Revisiting QCD-induced little inflation scenario

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arXiv: 2603. 29772

Outline

1. Introduction

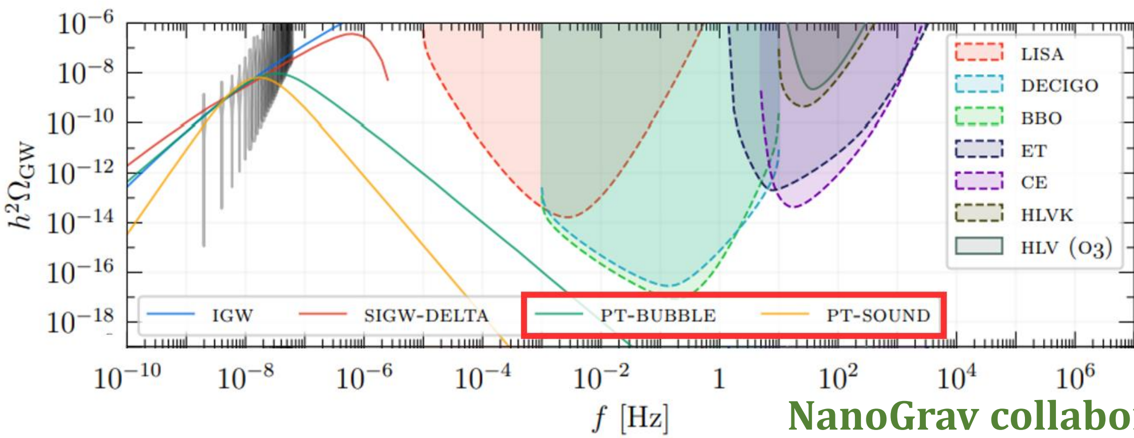
2. QCD-induced little inflation scenario

3. Chiral density wave phase

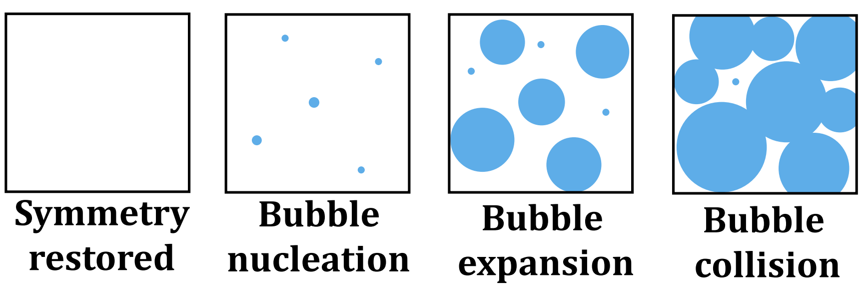
4. Summary and conclusion

Introduction

- Pulsar timing arrays reported strong evidence for a stochastic gravitational wave.



NanoGrav collaboration, 2306.16219



$\alpha > 1$

$\beta H^{-1} \lesssim 10$

$T_* \sim 10^2$ MeV

⇒ Can the QCD phase transition explain the PTA data?

Introduction

- In the standard cosmology, the QCD phase transition is crossover.
- **No bubbles → no GW signal**

$$\left(Y_B^{\rm obs} \sim 10^{-10} \Rightarrow \frac{\mu_B}{T} \sim 10^{-8} \text{ at high } T\right)$$

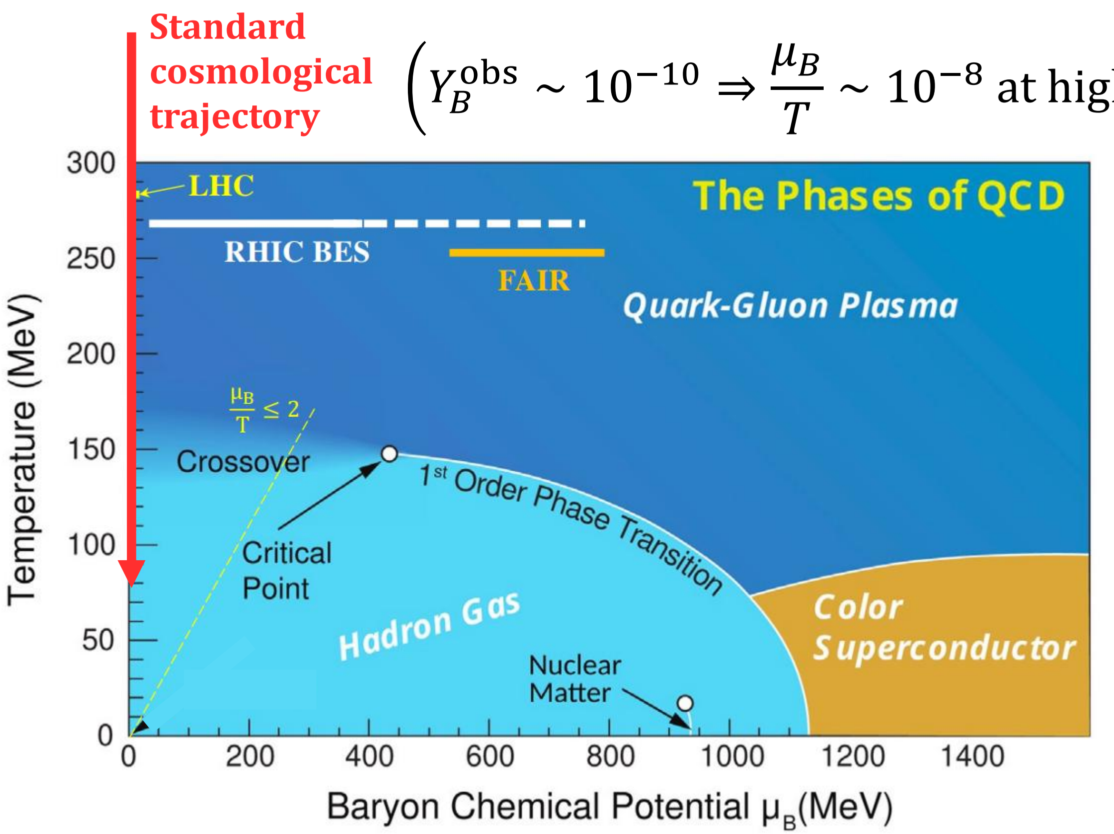
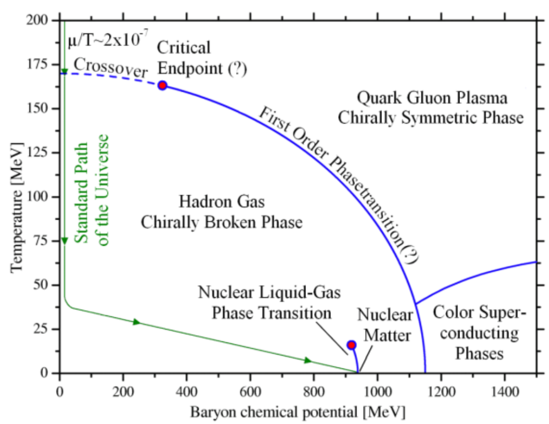


Fig. from 2303.02579

Introduction

- Different trajectory?
  - ✓ **Large baryon asymmetry** at high T
    - → strong supercooling through QCD PT
    - → current Universe with small baryon asymmetry

Kämpfer (1986)
Ng, Sze (1991)
Borghini et al. hep-ph/0001284
Boeckel, Schaffner-Bielich 0906.4520
Boeckel, Schaffner-Bielich 1105.0832



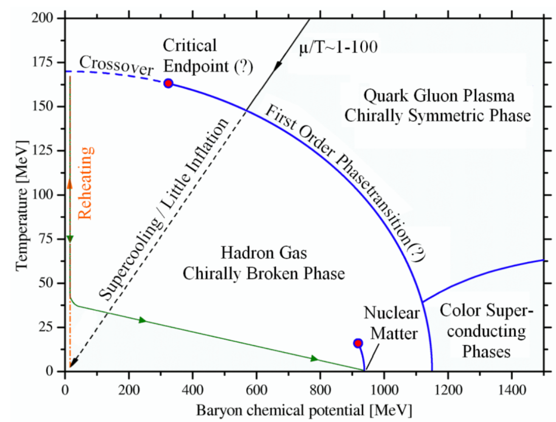


Fig. from
1105.0832

- ✓ Large lepton asymmetry
  - → chemical balance → large quark density
  - → first-order PT

not discussed
in this talk

Schwarz, Stuke, 0906.3434
Caprini et al. 1007.1218
Wygas et al. 1807.10815
Gao, Oldengott, 2106.11991

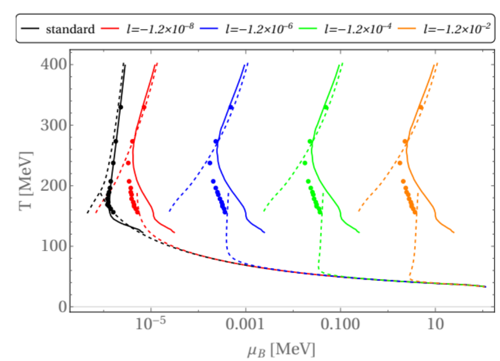


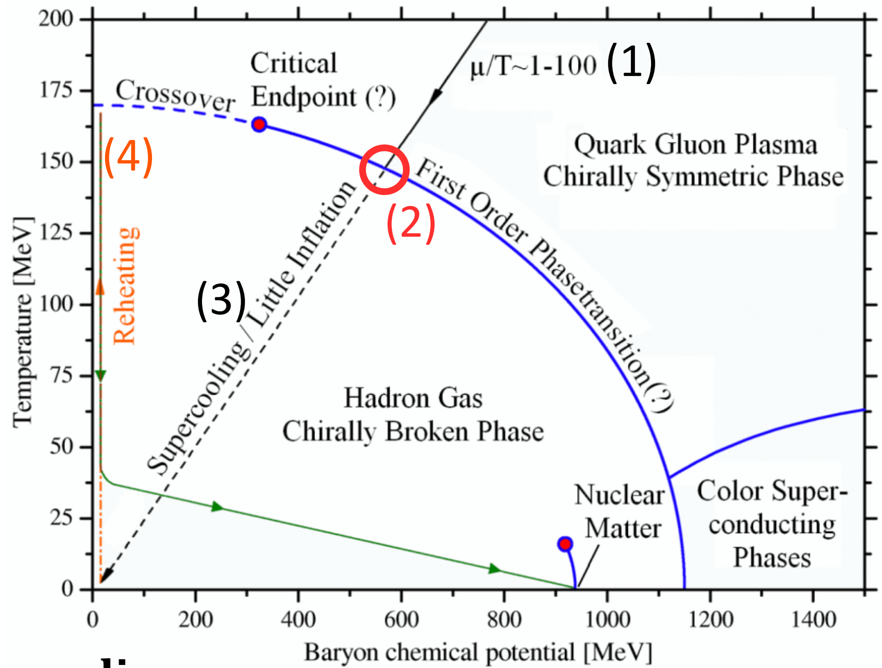
Fig. from
2106.11991

QCD-induced little inflation

- **The scenario**

(1) Start with a large baryon asymmetry generated at a high temperature.

(2) The QCD phase transition becomes first-order.



(3) The Universe undergoes a **strong supercooling**; **little inflation** is driven by the QCD vacuum energy domination.

(4) After the phase transition, latent heat is released (entropy is produced), and the baryon yield becomes the value observed today, $Y_B \sim 10^{-10}$.

QCD-induced little inflation

- **Non-trivial requirements**

  (1) Large baryon asymmetry generation

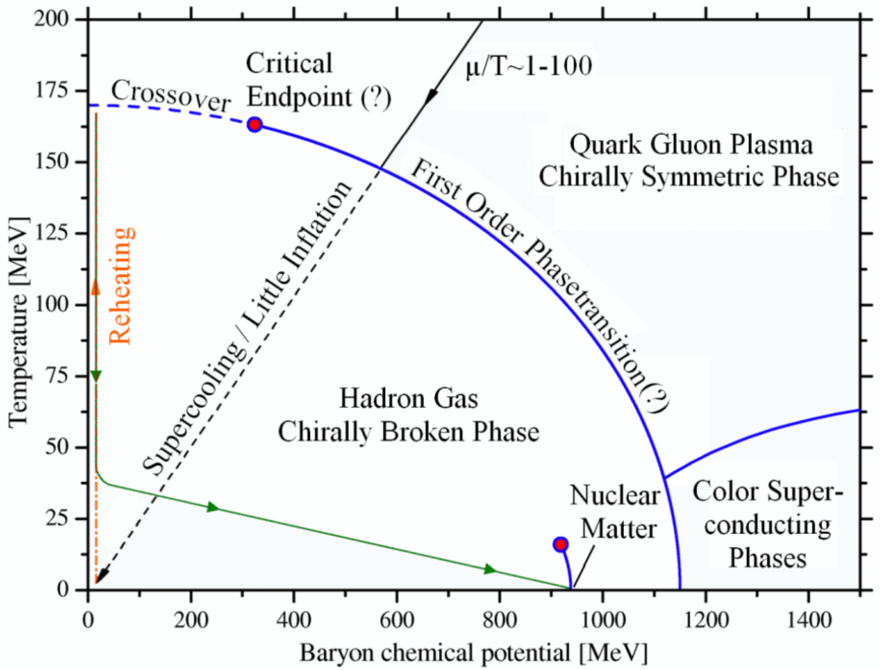
  $$Y_{B,\text{initial}} > O(10^{-2})$$

  (2) Supercooling must be strong enough

  to dilute away the baryon asymmetry

  $$Y_{B,\text{final}} \sim 10^{-10}$$

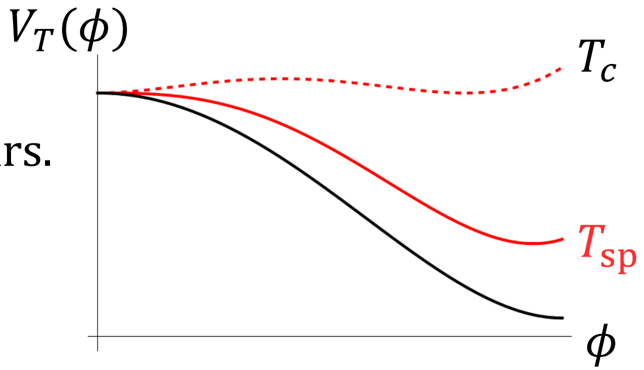
  $$(\mu_{B,\text{end}} \sim 1 \text{ MeV})$$



  (3) To explain the PTA signal, the transition has to be slow (i.e., $\beta/H < 10$).

- Each requirement is highly non-trivial to be realized, but let us focus on the point 2; whether QCD can provide such a strong supercooling, or not.

Spinodal temperature



- **The spinodal temperature ($T_{\rm sp}$)**: the temperature below which potential barrier disappears.

- $T_{\rm sp} = 0 \;\rightarrow\;$ the potential barrier lasts for any nonzero T, and the potential barrier size $\sim T$
  $\Rightarrow$ slow phase transition is possible $\rightarrow$ small β/H (good for the PTA signal)
  e.g. classically conformal models; may be realized in the gluon condensation
  Campbell, Ellis, Olive (1990)
  (model taken in Boeckel, Schaffner-Bielich)

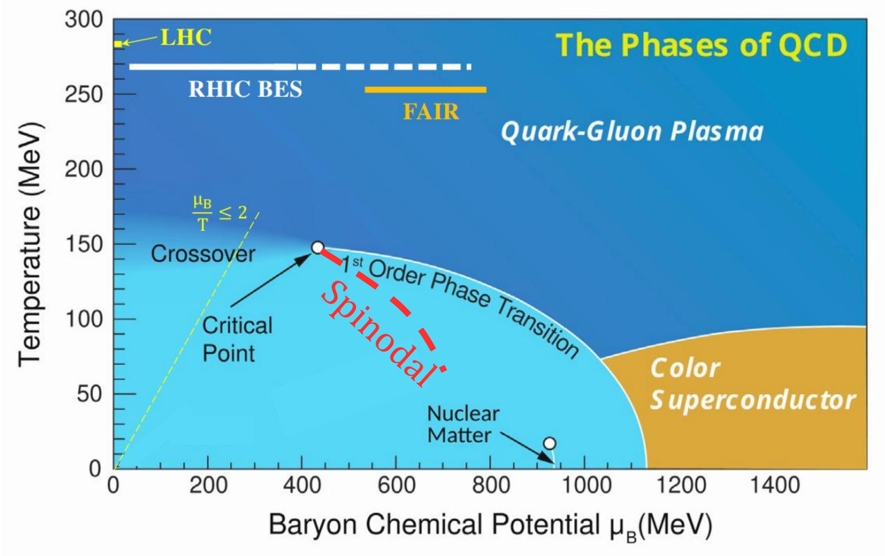
- $T_{\rm sp} \neq 0 \rightarrow$ potential barrier disappears at $T < T_{\rm sp}$
  $\Rightarrow$ The symmetric phase becomes unstable.
  Supercooling cannot be maintained below $T_{\rm sp}$

- No $T_{\rm sp} \;\rightarrow$ Potential barrier exists even at zero T.

Which case does QCD belong to?

Critical end point (CEP)

- In the QCD phase diagram, there is a critical end point (CEP).
- At CEP, the spinodal and critical lines should merge.



- Exact location of CEP is still unknown, but there are some bounds/evidence

$$\left(\frac{\mu_B}{T}\right)_{\text{CEP}} \gtrsim 2\,, \qquad (\mu_B)_{\text{CEP}} \gtrsim 450\ \text{MeV}, \qquad (\mu_B)_{\text{CEP}} \simeq 625\ \text{MeV}$$

Giordano et al. 2004.07066

Borsanyi et al. 2502.10267
Fraga et al. 1104.3755

Sorensen et al. 2405.10278

Original scenario (QGP → hadron phase) is not allowed in SM! (our conclusion 1)

Chiral density wave phase

- Any other QCD phase transition?
  - ✓ There is a phase with **inhomogeneous** chiral condensate at $\frac{\mu_B}{T} \gg 1$.
  - ✓ It is called **chiral density wave**, and it involves condensation of fermion-hole pairing near the Fermi surface, instead of fermion-antifermion pairing.

    → momentum scale $\simeq 2k_F$

    → modulation of chiral condensation along a direction chosen spontaneously

    → breaks rotational and translational symmetries
  - ✓ may be good because CDW phase can be already hadronized, and the zero baryon density is realized around $\mu_0 = 922$ MeV, not zero.
  - ✓ No proof in QCD, but many theoretical arguments in the large N_c limit.

    Deryagin, Grigoriev, Rubakov (1992)
    Shuster, Son, hep-ph/9905448
    Park et al., hep-ph/9910347
    Buballa, Carignano, 1406.1367
  - ✓ No first-principles framework (sign problem)

    → We take nucleon-meson model with isoscalar vector meson.

    Pitsinigkos and Schmitt, 2309.01603

Nucleon-meson model

Pitsinigkos and Schmitt, 2309.01603

- Lagrangian of the model is given by

$$\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm mes} + \mathcal{L}_{\rm int}$$

with

$$\mathcal{L}_{\rm bar} = \bar{\psi}(i\gamma^\mu \partial_\mu + \gamma^0 \mu_\psi)\psi, \qquad \psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \quad \mu_\psi = \begin{pmatrix} \mu_p & 0 \\ 0 & \mu_n \end{pmatrix} = \begin{pmatrix} \mu_B & 0 \\ 0 & \mu_B \end{pmatrix}$$

$$\mathcal{L}_{\rm mes} = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{4}{\rm Tr}\left[\partial_\mu \pi \partial^\mu \pi\right] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu + \frac{d}{4}(\omega_\mu\omega^\mu)^2 - \mathcal{U}(\sigma,\pi),$$

$$\left(\mathcal{U}(\sigma,\pi) = \sum_{n=1}^{4} \frac{a_n}{n!} \frac{(\sigma^2 + \pi_a\pi_a - f_\pi^2)^n}{2^n} - \epsilon\,(\sigma - f_\pi) \right)$$

$$\mathcal{L}_{\rm int} = -\bar{\psi}\left[g_\sigma(\sigma + i\gamma^5\pi) + g_\omega \gamma^\mu \omega_\mu\right]\psi$$

ϵ: chiral symmetry breaking parameter ($= f_\pi m_\pi^2$)

- CDW background ansatz

$$\sigma = \phi\cos(2\vec{q}\cdot\vec{x}),$$
$$\pi_3 = \phi\sin(2\vec{q}\cdot\vec{x}),$$
$$\pi_1 = \pi_2 = 0,$$

Nucleon-meson model

- Taking $\psi \to e^{-i\gamma^5 \tau_3 \vec{q}\cdot\vec{x}}\psi$

- $\mathcal{L}_{\text{eff}} = \bar{\psi}\left(i\gamma_\mu \partial^\mu + \gamma^0 \mu_* - M + \gamma^5 \vec{q}\cdot\vec{\gamma}\,\tau_3\right)\psi + \frac{m_\omega^2}{2}\omega^2 + \frac{d}{4}\omega^4 - U - \Delta U$

  ✓ Nucleon dispersion $E_k^{\pm} = \sqrt{\left(\sqrt{k_\ell^2 + M^2} \pm q\right)^2 + k_\perp^2}$ (±: spin)

  increases the net matter pressure/entropy as $\sim M^2 \log(\frac{\mu_* + k_F}{M})\, q^2$.

  ✓ $U(\phi) = \sum_{n=1}^{4} \frac{a_n}{n!} \frac{(\phi^2 - f_\pi^2)^n}{2^n} - \epsilon(\phi - f_\pi)$

  $\Delta U(\phi, q) = 2\phi^2 q^2 + (1 - \delta_{q0})\epsilon\phi.$

competition!

from kinetic term

from $\cos(2\vec{q}\cdot\vec{x})$ averaged spatially along $\vec{q}$ direction

Nucleon-meson model

- Model parameters: $\epsilon,\ g_\sigma,\ g_\omega,\ d,\ a_1,\ a_2,\ a_3,\ a_4$

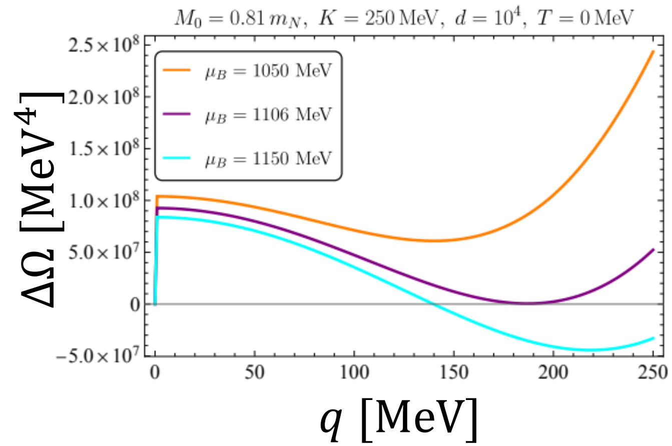
- Physical constraints: $f_\pi = 93$ MeV, $m_\pi = 139$ MeV, $m_N = 939$ MeV,
  $\mu_0 = 922.7$ MeV (saturation chemical potential),
  $n_0 = 0.153\ \mathrm{fm}^{-3}$(saturation density)
  $+$ consistency conditions

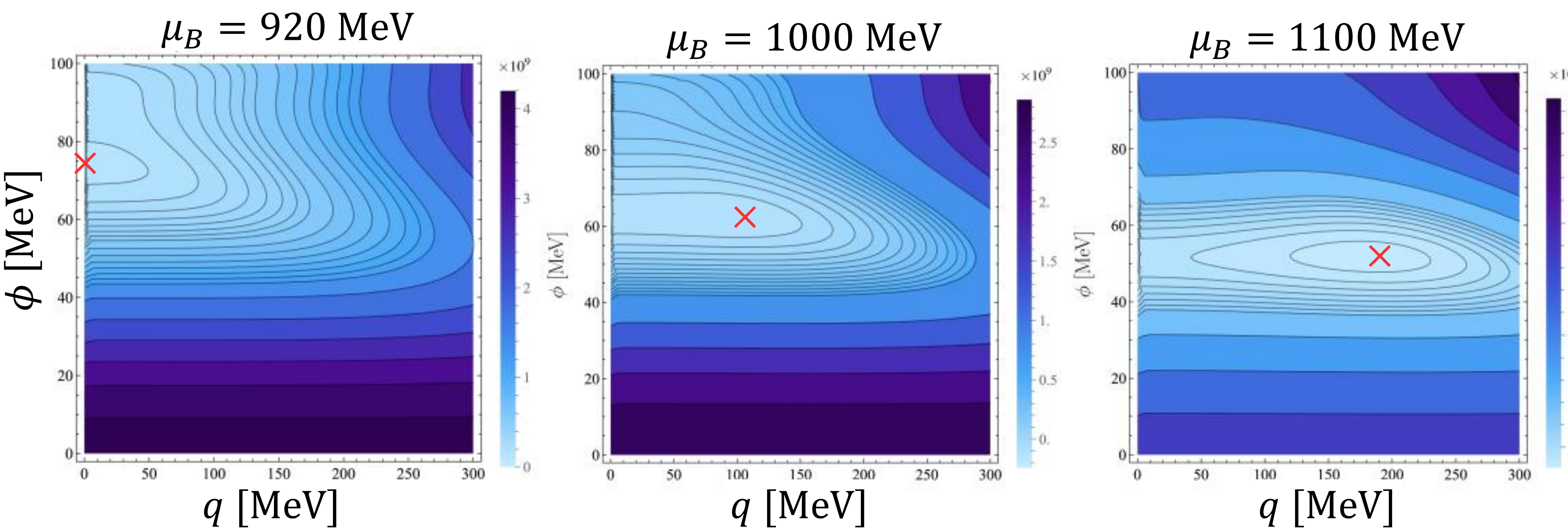
- Input (control) parameters:
  - ✓ M_0 (effective nucleon mass at saturation): $M_0 \sim 0.7 - 0.8\, m_N$
  - ✓ d (self quartic coupling of ω): $d < 10^4$ to have a consistent ω condensate
  - ✓ K (incompressibility or compression modulus): $K \sim 200 - 300$ MeV

Effective potential

- A local minimum at nonzero q arises at large μ_B.
- $q = 0$ minimum always exists due to the δ_{q0} term.

- Spinodal chemical potential at which the nonzero q minimum disappears.





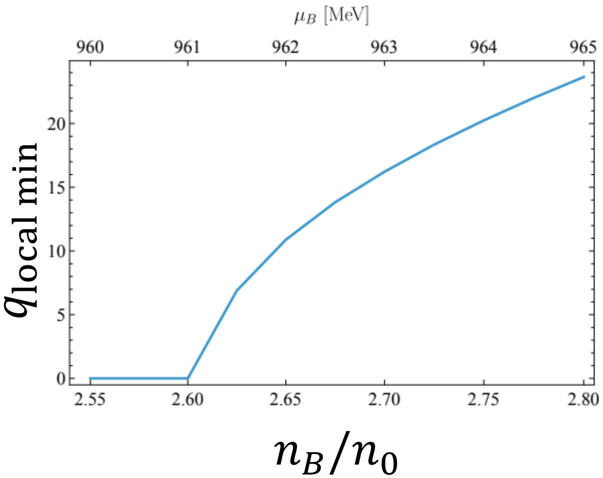
$T = 0$ (with $M_0 = 0.81\, m_N, K = 250$ MeV$, d = 10^4$)

Evolution of q

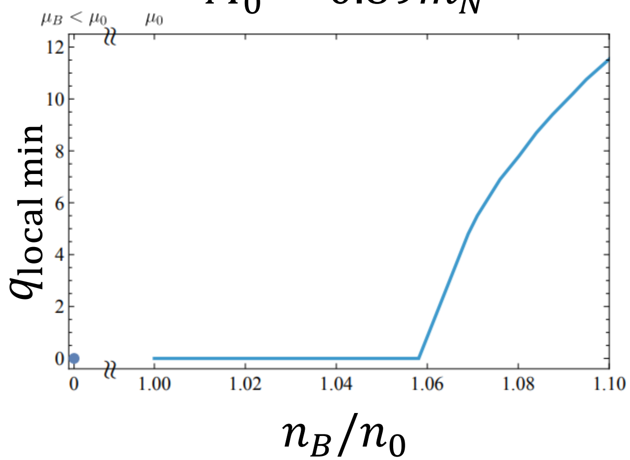
- As M_0 increases, the spinodal chemical potential decreases.
- When $M_0 \gtrsim 0.9\, m_N$, spinodal chemical potential disappears, CDW phase can survive until the liquid-gas transition.

(Such a large M_0 is disfavored, but not strictly ruled out because it's just a model study.)

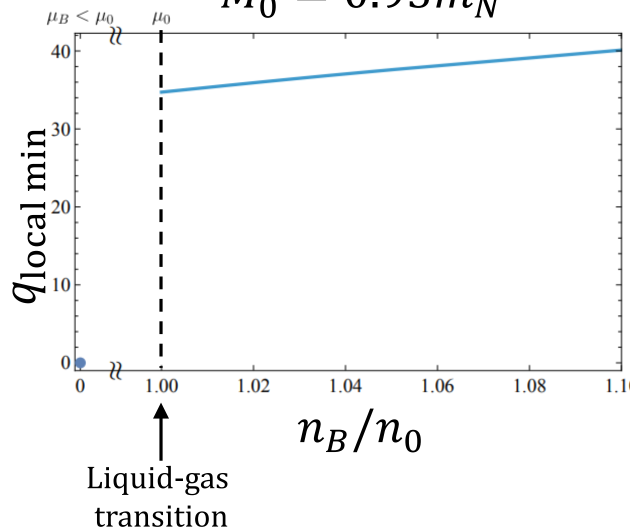
$M_0 = 0.81 m_N$



$M_0 = 0.89 m_N$



$M_0 = 0.93 m_N$

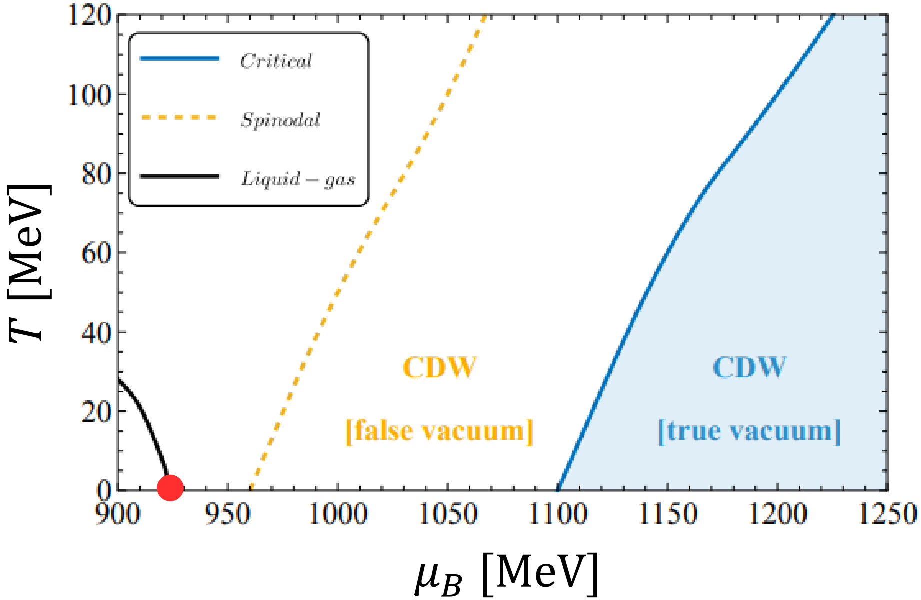


$T = 0$ with $K = 250$ MeV, $d = 10^4$

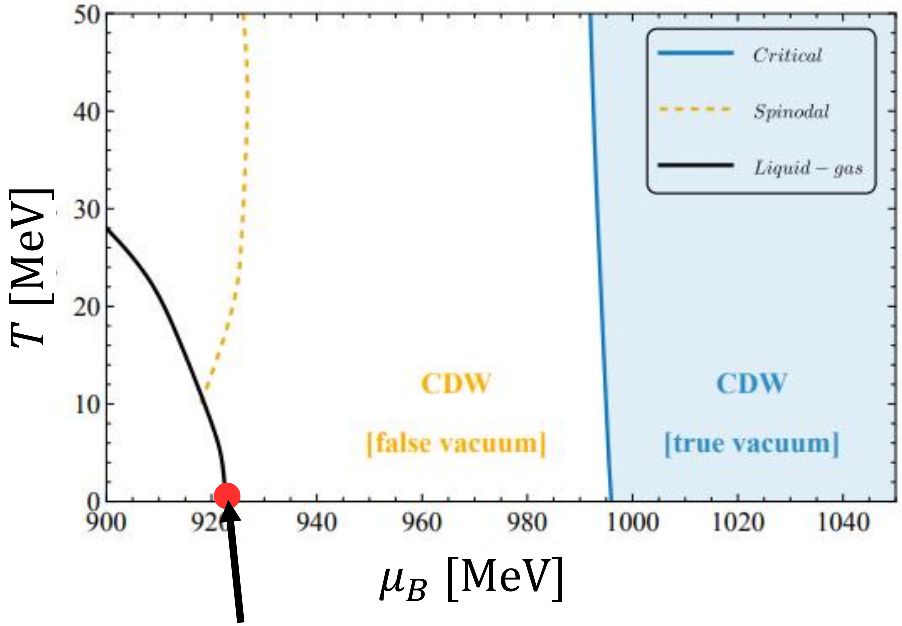
CDW in the phase diagram

- In the phase diagram, the spinodal line can meet the liquid-gas transition line if $M_0 \gtrsim 0.9\, m_N$.

$\boldsymbol{M_0 = 0.81\, m_N}$



$\boldsymbol{M_0 = 0.93\, m_N}$

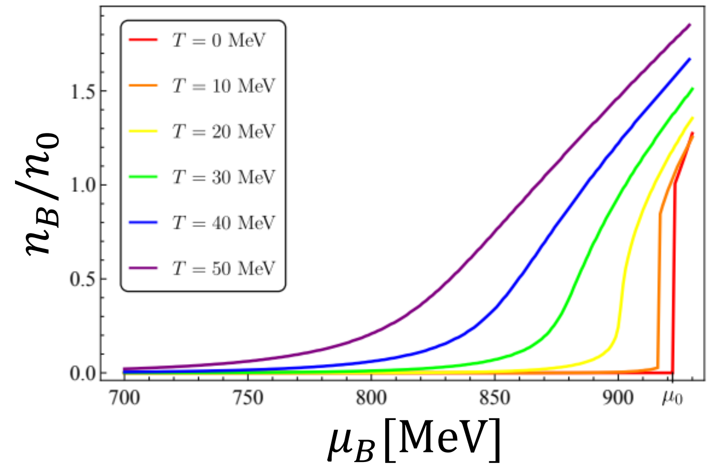


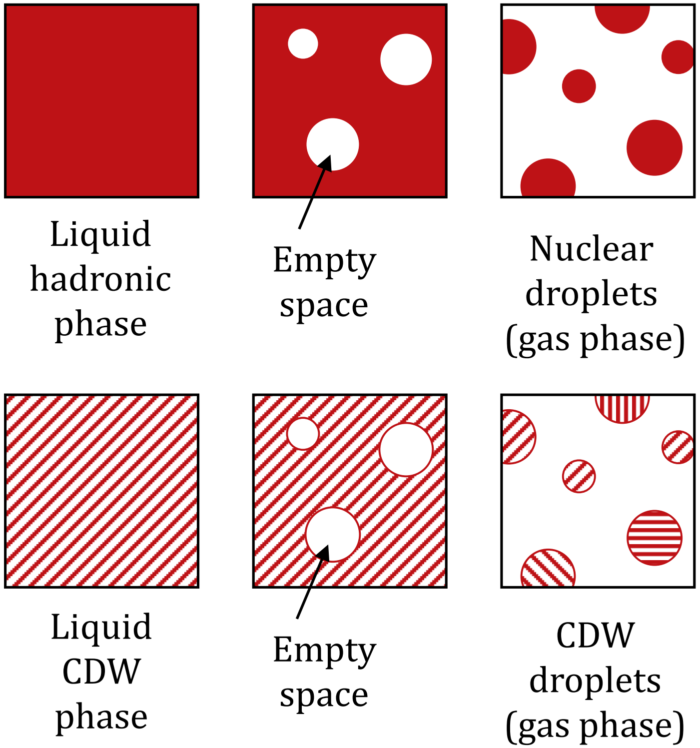
Saturation point (vacuum)

with $K = 250$ MeV, $d = 10^4$

The liquid-gas transition

- n_B is discontinuous around μ_0 if T is small.
- Fermi-Dirac distribution
$$\frac{1}{e^{(E-\mu)/T}+1} \rightarrow 0 \ \text{ as } T \rightarrow 0 \text{ for } \mu < M$$
- However, the baryon number is conserved, and n_B must change continuously ($\propto a^{-3}$) during the cosmological evolution.
- The transition involves nucleation of nuclear droplets (~nuclei) and they move apart from each other.
  $\rightarrow$ "Net" baryon density decreases continuously.
- Each droplet has $\mu_B \sim \mu_0$ and $n_B \sim n_0$.
- If CDW phase was supercooled, each droplet can stay in CDW phase, and carry the latent heat inside.





Baryon density and reheating

- Average volume of (CDW) droplet: V_d
- Number density of (CDW) droplet: n_d
- Baryon density in each droplet: $\mathrm{n_B} \sim n_0 = 0.153\ \mathrm{fm}^{-3}$ (saturation density)

- Net baryon density

$$n_B^\mathrm{net} = n_0 V_\mathrm{d}\, n_\mathrm{d}$$

- Reheating temperature

$$\rho_\mathrm{pl}(T_\mathrm{RH}) \sim \Delta\Omega(q_0) V_\mathrm{d}\, n_\mathrm{d}$$

$$\text{with } \Delta\Omega \sim (100\ \mathrm{MeV})^4$$

- Final baryon yield

$$Y_B = \frac{n_B^\mathrm{net}}{s(T_\mathrm{RH})} \sim \frac{n_0 T_\mathrm{RH}}{\Delta\Omega(q_0)} \sim 10^{-2}\left(\frac{T_\mathrm{RH}}{2\ \mathrm{MeV}}\right)$$

$Y_B \sim 10^{-10}$ requires $T_\mathrm{RH} \ll T_\mathrm{BBN} \Rightarrow$ **Ruled out by BBN and CMB !!**

➡ **CDW phase transition does not help!** (our conclusion 2)

Summary and conclusion

- The QCD-induced little inflation scenario had been a potential source of PTA GW signal, and we revisited this scenario.

- First, we argue that QGP → hadronic phase transition cannot provide a sufficiently strong supercooling for the observed baryon density. This is because the location of the critical endpoint in the QCD phase diagram is at $(\mu_B)_{\mathrm{CEP}} \gtrsim 450$ MeV.

- Next, we investigated CDW phase transition.
  - ✓ We find that the transition can be delayed until the liquid-gas transition, and thus supercooling may be strong enough.
  - ✓ However, the reheating temperature becomes too small, and it is ruled out by BBN and CMB.

- **We conclude that PTA GW signal cannot be explained by Standard Model QCD PTs with a large initial baryon density.**

Thank you!