Lecture on the knockout reaction theory

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Lecture at IBS

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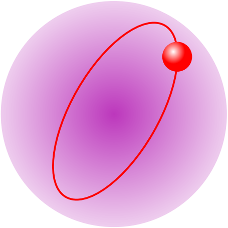
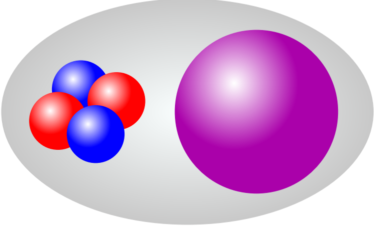
Single-particle and cluster nature

Single-particle



Clusters



- Independent nucleon picture
- Bound in a single-particle orbital
- Mean-field potential made by all nucleons

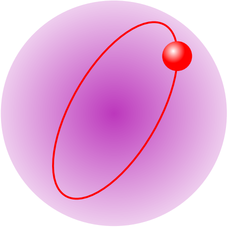
- Two or more subunits
- Nucleons are tightly bound to form a cluster
- Coupling between clusters are weak like a molecule

Note: These two states are not orthogonal, they overlap each other [1]

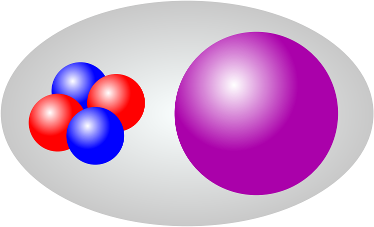
[1] B. F. Bayman and A. Bohr, Nucl. Phys. **9**, 596 (1958).

Single-particle and cluster nature

Single-particle



Clusters



Single-particle and cluster orbitals are characterized by their radial (n), orbital $(\boldsymbol{\ell})$, spin $(\boldsymbol{s})$ and total $\boldsymbol{j} = \boldsymbol{\ell} + \boldsymbol{s}$ quantum numbers.

Its amplitude

$$\varphi_{n\ell j}(R) = \langle \Phi_{\rm B} \,|\, \hat{a}_{n\ell j}(R) \,|\, \Phi_{\rm A} \rangle = \left\langle [\Phi_{\rm B} \otimes \phi(R)]_{nlj} \,\middle|\, \Phi_{\rm A} \right\rangle$$

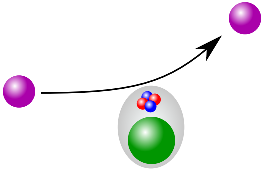
and the spectroscopic factor

$$S_{n\ell j} = \left| \langle \Phi_{\rm B} \,|\, \hat{a}_{n\ell j} \,|\, \Phi_{\rm A} \rangle \right|^2$$

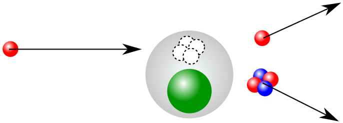
are the key quantities that characterize these nature.

Reaction probes for α clustering

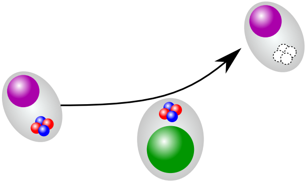
Inelastic: A(B, B)A*



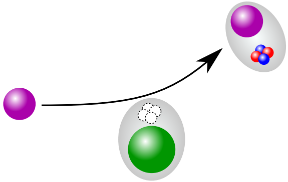
Knockout: A(p, pα)A-α



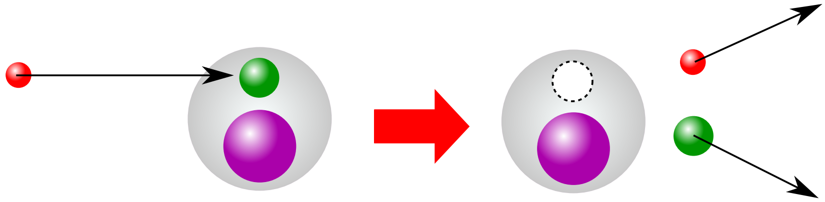
Stripping: A(B, B+α)A-α



Pickup: A(B, B-α)A+α

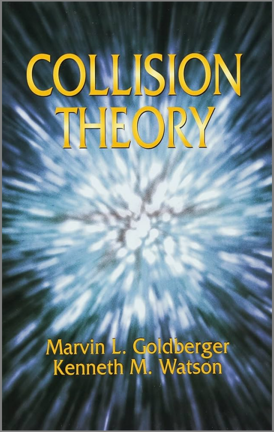


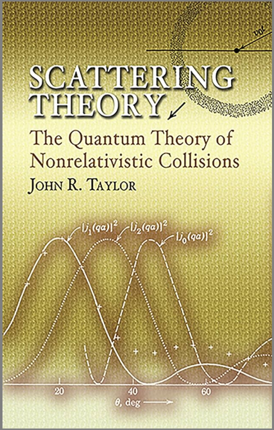
Particle knockout reaction

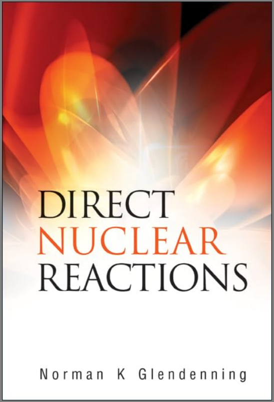


- One-step direct reaction with hundreds MeV incident energy
- Particle is knocked out by an impulse collision
- Reaction probability (cross section) is proportional to the particle probability
- Particle component only in the ground state of the target is probed
  - Little contribution from excited (resonance) states

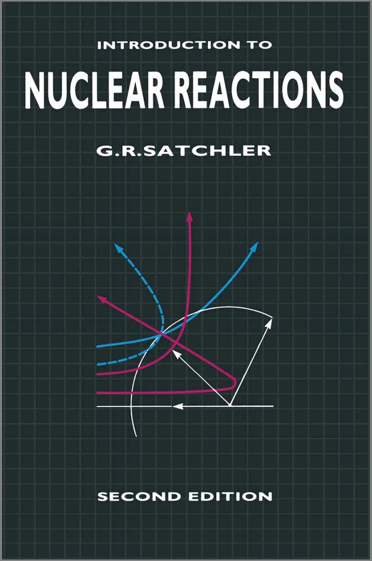
Direct nuclear reaction textbooks

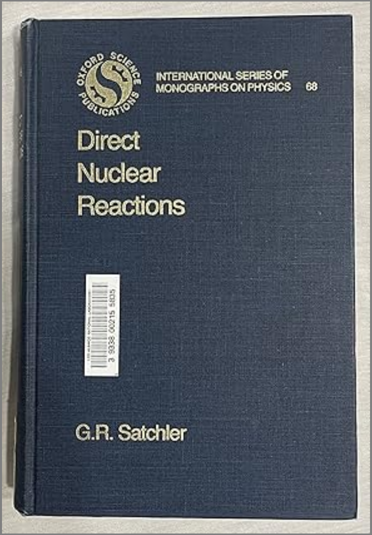




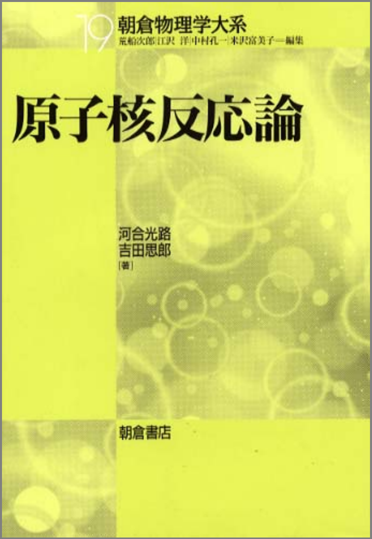


Direct nuclear reaction textbooks





Direct nuclear reaction textbooks



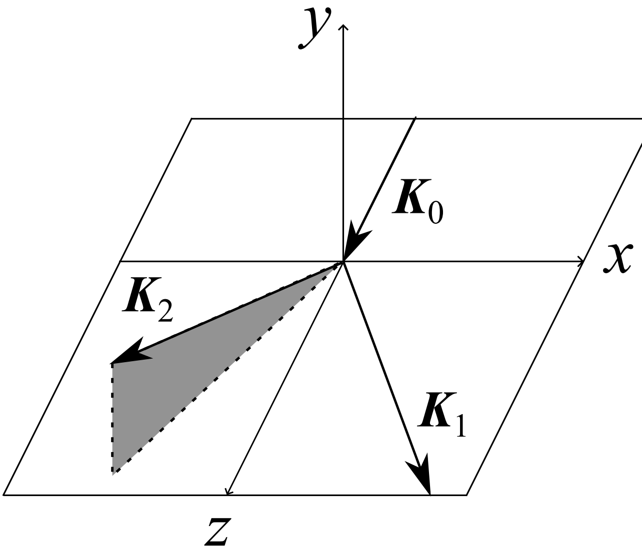
Knockout reaction references

- "Distorted-wave impulse-approximation calculations for quasifree cluster knockout reactions", N. S. Chant and P. G. Roos, PRC **15**, 57 (1977)
- "Spin orbit effects in quasifree knockout reactions", N. S. Chant and P. G. Roos, PRC **27**, 1060 (1983)
- "Proton-induced knockout reactions with polarized and unpolarized beams" T. Wakasa+, PPNP **96**, 32 (2017)
- "Quenching of single-particle strength from direct reactions with stable and rare-isotope beams" T. Aumann+, PPNP **118**, 103847 (2021)

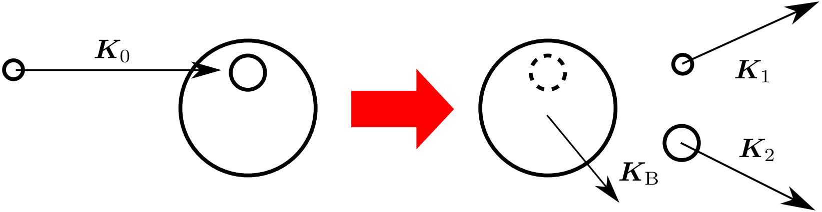
Knockout reaction references

- "PIKOE: A computer program for distorted-wave impulse approximation calculation for proton induced nucleon knockout reactions" K. Ogata+, CPC **297**, 109058 (2024)
- "On the Kinematics of (p, pX) Knockout Reactions in Normal and Inverse Kinematics" T. Uesaka, PTEP **2024**, 083D01 (2024)
- "New insight into α clustering from knockout reaction analysis" K. Yoshida, **Osaka University**, Ph.D. Thesis (2018)

Kinematics and the coordinate system



Knockout cross section



$$d\sigma = \frac{(2\pi)^4}{\hbar v} d\boldsymbol{K}_1 d\boldsymbol{K}_2 d\boldsymbol{K}_\mathrm{B} \underbrace{\delta(\boldsymbol{K}_i - \boldsymbol{K}_f)\delta(E_i - E_f)}_{\text{conservation law}} \quad \underbrace{\times |T|^2}_{\text{Transition amp.}}$$

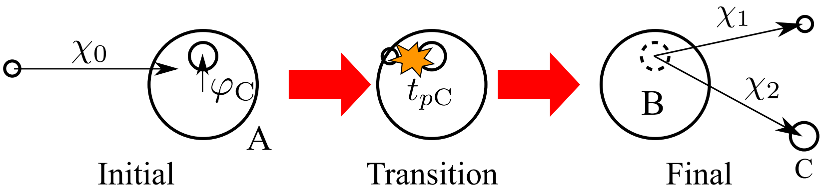
Knockout cross section (Triple differential cross section)

$$\frac{d\sigma}{dE_1 d\Omega_1 d\Omega_2} = \frac{(2\pi)^4}{\hbar v} F_\mathrm{kin} |T|^2$$

F_kin: Kinematical factor

$$F_\mathrm{kin} = \frac{E_1 K_1 E_2 K_2}{(\hbar c)^4} \left[1 + \frac{E_2}{E_\mathrm{B}} + \frac{E_2}{E_\mathrm{B}} \frac{(\boldsymbol{K}_1 - \boldsymbol{K}_0 - \boldsymbol{K}_\mathrm{A}) \cdot \boldsymbol{K}_2}{\boldsymbol{K}_2^2} \right]^{-1}$$

Reaction model: Distorted Wave Impulse Approximation



Transition matrix

$$T = \langle \chi_1 \chi_2 \Phi_{\mathrm{C}} \Phi_{\mathrm{B}} \,|\, t_{p\mathrm{C}} \,|\, \chi_0 \Phi_{\mathrm{A}} \rangle = \langle \chi_1 \chi_2 \,|\, t_{p\mathrm{C}} \,|\, \chi_0 \varphi_{\mathrm{C}} \rangle$$

χ_i: Distorted waves under optical potentials

$t_{p\mathrm{C}}$: p-C effective interaction in free space

φ_{C}: Cluster wave function $\langle [\Phi_{\mathrm{C}} \otimes \Phi_{\mathrm{B}}] \,|\, \Phi_{\mathrm{A}} \rangle$

Knockout cross section (Triple differential cross section)

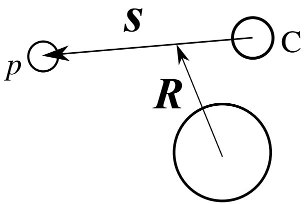
$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} \propto |T|^2$$

Plane-wave limit (PWIA)

$$
\begin{aligned}
T &\approx \langle \chi_1 \chi_2 \,|\, t_{p\mathrm{C}} \,|\, \chi_0 \varphi_\mathrm{C} \rangle \\
&\xrightarrow[\text{(P.W.)}]{} \left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}} \langle \boldsymbol{K}_1 + \boldsymbol{K}_2 - \boldsymbol{K}_0 \,|\, \varphi_\mathrm{C} \rangle_{\boldsymbol{R}} \\
&= \left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}} \langle -\boldsymbol{K}_\mathrm{B} \,|\, \varphi_\mathrm{C} \rangle_{\boldsymbol{R}} \qquad (\boldsymbol{K}_\mathrm{B} = \boldsymbol{K}_0 - \boldsymbol{K}_1 - \boldsymbol{K}_2) \\
&= \underbrace{\left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}}}_{p\text{-C collision}} \underbrace{\tilde{\varphi}_\mathrm{C}(\boldsymbol{k}_C)}_{\text{Structure}} \qquad (-\boldsymbol{K}_\mathrm{B} \approx \boldsymbol{k}_C)
\end{aligned}
$$

Knockout cross section

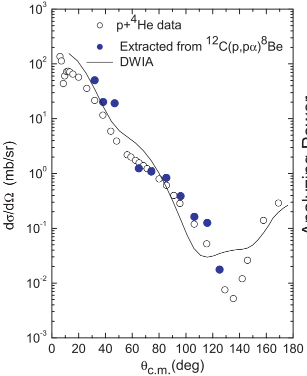
$$
|T|^2 \to \frac{d\sigma_{pC}}{d\Omega_{pC}} \, |\tilde{\varphi}_C(\boldsymbol{k}_C)|^2
$$

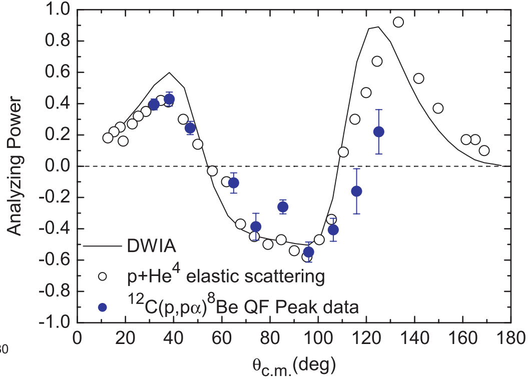


Height $\leftrightarrow$ Spectroscopic factor

Width $\leftrightarrow$ momentum distribution

Factorization: $^{12}\mathrm{C}(p, p\alpha)^{8}\mathrm{Be}$ at 100 MeV case





J. Mabiala+, PRC **79**, 054612 (2009)

Triple-differential cross section

Infinitesimal cross section of the three-body channel is given by

$$d\sigma = \frac{(2\pi)^4}{\hbar v_i} \underbrace{\boxed{d\boldsymbol{K}_1 d\boldsymbol{K}_2 d\boldsymbol{K}_\mathrm{B} \delta(E_f - E_i)\delta(\boldsymbol{K}_f - \boldsymbol{K}_i)}}_{\text{Conservation law}}$$

$$\times \underbrace{\boxed{\frac{1}{(2s_0+1)(2I_\mathrm{A}+1)}}}_{\text{spin average}} \sum_{\mu_1\mu_2\mu_\mathrm{B}\mu_0\mu_\mathrm{A}} \underbrace{\boxed{\left|T_{\mu_1\mu_2\mu_\mathrm{B}\mu_0\mu_\mathrm{A}}\right|^2}}_{T\text{-matrix}}. \qquad (1)$$

Note that the δ functions simply implies the conservation law. Only the Clebsch-Gordan coeff. in the T-matrix depends on μ_A and μ_B,

$$\sum_{\mu_\mathrm{A}\mu_\mathrm{B}} \left(j'\mu'_j I_\mathrm{B}\mu_\mathrm{B} | I_\mathrm{A}\mu_\mathrm{A}\right)\left(j\mu_j I_\mathrm{B}\mu_\mathrm{B} | I_\mathrm{A}\mu_\mathrm{A}\right) = \frac{2I_\mathrm{A}+1}{2j+1}\delta_{j'j}\delta_{\mu'_j\mu_j}, \quad (2)$$

$$d\sigma = \frac{(2\pi)^4}{\hbar v_i} d\boldsymbol{K}_1 d\boldsymbol{K}_2 \delta(E_f - E_i) \frac{1}{(2s_0+1)(2j+1)} \sum_{\mu_1\mu_2\mu_0\mu_\mathrm{j}} \left|T_{\mu_1\mu_2\mu_0\mu_\mathrm{j}}\right|^2. \quad (3)$$

Kinematical factor (Phase volume)

Doing some algebra, we can show that

$$d\boldsymbol{K}_1 d\boldsymbol{K}_2 \delta(E_f - E_i) = F_{\rm kin} dE_1 d\Omega_1 d\Omega_2 \tag{4}$$

$$F_{\rm kin} \equiv \frac{K_1 K_2 E_1 E_2}{(\hbar c)^4}\left[1 + \frac{E_2}{E_{\rm B}} - \frac{E_2}{E_{\rm B}}\frac{(\boldsymbol{K}_0 + \boldsymbol{K}_{\rm A} - \boldsymbol{K}_1)\cdot \boldsymbol{K}_2}{K_2^2}\right]^{-1} \tag{5}$$

Then

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = F_{\rm kin}\frac{(2\pi)^4}{\hbar v_i}\frac{1}{(2s_0+1)(2j+1)}\sum_{\mu_1\mu_2\mu_0\mu_{\rm j}}\left|T_{\mu_1\mu_2\mu_0\mu_{\rm j}}\right|^2. \tag{6}$$

The right hand side is usually evaluated in the center-of-mass frame. The Jacobian from G frame to X frame is defined by

$$\mathcal{J}_{XG} = \frac{E_1^{\rm G} E_2^{\rm G} E_{\rm B}^{\rm G}}{E_1^{\rm X} E_2^{\rm X} E_{\rm B}^{\rm X}}. \tag{7}$$

Knockout T-matrix: How we decouple the 3-body W.F.

The final state is difficult to handle, because it is a three-body system of 1, 2, and B. Taking the V-type coordinate, center-of-mass subtracted three body kinetic energy operator is given by

$$T_{\boldsymbol{r}_1} + T_{\boldsymbol{r}_2} + T_{\boldsymbol{r}_\mathrm{B}} - T_{\mathrm{c.m.}}$$

$$= -\frac{\hbar^2}{2\mu_{1\mathrm{B}}}\nabla^2_{\boldsymbol{R}_1} - \frac{\hbar^2}{2\mu_{2\mathrm{B}}}\nabla^2_{\boldsymbol{R}_2} - \boxed{\frac{\hbar^2}{m_\mathrm{B}}\boldsymbol{\nabla}_{\boldsymbol{R}_1}\cdot\boldsymbol{\nabla}_{\boldsymbol{R}_2}} \tag{8}$$

3-body dynamics

$$\equiv T_{\boldsymbol{R}_1} + T_{\boldsymbol{R}_2} + T_{\mathrm{coup.}} \tag{9}$$

The final-state three-body Schrödinger equation is given by

$$\left[T_{\boldsymbol{R}_1} + T_{\boldsymbol{R}_2} + T_{\mathrm{coup.}} + U_{1\mathrm{B}}(\boldsymbol{R}_1) + U_{2\mathrm{B}}(\boldsymbol{R}_2) - E_f\right]\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2) = 0,$$

$$E_f = \frac{\hbar^2}{2\mu_{1\mathrm{B}}}K_1^2 + \frac{\hbar^2}{2\mu_{2\mathrm{B}}}K_2^2 + \frac{\hbar^2}{m_\mathrm{B}}\boldsymbol{K}_1\cdot\boldsymbol{K}_2.$$

To reduce the three-body wave function into a product of two-body wave functions, we adopt the approximation [10]

$$T_{\text{coup.}} = -\frac{\hbar^2}{m_{\text{B}}}\boldsymbol{\nabla}_{\boldsymbol{R}_1} \cdot \boldsymbol{\nabla}_{\boldsymbol{R}_2} \approx \frac{\hbar^2}{m_{\text{B}}}\boldsymbol{K}_1 \cdot \boldsymbol{K}_2. \tag{10}$$

Hence, the final-state three-body wave function is decoupled as $\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2) \approx \chi_1(\boldsymbol{R}_1)\chi_2(\boldsymbol{R}_2)$ with

$$\left[T_{\boldsymbol{R}_1} + U_{1\text{B}}(\boldsymbol{R}_1) - \frac{\hbar^2 K_1^2}{2\mu_{1\text{B}}}\right]\chi_1(\boldsymbol{R}_1) = 0, \tag{11}$$

$$\left[T_{\boldsymbol{R}_2} + U_{2\text{B}}(\boldsymbol{R}_2) - \frac{\hbar^2 K_2^2}{2\mu_{2\text{B}}}\right]\chi_2(\boldsymbol{R}_2) = 0. \tag{12}$$

[10] R. D. Koshel, NPA **260**, 401 (1976).

Knockout T-matrix

Now the T-matrix has the form

$$T = \left\langle \chi_1^{(-)} \chi_2^{(-)} \Phi_{\mathrm{B}} \middle| v_{12} \middle| \chi_0^{(+)} \Phi_{\mathrm{A}} \right\rangle . \tag{13}$$

Now we assume

- B is inert and a spectator during the reaction.
- A is a bound state of B and particle 2.

Based on the Watson's multiple scattering theory,

- v_{12} can be replaced by the in-medium effective interaction, τ_{12}

and on top of that, we assume

- impulse approximation: τ_{12} can be approximated by a effective interaction in the free space, t_{12}

as discussed in the following.

Impulse approximation

Based on the multiple scattering theory [11–13], the effective interaction τ_{12} between particle 1 and 2 is constructed from a bare interaction v_{12} as

$$\tau_{12} = v_{12}(1 + G\tau_{12}), \tag{14}$$

$$G = (E - H + i\varepsilon)^{-1}. \tag{15}$$

Here, H is not the free Hamiltonian but it contains the interaction between the bound particle 2 and B, $V_{1\mathrm{B}}$. In the nucleus A rest frame,

$$G = (E_A - T_1 - T_2 - T_\mathrm{B} - V_{2\mathrm{B}} + i\varepsilon)^{-1}. \tag{16}$$

[11–13] K. M. Watson, Physical Review **89**, 575 (1953), N. C. Francis and K. M. Watson, Physical Review **92**, 291 (1953), W. B. Riesenfeld and K. M. Watson, Physical Review **102**, 1157 (1956).

Impulse approximation

$$G = (E_A - T_1 - T_2 - T_{\mathrm{B}} - V_{2\mathrm{B}} + i\varepsilon)^{-1}. \quad (17)$$

If the total energy in the A rest frame E_A is large enough compared to $V_{2\mathrm{B}}$, it can be neglected. Also, since $T_{\mathrm{B}} \approx T_1/(A-1)$, T_{B} can be neglected when A is large. Thus,

$$G \approx G_0 = (E_A - T_1 - T_2 + i\varepsilon)^{-1}. \quad (18)$$

This approximates τ_{12} as

$$\tau_{12} \approx t_{12} = v_{12}(1 + G_0 t_{12}) \quad (19)$$

and one can use the effective interaction in the free space as the transition interaction.

Knockout T-matrix

Now the T-matrix has the form (restoring the internal wave functions of the particle 0, 1 and 2),

$$T = \left\langle \chi_1^{(-)} \chi_2^{(-)} \Phi_1 \Phi_2 \Phi_{\mathrm{B}} \middle| t_{12} \middle| \chi_0^{(+)} \Phi_0 \Phi_{\mathrm{A}} \right\rangle . \tag{20}$$

We assumed that

- B is inert and a spectator during the reaction.
- A is a bound state of B and particle 2.

These conditions imply

$$\Phi_{\mathrm{A}} = [\Phi_{\mathrm{B}} \otimes \Phi_2]_{njlm} \tag{21}$$

$$T \approx \left\langle \chi_1^{(-)} \chi_2^{(-)} \Phi_1 \middle| t_{12} \middle| \chi_0^{(+)} \Phi_0 \varphi_{2,njlm} \right\rangle . \tag{22}$$

$$\varphi_{2,njlm} = \left\langle [\Phi_{\mathrm{B}} \otimes \Phi_2]_{njlm} \middle| \Phi_{\mathrm{A}} \right\rangle . \tag{23}$$

Note that if the incident particle 0 does not change its state, $\langle \Phi_1 \,|\, \Phi_0 \rangle = 1$.

Knockout T-matrix: Spin-dependent version

Applying the asymptotic momentum approx., T-matrix is factorized as (explained in detail later)

$$T = \left\langle \boldsymbol{\kappa}' \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa} \right\rangle \times \sum_m (l m s_\mathrm{N} \mu_\mathrm{N} | j \mu_j) \left\langle \chi_1^{(-)} \chi_2^{(-)} \,\middle|\, \chi_0^{(+)} \varphi_{2,nljm} \right\rangle \tag{24}$$

If I restore the spin degrees of freedom,

$$T_{\mu_1 \mu_2 \mu_0 \mu_j} = \sum_{\mu_1' \mu_2' \mu_0' \mu_\mathrm{N}} \underbrace{\left\langle \boldsymbol{\kappa}', \mu_1' \mu_2' \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0' \mu_\mathrm{N} \right\rangle}_{\text{elementary process}} \times \sum_m \underbrace{(l m s_\mathrm{N} \mu_\mathrm{N} | j \mu_j)}_{\text{LS coupling}} \underbrace{\left\langle \chi_{1,\mu_1' \mu_1}^{(-)} \chi_{2,\mu_2' \mu_2}^{(-)} \,\middle|\, \chi_{0,\mu_0' \mu_0}^{(+)} \varphi_{2,nljm} \right\rangle}_{\text{Reduced } T\text{-matrix}}$$

This is so-called the factorized DWIA, and $\left| T_{\mu_1 \mu_2 \mu_0 \mu_j} \right|^2$ is the essential quantity for the cross section, polarization obs., etc.

Knockout T-matrix: Simplification

A simpler form of the DWIA can be obtained by

- neglecting the spin-orbit part of the scattering potential

$$T_{\mu_1\mu_2\mu_0\mu_j} \to \sum_{\mu_\mathrm{N}} \left\langle \boldsymbol{\kappa}', \mu_1\mu_2 \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0\mu_\mathrm{N} \right\rangle \times \sum_{m} (lms_\mathrm{N}\mu_\mathrm{N}|j\mu_j) \left\langle \chi_1^{(-)}\chi_2^{(-)} \,\middle|\, \chi_0^{(+)}\varphi_{2,nljm} \right\rangle \quad (25)$$

- taking average over the spins in the elementary process

$$\sum_{\mu'_\mathrm{N}\mu_N} \left\langle \boldsymbol{\kappa}', \mu_1\mu_2 \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0\mu'_\mathrm{N} \right\rangle^\dagger \left\langle \boldsymbol{\kappa}', \mu_1\mu_2 \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0\mu_\mathrm{N} \right\rangle \approx \frac{1}{2s_\mathrm{N}+1} \sum_{\bar{\mu}_\mathrm{N}} \left|\left\langle \boldsymbol{\kappa}', \mu_1\mu_2 \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0\mu_\mathrm{N} \right\rangle\right|^2 \delta_{\mu'_\mathrm{N}\mu_\mathrm{N}} \quad (26)$$

so-called the cross section approximation form is obtained.

Knockout T-matrix: Simplification

$$
\begin{aligned}
&\sum_{\mu_1\mu_2\mu_0\mu_j} \left|T_{\mu_1\mu_2\mu_0\mu_j}\right|^2 \\
&= \sum_{\mu_1\mu_2\mu_0\mu_j} \sum_{\mu_\mathrm{N}\bar{\mu}_\mathrm{N}} \sum_{mm'} \frac{1}{2s_\mathrm{N}+1} \left|\left\langle \boldsymbol{\kappa}', \mu_1\mu_2 \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0\bar{\mu}_\mathrm{N} \right\rangle\right|^2 \\
&\times (lm's_\mathrm{N}\mu_\mathrm{N}|j\mu_j)(lms_\mathrm{N}\mu_\mathrm{N}|j\mu_j) \qquad \color{red}{\left(\to \frac{2j+1}{2l+1}\delta_{m'm} \text{ by } \sum_{\mu_\mathrm{N}\mu_j}\right)} \\
&\times \left\langle \chi_1^{(-)}\chi_2^{(-)} \,\middle|\, \chi_0^{(+)}\varphi_{2,nljm'} \right\rangle^\dagger \left\langle \chi_1^{(-)}\chi_2^{(-)} \,\middle|\, \chi_0^{(+)}\varphi_{2,nljm} \right\rangle \\
&= \sum_{\mu_1\mu_2\mu_0} \sum_{\bar{\mu}_\mathrm{N}} \sum_{m} \frac{1}{2s_\mathrm{N}+1} \frac{2j+1}{2l+1} \left|\left\langle \boldsymbol{\kappa}', \mu_1\mu_2 \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0\bar{\mu}_\mathrm{N} \right\rangle\right|^2 \\
&\times \left|\left\langle \chi_1^{(-)}\chi_2^{(-)} \,\middle|\, \chi_0^{(+)}\varphi_{2,nljm} \right\rangle\right|^2
\end{aligned}
\tag{27}
$$

Knockout T-matrix: Using elementary cross section

(Using the on-shell approximation) t-matrix element is related to the elementary cross section

$$\frac{d\sigma_{12}}{d\Omega_{12}}(E_{12},\theta_{12}) \propto (\text{some factor})\frac{1}{2s_0+1}\frac{1}{2s_\mathrm{N}+1} \times \sum_{\mu_1\mu_2\mu_0\bar{\mu}_\mathrm{N}} \left|\left\langle \boldsymbol{\kappa}', \mu_1\mu_2 \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0\bar{\mu}_\mathrm{N}\right\rangle\right|^2 \tag{28}$$

Then the TDX is given by

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = F_\mathrm{kin}\frac{(2\pi)^4}{\hbar v_i}\frac{(2\pi\hbar^2)^2}{\mu_{12}}\frac{1}{(2l+1)} \times \underbrace{\boxed{\frac{d\sigma_{12}}{d\Omega_{12}}(E_{12},\theta_{12})}}_{\text{elementary c.s.}} \sum_m \underbrace{\boxed{\left|\bar{T}_m\right|^2}}_{\text{reduced } T}, \tag{29}$$

$$\boxed{\bar{T}_m \equiv \left\langle \chi_1^{(-)}\chi_2^{(-)} \,\middle|\, \chi_0^{(+)}\varphi_{2,nljm}\right\rangle}$$

Knockout T-matrix: Two final forms

In summary, spin-dependnet form:

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = F_{\mathrm{kin}} \frac{(2\pi)^4}{\hbar v_i} \frac{1}{(2s_0+1)(2j+1)} \sum_{\mu_1\mu_2\mu_0\mu_{\mathrm{j}}} \left| T_{\mu_1\mu_2\mu_0\mu_{\mathrm{j}}} \right|^2 .$$

$$T_{\mu_1\mu_2\mu_0\mu_j} = \sum_{\mu_1'\mu_2'\mu_0'\mu_{\mathrm{N}}} \left\langle \boldsymbol{\kappa}', \mu_1'\mu_2' \,\middle|\, t_{12} \,\middle|\, \boldsymbol{\kappa}, \mu_0'\mu_{\mathrm{N}} \right\rangle$$

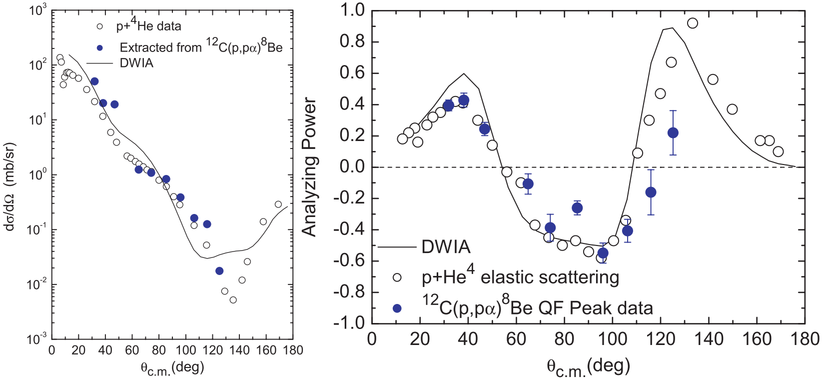
$$\times \sum_m (l m s_{\mathrm{N}} \mu_{\mathrm{N}} | j \mu_j) \left\langle \chi^{(-)}_{1,\mu_1'\mu_1} \chi^{(-)}_{2,\mu_2'\mu_2} \,\middle|\, \chi^{(+)}_{0,\mu_0'\mu_0} \varphi_{2,nljm} \right\rangle$$

Cross section factorized form:

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = F_{\mathrm{kin}} \frac{(2\pi)^4}{\hbar v_i} \frac{(2\pi\hbar^2)^2}{\mu_{12}} \frac{1}{(2l+1)} \frac{d\sigma_{12}}{d\Omega_{12}}(E_{12}, \theta_{12}) \sum_m \left| \bar{T}_m \right|^2 ,$$

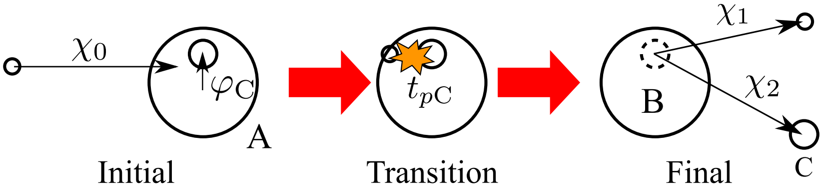
$$\bar{T}_m \equiv \left\langle \chi^{(-)}_1 \chi^{(-)}_2 \,\middle|\, \chi^{(+)}_0 \varphi_{2,nljm} \right\rangle$$

Factorization: $^{12}\mathrm{C}(p, p\alpha)^8\mathrm{Be}$ at 100 MeV case



J. Mabiala+, PRC **79**, 054612 (2009)

Reaction model: Distorted Wave Impulse Approximation



Transition matrix

$$T = \langle \chi_1 \chi_2 \Phi_C \Phi_B \,|\, t_{pC} \,|\, \chi_0 \Phi_A \rangle = \langle \chi_1 \chi_2 \,|\, t_{pC} \,|\, \chi_0 \varphi_C \rangle$$

χ_i: Distorted waves under optical potentials

t_{pC}: p-C effective interaction in free space

φ_C: Cluster wave function $\langle [\Phi_C \otimes \Phi_B] \,|\, \Phi_A \rangle$

Knockout cross section (Triple differential cross section)

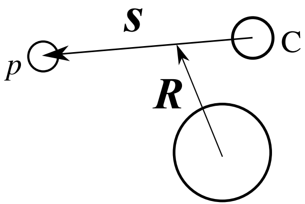
$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} \propto |T|^2$$

Plane-wave limit (PWIA)

$$
\begin{aligned}
T &\approx \langle \chi_1\chi_2 \,|\, t_{p\mathrm{C}} \,|\, \chi_0\varphi_\mathrm{C}\rangle \\
&\xrightarrow[\text{(P.W.)}]{} \left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}} \langle \boldsymbol{K}_1 + \boldsymbol{K}_2 - \boldsymbol{K}_0 \,|\, \varphi_\mathrm{C}\rangle_{\boldsymbol{R}} \\
&= \left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}} \langle -\boldsymbol{K}_\mathrm{B} \,|\, \varphi_\mathrm{C}\rangle_{\boldsymbol{R}} \qquad (\boldsymbol{K}_\mathrm{B} = \boldsymbol{K}_0 - \boldsymbol{K}_1 - \boldsymbol{K}_2) \\
&= \underbrace{\left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}}}_{p\text{-C collision}} \ \underbrace{\tilde{\varphi}_\mathrm{C}(\boldsymbol{k}_C)}_{\text{Structure}} \qquad (-\boldsymbol{K}_\mathrm{B} \approx \boldsymbol{k}_C)
\end{aligned}
$$

Knockout cross section

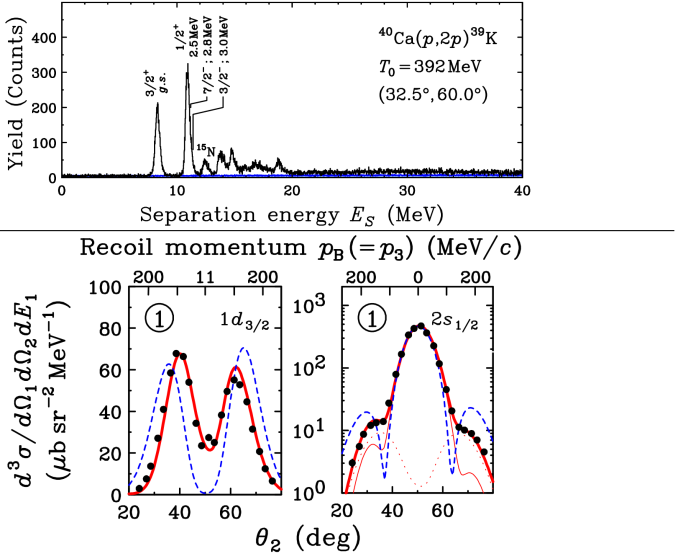
$$
|T|^2 \to \frac{d\sigma_{pC}}{d\Omega_{pC}} \, |\tilde{\varphi}_C(\boldsymbol{k}_C)|^2
$$



Height $\leftrightarrow$ Spectroscopic factor

Width $\leftrightarrow$ momentum distribution

Single-particle orbital and shape of cross section



Figs.: T. Wakasa+, PPNP **96**, 32 (2017)

Our knockout reaction code: PIKOE

Computer Physics Communications 297 (2024) 109058

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Computer Programs in Physics

PIKOE: A computer program for distorted-wave impulse approximation calculation for proton induced nucleon knockout reactions ☆

Kazuyuki Ogata [a,b,*], Kazuki Yoshida [c], Yoshiki Chazono [d]

[a] *Department of Physics, Kyushu University, Fukuoka 819-0395, Japan*
[b] *Research Center for Nuclear Physics, Osaka University, Ibaraki 567-0047, Japan*
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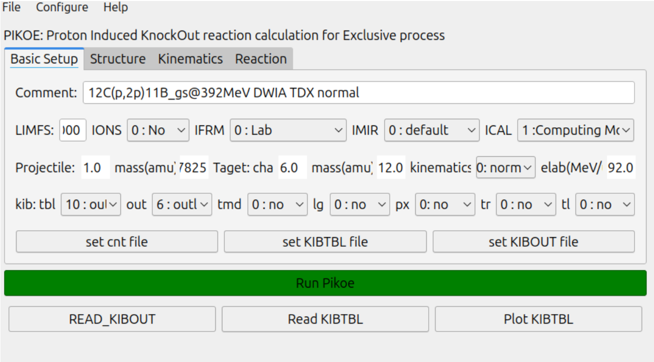
All the details can be found in Computer Physics Communications **297**, 2024, 109058, Kazuyuki Ogata, Kazuki Yoshida, Yoshiki Chazono.

See also PIKOE webpage.

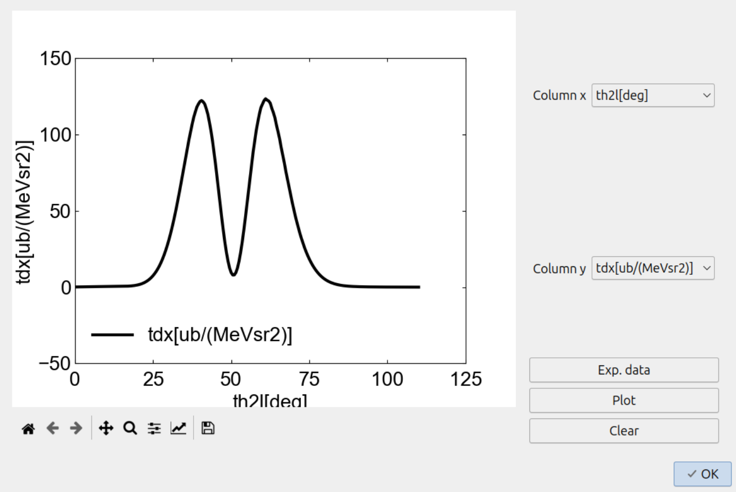
GUI version of PIKOE

- Available at github.com/alphacentaury-github/PIKOEGUI
- Under development by Dr. Young-Ho Song
- Lower barriers for new users!
- Set parameters/inputs and run the code in GUI
- Put less important parameters/inputs automatically



GUI version of PIKOE

- Quick plot of the results



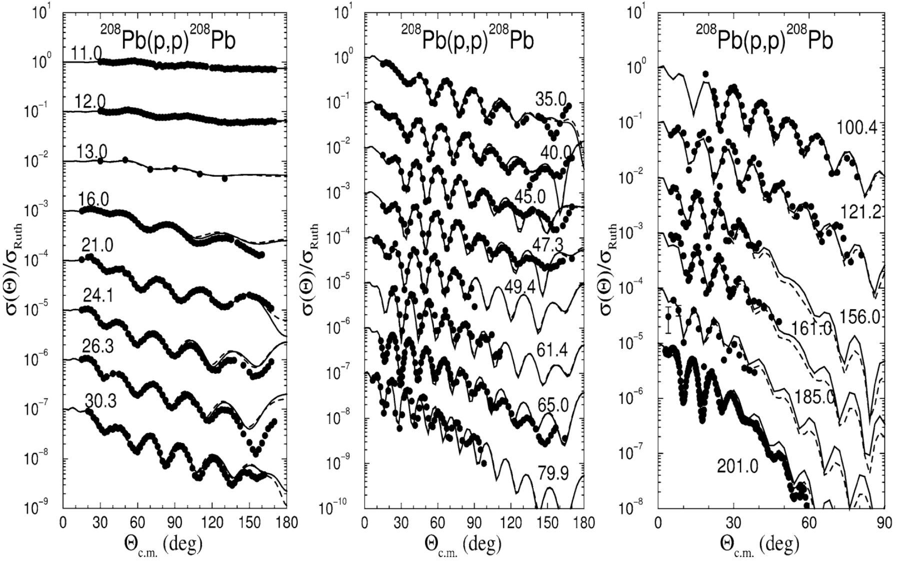
Optical potential and Distorted wave

The optical potential is a complex scattering potential U determined so that the scattering wave χ reproduce the elastic scattering:

$$\left[\frac{\hbar^2\nabla^2}{2\mu} + U(R) - E\right]\chi(\boldsymbol{R}) = 0.$$

- $U(R)$ is determined so as to reproduce the phase shift (S-matrix) of $\chi(\boldsymbol{R})$ reproduce the elastic scattering data.
- There are a lot of phenomenological optical potential fit for stable nuclei.
- There are also many microscopic approach based on NN interaction and NN degrees of freedom.

Example: p-$^{208}\mathrm{Pb}$ scattering and Koning-Delaroche pot.



Koning-Delaroche pot.

The Koning-Delaroche global optical potential (A. J. Koning and J. P. Delaroche, NPA **713**, 231 (2003)) has 6 terms,

$$\begin{aligned}\mathcal{U}(r,E) &= \mathcal{V}_V(r,E) - i\mathcal{W}_V(r,E) - i\mathcal{W}_D(r,E) \\ &\quad + \left[\mathcal{V}_{SO}(r,E) + i\mathcal{W}_{SO}(r,E)\right] \boldsymbol{\ell}\cdot\boldsymbol{\sigma} + \mathcal{V}_C(r). \end{aligned} \tag{30}$$

Using the Woods-Saxon form $f(r,R,a) = 1/(1+\exp\left[(r-R)/a\right])$, they are parametrized as

Re. Cent.: $\mathcal{V}_V(r,E) = V_V(E) f(r,R_V,a_V)$

Im. Cent.: $\mathcal{W}_V(r,E) = W_V(E) f(r,R_V,a_V)$

Im. Surf.: $\mathcal{W}_D(r,E) = -4a_D W_D(E) f'(r,R_V,a_V)$

Re. SO: $\mathcal{V}_{SO}(r,E) = V_{SO}(E)\left(\frac{\hbar}{m_\pi c}\right)^2 f'(r,R_{SO},a_{SO})/r$

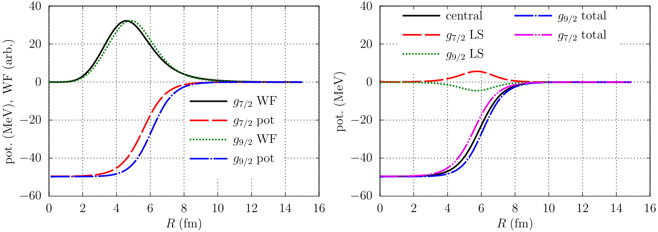
Im. SO: $\mathcal{W}_{SO}(r,E) = W_{SO}(E)\left(\frac{\hbar}{m_\pi c}\right)^2 f'(r,R_{SO},a_{SO})/r$

Coulomb: Uniformly charged sphere of radius R_C.

Single-particle wave function φ is a nucleon bound-state wave function in a mean-field potential V.

$$\left[-\frac{\hbar^2\nabla_{\boldsymbol{R}}^2}{2\mu}+V(R)-E\right]\varphi(\boldsymbol{R})=0.$$

- As a simplest one, Woods-Saxon shaped potential proposed by Bohr and Mottelson is implemented in PIKOE.
- The depth can be adjusted to reproduce the separation energy.



NN effective interaction and cross section

PHYSICAL REVIEW C VOLUME 31, NUMBER 2 FEBRUARY 1985

Nucleon-nucleon t-matrix interaction for scattering at intermediate energies

M. A. Franey
Department of Physics, University of Minnesota, Minneapolis, Minnesota 55455

W. G. Love
Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602
and Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545
(Received 9 October 1984)

- NN effective interaction and NN cross section by Franey and Love [15, 16] is implemented.

[15, 16] W. G. Love and M. A. Franey, PRC **24**, 1073 (1981), M. A. Franey and W. G. Love, PRC **31**, 488 (1985).

List of methods and ingredients for knockout reactions

Reaction theory

- Distorted wave impulse approximation (DWIA)
- Transfer-to-the-continuum (TC)
- Faddeev-AGS

Optical potentials and distorted waves

- Phenomenological optical potential (Elastic cross section data)
- Folding model
  - (in-medium) effective NN interaction + nuclear density
- Dispersive optical model
  - Consistent description of the s.p.w.f. and the distorted wave

Elementary process

- Franey-Love effective NN interaction (free space)
- In-medium NN interaction (requires density profile)
- Phase shift analysis of the NN scattering data

List of methods and ingredients for knockout reactions

Single-particle wave function / Cluster amplitude / Density profile

- Light mass region (*ab initio* theories)
  - No-core shell model
  - Gaussian expansion method
- Light to medium mass region
  - Shell model
  - Anti-symmetrized molecular dynamics (AMD)
  - THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function
  - Core + $ppnn$ 5-body model (α+^{12}C by W. Horiuchi)
- medium to heavy mass region
  - Generalized relativistic mean-field model (α density distribution on the surface of Sn isotopes by S. Typel)
  - Density functional theory (Hinohara and Nakatsukasa)

Distorted Wave Impulse Approximation (DWIA)

- Quasi-free scattering and distortion effect

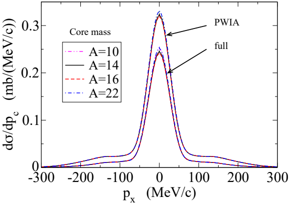
Quantum Transfer-to-the-Continuum model (QTC)

- Coupled channels problem with discretized continuum states

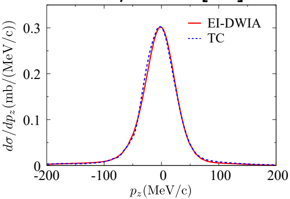
Faddeev theory

- Exact solution of the three-body scattering problem

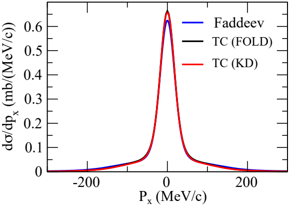
DWIA/Faddeev [17]



DWIA/QTC [18]



QTC/Faddeev [19]



[17] E. Cravo+, PRC **93**, 054612 (2016). [18] K. Yoshida+, PRC **97**, 024608 (2018). [19] M. Gómez-Ramos+, PRC **102**, 064613 (2020).

Nuclear shell and its evolution

Theoretical prediction: absence of the protons in the $1f_{7/2}$ orbit changes the neutron $1f_{5/2}$ orbit due to the lack of the tensor force

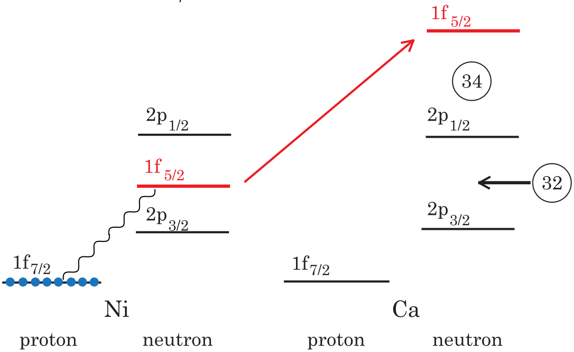
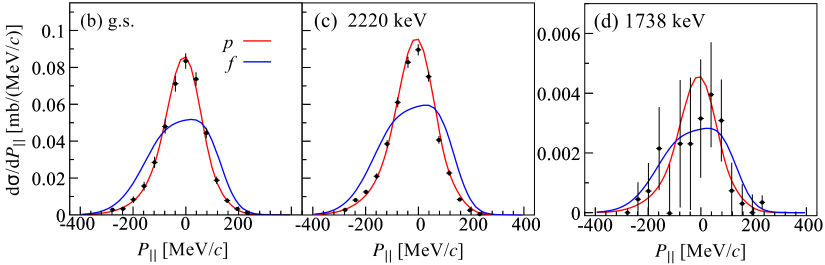


Figure from T. Otsuka and Y. Tsunoda
J. Phys. G: Nucl. Part. Phys. 43 024009 (2016).

Which orbit do neutrons occupy in $^{54}\mathrm{Ca}$ $(Z = 20,\ N = 34)$ system?

Shell Evolution And Search for Two-plus energies At RIBF

Neutron knockout reaction from ^{54}Ca [20]



$P_{\|}$: Longitudinal (beam direction) component of the momentum $\boldsymbol{Q}$

- Neutron is knocked out mostly from the p-orbit
- Six neutrons occupy the p-orbit about $85\ \%$
- f-orbit is far above p-orbit, making the $N = 34$ shell gap

[20] S. Chen+, PRL **123**, 142501 (2019).

The Maris effect

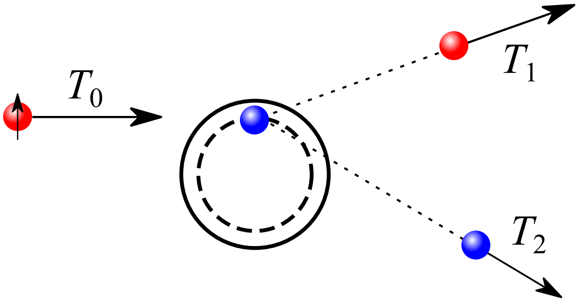
Vector analyzing power A_y: Definition

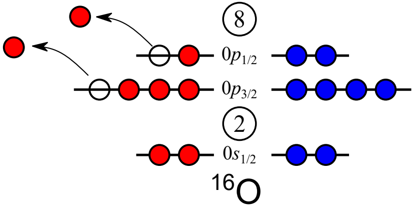
Vector analyzing power A_y of a spin $1/2$ incident particle is defined by

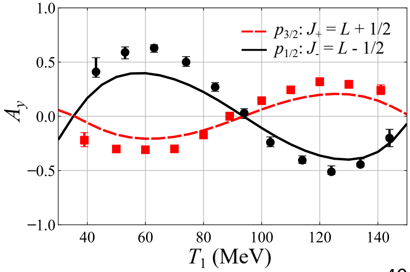
$$-1 \leq A_y = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \leq 1$$

- Cross section is a sum of cross sections with spin up (down) incident particle $\sigma = \sigma_+ + \sigma_-$
- $A_y = 1$ if σ_+ dominates the cross section ($\sigma = \sigma_+$ and $\sigma_- = 0$)
- $A_y = -1$ if σ_- dominates the cross section ($\sigma = \sigma_-$ and $\sigma_+ = 0$)

Vector analyzing power of $^{16}\mathrm{O}(p,2p)^{15}\mathrm{N}$ at 200 MeV

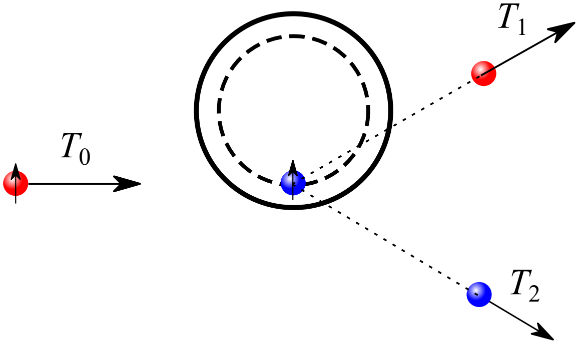






Data: P. Kitching+, NPA **340**, 423 (1980)

The Maris effect: determination of $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{s}$



With polarized beam and proper kinematics, we can distinguish the nucleon orbitals $J_+ = L + 1/2$ and $J_0 = L - 1/2$.

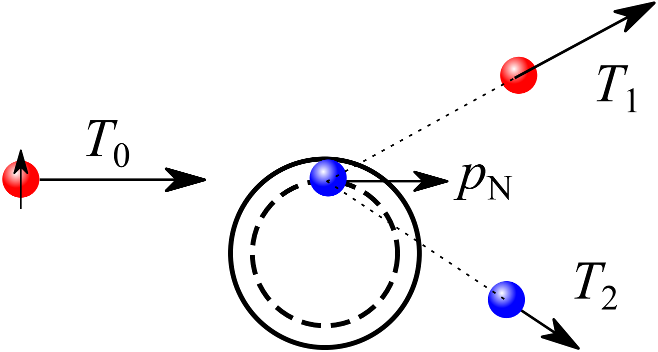
Theoretically predicted by

- G. Jacob and T. A. J. Maris, RMP **45**, 6 (1973)

and experimentally confirmed by

- P. Kitching+, PRL **37**, 1600 (1976), P. Kitching+, NPA **340**, 423 (1980)

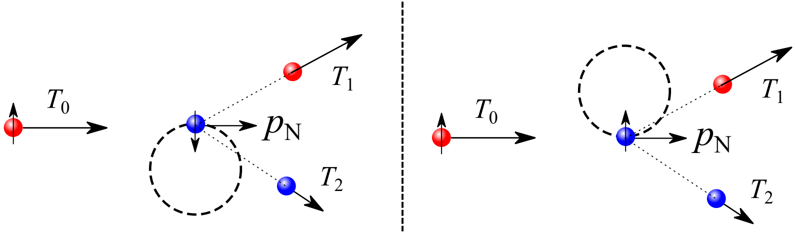
Maris effect



Important points

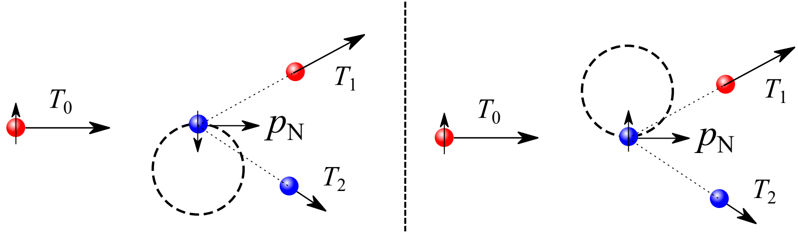
- Nucleon motion inside the target can be determined from the knockout reaction kinematics
- Imbalance between the emission energies $T_1 \gg T_2$
- Beam proton is polarized

Knockout from $J_+ = L + 1/2$

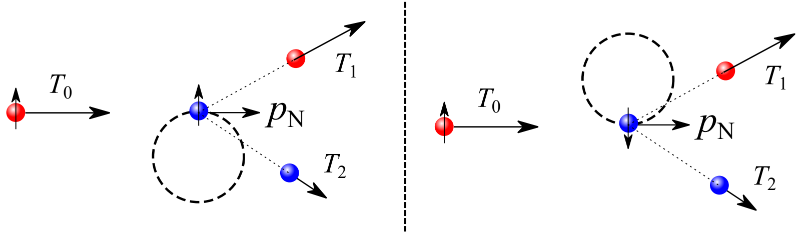


Maris effect

Knockout from $J_+ = L + 1/2$

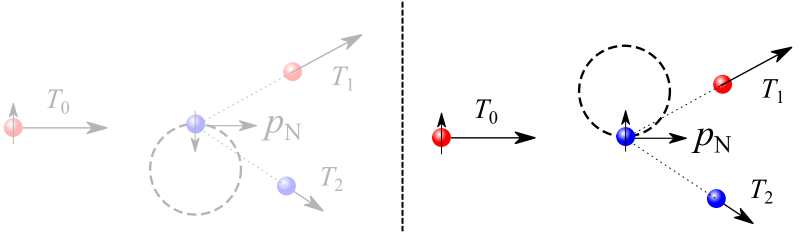


Knockout from $J_- = L - 1/2$

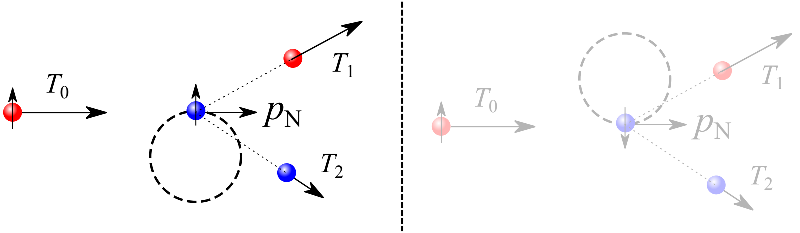


NN amplitude favors parallel configuration $|t_{++}| \gg |t_{+-}|$

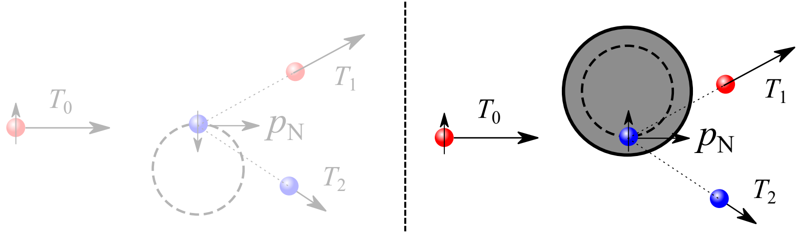
Knockout from $J_+ = L + 1/2$



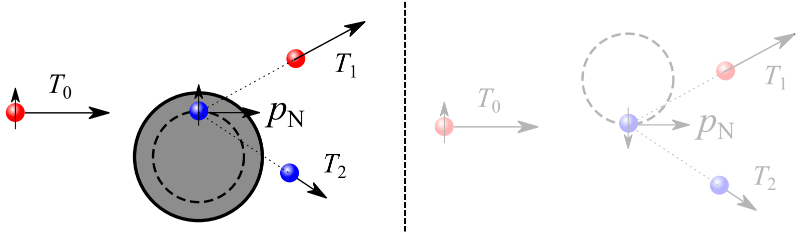
Knockout from $J_- = L - 1/2$



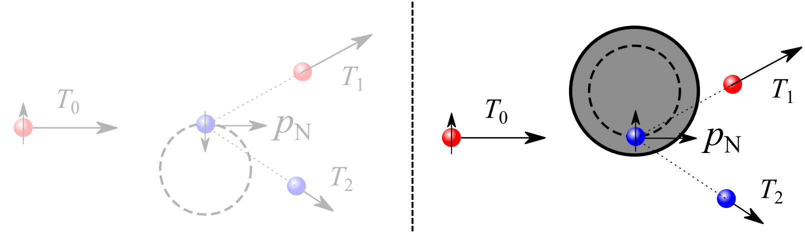
Knockout from $J_+ = L + 1/2$



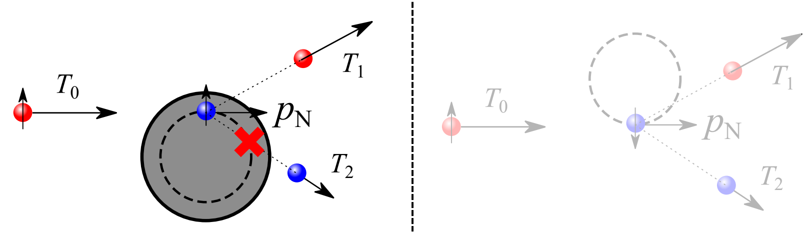
Knockout from $J_- = L - 1/2$



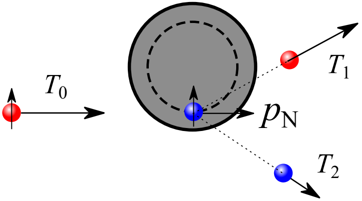
Knockout from $J_+ = L + 1/2$

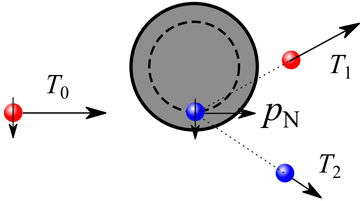


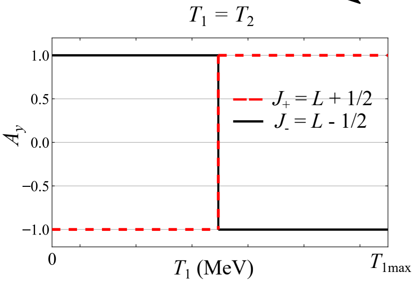
Knockout from $J_- = L - 1/2$



Knockout from $J_+ = L + 1/2$	Knockout from $J_- = L - 1/2$



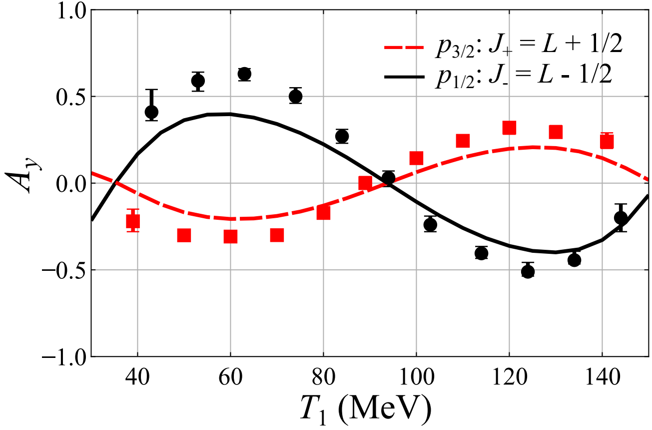




In the extreme case (strong absorption limit)

$$A_y = \begin{cases} 1 & \text{if} \quad T_1 > T_2 \\ -1 & \text{if} \quad T_1 < T_2 \end{cases}$$

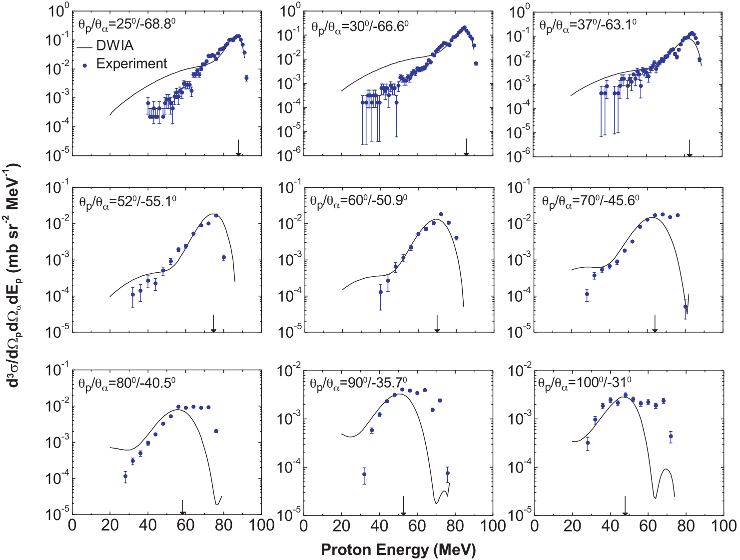
Vector analyzing power of $^{16}\mathrm{O}(p, 2p)^{15}\mathrm{N}$ at 200 MeV



Data: P. Kitching+, NPA **340**, 423 (1980)

α clustering in $^{12}\mathrm{C}$ ground state

Previous work: $^{12}\mathrm{C}(p, p\alpha)^{8}\mathrm{Be}$ at 100 MeV



J. Mabiala+, PRC **79**, 054612 (2009)

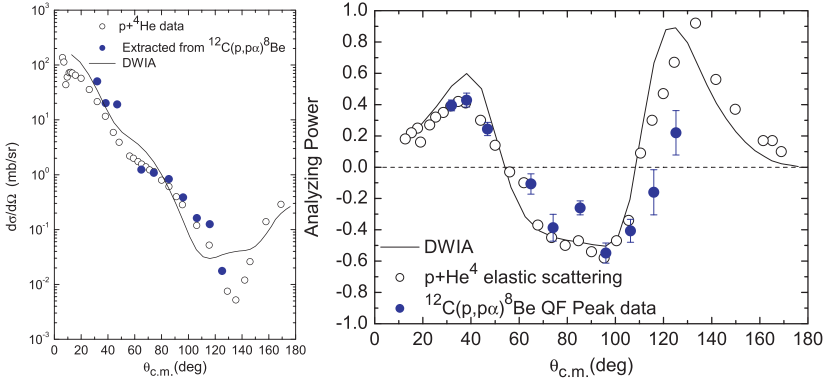
Previous work: $^{12}\mathrm{C}(p,p\alpha)^{8}\mathrm{Be}$ at 100 MeV

TABLE III. Spectroscopic factors for quasifree angle pairs extracted from the data. $\langle S_\alpha \rangle^{\mathrm{exp}}$ is a statistical average of all angles.

θ_p/θ_α (deg)	S_α	$\langle S_\alpha \rangle^{\mathrm{exp}}$	$S_\alpha^{\mathrm{theory}}$ (Ref. [18])
25/−68.8	0.59		
30/−66.6	0.93		
37/−63.1	0.65		
52/55.1	0.19		
60/−50.9	0.20	0.73 ± 0.49	0.56
70/−45.6	0.55		
80/−40.5	0.98		
90/−35.7	1.28		
100/−31.0	1.68		
110/−26.6	0.28		

J. Mabiala+, PRC **79**, 054612 (2009)
[18] D. Kurath, PRC **7**, 1390 (1973)

Previous work: $^{12}\mathrm{C}(p, p\alpha)^{8}\mathrm{Be}$ at 100 MeV



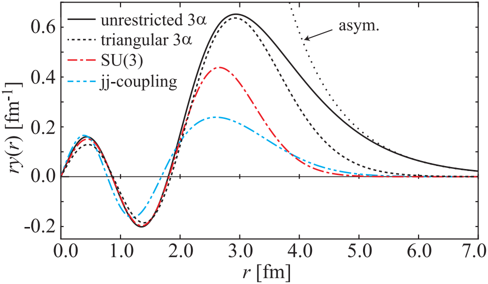
J. Mabiala+, PRC **79**, 054612 (2009)

3α (unrestricted) Triple α GCM calculation

3α (triangle) Triple α triangle shape

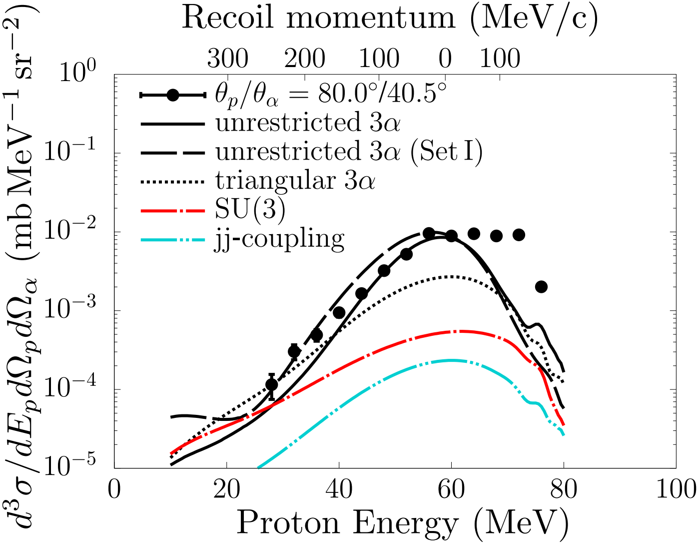
SU(3) SU(3) limit of the 3α model

jj-coupling jj-coupling shell model calculation



K. Yoshida and M. Kimura, arXiv:2606.31437 (2026)

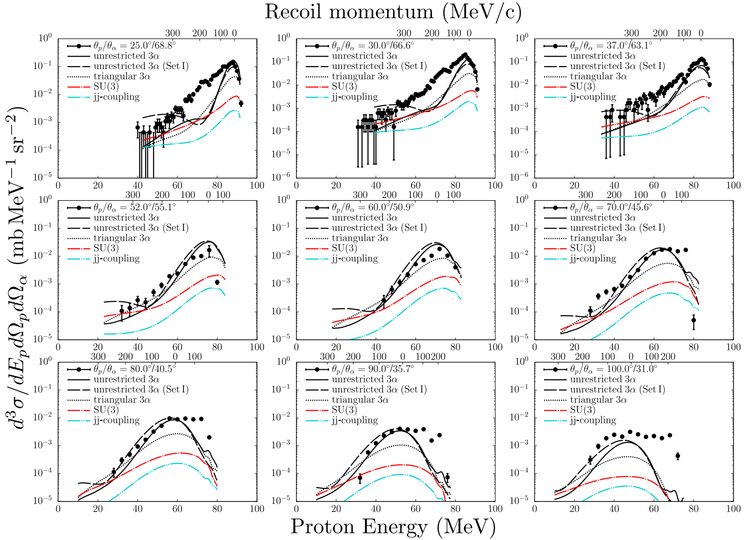
K. Yoshida and M. Kimura, arXiv:2606.31437 (2026)



Data: J. Mabiala+, PRC **79**, 054612 (2009)

Cross section results

K. Yoshida and M. Kimura, arXiv:2606.31437 (2026)



Data: J. Mabiala+, PRC **79**, 054612 (2009)

PHYSICAL REVIEW C **106**, 014621 (2022)

α knockout reaction as a new probe for α formation in α-decay nuclei

Kazuki Yoshida[1,*] and Junki Tanaka[2]

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(Received 14 November 2021; accepted 15 July 2022; published 25 July 2022)

Heavy and unstable: α formation on α-decay nuclei

α-decay lifetime and its width (independent of channel radius R)

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma},$$

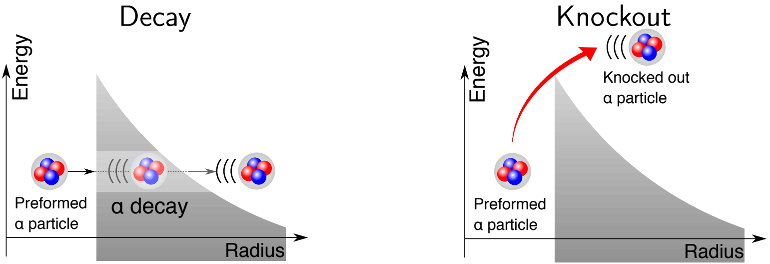
$$\Gamma = 2 \underbrace{\frac{kR}{F^2(kR) + G^2(kR)}}_{\text{Penetrability}} \underbrace{\frac{\hbar^2}{2\mu R} |RF(R)|^2}_{\text{reduced } \alpha \text{ width}}$$

$$= 2\, P(R)\, \gamma^2(R)$$

^{212}Po case, $T_{1/2} \sim 0.3\,\mu s$, which means $\Gamma \sim 1.5 \times 10^{-15}\,\mathrm{MeV}$ (cf. $Q_\alpha \sim 9.0\,\mathrm{MeV}$)

α knockout reaction from α-decay nuclei

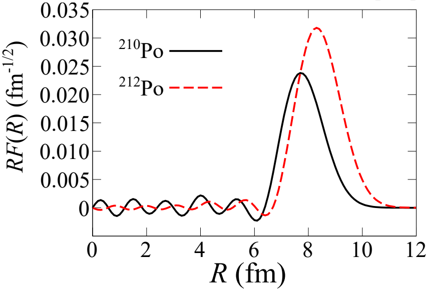
K. Yoshida and J. Tanaka, PRC **106**, 014621 (2022)



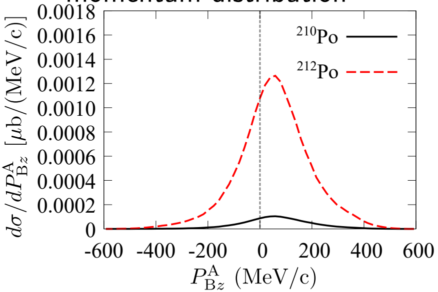
Knockout an α before it penetrates the barrier

- $T_{\text{decay}} \sim 0.3\ \mu s$ (^{212}Po), $T_{\text{knockout}} \sim 10^{-22}\ s \sim 30\ \text{fm}/c$
- Free from the penetration process, direct access to α amplitude
- Clean probe for the α component in the g.s.

α amplitude from decay study [27]



momentum distribution



Recoil momentum of the residue

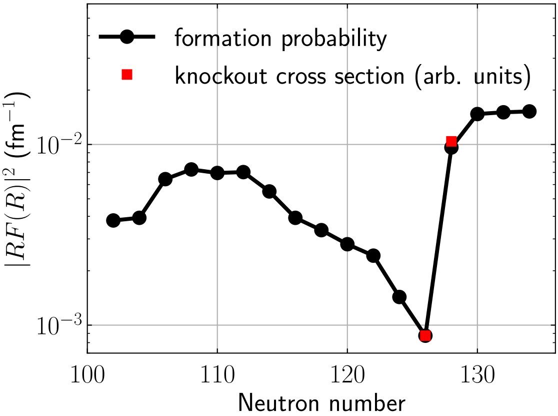
	S-factor	peak height	Cross section	$\lvert RF(R)\rvert^2$
^{212}Po / ^{210}Po	1.92	12.1	11.9	10.2

Difference is magnified by the peripherality of the reaction

$\rightarrow$ sensitive probe for preformed α particle on the surface

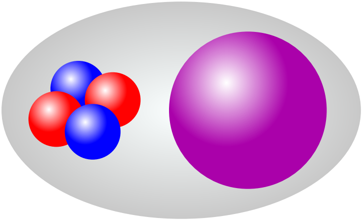
[27] C. Qi+, PRC **81**, 064319 (2010).

α knockout from ^{210,212}Po case



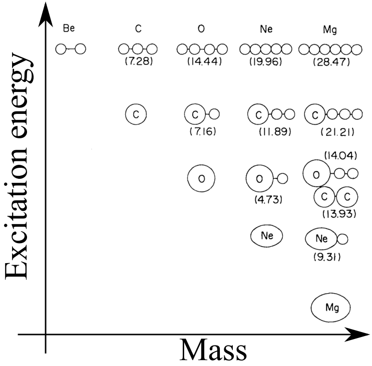
Data: A. N. Andreyev+, PRL **110**, 242502 (2013)

Difference is magnified by the peripherality of the reaction
$\rightarrow$ sensitive probe for preformed α particle on the surface



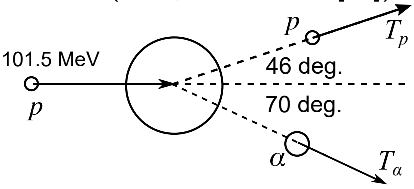
- Strong nucleon correlations makes a cluster
- Weak coupling between clusters
- What about the ground state?

Ikeda diagram [29]



[29] K. Ikeda+, PTP suppl. **E68**, 464 (1968).

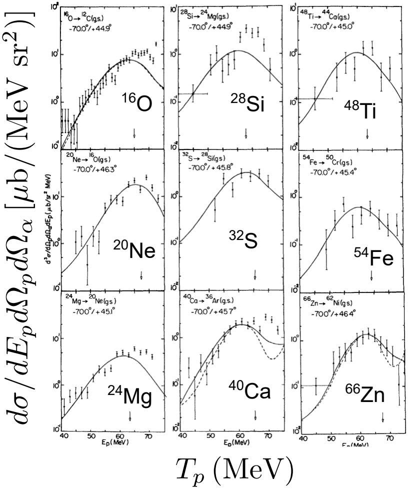
Systematic experiment of (p,$p\alpha$) reaction (Carey *et al.* 1984 [30])



Kinematical setup

Targets

- ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca, ^{48}Ti, ^{54}Fe, ^{66}Zn



[30] T. A. Carey+, PRC **29**, 1273 (1984).

PHYSICAL REVIEW C **100**, 044601 (2019)

Quantitative description of the ^{20}Ne(p, $p\alpha$)^{16}O reaction as a means of probing the surface α amplitude

Kazuki Yoshida,[1,2,*] Yohei Chiba,[3,4,2] Masaaki Kimura,[5,6,2] Yasutaka Taniguchi,[7,2]
Yoshiko Kanada-En'yo,[8,2] and Kazuyuki Ogata[2,3,4]

Antisymmetrized Molecular Dynamics (AMD)

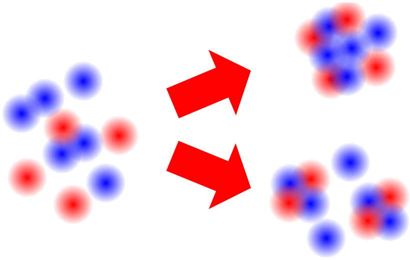
AMD wave function is a superposition of nucleon wave packets [31]

$$\varphi_i(\boldsymbol{r}_j) \propto \underbrace{\exp\left[-\nu\left(\boldsymbol{r}_j - \boldsymbol{Z}_i\right)^2\right]}_{\text{wave packet}} \times \underbrace{\left(\alpha_i \left|\uparrow\right\rangle + \beta_i \left|\downarrow\right\rangle\right)\eta_i}_{\text{spin-isospin}}$$

AMD wave function

$$\Psi_A^{\mathrm{AMD}} = \frac{1}{\sqrt{A!}}\mathcal{A}\left\{\varphi_1 \dots \varphi_A\right\}$$

NN interaction: Gogny D1S [32]



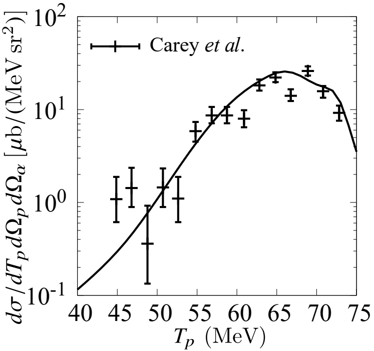
[31] Y. Kanada-En'yo+, PTEP **2012**, 01A202 (2012).
[32] J. F. Berger+, CPC **63**, 365 (1991).

$\alpha + ^{16}$O cluster state in ^{20}Ne$_{g.s.}$

Carey *et al.* (1984) [30].

- α cluster wave function by a Woods-Saxon potential
- $S_\alpha = 0.54$ (exp. + reaction)
- $S_\alpha = 0.18 – 0.23$ (Structure theory [33–35])

Inconsistent by a factor of two

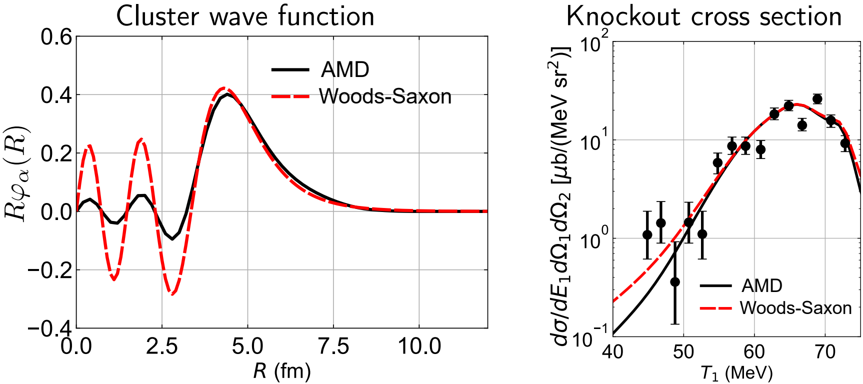


K. Yoshida *et al.* (2019) [36].

- DWIA + AMD wave function [37]
- $S_\alpha = 0.26$ (Consistent)

[30] T. A. Carey+, PRC **29**, 1273 (1984). [33] W. Chung+, PLB **79**, 381 (1978). [34] J. P. Draayer, NPA **237**, 157 (1975). [35] K. T. Hecht and D. Braunschweig, NPA **244**, 365 (1975). [36] K. Yoshida+, PRC **100**, 044601 (2019). [37] Y. Chiba and M. Kimura, PTEP **2017**, 053D01 (2017).

Peripherality of reaction and surface α amplitude



- Pauli principle is taken into account within the Antisymmetrized Molecular Dynamics (AMD) framework
- Both wave functions agree on the surface
- Knockout cross section is determined by the surface α amplitude, not the whole region (S-factor).

PHYSICAL REVIEW C **103**, L031305 (2021)

Letter

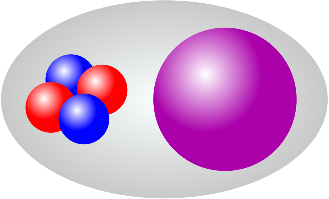
Unexpectedly enhanced α-particle preformation in ^{48}Ti probed by the $(p, p\alpha)$ reaction

Yasutaka Taniguchi,[1,2,*] Kazuki Yoshida,[3,2] Yohei Chiba,[4,5,2] Yoshiko Kanada-En'yo,[6,2]
Masaaki Kimura,[7,8,2] and Kazuyuki Ogata[2,4,5]
[1]*Department of Information Engineering, National Institute of Technology (KOSEN), Kagawa College, Mitoyo, Kagawa 769-1192, Japan*
[2]*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan*
[3]*Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan*
[4]*Department of Physics, Osaka City University, Osaka 558-8585, Japan*
[5]*Nambu Yoichiro Institute of Theoretical and Experimental Physics (NITEP), Osaka City University, Osaka 558-8585, Japan*
[6]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[7]*Department of Physics, Hokkaido University, Sapporo 060-0810, Japan*
[8]*Nuclear Reaction Data Centre, Hokkaido University, Sapporo 060-0810, Japan*

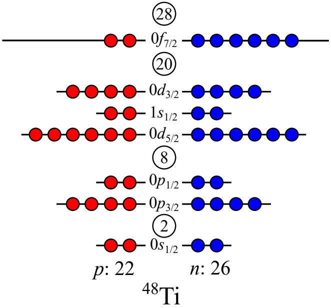
(Received 7 December 2020; accepted 16 March 2021; published 24 March 2021)

α-^{44}Ca cluster of ^{48}Ti

α-^{44}Ca cluster of ^{48}Ti and ^{48}Ti(p,$p\alpha$)^{44}Ca reaction [38]



α+^{44}Ca cluster of ^{48}Ti

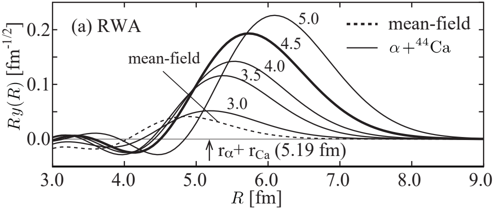


- $Z \neq N$ stable nucleus
- magic number 20 core $^{40}\mathrm{Ca}$ + (2p + 6n)
- Structure theories based on the nucleon degrees of freedom are available

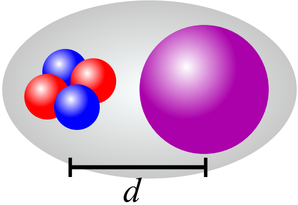
[38] Y. Taniguchi+, PRC **103**, L031305 (2021).

α+^{44}Ca cluster and ^{48}Ti(p,$p\alpha$)^{44}Ca reaction

Cluster amplitude



Inter-cluster distance d

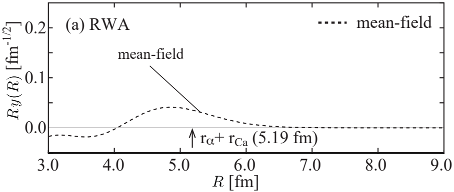


- Stricture theory based on the nucleon degrees of freedom (Antisymmetrized molecular dynamics: AMD)
- Constraint on the inter-cluster distance d

[38] Y. Taniguchi+, PRC **103**, L031305 (2021).
α-^{44}Ca potential:[39] T. Delbar+, PRC **18**, 1237 (1978).

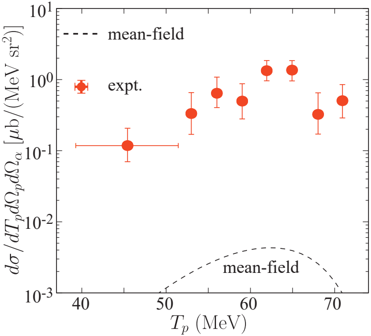
α+^{44}Ca cluster and ^{48}Ti(p,$p\alpha$)^{44}Ca reaction

Cluster amplitude



- Mean-field like structure fails to reproduce the reaction data

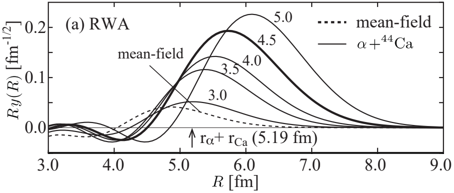
Knockout cross section



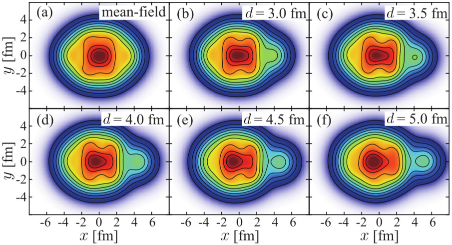
[38] Y. Taniguchi+, PRC **103**, L031305 (2021).

$\alpha+{}^{44}$Ca cluster and ^{48}Ti$(p,p\alpha){}^{44}$Ca reaction

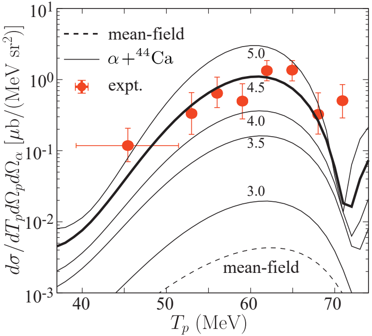
Cluster amplitude

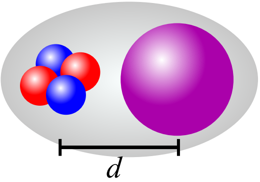


Density



Knockout cross section

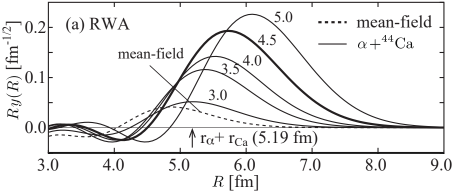




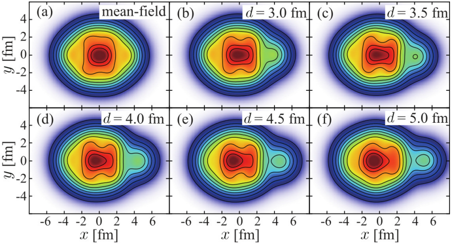
[38] Y. Taniguchi+, PRC **103**, L031305 (2021).

α+^{44}Ca cluster and ^{48}Ti(p,$p\alpha$)^{44}Ca reaction

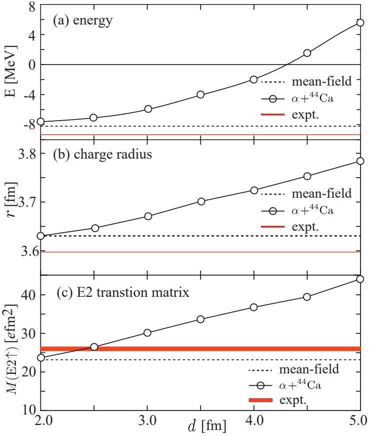
Cluster amplitude



Density



Energy, radius, M(E2)



Inter cluster distance

[38] Y. Taniguchi+, PRC **103**, L031305 (2021).

Is α-clustering universal?

Structure theory

- S. Typel, PRC **89**, 064321 (2014)
- α particle formation on Sn ($Z = 50$) surface

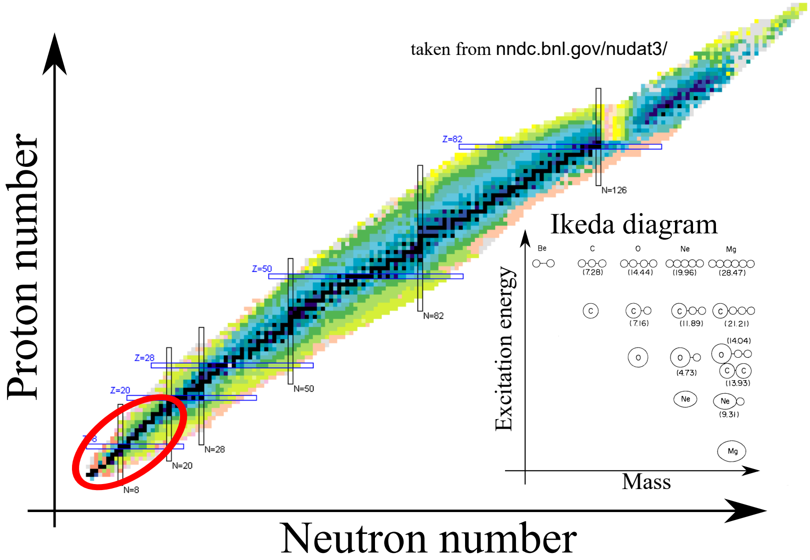
Reaction theory

- K. Yoshida$+$, PRC **94**, 044604 (2016)
- Proton induced α knockout reaction is suitable to investigate the α clustering on nuclear surface

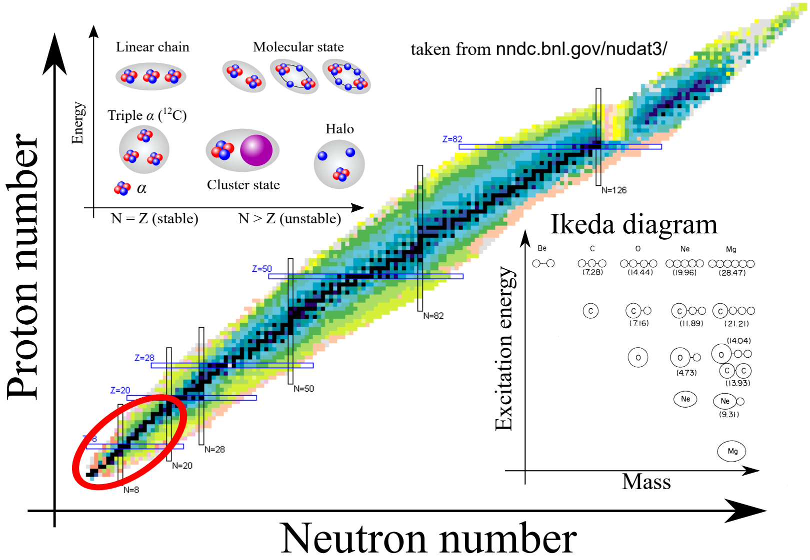
Experiment

- J. Tanaka and Z.-H. Yang et al, Science 371, 260 (2021)
- α clustering on Sn surface was quantitatively confirmed and isotope dependence well agreed the theoretical prediction

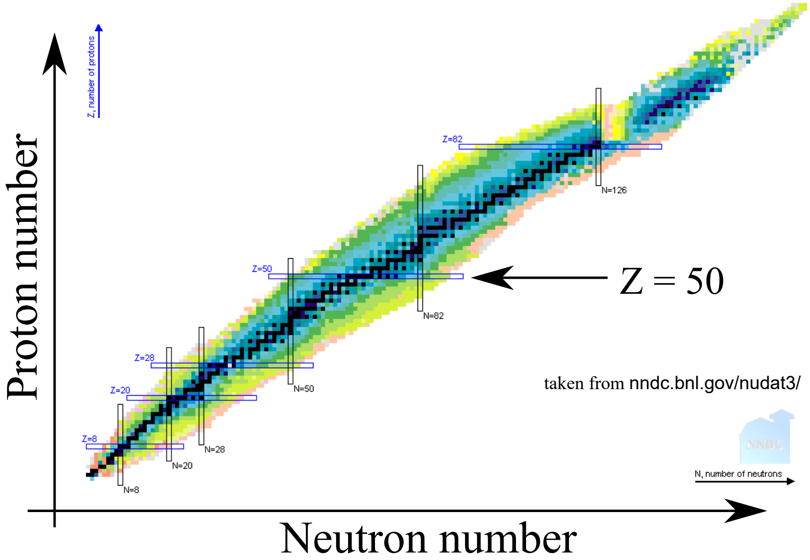
Is the α clustering universal?



Is the α clustering universal?

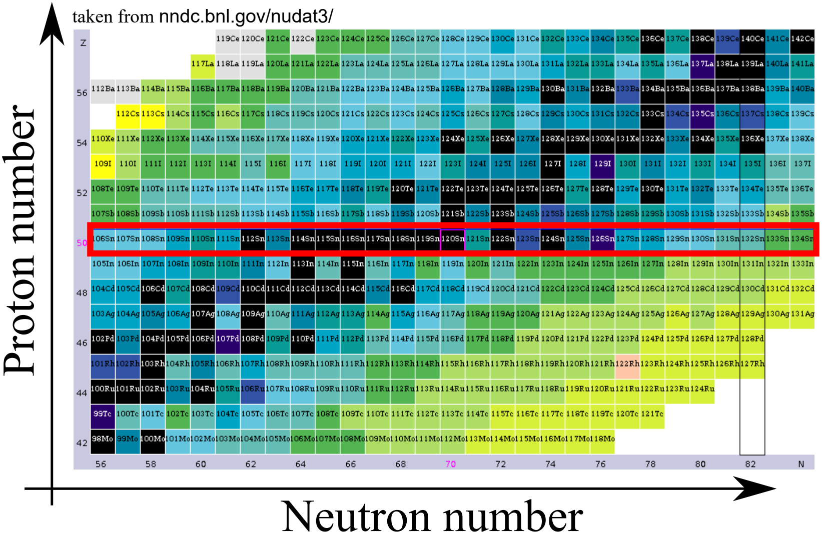


α particle on Sn surface



α particle on Sn surface

taken from nndc.bnl.gov/nudat3/



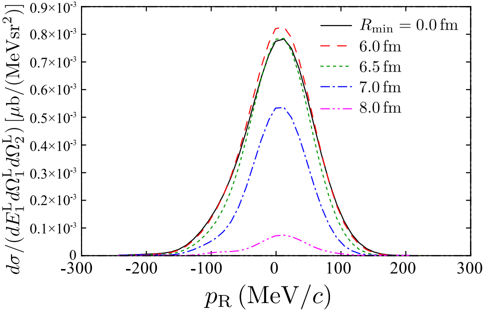
α-knockout reaction as a probe for α on nuclear surface

PHYSICAL REVIEW C **94**, 044604 (2016)

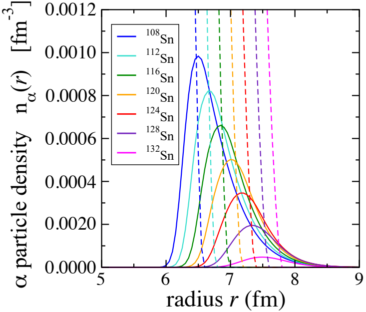
Investigating α clustering on the surface of ^{120}Sn via the $(p, p\alpha)$ reaction, and the validity of the factorization approximation

Kazuki Yoshida,* Kosho Minomo, and Kazuyuki Ogata
Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan
(Received 2 March 2016; revised manuscript received 15 July 2016; published 10 October 2016)

α knockout cross section

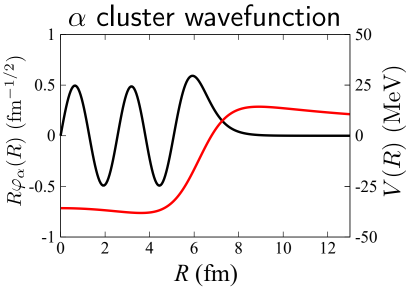


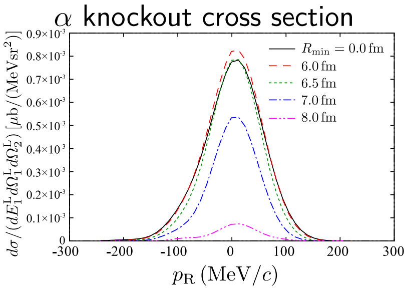
cf. α density by Typel



- Theoretical studies on the proton induced α knockout reaction from ^{120}Sn
- The reaction is sensitive surface region and only $r > 6$ fm region contributes to the reaction cross section

α-knockout reaction as a probe for α on nuclear surface





- Theoretical studies on the proton induced α knockout reaction from ^{120}Sn
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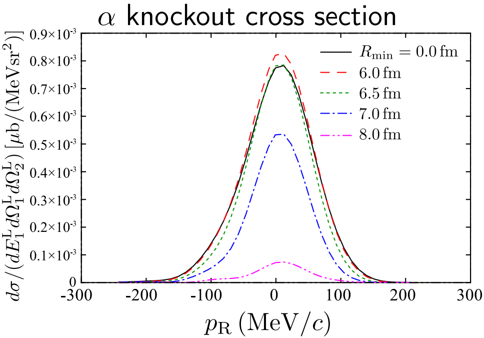
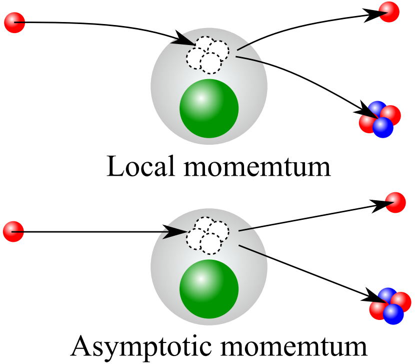
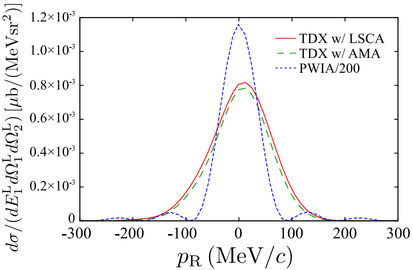
α-knockout reaction as a probe for α on nuclear surface



- Theoretical studies on the proton induced α knockout reaction from ^{120}Sn
- The reaction is sensitive surface region and only $r > 6$ fm region contributes to the reaction cross section

^{120}Sn(p,$p\alpha$)^{116}Cd reaction





- Local momenta can be approximated by asymptotic ones
  - Kinematics of the p-α collision is uniquely determined by the reaction kinematics
  - This is thanks to the peripherality of the reaction

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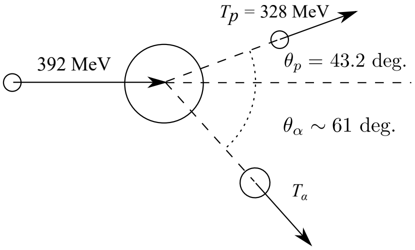
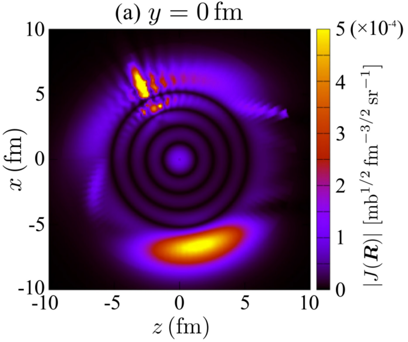
[41] K. Yoshida+, PRC **94**, 044604 (2016).

^{120}Sn$(p,p\alpha)^{116}$Cd reaction



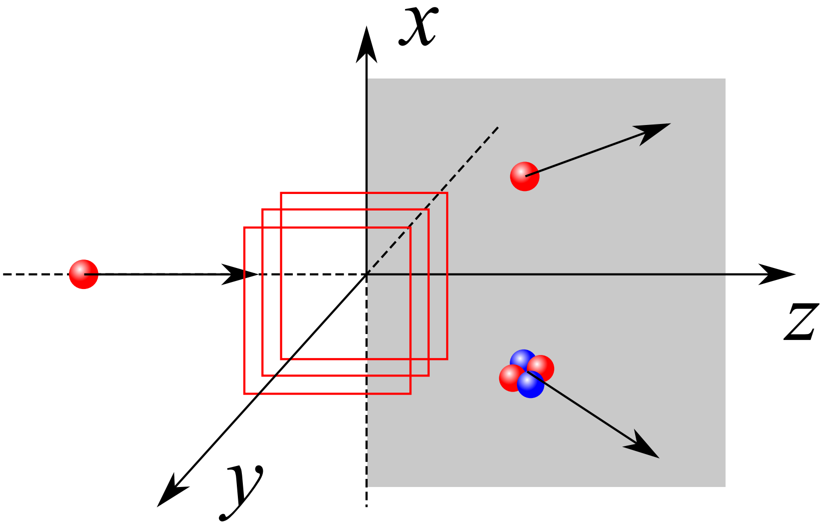
Reaction amplitude



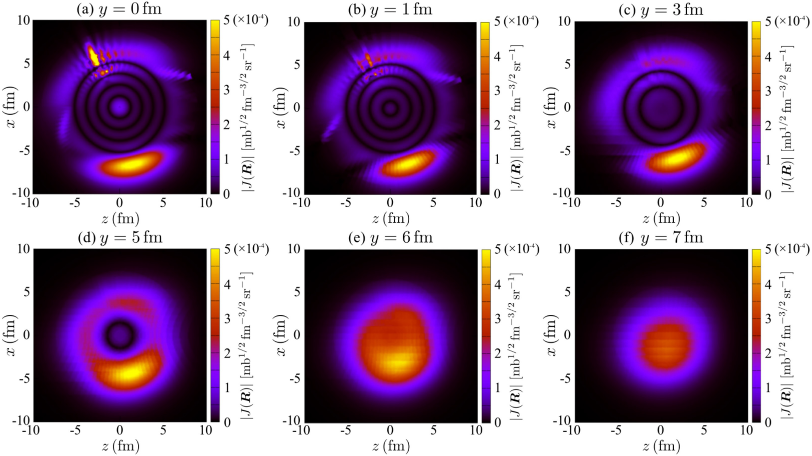
The α particle cannot go through the nuclear medium

$\rightarrow$ The reaction takes place at nuclear surface whare the α particle is emitted

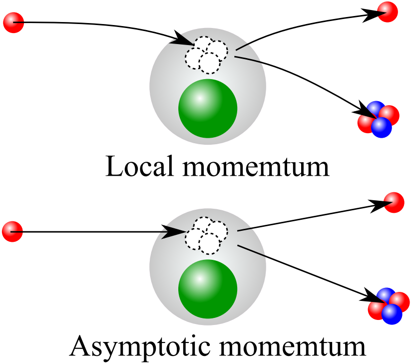
^{120}Sn(p,$p\alpha$)^{116}Cd reaction

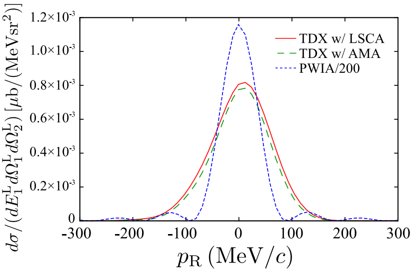


^{120}Sn(p,$p\alpha$)^{116}Cd reaction



^{120}Sn(p,$p\alpha$)^{116}Cd reaction





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[41] K. Yoshida+, PRC **94**, 044604 (2016).

Reference i

[1] B. F. Bayman and A. Bohr, Nucl. Phys. **9**, 596 (1958) (cit. on p. 3).

[2] N. S. Chant and P. G. Roos, PRC **15**, 57 (1977) (cit. on p. 10).

[3] N. S. Chant and P. G. Roos, PRC **27**, 1060 (1983) (cit. on p. 10).

[4] T. Wakasa+, PPNP **96**, 32 (2017) (cit. on pp. 10, 33).

[5] T. Aumann+, PPNP **118**, 103847 (2021) (cit. on p. 10).

[6] K. Ogata+, CPC **297**, 109058 (2024) (cit. on p. 11).

[7] T. Uesaka, PTEP **2024**, 083D01 (2024) (cit. on p. 11).

[8] K. Yoshida, **Osaka University**, Ph.D. Thesis (2018) (cit. on p. 11).

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