Model-independent approaches in the analysis of Dark Matter direct detection data

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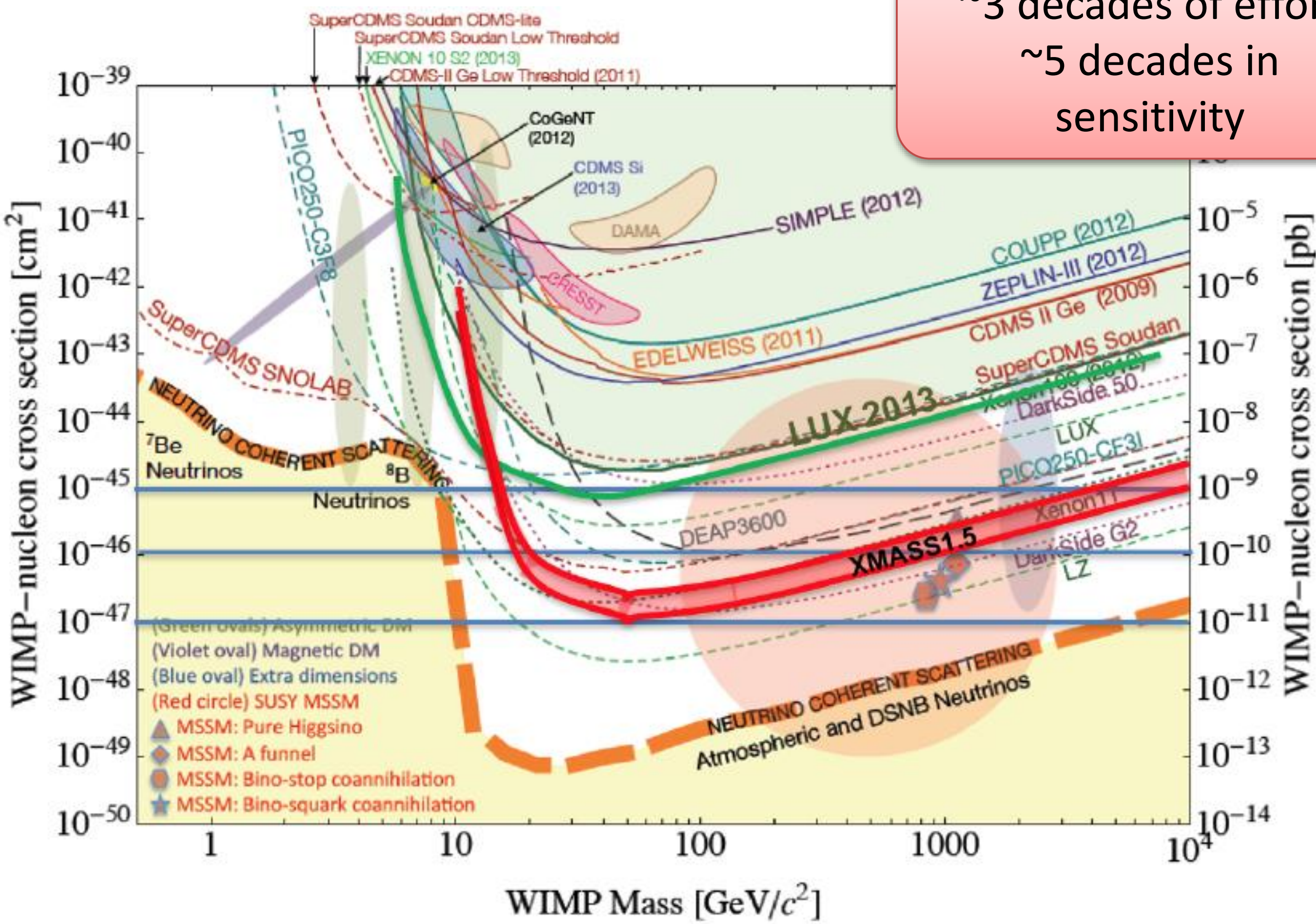
Plan of the talk

1) Short review of model-independent techniques to analyze WIMP direct detection data

2) Determination of WIMP velocity distribution f(v) from data vs. determination of observables for the most general f(v) → observables seen as generalized moments of f(v)

WIMP direct searches: spin-independent interaction+Maxwellian distribution

~3 decades of effort
~5 decades in sensitivity

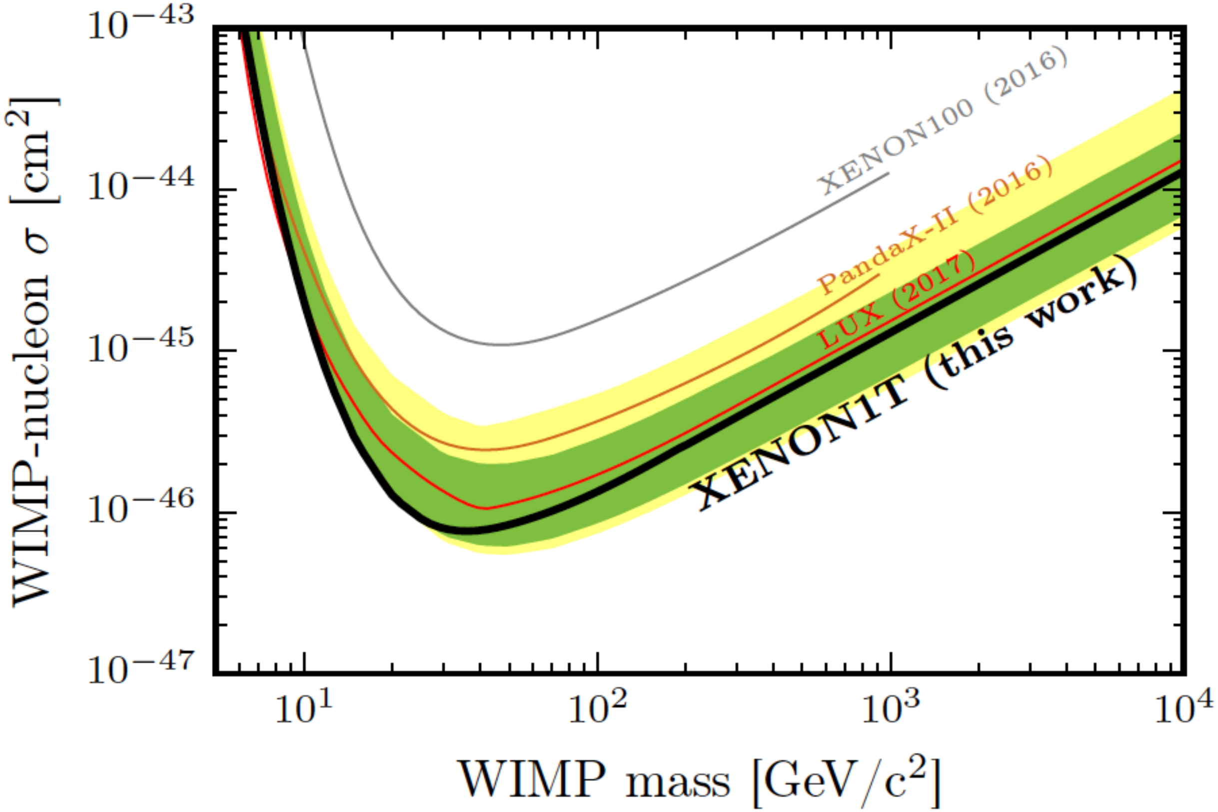


Will the race discover DM before eventually reaching the irreducible background of solar and atmospheric neutrinos???

(from Y. Suzuki talk @IDM 2016, July 2016)

First DM results from XENON1T

(complete LUX exposure reached in 34.2 live days)



Aprile et al, 1705.06655

TON-size era for WIMP direct detection has begun

N.B.: theoretical predictions for the WIMP direct detection rate depend on two main ingredients:

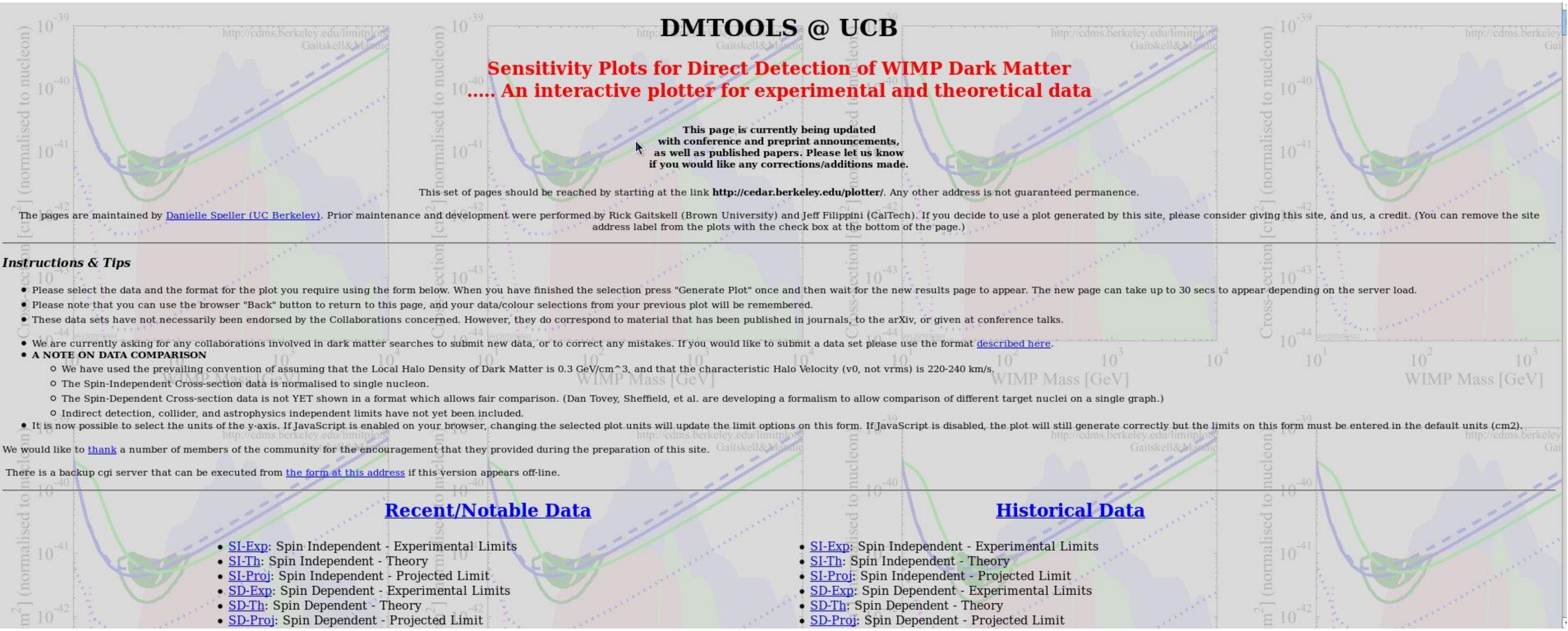
1) a scaling law for the cross section, in order to compare experiments using different targets

   Traditionally spin-independent cross section (proportional to (atomic mass number)^2) or spin-dependent cross section (proportional to the product $\boldsymbol{S}_{WIMP} \cdot \boldsymbol{S}_{nucleus}$) is assumed

2) a model for the velocity distribution of WIMPs

   Traditionally a Maxwellian distribution is assumed

Getting an updated mass-cross section plot has never been easier!



DMTOOLS @ UCB

Sensitivity Plots for Direct Detection of WIMP Dark Matter ..... An interactive plotter for experimental and theoretical data

This page is currently being updated with conference and preprint announcements, as well as published papers. Please let us know if you would like any corrections/additions made.

This set of pages should be reached by starting at the link **http://cedar.berkeley.edu/plotter/**. Any other address is not guaranteed permanence.

The pages are maintained by Danielle Speller (UC Berkeley). Prior maintenance and development were performed by Rick Gaitskell (Brown University) and Jeff Filippini (CalTech). If you decide to use a plot generated by this site, please consider giving this site, and us, a credit. (You can remove the site address label from the plots with the check box at the bottom of the page.)

Instructions & Tips

- Please select the data and the format for the plot you require using the form below. When you have finished the selection press "Generate Plot" once and then wait for the new results page to appear. The new page can take up to 30 secs to appear depending on the server load.
- Please note that you can use the browser "Back" button to return to this page, and your data/colour selections from your previous plot will be remembered.
- These data sets have not necessarily been endorsed by the Collaborations concerned. However, they do correspond to material that has been published in journals, to the arXiv, or given at conference talks.
- We are currently asking for any collaborations involved in dark matter searches to submit new data, or to correct any mistakes. If you would like to submit a data set please use the format described here.
- **A NOTE ON DATA COMPARISON**
  - We have used the prevailing convention of assuming that the Local Halo Density of Dark Matter is 0.3 GeV/cm^3, and that the characteristic Halo Velocity (v0, not vrms) is 220-240 km/s.
  - The Spin-Independent Cross-section data is normalised to single nucleon.
  - The Spin-Dependent Cross-section data is not YET shown in a format which allows fair comparison. (Dan Tovey, Sheffield, et al. are developing a formalism to allow comparison of different target nuclei on a single graph.)
  - Indirect detection, collider, and astrophysics independent limits have not yet been included.
- It is now possible to select the units of the y-axis. If JavaScript is enabled on your browser, changing the selected plot units will update the limit options on this form. If JavaScript is disabled, the plot will still generate correctly but the limits on this form must be entered in the default units (cm2).

We would like to thank a number of members of the community for the encouragement that they provided during the preparation of this site.

There is a backup cgi server that can be executed from the form at this address if this version appears off-line.

Recent/Notable Data

- SI-Exp: Spin Independent - Experimental Limits
- SI-Th: Spin Independent - Theory
- SI-Proj: Spin Independent - Projected Limit
- SD-Exp: Spin Dependent - Experimental Limits
- SD-Th: Spin Dependent - Theory
- SD-Proj: Spin Dependent - Projected Limit

Historical Data

- SI-Exp: Spin Independent - Experimental Limits
- SI-Th: Spin Independent - Theory
- SI-Proj: Spin Independent - Projected Limit
- SD-Exp: Spin Dependent - Experimental Limits
- SD-Th: Spin Dependent - Theory
- SD-Proj: Spin Dependent - Projected Limit

(http://cedar.berkeley.edu/plotter/)

…at least for the most common assumptions: spin-independent, spin-dependent interaction+ Maxwellian

Indeed, spin-independent and spin-dependent cross sections are predicted for the neutralino in supersymmetry and numerical simulations of galaxy formation support the choice of a Maxwellian for the velocity distributions.

However a bottom-up approach would also be desireable,especially if no hints come from high-energy physics about the fundamental properties of the WIMP particle. Indeed two questions arise:

- what is the most general class of scaling laws for a WIMP-nucleus cross section?
- the detailed merger history of the Milky Way is not known, allowing for the possibility of the presence of sizeable non–thermal components for which the density, direction and speed of WIMPs are hard to predict, *especially in the high velocity tail of the distribution*: do we need to assume a Maxwellian velocity distribution?

Recently both aspects have been addressed

Compatibility among different experiments (ex. DAMA/Libra vs. CoGeNT) can be verified without assuming any model for the halo

Write expected WIMP rate as:

$$\frac{dR}{dE_R} = \frac{\rho_\chi \sigma_n}{2m_\chi \mu_{n\chi}^2} \frac{C_T}{f_n^2} F^2(E_R)\epsilon(E_R) g(v_{\rm min}, t)$$

$F^2(E_R)$ is the form factor, and the function:

$$g(v_{\rm min}, t) = \int_{v_{\rm min}}^{\infty} \frac{f_{\rm local}(\vec{v}, t)}{v} d^3v$$

contains all the dependence on the halo model with:

$$v_{\rm min} = \sqrt{\frac{m_N E_R}{2\mu^2}}$$

So there is a one-to-one correspondence between the recoil energy E_R and v_{min}

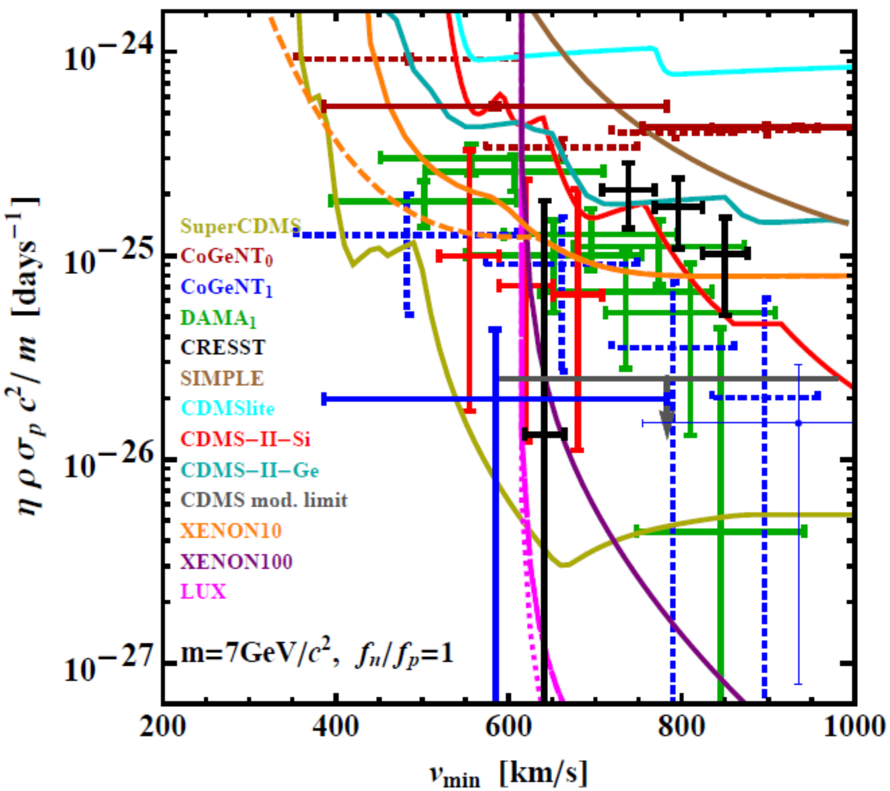
→ map the event rate expected in different experiments into the same intervals in v_{min} (P.J. Fox, J. Liu, N. Weiner, PRD83,103514 (2011))

In this way the dependence on the galactic model cancels out in the ratio of the expected count rates of the two experiments because they depend on the same integrals of $f_{local}(v)$

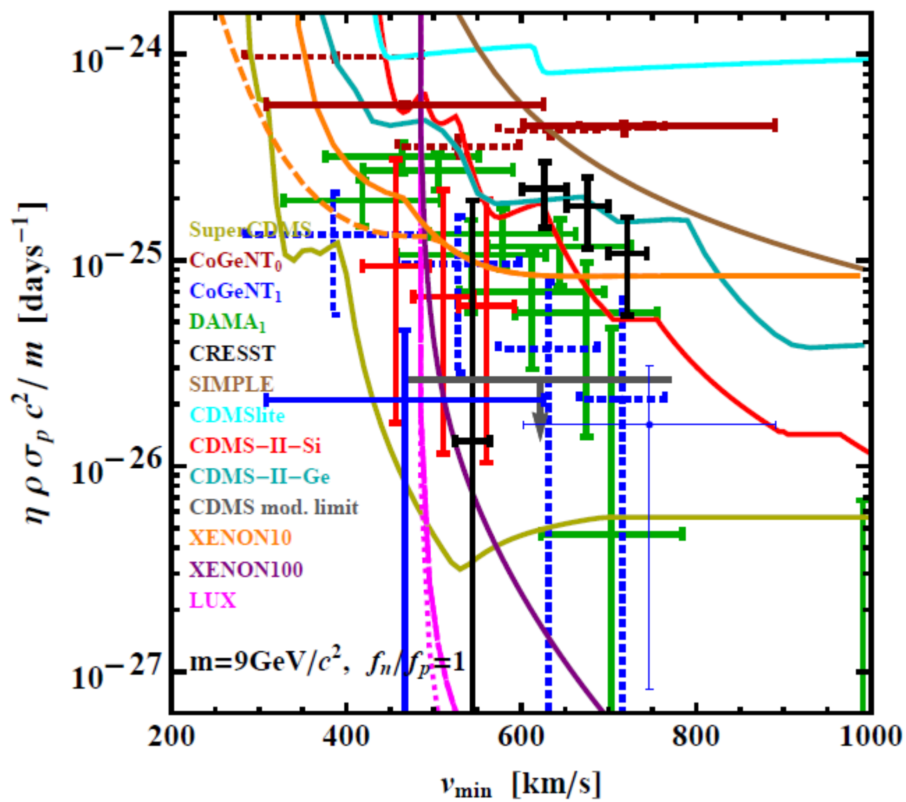
halo-independent analysis for elastic scattering

Del Nobile, Gelmini, Gondolo, Huh, arXiv:1405.5582

m_{WIMP}=7 GeV



m_{WIMP}=9 GeV



$$R^{\rm SI}_{[E'_1,E'_2]}(t) = \int_0^\infty {\rm d}v_{\rm min}\,\tilde\eta(v_{\rm min},t)\,\mathcal{R}^{\rm SI}_{[E'_1,E'_2]}(v_{\rm min})$$

$$\tilde\eta(v_{\rm min},t) \equiv \frac{\rho\sigma_p}{m}\int_{v\geqslant v_{\rm min}} {\rm d}^3v\,\frac{f(\mathbf{v},t)}{v}$$

$$\tilde\eta(v_{\rm min},t) \simeq \tilde\eta^0(v_{\rm min}) + \tilde\eta^1(v_{\rm min})\cos[\omega(t-t_0)]$$

N.B. : only halo dependence factorized. Results depend on assumptions on other quantities such as quenching factors, L_{eff}, Q_y etc.

$$\tilde{\eta}(v_{min}, t) \equiv \frac{\rho}{m_{WIMP}} \sigma_0 \eta(v_{min}, t)$$

- Annual modulation

Experimental data fits (DAMA, CoGeNT, KIMS) assume a sinusoidal behaviour:

$$\tilde{\eta}(v_{\rm min}, t) \simeq \tilde{\eta}^0(v_{\rm min}) + \tilde{\eta}^1(v_{\rm min}) \cos[\omega(t - t_0)]$$

The usual “halo-independent” approach to analyze yearly modulation data: factorize a modulated halo function $\tilde{\eta}_1$ with the only constraint $\tilde{\eta}_1 < \tilde{\eta}_0$.

(In the case of a Maxwellian typically $\tilde{\eta}_1 / \tilde{\eta}_0 \leq 0.07$)

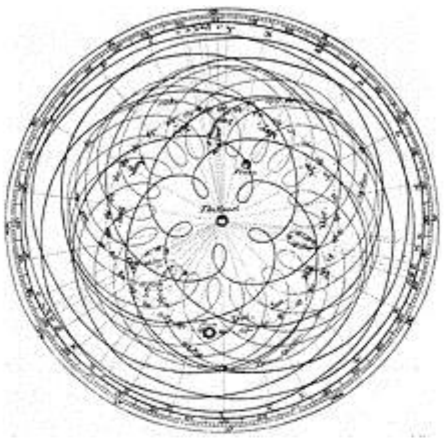
Standard lore: cannot predict $\tilde{\eta}_1 / \tilde{\eta}_0$ without a model for the velocity distribution. Is it really so? More on that later

Summarizing, the minimal requirements for halo functions $\eta_{0,1}$ are:

$$\tilde{\eta}_0(v_{\min,2}) \leq \tilde{\eta}_0(v_{\min,1}) \quad \text{if } v_{\min,2} > v_{\min,1}$$ (decreasing function)

$$\tilde{\eta}_1 \leq \tilde{\eta}_0 \quad \text{at the same } v_{\min}$$ (modulated part<100%)

$$\tilde{\eta}_0(v_{\min} \geq v_{\rm esc}) = 0.$$ (no bound WIMPs<escape velocity)



Several epicycles added to the usual scenario:

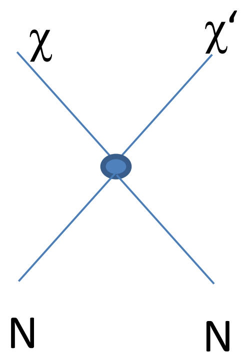
- Non-standard coupling
- Inelastic scattering
- Isospin violation
- ….

Inelastic Dark Matter

D. Tucker-Smith and N.Weiner, Phys.Rev.D 64, 043502 (2001), hep-ph/0101138

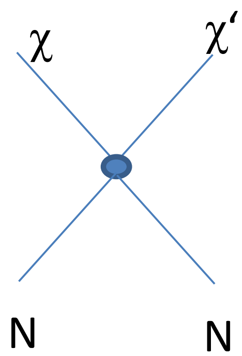
Two mass eigenstates χ and χ' very close in mass: $m_\chi - m_{\chi'} \equiv \delta$ with $\chi + N \rightarrow \chi + N$ forbidden

"Endothermic "scattering ($\delta>0$)



Kinetic energy needed to "overcome" step → rate no longer exponentially decaying with energy, maximum at finite energy E_*

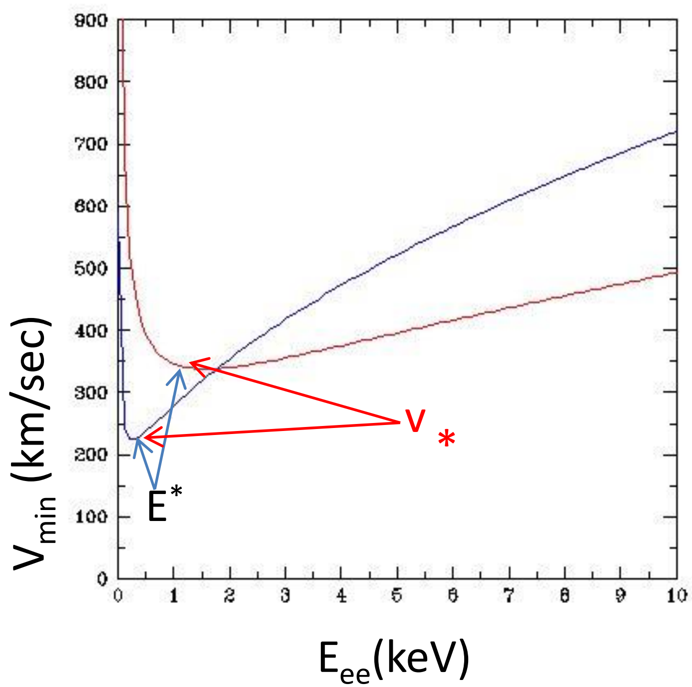
"Exothermic" scattering ($\delta<0$)



χ is metastable, δ energy deposited independently on initial kinetic energy (even for WIMPs at rest)

Inelastic DM and the halo-independent approach: recoil energy E_{ee} is no longer monotonically growing with v_{min} (energy E^* corresponds to minimal v_{min})

$$v_{min} = \frac{1}{\sqrt{2 m_N E_R}} \left(\frac{m_N E_R}{\mu} + \delta \right) = a\sqrt{E_r} + \frac{b}{\sqrt{E_R}}$$



N.B. for $\delta>0$ WIMPs need a minimal absolute incoming speed v_* to upscatter to the heavier state → vanishing rate if $v_* > v_{esc}$ (escape velocity)

Need to rebin the data in such a way that the relation between v_{min} and E_R is invertible in each bin (easy: just ensure that for all target nuclei E^* corresponds to one of the bin boundaries)

S. Scopel and K.H. Yoon, JCAP1408, 060 (2014) isospin violation (more properly: isovector interaction)

$$R = \sigma_p \sum_i \eta_i \frac{\mu_{A_i}^2}{\mu_p^2} I_{A_i} \left[Z + (A_i - Z) f_n / f_p \right]^2$$

(spin-independent cross section, same for other interactions)

sum over isotopes

Cancellation between f_p (WIMP-proton coupling) and f_n (WIMP-nucleon coupling) when $f_n/f_p \sim -Z/(A-Z)$ → can suppress the scattering cross section ona specific target (i.e. $f_n/f_p \sim -0.79$ for Germanium)

Minimal "degrading factors", i.e. maximal factors by which the reciprocal scaling law between two elements can be reduced (limited by multiple isotopes, one choice of f_n/f_p ratio cannot fit all)

Element	Xe	Ge	Si	Ca	W	Ne	C
Xe (54, *)	1.00	8.79	149.55	138.21	10.91	34.31	387.66
Ge (32, *)	22.43	1.00	68.35	63.14	130.45	15.53	176.47
Si (14, *)	172.27	30.77	1.00	1.06	757.44	1.06	2.67
Ca (20, *)	173.60	31.53	1.17	1.00	782.49	1.10	2.81
W (74, *)	2.98	13.88	177.46	166.15	1.00	41.64	466.75
Ne (10, *)	163.65	28.91	4.39	4.09	726.09	1.00	11.52
C (6, *)	176.35	32.13	1.07	1.02	789.59	1.12	1.00
I (53, 127)	1.94	5.51	127.04	118.35	20.68	28.92	326.95
Cs (55, 133)	1.16	7.15	139.65	127.61	12.32	31.88	355.27
O (8, 16)	178.40	32.13	1.08	1.03	789.90	1.13	1.01
Na (11, 23)	101.68	13.77	8.45	8.33	481.03	2.27	22.68
Ar (18, 36)	178.49	32.13	1.08	1.03	789.90	1.13	1.01
F (9, 19)	89.39	10.88	12.44	11.90	425.93	3.05	33.47

(J.L.Feng, J.Kumar, D.Marfatia and D.Sanford, Phys.Lett.B703, 124 (2011), 1102.4331)

On the most general WIMP-nucleus cross section
(i.e. beyond “spin-dependent” and “spin”independent”)

Most general approach: consider ALL possible NR couplings, including those depending on velocity and momentum

$$\mathcal{H} = \sum_i \left(c_i^0 + c_i^1 \tau_3\right) \mathcal{O}_i$$

τ_3=nuclear isospin operator, i.e.

$$c_i^{\rm p} = (c_i^0 + c_i^1)/2 \qquad \text{(proton)}$$
$$c_i^{\rm n} = (c_i^0 - c_i^1)/2 \qquad \text{(neutron)}$$

(if $c_i^p = c_i^n \rightarrow c_i^1 = 0$)

N.R. operators O_i guaranteed to be Hermitian if built out of the following four 3-vectors:

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^\perp, \quad \vec{S}_\chi, \quad \vec{S}_N$$

with:

$$\vec{v}^\perp = \vec{v} + \frac{\vec{q}}{2\mu_N}$$
$$\vec{v} \equiv \vec{v}_{\chi,\rm in} - \vec{v}_{N,\rm in}$$

$\Rightarrow$ $\vec{v}^\perp \cdot \vec{q} = 0$

$$\mathcal{O}_1 = 1_\chi 1_N,$$
$$\mathcal{O}_2 = (v^\perp)^2,$$
$$\mathcal{O}_3 = i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right),$$
$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N,$$
$$\mathcal{O}_5 = i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right),$$
$$\mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right)$$
$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp,$$
$$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp,$$
$$\mathcal{O}_9 = i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right),$$
$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N},$$
$$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}.$$

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542; N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

Additional operators that do not arise for traditional spin≤1 mediators:

$$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp),$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right),$$

$$\mathcal{O}_{14} = i\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)(\vec{S}_N \cdot \vec{v}^\perp),$$

$$\mathcal{O}_{15} = -\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left[(\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right]$$

$$\mathcal{O}_{16} = -\left[(\vec{S}_\chi \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right]\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right)$$

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

In the expected rate WIMP physics (encoded in the R functions that depend on the c_i couplings) and the nuclear physics (contained in 8 (6+2) response functions W factorize in a simple way:

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}E_R} = \sum_T \frac{\mathrm{d}\mathcal{R}_T}{\mathrm{d}E_R} \equiv \sum_T \xi_T \frac{\rho_\chi}{2\pi m_\chi} \int_{v>v_{\min}(q)} \frac{f(\vec{v}+\vec{v}_e(t))}{v} P_{\rm tot}(v^2,q^2)\, d^3v$$

$$\begin{aligned} P_{\rm tot}(v^2,q^2) &= \frac{4\pi}{2j_N+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \Bigg[R_M^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_M^{\tau\tau'}(y) \\ &+ R_{\Sigma''}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Sigma''}^{\tau\tau'}(y) + R_{\Sigma'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Sigma'}^{\tau\tau'}(y) \Bigg] \\ &+ \frac{q^2}{m_N^2} \Bigg[R_{\Phi''}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Phi''}^{\tau\tau'}(y) + R_{\Phi'' M}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Phi'' M}^{\tau\tau'}(y) \\ &+ R_{\tilde{\Phi}'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\tilde{\Phi}'}^{\tau\tau'}(y) + R_{\Delta}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Delta}^{\tau\tau'}(y) \\ &+ R_{\Delta\Sigma'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Delta\Sigma'}^{\tau\tau'}(y) \Bigg] \Bigg\}, \end{aligned}$$

N.B.: besides usual spin-independent and spin-dependent terms new contributions arise, with explicit dependences on the transferred momentum q and the WIMP incoming velocity

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

WIMPs response funtions

$$R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right]$$

$$R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2}$$

$$R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2}$$

$$R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2}$$

$$R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right]$$

$$R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right]$$

$$R_\Delta^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2}$$

$$R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}.$$

general form:

$$R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'}\frac{(v_T^\perp)^2}{c^2} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'}\frac{v_T^2 - v_{min}^2}{c^2}$$

Nuclear response functions

Assuming one-body dark matter-nucleon interactions, the Hamiltonian density for dark matter-nucleus interactions is:

$$\begin{aligned}\mathcal{H}_{ET}(\vec{x}) \;=\; & \sum_{i=1}^{A} l_0(i)\ \delta(\vec{x}-\vec{x}_i) + \sum_{i=1}^{A} l_0^A(i)\ \frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i)+\delta(\vec{x}-\vec{x}_i)\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right] \\ + & \sum_{i=1}^{A} \vec{l}_5(i)\cdot\vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i) + \sum_{i=1}^{A} \vec{l}_M(i)\cdot\frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i\delta(\vec{x}-\vec{x}_i)+\delta(\vec{x}-\vec{x}_i)\frac{1}{i}\overrightarrow{\nabla}_i\right] \\ + & \sum_{i=1}^{A} \vec{l}_E(i)\cdot\frac{1}{2M}\left[\overleftarrow{\nabla}_i\times\vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i)+\delta(\vec{x}-\vec{x}_i)\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right]\end{aligned}$$

So the WIMP-nucleus Hamiltonian has the general form:

$$\int d\vec{x}\ e^{-i\vec{q}\cdot\vec{x}}\ \left[l_0\langle J_iM_i|\hat{\rho}(\vec{x})|J_iM_i\rangle - \vec{l}\cdot\langle J_iM_i|\hat{\vec{j}}(\vec{x})|J_iM_i\rangle\right]$$

With:

$$e^{i\vec{q}\cdot\vec{x}_i} \;=\; \sum_{J=0}^{\infty}\sqrt{4\pi}\ [J]\ i^J j_J(qx_i)Y_{J0}(\Omega_{x_i})$$

$$\hat{e}_\lambda e^{i\vec{q}\cdot\vec{x}_i} \;=\; \begin{cases}\sum_{J=0}^{\infty}\sqrt{4\pi}\ [J]\ i^{J-1}\frac{\vec{\nabla}_i}{q}j_J(qx_i)Y_{J0}(\Omega_{x_i}), & \lambda=0\\ \sum_{J\geq 1}^{\infty}\sqrt{2\pi}\ [J]\ i^{J-2}\left[\lambda j_J(qx_i)\vec{Y}_{JJ1}^{\lambda}(\Omega_{x_i})+\frac{\vec{\nabla}_i}{q}\times j_J(qx_i)\vec{Y}_{JJ1}^{\lambda}(\Omega_{x_i})\right], & \lambda=\pm1\end{cases}$$

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

which depends on the expectations of six distinct nuclear response functions, defined as:

$$M_{JM}(q\vec{x})$$

$$\Delta_{JM}(q\vec{x}) \equiv \vec{M}_{JJ}^{M}(q\vec{x}) \cdot \frac{1}{q}\vec{\nabla}$$

$$\Sigma'_{JM}(q\vec{x}) \equiv -i\left\{\frac{1}{q}\vec{\nabla} \times \vec{M}_{JJ}^{M}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{-\sqrt{J}\ \vec{M}_{JJ+1}^{M}(q\vec{x}) + \sqrt{J+1}\ \vec{M}_{JJ-1}^{M}(q\vec{x})\right\} \cdot \vec{\sigma}$$

$$\Sigma''_{JM}(q\vec{x}) \equiv \left\{\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{\sqrt{J+1}\ \vec{M}_{JJ+1}^{M}(q\vec{x}) + \sqrt{J}\ \vec{M}_{JJ-1}^{M}(q\vec{x})\right\} \cdot \vec{\sigma}$$

$$\tilde{\Phi}'_{JM}(q\vec{x}) \equiv \left(\frac{1}{q}\vec{\nabla} \times \vec{M}_{JJ}^{M}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right) + \frac{1}{2}\vec{M}_{JJ}^{M}(q\vec{x}) \cdot \vec{\sigma}$$

$$\Phi''_{JM}(q\vec{x}) \equiv i\left(\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right)$$

with $M_{JM} = j_J Y_{JM}$ Bessel spherical harmonics and $\vec{M}^{M}_{JL} = j_J \vec{Y}_{JM}$ vector spherical harmonics.

- **M**= vector-charge (scalar, **usual spin-independent part**, non-vanishing for all nuclei)
- **Φ''**=vector-longitudinal, related to spin-orpit coupling $\sigma \cdot l$ (also spin-independent, non-vanishing for all nuclei)
- **Σ'** and **Σ''** = associated to longitudinal and transverse components of nuclear spin, **their sum is the usual spin-dependent interaction**, require nuclear spin j>0
- **Δ**=associated to the orbital angular momentum operator l, also requires j>0
- $\tilde{\Phi}'$= related to a vector-longitudinal operator that transforms as a tensor under rotations, requires j>1/2

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

Squaring the amplitude get the following nuclear response functions:

$$W_O^{\tau\tau'}(y) \equiv \sum_{J=0,2,\ldots}^{\infty} \langle j_N||O_{J;\tau}(q)||j_N\rangle\langle j_N||O_{J;\tau'}(q)||j_N\rangle \text{ for } O = M, \Phi'',$$

$$W_O^{\tau\tau'}(y) \equiv \sum_{J=1,3,\ldots}^{\infty} \langle j_N||O_{J;\tau}(q)||j_N\rangle\langle j_N||O_{J;\tau'}(q)||j_N\rangle \text{ for } O = \Sigma'', \Sigma', \Delta,$$

$$W_{\tilde{\Phi}'}^{\tau\tau'}(y) = \sum_{J=2,4,\ldots}^{\infty} \langle j_N||\tilde{\Phi}'_{J;\tau}(q)||j_N\rangle\langle j_N||\tilde{\Phi}'_{J;\tau'}(q)||j_N\rangle,$$

$$W_{\Phi''M}^{\tau\tau'}(y) = \sum_{J=0,2,\ldots}^{\infty} \langle j_N||\Phi''_{J;\tau}(q)||j_N\rangle\langle j_N||M_{J;\tau'}(q)||j_N\rangle,$$

(interference terms)

$$W_{\Delta\Sigma'}^{\tau\tau'}(y) = \sum_{J=1,3,\ldots}^{\infty} \langle j_N||\Delta_{J;\tau}(q)||j_N\rangle\langle j_N||\Sigma'_{J;\tau'}(q)||j_N\rangle.$$

These 8 (6+2 interferences) W nuclear response functions have been calculated for most nuclei using a numerical (truncated) harmonic potential shell model (Fitzpatrick et al., JCAP 1302 1302(2013), Catena and Schwabe, JCAP 1504 no. 04, 042 (2015)) with oscillator parameter:

$$b[\text{fm}] = \sqrt{41.467/(45A^{-1/3} - 25A^{-2/3})}$$

$$y = (qb/2)^2$$

P.S.: for a WIMP with spin>1/2 additional operators arise. They can be obtained as the spherical components of generic powers of q,$v_\perp$ S_χ and S_N However, in the limit of one-nucleon interaction and including only terms linear in $v_\perp$ the number of operators remains finite *for any WIMP spin.*

Moreover no additional nuclear response functions are needed.

$$\mathcal{O}_{17} = \frac{i}{2m_N}\left[(\mathbf{S}\cdot\mathbf{q})(\mathbf{S}\cdot\mathbf{v}^\perp) + (\mathbf{S}\cdot\mathbf{v}^\perp)(\mathbf{S}\cdot\mathbf{q})\right],$$

$$\mathcal{O}_{18} = \frac{i}{4m_N}\left[(\mathbf{S}\cdot\mathbf{q})(\mathbf{S}\cdot\boldsymbol{\sigma}) + (\mathbf{S}\cdot\boldsymbol{\sigma})(\mathbf{S}\cdot\mathbf{q})\right],$$

$$\mathcal{O}_{19} = \frac{i}{4m_N}\left[(\mathbf{S}\cdot\mathbf{q})(\mathbf{S}\cdot(\mathbf{v}^\perp\times\boldsymbol{\sigma})) + (\mathbf{S}\cdot(\mathbf{v}^\perp\times\boldsymbol{\sigma}))(\mathbf{S}\cdot\mathbf{q})\right],$$

$$\mathcal{O}_{20} = \frac{1}{m_N^2}(\mathbf{S}\cdot\mathbf{q})^2,$$

$$\mathcal{O}_{21} = \frac{1}{2m_N^2}(\mathbf{S}\cdot\mathbf{q})^2(\mathbf{v}^\perp\cdot\boldsymbol{\sigma}),$$

$$\mathcal{O}_{22} = \frac{1}{2m_N^2}\left[(\mathbf{S}\times\mathbf{q}\cdot\mathbf{v}^\perp)(\mathbf{S}\cdot\mathbf{q}) + (\mathbf{S}\cdot\mathbf{q})(\mathbf{S}\times\mathbf{q}\cdot\mathbf{v}^\perp)\right],$$

$$\mathcal{O}_{23} = \frac{1}{4m_N^2}\left[(\mathbf{S}\times\mathbf{q}\cdot\boldsymbol{\sigma})(\mathbf{S}\cdot\mathbf{q}) + (\mathbf{S}\cdot\mathbf{q})(\mathbf{S}\times\mathbf{q}\cdot\boldsymbol{\sigma})\right],$$

$$\mathcal{O}_{24} = \frac{1}{4m_N^2}\left[(\mathbf{S}\times\mathbf{q})\cdot(\mathbf{v}^\perp\times\boldsymbol{\sigma})(\mathbf{S}\cdot\mathbf{q}) + (\mathbf{S}\cdot\mathbf{q})(\mathbf{S}\times\mathbf{q})\cdot(\mathbf{v}^\perp\times\boldsymbol{\sigma})\right],$$

$$\mathcal{O}_{25} = \frac{1}{2m_N^2}\left[(\mathbf{S}\cdot\mathbf{v}^\perp)(\mathbf{S}\cdot\mathbf{q})^2 + (\mathbf{S}\cdot\mathbf{q})^2(\mathbf{S}\cdot\mathbf{v}^\perp)\right],$$

$$\mathcal{O}_{26} = \frac{1}{4m_N^2}\left[(\mathbf{S}\cdot\boldsymbol{\sigma})(\mathbf{S}\cdot\mathbf{q})^2 + (\mathbf{S}\cdot\mathbf{q})^2(\mathbf{S}\cdot\boldsymbol{\sigma})\right],$$

$$\mathcal{O}_{27} = \frac{1}{4m_N^2}\left[(\mathbf{S}\cdot\mathbf{v}^\perp\times\boldsymbol{\sigma})(\mathbf{S}\cdot\mathbf{q})^2 + (\mathbf{S}\cdot\mathbf{q})^2(\mathbf{S}\cdot\mathbf{v}^\perp\times\boldsymbol{\sigma})\right].$$

$\sigma = S_N$, $S = S_\chi$

Operators are numbered from 1 to 27, but O_2 depends on v^2 and O_{16} can be expressed as a linear combination of O_{12} and O_{15} → 27-2=**25** operators

P. Gondolo, S.S., in progress

General expressions for expected rates

N.B. : need to introduce 3 different energies:

E_R=nuclear recoil energy;

E_{ee}=electron-equivalent energy,

E'=visible energy.

The expected rate in a direct detection experiment for the energy bin $E'_1 < E' < E'_2$ is given by:

$$R_{[E'_1,E'_2]} = MT \int_{E'_1}^{E'_2} \frac{dR}{dE'} \, dE'$$

$$\frac{dR}{dE'} = \int_0^\infty \frac{dR}{dE_{ee}} \mathcal{G}(E', E_{ee}) \epsilon(E') \, dE_{ee}$$

$$E_{ee} = q(E_R) E_R,$$

$q(E_R)$ =quenching factor (fraction of recoil energy into ionization/scintillation), ε=acceptance/cuts efficiency, $G(E',E_{ee})$=energy resolution, and

$$\frac{dR}{dE_R} = N_T \int_{v_{min}}^{v_{esc}} \frac{\rho_\chi}{m_\chi} v \frac{d\sigma}{dE_R} \, f(v) dv$$

The differential cross section is given by:

$$\frac{d\sigma}{dE_R} = \frac{1}{10^6}\frac{2m_N}{4\pi}\frac{c^2}{v^2}\left[\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\sum_{spin}|\mathcal{M}|^2\right]$$

with:

$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\sum_{spin}|\mathcal{M}|^2 = \frac{4\pi}{2j_N+1}\sum_{\tau\tau'}\sum_{l} R_l^{\tau\tau'} W_l^{\tau\tau'}$$

WIMP response function ($R_l^{\tau\tau'}$) nuclear response function ($W_l^{\tau\tau'}$)

where the R's contains a sum over all the couplings of the non-relativistic effective lagrangian

$$\mathcal{H} = \sum_{\alpha=n,p}\sum_{i=1}^{27} c_i^\alpha \mathcal{O}_i^\alpha, \quad c_2 = c_{16} = 0$$

$$c_0^i = \frac{c_i^p + c_i^n}{2}, \; c_1^i = \frac{c_i^p - c_i^n}{2},$$

while the W's are the corresponding 8 nuclear response functions (including interferences)

To make contact to the previous formalism and in case of a single coupling $c_i \neq 0$ it is possible to factorize the conventional WIMP-proton cross section:

$$\sigma_p = (c_i^p)^2 \frac{\mu_{\chi N}^2}{\pi}$$

and get for the rate the expression:

$$\int_0^\infty dE_R \mathcal{R}(E_R)\{\tilde{\eta}[v_{min}(E_R)]c^2\} = \int_0^\infty dv_{min} \mathcal{R}(v_{min})\{\tilde{\eta}(v_{min})c^2\}$$

$$\mathcal{R}(v_{min}) = \mathcal{R}(E_R)\frac{dE_R}{dv_{min}}$$

$$\tilde{\eta}(v_{min}) = \frac{\rho_\chi}{m_\chi}\sigma_p \eta(v_{min})$$

"generalized halo function"

$$\eta(v_{min}) \equiv \int_{v_{min}} \frac{f(v)}{v} dv$$

"halo function"

all the experimental quantities (energy resolution, efficiency, quenching, etc) are contained in the response function R:

$$\mathcal{R}(E_R) = N_T M T \frac{1}{10^6} \frac{m_N}{2\mu_{\chi\mathcal{N}}^2} \int_{E_1'}^{E_2'} dE' \epsilon(E') \mathcal{G}[E', q(E_R) E_R] \frac{1}{2j_\chi + 1} \frac{1}{2j_N + 1} \sum_{spin} \left| \mathcal{M}\left(c_i^p = 1, r = \frac{c_i^n}{c_i^p} \right) \right|^2$$

that depends also on the ratio $r=c_n/c_p$. The isospin rotation can be easily done since the response function for a generic value of $r=c_n/c_p$ is a linear combination of the three response functions for r=-1,0,1:

$$\mathcal{R}(E_R, r) = \frac{r(r+1)}{2} \mathcal{R}(E_R, r=1) + \frac{r(r-1)}{2} \mathcal{R}(E_R, r=-1) + (1-r^2) \mathcal{R}(E_R, r=0)$$

N.B. the previous derivation requires no explicit velocity dependence in the cross section. However the same form is retained also in the case of an explicit velocity dependence in the cross section.

Take out the velocity integral, and write the expected number of events as:

$$N(t)_{[E_1',E_2']} = \int_0^\infty \mathcal{H}_{[E_1',E_2']}(v)\, f(v,t)\, dv$$

where the response function contains all the dependences on the cross section and the experimental quantities. By setting:

$$f(v,t) \equiv -v\frac{\partial \eta(v,t)}{\partial v}$$

integrating by parts and incorporating as usual the point-like cross section and the local density in the definition of the halo function leads to:

$$N_{[E_1',E_2']}(v)(t) = \int_0^\infty \mathcal{R}_{[E_1',E_2']}(v)\, \tilde{\eta}(v,t)\, dv$$

This expression looks pretty much the same as the previous one (with $v_{min} \rightarrow v$) but *is valid in principle for any velocity dependence in the cross section*.

So two alternative expressions for the expected rate:

$$N_{[E_1',E_2']}(v)(t) = \int_0^\infty \mathcal{R}_{[E_1',E_2']}(v)\,\tilde{\eta}(v,t)\,dv$$

$$N(t)_{[E_1',E_2']} = \int_0^\infty \mathcal{H}_{[E_1',E_2']}(v)\,f(v,t)\,dv$$

N.B: mathematically, expected rates can be seen a "generalized moments" of the distribution f(v), where "ordinary" moments are integrals

$$\int v^n f(v)\,dv$$

The two different response functions are related by:

$$\mathcal{R}_{[E_1',E_2']}(v) = \frac{\partial}{\partial v}\left[v\mathcal{H}_{[E_1',E_2']}(v)\right], \qquad \mathcal{H}_{[E_1',E_2']}(v) = \frac{1}{v}\int_0^v \mathcal{R}_{[E_1',E_2']}(v')\,dv'$$

If a large number of evaluations of the response function is needed (as in the case of a Monte Carlo simulation) it is convenient to tabulate the integrated response function:

$$\bar{\mathcal{R}}(E_R) \equiv \int_0^{E_R} \mu_{\chi N}^2 \mathcal{R}(E')\, dE'$$

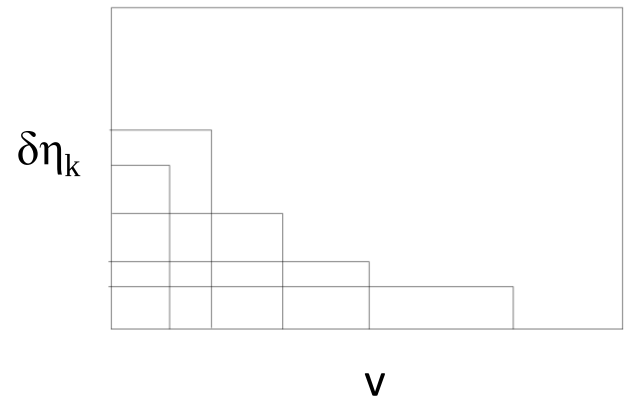
that does not depend on the WIMP mass or mass splitting, and calculate:

$$\mathcal{H}(v) = \frac{1}{v} \frac{1}{\mu_{\chi N}^2} \bar{\mathcal{R}}[E_R(v)]$$

by interpolation.

Another approach is to parameterize the halo functions η (both for the average rate and for its modulated part) in terms of a sum of a "large-enough" number N of streams:

$$\tilde{\eta}_{0,1}(v) = \sum_{k=1}^{N} \delta\tilde{\eta}_{0,1}^{k} \theta(v_k - v)$$

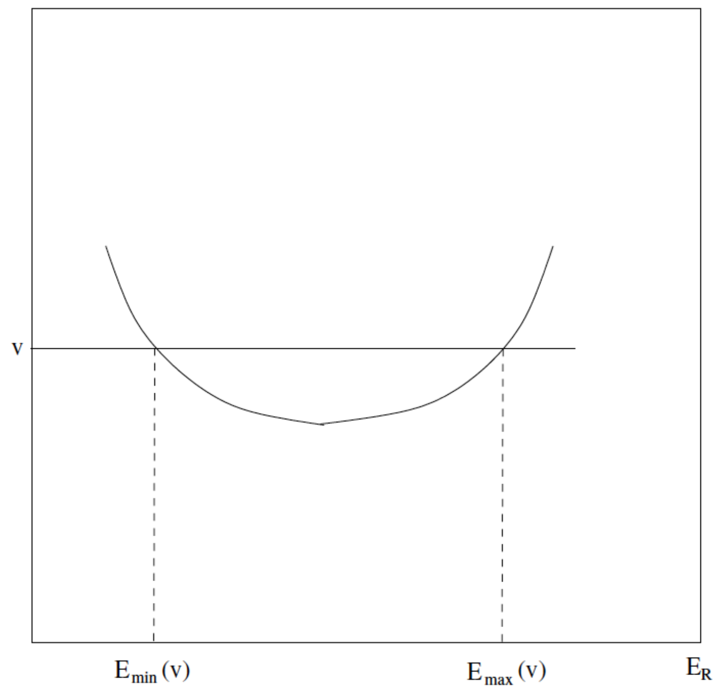


in this case the rate can still be expressed in terms of interpolations of the $\bar{R}$ function:

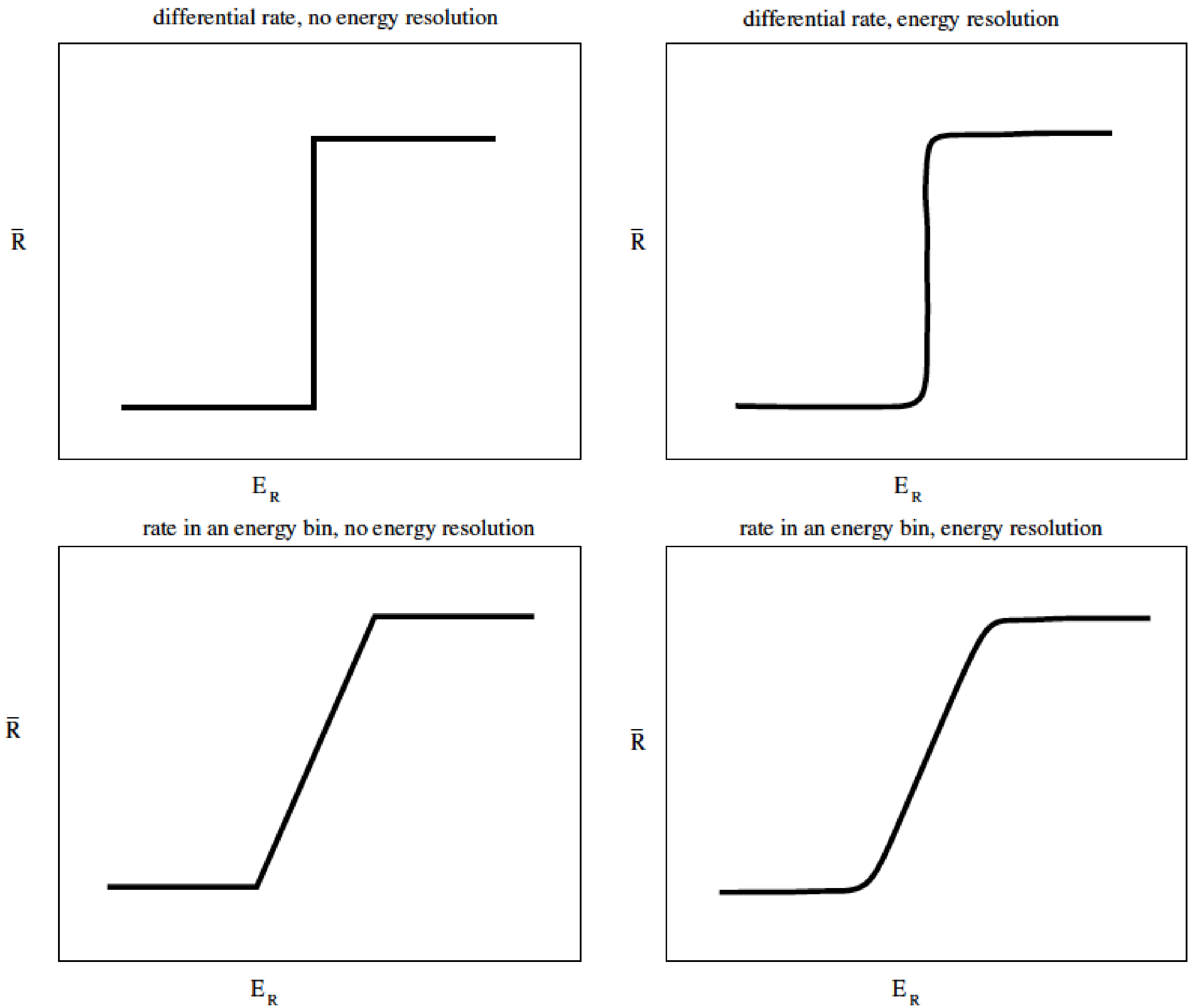
$$R = \int_0^{v_{esc}} d\,v \mathcal{R}[E_R(v)]\tilde{\eta}(v) = \sum_{k=1}^{N} \delta\tilde{\eta}_{0,1}^k \int_0^{v_{esc}} d\,v \mathcal{R}[E_R(v)]$$

$$= \frac{1}{\mu_{\chi N}^2} \sum_{k=1}^{N} \delta\tilde{\eta}_{0,1}^k \left[\bar{\mathcal{R}}(E_{max}(v_k, m_\chi, \delta)) - \bar{\mathcal{R}}(E_{min}(v_k, m_\chi, \delta))\right]$$

where the above expression includes the possibility of inelastic scattering



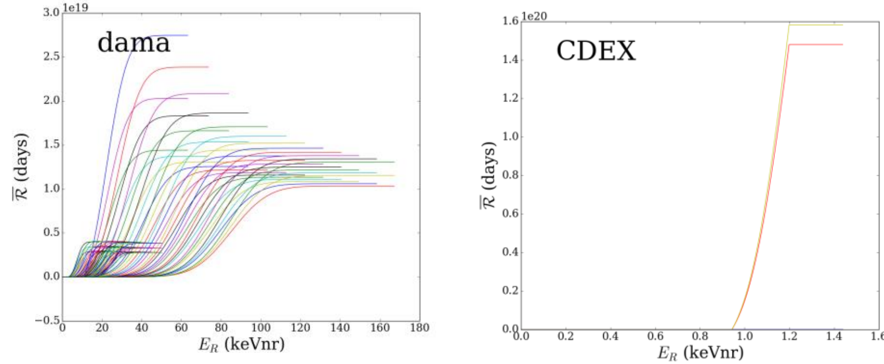
Schematic behaviors of the $\bar{R}$ function

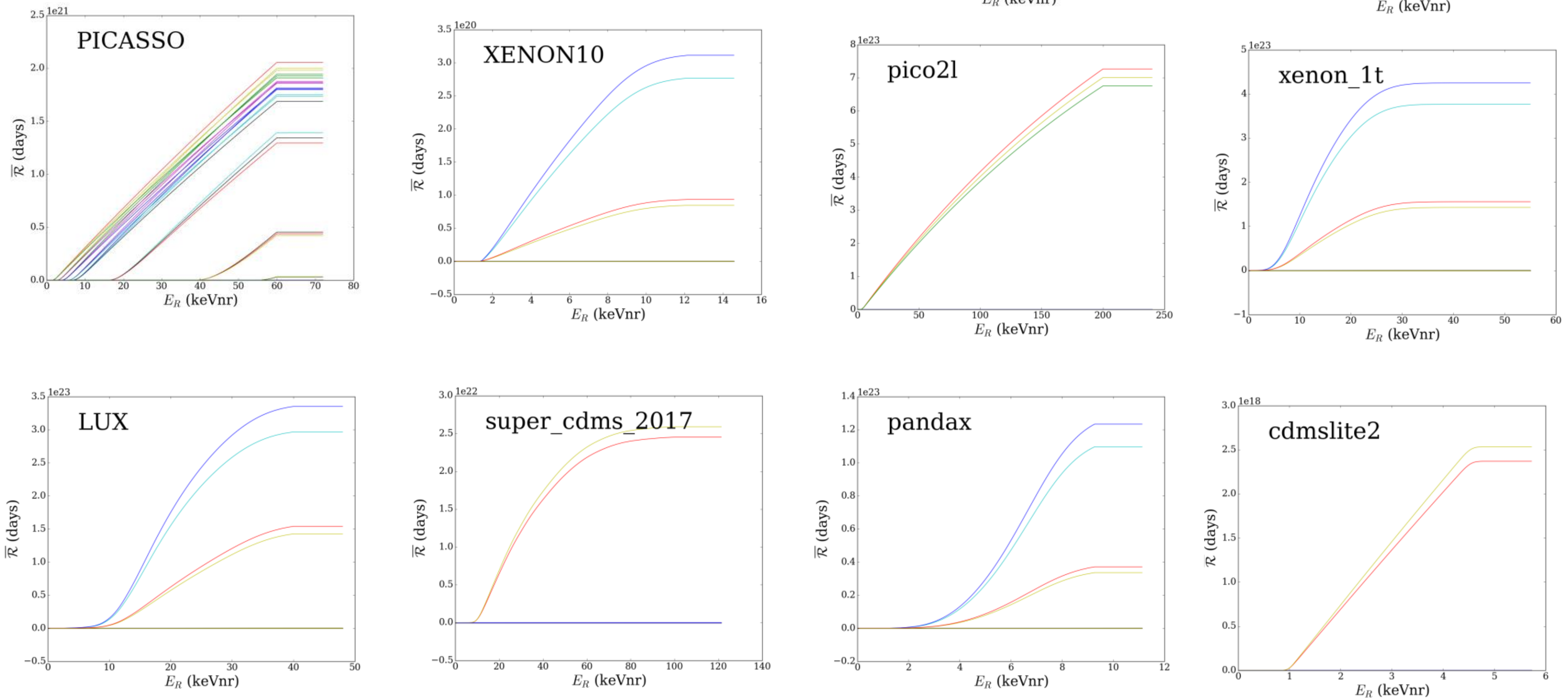


c_4 (spin-dependent interaction)

Can tabulate full calculation of $\overline{R}$ response function for each:

1) Experiment
2) Energy bin/energy threshold/energy value
3) Isospin value (c_n/c_p=-1,0,1)
4) Nuclear target (including all stable isotopes)
5) Effective coupling





S.S., J.H.Yoon, S.H. Kang, in progress

Several attempts in the literature to determine the velocity distribution from the data, sampling f(v) in a finite number of velocity bins $N_{streams}$ and using a maximum-likelihood method
(B. Feldstein, F. Kahlhoefer, JCAP 1408 (2014) 065; JCAP 1412 (2014) 12, 052. Alejandro Ibarra, Andreas Rappelt, arXiv:1703.09168)

In this case $f(v) \rightarrow \lambda_i \, \delta(v-v_i)$ with $\Sigma_i \lambda_i=1$ with $i=1,N_{streams}$ and the likelihood $L(S^k)$ depends on the direct detection theoretical predictions S^k with $k=1,N_{exp}$ and S^k in the form:

$$S^k = \sum_{i=1}^{N} \lambda_i H_i^k$$

with H_i^k fixed constants determined by experimental properties (response functions).
The problem of this approach is that in general $N_{streams} << N_{exp}$ (actually, the idea it to take $N_{streams} \rightarrow \infty$) so fixing N_{exp} experimental values leaves a large degeneracy in the λ's.

The WIMP velocity distribution f(v) can also be inferred from estimations of the DM density profile in our Galaxy $\rho_{DM}(r)$ (either through measurements of the galactic rotational velocity or by simulations of Galacy formation. Both quantities depend on the same 6-dimensional distribution F:

$$F(\vec{r}, \vec{v})$$

In particular:

$$f(\vec{v}) = F(\vec{r}_0, \vec{v})$$

(r_0=Earth's position)

while:

$$\rho_{DM}(\vec{r}) = \int d^3v F(\vec{r}, \vec{v})$$

← to get F need to invert this equation, which is degenerate

ADD INFORMATION: SIMMETRIES

For instance, assuming spherical symmetry by Jean's theorem F can only depend on v through the only integral of motion, the total mechanical energy E=T+V

⇒ $$F(\vec{r}, \vec{v}) = F(E)$$

This allows to get Eddington's equation, i.e. a determination of F as a functional of the DM density profile

Eddington's equation

$$F(\epsilon) = \frac{1}{\sqrt{8}\pi^2} = \frac{d}{d\epsilon}\int_0^{\epsilon} \frac{d\rho}{d\psi}\frac{d\psi}{\sqrt{\epsilon - \psi}}$$

$\epsilon = \psi - \frac{1}{2}v^2$ =mechanical energy per unit mass

input: density profile
output: velocity distribution function

Consistency check: given a density profile ρ(r) the necessary condition F(ε)>0 is not guaranteed (not every density profile is actually consistent to a steady-state solution)

Alternative approach are being attempted: for instance, can get f(v) from a Bayesian analysis → break the degeneracy by choosing a prior distribution for f(v) and get a well defined posterior. Of course quantifying our prior knowledge of f(v) is a sticky issue…

Can assume that our prior knowledge of f(v) favours a (truncated) maxwellian m(v). Find the distribution f(v) that maximizes the *relative entropy*

$$S[f, m] = -\int f(v) \ln\left(\frac{f(v)}{m(v)}\right) dv$$

subject to the experimental constraints:

$$\int_{\mathbf{v}} f(v) w_i(v) dv = \mu_i$$

Using Lagrange multipliers f turns out to be:

$$f(v) \propto m(v) \exp\left(\sum_i \lambda_i w_i(v)\right)$$

A. Fowlie, JCAP10(2017)002

In this way the prior for f is:

$$p(f \,|\, \text{prior knowledge}) \propto e^{\beta S[f,m]}$$

where β is a regularization parameter that describes the strength of our prior information:

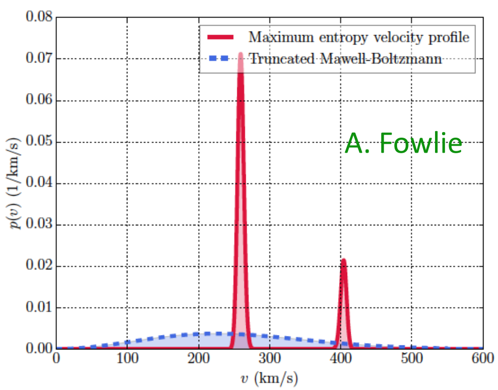
β=0 : no prior information
Minimal entropy

β=∞ : strong belief in a Maxwellian and large entropy (f=m irrespective of the data)

Crucially, for $\beta \neq 0$ the posterior probability is a strictly convex function → unique minimum→ degeneracy broken

Example: fit to the DAMA modulation effect:

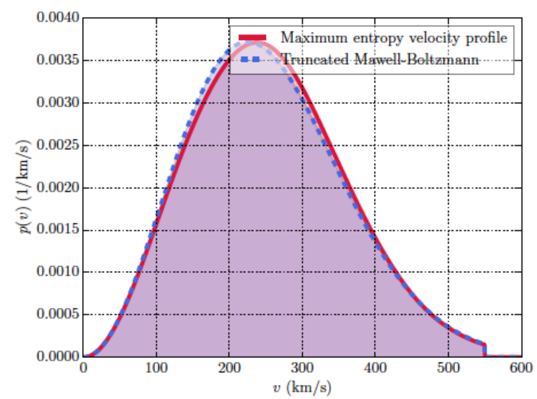
maximum entropy profile



(a) $\beta = 0.$

Continuous "morphing" driven by β from a minimal-entropy solution (two streams at v~250 km/s and v~400 km/s) to a maximal-entropy solution (Maxwellian)

maximum entropy profile



(d) $\beta = 100.$

(isotropic f(v) assumed)

A. Fowlie, JCAP10(2017)002

The bottom line: getting f(v) from experimental data is an "ill-posed" or under-constrained problem (mathematically, it boils down to determining f(v) from its "generalized moments"): need additional assumptions to break the degeneracy

An alternative approach: give up the determination of f(v) and instead integrate out f(v) as a *nuisance* parameter and look for the **maximal range** of any other observable calculated in terms of f(v) (P. Gondolo, S. Scopel, JCAP 1709 (2017) no.09, 032 arXiv:1703.08942).

is it possible? (in principle f(v) is a function of an infinite number of nuisance parameters (i.e. the superposition of an infinite number of streams):

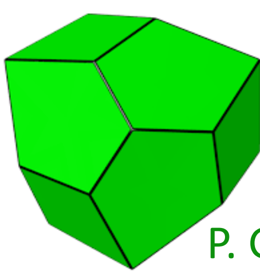
$$f(\vec{u}) = \int d^3u' f(\vec{u}')\delta(\vec{u} - \vec{u}')$$

Consider the case of a finite number of streams $N_{Streams}$

Fixing N_{exp} observations with $N_{streams}$ unknown λ's corresponds to solving the linear system:

$$
\left\{
\begin{array}{rcl}
\displaystyle\sum_{i=1}^{N_{streams}} H_i^1 \lambda_i & = & N_{exp}^1 \\
 & \cdots & \\
\displaystyle\sum_{i=1}^{N_{streams}} H_i^{N_{exp}} \lambda_i & = & N_{exp}^{N_{exp}} \\
\displaystyle\sum_{i=1} \lambda_i & = & 1 \\
\lambda_i > 0 & &
\end{array}
\right.
$$

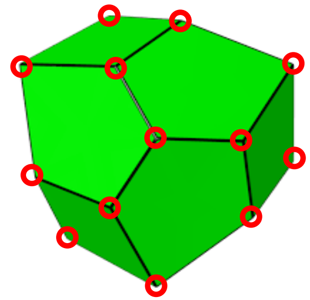
The system is unconstrained ($N_{exp} < N_{streams}$) so it singles out a polyhedron (i.e. the subset of a hyperplane when $0< \lambda_i<1$) of dimension $N_{streams}$ - N_{exp} -1 in the $N_{streams}$ -dimensional space of the λ's



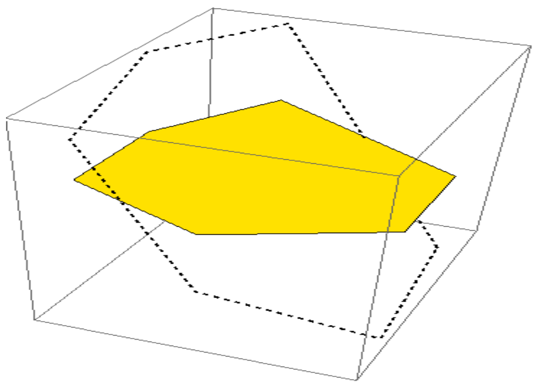
P. Gondolo, S. Scopel, JCAP 1709 (2017) no.09, 032 arXiv:1703.08942

P. Gondolo, S. Scopel, JCAP 1709 (2017) no.09, 032 arXiv:1703.08942

The vertices of the polyhedron are found by fixing to zero $N_{streams}$ - N_{exp} -1 of the λ's and solving the remaining system of N_{exp} +1 unknowns and N_{exp} +1 constraints



An intuitive theorem: given a linear function $W=\Sigma_i \lambda_i W_i$ defined on a polyhedron its maximum and minimum must correspond to one of the vertices



So to find the extrema of W need only to check the vertices – in practice this means to always consider N_{exp} +1 λ's at a time (setting the other λ's to zero)

The problem is N_{exp} +1 dimensional, not $N_{streams}$ dimensional, and since $N_{exp} << N_{streams}$ this implies a huge simplification!

This theorem is also valid in the space of continuous functions:

- given the $N+1$ known functions $g^i(x)$ $(i = 1, ..., N)$ and $h(x)$ and the unknown function $f(x)$, all defined in the same domain, the N constraints:

$$I_g^i = \int_0^\infty g^i(x)\, f(x)\, dx,\ i = 1, ...N,$$

imply that the extreme values of the integral:

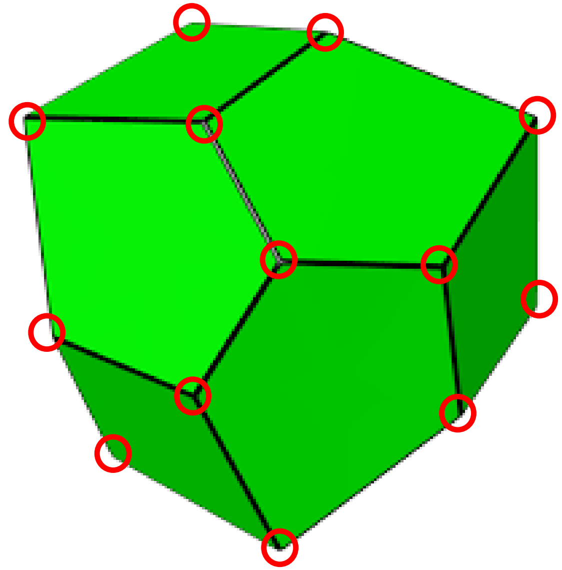
$$I_h = \int_0^\infty h(x)\, f(x)\, dx,$$

are obtained by expressing the unknown function $f(x)$ in terms of the N parametrizations:

$$f_n(x) = \sum_{j=1}^{n} \lambda_j\, \delta(x - x_j),\ n = 1, ..., N,$$

with $\sum_{i=1}^{n} \lambda_i$=1 and n=1,..,N.

I. Pinelis, *"On the extreme points of moments sets"*, Math. Meth. Oper. Res. 83 (2016) 325–349, [arXiv:1204.0249; H. P. Mulholland and P. Rogers, *"Representation theorems for distribution functions"*, Proc. of London Math. Society s3-8(2) (1958) 177–223.



In the continuum case the theorem reduces an extremization problem in infinite dimensions (the moment set) into an extremization problem in a finite number of dimensions (the space of extreme distributions, which has dimension at most $(1+d)N'$, where N=N+1 is the number of moment conditions and d is the dimensionality of the velocity space).

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In practice, this means that, at fixed n, the maximal range of the I_g integral is swept by the λ_j , x_j parameters that satisfy the n constraints with $f_n(x)$ given by the superposition of n streams, i.e. the system of n + 1 linear equations:

$$\sum_{j=1}^{n} \lambda_j g^i(x - x_j) = I_f^i, \quad i = 1, n$$

$$\sum_{k=1}^{n} \lambda_k = 1, \ \lambda_k > 0.$$

The full range of I_g is then obtained by combining the N intervals at fixed n.

Direct application to the analysis of direct detection data: given n experimental measurements any other quantity of the form

$$A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$$

can be bracketed for *any* f(v).

Recap:

- Given n independent direct detection measurements a parameterization of the velocity distribution in terms of n streams, combined with analogous parameterizations for n-1,n-2,…1 brackets *any* observable of the form $A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$ where only A(v) is known.

Where do we get from here?

Halo-independent yearly-modulated fractions

Due to the rotation of the Earth around the Sun the signal in a direct detection experiment depends on time. Assuming that the only time dependence is due to the boost from the Galactic to the Lab rest frame:

$$S(t)_{[E'_1,E'_2]} = \int \mathcal{H}_{[E'_1,E'_2]}(v)\, f(v,t)\, dv =$$

$$S(t)_{0,[E'_1,E'_2]} + S_{m,[E'_1,E'_2]} \cos\left[\frac{2\pi}{365\,\text{days}}(t-t_0)\right] =$$

$$\int \mathcal{R}_{[E'_1,E'_2]}(v)\tilde{\eta}(v,t)\, dv =$$

$$\int \mathcal{R}_{[E'_1,E'_2]}(v) \left\{\tilde{\eta}_0(v) + \tilde{\eta}_1(v)\cos\left[\frac{2\pi}{365\,\text{days}}(t-t_0)\right]\right\} dv$$

Standard lore: need to know explicitly f(v) to get the modulated fraction $\tilde{\eta}_1(v)/\tilde{\eta}_0(v)$ (ex: <10% for a Maxwellian)

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The problem: how to estimate the time dependence (yearly modulation) of the signal from the time dependence of an unknown quantity [f(v)]?

Very simple solution: a change of variable

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$$f(\vec{v}, t) = f_{gal}\left(\vec{u} = \vec{v} + \vec{v}_\odot + \vec{v}_\oplus(t)\right)$$

Lab rest frame

Galaxy rest frame

velocity of the Sun

velocity of the Earth

Change integration variable from v (lab frame) to u (Galactic frame):

$$S_{[E_1', E_2']}(t) = \int \mathcal{H}_{[E_1', E_2']}\left(\vec{u} - \vec{v}_\odot - \vec{v}_\oplus(t)\right)\, f_{gal}(\vec{u})\, d^3u$$

N.B. : the time dependence is now is only in the response function

The unmodulated and modulated parts are obtained via a Fourier time analysis:

$$S_{0,[E_1', E_2']} = \frac{1}{T}\int_0^T dt, S_{[E_1', E_2']}(t)$$

$$S_{m,[E_1', E_2']} = \frac{1}{T}\int_0^T dt, \cos\left[\frac{2\pi}{365}(t - t_0)\right] S_{[E_1', E_2']}(t)$$

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N.B. present detectors are not sensitive to directionality (they only measure the nuclear recoil energy, that depends on the WIMP's speed *in the lab rest frame*) → isotropic response functions H(v) in the lab rest frame

However, when making the change of variable from v (lab frame) to u (galactic rest frame) the detector's response functions $H(\vec{u})$ are non longer necessarily isotropic

In the following we will assume that f(u) is isotropic → in that case can average the response functions over angles:

$$\mathcal{H}_{[E_1',E_2']}(u) \equiv \frac{1}{4\pi}\int d\Omega\, \mathcal{H}_{[E_1',E_2']}(\vec{u})$$

In this case the case:

$$S_{0,[E_1',E_2']} = \int \mathcal{H}_{0,[E_1',E_2']}(u)\, f_{gal}(u)\, du$$
$$S_{m,[E_1',E_2']} = \int \mathcal{H}_{m,[E_1',E_2']}(u)\, f_{gal}(u)\, du$$

$$u \equiv |\vec{u}|$$

with:

$$\mathcal{H}_{0,\mathrm{gal}}(u) = \frac{1}{4\pi}\int d\Omega_u \, \frac{1}{T}\int_0^T dt\, \mathcal{H}\left(|\vec{u}-\vec{v}|\right)$$

$$\mathcal{H}_{m,\mathrm{gal}}(u) = \frac{1}{4\pi}\int d\Omega_u \, \frac{1}{T}\int_0^T dt\, \cos\left[\frac{2\pi}{365}(t-t_0)\right]\mathcal{H}\left(|\vec{u}-\vec{v}|\right)$$

N.B. The modulated amplitude depends on the cosine transform of the response function, which is completely known→ modulation as a property of the detector

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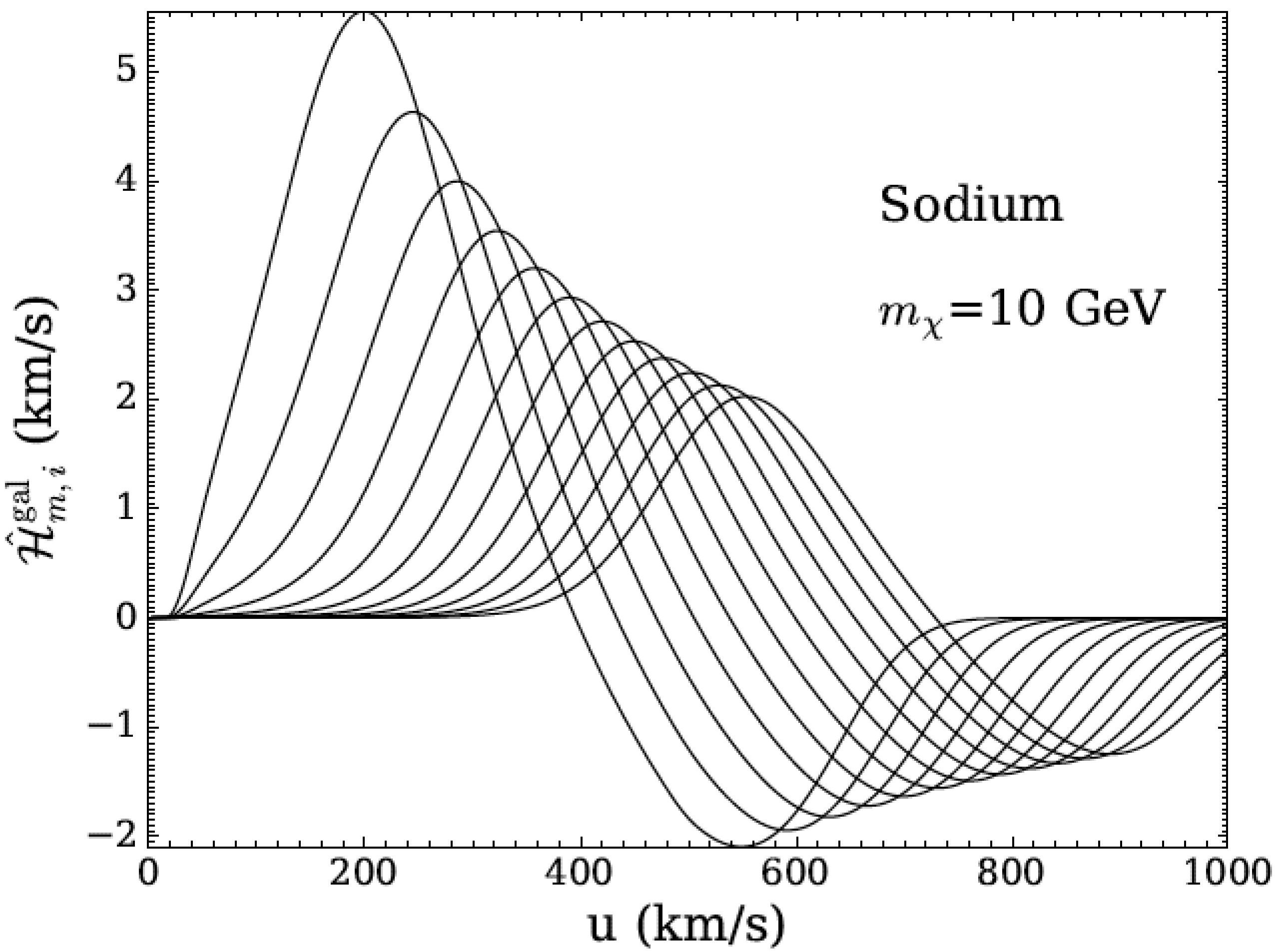
Angle-averaged modulated response functions in DAMA (galactic rest frame)



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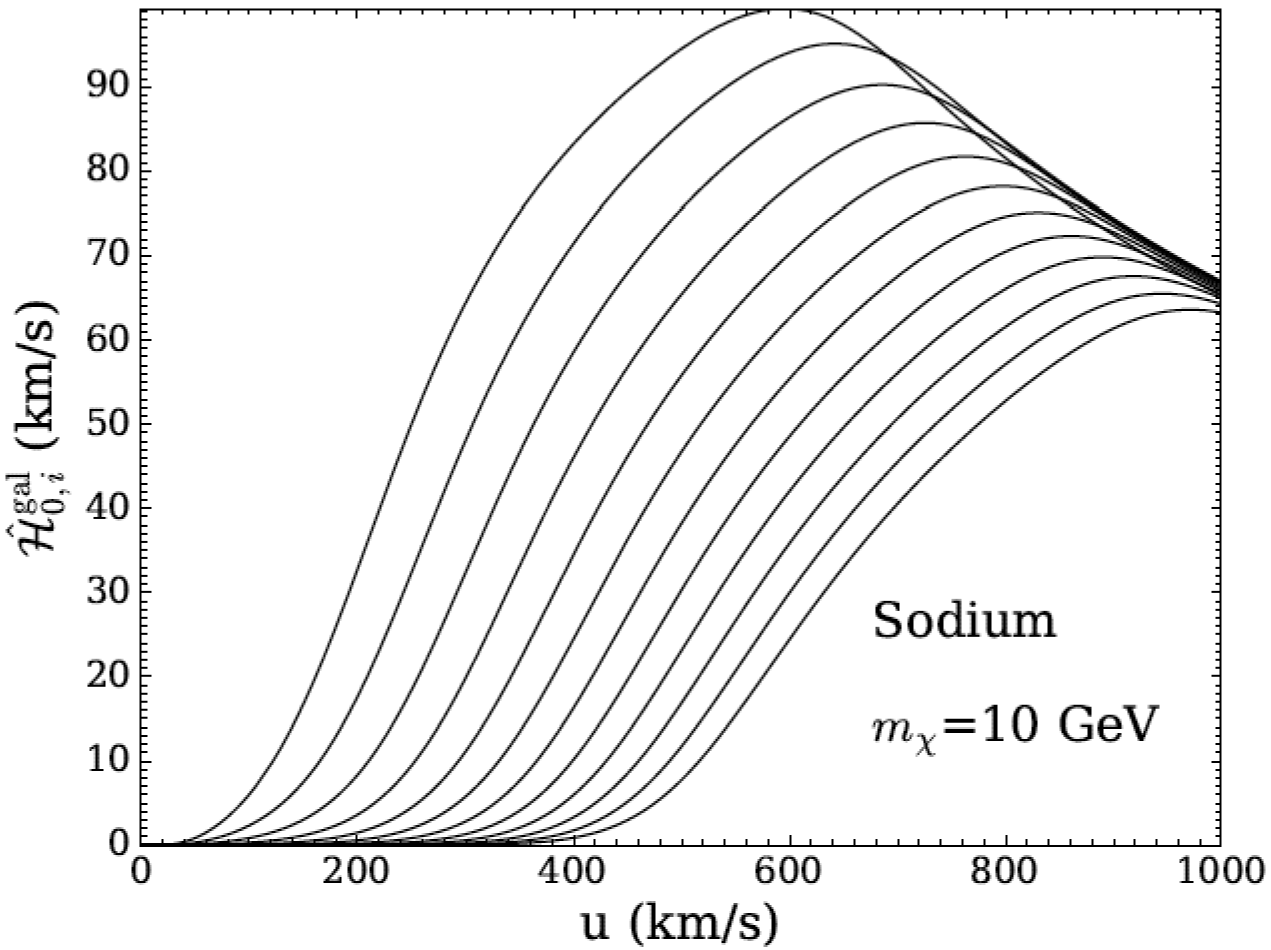
Angle-averaged unmodulated response functions in DAMA (galactic rest frame)



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S_0 and S_m are both given by the integral of a know response function times *the same* unknown f(u) → use theorem on extreme distributions to profile out the unmodulated amplitudes in DAMA starting from measured modulated amplitudes

$$-2\ln\mathcal{L} = \sum_{k=1}^{12}\left(\frac{S_m^k - S_{m,exp}^k}{\sigma_k}\right)^2$$

$$f_{gal}(u) = \sum_{k=1}^{N}\lambda_k\delta(u-u_k), \quad N = 1, 12$$

N.B.: The velocity distribution is a *nuisance* parameter, manageable because we need it only on the boundary, where the dimensionality is reduced

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According to the previous theorem for any choice of the quantities S_m^k (k=1,…,N) any other quantity of the form $A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$ can be bracketed for any f(v). Actually, fixing S_m^k fixes also the Lagrangian L, *so this is also true for a fixed value of L*

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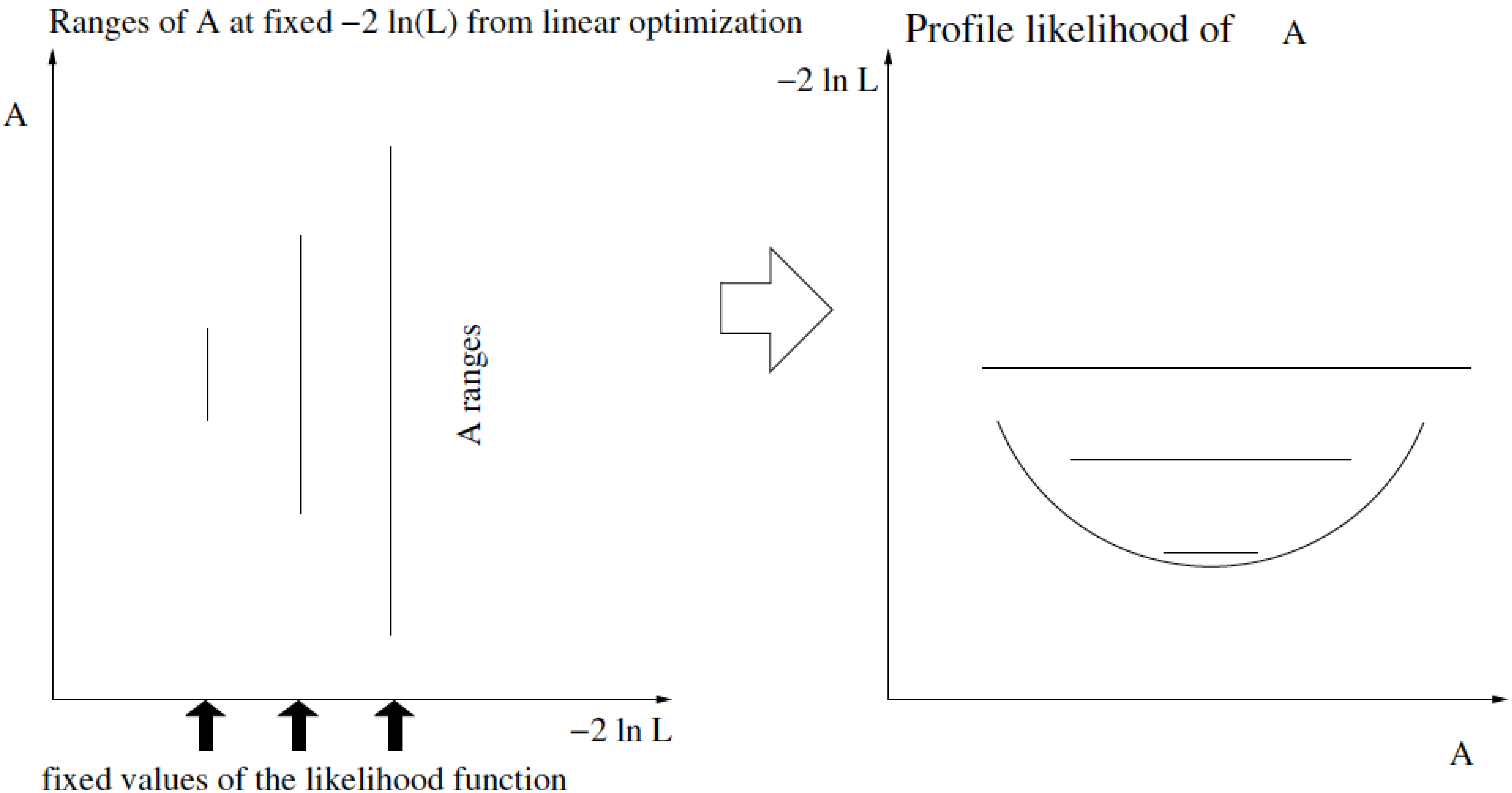
If we can bracket the full range of A at fixed L by turning the plot 90 degrees we can get the profile-likelihood of A



The n-sigma range of A is obtained by taking the points with $L-L_{min}<n^2$

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N.B. the new physics is contained in the cross section σ, which is a normalizing factor (together with the WIMP local density ρ_χ) in the response functions. In particular, in the lab rest frame:

$$\mathcal{H}_i(v) = \frac{N_T}{M_{\rm det}\,\Delta E}\,\frac{\rho_\chi}{m_\chi}\,\sigma_{\chi T}\,\widehat{\mathcal{H}}_i(v),$$

$$\widehat{\mathcal{H}}_i(v) = \frac{v}{E_R^{\rm max}(v)}\int_0^{E_R^{\rm max}(v)} dE_R\,F(E_R,v)\int_{E_{ee,i}}^{E_{ee,i+1}} dE_{ee}\,\mathcal{G}_T(E_{ee},E_R)\,\epsilon(E_{ee})$$

nuclear form factor

energy resolution

Since, for an extreme distribution:

$$S_{0,[E_1',E_2']} = \sum \lambda_k \mathcal{H}_{0,[E_1',E_2']}(u_k)$$

$$S_{m,[E_1',E_2']} = \sum \lambda_k \mathcal{H}_{m,[E_1',E_2']}(u_k)$$

a convenient way is to normalize the streams to σ (at fixed local DM density):

$$\sigma = \sum_k \lambda_k$$

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Problem: how do we sample the parameter space with N=1,2,3,4…,N streams?
According to the theorem the full range of $S_{0,[E_1',E_2']}$ is spanned by using:

$$S_{0,[E_1',E_2']} = \sum_{k=1}^{m} \lambda_k \mathcal{H}_{0,[E_1',E_2']}(u_k) \quad m = 1,2,...,N$$

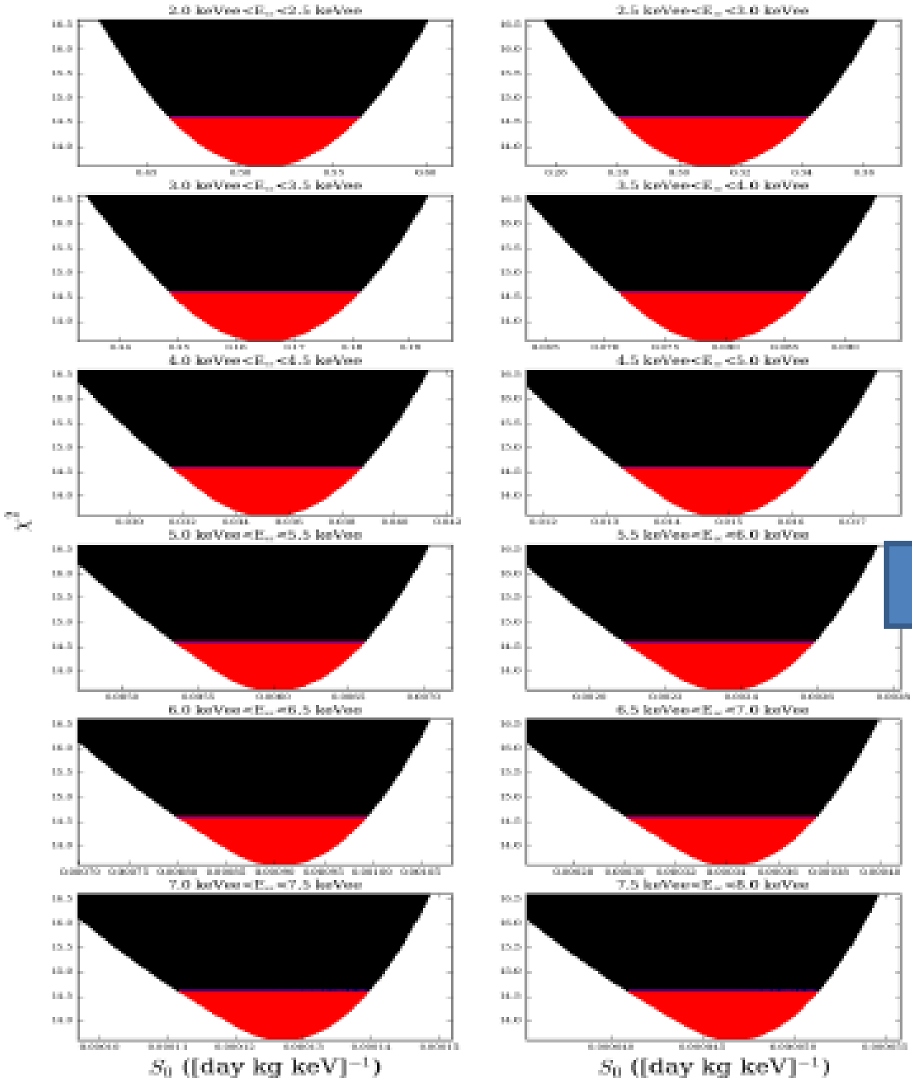
Need to do that numerically.

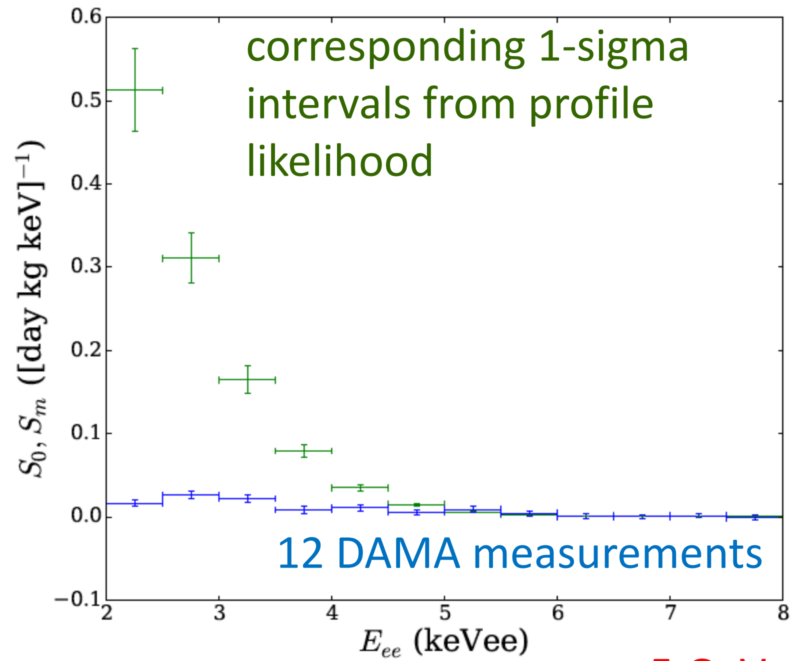
Suitable for a Markov Chain sampling. Two advantages:

- the sampling is driven by the Likelihood itself, don't waste time in low-probability regions
- Perfect for profiling, highest density of points where -2 ln L is minimal

Can use a Markov–Chain Montecarlo code[*] to generate large sets {v} of v_k velocities and {λ} of λ_k coefficients for $1 \le k \le m$ and $1 \le m \le N+1=4$ to calculate both -2 ln L and $\langle \tilde{\eta}(v_{min}) \rangle$

[*]emcee, D. Foreman-Mackey, D. W. Hogg, D. Lang, and J. Goodman, emcee: The mcmc hammer, Publications of the Astronomical Society of the Pacific 125 (2013), no. 925 306.





m_χ=5 GeV

5×10^6 points Markov chain, 250 independent walkers Metropolis-Hastings sampler

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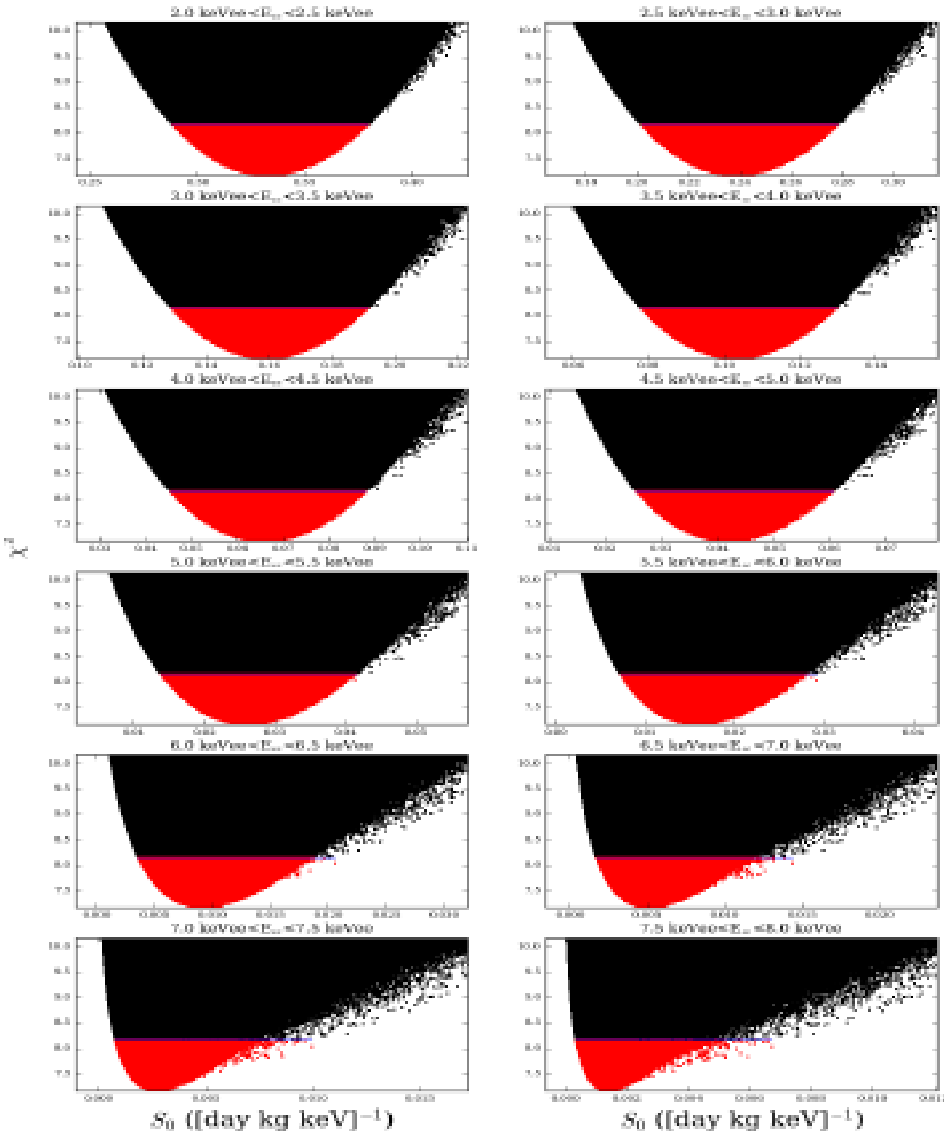

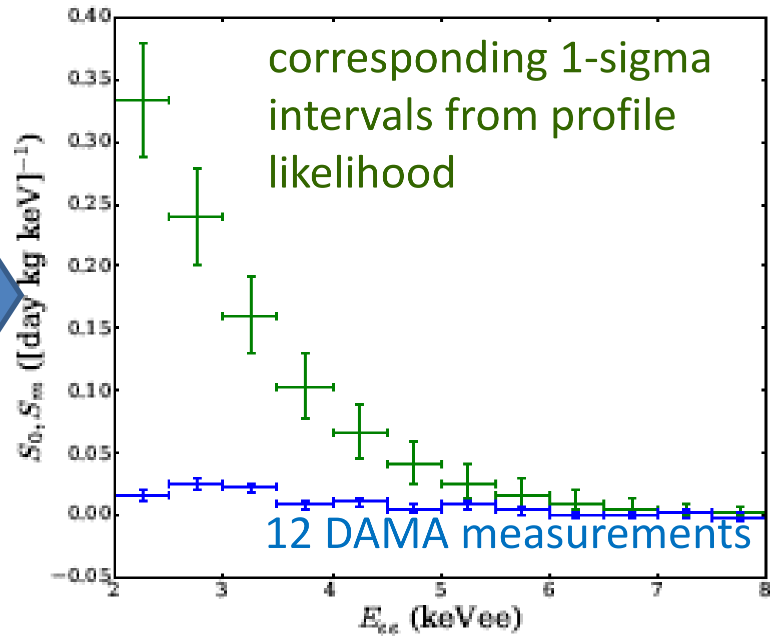

m_χ=10 GeV

5x10[6] points Markov chain, 250 independent walkers Metropolis-Hastings sampler

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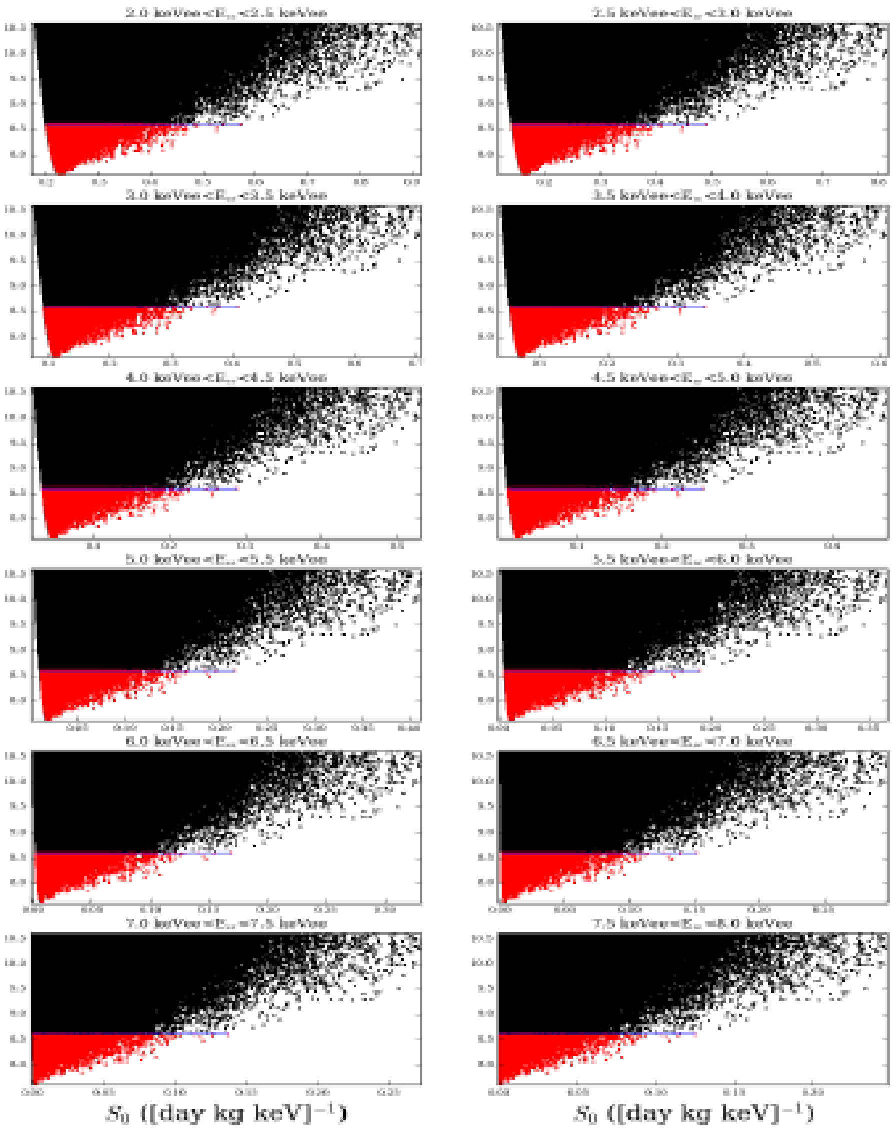

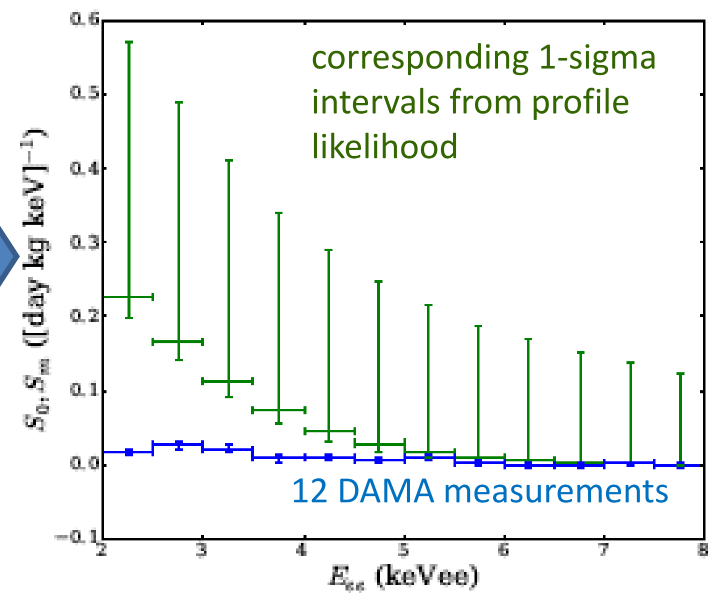

m_χ=15 GeV

5×10^6 points Markov chain, 250 independent walkers Metropolis-Hastings sampler

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Disentangled from the background for
any isotropic velocity distribution

E_i [keVee]	$S_{m,i}$	$S_{0,i}$ $m_\chi = 5$ GeV	$S_{0,i}$ $m_\chi = 10$ GeV	$S_{0,i}$ $m_\chi = 15$ GeV	$B_i + S_i$
2.0–2.5	0.0161(39)	$0.513^{+0.051}_{-0.050}$	$0.333^{+0.046}_{-0.045}$	$0.227^{+0.35}_{-0.027}$	1.029
2.5–3.0	0.0260(44)	$0.311^{+0.031}_{-0.030}$	$0.239^{+0.039}_{-0.037}$	$0.165^{+0.33}_{-0.024}$	1.228
3.0–3.5	0.0219(44)	$0.165^{+0.016}_{-0.016}$	$0.160^{+0.032}_{-0.030}$	$0.112^{+0.30}_{-0.020}$	1.294
3.5–4.0	0.0084(40)	$0.0791^{+0.0078}_{-0.0078}$	$0.103^{+0.027}_{-0.025}$	$0.073^{+0.27}_{-0.017}$	1.140
4.0–4.5	0.0107(36)	$0.0352^{+0.0035}_{-0.0036}$	$0.066^{+0.022}_{-0.021}$	$0.047^{+0.24}_{-0.014}$	0.956
4.5–5.0	0.0054(32)	$0.0148^{+0.0015}_{-0.0016}$	$0.042^{+0.019}_{-0.016}$	$0.030^{+0.22}_{-0.011}$	0.853
5.0–5.5	0.0089(32)	$0.00600^{+0.0059}_{-0.0065}$	$0.026^{+0.016}_{-0.012}$	$0.018^{+0.20}_{-0.0084}$	0.868
5.5–6.0	0.0039(31)	$0.00236^{+0.00023}_{-0.00026}$	$0.0156^{+0.013}_{-0.0084}$	$0.011^{+0.18}_{-0.0059}$	0.853
6.0–6.5	0.00018(308)	$9.04^{+0.89}_{-1.03} \times 10^{-4}$	$0.0090^{+0.011}_{-0.0054}$	$0.0064^{+0.16}_{-0.0038}$	0.868
6.5–7.0	0.00018(281)	$3.41^{+0.34}_{-0.40} \times 10^{-4}$	$0.0050^{+0.0094}_{-0.0033}$	$0.0035^{+0.15}_{-0.0023}$	0.860
7.0–7.5	0.0015(28)	$1.27^{+0.12}_{-0.15} \times 10^{-4}$	$0.0026^{+0.0073}_{-0.0018}$	$0.0019^{+0.13}_{-0.0013}$	0.860
7.5–8.0	−0.0013(29)	$4.67^{+0.46}_{-0.56} \times 10^{-5}$	$0.0013^{+0.0054}_{-0.0010}$	$0.95^{+123}_{-0.71} \times 10^{-3}$	0.890

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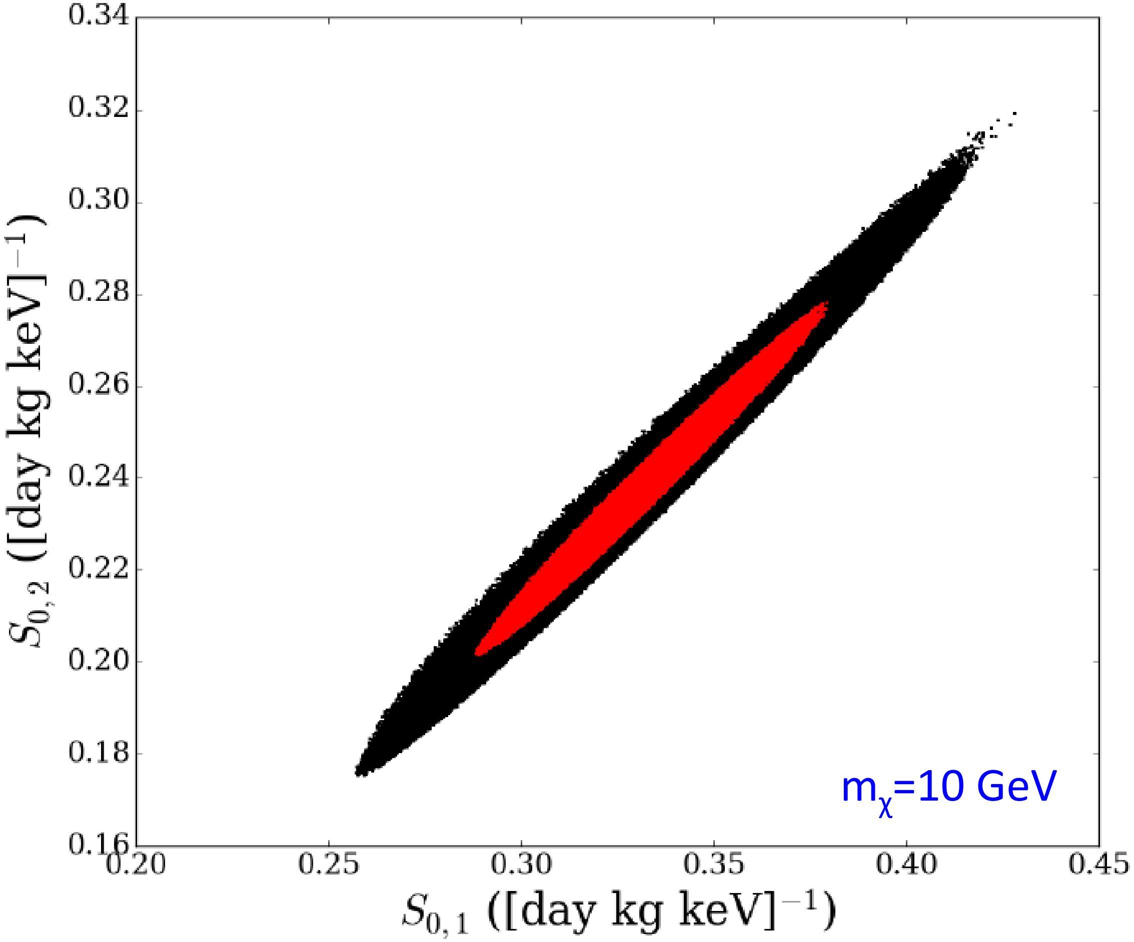
Example of correlation ellipsoid among different $S_{0,i}$



- $-2\Delta \ln \mathcal{L}_p(\{S_{0,i}\}) \leq 1$
- $-2\Delta \ln \mathcal{L}_p(\{S_{0,i}\}) \leq 3$

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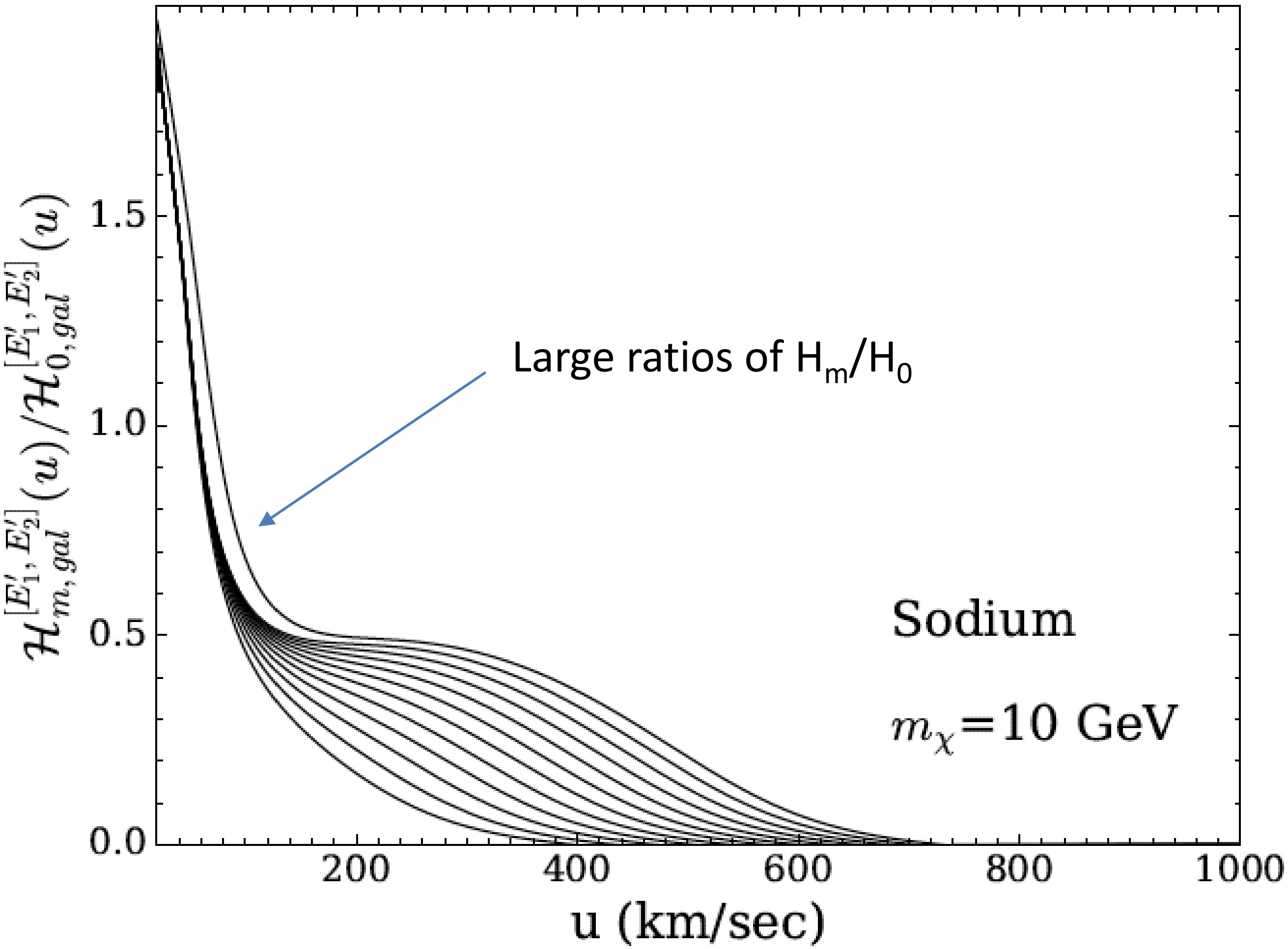
1-sigma ranges for modulation fractions:

m_χ=5: $0.03<S_m/S_0<0.13$; m_χ=10: $0.05<S_m/S_0<0.13$; m_χ=20: $0.07<S_m/S_0<0.19$

N.B. Non-isotropic distributions can easily predict larger modulation fractions (up to 100 %).

However, also the space of isotropic f(u) contains large modulation solutions, which however are disfavored by the data



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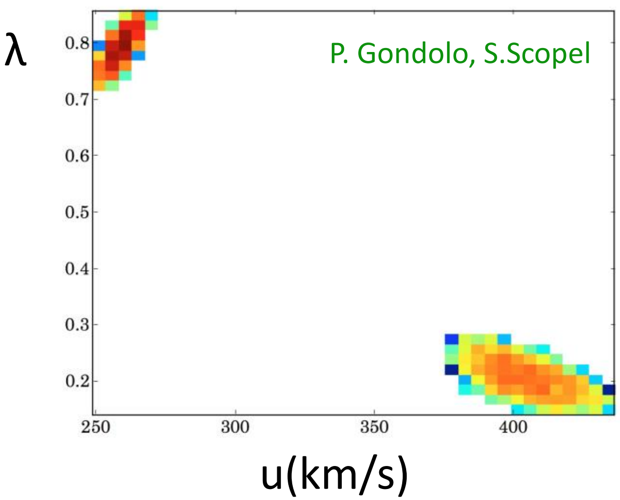
A few comments:

- Cannot adopt this procedure to get a best-likelihood determination of f(v) - the streams parameterization of f(v) is only valid on the boundaries – unless *assume* mimimal-entropy solution à-la Fowlie (regularization parameter $\beta=0$)
- Mathematically and statistically sound procedure, with many potential applications.
- Straightforward generalization to non-isotropic velocity distribution – no conceptual problems, but numerically more challenging.

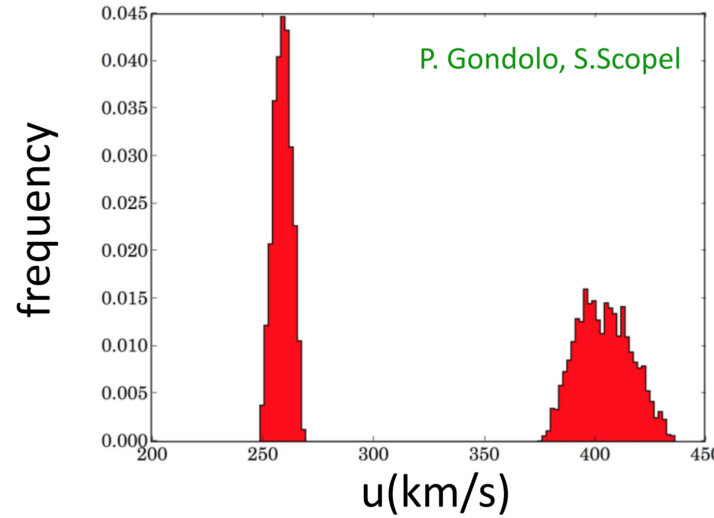
This procedure leads also to a determination of the velocity distribution**, but only to the solution with minimal entropy**. Indeed:

1) in our Markov Chain the absolute minimum of the chi square corresponds to 2 streams
2) Excellent agreement with Minimal Entropy result from Fowlie (JCAP(2017)002) → our result corresponds to β =0 (so, indeed, when β is fixed the f(v) is determined)

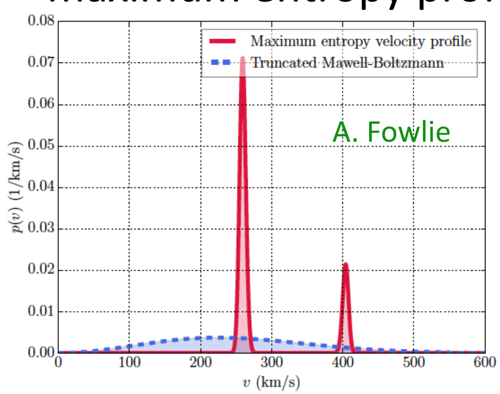
posterior, 2 streams



posterior, 2 streams

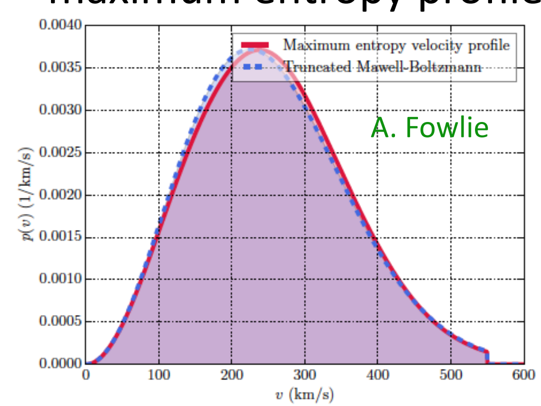


maximum entropy profile



(a) $\beta = 0.$

maximum entropy profile



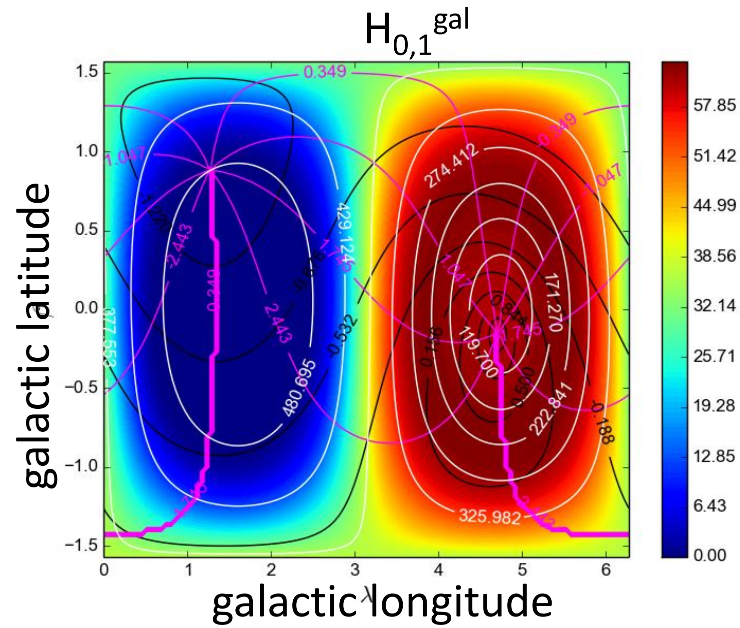
(d) $\beta = 100.$

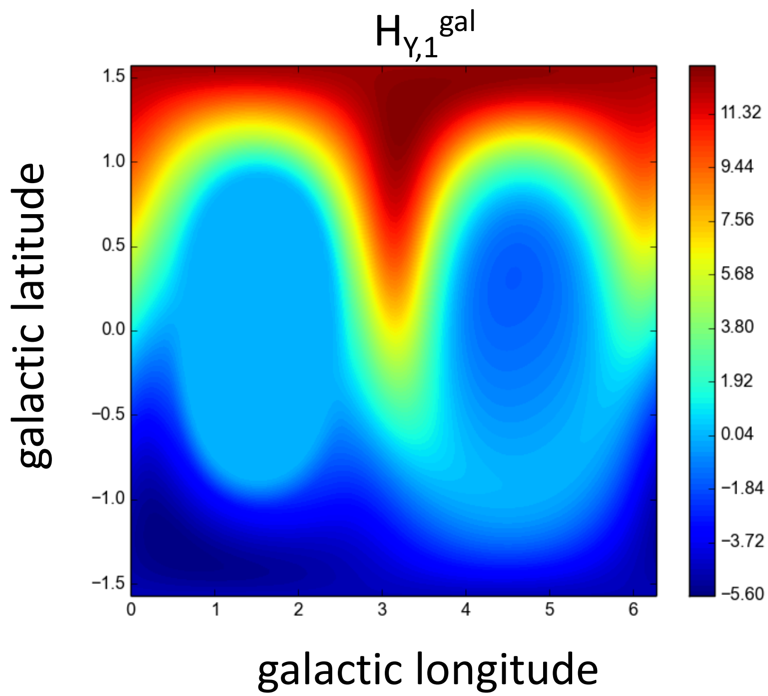
The next step: 3-d response functions for non-isotropic velocity distributions

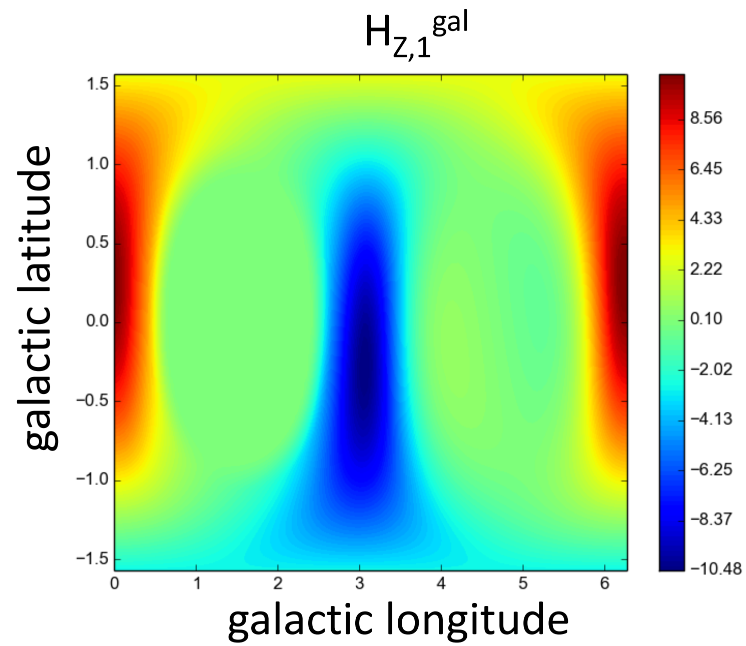
$$\mathcal{H}^{\rm gal}_{0,i}(\mathbf{u}) = \frac{1}{T}\int_0^T dt\, \mathcal{H}_i(|\mathbf{u} - \mathbf{v}_\odot - \mathbf{v}_\oplus(t)|),$$

$$\mathcal{H}^{\rm gal}_{Y,i}(\mathbf{u}) = \frac{2}{T}\int_0^T dt\, \cos[\omega(t-t_0)]\, \mathcal{H}_i(|\mathbf{u} - \mathbf{v}_\odot - \mathbf{v}_\oplus(t)|).$$

$$\mathcal{H}^{\rm gal}_{Z,i}(\mathbf{u}) = \frac{2}{T}\int_0^T dt\, \sin[\omega(t-t_0)]\, \mathcal{H}_i(|\mathbf{u} - \mathbf{v}_\odot - \mathbf{v}_\oplus(t)|).$$







In all plots: |u|=300 km/s

P. Gondolo, S.S. , G. Tomar, in progress

Also the halo-function η can be treated as a nuisance parameter

$$R = \int_0^{v_{esc}} d\,v \mathcal{R}[E_R(v)]\tilde{\eta}(v) = \sum_{k=1}^{N} \delta\tilde{\eta}_{0,1}^{k} \int_0^{v_{esc}} d\,v \mathcal{R}[E_R(v)]$$

$$= \frac{1}{\mu_{\chi N}^2} \sum_{k=1}^{N} \delta\tilde{\eta}_{0,1}^{k} \left[\bar{\mathcal{R}}(E_{max}(v_k, m_\chi, \delta)) - \bar{\mathcal{R}}(E_{min}(v_k, m_\chi, \delta))\right]$$

nuisance parameters

One explicit example: a frequentist analysis of pSIDM (proton-philic Spin Inelastic Dark Matter)

One of the most popular scenarios for WIMP-nucleus scattering is a spin-dependent interaction where the WIMP particle is a χ fermion (either Dirac or Majorana) that recoils through its coupling to the spin of nucleons N=p,n:

$$\mathcal{L}_{int} \propto \vec{S}_\chi \cdot \vec{S}_N = c^p \vec{S}_\chi \cdot \vec{S}_p + c^n \vec{S}_\chi \cdot \vec{S}_n$$

(for instance, predicted by supersymmetry when the WIMP is a neutralino that couples to quarks via Z-boson or squark exchange)

A few facts of life:

Nuclear spin is mostly carried by odd-numbered nucleons. Even-even isotopes carry no spin.

• the DAMA effect is measured with Sodium Iodide. Both Na and I have spin **carried by an unpaired proton**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{23}Na	3/2	11	12	100 %
^{127}I	5/2	53	74	100 %

Germanium experiments carry only a very small amount of ^{73}Ge, the only isotope with spin, **carried by an unpaired neutron**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{73}Ge	9/2	32	41	7.7 %

Xenon experiment contain two isotopes with spin, **both carried mostly by an unpaired neutron**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{129}Xe	½	54	75	26%
^{131}Xe	3/2	54	77	21%

→several authors have considered the possibility that $c_n \ll c_p$: in this case the WIMP particle is seen by DAMA but does not scatter on xenon and germanium detectors

However another class of Dark Matter experiments (superheated droplet detector and bubble chambers) **all use nuclear targets with an unpaired proton**:

Experiment	Target	Type	Energy thresholds (keV)	Exposition (kg day)
SIMPLE	$C_2Cl\,F_5$	superheated droplets	7.8	6.71
COUPP	$C\,F_3\,I$	bubble chamber	7.8, 11, 15.5	55.8, 70, 311.7
PICASSO	$C_3\,F_8$	bubble chamber	1.7, 2.9, 4.1, 5.8, 6.9, 16.3, 39, 55	114
PICO-2L	$C_3\,F_8$	bubble chamber	3.2, 4.4, 6.1, 8.1	74.8, 16.8, 82.2, 37.8

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{19}F	1/2	9	10	100
^{35}Cl	3/2	17	18	75.77 %
^{37}Cl	3/2	17	20	24.23 %
^{127}I	5/2	53	74	100

These experiments are sensitive to c_p , so for $c_n \ll c_p$ spin-dependent scatterings on Fluorine have been shown to lead to tension with the DAMA (C. Amole et al., (PICO Coll.) PLB711, 153(2012), E. Del Nobile, G.B. Gelmini, A. Georgescu and J.H. Huh, 1502.07682)

N.B. All only sensitive to the energy threshold, which for bubble and droplets nucleation is controlled by the pressure of the liquid

S.Scopel and K.Yoon, JCAP 1602(2016)050

Evading Fluorine constraints for a WIMP with spin-dependent coupling to protons: inelastic scattering (proton-philic Spin-dependent IDM, pSIDM)

$$v_{\min} = \frac{1}{\sqrt{2 m_N E_R}} \left| \frac{m_N E_R}{\mu_{\chi N}} + \delta \right|$$

$v_{min} > v^*_{min}$ $\qquad$ $$v^*_{\min} = \sqrt{\frac{2\delta}{\mu_{\chi N}}}$$

$A_{sodium}=23$ $\qquad$ $A_{Fluorine}=19$

$m_{Sodium} > m_{Fluorine} \rightarrow \mu_{\chi N}^{Sodium} > \mu_{\chi N}^{Sodium}$

$\rightarrow v^{*Sodium}_{min} < v^{*Fluorine}_{min}$

what if $v^{*Sodium}_{min} < v_{esc} < v^{*Fluorine}_{min}$?

(N.B. v_{esc} in lab frame)

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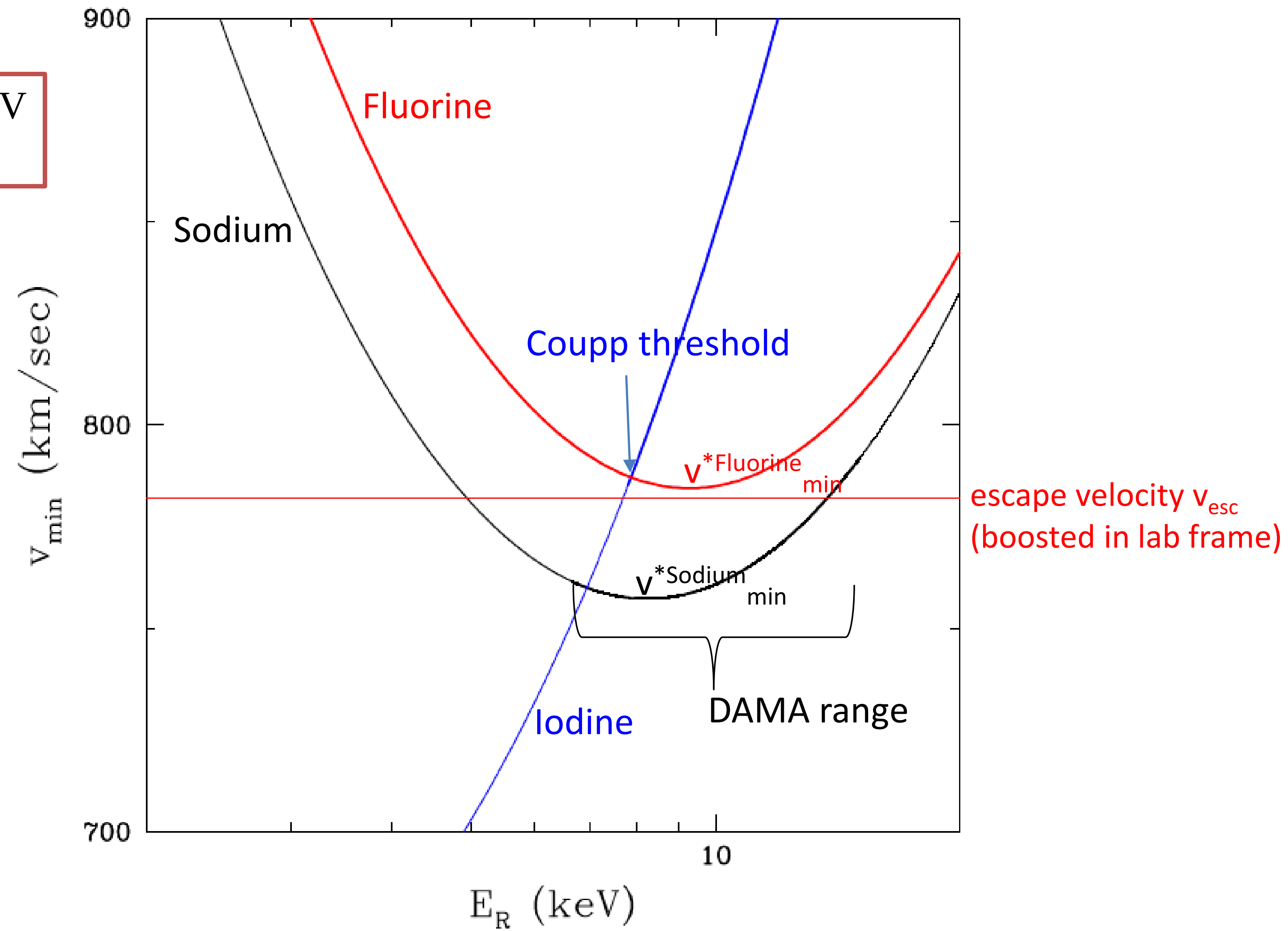
m_χ = 11.4 GeV
δ=23.7 keV



depending on m_χ and δ, can drive Fluorine (and Iodine in COUPP) beyond v_{esc} while Sodium remains below → no constraint on DAMA from droplet detectors and bubble chambers

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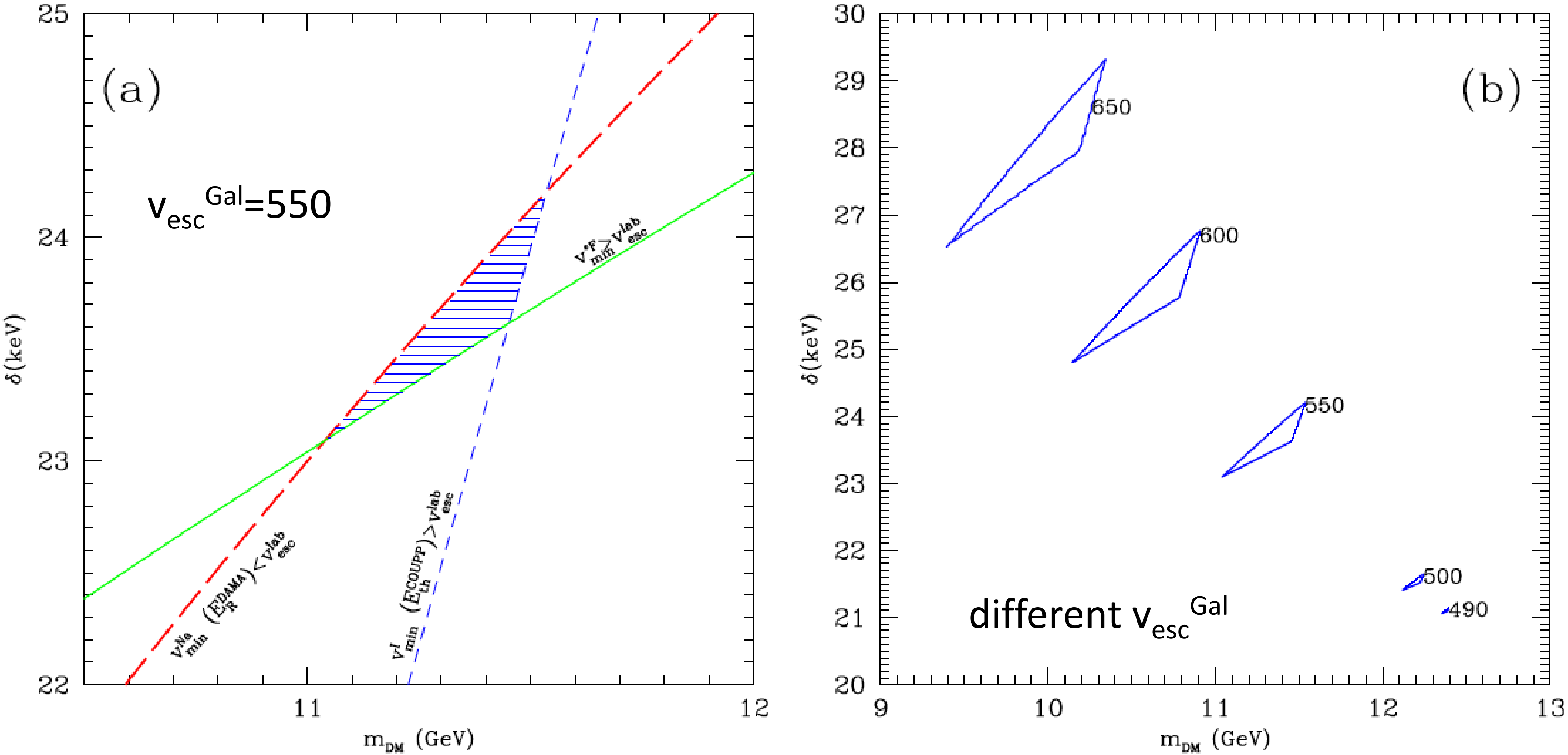

very tuned region. but this is just kinematics

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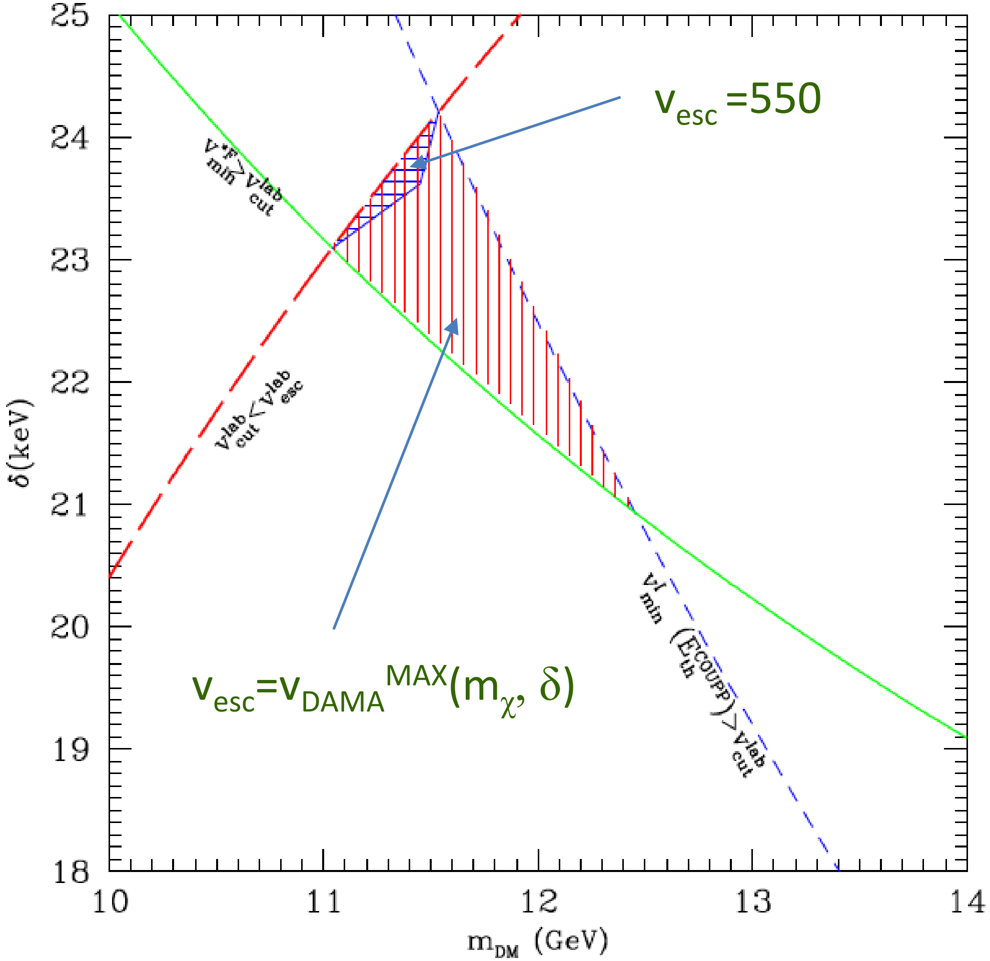
taking $v_{esc}=v_{DAMA}^{MAX}(m_{\chi}, \delta)$ the kinematic region enlarges considerably



Construct a likelihood function fixing $c_i=c_4$ (spin-dependent interaction.
Including the DAMA modulation amplitudes and all constraints
In the likelihood both the non-modulated halo function η_0 and the modulated one η_1 are parameterized as:

$$\tilde{\eta}_{0,1}(v) = \sum_{k=1}^{N} \delta\tilde{\eta}_{0,1}^{k} \theta(v_k - v)$$

with the constraints:

$$\tilde{\eta}_0(v_{\rm min,2}) \le \tilde{\eta}_0(v_{\rm min,1}) \quad \text{if } v_{\rm min,2} > v_{\rm min,1}$$ (decreasing function)

$$\tilde{\eta}_1 \le \tilde{\eta}_0 \quad \text{at the same } v_{\rm min}$$ (modulated part<100%)

$$\tilde{\eta}_0(v_{\rm min} \ge v_{\rm esc}) = 0.$$ (no bound WIMPs<escape velocity)

Also backgrounds b_i are taken as free parameters with the only requirement of positivity.
Then using:

$$\mathcal{L}(m_\chi, \delta, r = c_4^n / c_4^p, \delta\tilde{\eta}_0^k, \delta\tilde{\eta}_1^k, b_i)$$

sample the parameter space with a Markov-Chain Montecarlo to profile the WIMP parameters m_χ, δ, $r=c^n_4/c^p_4$

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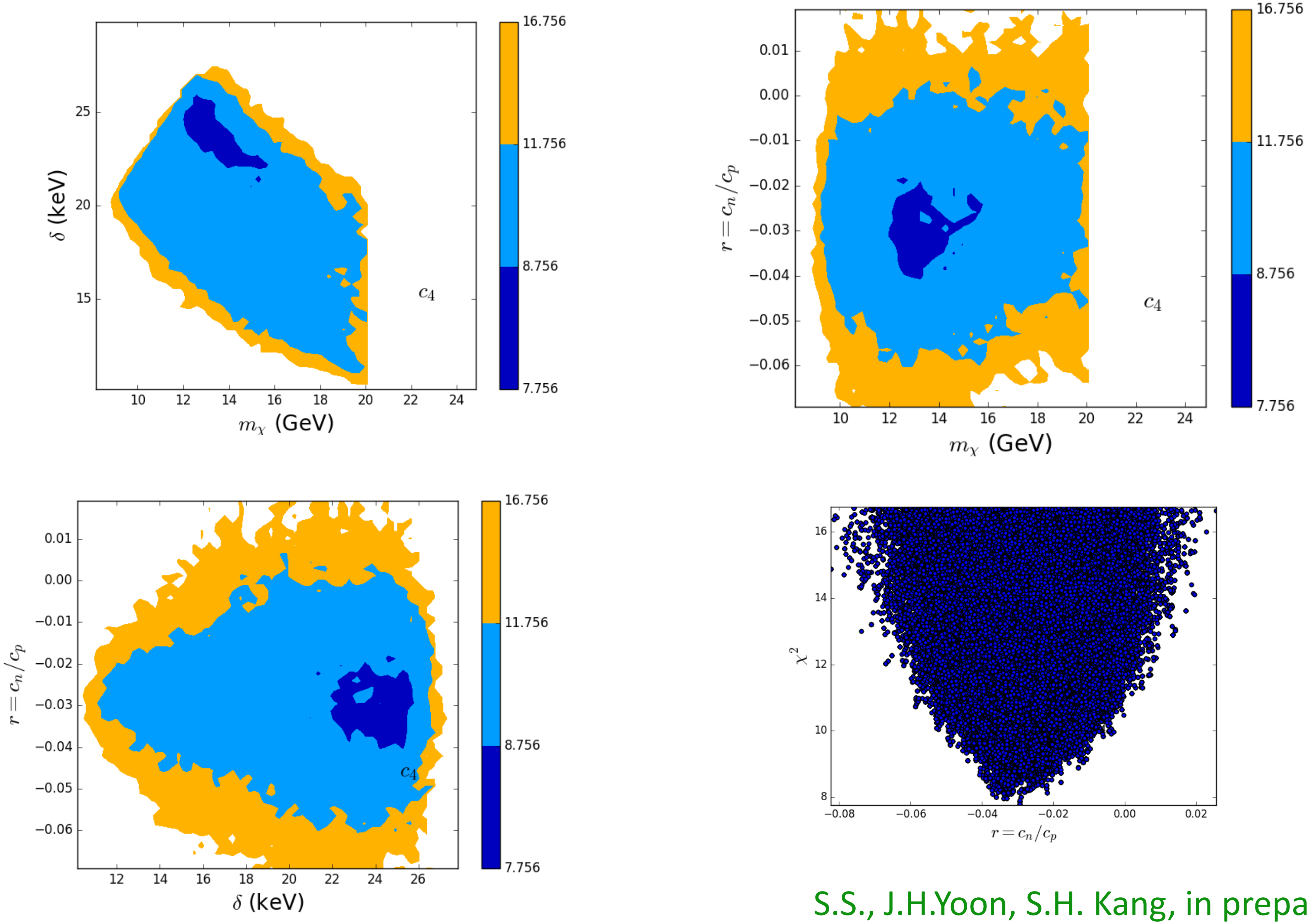
Contour plot for absolute minimum of chi square (integrating over # of streams)



S.S., J.H.Yoon, S.H. Kang, in preparation

Conclusions

- A model independent analysis of direct detection data implies:

1) using non-relativistic effective theory which introduces new response functions with explicit dependence on the transferred momentum and the WIMP incoming velocity
2) factorizing the halo-function dependence
3) allowing for inelastic scattering
4) allowing for isovector couplings

- In this way a much wider parameter space opens up.

Theoretical predictions for WIMP signals depend on integrals of the velocity distribution f(v) (**generalized moments**)→ cannot be directly inverted using exp data to determine the velocity distribution (degenerate problem, integration implies information loss)

An alternative approach: treat f(v) (or the halo function η) as a nuisance parameter and profile the other WIMP parameters (couplings, mass, etc) or other observables.