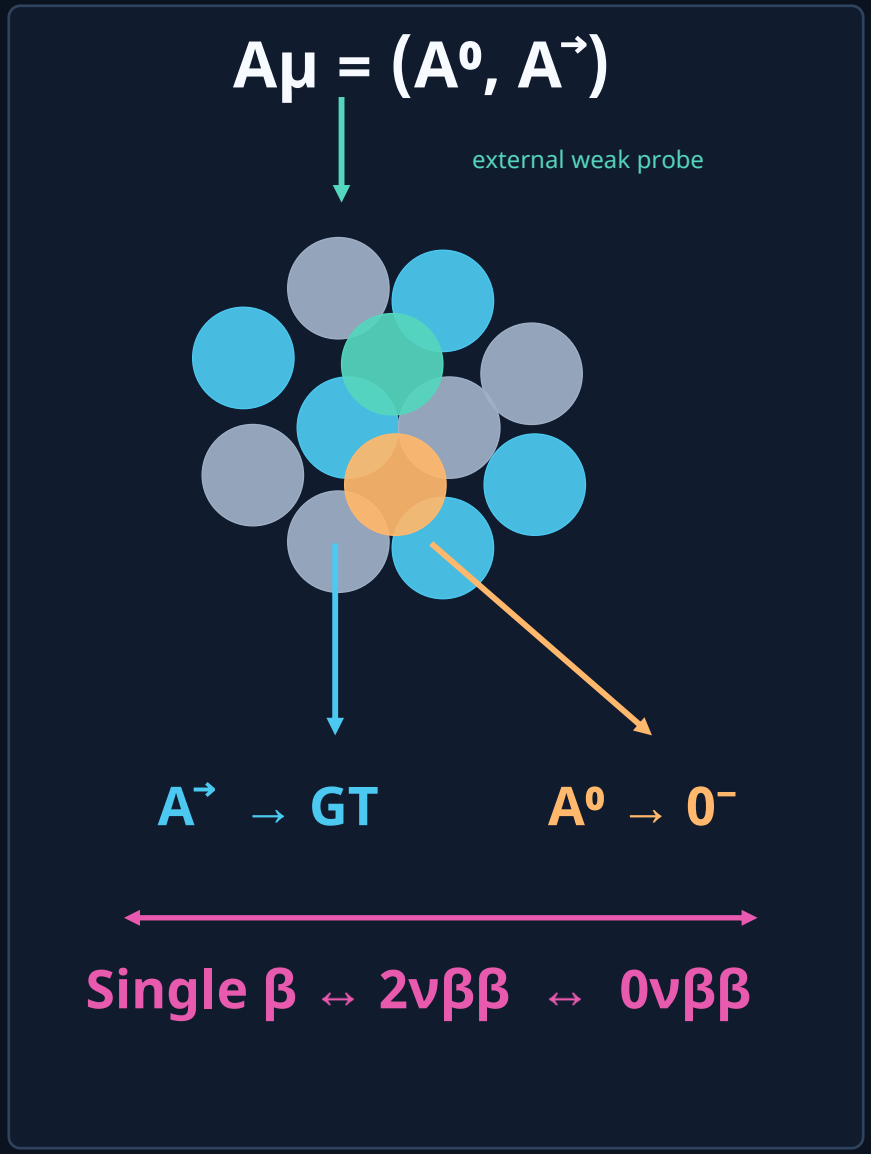
NUCLEAR WEAK CURRENT

Nuclear Axial-Vector Coupling Constant g_a

Axial charge and its role in beta and double-beta decay

- A systematic comparison of Gamow–Teller quenching, axial-charge enhancement, 2νββ, and 0νββ -



Myung-Ki Cheoun, Soongsil Univ. & OMEG(Origin of Matter and Evolution of Galaxies) Institute

The entire talk in one slide

Single beta and Gamow–Teller

$$A_{1b} \simeq g_A \boldsymbol{\sigma}\tau^{\pm}$$

- Spatial axial current
- Allowed, low-q transitions
- Often appears quenched in restricted models

Axial charge

$$A^0_{1b} \sim g_A \boldsymbol{\sigma}\cdot\boldsymbol{p}/m_N$$

- Time component of the axial current
- Rank-zero, parity changing
- Soft-pion two-body term can enhance it

Double-beta decay

$$2\nu : GT \times GT \qquad 0\nu : J^\mu J_\mu$$

- 2νββ probes low-q GT vertices
- 0νββ probes **finite-q current–current operators**
- No universal quenching factor covers both

Central message: effective couplings are operator-, momentum-, model-, and nucleus-dependent.

Sources: Refs. [3–7, 9–13]

Roadmap

01 — Language & current decomposition

02 — Single beta & Gamow–Teller quenching and ^{54}K

03 — Axial-charge enhancement

04 — 2νββ and 0νββ

05 — Implementation, uncertainties, conclusions

Backup slides contain conventions and references.

Charged weak current at the nucleon level

$$A^0(\mathbf{x}) \ : \ \text{axial-charge density}$$

$$Q_5 = \int d^3x \, A^0(\mathbf{x}) \ : \ \text{axial charge}$$

$$J^\mu_{CC} = V^\mu - A^\mu$$

$$\langle p|A^\mu|n\rangle = \bar{u}_p \left[G_A(q^2)\gamma^\mu\gamma_5 + \frac{G_P(q^2)}{2m_N} q^\mu \gamma_5 \right] u_n$$

Form factors

- $G_a(q^2)$: axial
- $G_p(q^2)$: induced pseudoscalar
- $g_a \equiv G_a(0)$

$|g_a| \approx 1.275$ for a free nucleon

$$g_A \neq A^0$$

What changes in a nucleus?

- **Wave functions** — correlations, truncations
- **Operators** — two-body currents, evolution
- **Kinematics** — q, energy transfer, multipoles

An “effective g_a” may absorb all three.

$$\Gamma^\mu(q) = g_V\gamma^\mu - g_A\gamma^\mu\gamma_5 + i[g_M/(2m_N)]\sigma^{\mu\nu}q_\nu - g_P q^\mu\gamma_5$$

Nonrelativistic reduction: $A^{\rightarrow}$ is leading, A^0 is recoil order

Spatial axial current

$$\mathbf{A}_{1b}^{\pm} = g_A \sum_{i=1}^{A} \boldsymbol{\sigma}_i \tau_i^{\pm} + \cdots$$

Leading order **O(1)**

- Allowed Gamow–Teller transitions
- No parity change
- Dominant in ordinary β decay and 2νββ

$$\boldsymbol{A}_{1b}(\boldsymbol{r}) = \sum_{j=1}^{A} \delta(\boldsymbol{r} - \boldsymbol{r}_j) \left[g_A(q^2)\boldsymbol{\sigma}_j - g_P(q^2) \frac{\boldsymbol{q}(\boldsymbol{\sigma}_j \cdot \boldsymbol{q})}{2m_N} \right] \tau_j^{\pm},$$

Axial charge A^0

$$A_{1b}^{0,\pm} = -\frac{g_A}{2m_N} \sum_{i=1}^{A} \tau_i^{\pm} \boldsymbol{\sigma}_i \cdot (\mathbf{p}_i + \mathbf{p}_i') + \cdots$$

Recoil order **O(p/m_n)**

- Rank-zero pseudoscalar operator
- Parity-changing **first-forbidden transitions**
- Small one-body denominator enables large relative 2BC

$$A_{1b}^{0}(\boldsymbol{r}) = \sum_{j=1}^{A} \delta(\boldsymbol{r} - \boldsymbol{r}_j) \left[g_A(q^2) \frac{\boldsymbol{\sigma}_j \cdot (\boldsymbol{p}_j' + \boldsymbol{p}_j)}{2m_N} - g_P(q^2) \frac{q_0(\boldsymbol{\sigma}_j \cdot \boldsymbol{q})}{2m_N} \right] \tau_j^{\pm}.$$

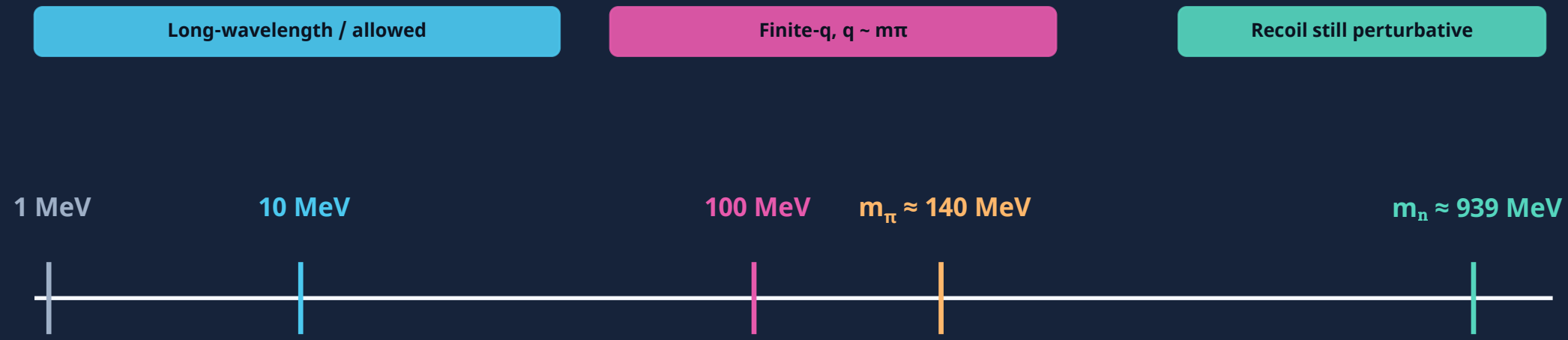
Multipoles, parity, and selection rules

Current component	Long-wavelength operator	Typical J^π	Representative process
Spatial $\vec{A}$	$\sigma\,\tau\pm$	1^+	Allowed GT β decay; 2νββ
Axial charge A^0	$\sigma\cdot p\,\tau\pm / m_n$	0^-	Rank-zero first-forbidden β decay
Finite-q $\vec{A}$ L, E, M	spin–dipole and higher multipoles	$0^-, 1^-, 2^-, \ldots$	0νββ multipole sum

Important distinction

Negative-parity multipoles in 0νββ do not imply that the axial charge A^0 dominates; finite-q spatial currents also generate them.

Momentum transfer selects the relevant operator



0νββ is not a zero-momentum extrapolation of ordinary β decay.

Allowed GT transitions and the definition of B(GT)

$$\hat{O}^{\pm}_{GT} = \sum_{i=1}^{A} \boldsymbol{\sigma}_i \tau_i^{\pm}$$

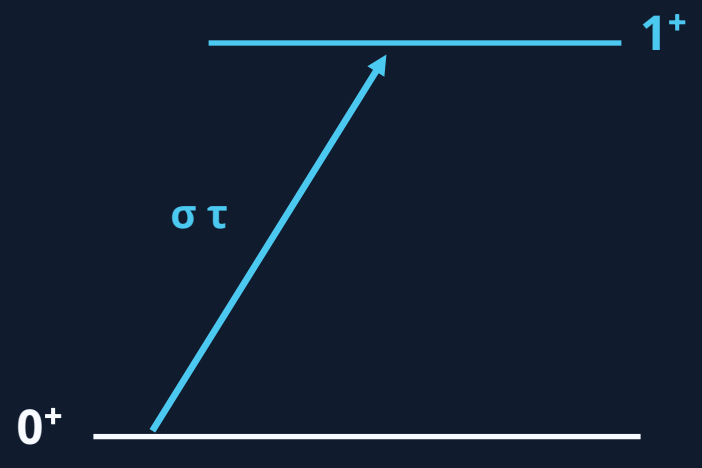
$$B(\mathrm{GT}) = \frac{1}{2J_i + 1} \left| \left\langle f \left\| \hat{O}^{\pm}_{GT} \right\| i \right\rangle \right|^2$$

Allowed selection rules

- ΔJ = 0, ±1 (but not 0 → 0)
- No parity change
- Strength can fragment over many 1^+ states

Energy-level picture

The measured ft value converts decay probability into B(GT), once the branch and phase space are known.



What does a quenching factor actually mean?

$$g_A^{\text{eff}} = q\, g_A^{\text{free}}$$

$$M_{GT}^{\text{eff}} = qM_{GT}^{\text{calc}}, \qquad B(GT)_{\text{eff}} = q^2 B(GT)_{\text{calc}}$$

For q = 0.75

75%
matrix element

56.3%
strength B(GT)

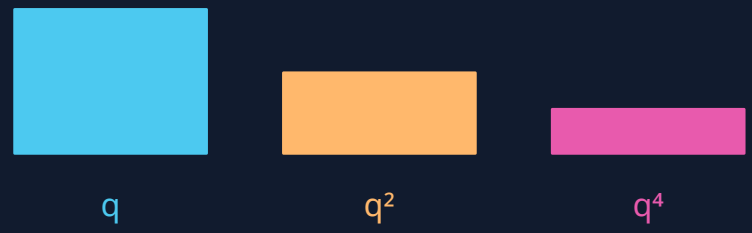
Rate sensitivity

Amplitude

Single-vertex strength

Two-vertex rate (fixed NME)

The q⁴ rule is only a sensitivity estimate when the nuclear matrix element itself is held fixed.



Quenching is a bookkeeping result of missing physics

01 **Model-space truncation** Core polarization, 2p–2h and higher excitations redistribute GT strength.

02 **Wave-function / Hamiltonian errors** Single-particle gaps, tensor and spin–isospin correlations, pairing, deformation.

03 **Operator truncation** Two-body axial currents and induced many-body terms are **omitted from a one-body GT operator**.

Fitted q

↓

absorbs all three

Hence q is nucleus-, model-, operator-, and momentum-dependent.

Chiral two-body spatial axial current

One-body current

$\vec{A}$

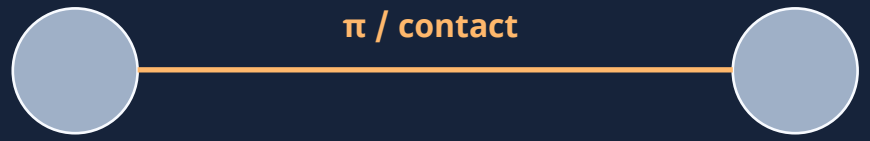
spectator nucleon

$$A_{1b} \propto g_A \boldsymbol{\sigma}\tau$$

$$\mathbf{A}_{1b}^{\pm} = g_A \sum_{i=1}^{A} \boldsymbol{\sigma}_i \tau_i^{\pm} + \cdots$$

Two-body current

$\vec{A}$



$$A = A_{1b} + A_{2b} + \cdots$$

Normal ordering can make A_{2b} look like $\delta g_a(q,\rho)$, but the correction is not universal.

$$A^0_{2b,\pi} \sim \frac{g_A}{4f_\pi^2} \sum_{i<j} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^{\pm} (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \cdot \boldsymbol{\nabla}_{ij} Y_\pi(r_{ij})$$

Phenomenological q versus a consistently evolved operator

Phenomenological route

$$\hat{O}_{GT}^{\mathrm{eff}} = q\,\hat{O}_{GT}^{1b}$$

- Fit q in a mass region
- Efficient and practical
- Mixes operator and wave-function errors
- Transfer to a different q or nucleus is uncertain

Microscopic route

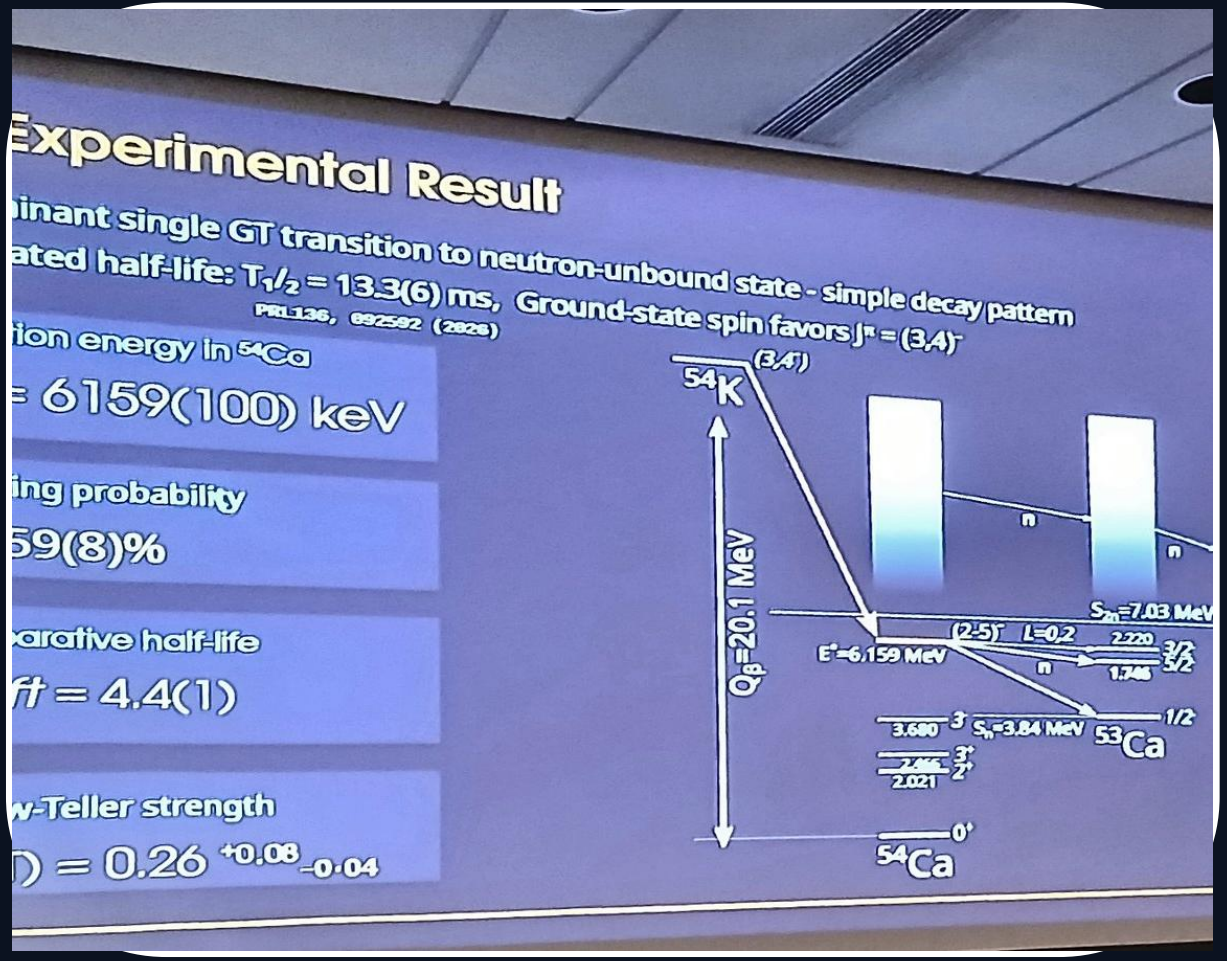
$H_{\mathrm{eff}},\ O_{\mathrm{eff}}$ from the same EFT/evolution

- Correlations included in the many-body method
- Two-body current retained explicitly
- Induced many-body operators must be truncated
- Residual method and EFT uncertainties remain

Agreement without an extra q does not mean “no quenching”; it means the renormalization is carried by explicit physics.

very neutron-rich !!

Case study: neutron-rich ^{54}K → ^{54}Ca*



Conference-slide photograph: key experimental result

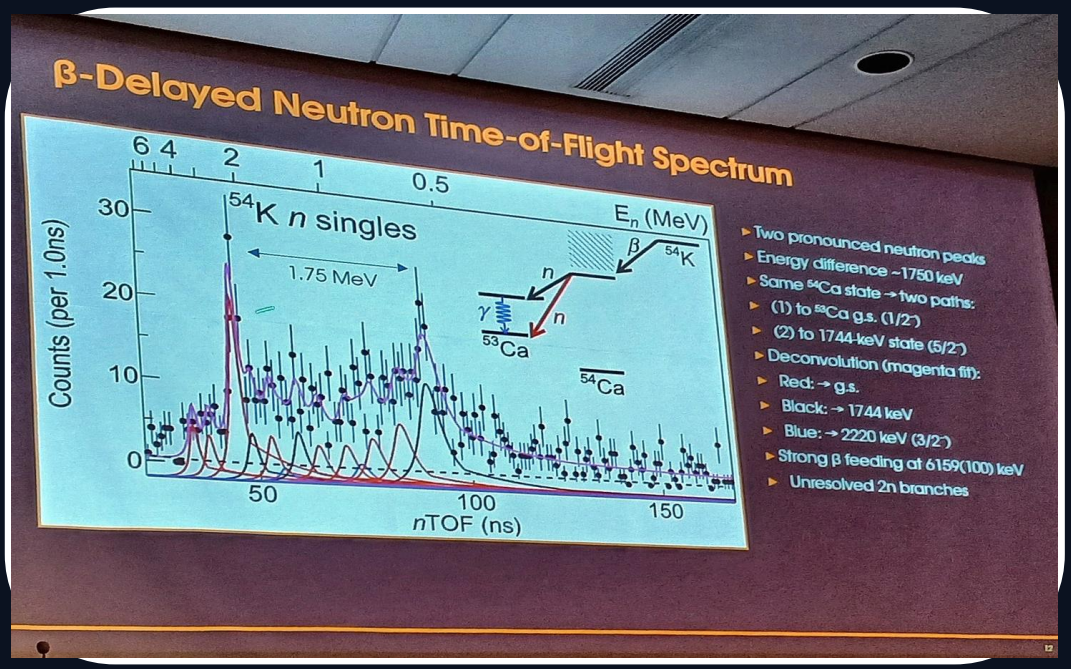
Why this decay is a useful benchmark

- A dominant β branch feeds one neutron-unbound state in ^{54}Ca.
- B(GT) can be constrained by β-delayed neutron and γ spectroscopy.
- The decay probes a neutron-rich, multi-shell regime beyond standard stable-nucleus benchmarks.
- LSSM and VS-IMSRG can be compared against the same strong transition.

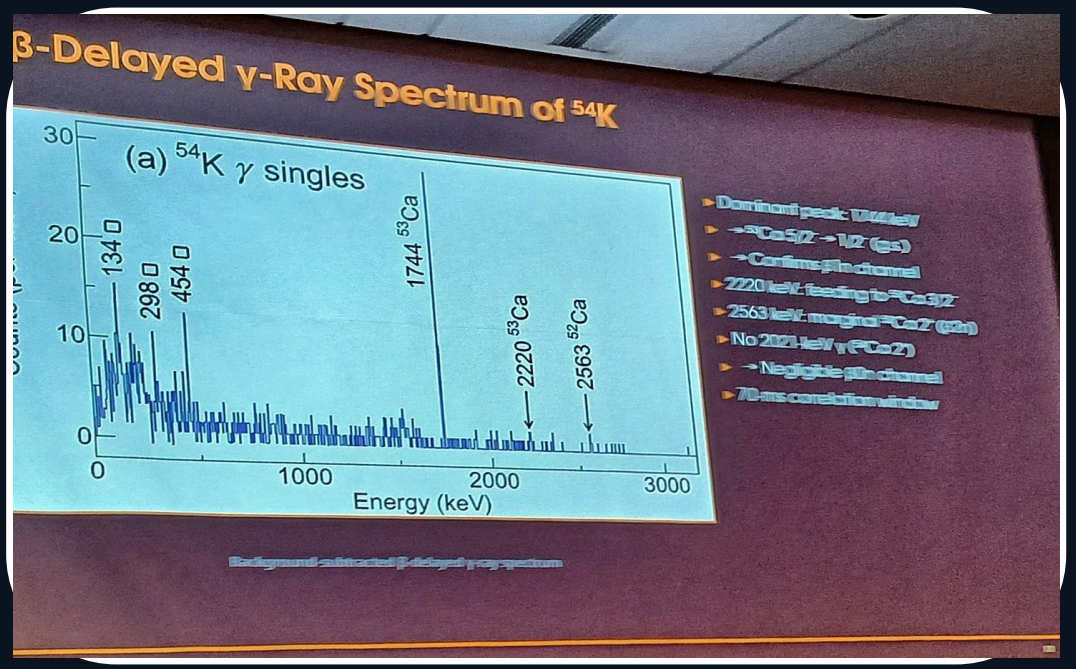
Focus of the case study

Can explicit 2BC replace the empirical q ≈ 0.75?

β-delayed neutron and γ spectroscopy reconstruct the decay path



β-delayed neutron time-of-flight spectrum



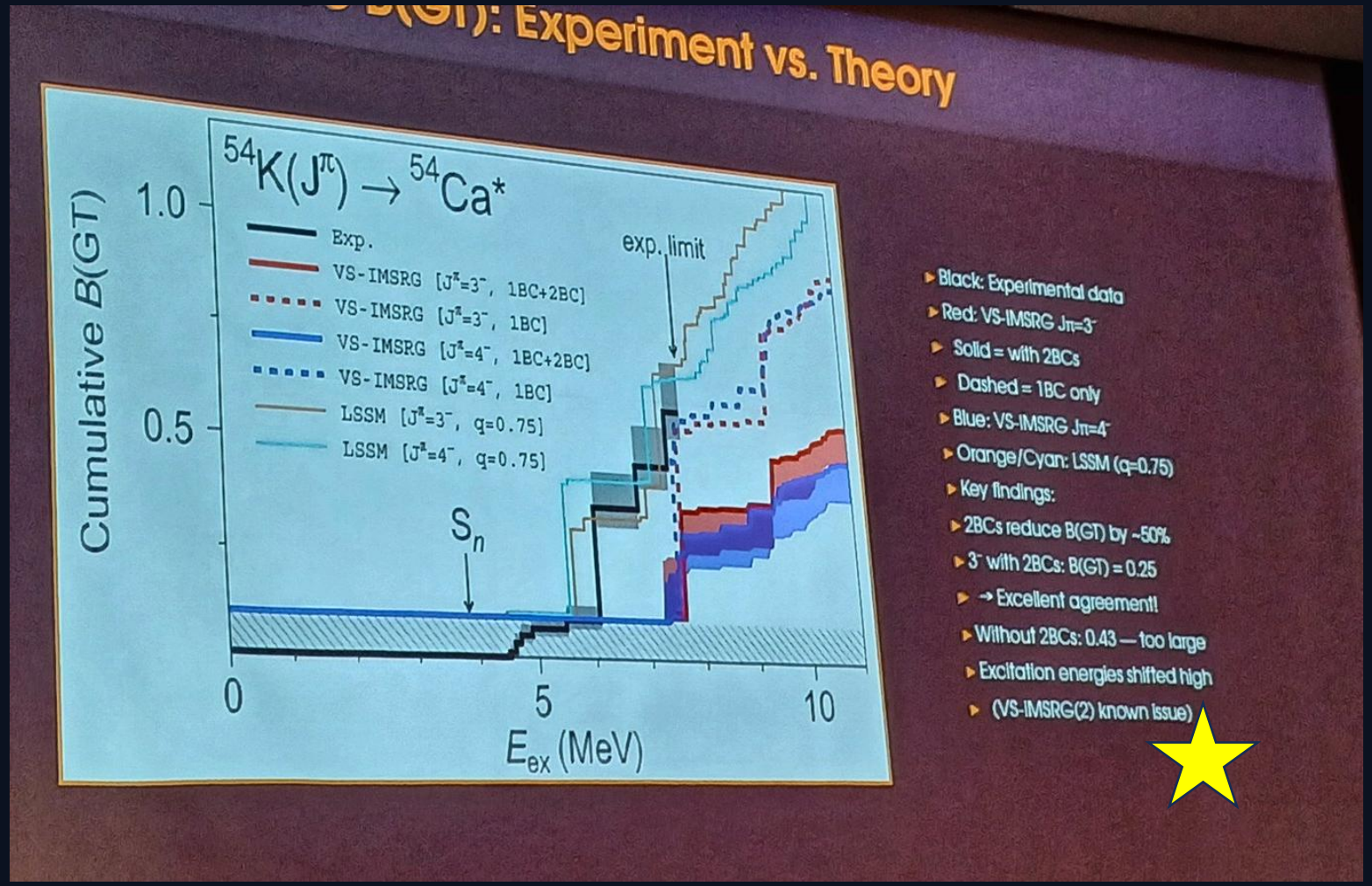
β-delayed γ-ray spectrum

^{54}K —β⁻→ ^{54}Ca* 6.159 MeV → ^{53}Ca g.s. + n

^{54}K —β⁻→ ^{54}Ca* 6.159 MeV → ^{53}Ca*(1744 keV) + n → 1744-keV γ

Two neutron peaks separated by ≈ 1.75 MeV are naturally interpreted as feeding different ^{53}Ca final states.

VS-IMSRG: the two-body current moves theory onto the data



Conference-slide photograph: 1BC versus 1BC+2BC

Dominant J = 3, negative-parity assignment

1BC only	**0.43**
+ 2BC	**0.25**
Experiment	**0.26**

Strength reduction ≈ 42%
Equivalent amplitude q ≈ √(0.25/0.43) ≈ 0.76

User-supplied presentation; compare Refs. [3,4]

What the ^{54}K benchmark does—and does not—show

01 Microscopic origin

An explicit chiral two-body axial current accounts for a large part of the empirical GT quenching.

02 Quantitative correspondence

The 0.43 → 0.25 strength change corresponds to an amplitude factor ≈ 0.76.

03 Operator ≠ spectrum

The strength improves while excitation energies remain shifted high; the Hamiltonian/many-body truncation still matters.

04 No universal transfer

A low-q result cannot be inserted wholesale into the finite-q 0νββ operator.

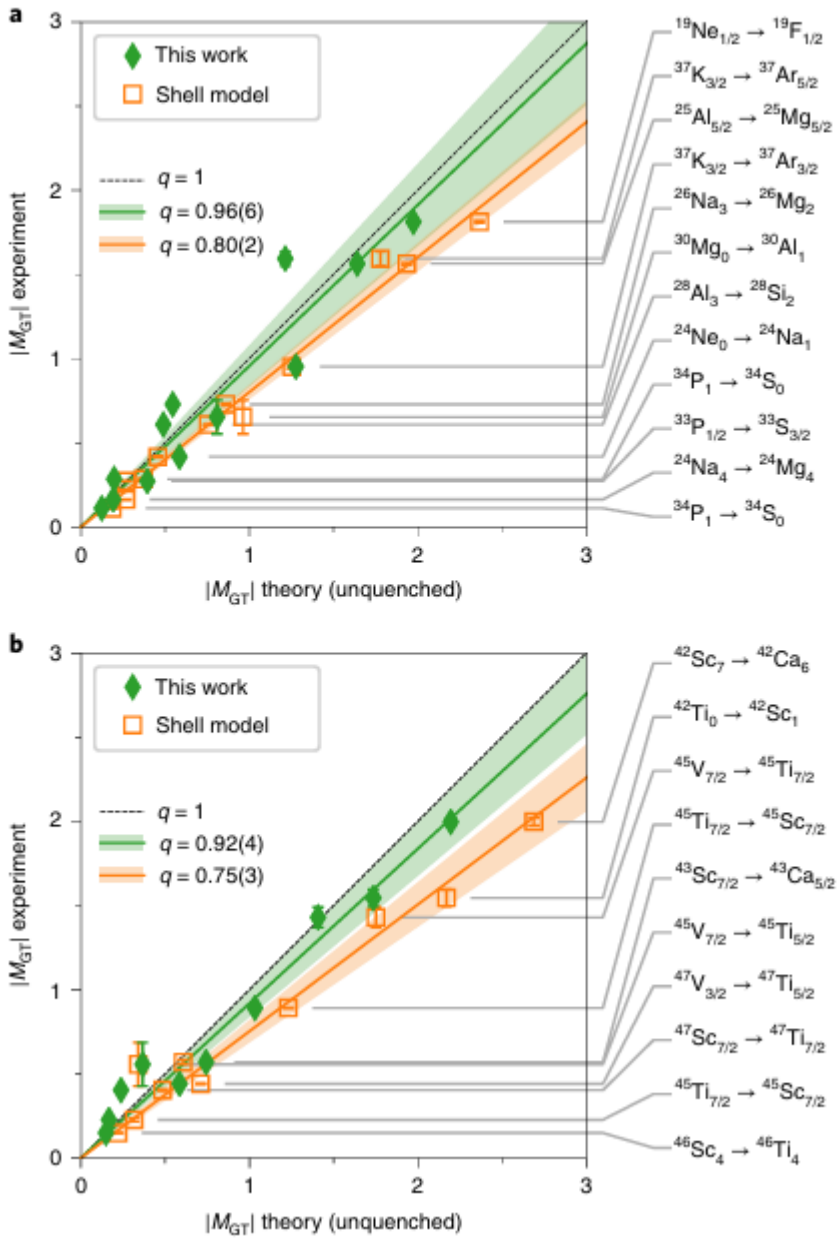
The case supports a modern operator-based interpretation of quenching, not a universal in-medium value of g_a.

Global GT matrix-element systematics

Fig. 3 | Gamow–Teller strengths in medium-mass nuclei. Comparison of experimental[30] and theoretical Gamow–Teller matrix elements for medium-mass nuclei. **a**,**b**, Plots of Gamow–Teller matrix elements: *sd*-shell (**a**) and lower *pf*-shell (**b**). Theoretical results were obtained using phenomenological shell-model interactions[16,31] with an unquenched standard Gamow–Teller $\sigma\tau$ operator (open orange squares), and using the VS-IMSRG approach with the NN-N^{4}LO + 3N$_{\text{ind}}$ interaction and consistently evolved Gamow–Teller operator plus 2BCs (filled green diamonds). The linear fits show the resulting quenching factor q given in the panels, and shaded bands indicate one standard deviation from the average quenching factor. Experimental uncertainties, taken from ref. [30], are shown as vertical error bars.

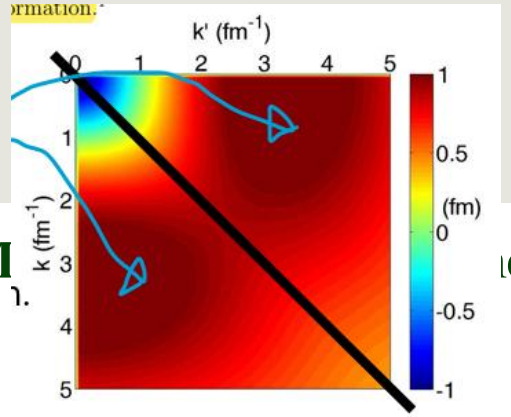
- Open squares: unquenched shell model → near q ≈ 0.75 – 0.80
- Filled squares: VS-IMSRG with 1BC+2BC → closer to q = 1
- ^{54}K extends the test to a neutron-rich multi-shell system

LETTERS



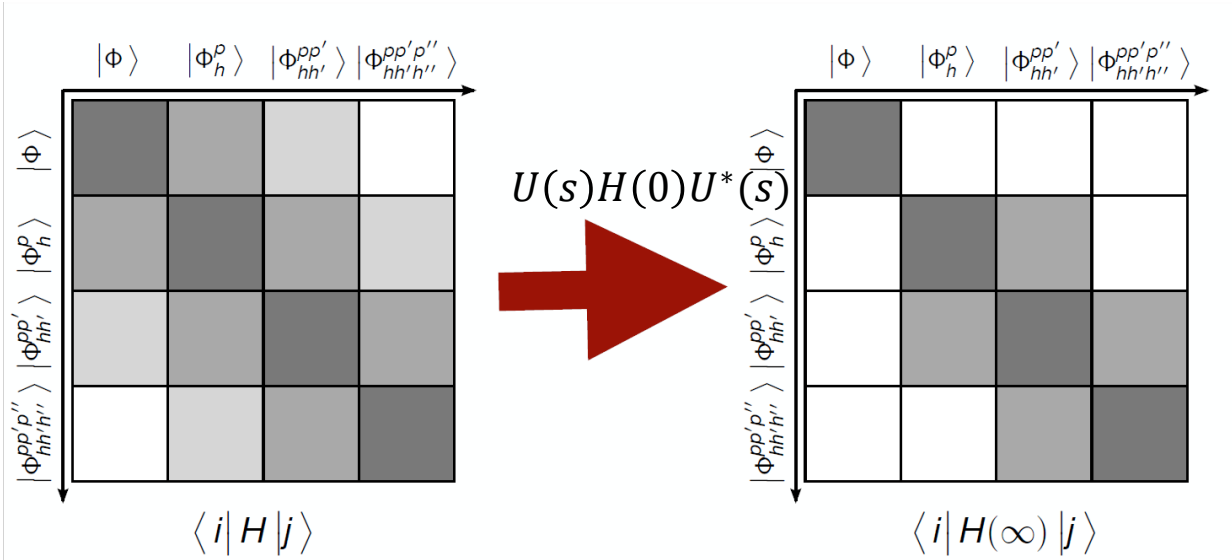
Gysbers et al., Nat. Phys. 15 (2019); Stroberg, Particles 4 (2021); ^{54}K presentation

What is the IMSRG?



The In-Medium Similarity Renormalization Group (IM **ethod** for computing the properties of nuclei.

The IMSRG is constructed in the normal-ordered operator basis of the many-body Hamiltonian; the IMSRG flow **decouples a target reference state from excitations in the many-body basis via a continuous, unitary transformation [2]** which effectively block-diagonalizes the Hamiltonian.



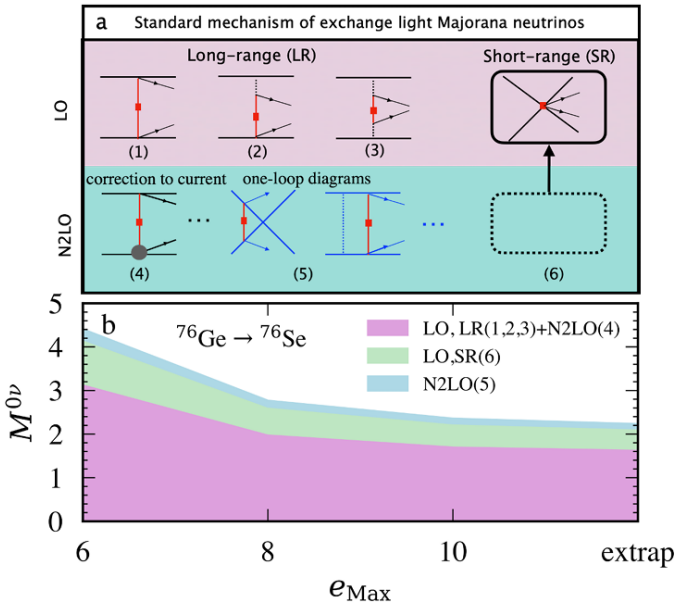


FIG. 1. **Hierarchy of contributions to $0\nu\beta\beta$ decay in chiral EFT, assuming light Majorana neutrino exchange.** **a**, Diagrammatic contributions at different orders. **b**, The convergence of the NMEs for ^{76}Ge → ^{76}Se using LO and N2LO transition operators. The red square indicates an insertion of LNV Majorana neutrino mass term $\langle m_{\beta\beta}\rangle$, and the gray circle represents corrections to the single-nucleon current parameterized in terms of form factors. The results are obtained from IM-GCM calculations using the EM1.8/2.0 chiral NN+3N interaction [33] as a function of e_{Max}.

[2] Hergert et al. 2016

This is not the last story,

but the least for double beta decay !!

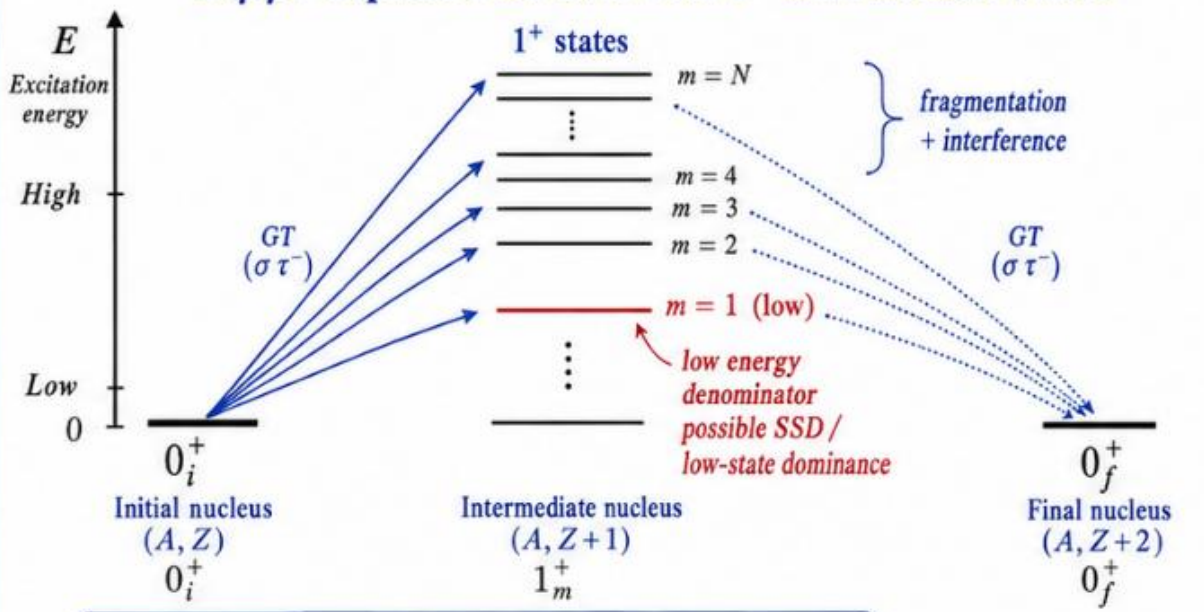
GT quenching and axial-charge enhancement are not contradictory

Feature	Gamow–Teller	Axial charge
Lorentz component	Spatial $\vec{A}$	Time A^0
Leading one-body operator	$g_a\,\sigma\,\tau$	$g_a\,\sigma\cdot p\,\tau / m_n$
Power counting	O(1)	$O(p/m_n)$
Parity	unchanged	changed
Typical **two-body** effect	often reduces low-q GT matrix elements	often **adds constructively** to rank-zero FF amplitudes
Phenomenology	$q \approx 0.7–0.8$ in restricted models	$\varepsilon_{(}A^0_{)} \approx 1.6–2$ in selected regions

Same fundamental g_a, different operators and different chiral power counting.

Why intermediate states matter in $2\nu\beta\beta$ and $0\nu\beta\beta$

$2\nu\beta\beta$: explicit sum over real 1^+ intermediate states



$$M_{GT}^{(2\nu)} = \sum_m \frac{\langle 0_f^+ \| \sigma\tau^- \| 1_m^+ \rangle \langle 1_m^+ \| \sigma\tau^- \| 0_i^+ \rangle}{D_m}$$

$$D_m = E_m - \frac{M_i + M_f}{2}$$

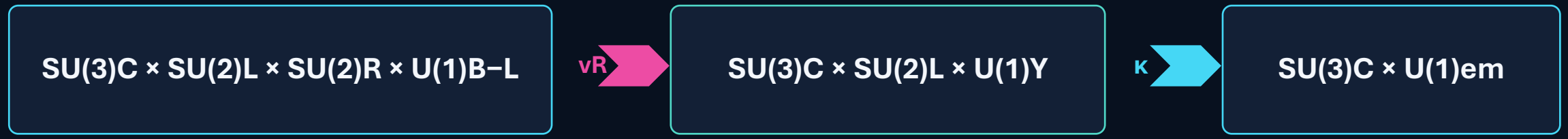
Only 1^+ states contribute for $0^+ \to 0^+$ $2\nu\beta\beta$

Decay rate is highly sensitive to individual low-lying 1^+ states and their signs.

Feature	$2\nu\beta\beta$
Intermediate states	explicit real 1^+ states
Momentum transfer	$q \sim$ few MeV
Dominant operator content	low-q GT $(\sigma\tau)$
Sensitivity	*strongly sensitive to low-lying 1^+ spectrum*

Left–Right Symmetry and neutrinoless double-β decay

LL, LR, and RR current structures



LL eL– eL–

Light ν exchange
Heavy N via WL mixing

same chirality • mass dependent

LR eL– eR–

λ mechanism
η mechanism

opposite chirality • momentum dependent

RR eR– eR–

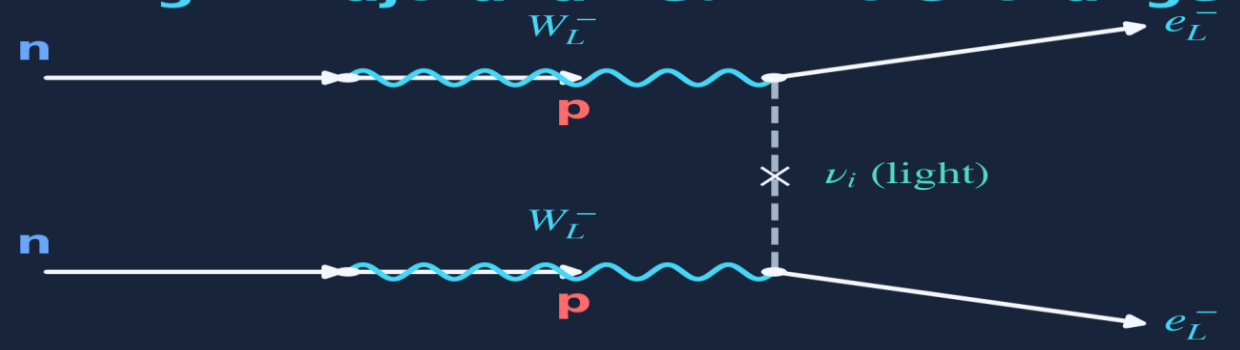
Heavy N exchange
ΔR–– exchange

same chirality • right-handed sector

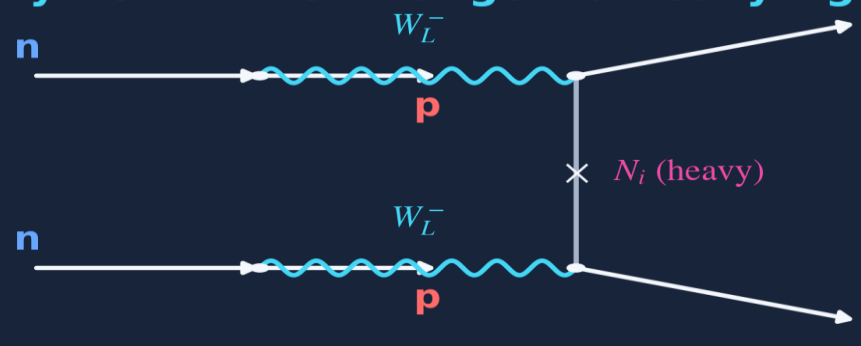
A single gauge extension naturally links parity restoration, Majorana masses, WR, and several competing 0νββ mechanisms.

Two left-handed currents: light and heavy Majorana exchange

LL: light Majorana-neutrino exchange



LL: heavy-neutrino exchange via heavy-light mixing



eL− eL− • same chirality

$$\mathcal{A}_\nu \simeq G_F^2 \frac{\sum_i U_{ei}^2 m_i}{q^2}, \qquad \eta_\nu = \frac{\sum_i U_{ei}^2 m_i}{m_e}$$

$$\mathcal{A}_N^L \simeq -G_F^2 \sum_i \frac{S_{ei}^2}{M_i}, \qquad \eta_N^L = m_p \sum_i \frac{S_{ei}^2}{M_i}$$

Nuclear matrix elements

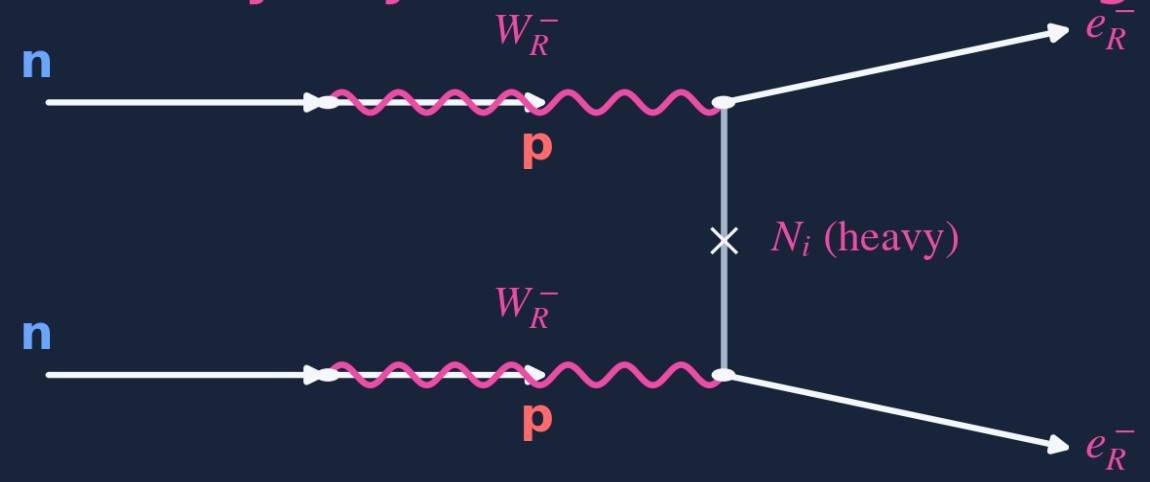
$$M_X^{0\nu} = M_{GT,X}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{F,X}^{0\nu} + M_{T,X}^{0\nu}, \qquad X = \nu, N$$

- X = v: long-range neutrino potential, q ~ 100 MeV
- X = N: short-range potential when Mi ≫ q
- The Fermi–GT–tensor spin decomposition is common, but the radial potentials are not

The familiar MGT − (gV/gA)²MF + MT formula belongs to the LL light-neutrino mechanism—and has a short-range analogue for heavy N.

Two right-handed currents: the heavy-neutrino channel

RR: heavy Majorana-neutrino exchange



eR− eR− • short range

$$\mathcal{A}_N^R \simeq G_F^2 \left(\frac{g_R}{g_L}\right)^4 \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \sum_i \frac{V_{ei}^{*2}}{M_i}$$

$$\eta_N^R = m_p \left(\frac{g_R}{g_L}\right)^4 \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \left| \sum_i \frac{V_{ei}^{*2}}{M_i} \right|$$

- Strong suppression by (MWL/MWR)^4
- Heavy-neutrino propagator gives 1/Mi
- NME is especially sensitive to short internucleon distances and short-range correlations

Additional RR topology: doubly charged triplet

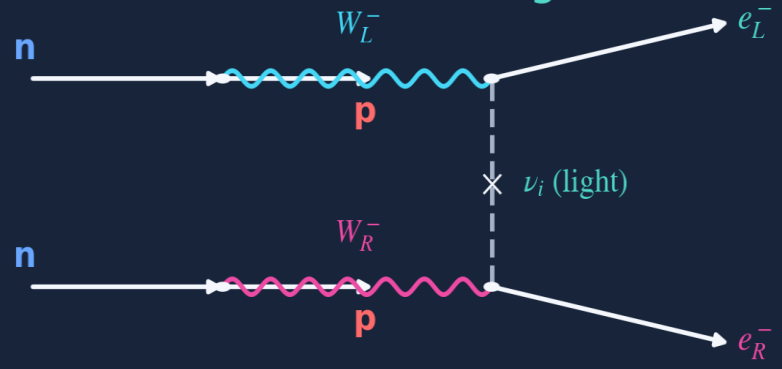
$$\eta_{\Delta_R} \propto m_p \left(\frac{g_R}{g_L}\right)^4 \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \frac{\sum_i V_{ei}^2 M_i}{M_{\Delta_R^{--}}^2}$$

Collider complementarity: pp → WR → eN → eejj

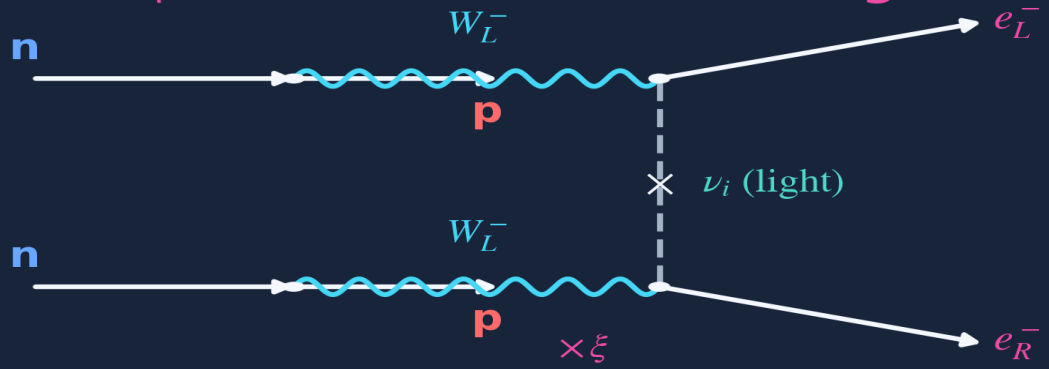
RR decay probes the right-handed scale directly through MWR, heavy-neutrino masses, and the right-handed mixing matrix V.

Opposite-helicity electrons: the λ and η mechanisms

λ mechanism: one left- and one right-handed current



η mechanism: W_L–W_R mixing



$$\eta_\lambda = \left(\frac{g_R}{g_L}\right)^2 \left(\frac{M_{W_L}}{M_{W_R}}\right)^2 \left|\sum_i U_{ei} T_{ei}^*\right|$$

$$\eta_\eta = \tan\xi \left|\sum_i U_{ei} T_{ei}^*\right|$$

$$\mathcal{A}_{LR} \propto G_F^2 \frac{1}{q} \sum_i U_{ei} T_{ei}^*$$

Nuclear operators are not the standard mass-mechanism NME

$$M\omega\pm \;\bullet\; Mq\pm \;\bullet\; MP \;\bullet\; MR$$

- 2024 LSSM: η is dominated by the weak-magnetism R term; p-wave terms are suppressed
- For λ, the ω term leads and the q term adds about 50%, with important MM contributions

λ comes from one WL and one WR propagator; η requires WL–WR mixing. Both are long-range 1/q mechanisms.

0νββ: a finite-q current–current operator

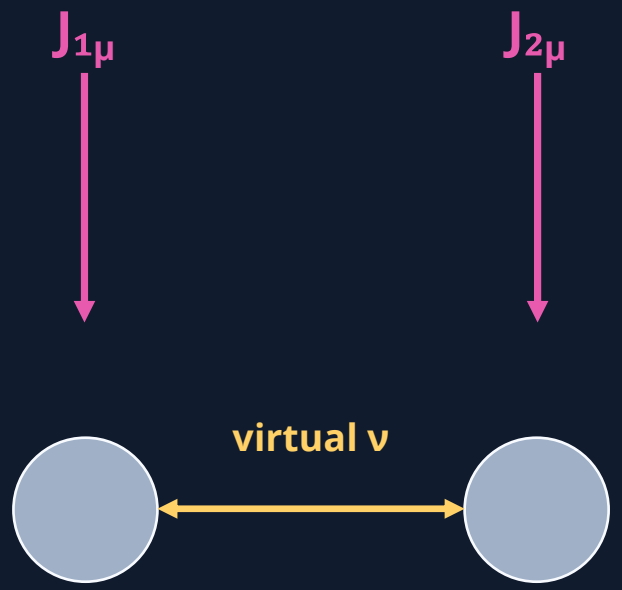
$$\hat{O}^{0\nu} \sim \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}}{q(q+\bar{E})} J_1^{\mu}(\boldsymbol{q}) J_{2\mu}(-\boldsymbol{q})$$

- Virtual-neutrino momentum is typically q ~ 100 MeV.
- Closure is useful because q is large compared with nuclear excitation energies.
- Many J^π multipoles contribute, not only 1^+.
- Finite nucleon size, $G_p(q^2)$, weak magnetism, and short-range physics matter.

$$\mathcal{J}_a^0(\boldsymbol{q}) \simeq g_V(q^2), \tag{102}$$

$$\mathcal{J}_a(\boldsymbol{q}) \simeq g_A(q^2)\boldsymbol{\sigma}_a - g_P(q^2)\frac{\boldsymbol{q}(\boldsymbol{\sigma}_a\cdot\boldsymbol{q})}{2m_N} + ig_M(q^2)\frac{\boldsymbol{\sigma}_a\times\boldsymbol{q}}{2m_N}. \tag{103}$$

$$\mathcal{J}_a^{\mu}\mathcal{J}_{b,\mu} = \mathcal{J}_a^0\mathcal{J}_b^0 - \mathcal{J}_a\cdot\mathcal{J}_b \tag{104}$$



The operator probes **correlated nucleon pairs at distances of order 1–2 fm.**

$$\widehat{O}^{0\nu}_{ab} = \tau_a^+\tau_b^+[H_F + H_{GT}\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b + H_T S_{ab}] \tag{75}$$

D: Reduce the two currents into spin-isospin operators → M: separates Fermi, GT, and tensor channels

$$S_{ab} = 3(\boldsymbol{\sigma}_a\cdot\hat{r}_{ab})(\boldsymbol{\sigma}_b\cdot\hat{r}_{ab}) - \boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b \tag{76}$$

D: Project the rank-two spin tensor along r_ab → M: defines the tensor operator S_ab

Refs. [9,12,13]

Fourier transformation defines the neutrino potentials

$$H_\nu(r) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}}/[q(q+\overline{E})] \tag{70}$$

D: Fourier-transform 1/[q(q+E-bar)] → M: defines the coordinate-space neutrino potential

$$H_\nu(r) = \frac{1}{2\pi^2}\int_0^\infty dq[q/(q+\overline{E})]j_0(qr)$$

D: Perform the angular integral for the central channel → M: reduces d^3q to the radial interval $0 \le q < \infty$

$$H_K(r) = \frac{2R_A}{\pi}\int_0^\infty dq[q/(q+\overline{E})]j_{L_K}(qr)h_K(q^2)$$

D: Include form factors and spherical Bessel multipoles → M: defines channel-dependent potentials H_K

$$\mathfrak{h}_{GT}(q^2) = g_A^2(q^2)\left[1 - \frac{2}{3}x(q) + \frac{1}{3}x^2(q)\right] + \frac{2}{3}g_M^2(q^2)\frac{q^2}{4m_N^2}$$

$$\mathfrak{h}_T(q^2) = g_A^2(q^2)\left[\frac{2}{3}x(q) - \frac{1}{3}x^2(q)\right] + \frac{1}{3}g_M^2(q^2)\frac{q^2}{4m_N^2}$$

$$x(q) \equiv \frac{q^2}{q^2+m_\pi^2},$$

$$R_A = 1.2A^{1/3}\,\mathrm{fm} \tag{73}$$

D: Use the empirical nuclear-radius parametrization → M: sets the conventional dimensionless NME scale

$$L_F = L_{GT} = 0, L_T = 2 \tag{74}$$

D: Identify central and tensor angular momenta → M: selects j_0 for F/GT and j_2 for …

Fermi, Gamow–Teller, and tensor components

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_{F} + M^{0\nu}_{T}$$

Fermi

$$\tau_a^+ \tau_b^+ \, H_F(r_{ab})$$

Dominantly vector-charge × vector-charge

Gamow–Teller

$$\tau_a^+ \tau_b^+ \, H_{GT}(r_{ab}) \, \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$$

Dominantly spatial axial-current × axial-current

Tensor

$$\tau_a^+ \tau_b^+ \, H_T(r_{ab}) \, S_{ab}$$

Finite-q spin tensor from longitudinal/transverse structure

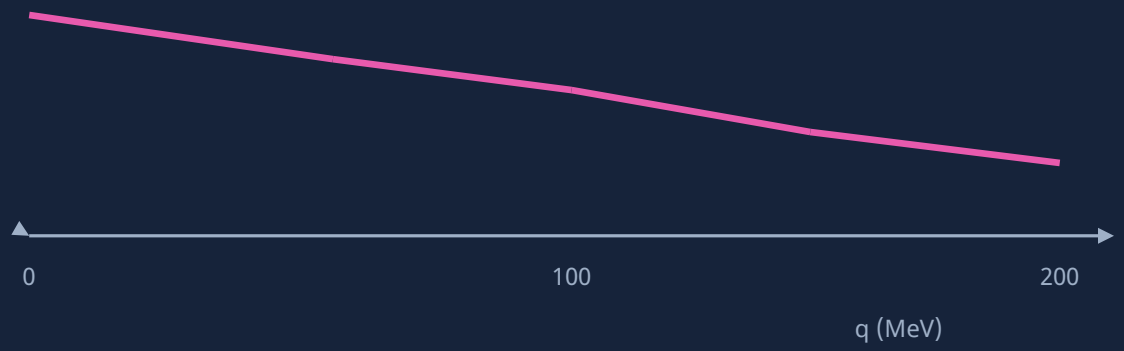
Conventions differ in where g_a and $(g_v/g_a)^2$ are placed; compare definitions before comparing numerical NMEs.

Induced pseudoscalar coupling reshapes the longitudinal spatial current

$$\boldsymbol{A} = \boldsymbol{A}_T + \boldsymbol{A}_L$$

$$\boldsymbol{A}_L \simeq g_A \frac{m_\pi^2}{q^2 + m_\pi^2}\, \hat{\boldsymbol{q}}(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}})\,\tau^+$$

Pion-pole factor $m_\pi^2/(q^2 + m_\pi^2)$



0νββ current decomposition

- **AA** — axial–axial
- **AP** — axial–pseudoscalar
- **PP** — pseudoscalar–pseudoscalar
- **MM** — weak magnetism

These terms can alter the light-neutrino NME by tens of percent. They are not axial-charge enhancement.

Direct A⁰A⁰ in 0νββ is usually subleading

Lorentz contraction

$$A_1^\mu A_{2\mu} = A_1^0 A_2^0 - \boldsymbol{A}_1 \cdot \boldsymbol{A}_2$$

O[(p/m$_n$)²]
direct A⁰A⁰

O(1)
$\vec{A}_1 \cdot \vec{A}_2$

The time component of the pseudoscalar term is further reduced by $q^0 \ll |\vec{q}|$ in typical 0νββ kinematics.

Two common misinterpretations

1 **"G$_p$ is important, therefore A⁰ is important."**

No: the dominant G$_p$ effect is in the longitudinal spatial current.

2 **"Negative-parity multipoles are axial charge."**

No: finite-q spatial axial multipoles also generate 0^-, 1^-, 2^-, … contributions.

Two-body currents and the ΔL = 2 short-range operator

Two-body axial current

$$(A_{1b} + A_{2b})(A_{1b} + A_{2b}) = A_{1b}A_{1b} + A_{1b}A_{2b} + A_{2b}A_{1b} + \cdots$$

- Generates effective two- and three-nucleon decay operators
- Normal ordering simplifies the problem but introduces reference dependence
- Corrections are momentum dependent: $\delta g_a(q,\rho)$, $\delta g_p(q,\rho)$
- Low-q GT quenching cannot be transferred unchanged to q ≈ 100 MeV

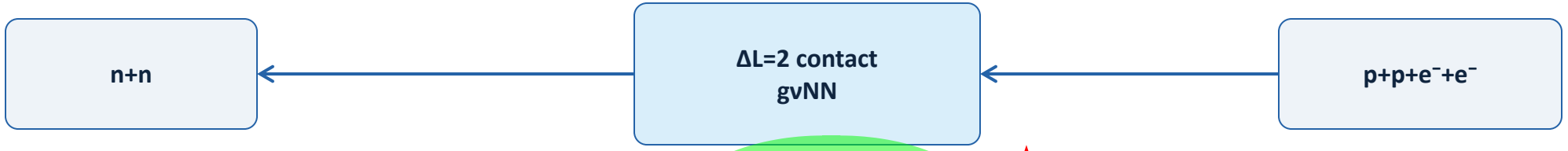
Genuine lepton-number-violating contact

$$nn \to pp\, e^- e^- : qquad \hat{O}^{0\nu}_{\text{long range}} + g_\nu^{NN} \hat{O}^{0\nu}_{\text{contact}}$$

- Required to renormalize the two-nucleon amplitude in chiral EFT
- Its coupling is not determined by ordinary β decay or g_a quenching
- Needs EFT matching, lattice-QCD input, or related observables

Do not identify the LNV contact operator with a phenomenological short-range-correlation function.

Renormalization promotes an NNNNee contact term to leading order



$$M_\nu^{0\nu} = M_{\nu,\mathrm{long}}^{0\nu} + M_{\nu,\mathrm{short}}^{0\nu} \tag{84}$$

D: Renormalize long-distance exchange with a contact operator → M: splits the observable into long- and short-range pieces

$$M_{\mathrm{long}}^{0\nu} = M_{GT}^{0\nu} - (g_V/g_A)^2 M_F^{0\nu} + M_T^{0\nu} \tag{80}$$

D: Take matrix elements of the three channels → M: gives the standard long-range NME combination

$$\mathcal{M}_{GT} = M_{GT}^{AA} + M_{GT}^{AP} + M_{GT}^{PP} + M_{GT}^{MM} \tag{81}$$

D: Expand the GT current product by nucleon form factors → M: resolves AA, AP, PP, and MM pieces

$$\mathcal{M}_T = M_T^{AP} + M_T^{PP} + M_T^{MM} \tag{82}$$

D: Apply the same reduction to the tensor channel → M: identifies AP, PP, and MM tensor terms

$$\mathcal{M}_\nu^{(3)} = -V_{ud}^2[-M_F/g_A^2 + \mathcal{M}_{GT} + \mathcal{M}_T + 2m_\pi^2 g_\nu^{NN} M_{F,\mathrm{sd}}/g_A^2] \tag{85}$$

D: Match the leading chiral operators onto nuclear NMEs → M: gives the renormalized dimension-three mass mechanism

The observable sum must be regulator independent.

Systematic comparison: 2νββ versus 0νββ

Feature	2νββ	0νββ (light-Majorana mechanism)
Final leptons	$2\,e^- + 2\,\bar{\nu}_e$	$2\,e^-$
Typical momentum	few MeV at each vertex	~100 MeV virtual momentum
Intermediate states	real odd–odd states	closure over many multipoles
Dominant axial structure	GT × GT through 1^+	finite-q spatial axial current → GT and tensor
Role of A^0	possible 0^- double-first-forbidden correction	direct A^0A^0 usually recoil-order
Induced pseudoscalar	small in allowed limit	important longitudinal correction
Two-body current	low-q GT renormalization	momentum-dependent current and many-body operator
Unique short-range issue	none beyond ordinary weak operators	leading ΔL=2 NN contact required

Refs. [9–13]

Recommended implementation by many-body method

Shell model

- State the valence space, core, truncation, and interaction
- Use nonclosure for 2νββ whenever possible
- Treat A^0 as a separate effective operator
- Use full q-dependent one-body current for 0νββ

QRPA / EDF

- Constrain g_{pp} and pairing with multiple observables
- Avoid degeneracy between fitted g_a and interaction parameters
- Compute the complete multipole sum at finite q
- Track RPA correlations separately from extra quenching

Ab initio

- Derive H and electroweak currents in one EFT scheme
- Evolve transition operators consistently
- Estimate omitted induced many-body terms
- Treat long-range and contact 0νββ operators with the same regulator

Always report the Hamiltonian, model space, current operator, regulator, and convention for g_a in the NME.

Five double-counting traps

Explicit 2BC + phenomenological q	Do not multiply an already corrected GT operator by the fitted quenching factor again.
Explicit soft-pion A^0 + $\varepsilon_{\zeta}A^0_{\zeta}$	The empirical enhancement should not be added on top of the same exchange current.
RPA spin–isospin suppression + density-dependent g_a	Check whether the same correlations are encoded twice.
Pion-pole G_p + axial-charge enhancement	They modify different Lorentz components.
SRC function + EFT ΔL=2 contact	They are conceptually distinct and not interchangeable.

A fitted effective coupling is safe only when the missing physics it represents is clearly identified.

Five take-home messages

1. **g_a is fundamental; $g_a$eff is not universal** — Effective values depend on the nuclear model, operator, nucleus, and momentum transfer.

2. **GT quenching concerns the spatial current** — Restricted calculations often require q ≈ 0.7–0.8; explicit correlations and 2BC can carry much of this renormalization.

3. **Axial-charge enhancement concerns A^0** — A recoil-suppressed one-body operator receives a chiral-filtered soft-pion two-body correction.

4. **2νββ and 0νββ probe different regimes** — 2νββ is GT × GT at low q; 0νββ is a finite-q current–current problem with many multipoles.

5. **Consistency beats a universal quenching factor** — Use q-dependent one- and two-body currents, document conventions, and avoid double counting.

崇實大學校

Thanks for your attention !!