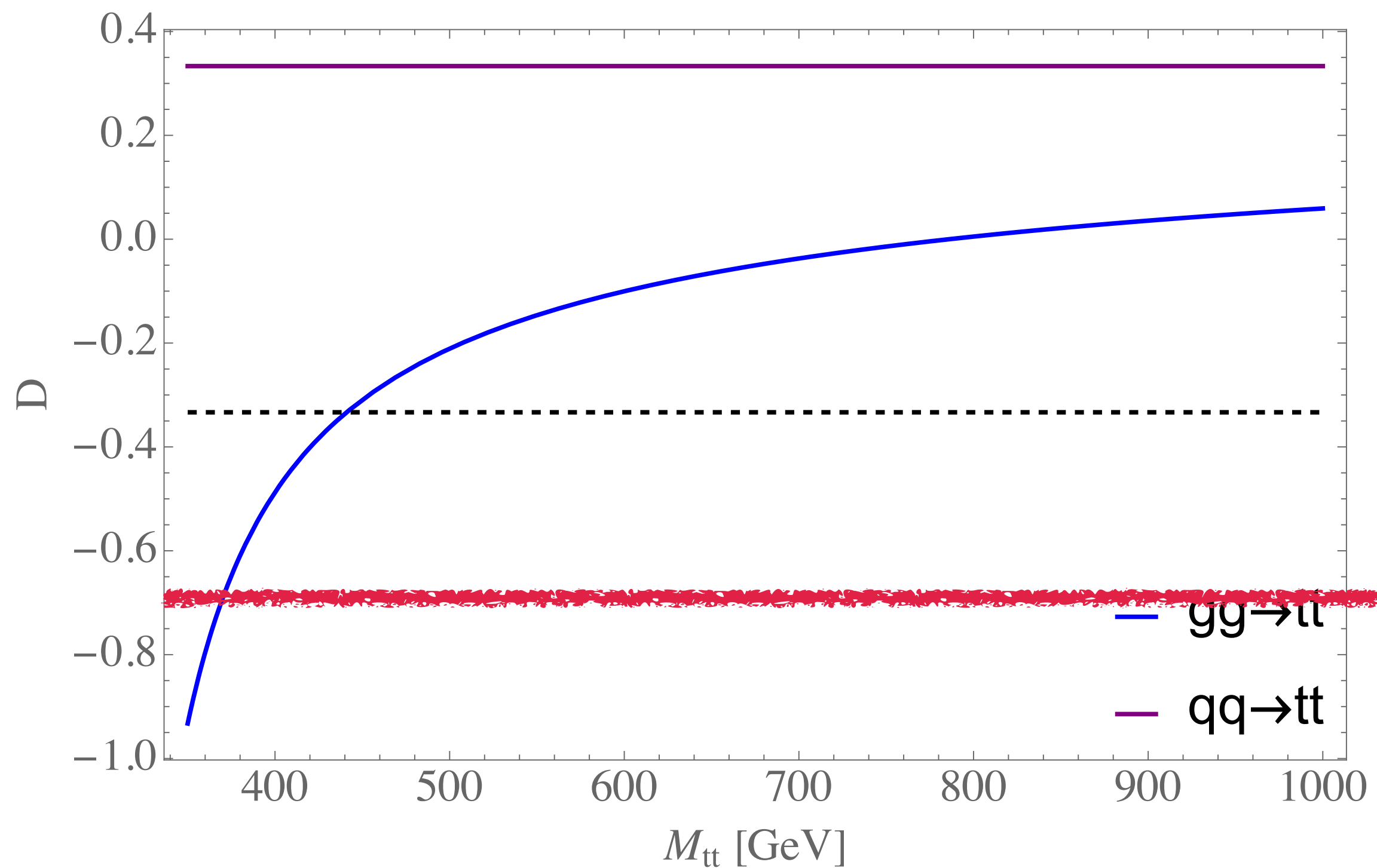
$$D_{\min} < -1/3 \quad (\text{entanglement})$$

$$D_{\min} < -\frac{1}{\sqrt{2}} \quad (\text{Bell violation})$$

Bell's inequality violation

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\left| \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2) \right| > 2.$$

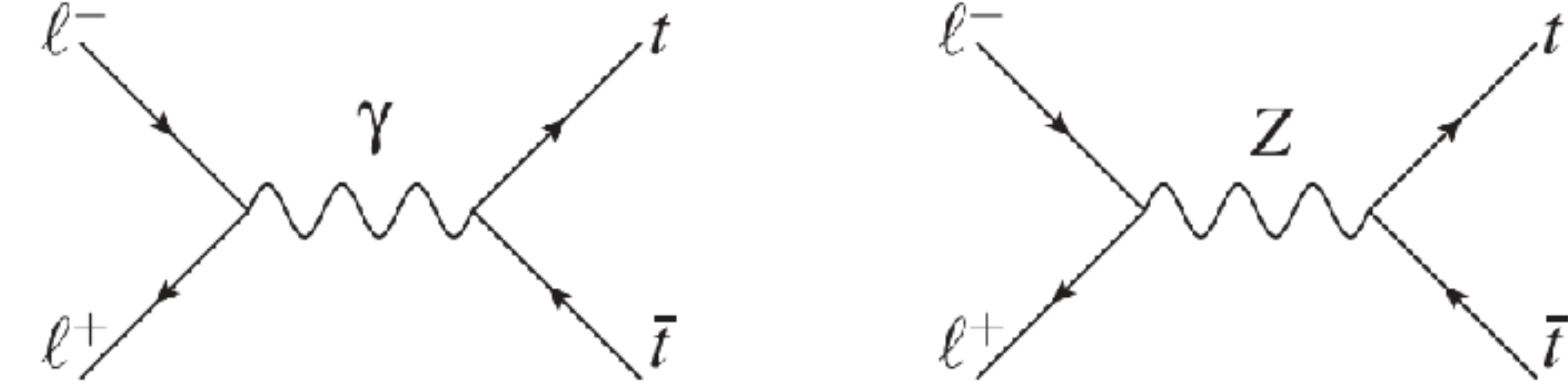


$$D_{\min} < -1/3 \quad (\text{entanglement})$$
$$D_{\min} < -\frac{1}{\sqrt{2}} \quad (\text{Bell violation})$$

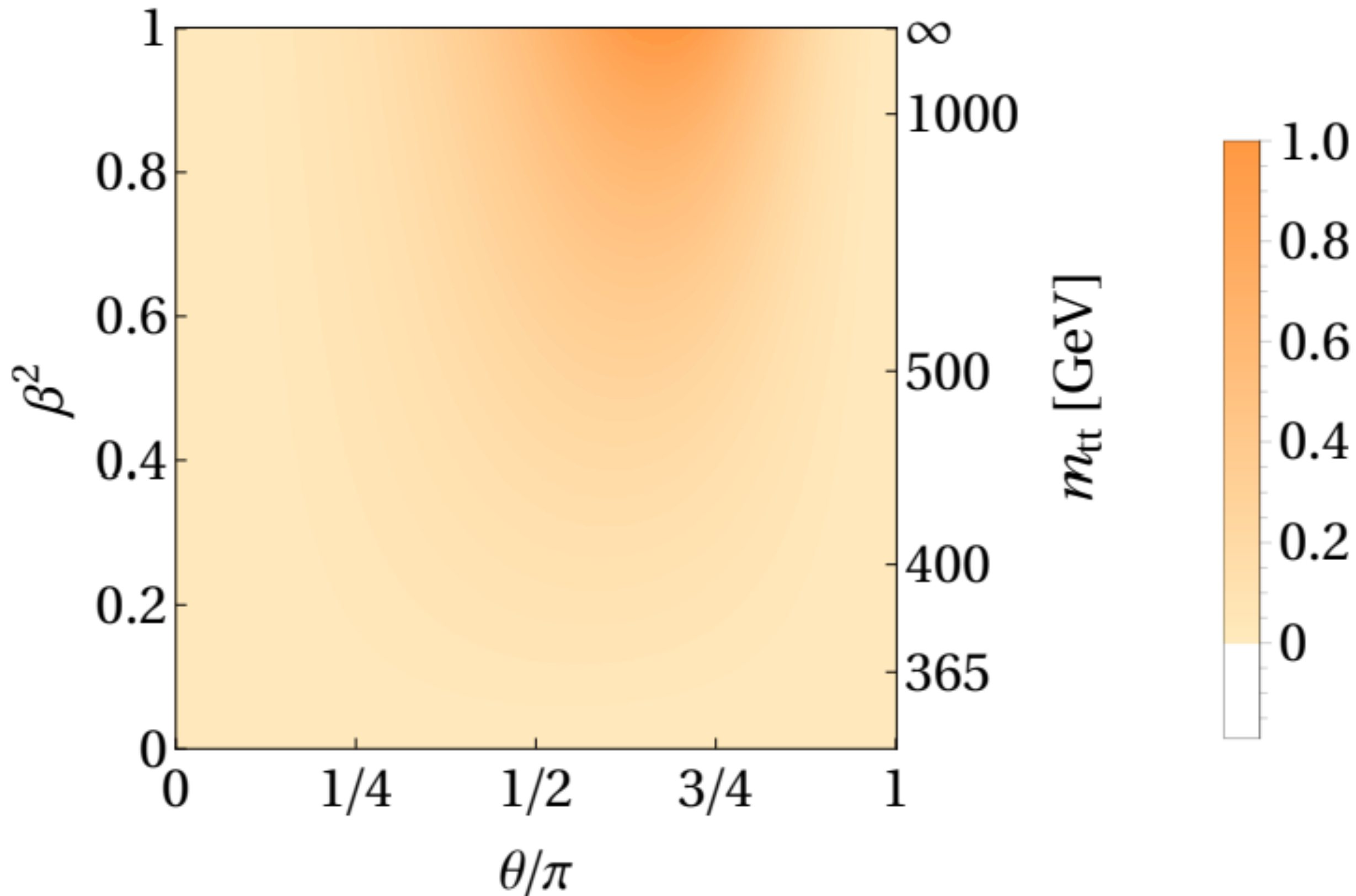
Top quark pairs @ Lepton Collider

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\hat{C}_{ij} = \begin{pmatrix} \hat{C}_{13}[+++]-\hat{C}_{15}[+++] & -\hat{C}_{14}[- - +]-\hat{C}_{16}[- + -] & -\hat{C}_{10}[- - +]-\hat{C}_{12}[- + -] \\ \hat{C}_{14}[- - +]-\hat{C}_{16}[- + -] & \hat{C}_{13}[+++]+\hat{C}_{15}[+++] & \hat{C}_9[+ - -]+\hat{C}_{11}[+++] \\ \hat{C}_{10}[- - +]-\hat{C}_{12}[- + -] & -\hat{C}_9[+ - -]+\hat{C}_{11}[+++] & -\hat{C}_1[+++]+\hat{C}_3[+++] \end{pmatrix}$$



Concurrence



$$\begin{cases} A^{[0]} = F^{[0]} (\beta^2 c_\theta^2 - \beta^2 + 2), \\ \tilde{C}_{kk}^{[0]} = F^{[0]} [\beta^2 - (\beta^2 - 2) c_\theta^2], \\ \tilde{C}_{kr}^{[0]} = F^{[0]} 2\sqrt{1 - \beta^2} c_\theta s_\theta, \\ \tilde{C}_{rr}^{[0]} = F^{[0]} s_\theta^2 (2 - \beta^2), \\ \tilde{C}_{nn}^{[0]} = -F^{[0]} \beta^2 s_\theta^2, \end{cases} \quad \begin{cases} A^{[1]} = 2 F^{[1]} c_\theta, \\ \tilde{C}_{kk}^{[1]} = 2 F^{[1]} c_\theta, \\ \tilde{C}_{kr}^{[1]} = F^{[1]} \sqrt{1 - \beta^2} s_\theta, \\ \tilde{C}_{rr}^{[1]} = 0, \\ \tilde{C}_{nn}^{[1]} = 0, \end{cases} \quad \begin{cases} A^{[2]} = F^{[2]} (1 + c_\theta^2), \\ \tilde{C}_{kk}^{[2]} = F^{[2]} (1 + c_\theta^2), \\ \tilde{C}_{kr}^{[2]} = 0, \\ \tilde{C}_{rr}^{[2]} = -F^{[2]} s_\theta^2, \\ \tilde{C}_{nn}^{[2]} = F^{[2]} s_\theta^2. \end{cases}$$

$$F^{[0]} = 48e^4 \left(Q_t^2 Q_\ell^2 + 2 \operatorname{Re} \frac{4Q_t Q_\ell g_{Vt} g_{V\ell} m_t^2}{D_Z} + \frac{16g_{Vt}^2 m_t^4 (g_{V\ell}^2 + g_{A\ell}^2)}{|D_Z|^2} \right),$$

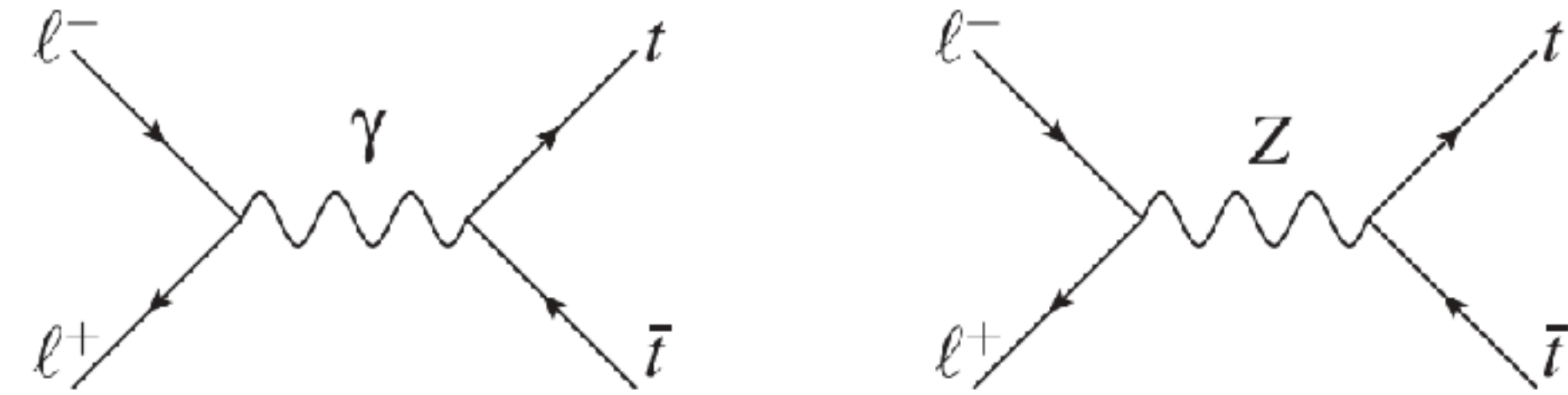
$$F^{[1]} = 192e^4 g_{At} g_{A\ell} m_t^2 \beta \left(\frac{16g_{Vt} g_{V\ell} m_t^2}{|D_Z|^2} + 2 \operatorname{Re} \frac{Q_t Q_\ell}{D_Z} \right),$$

$$F^{[2]} = \frac{768e^4 g_{At}^2 m_t^4 \beta^2 (g_{V\ell}^2 + g_{A\ell}^2)}{|D_Z|^2},$$

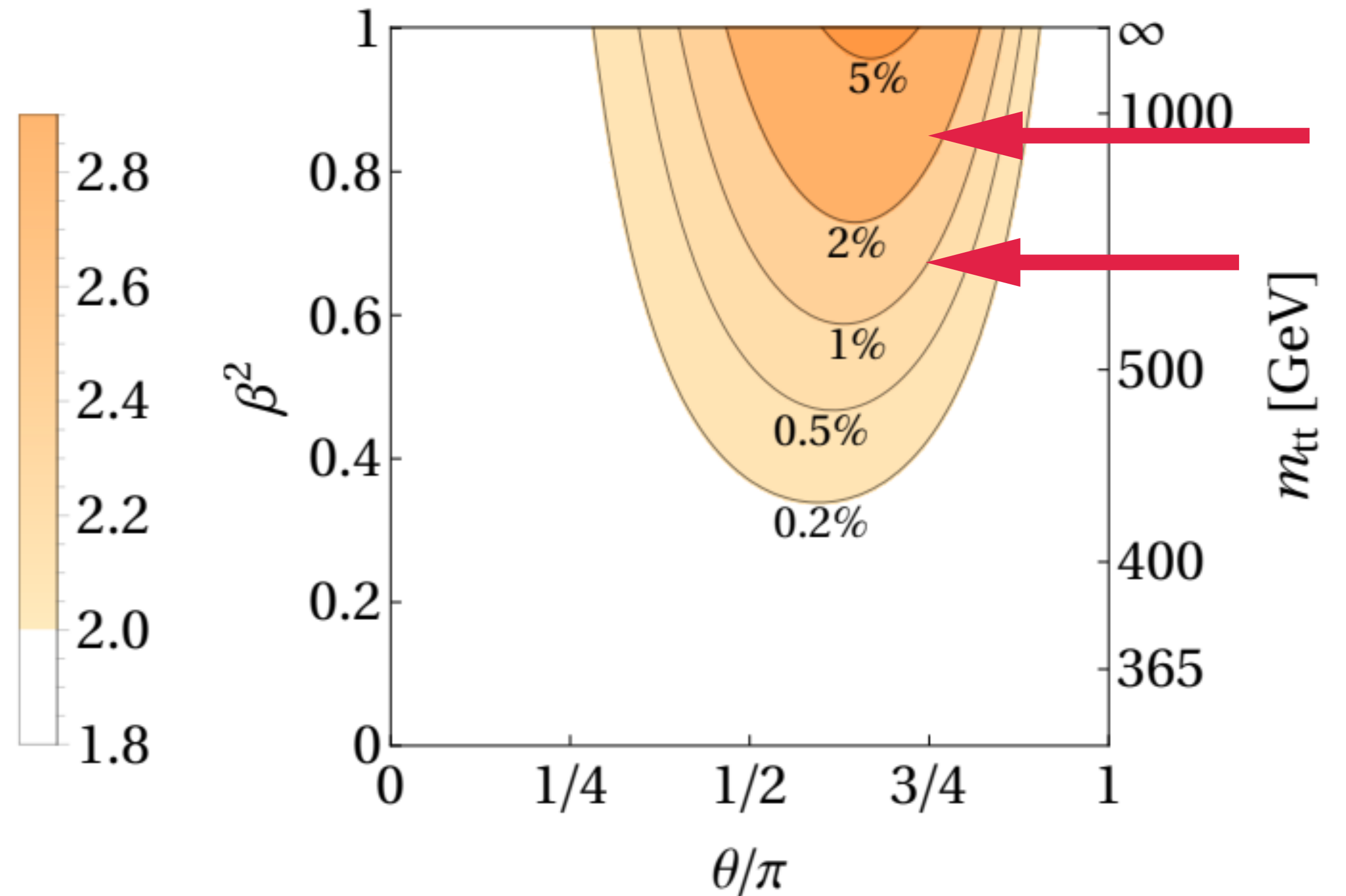
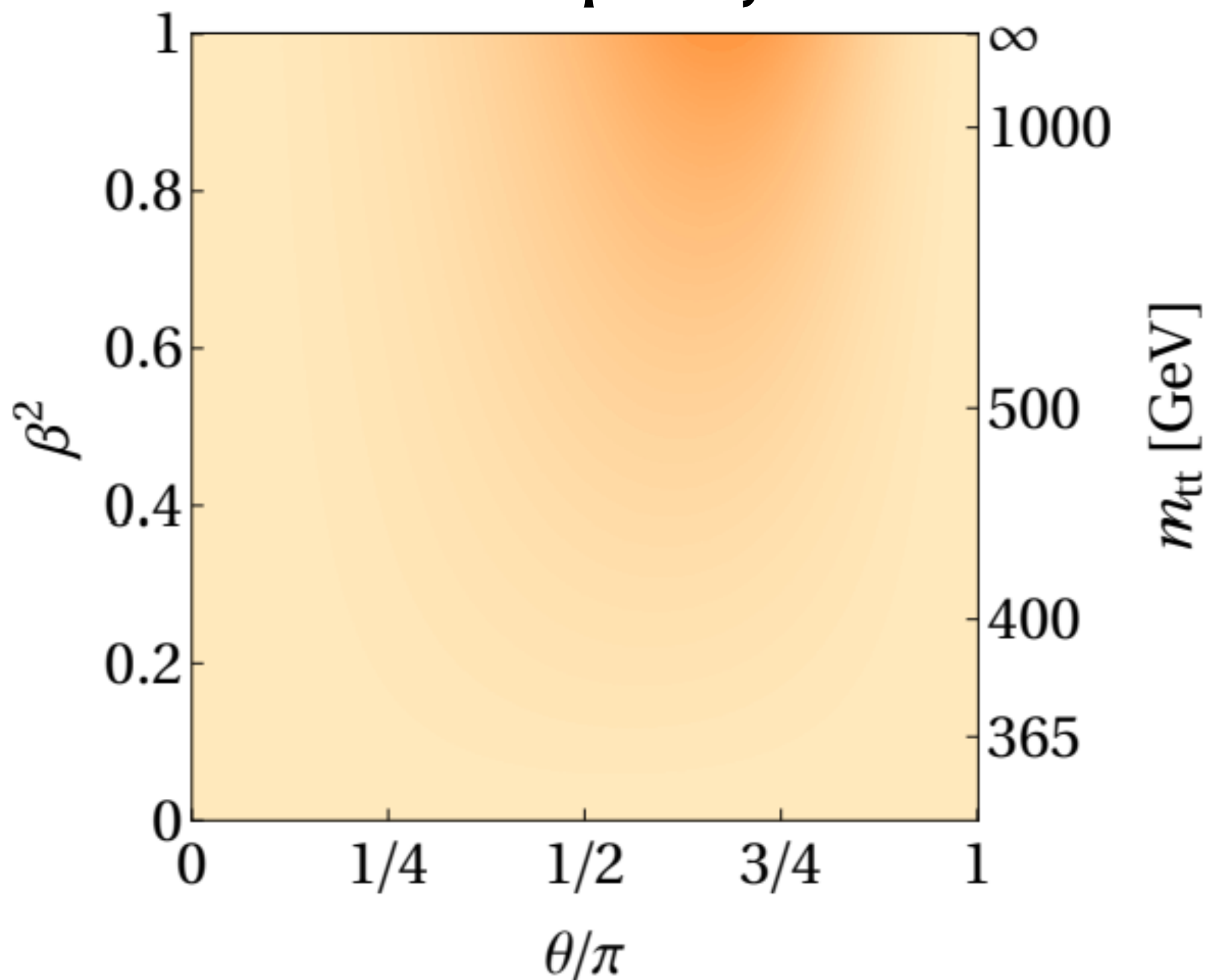
$$D_Z = c_w^2 s_w^2 (4m_t^2 - (1 - \beta^2) m_Z^2)$$

Top quark pairs @ Lepton Collider

[Maltoni, Severi, Tentori, Vryonidoud '24]



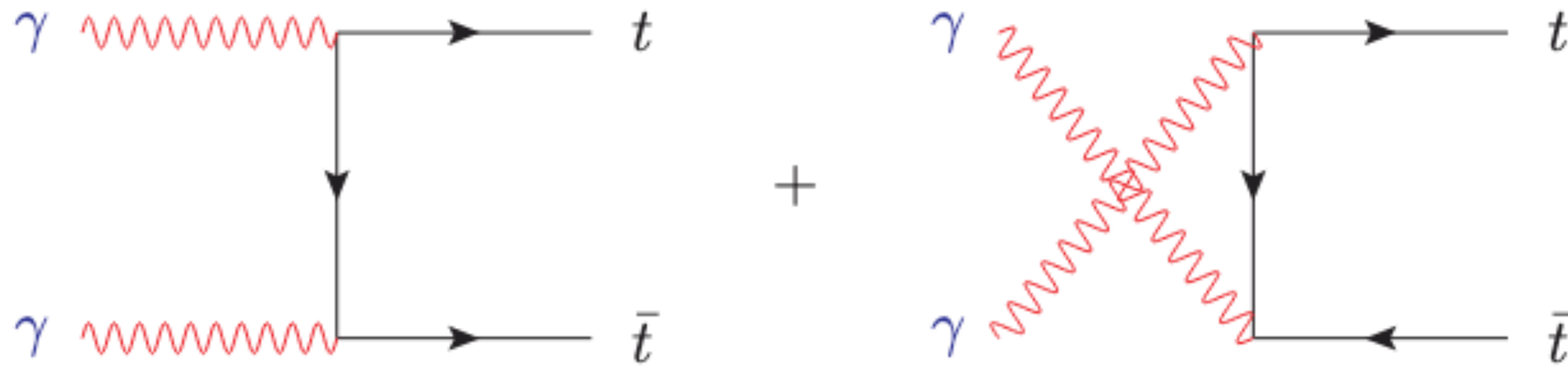
Bell's inequality violation



Top quark pairs @ Photon Collider

- We can also consider the process $\gamma\gamma \rightarrow t\bar{t}$.

The process is very similar to the process $gg \rightarrow t\bar{t}$.



- Also independent with the process $l^+l^- \rightarrow t\bar{t}$.

$$F_\gamma = \frac{64\pi^2\alpha^2 Q_t^4}{(1 - \beta^2 z^2)^2}, \quad \tilde{B}_i^\pm = 0,$$

$$\tilde{A}^{\gamma\gamma} = F_\gamma(1 + 2\beta^2(1 - z^2) - \beta^4(2 - 2z^2 + z^4)),$$

$$\tilde{C}_{nn}^{\gamma\gamma} = F_\gamma(1 + 2\beta^2(1 - z^2) + \beta^4(2 - 2z^2 + z^4)),$$

$$\tilde{C}_{kk}^{\gamma\gamma} = -F_\gamma(1 - 2z^2(1 - z^2)\beta^2 - \beta^4(2 - 2z^2 + z^4)),$$

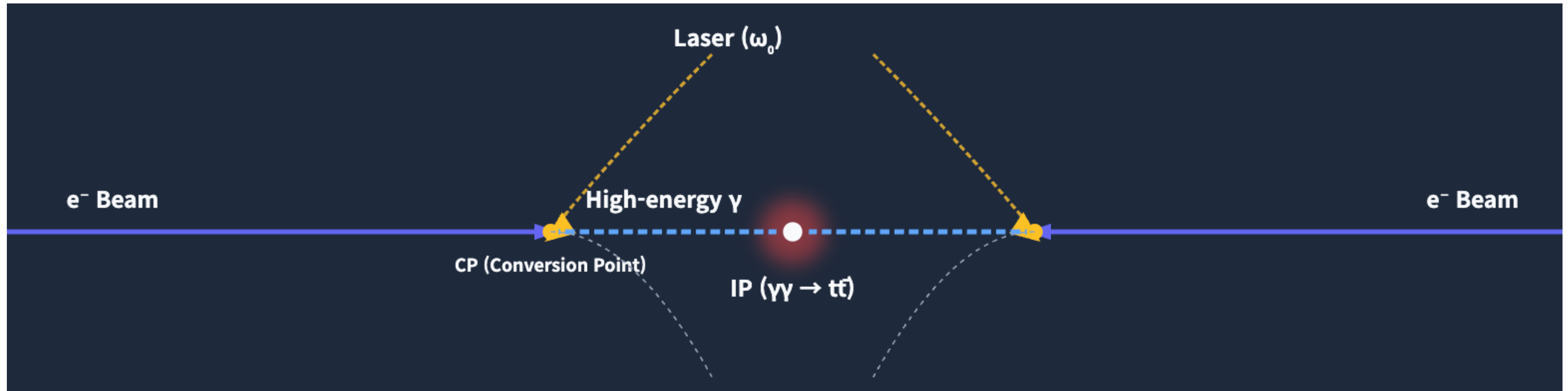
$$\tilde{C}_{rr}^{\gamma\gamma} = -F_\gamma(1 - \beta^2(2 - \beta^2)(2 - 2z^2 + z^4)),$$

$$\tilde{C}_{rk}^{\gamma\gamma} = \tilde{C}_{kr}^{\gamma\gamma} = F_\gamma 2z(1 - z^2)^{3/2}\beta^2\sqrt{1 - \beta^2},$$

$$\tilde{C}_{nr}^{\gamma\gamma} = \tilde{C}_{rn}^{\gamma\gamma} = \tilde{C}_{nk}^{\gamma\gamma} = \tilde{C}_{kn}^{\gamma\gamma} = 0,$$

Top quark pairs @ Photon Collider

[Ginzburg, Kotkin, Serbo, Telnov '83]



- It is well established that at a future linear e^+e^- collider, the Compton backscattering of the laser light with the $e^\pm$ beam can provide very high-energy photons.
- Especially, top-quark pair production is possible with large rates in (un)polarized photon-photon fusion.

Top quark pairs @ Photon Collider

[Ginzburg, Kotkin, Serbo, Telnov '83]

- The $\gamma\gamma$ luminosity of the two colliding photons with helicity (λ_1, λ_2) is

$$\frac{1}{\mathcal{L}_{ee}} \frac{d\mathcal{L}_{\gamma\gamma}^{\lambda_1\lambda_2}}{d\tau} = \left(\langle 00 \rangle_\tau + \lambda_1 \langle 20 \rangle_\tau + \lambda_2 \langle 02 \rangle_\tau + \lambda_1 \lambda_2 \langle 22 \rangle_\tau \right) \left(\frac{2\pi\alpha^2}{\sigma_c^{\gamma_1} x M_e^2} \right) \left(\frac{2\pi\alpha^2}{\sigma_c^{\gamma_2} x M_e^2} \right)$$

with $\tau = \hat{s}/s$.

$$\sigma_c^{\gamma_1} = \sigma_c^0 + P_e P_c \sigma_c^1; \quad \sigma_c^{\gamma_2} = \sigma_c^0 + \tilde{P}_e \tilde{P}_c \sigma_c^1. \quad \langle ij \rangle_\tau \equiv \int_{\tau/y_{\max}}^{y_{\max}} \frac{dy}{y} f_i(y) \tilde{f}_j(\tau/y),$$

$$\sigma_c^0 = \frac{2\pi\alpha^2}{x M_e^2} \left[\left(1 - \frac{4}{x} - \frac{8}{x^2} \right) \log(x+1) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(x+1)^2} \right],$$

$$\sigma_c^1 = \frac{2\pi\alpha^2}{x M_e^2} \left[\left(1 + \frac{2}{x} \right) \log(x+1) - \frac{5}{2} + \frac{1}{x+1} - \frac{1}{2(x+1)^2} \right].$$

$$f_0(y) = \frac{1}{1-y} + (1-y) - 4r(1-r) - rx(2r-1)(2-y) P_e P_c,$$

$$f_2(y) = rx [1 + (1-y)(2r-1)^2] P_e - (2r-1) \left(\frac{1}{1-y} + 1-y \right) P_c;$$

$$\tilde{f}_0(\tau/y) = \frac{1}{1-\tau/y} + (1-\tau/y) - 4\tilde{r}(1-\tilde{r}) - \tilde{r}x(2\tilde{r}-1)(2-\tau/y) \tilde{P}_e \tilde{P}_c,$$

$$\tilde{f}_2(\tau/y) = \tilde{r}x [1 + (1-\tau/y)(2\tilde{r}-1)^2] \tilde{P}_e - (2\tilde{r}-1) \left(\frac{1}{1-\tau/y} + 1-\tau/y \right) \tilde{P}_c,$$

x is machine parameter

$$x = \frac{4E_b\omega_0}{M_e^2} = 15.3 \left(\frac{E_b}{\text{TeV}} \right) \left(\frac{\omega_0}{\text{eV}} \right), \quad = 4.8$$

y is fraction of the e^- to photon

$$r = \frac{y}{x(1-y)}, \quad \tilde{r} = \frac{\tau/y}{x(1-\tau/y)}.$$

Top quark pairs @ Photon Collider

- The $\gamma\gamma$ luminosity weighted polarization coefficients

$$\hat{C}_i = \sum_{\lambda_1, \lambda_2 = \pm} c_i^{\lambda_1 \lambda_2}, \quad \longrightarrow \quad \hat{C}_i^w \equiv \sum_{\lambda_1, \lambda_2 = \pm} w^{\lambda_1 \lambda_2} c_i^{\lambda_1 \lambda_2},$$

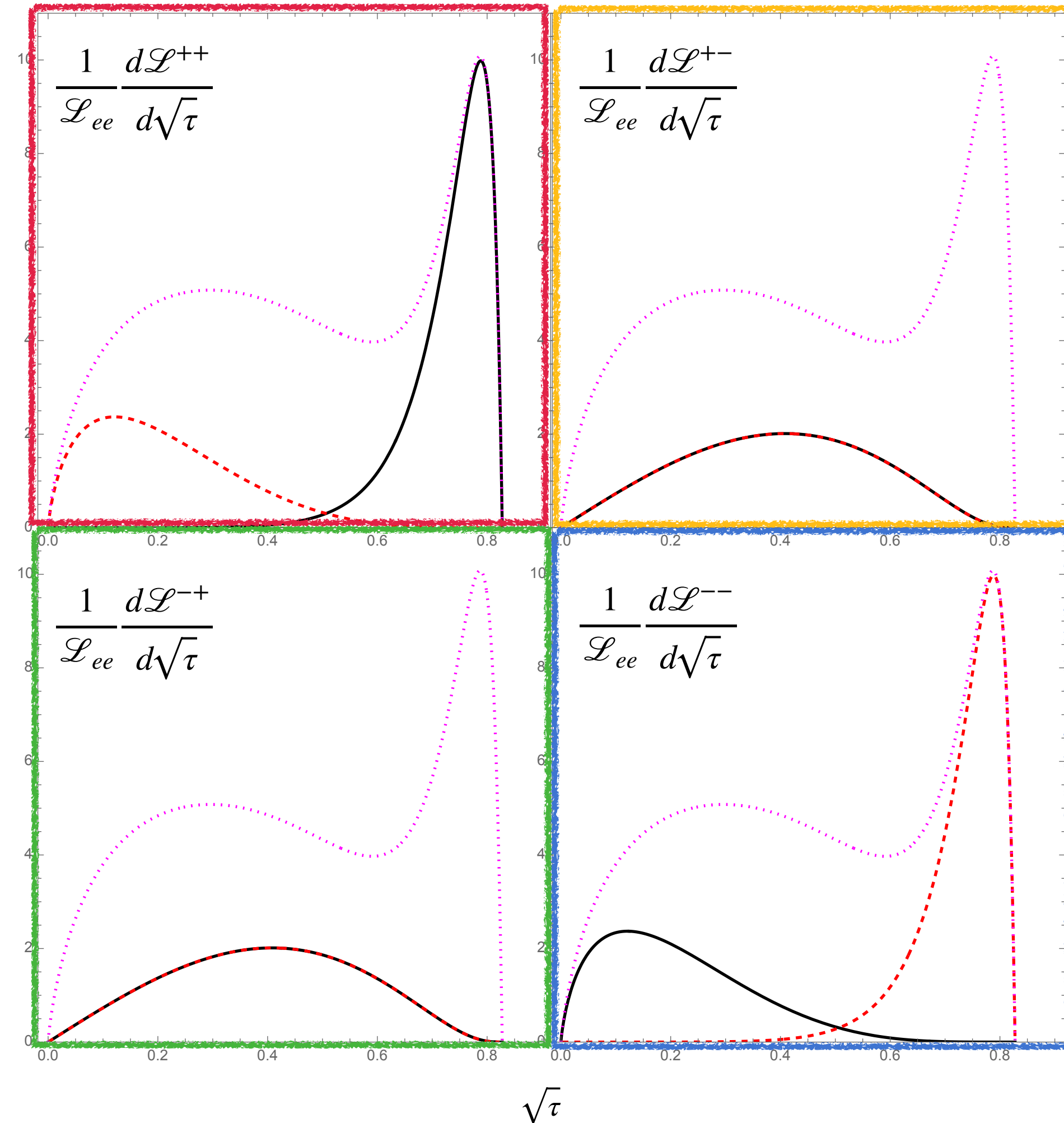
$$w^{\lambda_1 \lambda_2} \equiv \frac{L^{\lambda_1 \lambda_2}}{\sum_{\lambda_1, \lambda_2 = \pm} L^{\lambda_1 \lambda_2}}, \quad L^{\lambda_1 \lambda_2} \equiv \frac{1}{\mathcal{L}_{ee}} \frac{d\mathcal{L}_{\gamma\gamma}^{\lambda_1 \lambda_2}}{d\sqrt{\tau}}.$$

$$\frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm} L^{\lambda_1 \lambda_2} = \frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm} \frac{1}{\mathcal{L}_{ee}} \frac{d\mathcal{L}_{\gamma\gamma}^{\lambda_1 \lambda_2}}{d\sqrt{\tau}} \xrightarrow{P_e P_c = \bar{P}_e \bar{P}_c = 0} \frac{1}{\mathcal{L}_{ee}} \frac{d\mathcal{L}_{\gamma\gamma}^{\text{unp}}}{d\sqrt{\tau}}.$$

Top quark pairs @ Photon Collider

[Ginzburg, Kotkin, Serbo, Telnov '83]

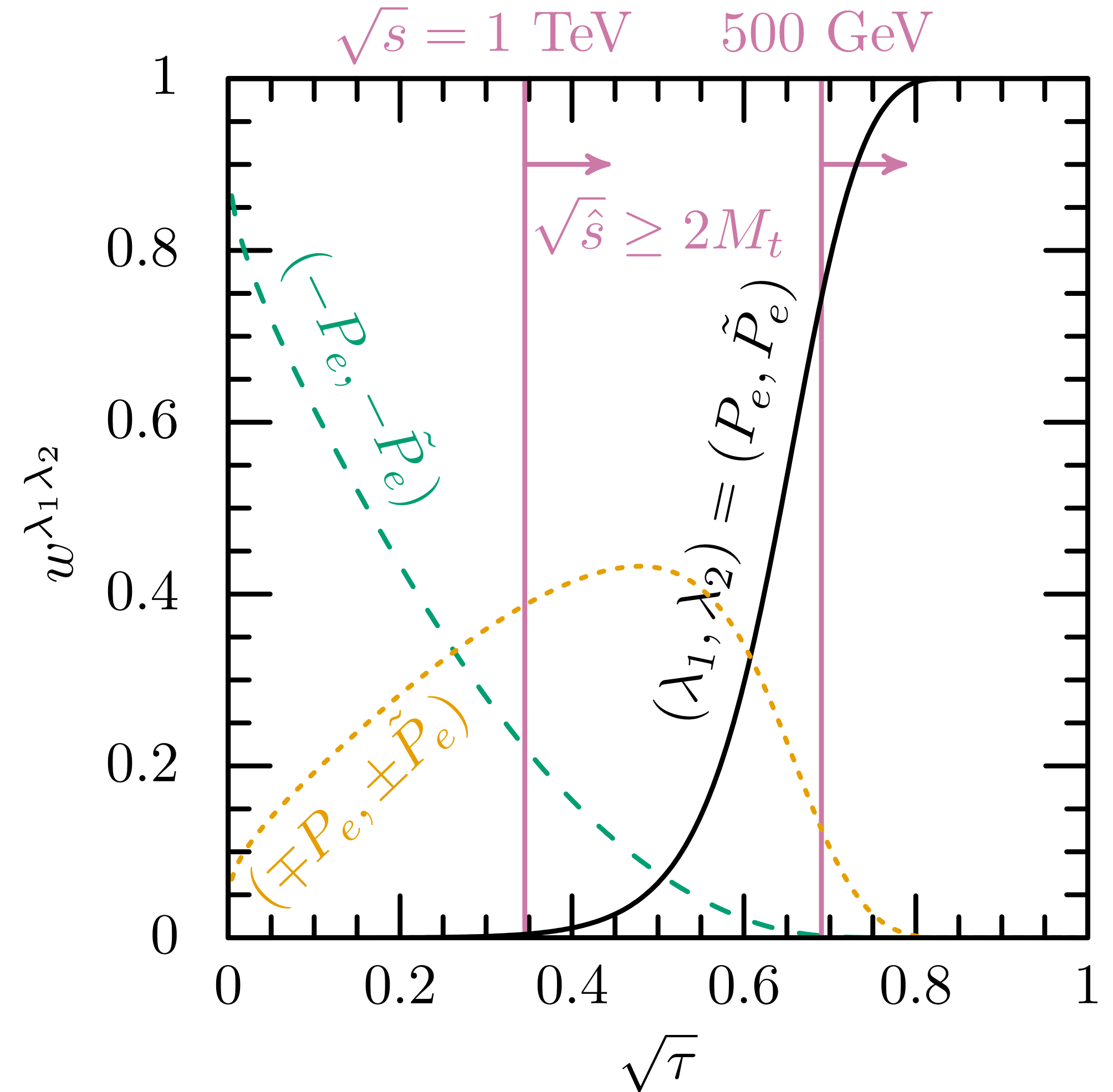
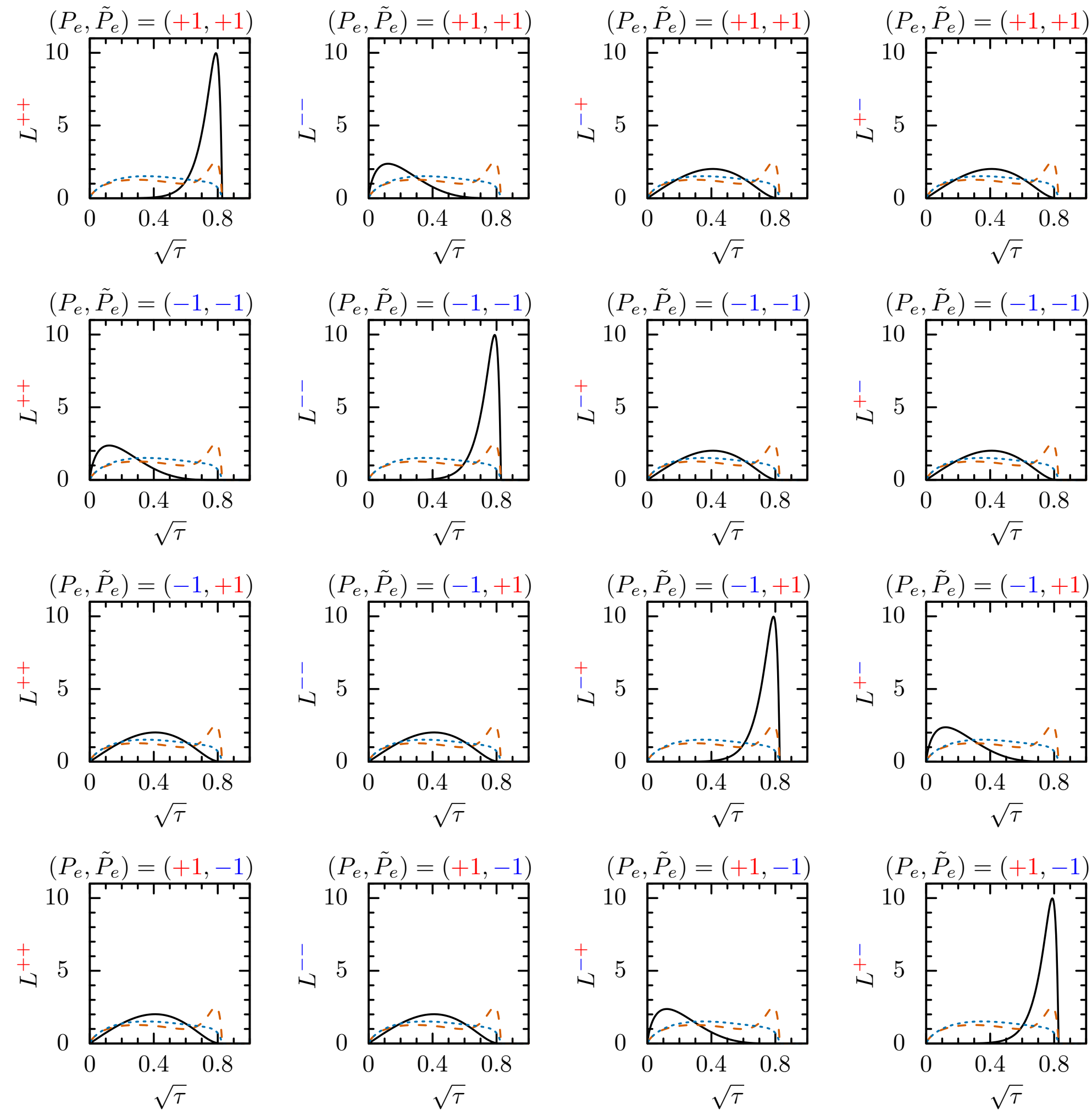
- For the polarized laser beams we can almost extract corresponding polarization of initial photon.



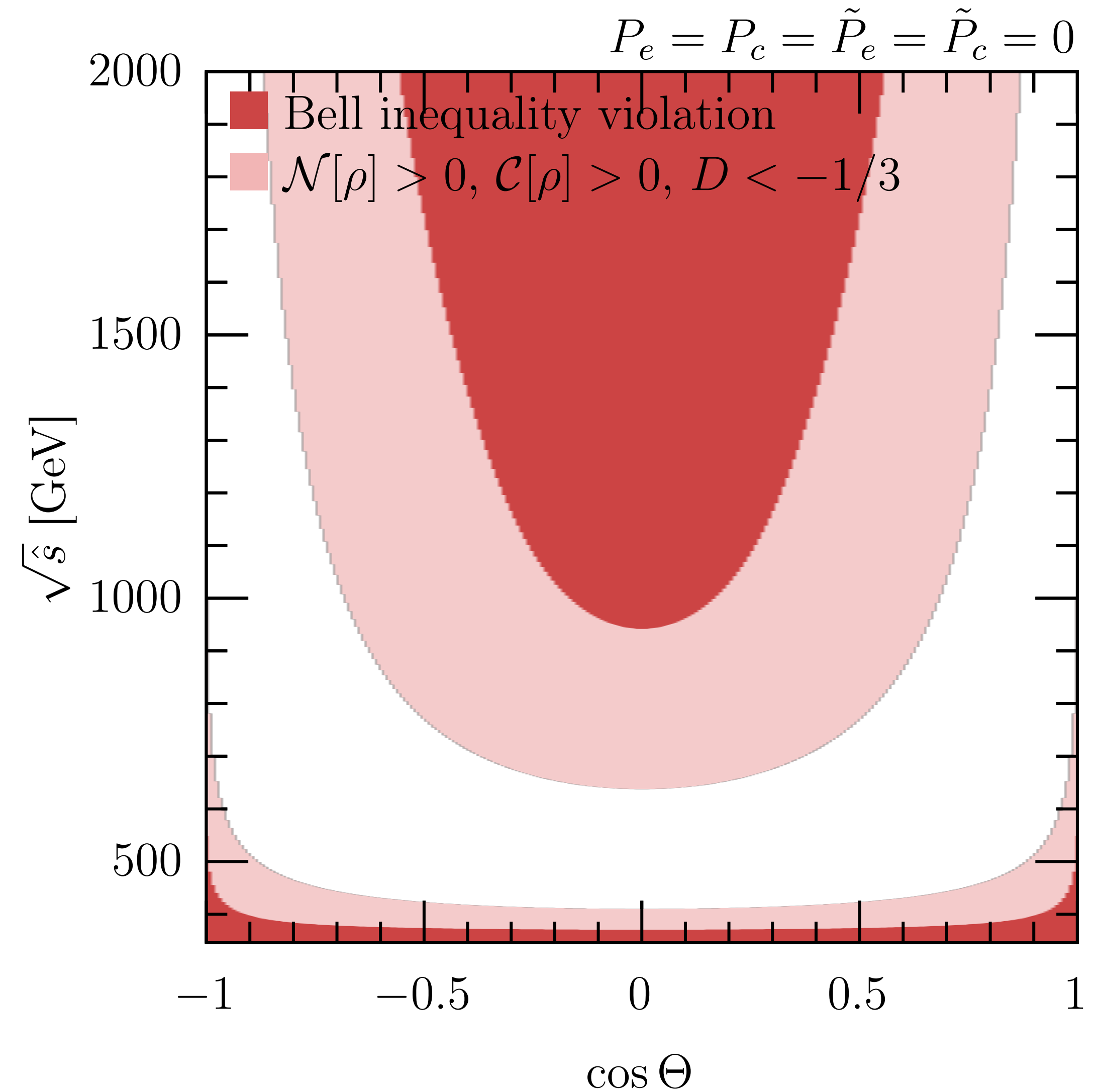
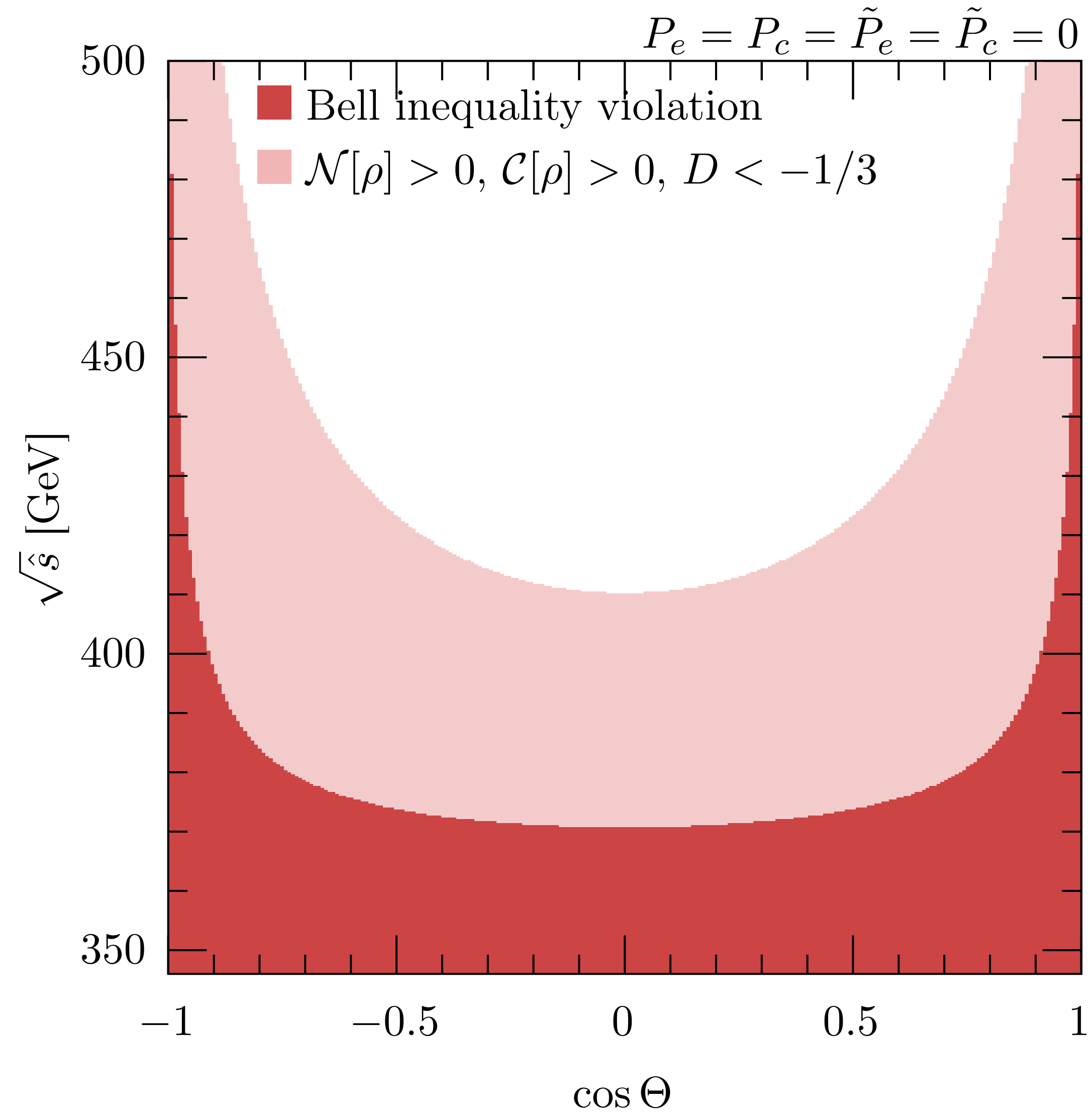
$(\lambda_1 \lambda_2) \downarrow (\sigma \bar{\sigma}) \rightarrow$	$\langle ++ \rangle$	$\langle -- \rangle$	$\langle -+ \rangle$	$\langle +- \rangle$
$\langle ++ \rangle$	$\frac{2M_t}{\sqrt{\hat{s}}} [1 + \beta]$	$\frac{2M_t}{\sqrt{\hat{s}}} [1 - \beta]$	0	0
$\langle -- \rangle$	$\frac{2M_t}{\sqrt{\hat{s}}} [-1 + \beta]$	$\frac{2M_t}{\sqrt{\hat{s}}} [-1 - \beta]$	0	0
$\langle -+ \rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$-\beta \sin \Theta (\cos \Theta + 1)$	$-\beta \sin \Theta (\cos \Theta - 1)$
$\langle +- \rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$\frac{2M_t}{\sqrt{\hat{s}}} \beta \sin^2 \Theta$	$-\beta \sin \Theta (\cos \Theta - 1)$	$-\beta \sin \Theta (\cos \Theta + 1)$

Top quark pairs @ Photon Collider

- The $\gamma\gamma$ luminosity of the two colliding photons with helicity (λ_1, λ_2) for $P_e P_c = \tilde{P}_e \tilde{P}_c = -1$



Top quark pairs @ Photon Collider



Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1 \lambda_2) \downarrow (\sigma \bar{\sigma}) \rightarrow$	$\langle ++ \rangle$	$\langle -- \rangle$	$\langle -+ \rangle$	$\langle +- \rangle$
$\langle ++ \rangle$	$\frac{2M_t}{\sqrt{s}} [1 + \beta]$	$\frac{2M_t}{\sqrt{s}} [1 - \beta]$	0	0

$$\rho = \frac{1}{2(1 + \beta^2)} \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & (1 + \beta)^2 & -(1 - \beta^2) & 0 \\ \hline 0 & -(1 - \beta^2) & (1 - \beta)^2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right); \quad \rho^{T_2} = \frac{1}{2(1 + \beta^2)} \left(\begin{array}{cc|cc} 0 & 0 & 0 & -(1 - \beta^2) \\ 0 & (1 + \beta)^2 & 0 & 0 \\ \hline 0 & 0 & (1 - \beta)^2 & 0 \\ -(1 - \beta^2) & 0 & 0 & 0 \end{array} \right)$$

$$C[\rho] = \frac{1 - \beta^2}{1 + \beta^2} > 0$$

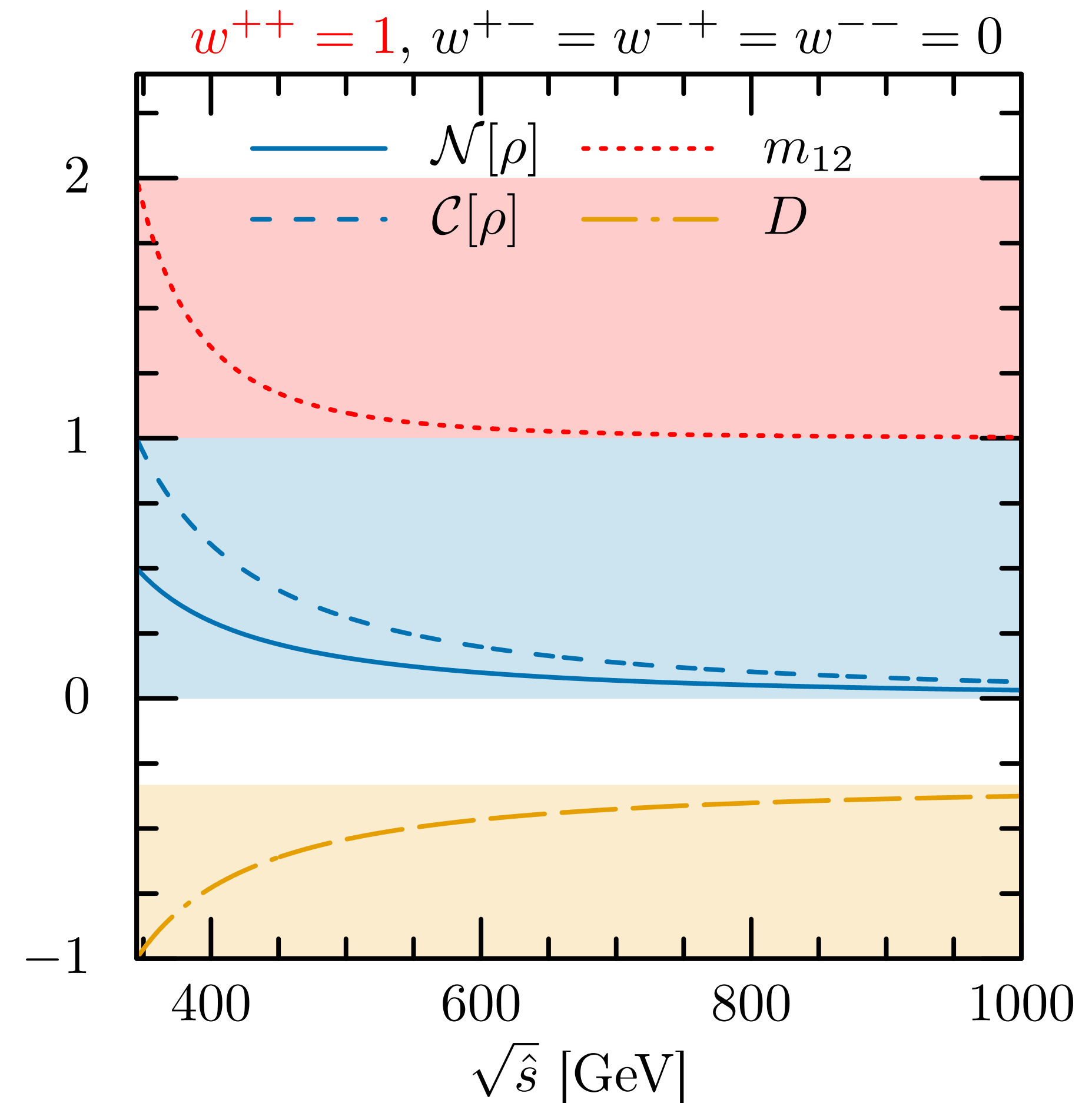
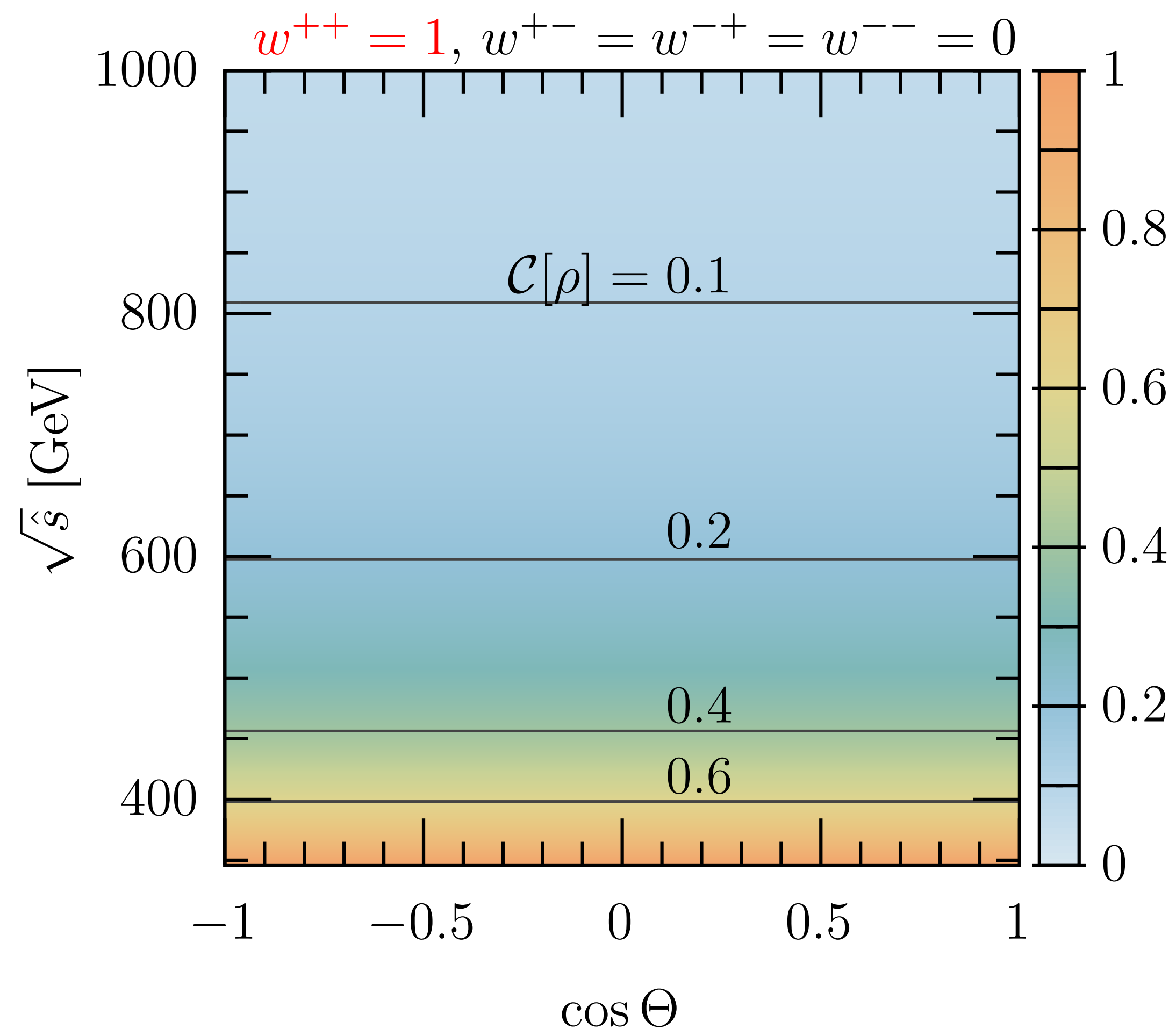
$$m_{12} = 1 + \frac{(1 - \beta^2)^2}{(1 + \beta^2)^2} > 1$$

Entire region is Entangled and violate CHSH inequality

Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1 \lambda_2) \downarrow (\sigma \bar{\sigma}) \rightarrow$	$\langle ++ \rangle$	$\langle -- \rangle$	$\langle -+ \rangle$	$\langle +- \rangle$
$\langle ++ \rangle$	$\frac{2M_t}{\sqrt{s}} [1 + \beta]$	$\frac{2M_t}{\sqrt{s}} [1 - \beta]$	0	0



Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1 \lambda_2) \downarrow (\sigma \bar{\sigma}) \rightarrow$	$\langle ++ \rangle$	$\langle -- \rangle$	$\langle -+ \rangle$	$\langle +- \rangle$
$\langle +- \rangle$	$-\frac{2M_t}{\sqrt{s}} \beta \sin^2 \Theta$	$\frac{2M_t}{\sqrt{s}} \beta \sin^2 \Theta$	$-\beta \sin \Theta (\cos \Theta - 1)$	$-\beta \sin \Theta (\cos \Theta + 1)$

$$\mathcal{C}[\rho]^{(+-/-+)} = \frac{(1 - z^2)\beta^2}{2 - (1 - z^2)\beta^2} > 0$$

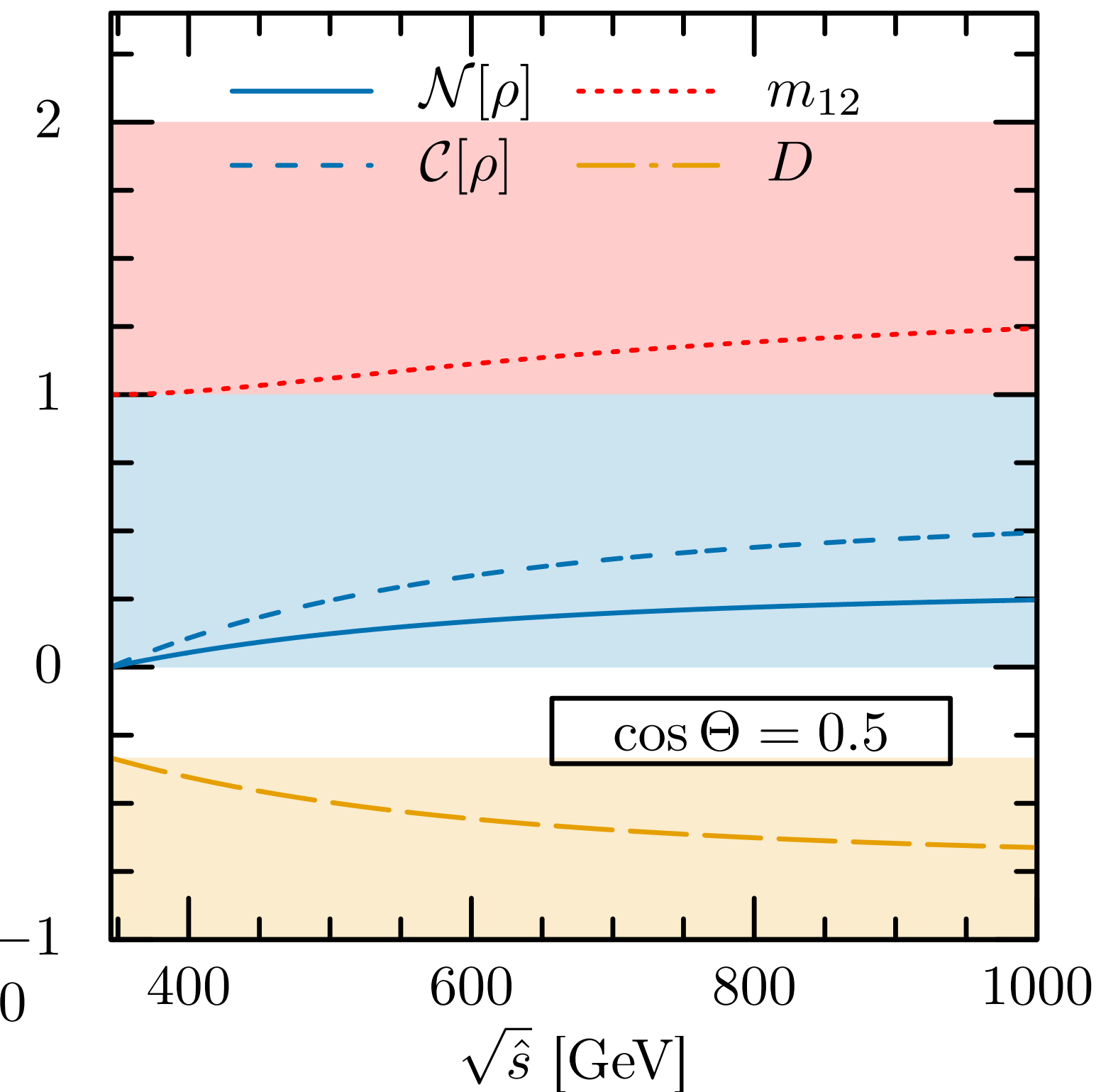
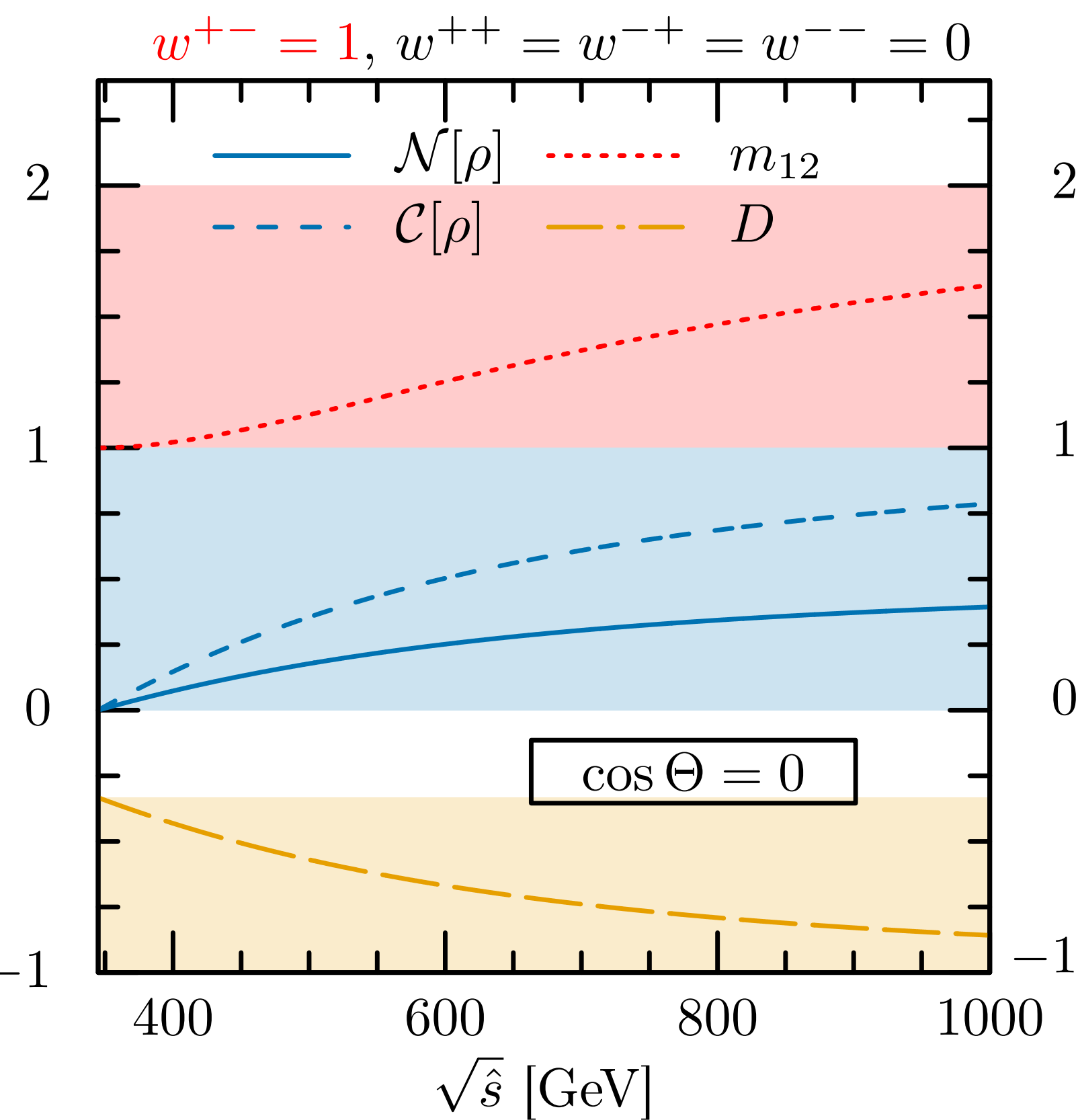
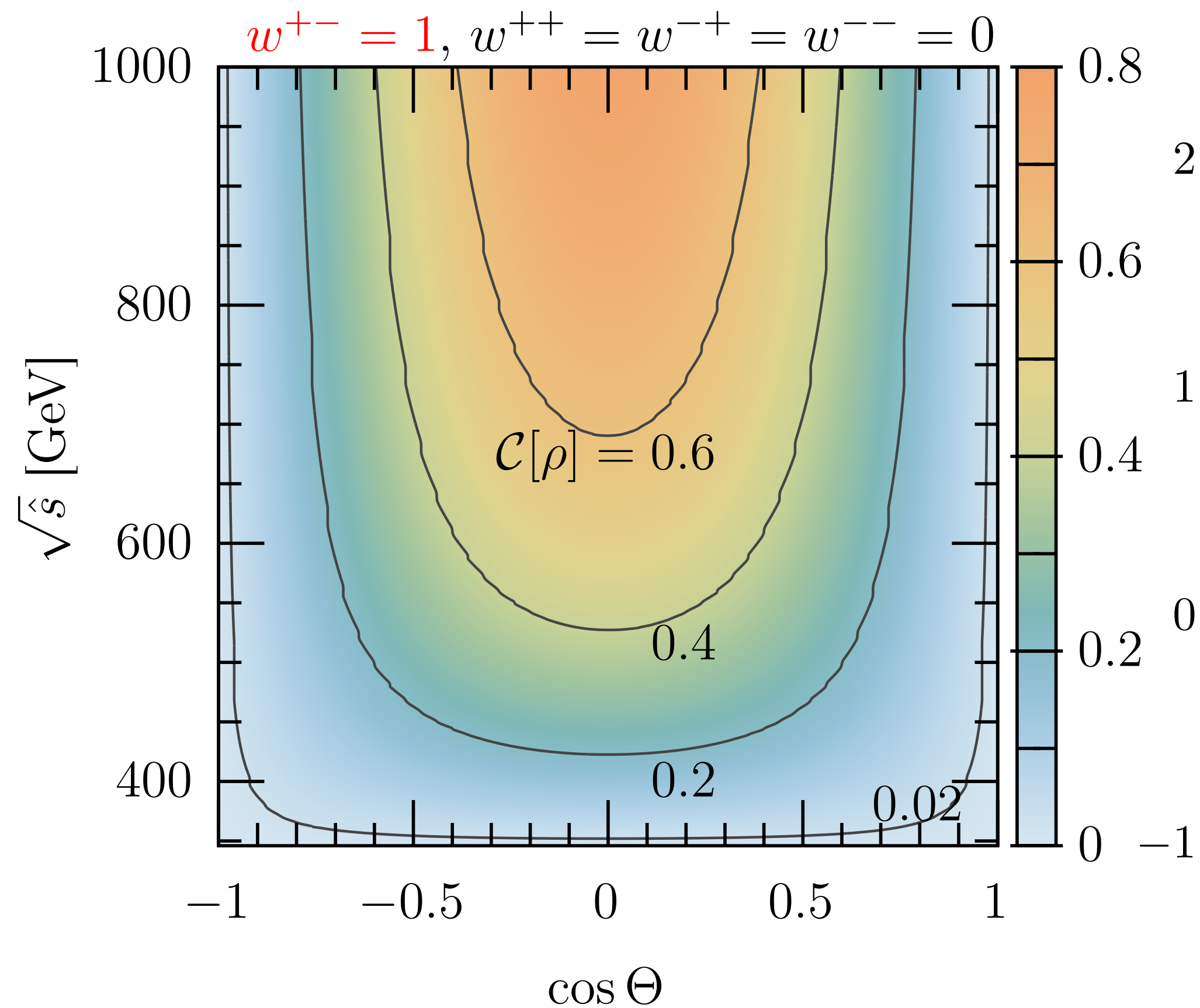
$$m_{12}^{(+-/-+)} = 1 + \frac{(1 - z^2)^2 \beta^4}{(2 - (1 - z^2)\beta^2)^2}$$

Entire region is Entangled and violate CHSH inequality

Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1 \lambda_2) \downarrow (\sigma \bar{\sigma}) \rightarrow$	$\langle ++ \rangle$	$\langle -- \rangle$	$\langle -+ \rangle$	$\langle +- \rangle$
$\langle +- \rangle$	$-\frac{2M_t}{\sqrt{s}}\beta \sin^2 \Theta$	$\frac{2M_t}{\sqrt{s}}\beta \sin^2 \Theta$	$-\beta \sin \Theta (\cos \Theta - 1)$	$-\beta \sin \Theta (\cos \Theta + 1)$

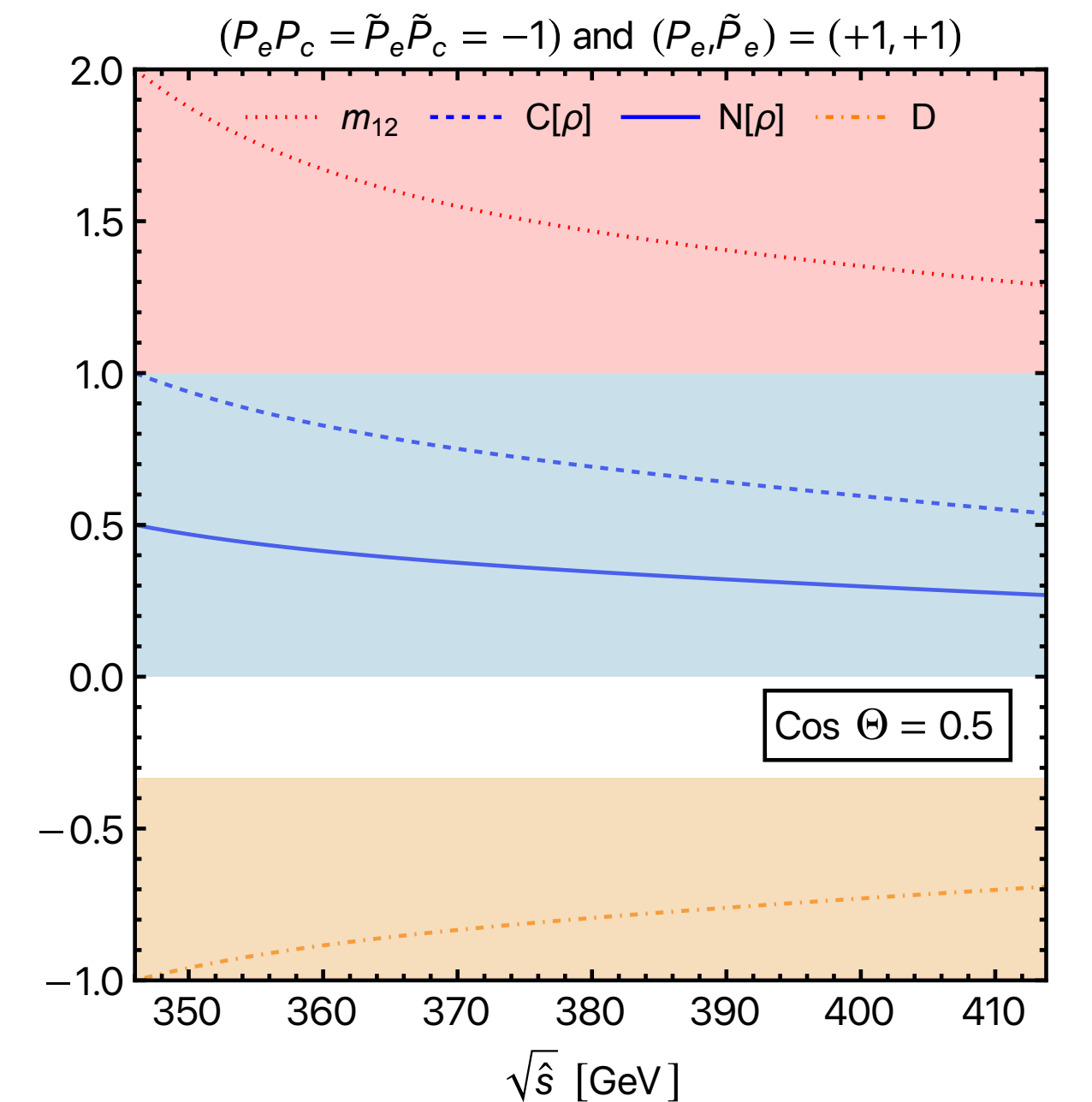
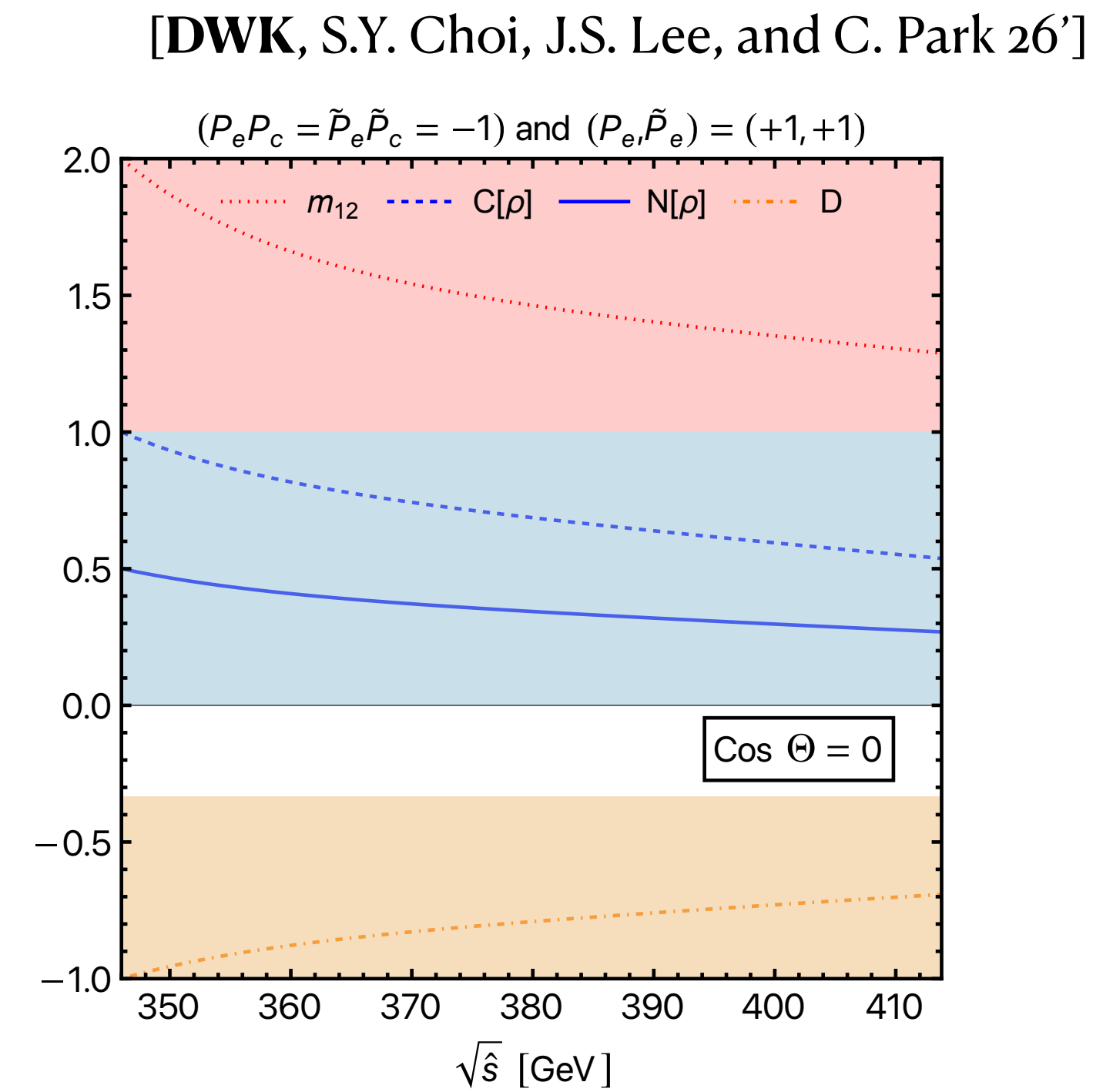
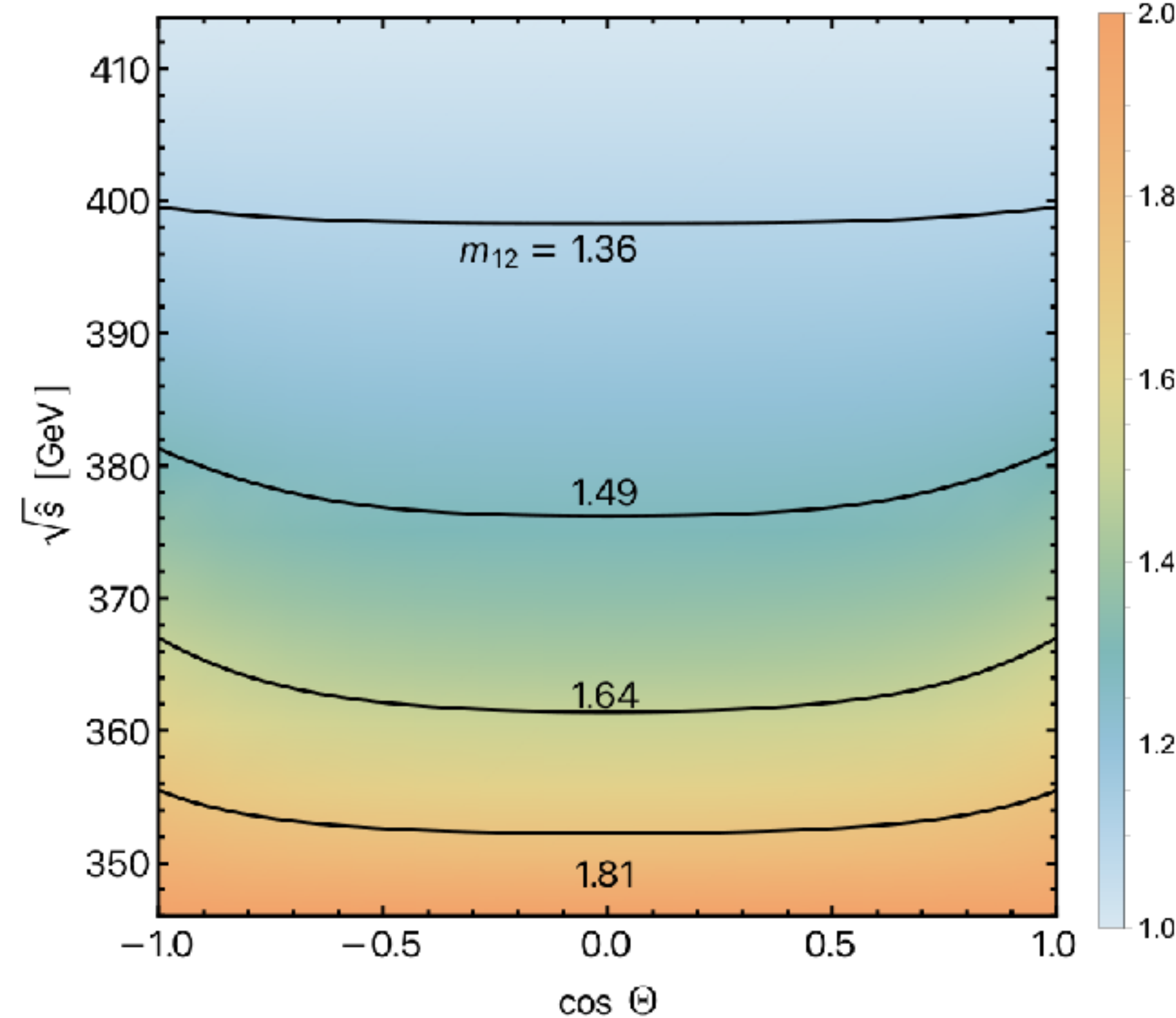
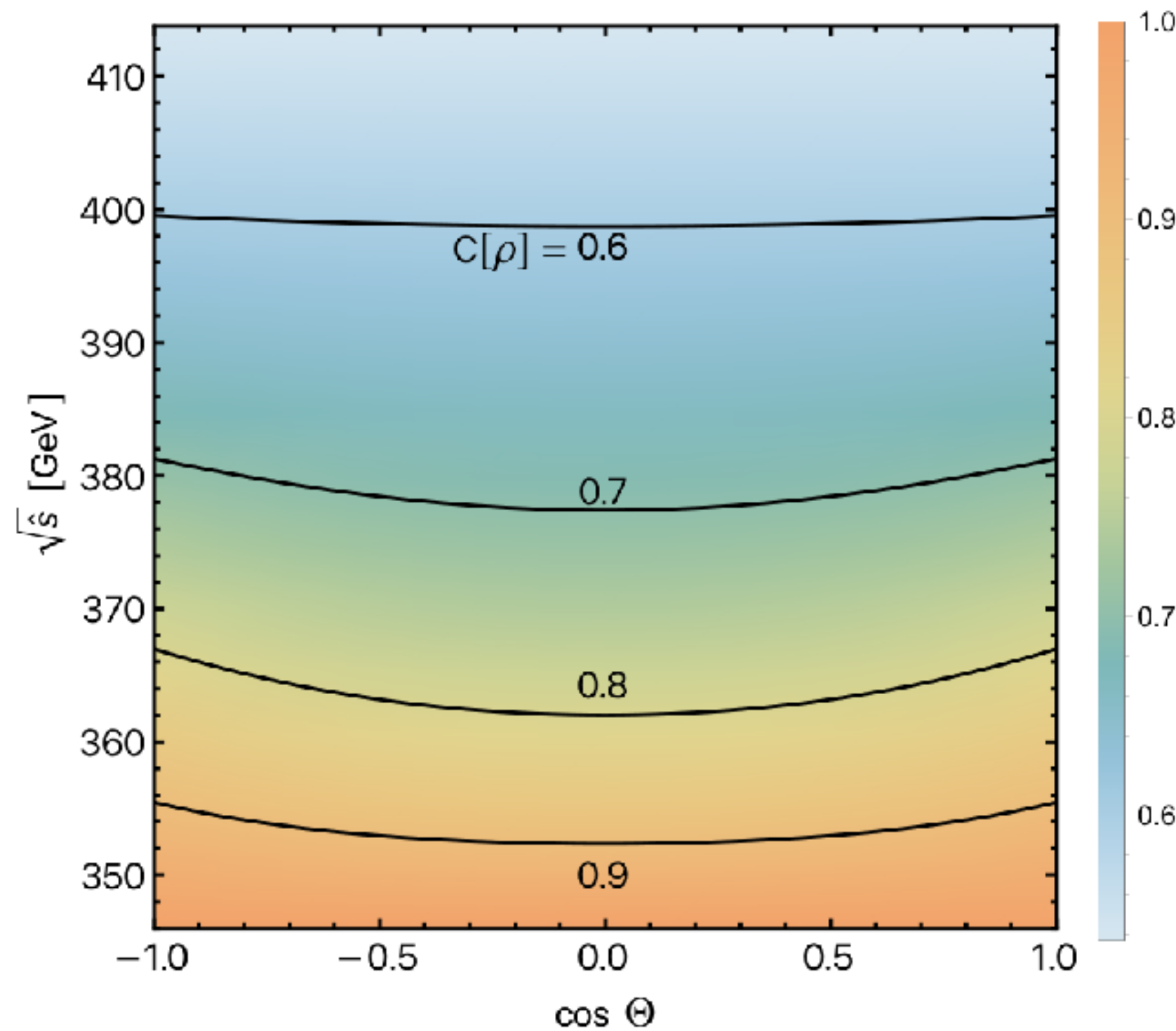


Top quark pairs @ Photon Collider

- Polarized colliding photons, $\sqrt{\hat{s}} = 500 \text{ GeV}$

$$C[\rho] \approx \frac{1 - \beta^2}{1 + \beta^2} > 0$$

$$m_{12} \approx 1 + \frac{(1 - \beta^2)^2}{(1 + \beta^2)^2} > 1$$

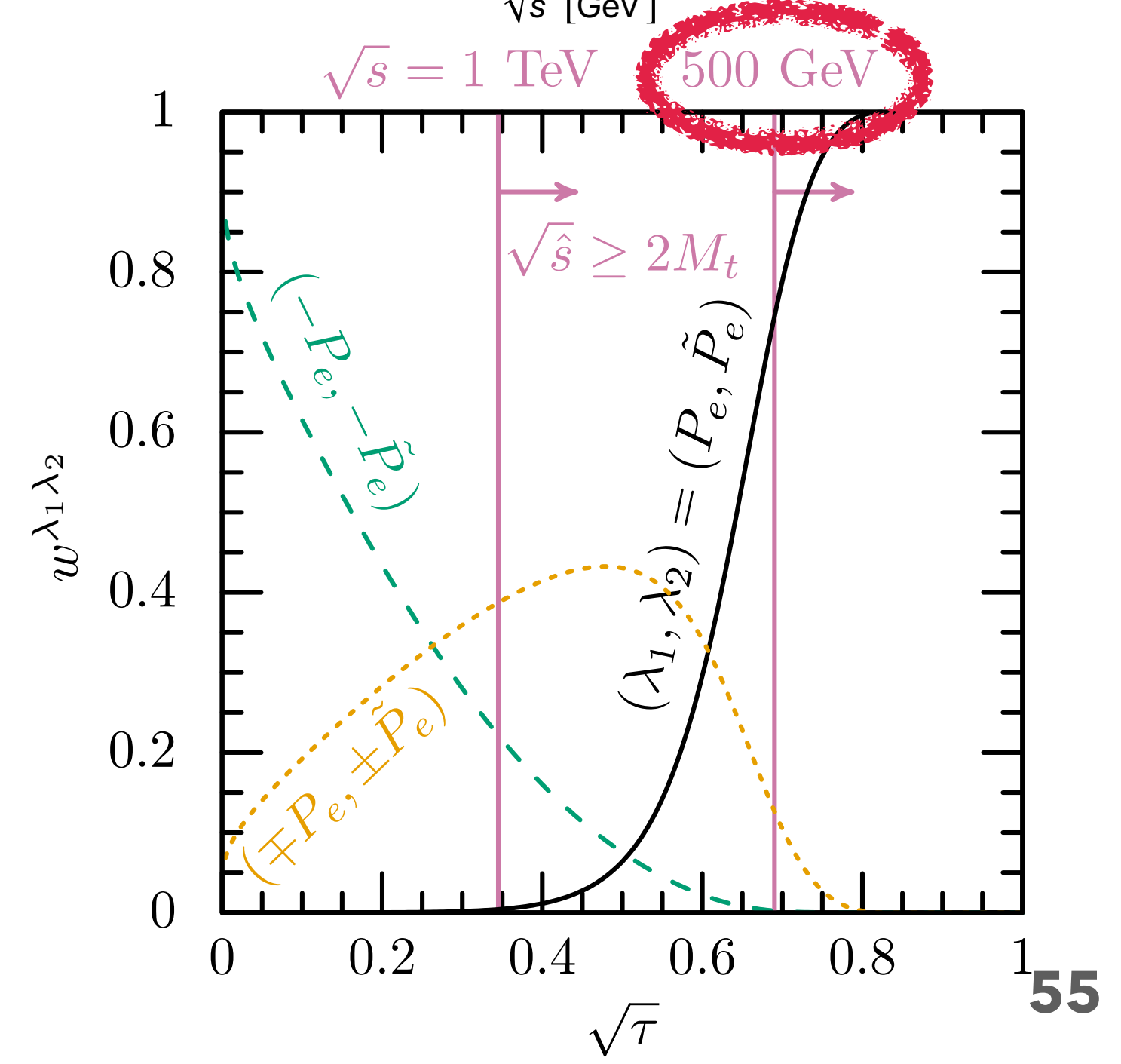
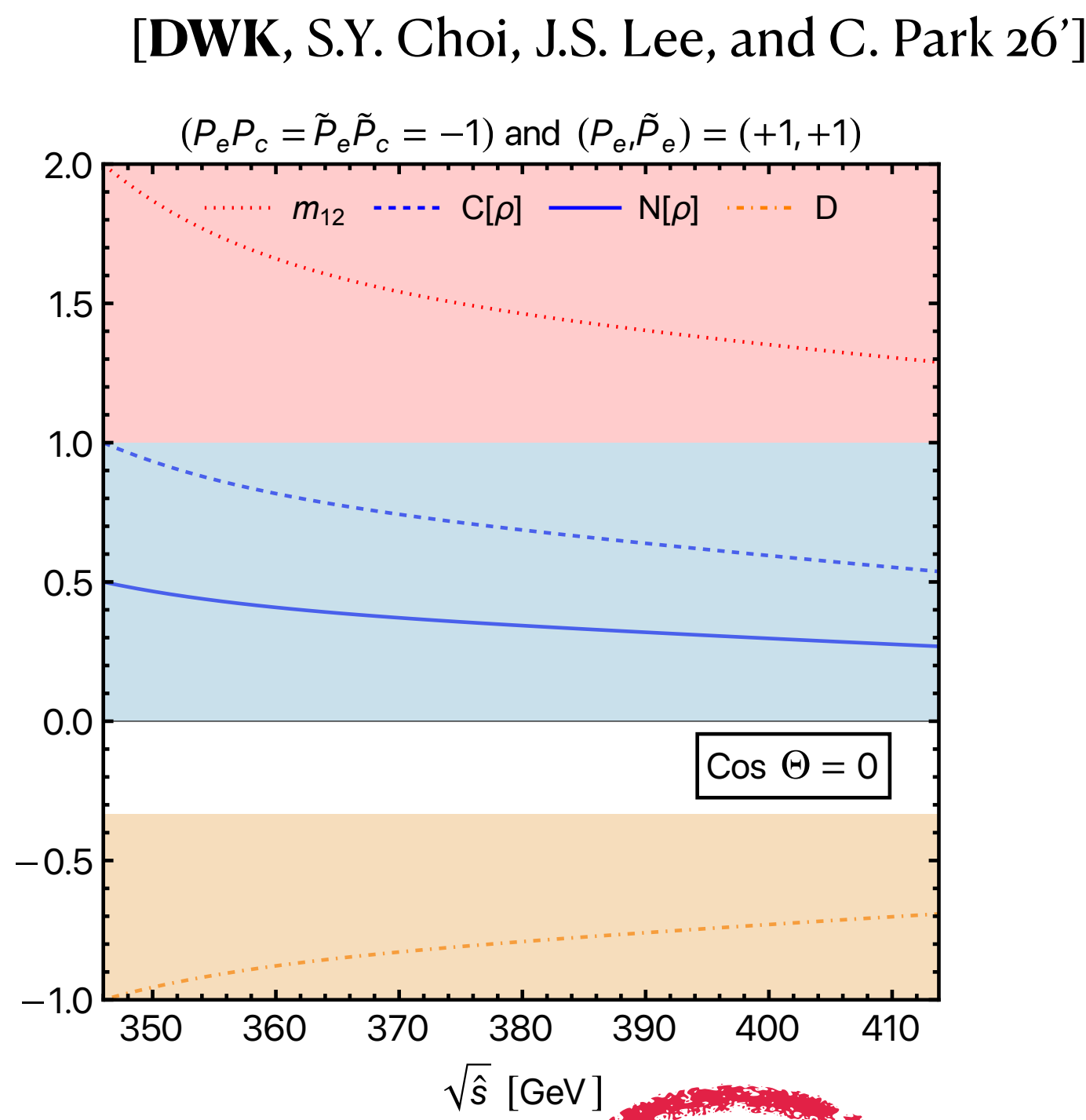
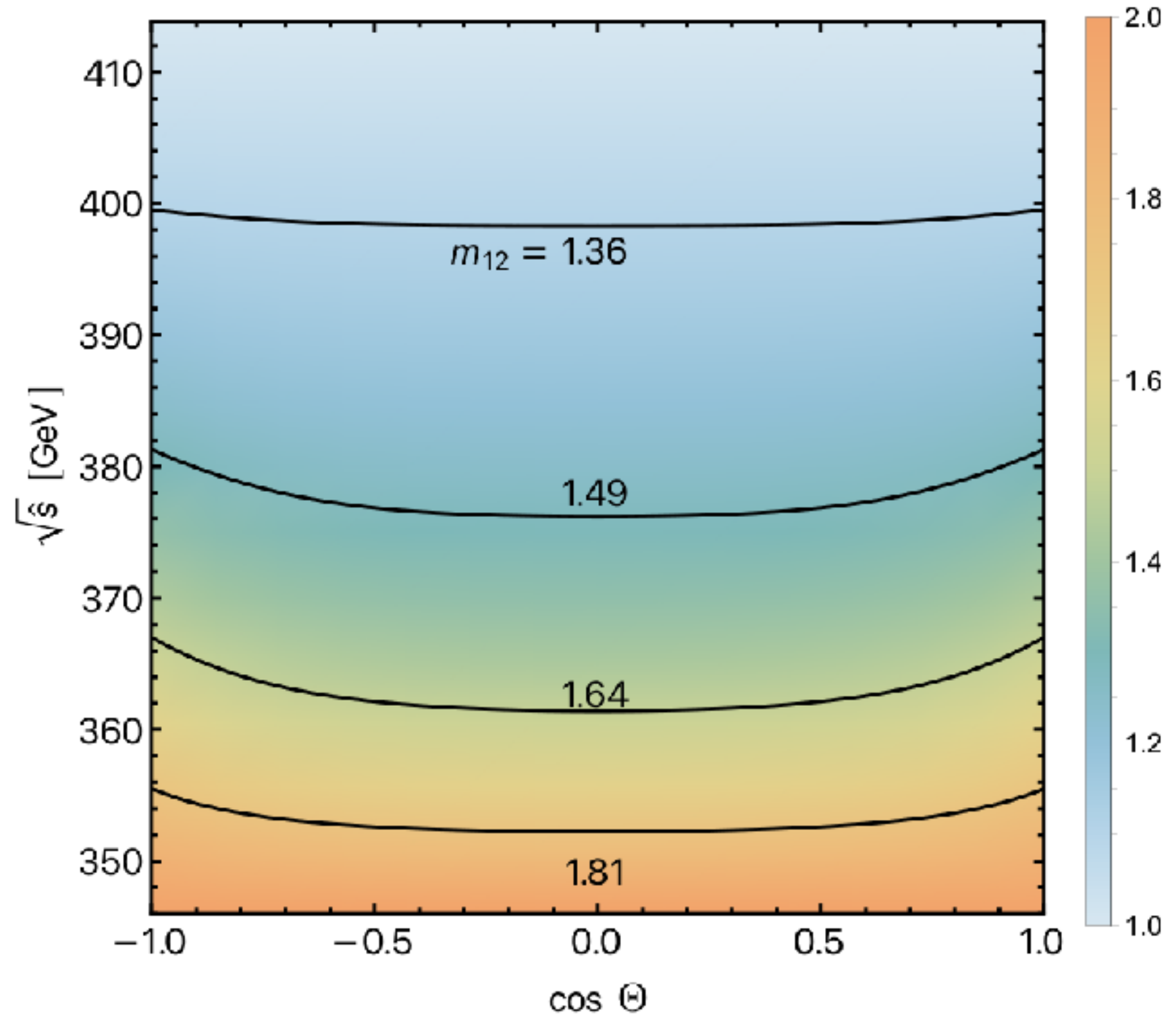
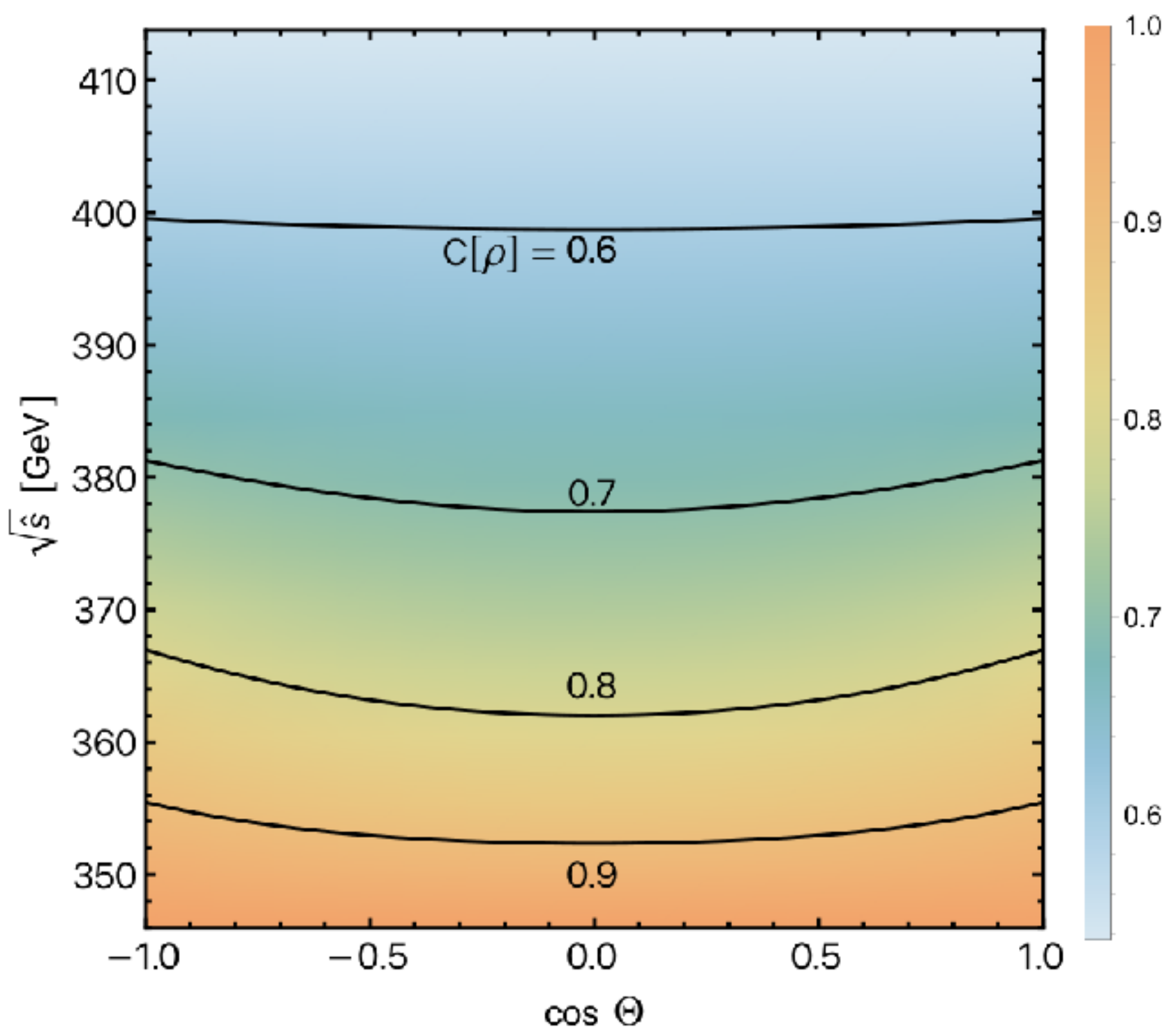


Top quark pairs @ Photon Collider

- Polarized colliding photons, $\sqrt{\hat{s}} = 500 \text{ GeV}$

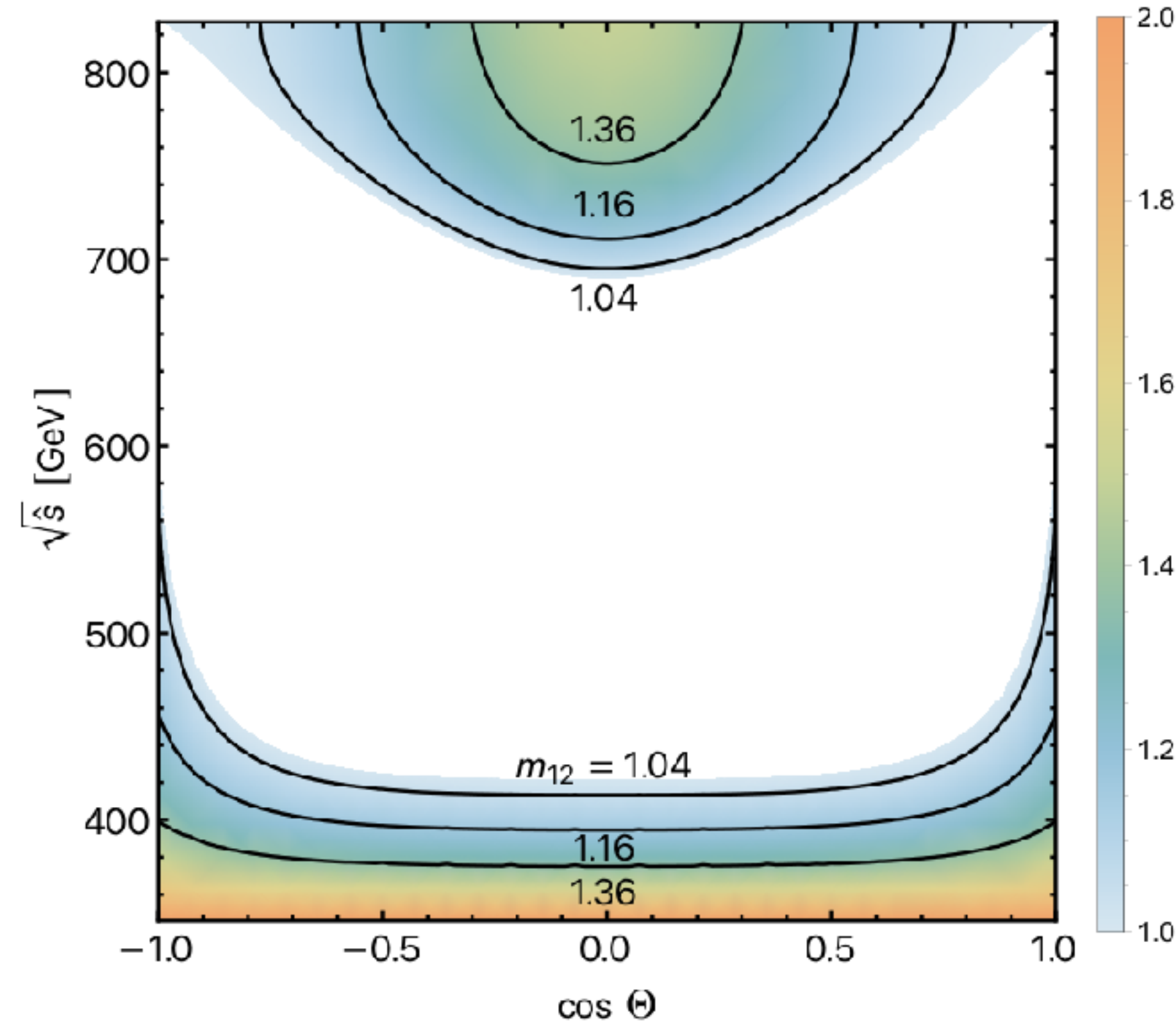
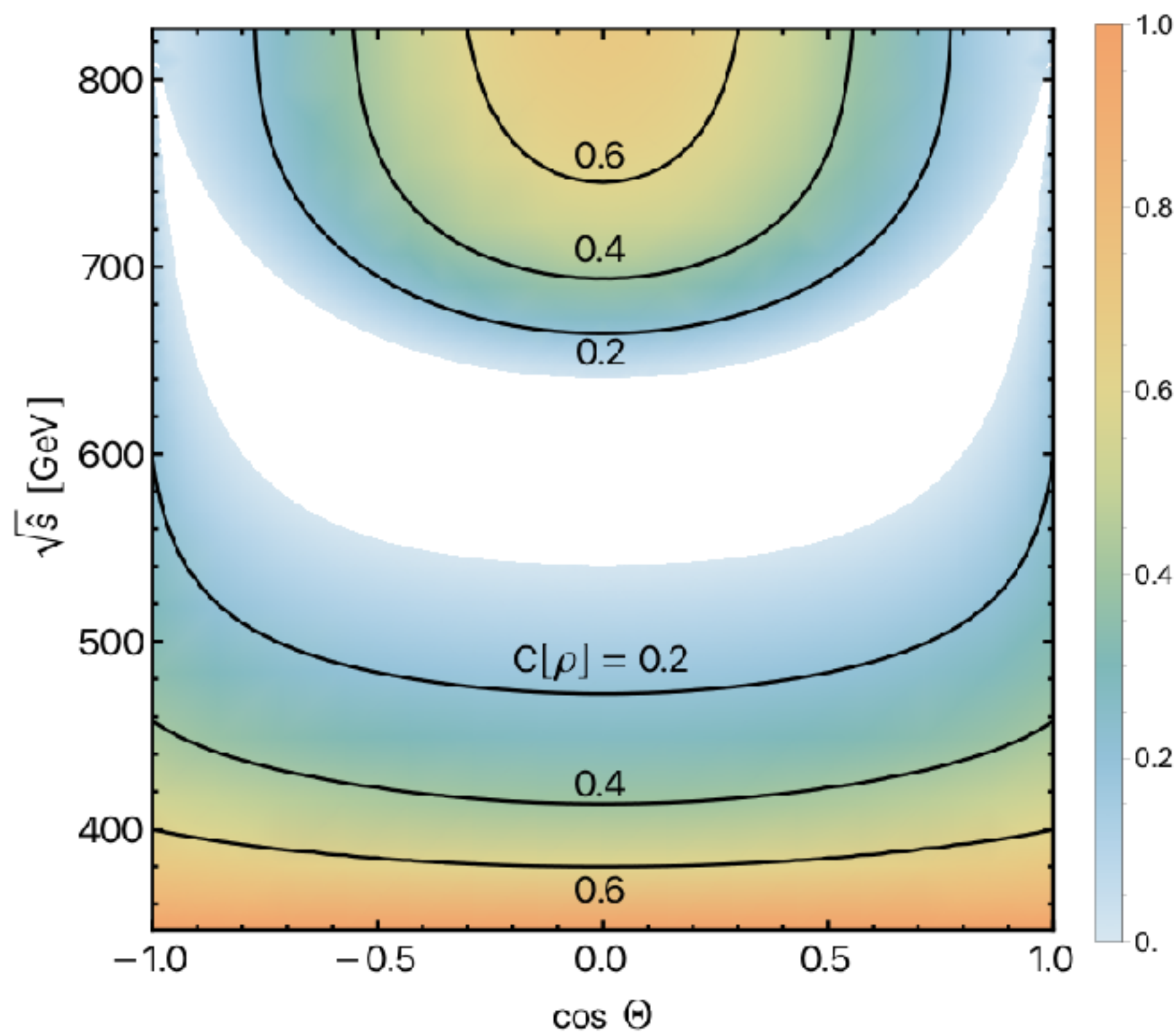
$$C[\rho] \approx \frac{1 - \beta^2}{1 + \beta^2} > 0$$

$$m_{12} \approx 1 + \frac{(1 - \beta^2)^2}{(1 + \beta^2)^2} > 1$$

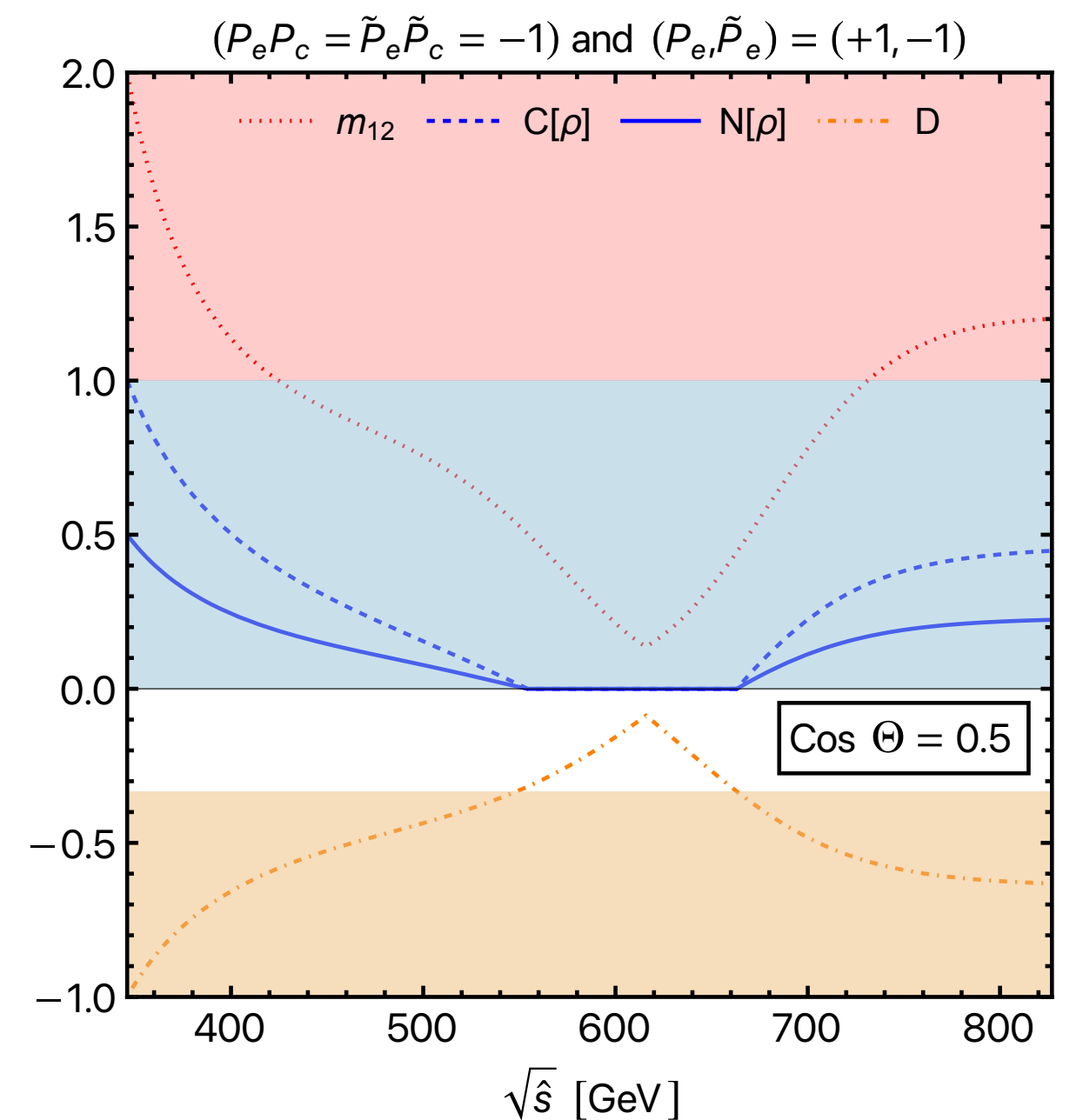
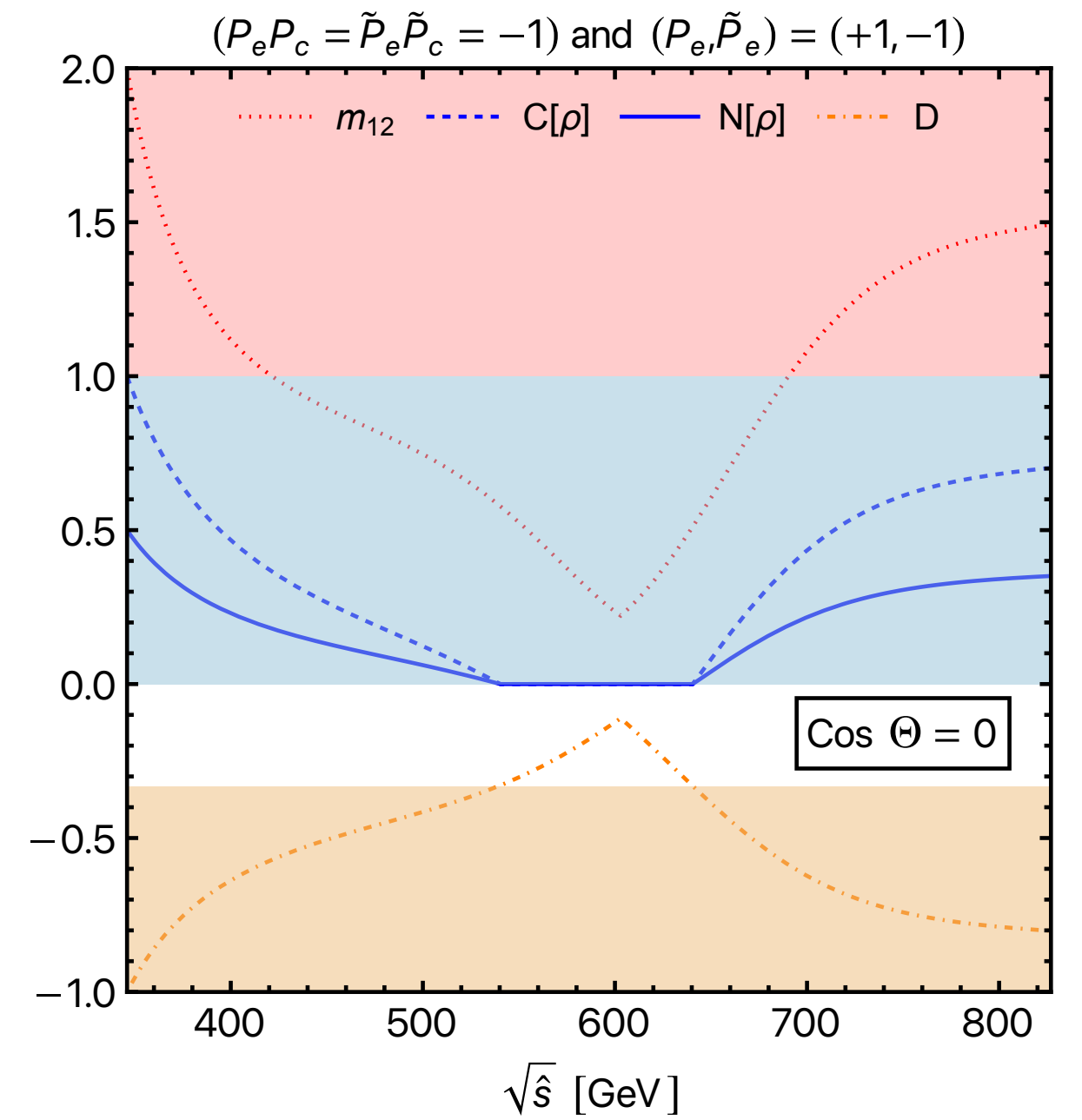


Top quark pairs @ Photon Collider

- Polarized colliding photons, $\sqrt{\hat{s}} = 1 \text{ TeV}$



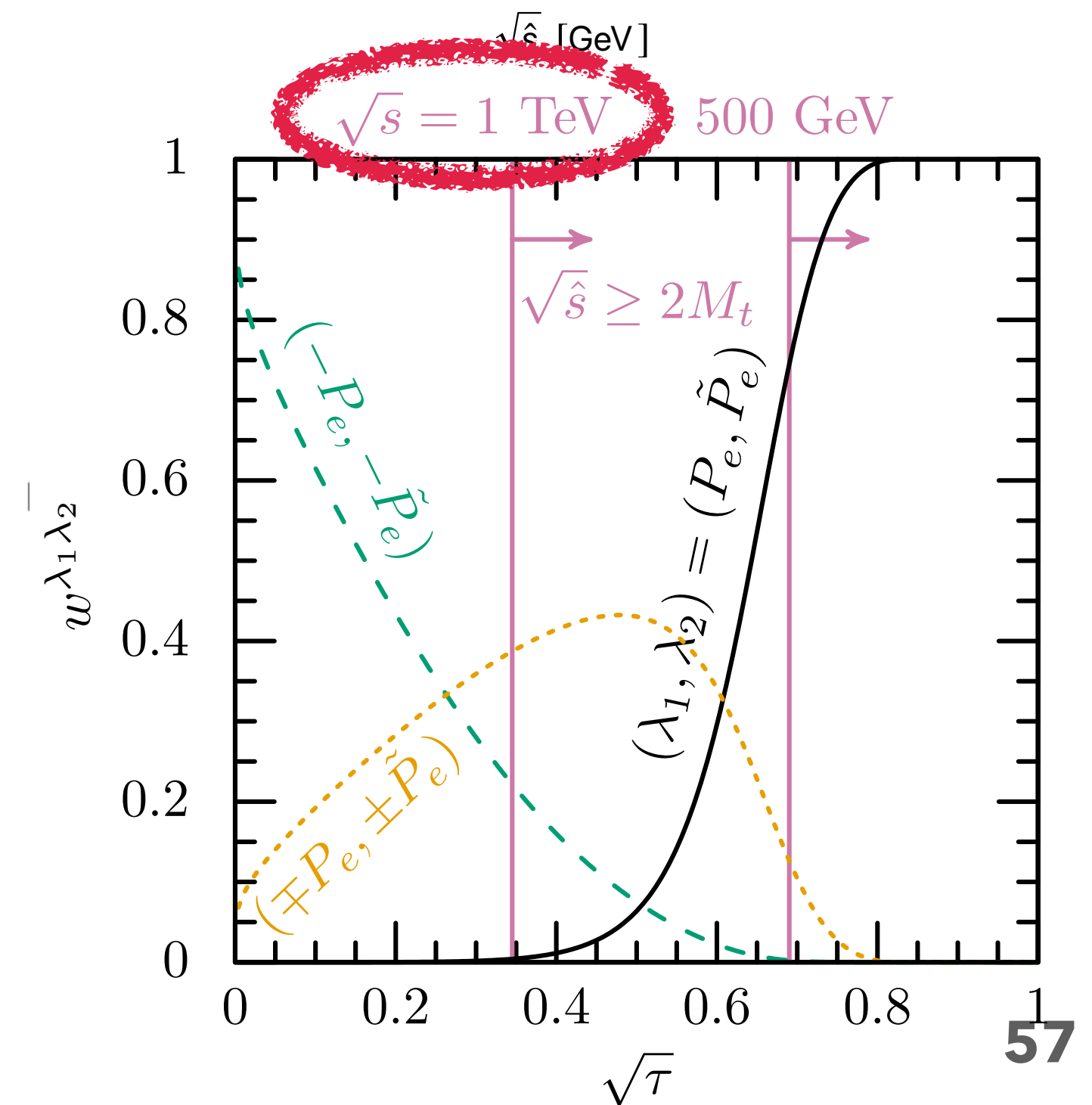
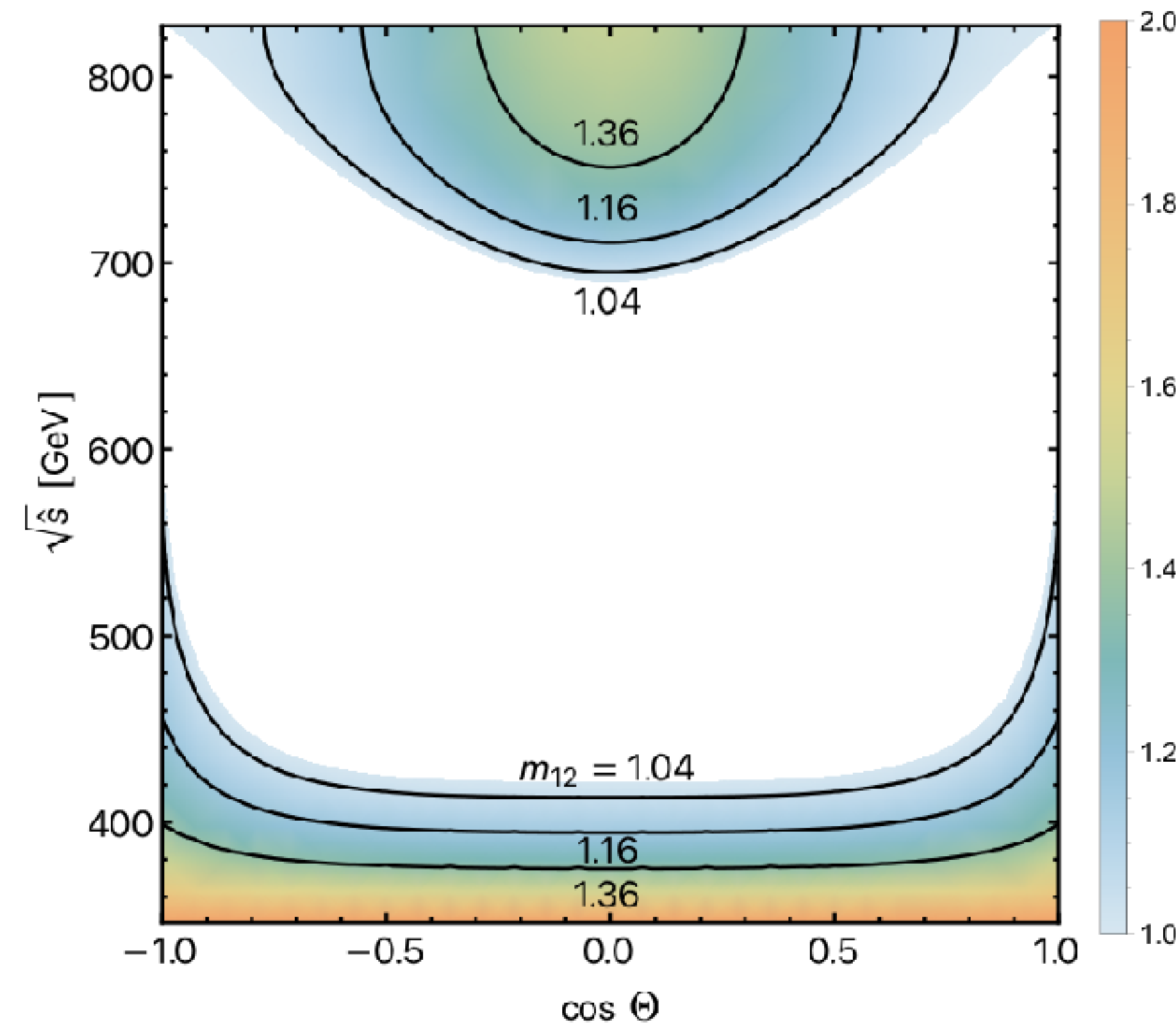
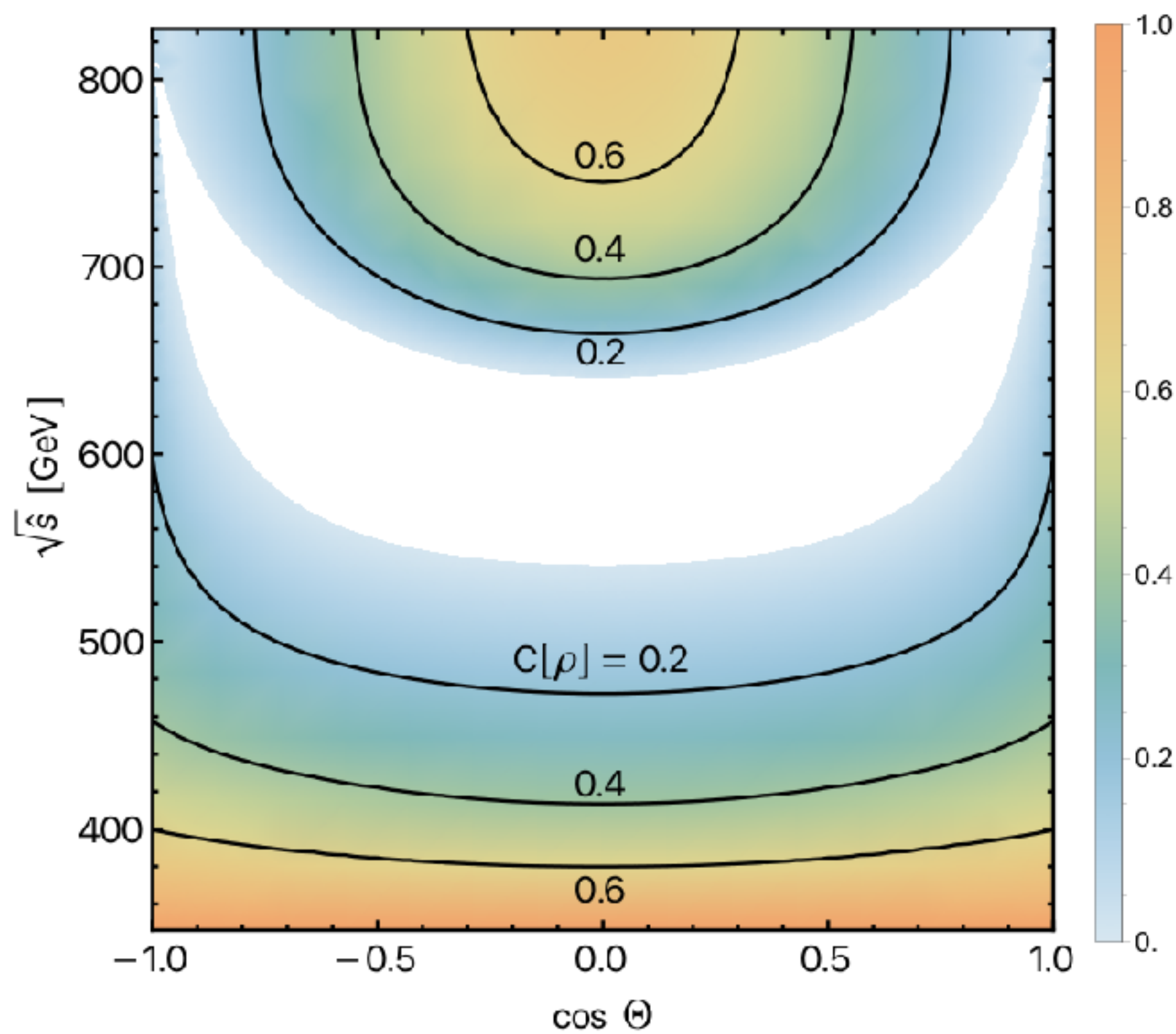
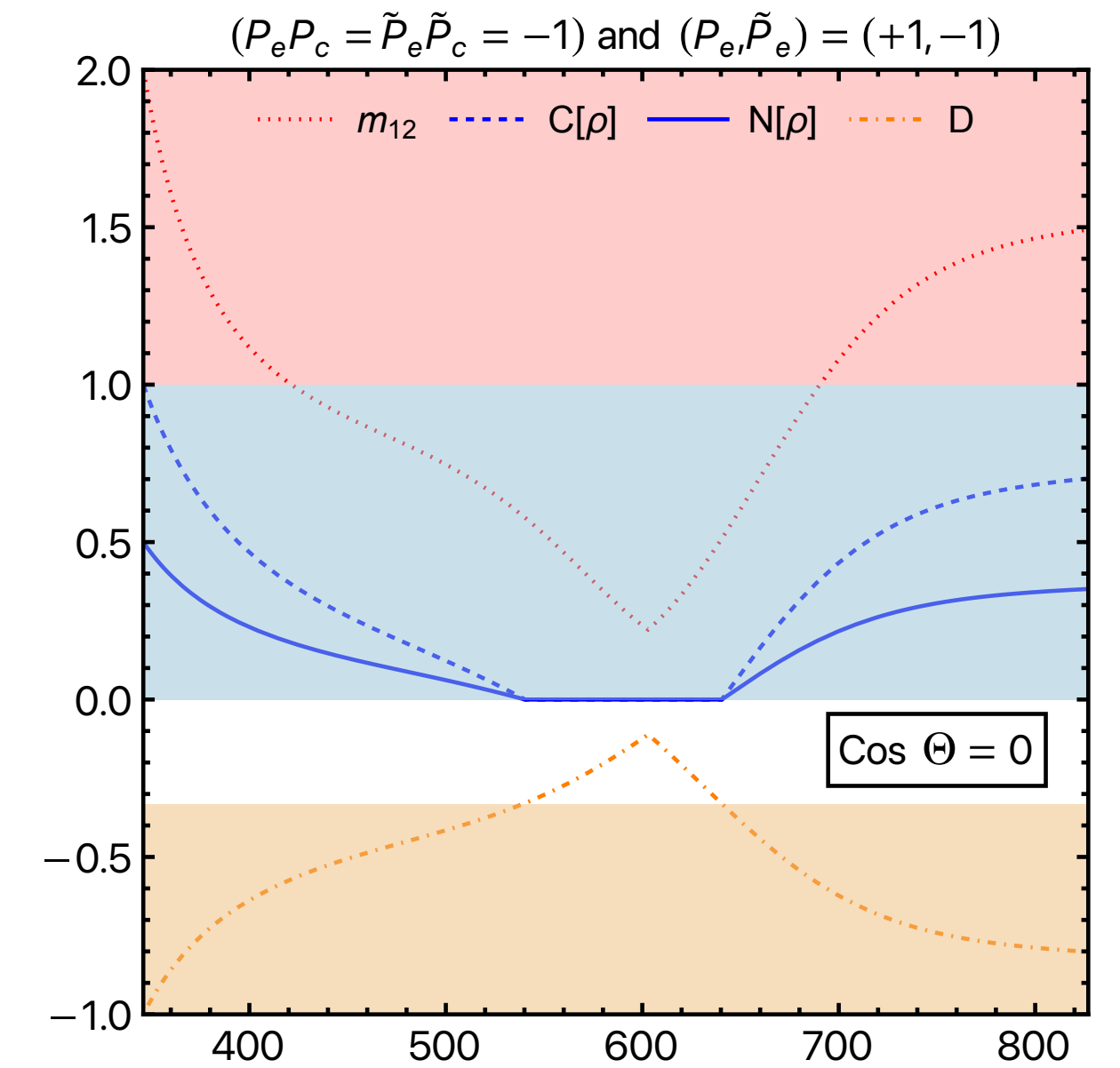
[DWK, S.Y. Choi, J.S. Lee, and C. Park 26']



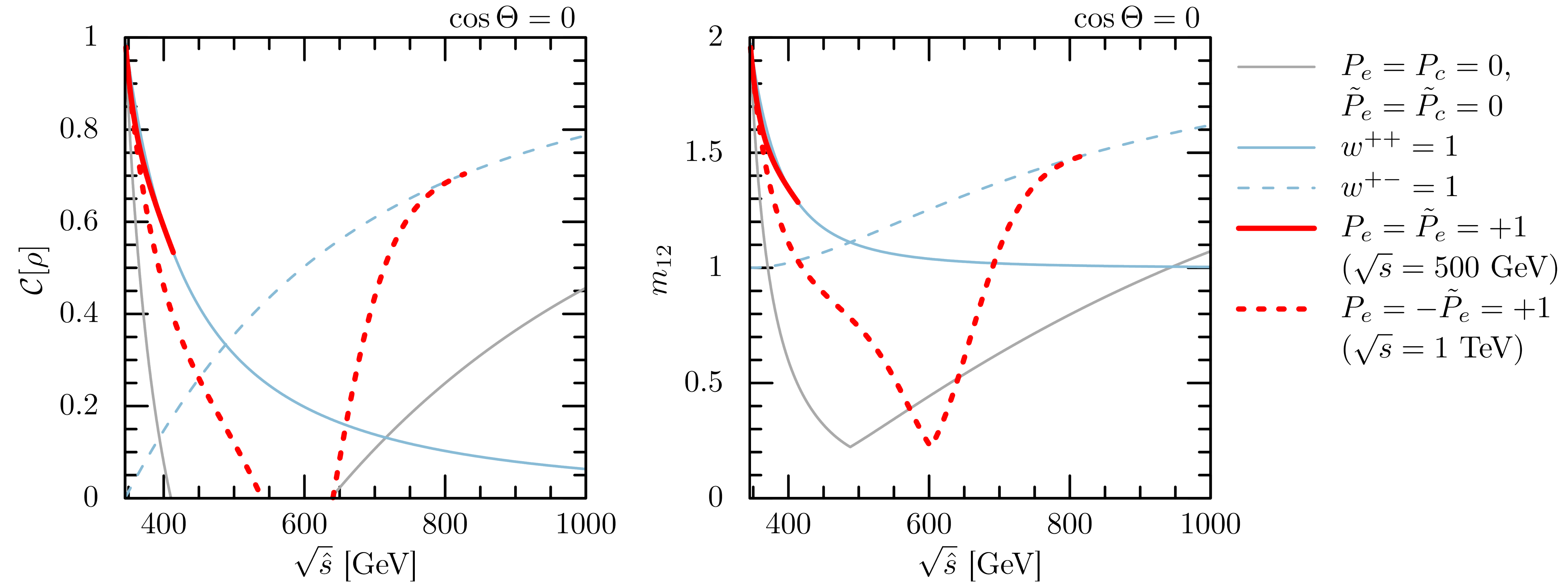
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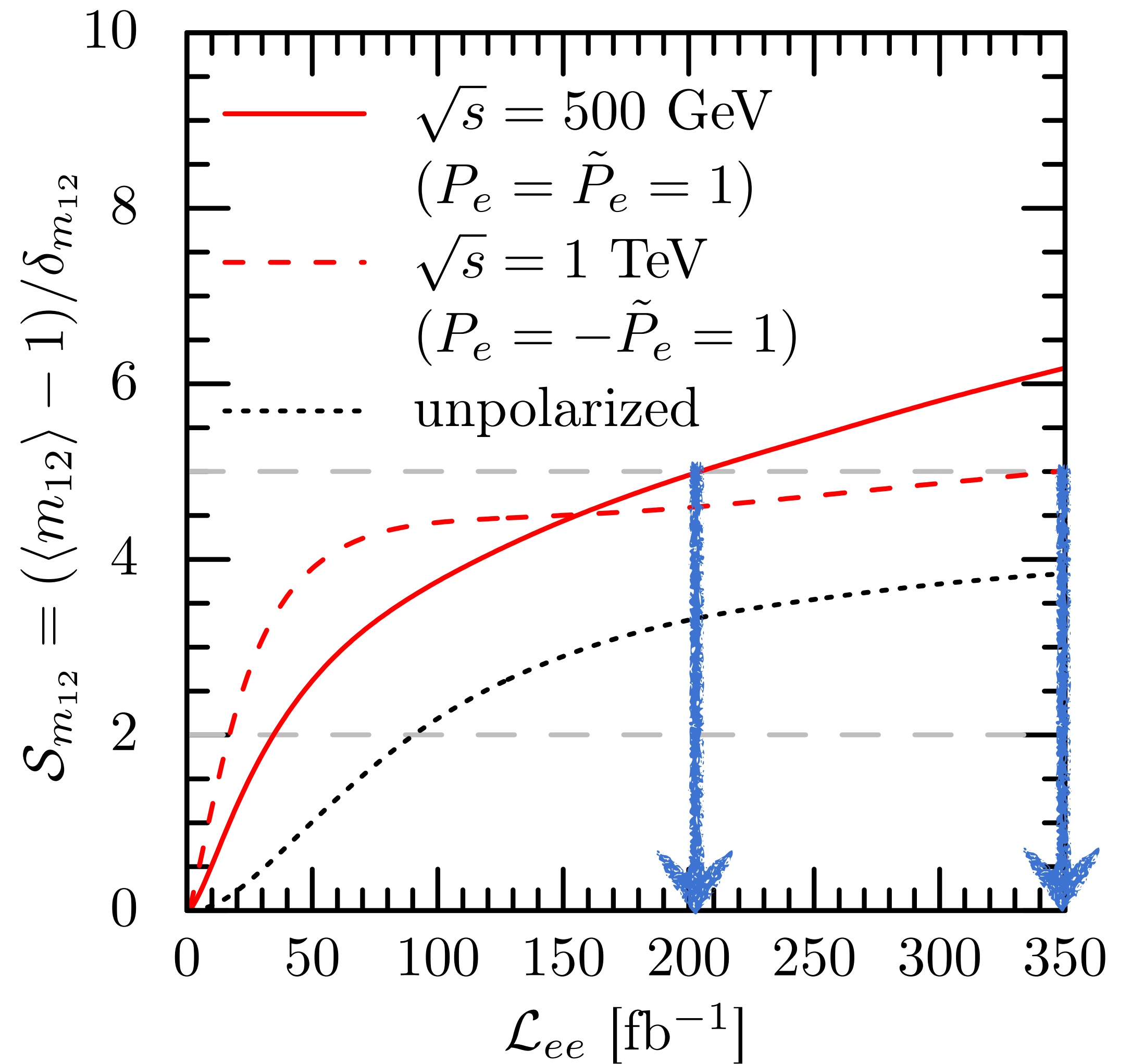
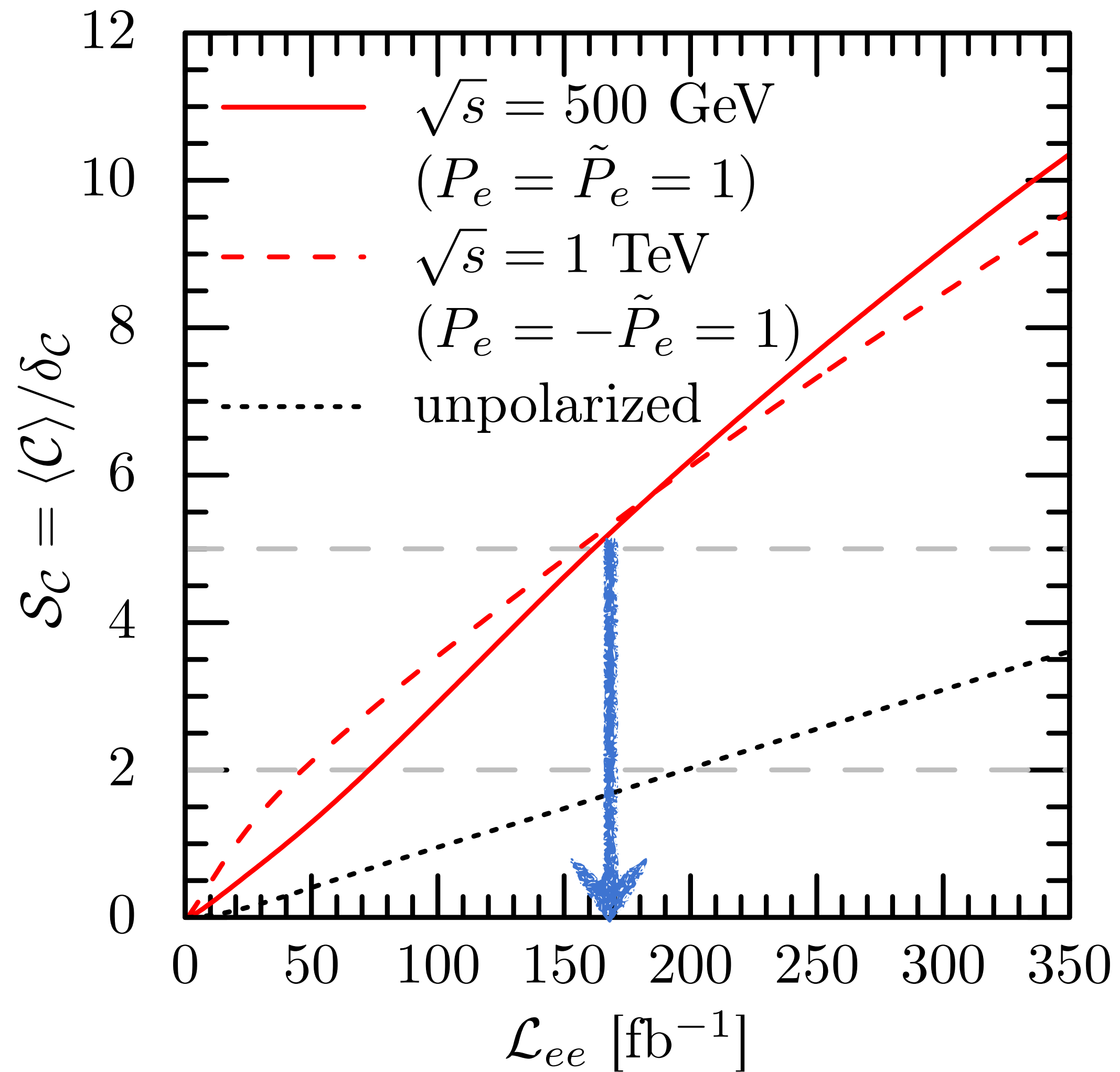
[DWK, S.Y. Choi, J.S. Lee, and C. Park 26']



Top quark pairs @ Photon Collider



Top quark pairs @ Photon Collider



Conclusion

- The **quantum observables**, constructed by **Polarization** and **Spin correlations** have become a powerful tools for BSM searches and precision measurements.
- We present a **general formalism** to construct **the spin density matrix** of a two qubit system at colliders from the **amplitude level**, taking account of **P**, **CP**, and **CPT** properties.

Polarization coefficient	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$	$\hat{C}_6$	$\hat{C}_7$	$\hat{C}_8$
[P, CP, CPT] Parities	[+ + +]	[- - -]	[+ + +]	[- + +]	[- + +]	[+ + -]	[- - -]	[+ - +]
Physical quantity	$\frac{\hat{A} - \hat{C}_{kk}}{2}$	$\frac{\hat{B}_k^+ - \hat{B}_k^-}{2}$	$\frac{\hat{A} + \hat{C}_{kk}}{2}$	$\frac{\hat{B}_k^+ + \hat{B}_k^-}{2}$	$-\frac{\hat{B}_r^+ + \hat{B}_r^-}{2}$	$\frac{\hat{B}_n^+ + \hat{B}_n^-}{2}$	$-\frac{\hat{B}_r^+ - \hat{B}_r^-}{2}$	$\frac{\hat{B}_n^+ - \hat{B}_n^-}{2}$
Polarization coefficient	$\hat{C}_9$	$\hat{C}_{10}$	$\hat{C}_{11}$	$\hat{C}_{12}$	$\hat{C}_{13}$	$\hat{C}_{14}$	$\hat{C}_{15}$	$\hat{C}_{16}$
[P, CP, CPT] Parities	[+ - -]	[- - +]	[+ + +]	[- + -]	[+ + +]	[- - +]	[+ + +]	[- + -]
Physical quantity	$\frac{\hat{C}_{rk} - \hat{C}_{kr}}{2}$	$\frac{\hat{C}_{kn} - \hat{C}_{nk}}{2}$	$\frac{\hat{C}_{rk} + \hat{C}_{kr}}{2}$	$-\frac{\hat{C}_{kn} + \hat{C}_{nk}}{2}$	$\frac{\hat{C}_{nn} + \hat{C}_{rr}}{2}$	$\frac{\hat{C}_{rn} - \hat{C}_{nr}}{2}$	$\frac{\hat{C}_{rr} - \hat{C}_{nn}}{2}$	$-\frac{\hat{C}_{rn} + \hat{C}_{nr}}{2}$

Conclusion

- The **quantum observables**, constructed by **Polarization** and **Spin correlations** have become a powerful tools for BSM searches and precision measurements.
- We present a **general formalism** to construct **the spin density matrix** of a two qubit system at colliders from the **amplitude level**, taking account of **P**, **CP**, and **CPT** properties.
- We show that a Photon collider with polarization control, we could also check **Bell's inequality violation** at high energy collider!
- $\tau^+\tau^-$, ZZ , W^+W^- , ... also suitable subject to study further.