Suppressing Extra-Dimensional Axion Isocurvature Dynamically

Work in collaboration with T. Gherghetta, arXiv: 2607.09969

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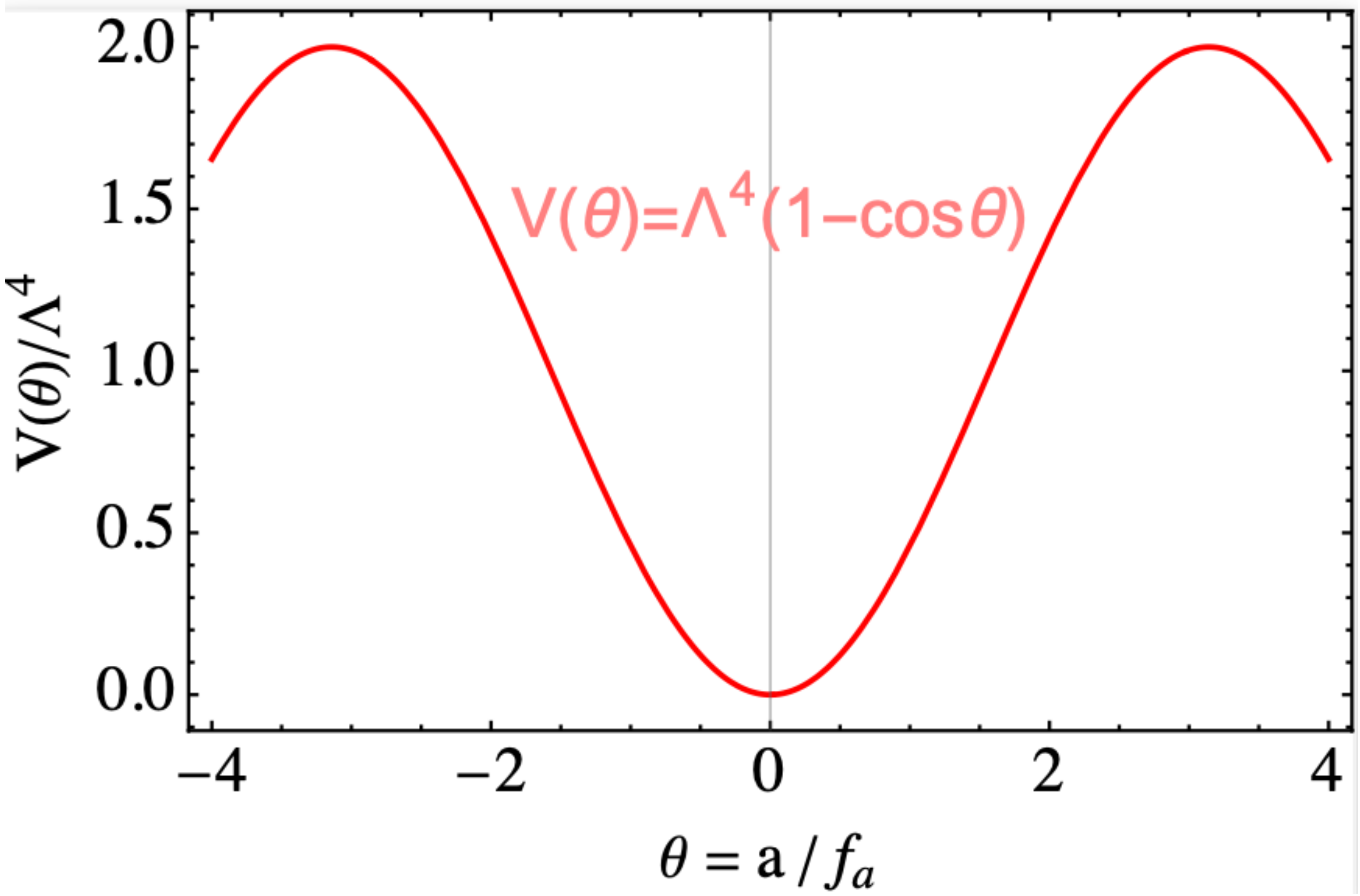
Outline

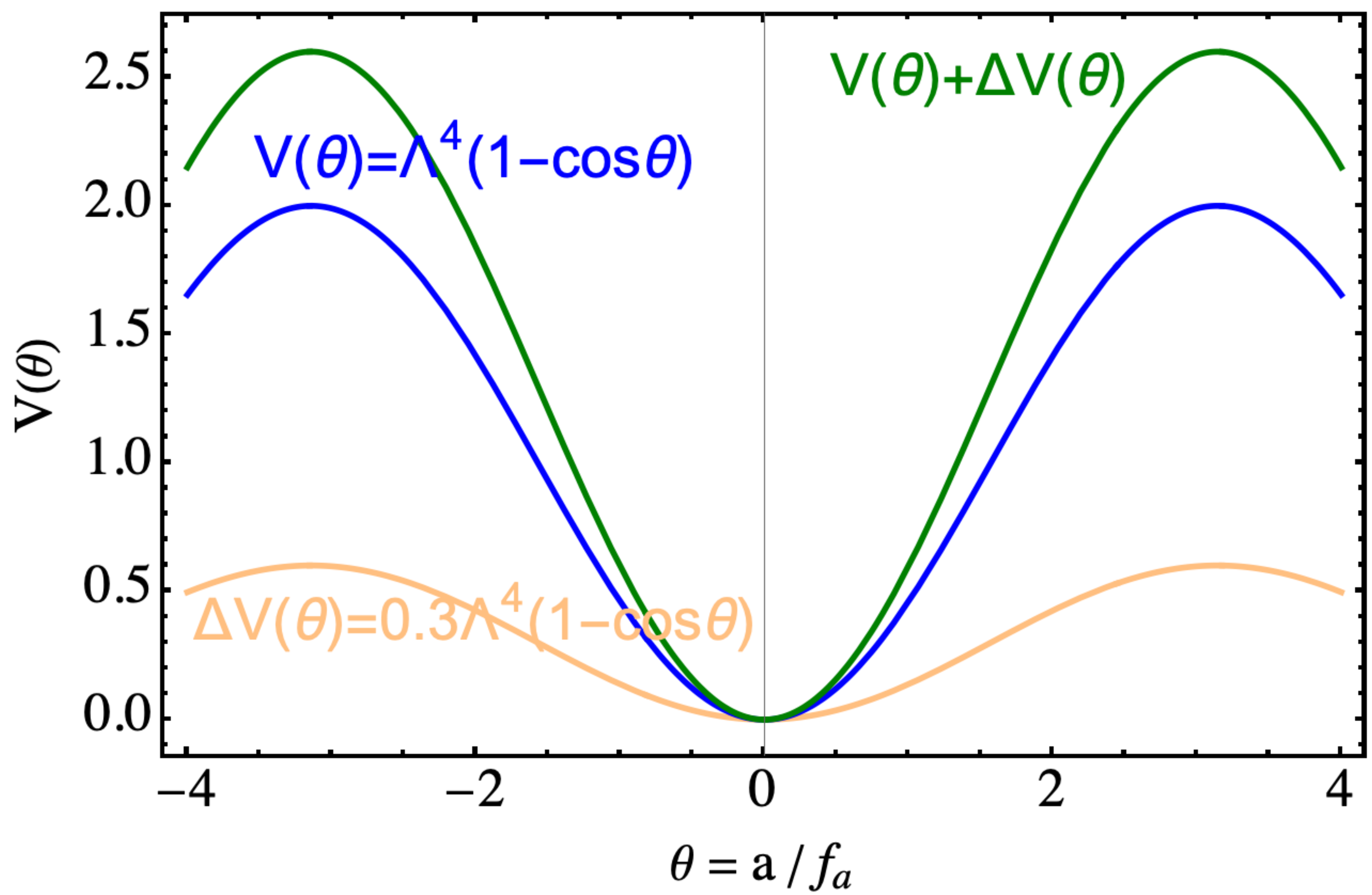
- Why ExtraD axion?
- Axion isocurvature perturbation
- Making ExtraD axion decay constant dynamical
- Conclusion

QCD axion quality problem in PQ picture



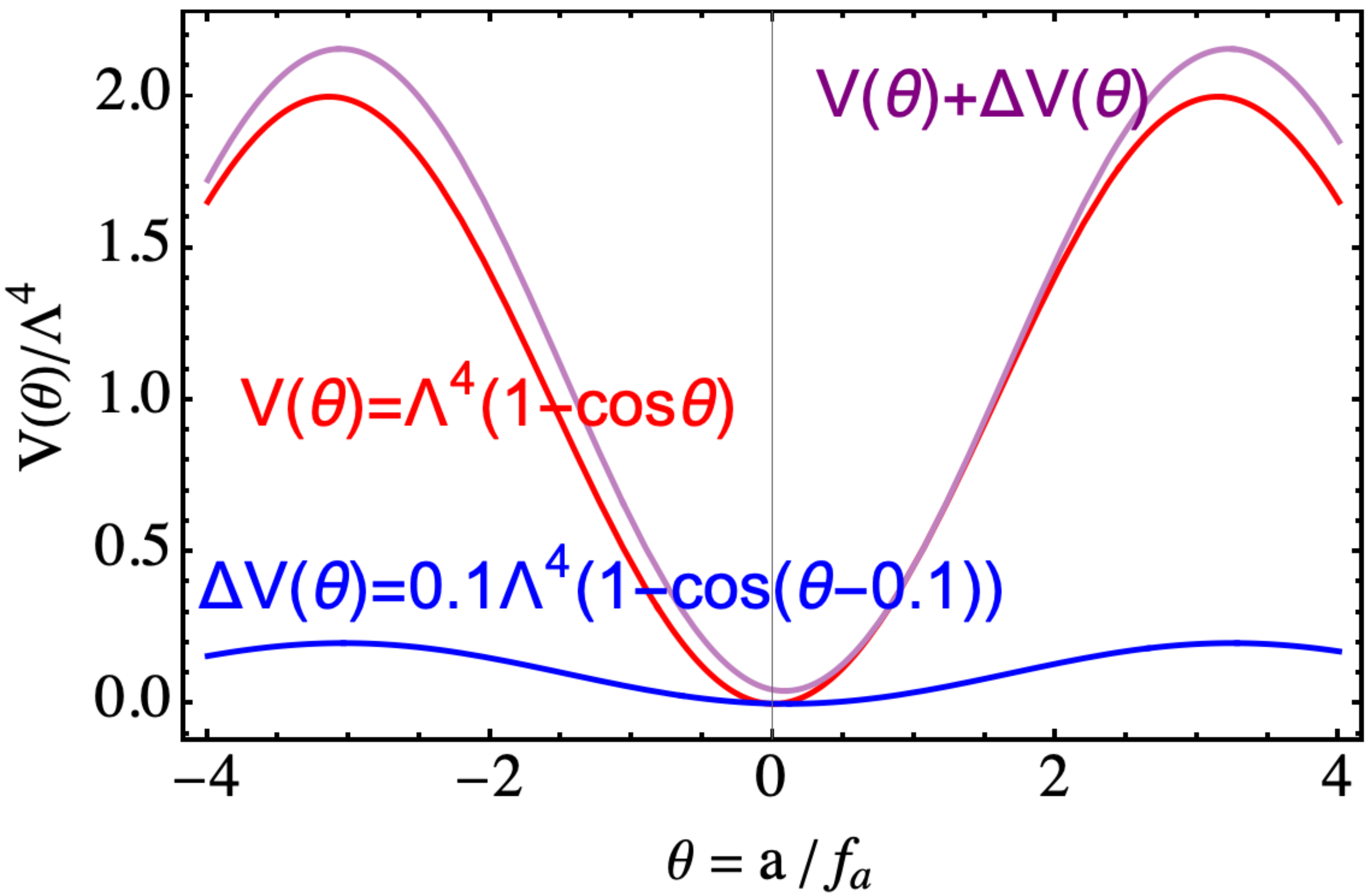
Good sol to strong CP problem iff $\theta_{\rm min} < 10^{-10}$

QCD axion quality problem in PQ picture



When $\Delta V(\theta)$ is aligned with $V(\theta)$, height of ΔV does not matter

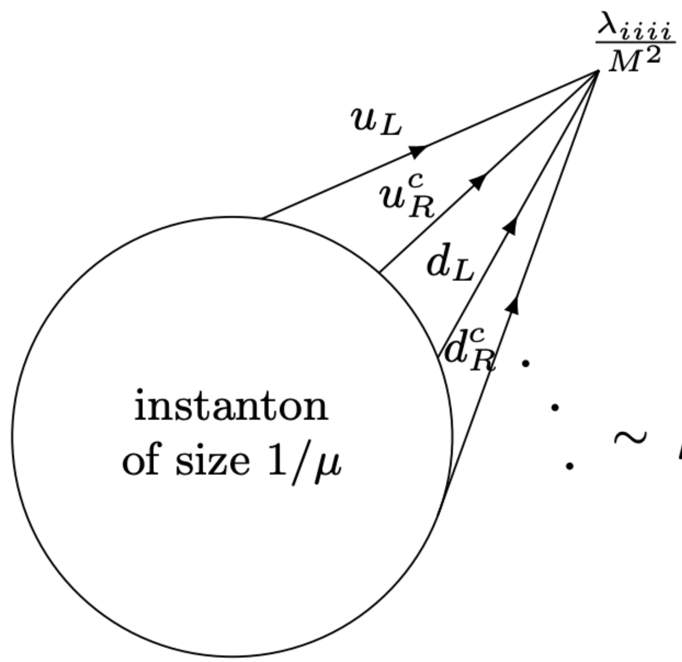
QCD axion quality problem in PQ picture



If $\Delta V(\theta)$ is misaligned with $V(\theta)$ and

$\theta_{min} \sim \Delta V / V > 10^{-10}$, axion is meaningless

- Coefficient of an operator closing fermion zero modes is generally complex



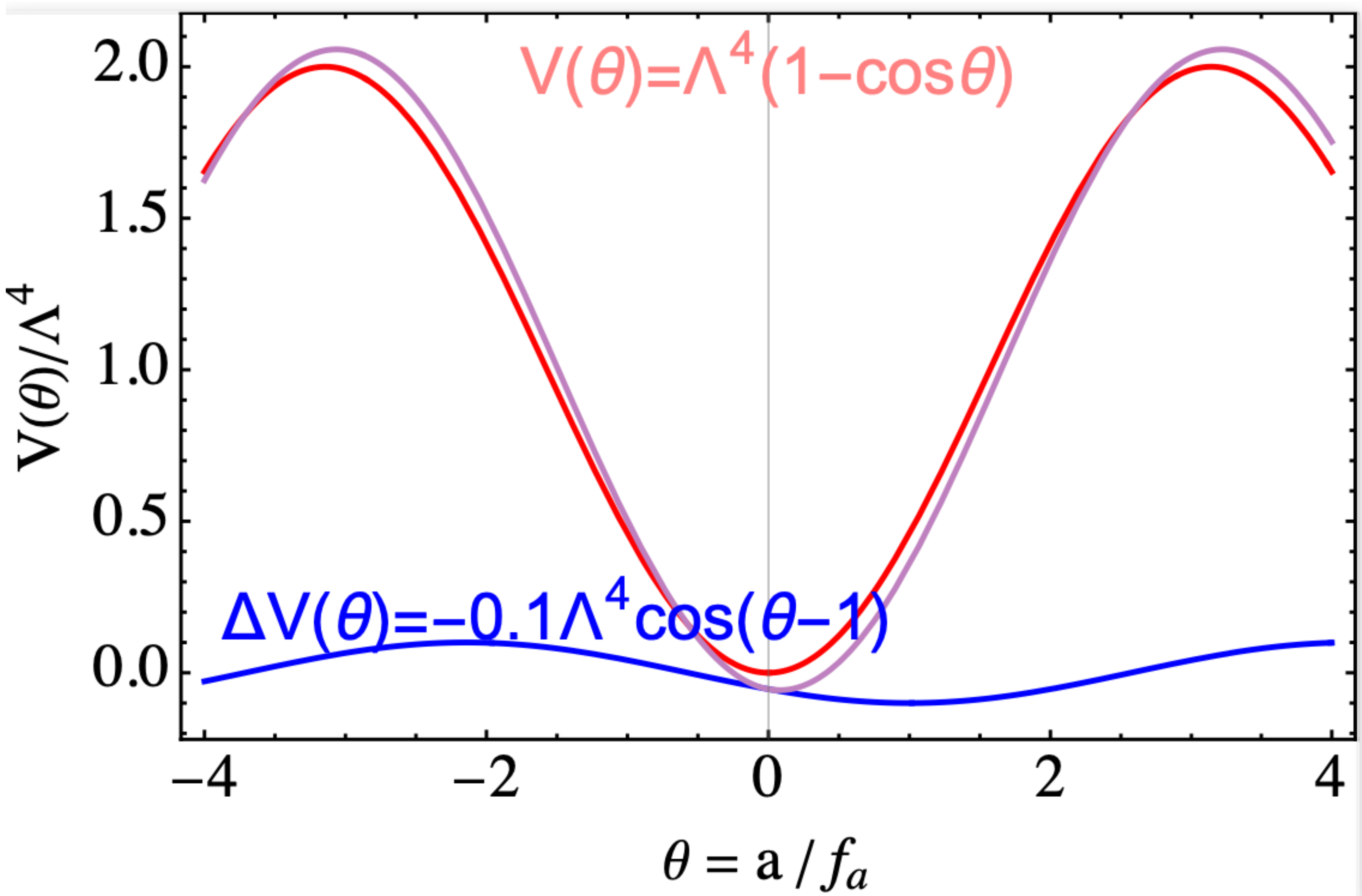
$$\sim \mu^4 \prod_{i=1}^{3} \frac{\mu^2 \lambda_{iiii}}{M^2} e^{-\frac{8\pi}{g^2(\mu)} - i\theta}$$

- Coefficient of a global symmetry breaking non-renormalizable operator is complex

Kamionkowski, March-Russell 92
Barr, Seckel 92

$$c \frac{\Phi^5}{M_P} + c^* \frac{\Phi^{\dagger\, 5}}{M_P}$$

QCD axion quality problem in PQ picture



How to suppress $\theta_{\min} \sim \Delta V / V$?

ExtraD Axion

See, e.g. Reece 2406.08543

- If axion can appear without a global U(1) and a complex scalar, then we may avoid the second source of quality problem.

- Consider 4+n dim spacetime $M = X \times_w Y$

- In extraD theories with warped compactification,

$$\mathrm{d}s^2 = w(y)\,\mathrm{d}s_X^2 + \mathrm{d}s_Y^2.$$

given a basis of cohomology class for $H^p(Y,Z)$,
p-form gauge field can be expanded in harmonic forms

$$\boxed{C^{(p)}(x,y) = \sum_{i=1}^{b_p(Y)} \theta^i(x)\widehat{\omega}_i^{(p)}(y)}$$

$$\widehat{\omega}_i^{(p)}(y) \in [\omega_i^{(p)}]$$

$$\mathrm{d}\widehat{\omega}_i^{(p)} = 0, \quad \mathrm{d}\star\widehat{\omega}_i^{(p)} = 0$$

ExtraD Axion

See, e.g. Reece 2406.08543

- Can choose basis in cohomology and homology to be related such that

$$\boxed{\int_{\widehat{\Sigma}_i^{(p)}} \widehat{\omega}_j^{(p)} = \delta_i^j}$$

$$\widehat{\omega}_j^{(p)} \in [\omega_j^{(p)}]$$

$$\widehat{\Sigma}_i^{(p)} \in [\Sigma_i^{(p)}]$$

- 4D axion fields from integrals of $C^{(p)}$ over p-cycles

$$\boxed{\theta^i = \int_{\Sigma_i^{(p)}} C^{(p)}}$$

ExtraD Axion

See, e.g. Reece 2406.08543

- From kinetic term,

$$-\int_M \frac{1}{2g_p^2} \mathrm{d}C^{(p)}(x,y) \wedge \star \mathrm{d}C^{(p)}(x,y)$$

$$\boxed{C^{(p)}(x,y) = \sum_{i=1}^{b_p(Y)} \theta^i(x) \widehat{\omega}_i^{(p)}(y)}$$

$$\downarrow$$

$$\sum_{i,j=1}^{b_p(Y)} \int_X \kappa_{ij} \left(-\frac{1}{2} \mathrm{d}\theta^i(x) \wedge \star_X \mathrm{d}\theta^j(x) \right)$$

- Decay constant determined by g_p and Vol[Y]

$$\kappa_{ij} \equiv \int_Y \frac{w(y)}{g_p^2} \widehat{\omega}_i^{(p)}(y) \wedge \star_Y \widehat{\omega}_j^{(p)} \longrightarrow f_i \equiv \sqrt{\kappa_{ii}}$$

ExtraD Axion

See, e.g. Reece 2406.08543

- Axion mass comes from terms including $C^{(p)}$ without derivative
- Charged object, gauge theory instanton…
- Electrically charged object under $C^{(p)}$: (p-1) brane
- On the worldvolume $\Gamma^{(p)}$
  (p-1) brane of charge q couples under $C^{(p)}$ via

$$\int_{\Gamma^{(p)}} q\, C^{(p)}$$

ExtraD Axion

See, e.g. Reece 2406.08543

- The integral $\int_{\Gamma^{(p)}} q\, C^{(p)}$ is non-zero
  only when $\Gamma^{(p)}$ extends along internal direction within Y
  → appears completely localized in 4D
  → Euclidean brane instanton

- Brane action

$$S_{\text{brane}} = \int_{\Gamma^{(p)}} \left(\mathcal{T}_{(p)} \star_{\Gamma} 1 + \mathrm{i} q\, C^{(p)} \right)$$

- Vacuum to vacuum amplitude gives

$$V_{\text{eff}}^{(\text{brane})}(\{\theta^i\}) = \left(M^4 \mathrm{e}^{-\mathcal{T}_{(p)}\, \mathrm{Vol}(\Gamma^{(p)})} \mathrm{e}^{\mathrm{i} q m_j \theta^j} + \text{h.c.} \right) \qquad [\Gamma^{(p)}] = \textstyle\sum_j m_j [\Sigma_j^{(p)}]$$

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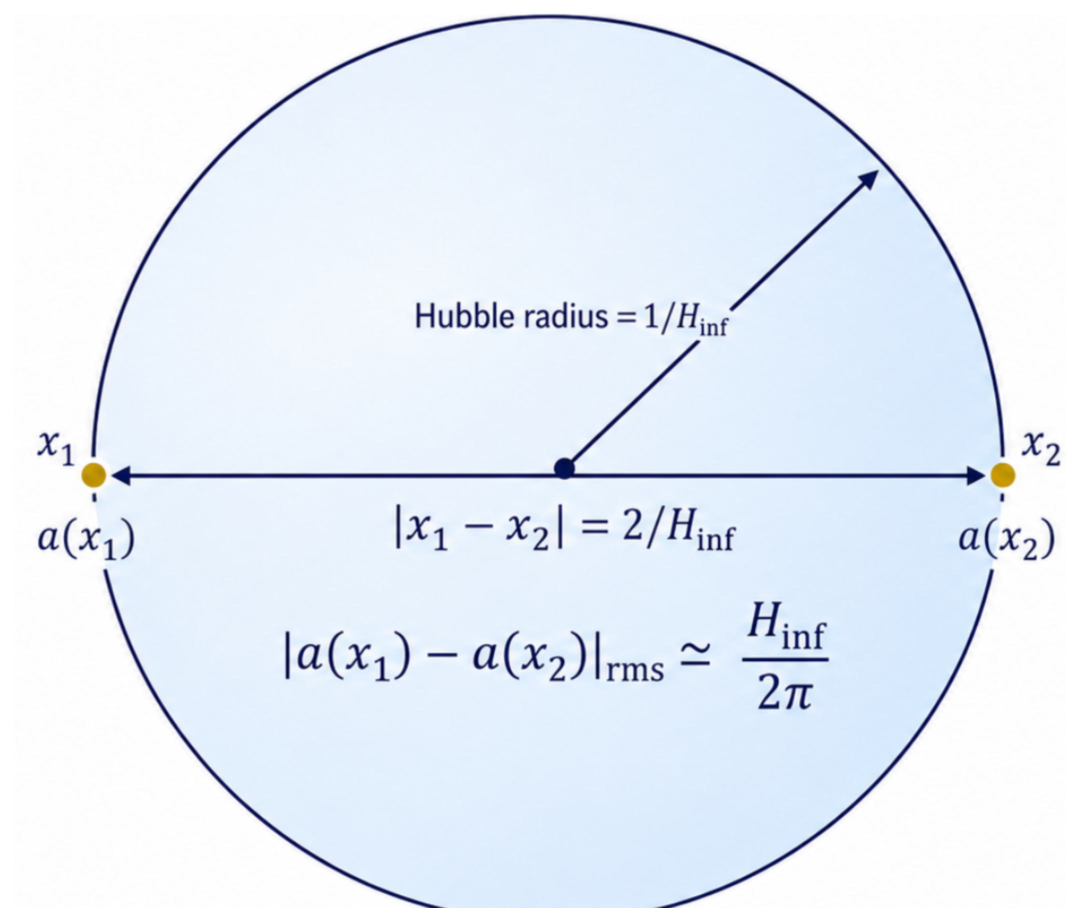
Take the log of axion quality problem for $T_{(p)}\mathrm{Vol}(\Gamma^{(p)}) >> 1$

Isocurvature perturbation

- However, differing from field theoretic axions, extraD axions are always present during inflation

- For $m<<H_{\rm inf}$, typical axion field variation on the length scale $\sim 1/H_{\rm inf}$ is

$$\Delta a_{H^{-1}} \sim \frac{H_{\rm inf}}{2\pi}$$



Isocurvature perturbation

- However, differing from field theoretic axions, extraD axions are always present during inflation

- For $m \ll H_{\rm inf}$, typical axion field variation on the length scale $\sim 1/H_{\rm inf}$ is

$$\delta a_i = \frac{H_{\rm inf}}{2\pi}$$

- What remains conserved since inflation

$$\frac{\delta \rho_a}{\rho_a} \simeq 2 \frac{\delta a_i}{a_i}$$

Isocurvature perturbation

- Axion fluctuation varies independently of $\rho_{tot}(t)$
  → isocurvature

- Gauge invariant axion-radiation entropy perturbation

$$S_{a\gamma} \equiv 3(\zeta_a - \zeta_\gamma) = \frac{\delta\rho_a}{\rho_a} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}$$

Isocurvature perturbation

- Axion fluctuation varies independently of $\rho_{tot}(t)$
  → isocurvature

- On hypersurface with $\delta\rho_\gamma=0$

$$S_{a\gamma} = \frac{\delta\rho_a}{\rho_a} = \frac{2\delta\theta_{a,i}}{\theta_{a,i}}$$

- Power spectrum at CMB scale $\mathcal{P}_{\rm iso}(k_{\rm CMB}) = \left(\frac{\Omega_a}{\Omega_{\rm cdm}}\right)^2 \left(\frac{H_I}{\pi f_I \theta_{a,i}}\right)^2$

$$\beta_{\rm iso} \equiv \frac{\mathcal{P}_{\rm iso}(k_{\rm CMB})}{\mathcal{P}_\zeta(k_{\rm CMB}) + \mathcal{P}_{\rm iso}(k_{\rm CMB})} < 0.036 \quad \longrightarrow \quad \boxed{\mathcal{P}_{\rm iso}(k_{\rm CMB}) \lesssim 10^{-11}}$$

Isocurvature problem?

- Satisfying constraint

$$\mathcal{P}_{\rm iso}(k_{\rm CMB}) \lesssim 10^{-11} \qquad\qquad \mathcal{P}_{\rm iso}(k_{\rm CMB}) = \left(\frac{\Omega_a}{\Omega_{\rm cdm}}\right)^2 \left(\frac{H_I}{\pi f_I \theta_{a,i}}\right)^2$$

requires dilution of axion dark matter
or large hierarchy between H_I and f_I
or suppress $\delta\theta_a$ with $m_{a,\rm inf} >> H_{\rm inf}$

- For $f_a \sim 10^{12}$ GeV,

→ severe constraint on Hubble
→ not a problem, but requires extremely flat potential

$$\boxed{H_I \lesssim \mathcal{O}(10^7)\,\mathrm{GeV}}$$

$$\boxed{\mathcal{P}_\zeta(k_{\rm CMB}) \sim \frac{H_I^2}{\epsilon_\chi M_P^2}, \qquad \epsilon_\chi \equiv \frac{M_P^2}{2}\left(\frac{V'(\chi)}{V(\chi)}\right)^2}$$

Outline

- Why ExtraD axion?

- Axion isocurvature perturbation

- Making decay constant of ExtraD axion dynamical
  → temporary large hierarchy between H_I and f_I

- Conclusion

Set-up

- In 5D, when we assume a bulk $U(1)_C$ gauge symmetry
- zero mode of C_5
  → 4D axion identified with the Wilson loop
  of 5D gauge field around the compact dimension

K. Choi 2004

$$\theta_a(x) = \frac{a(x)}{f_a} = \oint dy\, C_5(x,y) \qquad f_a = \sqrt{\frac{k}{4g_{5C}^2}\,\frac{1}{e^{2\pi k r_c}-1}}$$

- Warped orbifold extraD

Randall, Sundrum 1999

$$S_{5D} \supset \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{g}\left(-\frac{1}{2}M_5^3 R - \Lambda_5\right) - \sum_{i=\mathrm{UV,IR}} \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-g_i}\, T_{i,c}\, \delta(\phi - \phi_i)$$

$$\Lambda_5 = -6M_5^3 k^2$$

$$\Lambda_5 = -kT_{\mathrm{UV},c} = kT_{\mathrm{IR},c}$$

$$\boxed{ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2 = e^{-2ky}dx^2 - dy^2}$$

Set-up

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$$\theta_a(x) = \frac{a(x)}{f_a} = \oint dy\, C_5(x,y) \qquad f_a = \sqrt{\frac{k}{4g_{5C}^2} \boxed{\frac{1}{e^{2\pi k r_c} - 1}}}$$

- Dynamical r_c → dynamical f_a

- Smaller r_c → larger f_a during inflation

Radion dynamics

- In RS, the typical stabilization mechanism for inter-brane separation → Goldberger-Wise

Goldberger, Wise 1999

$$S_{\rm GW} = \int d^4x dy \sqrt{g}\left[\frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi - \frac{1}{2}m_\Phi^2\Phi^2\right]$$
$$- \sum_{i={\rm UV,IR}}\int d^4x\int_{-\pi}^{\pi} d\phi\sqrt{-g_i}\,\lambda_i(\Phi^2 - v_i^2)^2\delta(\phi-\phi_i)$$

$$\Phi(y_{\rm IR}) = v_{\rm IR} \equiv c_{\rm IR}k^{\frac{3}{2}}, \quad \Phi(y_{\rm UV}) = v_{\rm UV} \equiv c_{\rm UV}k^{\frac{3}{2}}$$

- Integrating out Φ gives a potential for radion (σ) parametrizing "size of extraD"

$$\sigma(x) \equiv \sqrt{\frac{6M_5^3}{k}}e^{-\pi kr(x)} \equiv Fe^{-\pi kr(x)}$$

Radion dynamics

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Goldberger, Wise 1999

$$
\begin{aligned}
S_{\rm GW} &= \int d^4x dy \sqrt{g}\left[\frac{1}{2} g^{MN}\partial_M\Phi\partial_N\Phi - \frac{1}{2}m_\Phi^2\Phi^2\right] \\
&- \sum_{i={\rm UV,IR}} \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-g_i}\,\lambda_i(\Phi^2 - v_i^2)^2\delta(\phi-\phi_i)
\end{aligned}
$$

$$
\Phi(y_{\rm IR}) = v_{\rm IR} \equiv c_{\rm IR} k^{\frac{3}{2}}, \quad \Phi(y_{\rm UV}) = v_{\rm UV} \equiv c_{\rm UV} k^{\frac{3}{2}}
$$

- Integrating out Φ gives a potential for radion (σ) parametrizing “size of extraD”

$$
\alpha \equiv m_\Phi^2/(4k^2)
$$

$$
\begin{aligned}
V_{\rm GW}(\sigma) &= \alpha c_{\rm UV}^2 k^4 - \alpha c_{\rm IR}^2 k^4 \left(\frac{\sigma}{F}\right)^4 \\
&+ 2(2+\alpha)k^4\left(\frac{\sigma}{F}\right)^4\left[c_{\rm IR} - c_{\rm UV}\left(\frac{\sigma}{F}\right)^\alpha\right]^2
\end{aligned}
$$

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$$

$$
\Phi(y_{\rm IR}) = v_{\rm IR} \equiv c_{\rm IR}k^{\frac{3}{2}}, \quad \Phi(y_{\rm UV}) = v_{\rm UV} \equiv c_{\rm UV}k^{\frac{3}{2}}
$$

- Stabilized at

$$
\boxed{\langle\sigma(x)\rangle \equiv \sigma_{\rm min} \simeq F\left(\frac{v_{\rm IR}}{v_{\rm UV}}\right)^{\frac{1}{\alpha}}} \quad \longrightarrow \quad \boxed{\pi r_c = -\frac{1}{k}\ln\left(\frac{\sigma_{\rm min}}{F}\right) \simeq \frac{1}{k\alpha}\ln\left(\frac{v_{\rm UV}}{v_{\rm IR}}\right)}
$$

5D inflation

Kaloper 1999
Nihei 1999
Kim, Kim 1999
DeWolfe, Freedman, Gubser, Karch 1999

- With intro of detuning of brane tensions

$$\delta T^{\rm dS}_{\rm UV} = 12 M_5^3 k e^{-2ky_c} \simeq 3 M_4^2 \widetilde{H}^2$$

$$\delta T^{\rm dS}_{\rm IR} = -12 M_5^3 k e^{-2k(y_c - y_{\rm IR})} \simeq -3 M_4^2 \widetilde{H}^2 e^{2ky_{\rm IR}}$$

5D gravity admits the geometry with inflating 4D slices

$$\boxed{ds^2 = \widetilde{a}^2(y)(dt^2 - \widetilde{b}^2(t)d\mathbf{x}^2) - dy^2}$$

$$\widetilde{a}(y) = \frac{\widetilde{H}}{k}\sinh[k(y_c - |y|)] \qquad \frac{\dot{\widetilde{b}}(t)}{\widetilde{b}(t)} = \widetilde{H} = \text{const} \quad \Rightarrow \quad \widetilde{b}(t) \propto e^{\widetilde{H}t}$$

5D inflation

Kaloper 1999
Nihei 1999
Kim, Kim 1999
DeWolfe, Freedman, Gubser, Karch 1999

- These detunings

$$\delta T^{\rm dS}_{\rm UV} = 12 M_5^3 k e^{-2ky_c} \simeq 3 M_4^2 \widetilde{H}^2$$

$$\delta T^{\rm dS}_{\rm IR} = -12 M_5^3 k e^{-2k(y_c - y_{\rm IR})} \simeq -3 M_4^2 \widetilde{H}^2 e^{2ky_{\rm IR}}$$

in 4D, appear as

$$V_{\rm 4D}(\chi, \sigma) \supset \delta T^{\rm dS}_{\rm UV} + \delta T^{\rm dS}_{\rm IR} \left(\frac{\sigma}{F}\right)^4$$

Smaller extraD size during inflation

- Consider inflaton sector on UV brane

- And inflaton coupling to Φ via

$$S_{\Phi\chi,\mathrm{UV}} = \int d^4x\, d\phi \sqrt{-g_{\mathrm{UV}}} \Big[\frac{1}{2} g_{\mathrm{UV}}^{MN} \partial_M \chi \partial_N \chi - V_{\mathrm{inf}}(\chi)$$
$$-\lambda_{\mathrm{UV}} \left(\Phi^2 - c_V^2 V_{\mathrm{inf}}(\chi)^{\frac{3}{4}} - v_{\mathrm{UV}}^2 \right)^2 \Big] \delta(\phi - \phi_{\mathrm{UV}}), \qquad (28)$$

- For $\alpha<0$ and $v_{IR}>v_{UV}$, in 4D radion potential is stabilized at

$$\sigma_{\min} \simeq F \left(\frac{v_{\mathrm{UV}} + c_V V_{\mathrm{inf}}(\chi)^{\frac{3}{8}}}{v_{\mathrm{IR}}} \right)^{\frac{1}{|\alpha|}}$$

Smaller extraD size during inflation

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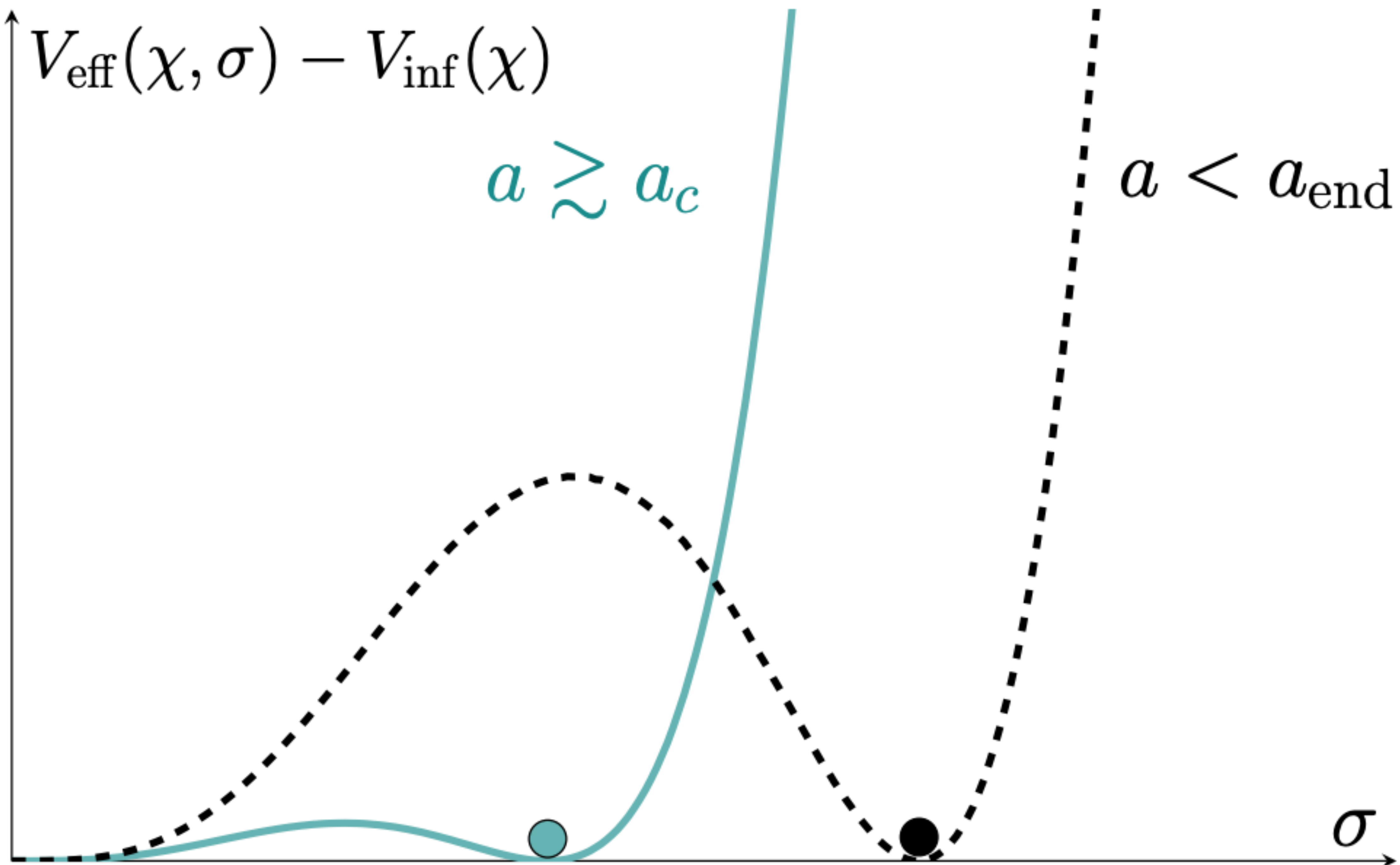
- And inflaton coupling to Φ via

$$S_{\Phi\chi,\mathrm{UV}} = \int d^4x \, d\phi \sqrt{-g_{\mathrm{UV}}} \Big[\frac{1}{2} g_{\mathrm{UV}}^{MN} \partial_M \chi \partial_N \chi - V_{\mathrm{inf}}(\chi) \\ -\lambda_{\mathrm{UV}} \left(\Phi^2 - c_V^2 V_{\mathrm{inf}}(\chi)^{\frac{3}{4}} - v_{\mathrm{UV}}^2 \right)^2 \Big] \delta(\phi - \phi_{\mathrm{UV}}), \quad (28)$$

- If $V(\chi)^{3/8} > v_{UV}$,

$$e^{-kd_{\mathrm{inf}}} = \left(\frac{c_V V_{\mathrm{inf}}(\chi)^{\frac{3}{8}}}{v_{\mathrm{IR}}} \right)^{\frac{1}{|\alpha|}} \gg e^{-kd_{\mathrm{post}}} = \left(\frac{v_{\mathrm{UV}}}{v_{\mathrm{IR}}} \right)^{\frac{1}{|\alpha|}}$$



$V_{\rm eff}(\chi,\sigma) - V_{\rm inf}(\chi)$
$a \gtrsim a_c$
$a < a_{\rm end}$
σ

Smaller extraD size during inflation

- With brane tensions & modified Goldberger-Wise action, can obtain 4D effective scalar potential

- Write parameters in terms of powers of k

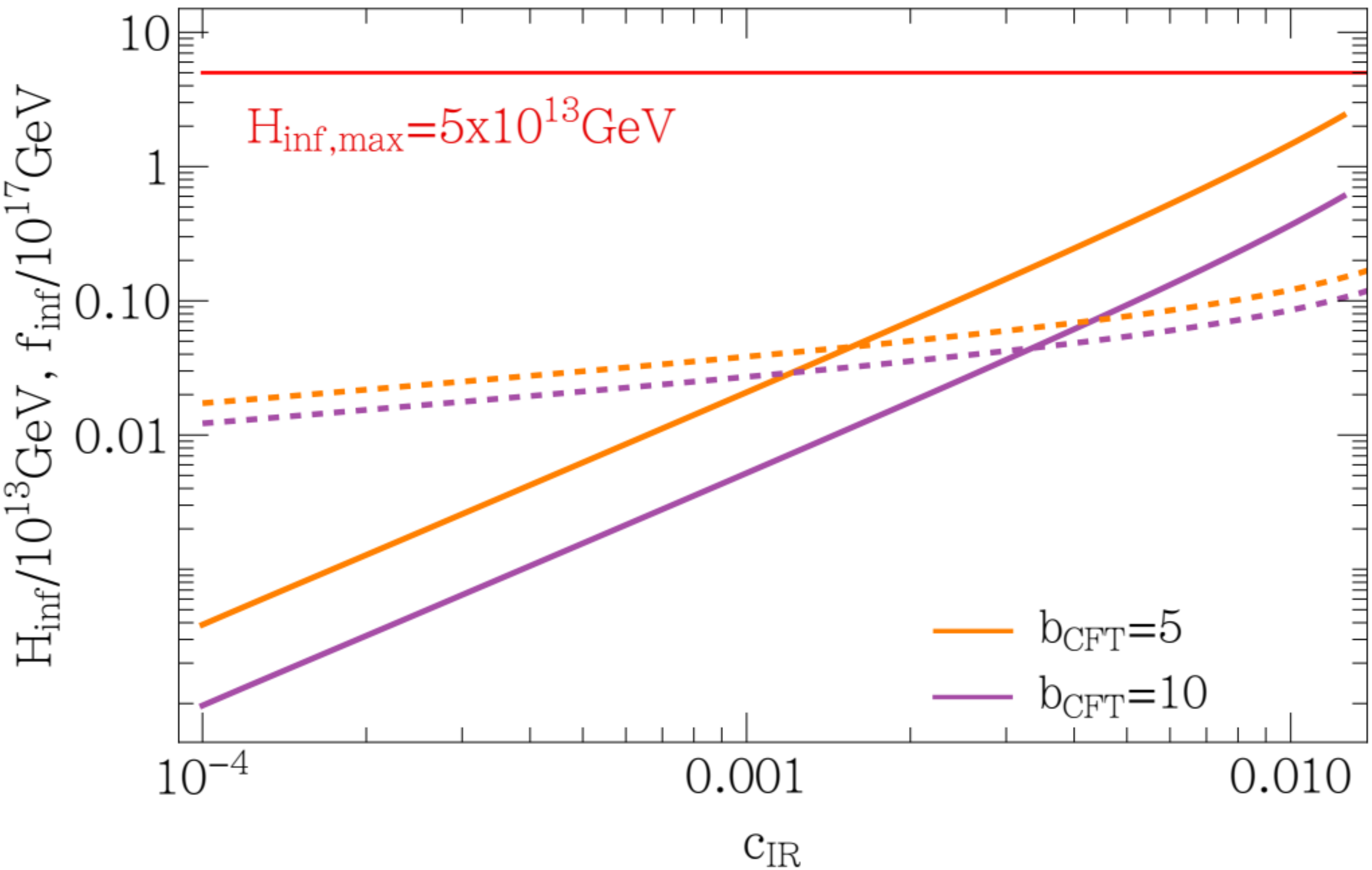
$$v_{\rm IR} \equiv c_{\rm IR} k^{\frac{3}{2}} \qquad v_{\rm UV} \equiv c_{\rm UV} k^{\frac{3}{2}} \qquad V_{\rm inf}(\chi) = c_{\rm inf}^{8/3} k^4$$

- Can numerically find c_{inf} and a warp factor for each c_{IR}

- Should ensure
(1) heavy enough radion $m_\sigma > H_{inf}$
(2) Inflation is not spoiled

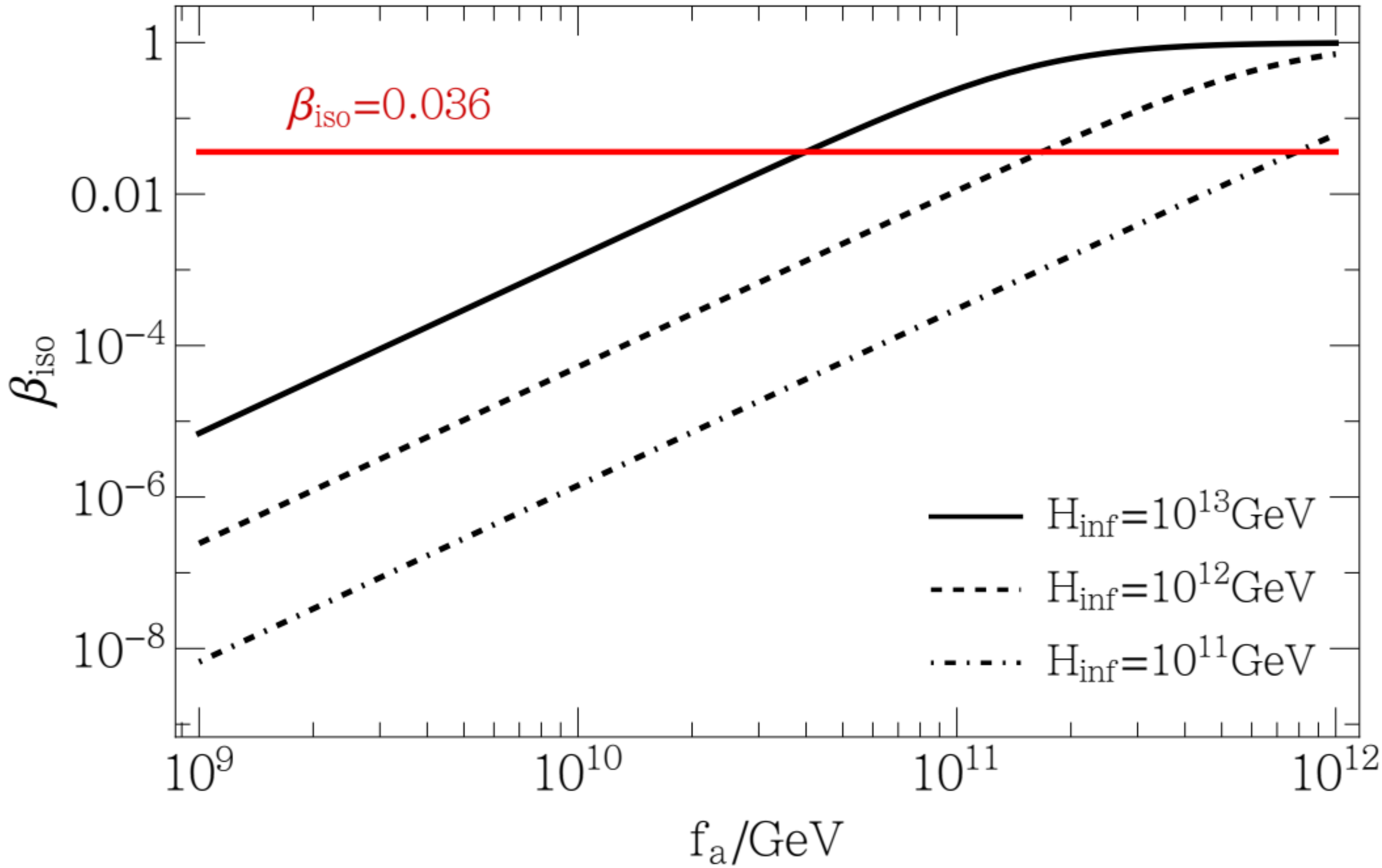
$$\epsilon_{\rm inf} = (0.8 - 0.9)\, \epsilon_{\chi,0}\,, \quad \eta_{\rm inf} = (0.9 - 0.95)\, \eta_{\chi,0}$$

Larger f_a during inflation



- For $H_{inf}=O(10^{13})$GeV, $f_{a,inf}=O(10^{16})$GeV becomes possible

Isocurvature suppression



- f_a=O(10^{11})GeV compatible with H_{inf}=O(10^{12})GeV

Avoiding domain walls on small scale

Kobayashi, Takahashi 2004

- When there is a rapid transition from $f_{\rm inf}$ to f_a,
small scale fluctuations can be enhanced for modes $k > k_*$

$$k_* = \sqrt{2n - \frac{3}{2}}\,(aH)\left(\frac{f}{f_{\rm inf}}\right)^{1-\frac{5}{4n}}$$

- If $f \sim a^{-n}$, for modes that re-enter the horizon at a_{fa},

$$\mathcal{P}_{\rm iso}^{\frac{1}{2}}(a, k_{f_a}) \simeq \frac{H_{\rm inf}}{2\pi f_a}\left(\frac{f_{\rm inf}}{f_a}\right)^{1-\frac{5}{2n}}\frac{a_{f_a}}{a}$$

- Should this be greater than 2π at axion mass generation,
axion domain wall may form for $k>k_{fa}$

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Kobayashi, Takahashi 2004

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- In our scenario,

$$f \propto \sigma \propto \left(V_{\rm inf}(\chi)^{\frac{3}{8}}\right)^{\frac{1}{|\alpha|}} \propto a^{-\frac{9}{8|\alpha|}} \qquad a_{f_a} \simeq a_{\rm RH}$$

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- In our scenario,

$$\mathcal{P}_{\rm iso}^{\frac{1}{2}}(a_m, k_{f_a}) \lesssim \frac{3\times 10^5\,{\rm GeV}}{T_{\rm RH}}$$

Summary

- ExtraD axion is charming possibility

- It is present during inflation
  → should satisfy the upper bound on β_{iso}

- ExtraD axion decay constant depends on
  gauge coupling and volume of extradimensions

- Non-trivial interaction btw radion and inflaton may induce time-varying axion decay constant
  → may help suppress axion angle fluctuation during inflation