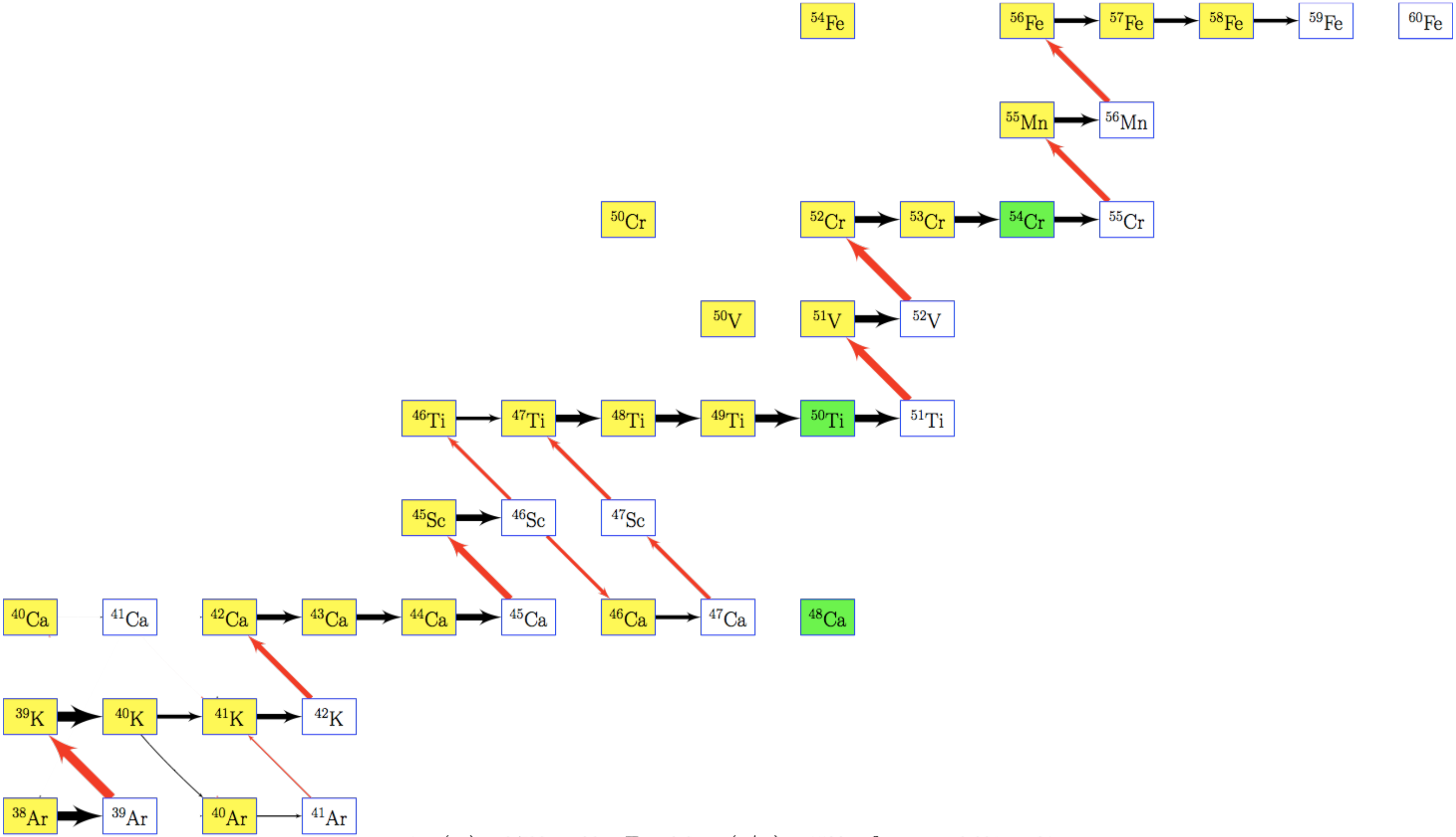
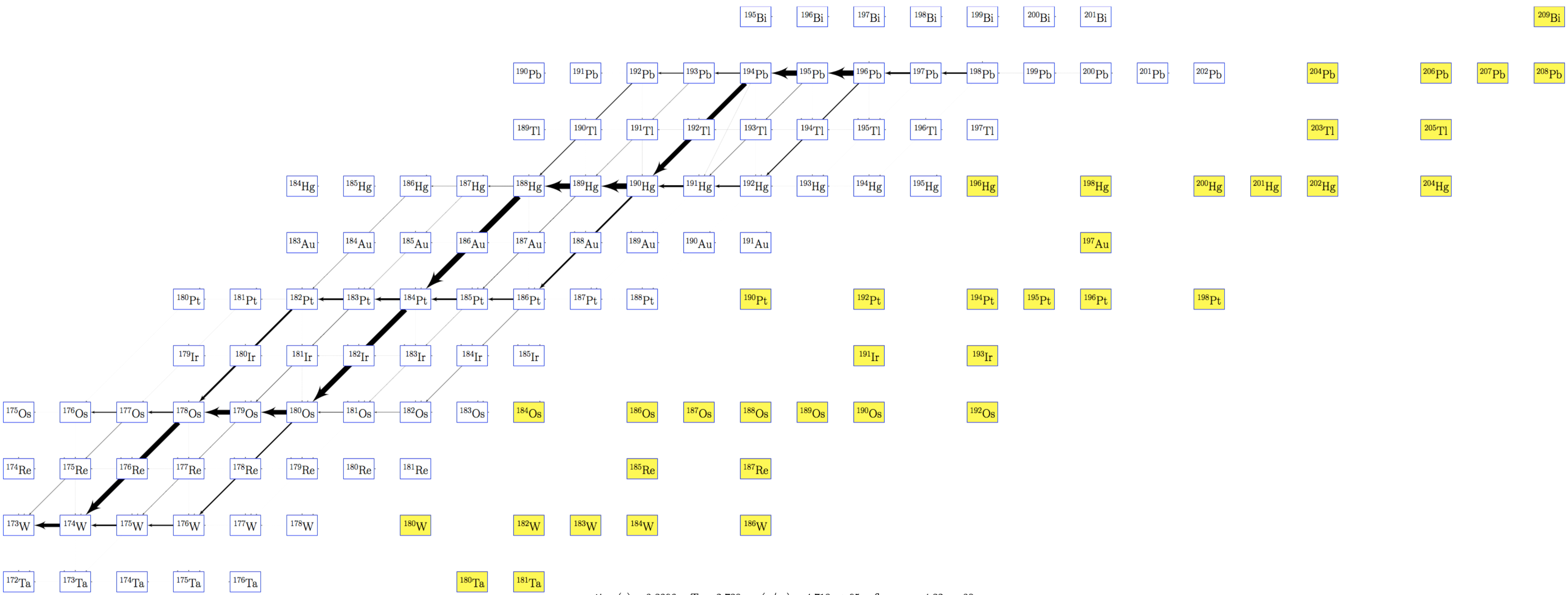
Understanding Network Flows with Branchings on Digraphs

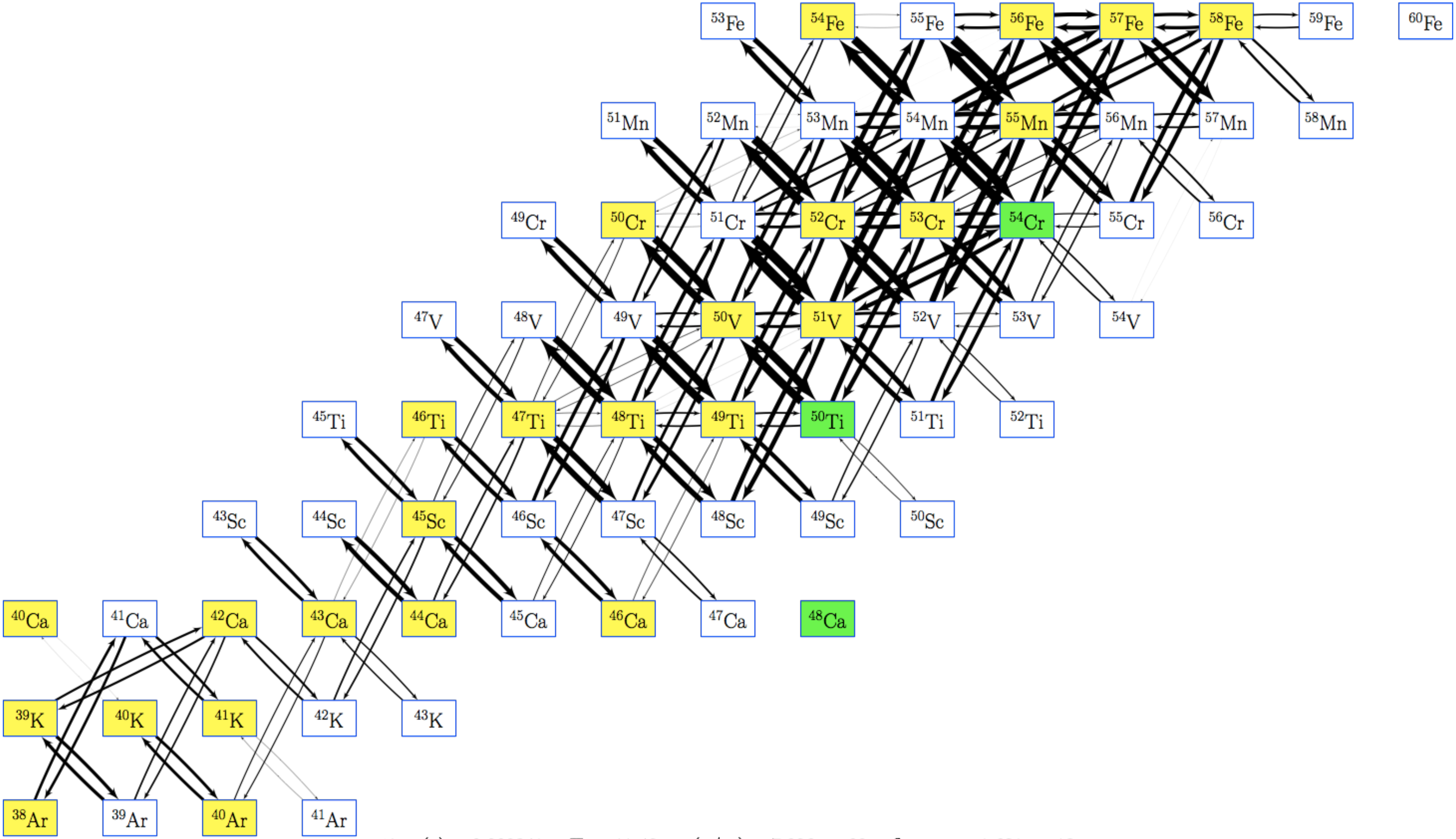
Bradley S. Meyer
Clemson University



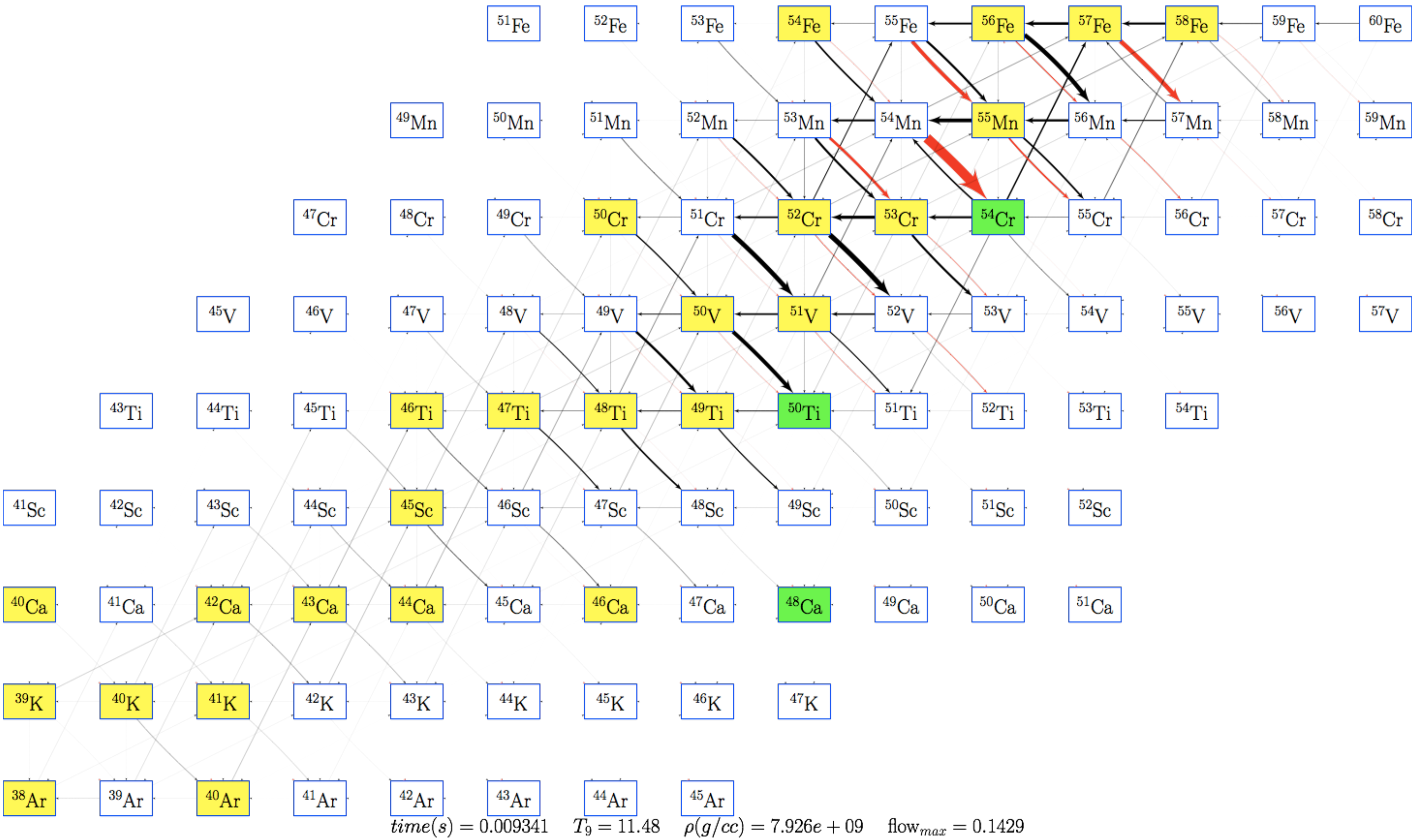
$time(yr) = 3.798e + 06 \quad T_9 = 0.2 \quad \rho(g/cc) = 1500 \quad \text{flow}_{max} = 3.981e - 21$



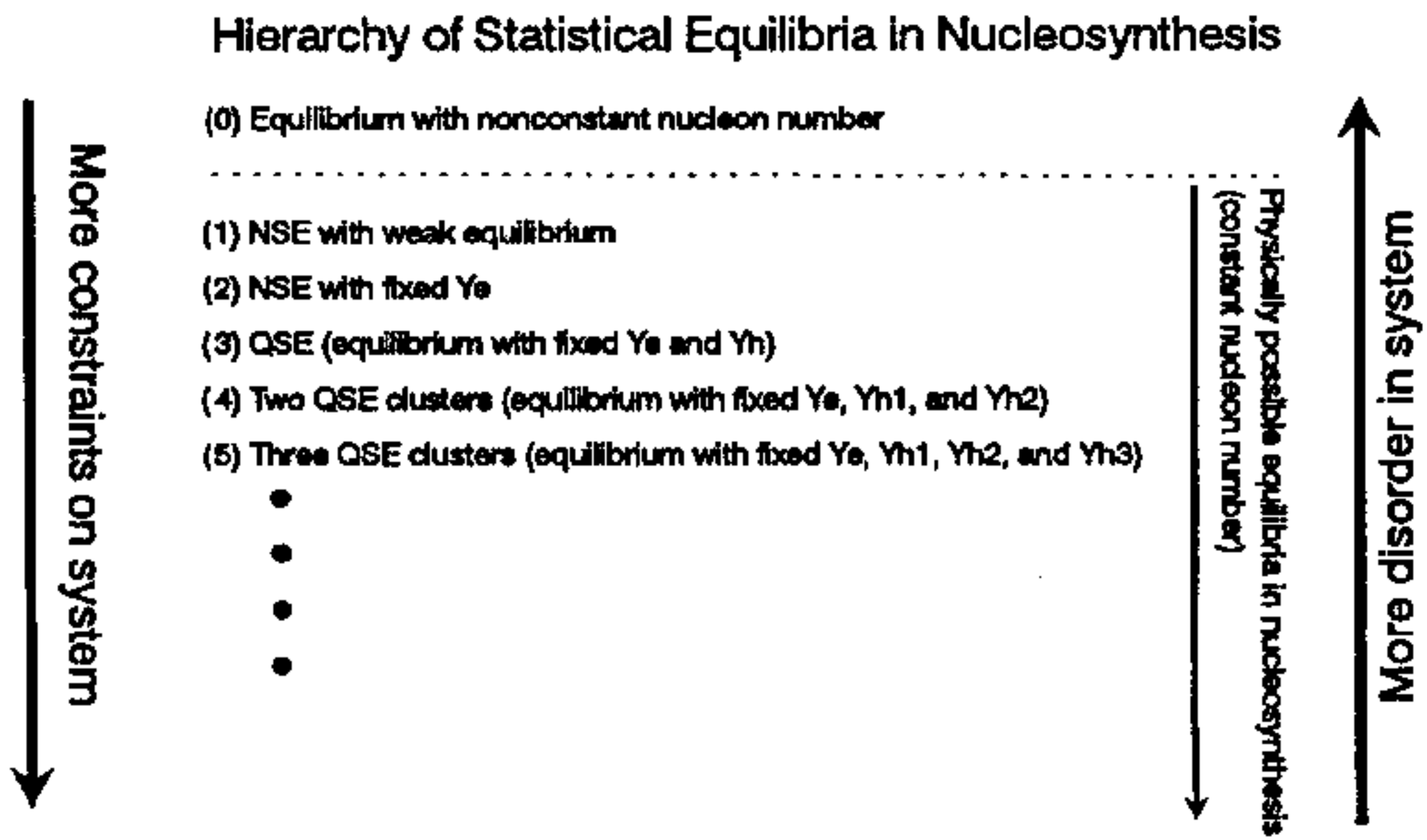
$time(s) = 0.3906 \quad T_9 = 2.738 \quad \rho(g/cc) = 4.718e+05 \quad \text{flow}_{max} = 4.32e-08$

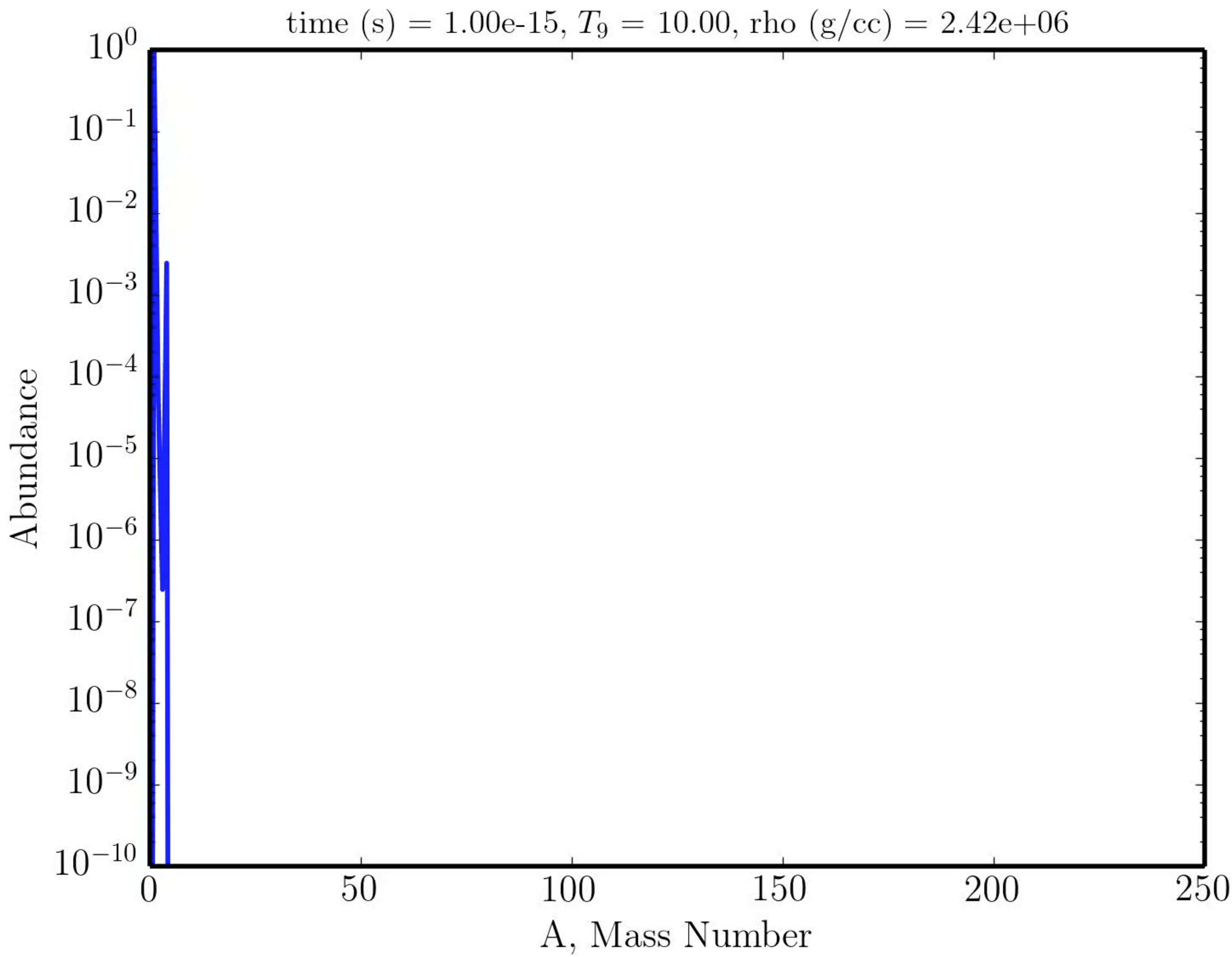


$time(s) = 0.009341 \quad T_9 = 11.48 \quad \rho(g/cc) = 7.926e+09 \quad \text{flow}_{max} = 1.631e+12$

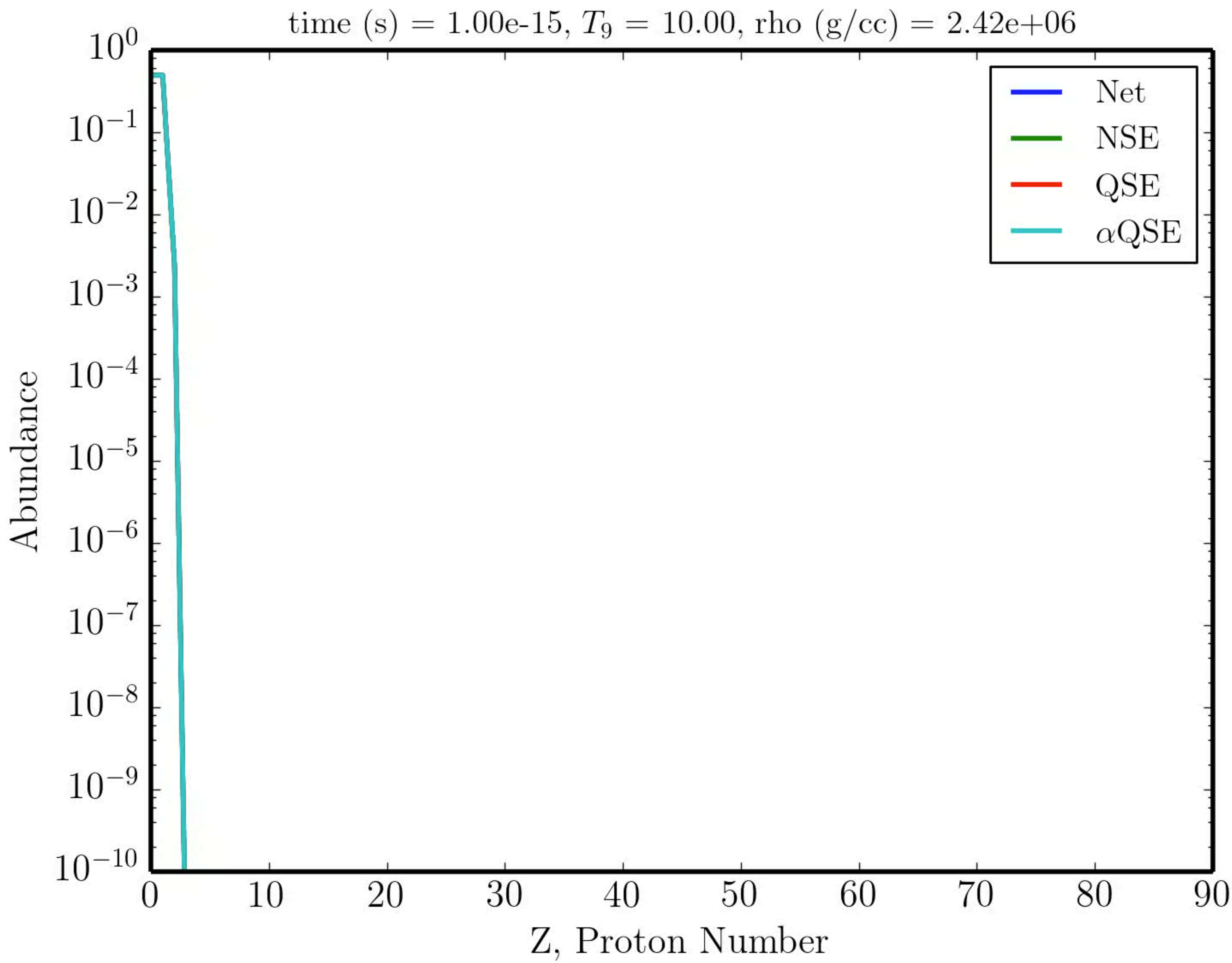


^{51}Fe ^{52}Fe ^{53}Fe ^{54}Fe ^{55}Fe ^{56}Fe ^{57}Fe ^{58}Fe ^{59}Fe ^{60}Fe
^{49}Mn ^{50}Mn ^{51}Mn ^{52}Mn ^{53}Mn ^{54}Mn ^{55}Mn ^{56}Mn ^{57}Mn ^{58}Mn ^{59}Mn
^{47}Cr ^{48}Cr ^{49}Cr ^{50}Cr ^{51}Cr ^{52}Cr ^{53}Cr ^{54}Cr ^{55}Cr ^{56}Cr ^{57}Cr ^{58}Cr
^{45}V ^{46}V ^{47}V ^{48}V ^{49}V ^{50}V ^{51}V ^{52}V ^{53}V ^{54}V ^{55}V ^{56}V ^{57}V
^{43}Ti ^{44}Ti ^{45}Ti ^{46}Ti ^{47}Ti ^{48}Ti ^{49}Ti ^{50}Ti ^{51}Ti ^{52}Ti ^{53}Ti ^{54}Ti
^{41}Sc ^{42}Sc ^{43}Sc ^{44}Sc ^{45}Sc ^{46}Sc ^{47}Sc ^{48}Sc ^{49}Sc ^{50}Sc ^{51}Sc ^{52}Sc
^{40}Ca ^{41}Ca ^{42}Ca ^{43}Ca ^{44}Ca ^{45}Ca ^{46}Ca ^{47}Ca ^{48}Ca ^{49}Ca ^{50}Ca ^{51}Ca
^{39}K ^{40}K ^{41}K ^{42}K ^{43}K ^{44}K ^{45}K ^{46}K ^{47}K
^{38}Ar ^{39}Ar ^{40}Ar ^{41}Ar ^{42}Ar ^{43}Ar ^{44}Ar ^{45}Ar
$time(s) = 0.009341$ $T_9 = 11.48$ $\rho(g/cc) = 7.926e+09$ $\mathrm{flow}_{max} = 0.1429$

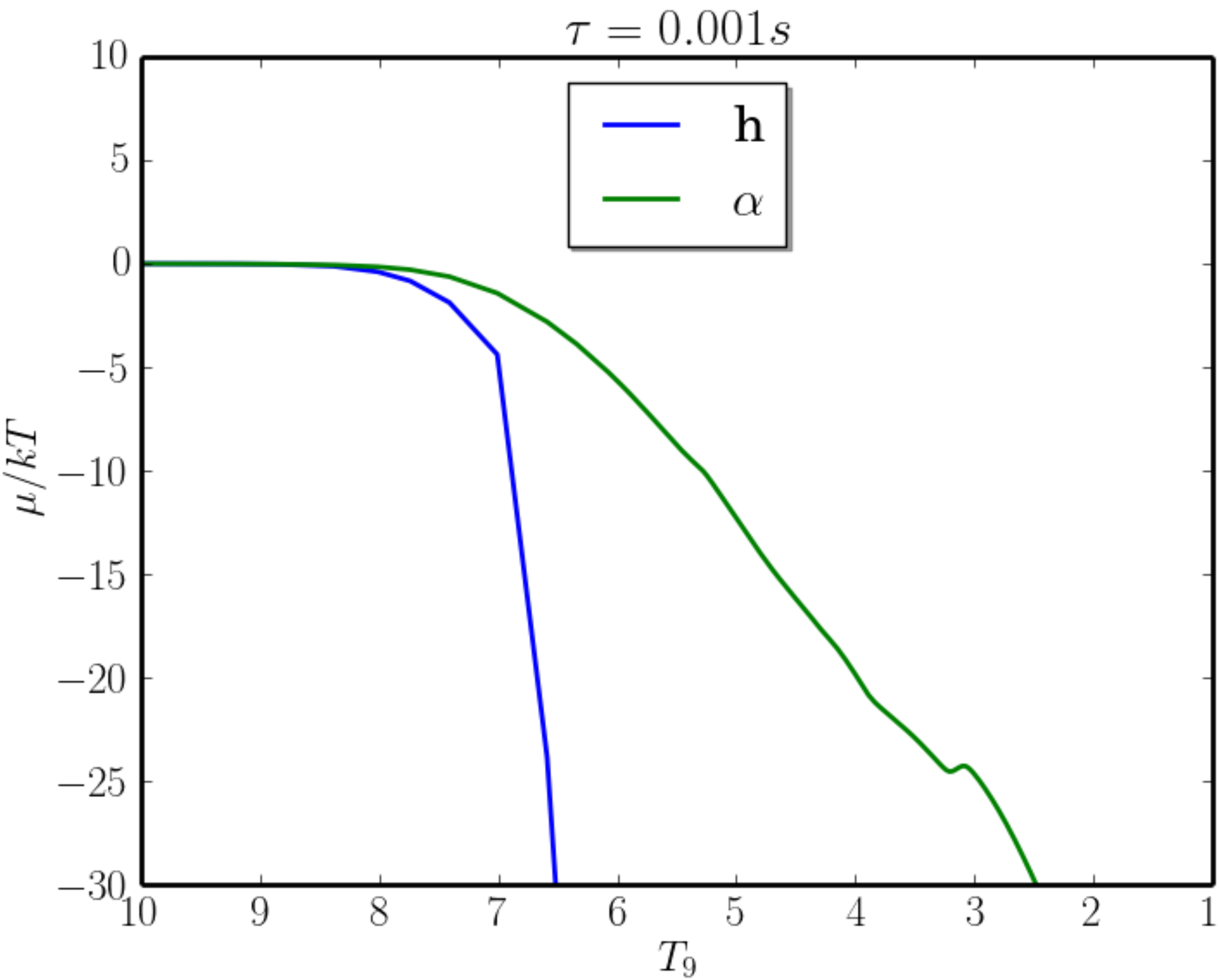

Hierarchy of Statistical Equilibria in Nucleosynthesis
(0) Equilibrium with nonconstant nucleon number
(1) NSE with weak equilibrium
(2) NSE with fixed Ye
(3) QSE (equilibrium with fixed Ye and Yh)
(4) Two QSE clusters (equilibrium with fixed Ye, Yh1, and Yh2)
(5) Three QSE clusters (equilibrium with fixed Ye, Yh1, Yh2, and Yh3)
More constraints on system
Physically possible equilibria in nucleosynthesis (constant nucleon number)
More disorder in system



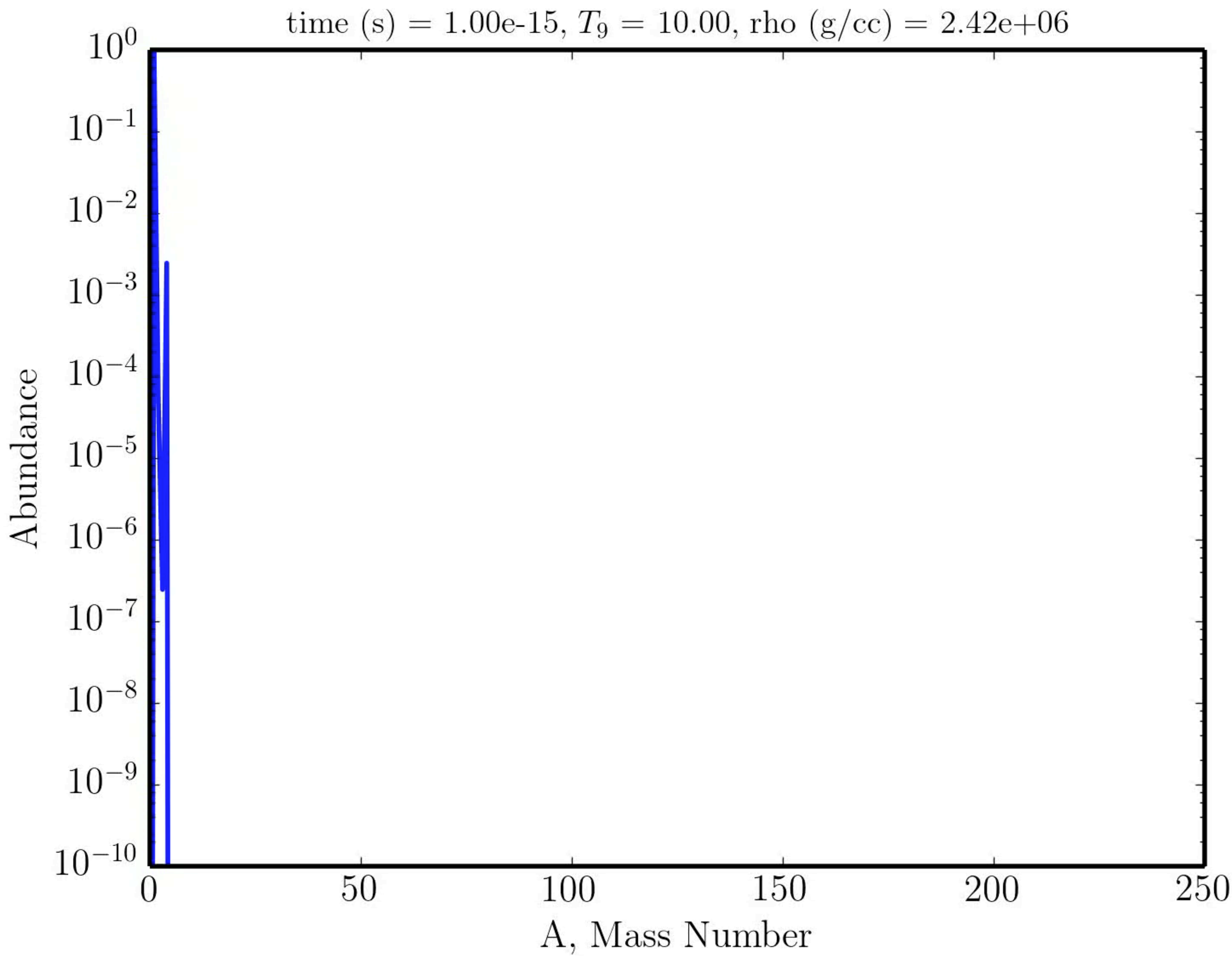
time (s) = 1.00e-15, T_9 = 10.00, rho (g/cc) = 2.42e+06
Abundance
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0
50
100
150
200
250
A, Mass Number



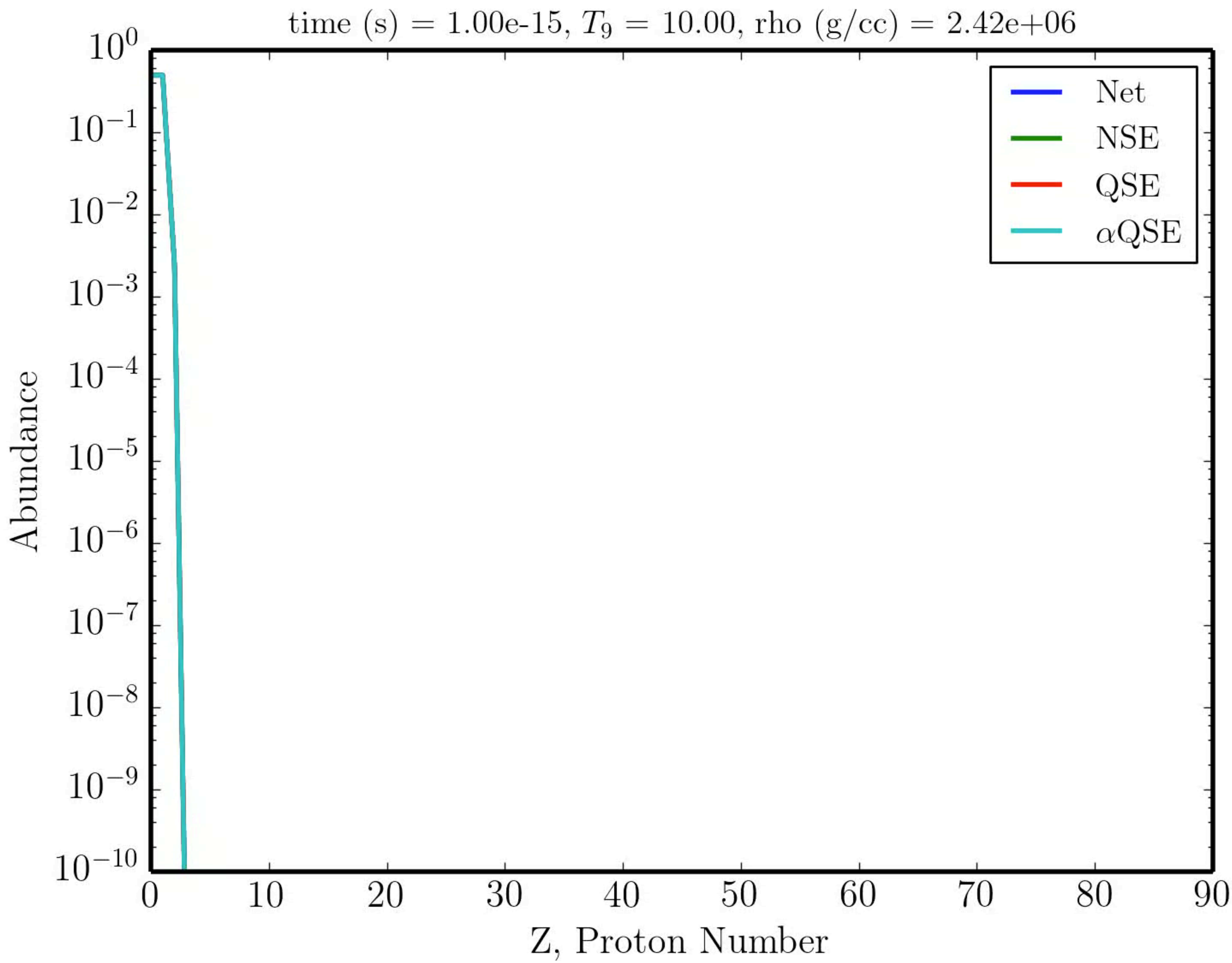
time (s) = 1.00e-15, T_9 = 10.00, rho (g/cc) = 2.42e+06
Net
NSE
QSE
αQSE
Abundance
Z, Proton Number



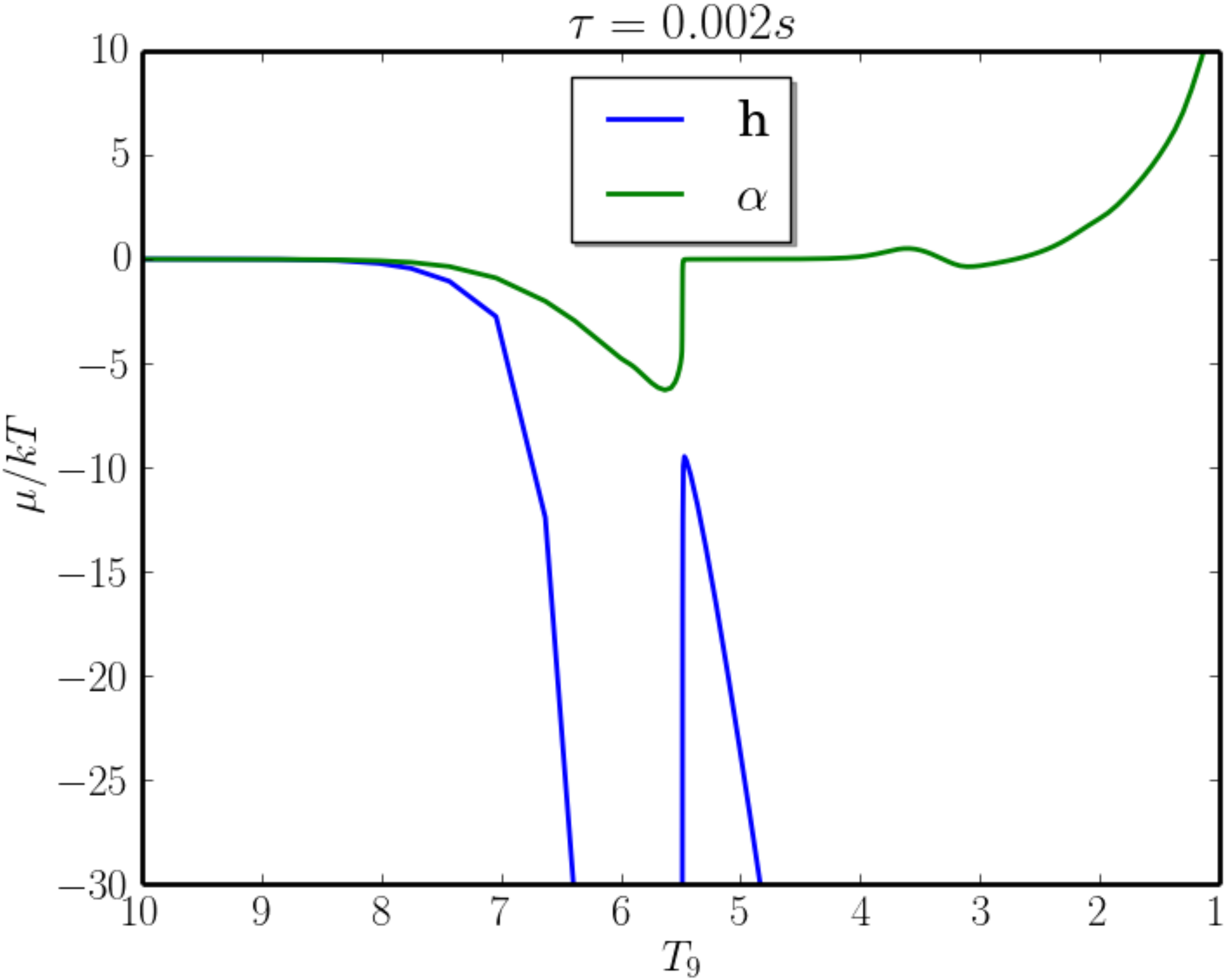
$\tau = 0.001s$
h
α
μ/kT
10
5
0
−5
−10
−15
−20
−25
−30
10
9
8
7
6
5
4
3
2
1
T_9



time (s) = 1.00e-15, T_9 = 10.00, rho (g/cc) = 2.42e+06
Abundance
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0
50
100
150
200
250
A, Mass Number



time (s) = 1.00e-15, T_9 = 10.00, rho (g/cc) = 2.42e+06
Abundance
Z, Proton Number
Net
NSE
QSE
αQSE
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0
10
20
30
40
50
60
70
80
90



$\tau = 0.002s$
h
α
μ/kT
T_9
10
5
0
−5
−10
−15
−20
−25
−30
10
9
8
7
6
5
4
3
2
1

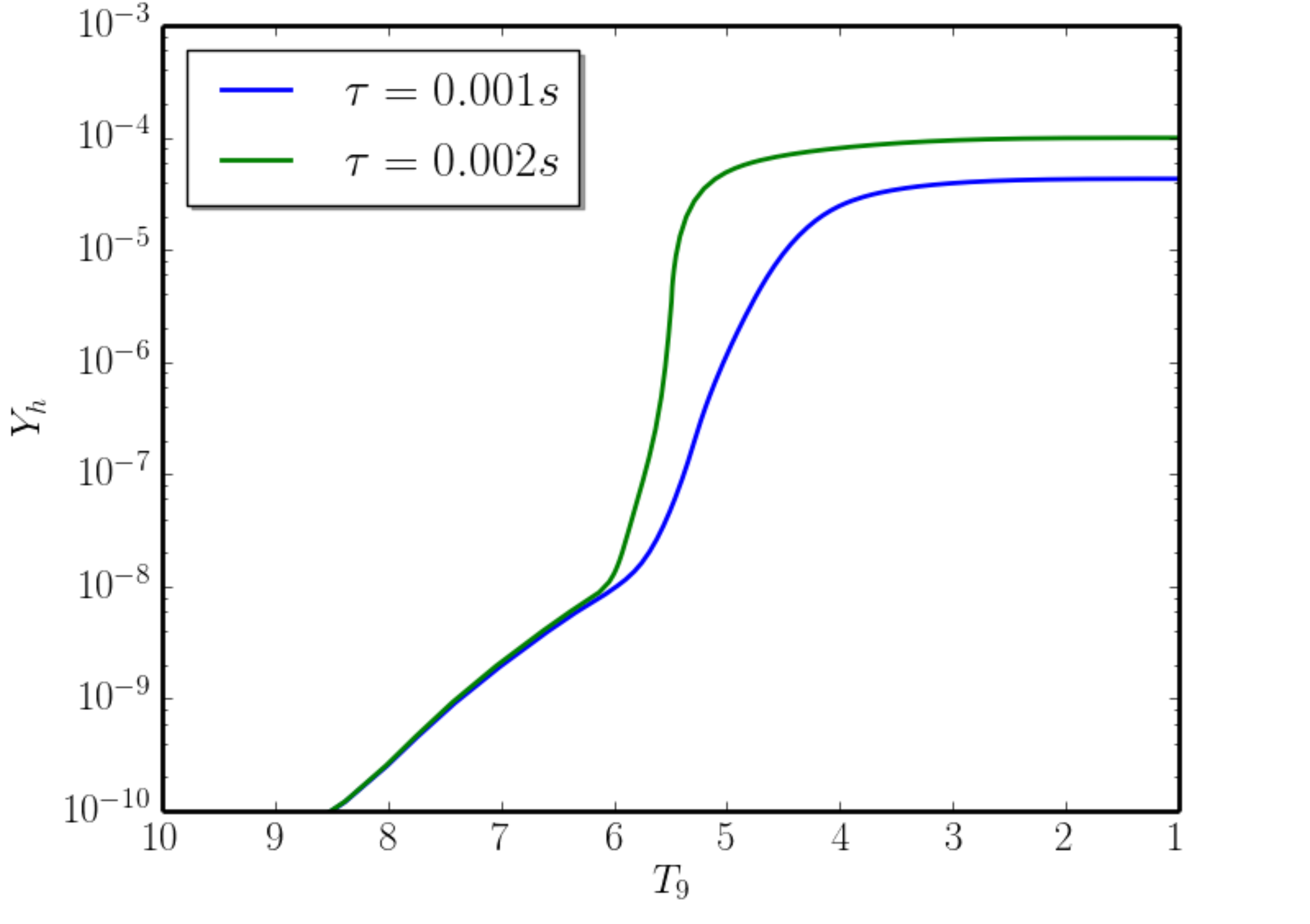
$p(n,g)d(n,g)t(p,g)^4He$

$\Rightarrow p + p + n + n \rightarrow a$

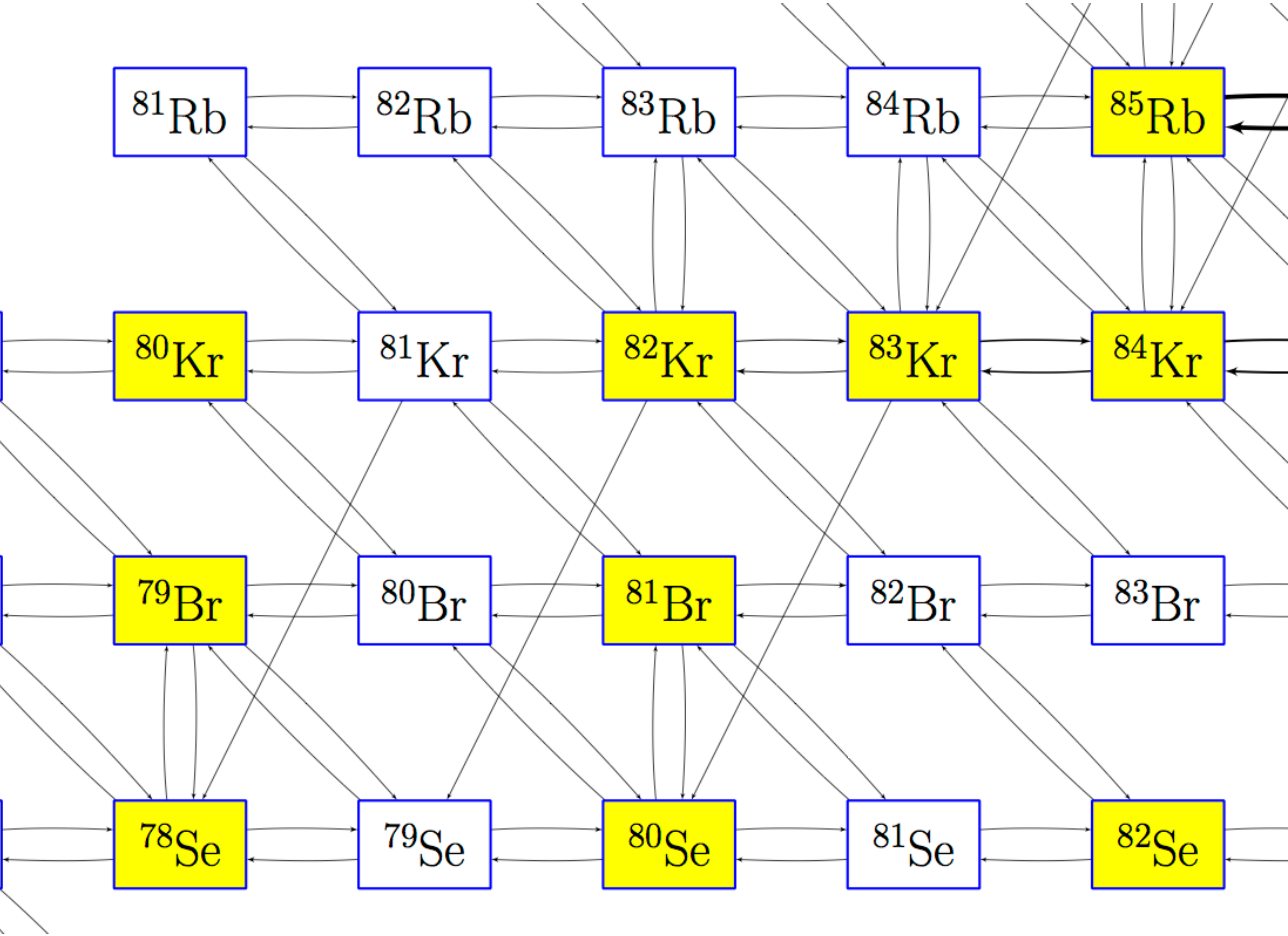
~~$p(n,g)d(n,g)t(p,g)^{4}He$~~

$^{56}Fe(p,g)57Co(p,g)^{58}Ni(n,g)^{59}Ni(n,a)^{56}Fe$

$\Rightarrow p + p + n + n \rightarrow a$

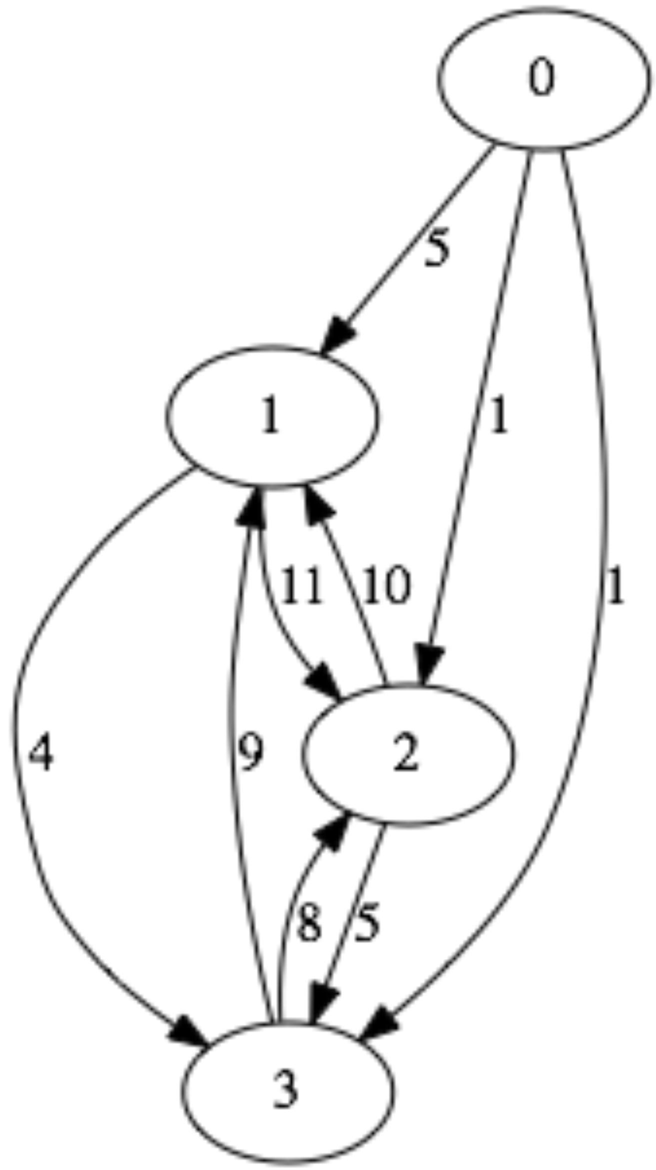


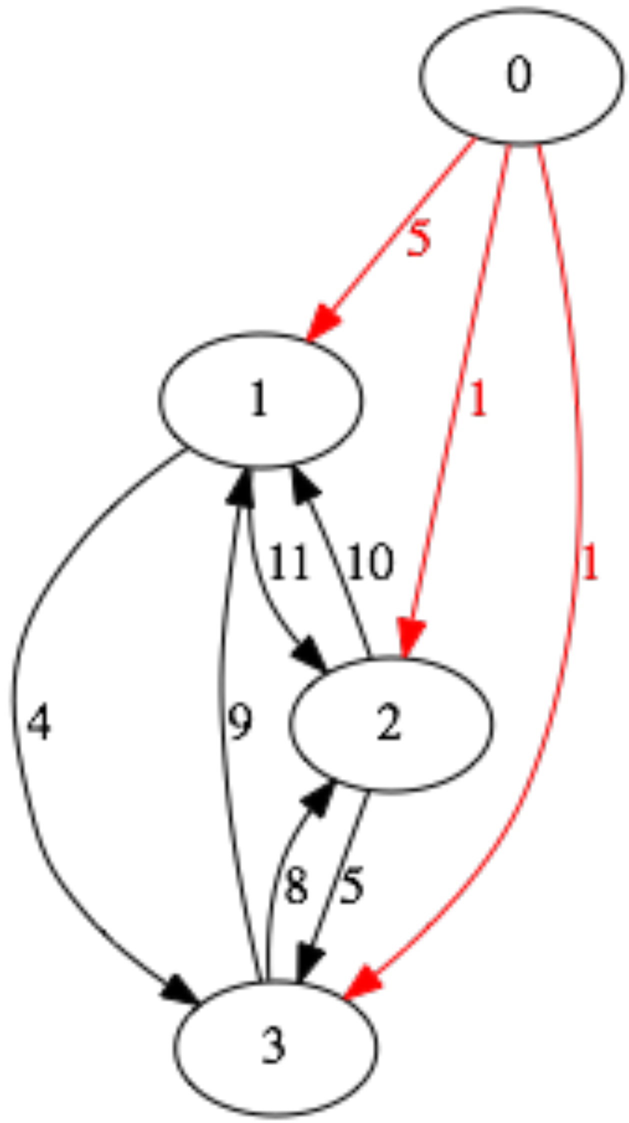
$\tau = 0.001s$
$\tau = 0.002s$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
Y_h
10 9 8 7 6 5 4 3 2 1
T_9



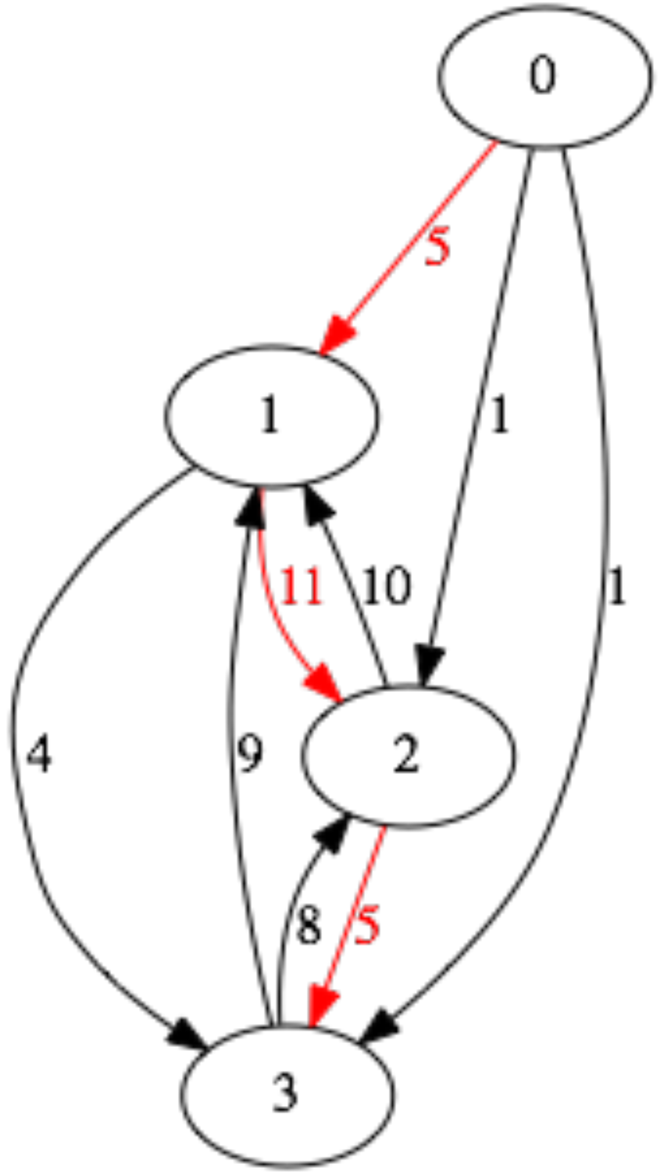
^{81}Rb
^{82}Rb
^{83}Rb
^{84}Rb
^{85}Rb
^{80}Kr
^{81}Kr
^{82}Kr
^{83}Kr
^{84}Kr
^{79}Br
^{80}Br
^{81}Br
^{82}Br
^{83}Br
^{78}Se
^{79}Se
^{80}Se
^{81}Se
^{82}Se

Dr. Oh-Hyun Kwon

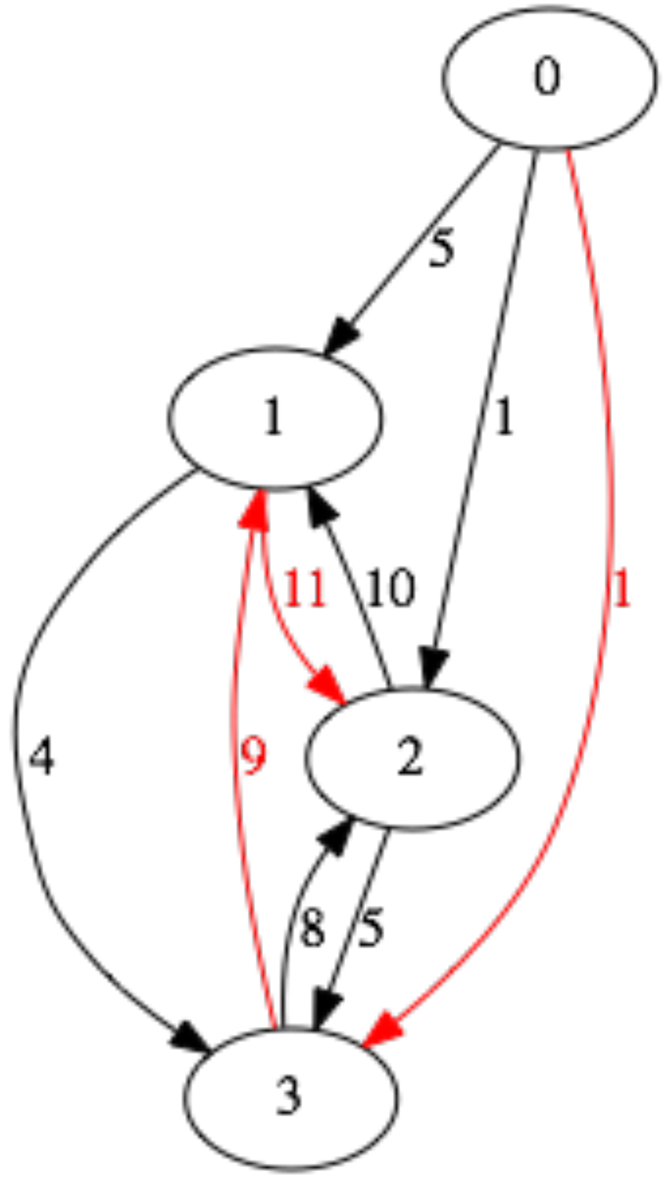

0
5
1
1
1
11
10
4
9
2
8
5
3



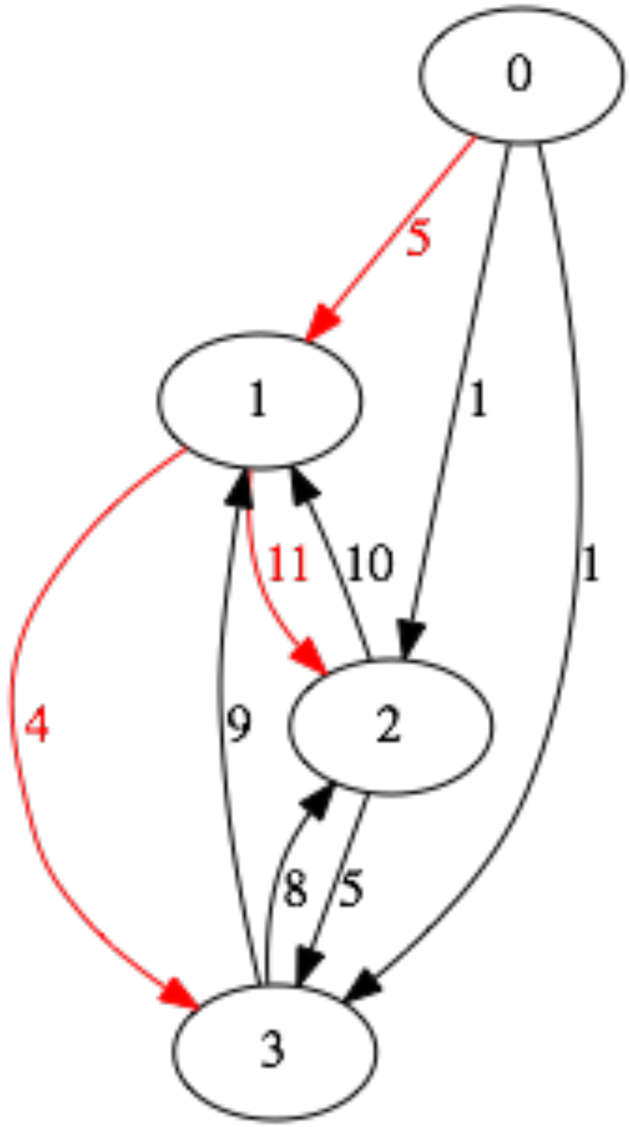
Branching Weight = 7



Branching Weight = 21



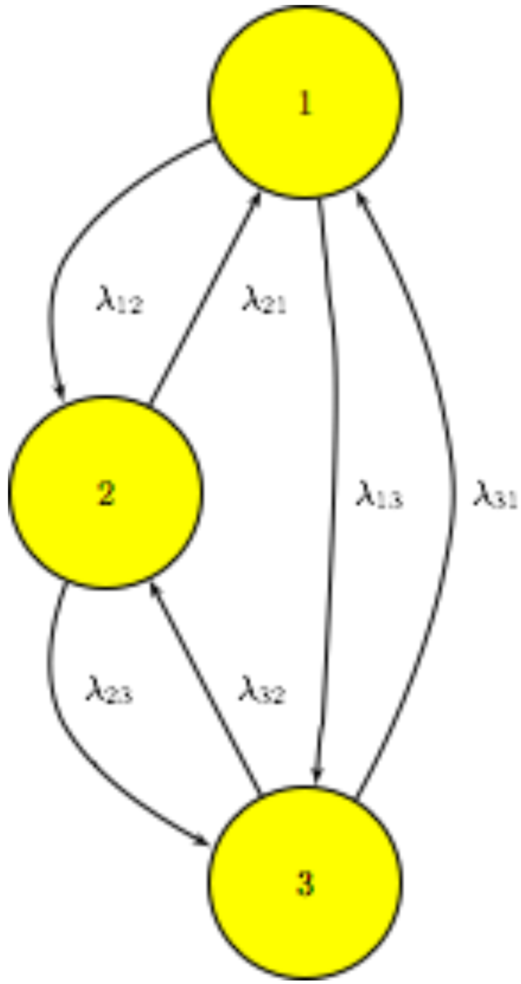
Branching Weight = 21

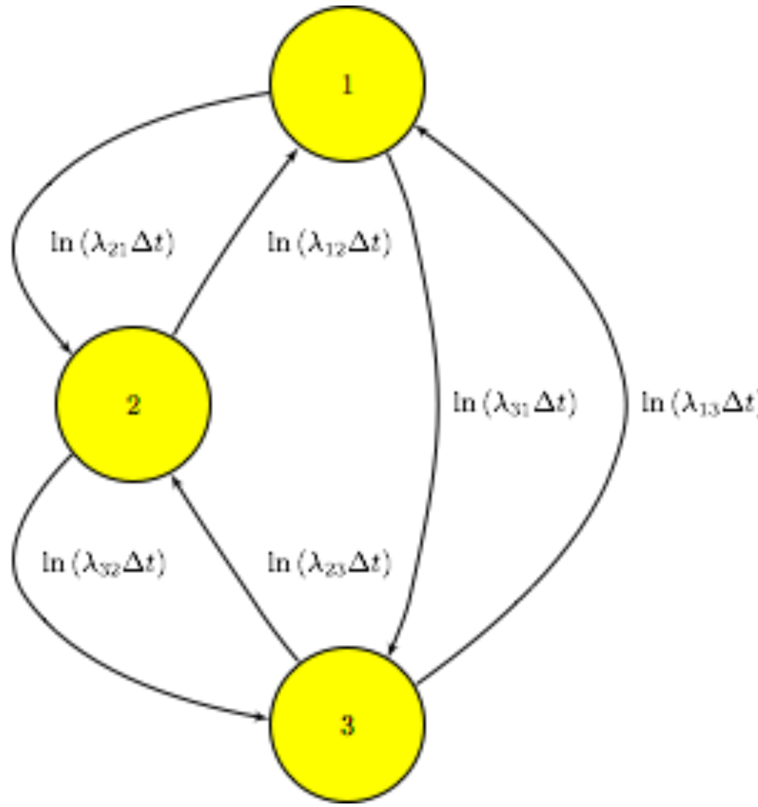


Branching Weight = 20

Matrix-Forest Theorems

- Relate matrix properties to branchings on directed graphs.
- Tutte (1984), Moon (1994), Chebotarev and Shamis (2006), Meyer and Wang (in prep).


1
2
3
λ12
λ21
λ13
λ31
λ23
λ32

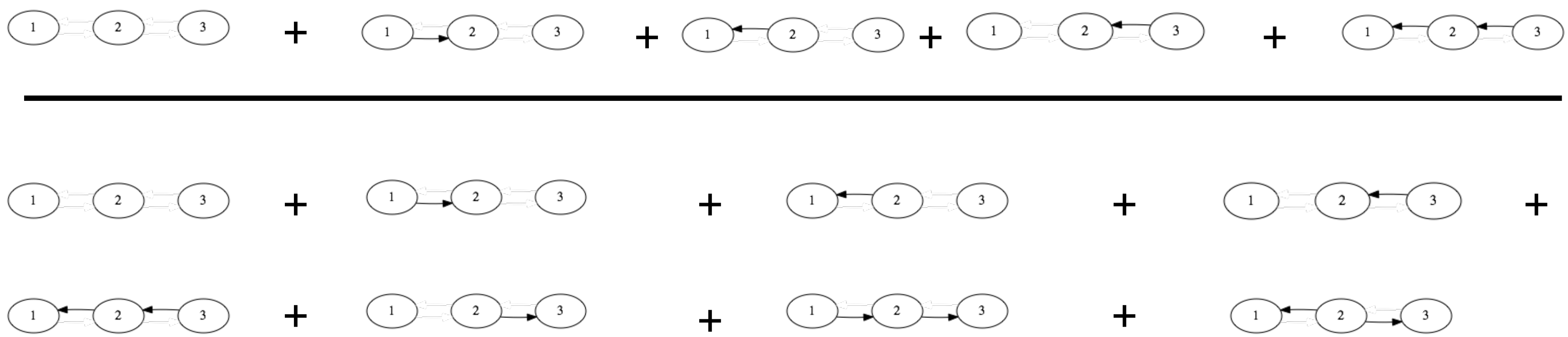

1
2
3
$\ln(\lambda_{21}\Delta t)$
$\ln(\lambda_{12}\Delta t)$
$\ln(\lambda_{31}\Delta t)$
$\ln(\lambda_{13}\Delta t)$
$\ln(\lambda_{32}\Delta t)$
$\ln(\lambda_{23}\Delta t)$

$$Y_i(t + \Delta t) = \frac{\sum_{B:(i)} e^{w(B)} \sum_{j \in B} Y_j(t)}{\sum_B e^{w(B)}}$$

$$\lambda_{j,i}^{eff} = \frac{1}{\Delta t} \frac{\sum_{B:(i)\to j} e^{w(B)}}{\sum_{B} e^{w(B)}}$$

1
2
3

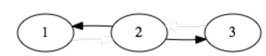
$Y_3(t + \Delta t)$



$Y_3(t + \Delta t)$ -> equilibrium

+

+



Number of Branchings in a complete simple graph of order n

$$(n+1)^{(n-1)}$$

Number of Branchings in a complete simple graph of order n

$$(n+1)^{(n-1)}$$

n = 3: 16

n = 4: 125

…

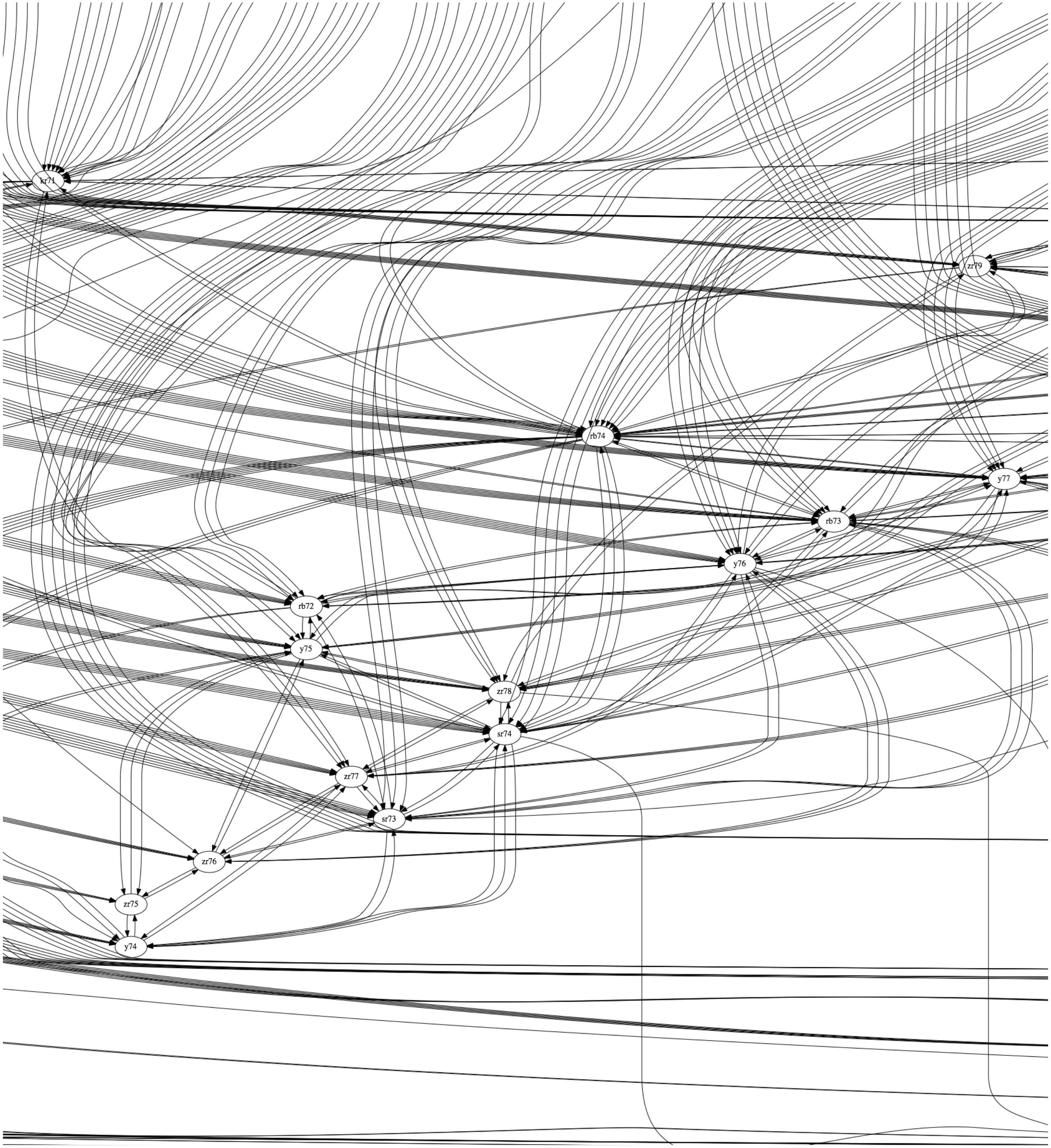
n = 9: 10^8

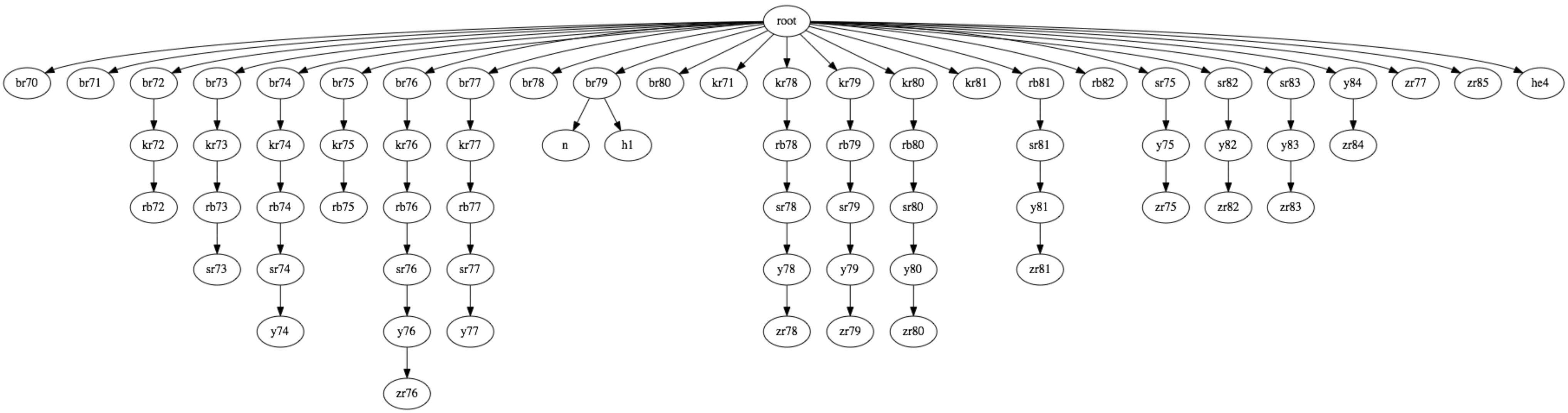
…

n = 99: 10^{196}

k-th Best Algorithms

- Optimal Branching Algorithm (Edmonds 1967 & Chi and Liu 1965)
- kth Best Branching (Camerini et al. 1980)
- $O(km \ln(n))$


kr71
zr79
rb74
y77
rb73
y76
rb72
y75
zr78
sr74
zr77
sr73
zr76
zr75
y74


root
br70
br71
br72
kr72
rb72
br73
kr73
rb73
sr73
br74
kr74
rb74
sr74
y74
br75
kr75
rb75
br76
kr76
rb76
sr76
y76
zr76
br77
kr77
rb77
sr77
y77
br78
br79
n
h1
br80
kr71
kr78
rb78
sr78
y78
zr78
kr79
rb79
sr79
y79
zr79
kr80
rb80
sr80
y80
zr80
kr81
rb81
sr81
y81
zr81
rb82
sr75
y75
zr75
sr82
y82
zr82
sr83
y83
zr83
y84
zr84
zr77
zr85
he4

By a nose

$^{78}Se(p,n)^{78}Br(n,g)^{79}Br(p,n)^{79}Kr(n,g)^{80}Kr(n,g)^{81}Kr(n,a)^{78}Se$

Why Might You Care?

- Better picture of the evolution of abundances in a nuclear reaction network, especially between rungs of the hierarchy of statistical equilibria
- Insight into which reaction rates or other nuclear properties govern the abundance evolution in reaction networks

http://nucnet-tools.sf.net