NUCLEAR REACTIONS

Direct nuclear reactions with RIB (part 3)

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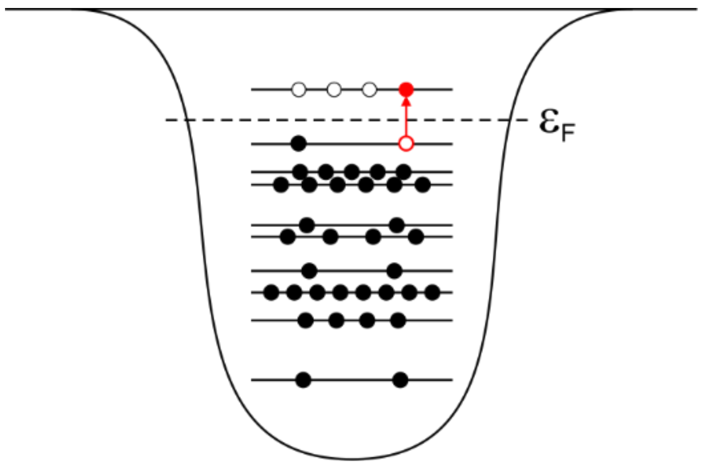
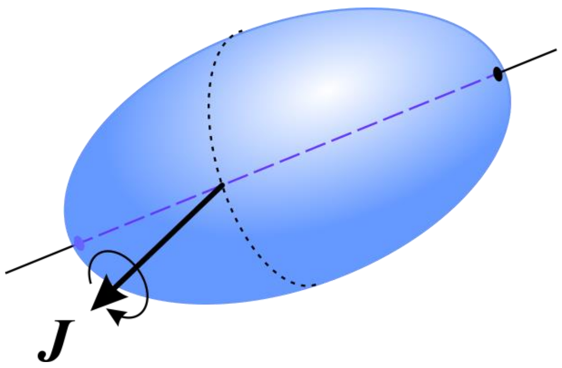
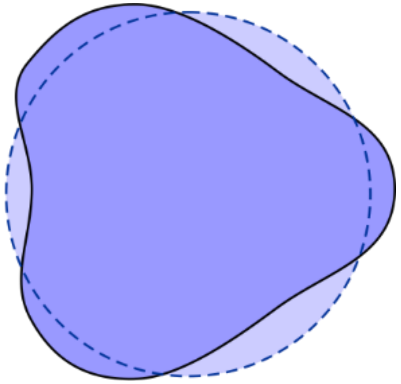
INELASTIC NUCLEAR SCATTERING

Types of inelastic nuclear excitation:

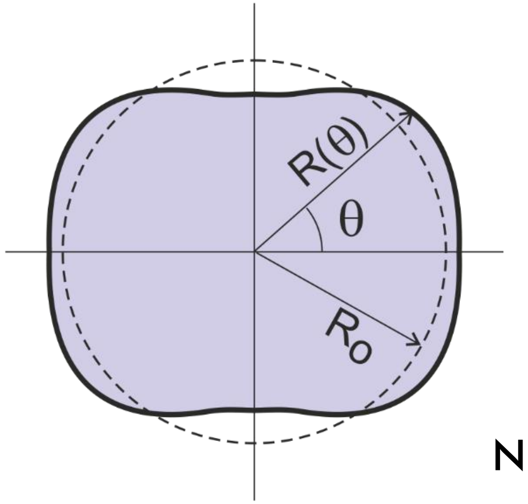
1. Excitation of one-particle-states.
2. Excitation of collective state: surface vibration or/and rotation







NUCLEAR SHAPE



Nuclear shape parametrization

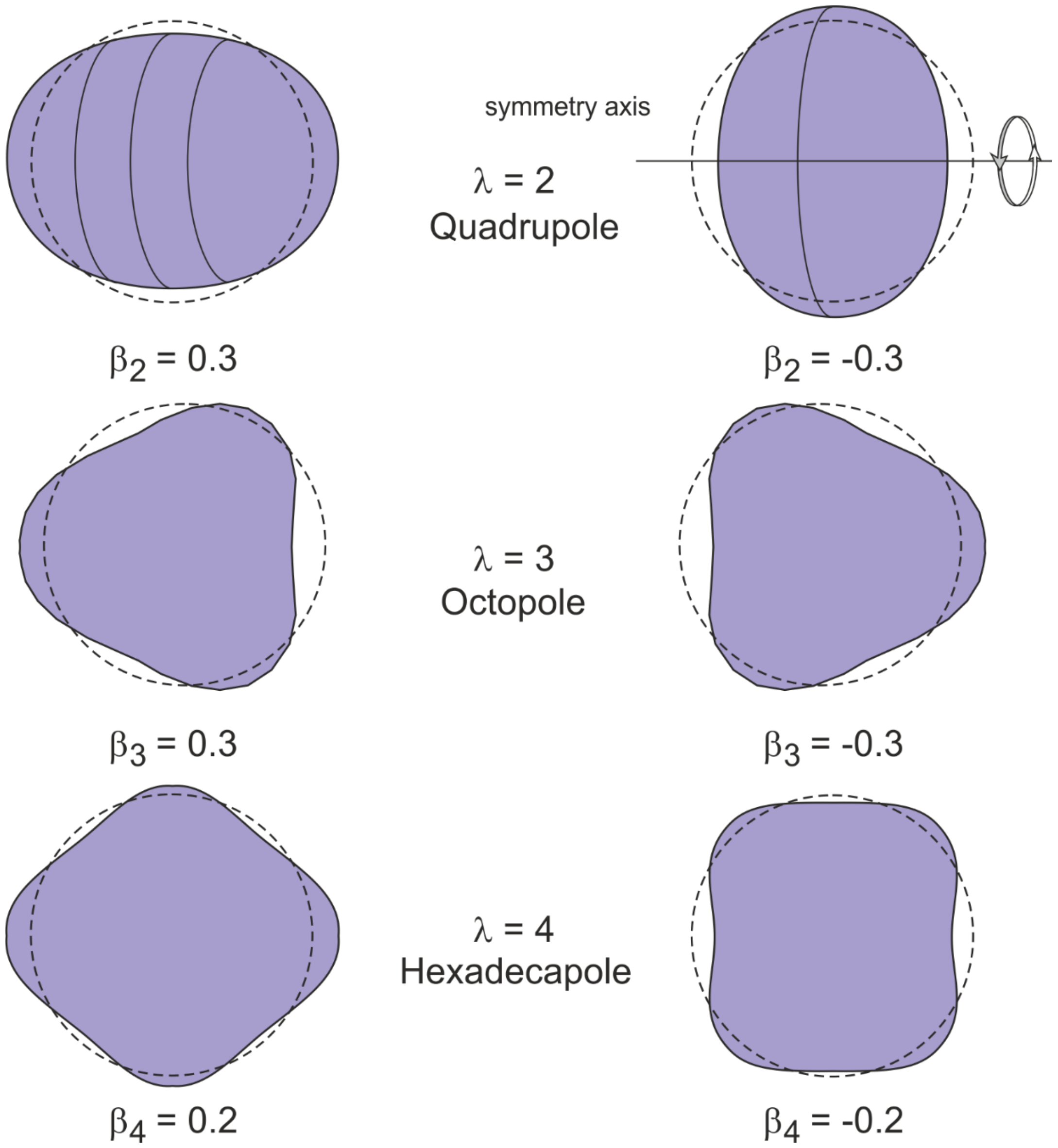
$$R(\Omega) = R_0\left[1 + \sum_{\lambda\geq 2,\mu} \beta_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi)\right] = R_0 + \delta R(\vec{\beta},\Omega)$$

Static deformation:
multipole electric moment

$$Q_\lambda = \frac{3ZR_0^\lambda}{\sqrt{\pi(2\lambda+1)}}\beta_\lambda^{g.s.}$$

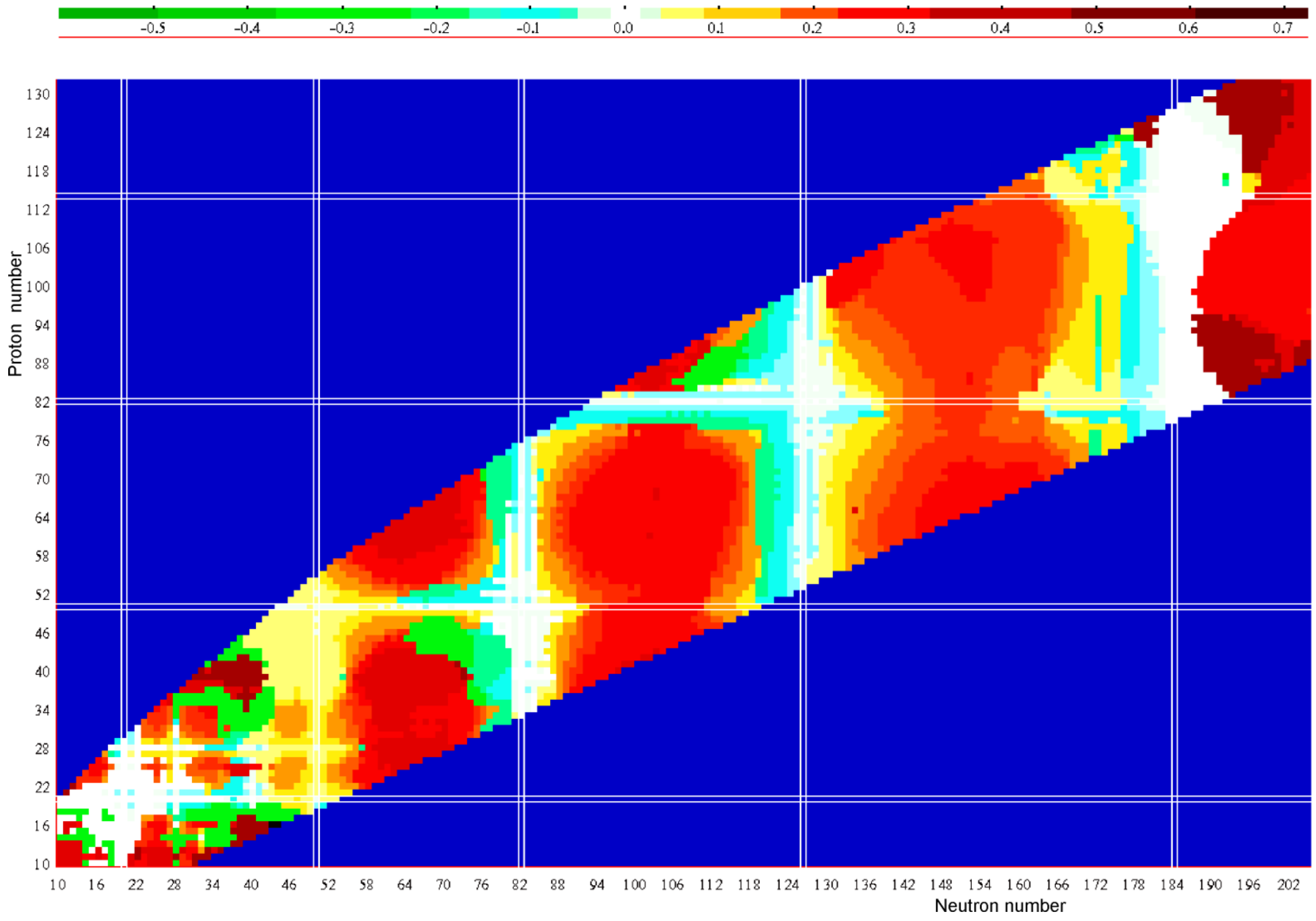
Dynamical deformation:
zero vibration amplitude

$$\left\langle \beta_\lambda^0 \right\rangle = \frac{4\pi}{3ZR_0^\lambda}\left(\frac{B(E\lambda)}{e^2}\right)^{1/2}$$



NUCLEAR SHAPE: DEFORMATION

P. Moller et al., 1995

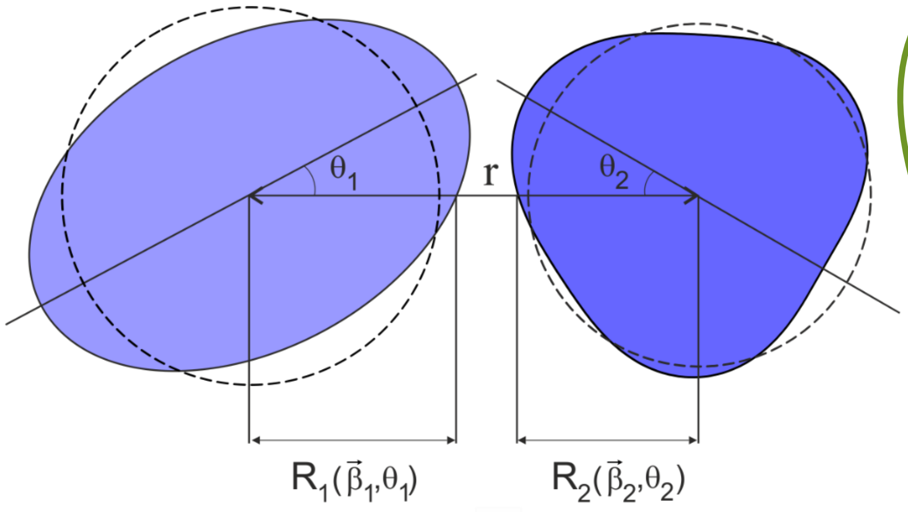


DWBA FOR INELASTIC SCATTERING

Reaction: a + A → a + A*

Differential cross section:

$$\frac{d\sigma_{fi}}{d\Omega}(\theta) = \frac{1}{(2\pi)^2}\frac{\mu^2}{\hbar^4}\frac{k_f}{k_i}\left|T_{fi}(\mathbf{k}_f,\mathbf{k}_i)\right|^2$$



DWBA amplitude:

$$T_{fi}^{DW}(\mathbf{k}_f,\mathbf{k}_i) \approx \left\langle \Psi_{\mathbf{k}_f}^{(-)}(\mathbf{R}_f)\varphi_A^{j'm'}(\mathbf{r}_f)\middle|\Delta V\middle|\varphi_A^{jm}(\mathbf{r}_i)\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{R}_i)\right\rangle$$

Nucleus-nucleus interaction potential:

$$U(r) = U_C(r) + U_N(r)$$

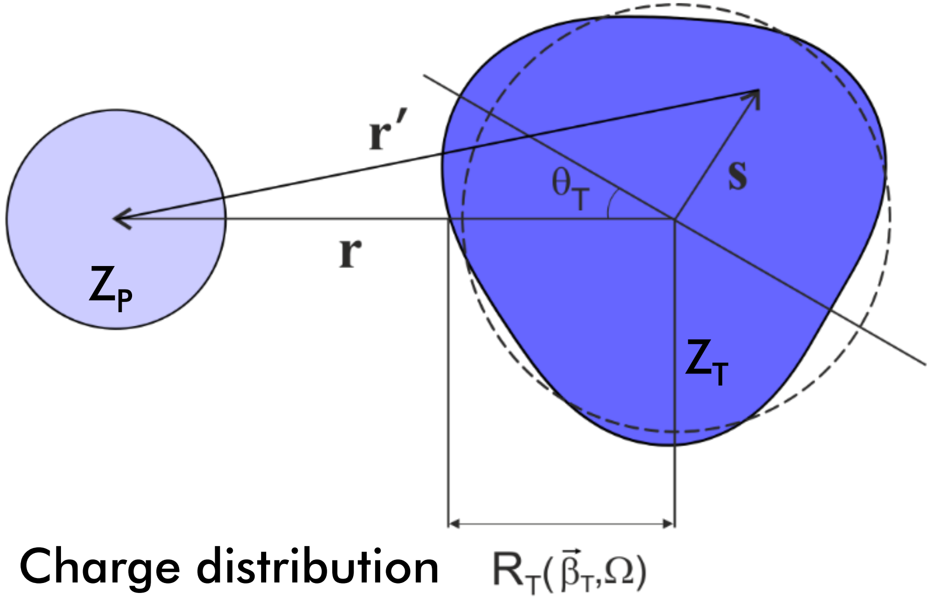
$$U_N(r) = \frac{V_0}{1+\exp\dfrac{r-\sum_i R_i(\beta_i,\theta_i)}{a}} = U_0(r) - \delta R\frac{dU}{dr} + \cdots$$

$$U_N(r) \approx U_0(r) - R_0\frac{dU_0}{dr}\sum_{\lambda\mu}\beta_{\lambda\mu}Y_{\lambda\mu}(\Omega) + \cdots$$

Optical potential

Distortion potential, leading to inelastic excitation

DEFORMED COULOMB POTENTIAL



Charge distribution

$$U_C(r,\vec{\beta}_2) = Z_P e \int d\Omega_s \int_0^{R_T(\vec{\beta}_T,\Omega_s)} s^2 ds \, \frac{\rho_0^C}{|\mathbf{r}-\mathbf{s}|}$$

Use partial decomposition of the function

$$\frac{1}{|\mathbf{r}-\mathbf{s}|} = \sum_{lm} \frac{4\pi}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{lm}^*(\Omega_s) Y_{lm}(\Omega)$$

And perform the integration one obtains:

$$U_C(r,\vec{\beta}_2) = Z_P Z_T e^2 \begin{cases} \dfrac{1}{2R_C}\left[3 - \dfrac{r^2}{R_C^2}\right], & r < R_C \\ \dfrac{1}{r}, & r > R_C \end{cases} + \frac{3Z_P Z_T e^2}{R_C} \sum_{lm} \frac{\beta_l^C Y_{lm}(\Omega)}{2l+1} \begin{cases} \dfrac{r^l}{R_C^l}, & r < R_C \\ \dfrac{R_C^{l+1}}{r^{l+1}}, & r > R_C \end{cases}$$

Coulomb part of the optical potential Coulomb addition to the distortion potential

VIBRATIONAL EXCITATIONS

Vibration: harmonic oscillator $$\sum_{\lambda\mu}\left[\frac{B_\lambda}{2}\left|\dot{\beta}_{\lambda\mu}\right|^2+\frac{C_\lambda}{2}\beta_{\lambda\mu}^2\right]\left|njm\right\rangle=\sum_{\lambda\mu}\hbar\omega_\lambda\left(n+\frac{1}{2}\right)\left|njm\right\rangle$$

Coupling potential: $$\Delta V(\mathbf{r},\beta)=\Delta V_C(\mathbf{r},\beta)+\Delta V_N(\mathbf{r},\beta)=\sum_{\lambda\mu}\ v_\lambda^C(r)+v_\lambda^N(r)\ \ \beta_{\lambda\mu}Y_{\lambda\mu}(\Omega)$$

Transition matrix element: $$T_{fi}\sim\left\langle n'j'm'\middle|\Delta V\middle|njm\right\rangle=\sum_\lambda\mathbb{C}\left\langle n'j'\middle\|\beta_\lambda\middle\|nj\right\rangle\ v_\lambda^C(r)+v_\lambda^N(r)$$

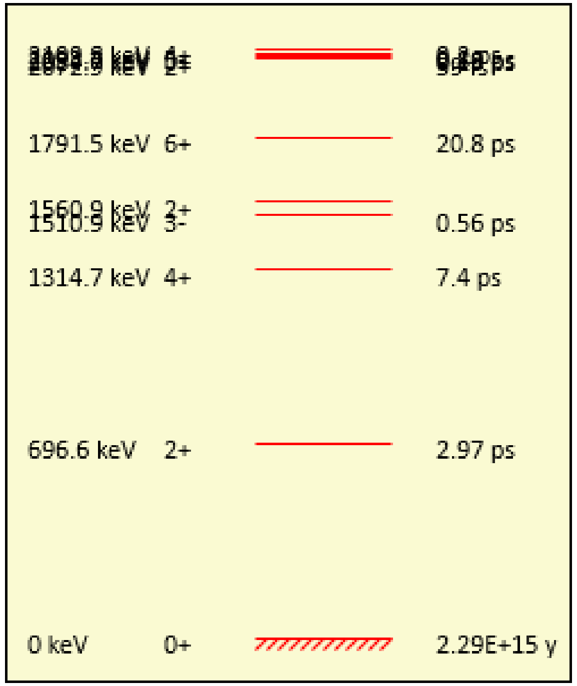
$$\left\langle 1j\middle\|\beta_\lambda\middle\|00\right\rangle=(-)^j\delta_{j\lambda}\sqrt{2j+1}\,\frac{4\pi}{3ZeR_C^j}\sqrt{B(E\lambda,0\to1)},$$

$$B(E\lambda,0\to1)=\left(\frac{3}{4\pi}ZeR_C^\lambda\right)^2\frac{\hbar}{2B_\lambda\omega_\lambda},\omega_\lambda=\sqrt{\frac{C_\lambda}{B_\lambda}}$$

here B(Eλ) is the reduced electric multipole transition probability, which determines the strength of γ-transitions of multypolarity λ between two nuclear states.

$$\frac{d\sigma}{d\Omega}(\lambda\leftarrow0)=\left(\frac{m}{2\pi\hbar^2}\right)^2\frac{k'}{k}\frac{\beta_{0\lambda}^2R_0^2}{2\lambda+1}\sum_\mu\left|\int\chi_{k'}^{(-)*}(\mathbf{r})\left(v_\lambda^C(r)-\frac{dU}{dr}\right)Y_{\lambda\mu}(\Omega)\chi_k^{(+)}(\mathbf{r})d\mathbf{r}\right|^2$$

EXAMPLE



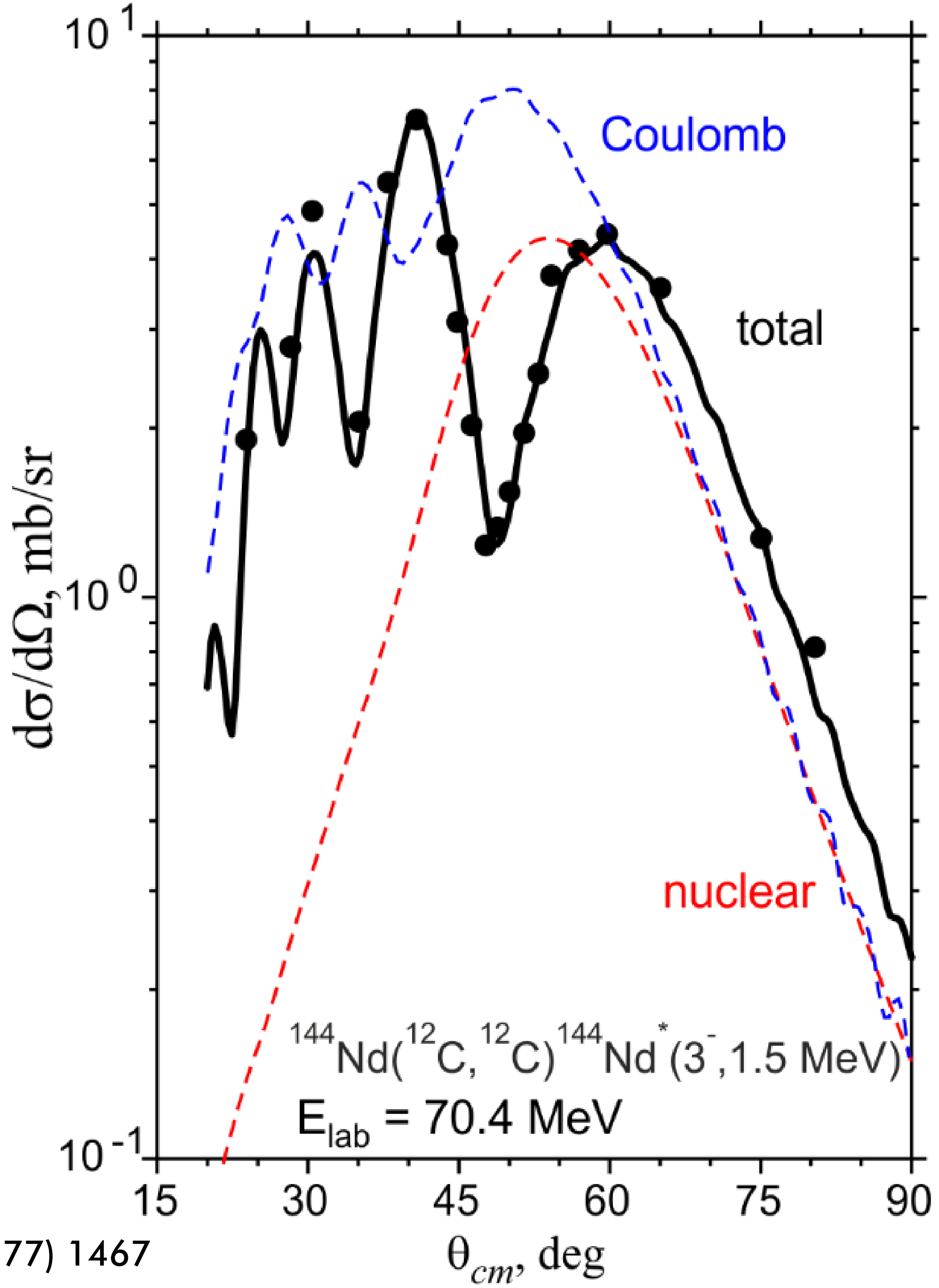
^{144}Nd

Available information:

- ❑ No static deformation
- ❑ B(E3) = 0.263 e^2b^3
- ❑ β_{03} = 0.114

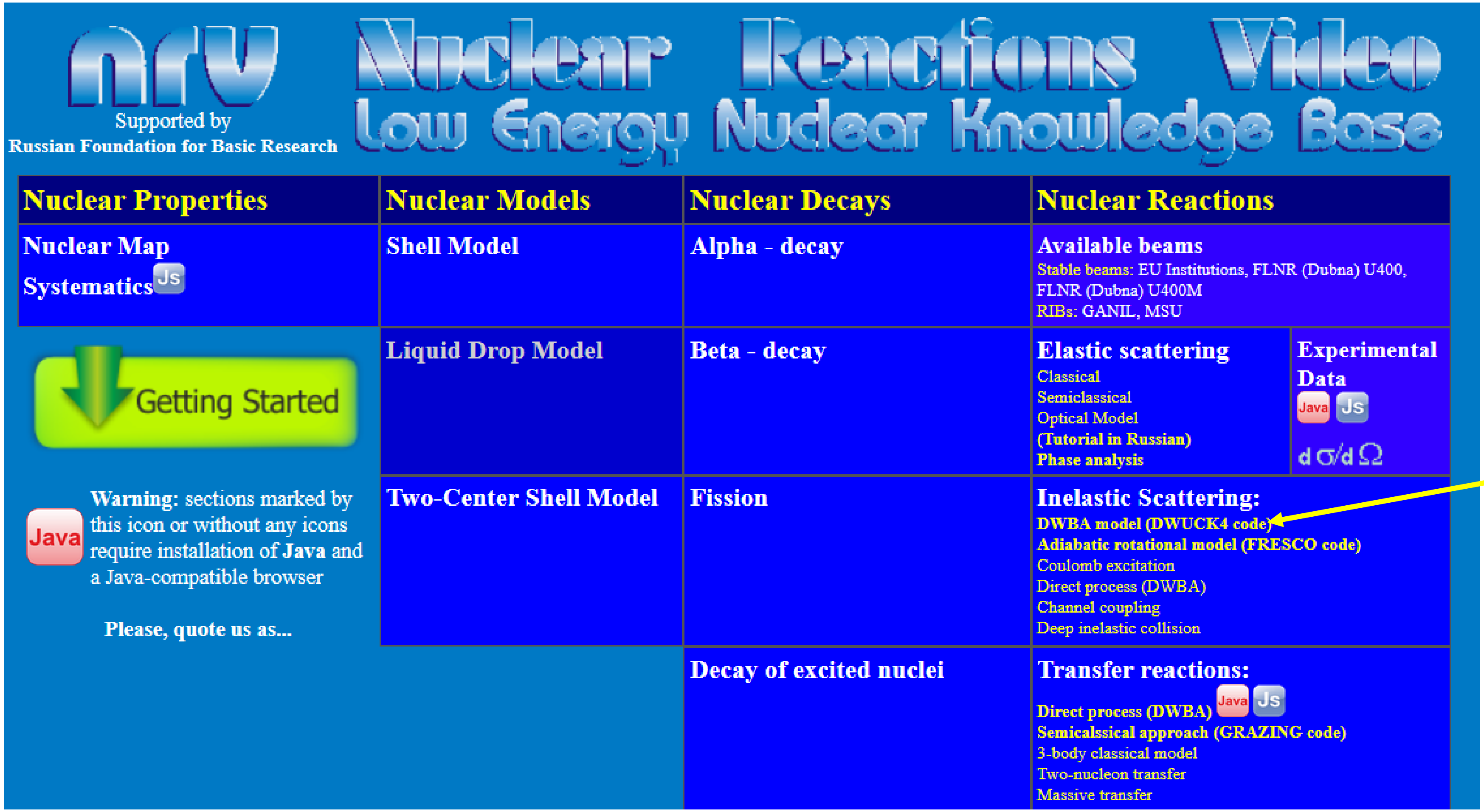
Extracted parameters:

- ❑ $\beta_{03,\ Nucl}$ = 0.1
- ❑ $\beta_{03,\ Coul}$ = 0.115



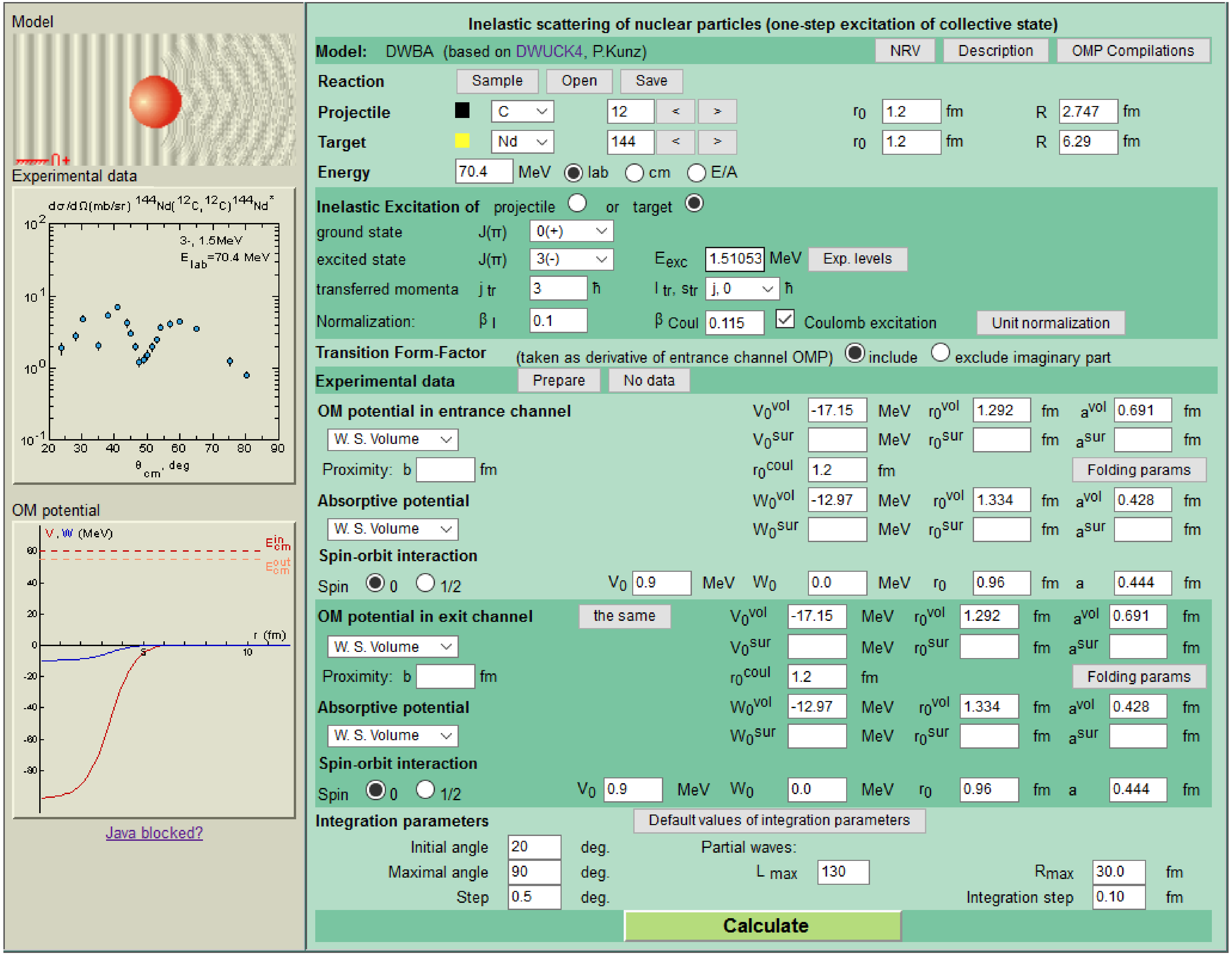
Exp. D.L. Hillis et al. Phys. Rev. C16 (1977) 1467

NRV DWBA FOR INELASTIC SCATTERING



Click here!

The 2nd RISP Intensive Program on «Rare Isotope Physics», 2017



Set your reaction

Set properties of excited state

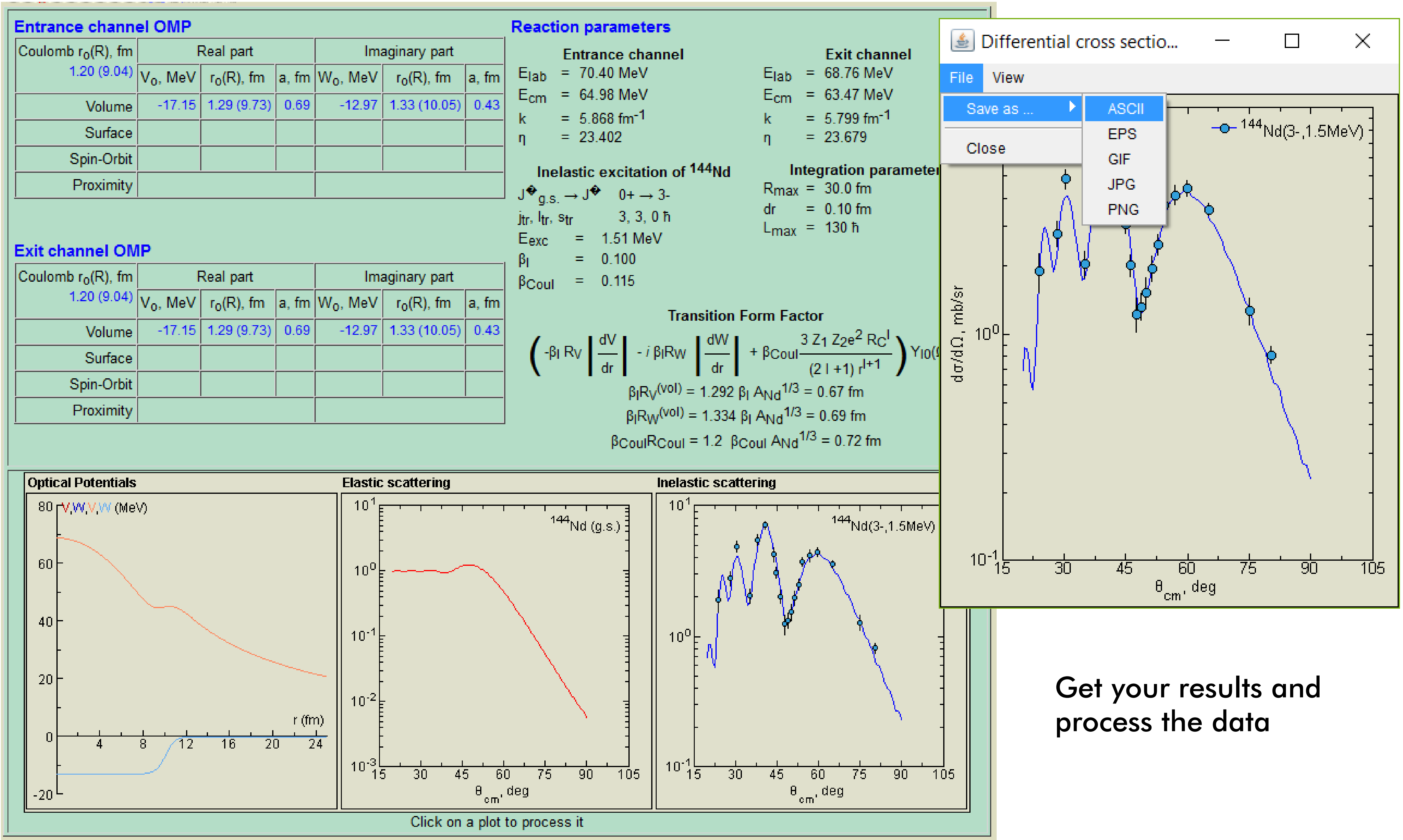
Set exp. data

Set OMP for in-channel

Set OMP for out-channel

Set integration params

The 2nd RISP Intensive Program on «Rare Isotope Physics», 2017

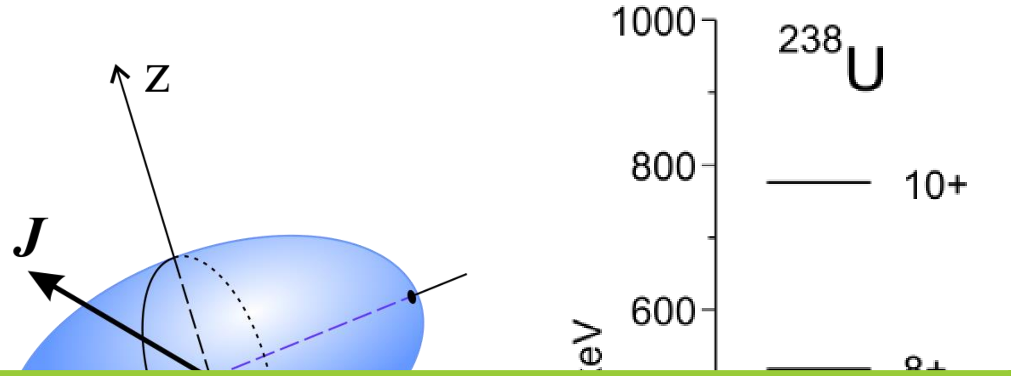


Get your results and process the data

ROTATIONAL EXCITATIONS

Rotation: quantum axially symmetric rotator

Schrodinger equation $\frac{\hbar^2}{2\Im}\mathbf{I}^2\left|JM\right\rangle = \frac{\hbar^2}{2\Im}J(J+1)\left|JM\right\rangle$



Jπ	0+	2+	4+	6+
E_j	0	$6\hbar^2/2\Im$	$20\hbar^2/2\Im$	$42\hbar^2/2\Im$
E_j/E_2	0	1	3.33	7
$E_j(^{238}U)$, keV	0	44.9	148.4	307.2
$E_j/E_2(^{238}U)$	0	1	3.30	6.84

Moment of inertia:

1 $$\Im_{rig} = \int r^2\rho(r,\theta,\phi)dV =$$

$$= \rho_0\int_{-1}^{1} d\cos\theta\int_0^{2\pi} d\phi \int_0^{R(\theta,\phi)} r^4 dr \approx \frac{2}{5}m_N r_0^2 A^{5/3}(1+0.31\beta_2) =$$

$$= 0.0138A^{5/3}(1+0.31\beta_2)\left[\frac{\hbar^2}{MeV}\right]$$

$$\Im_{rig}(^{238}U) = 148\frac{\hbar^2}{MeV}$$

2 $$\Im(^{238}U) = 3\frac{\hbar^2}{E_{2+}} = 66.8\frac{\hbar^2}{MeV}$$

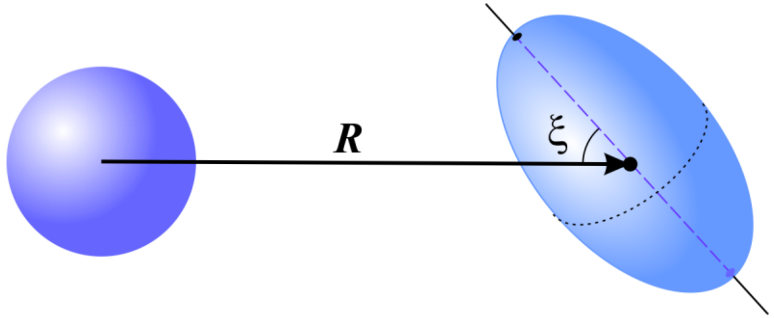
$$\beta_2(^{238}U) = 0.24$$

Nucleus is not a rigid body!

3 $$\Im_{LD}(^{238}U) = \frac{45}{16\pi}\Im_{rig}\beta^2 = 7.6\frac{\hbar^2}{MeV}$$

Nucleus does not rotate like irrotational flow!

COUPLED CHANNELS



Hamiltonian describes two-nucleus system where one nuclei has internal structure

$$\hat{\mathbf{H}} = \hat{t}_R + \hat{h}_\xi + \hat{V}(R,\xi) \qquad \hat{\mathbf{H}}\Psi(\mathbf{R},\xi) = E\Psi(\mathbf{R},\xi)$$

$\hat{h}_\xi \varphi_i(\xi) = \varepsilon_i \varphi_i(\xi)$ here wave functions $\varphi_i(\xi)$ form the basis

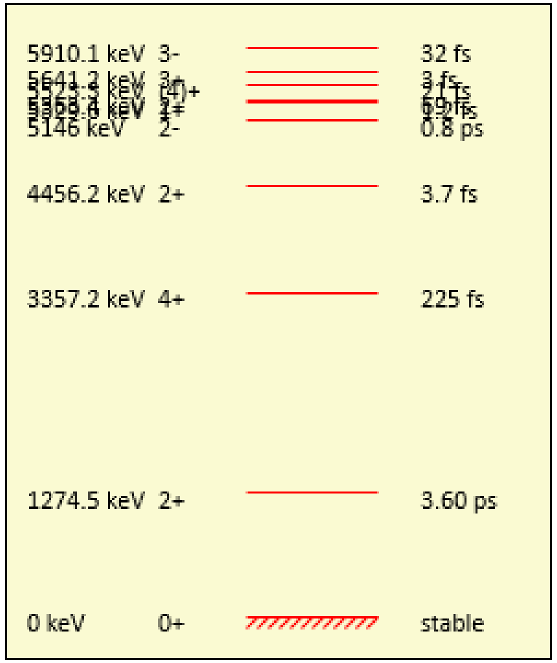
$\Psi(\mathbf{R},\xi) = \sum_i \psi_i(\mathbf{R})\varphi_i(\xi)$ decomposition of the total wave function over the basis $\varphi_i(\xi)$

Set of coupled equations:

$$\hat{t}_R + \varepsilon_i \;\; \psi_i(\mathbf{R}) + \sum_j \hat{V}_{ij}(\mathbf{R})\psi_j(\mathbf{R}) = E\psi_i(\mathbf{R}), \qquad \psi_i(\mathbf{R}) \to \delta_{i1} e^{ik_1 R} + f_{i1}(\theta)\frac{e^{ik_i R}}{R}, \qquad i = 1,2,\ldots$$

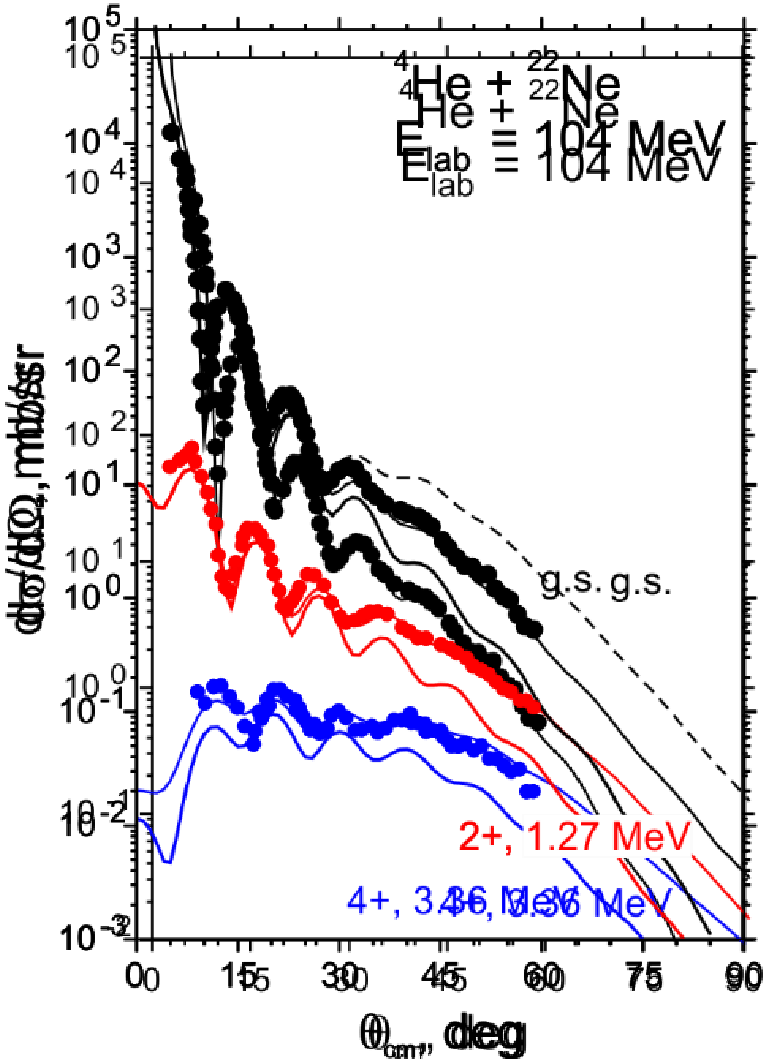
$\hat{V}_{ij}(\mathbf{R}) = \underbrace{\int \varphi_i^*(\xi)\hat{V}(\mathbf{R},\xi)\varphi_j(\xi)d\xi}_{\text{DWBA transition form-factor}}$ matrix elements that couple different reaction channels

EXAMPLE: ^{4}He + ^{20}Ne (Coupled channels)



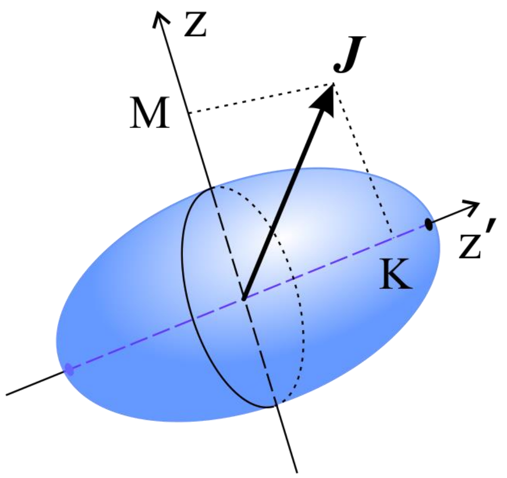
^{22}Ne

P. Moller: β_2 = 0.384, β_4=0.09



OMP	OM / CC
V_0, MeV	112.72
r_V, fm	0.789
a_V, fm	0.76
W_0, MeV	20.83 / 15.7
r_W, fm	1.09
a_W, fm	0.56

ROTATIONAL EXCITATIONS



For nuclei with odd number of either protons or neutrons two contributions to the total angular momentum J can be distinguished

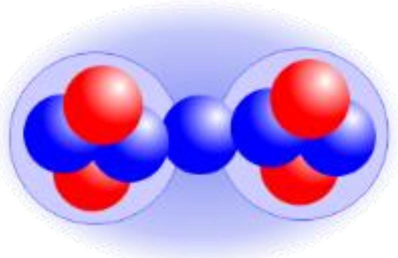
- ❑ the collective contribution from the rotation of even-even core
- ❑ 2 the single-particle contribution from the valence single nucleon.

$\left|JMK\right\rangle$ one takes into account also projection K onto the internal symmetry axis ("strong coupling limit"&Nilsson model)

Number of rotational bands equals to the number of different projection K. Both, even and odd spins are allowed within each band.

The smallest value of J for the band is K.
The lowest energy state (with spin) is called the band head.

The rotational energies are give by $$E = E_0 + \frac{\hbar^2}{2\Im_K} J(J+1) - K^2$$



^{9}Be (3/2-)

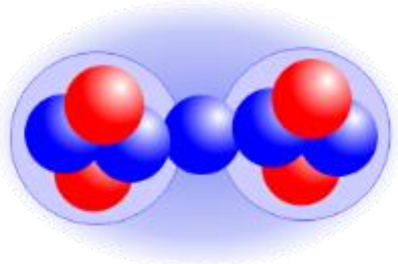
PRACTICES:

Apply Optical model and Coupled channel approach to describe data for inelastic scattering reaction d(19.5 MeV) + ^{9}Be

Elastic and inelastic data:

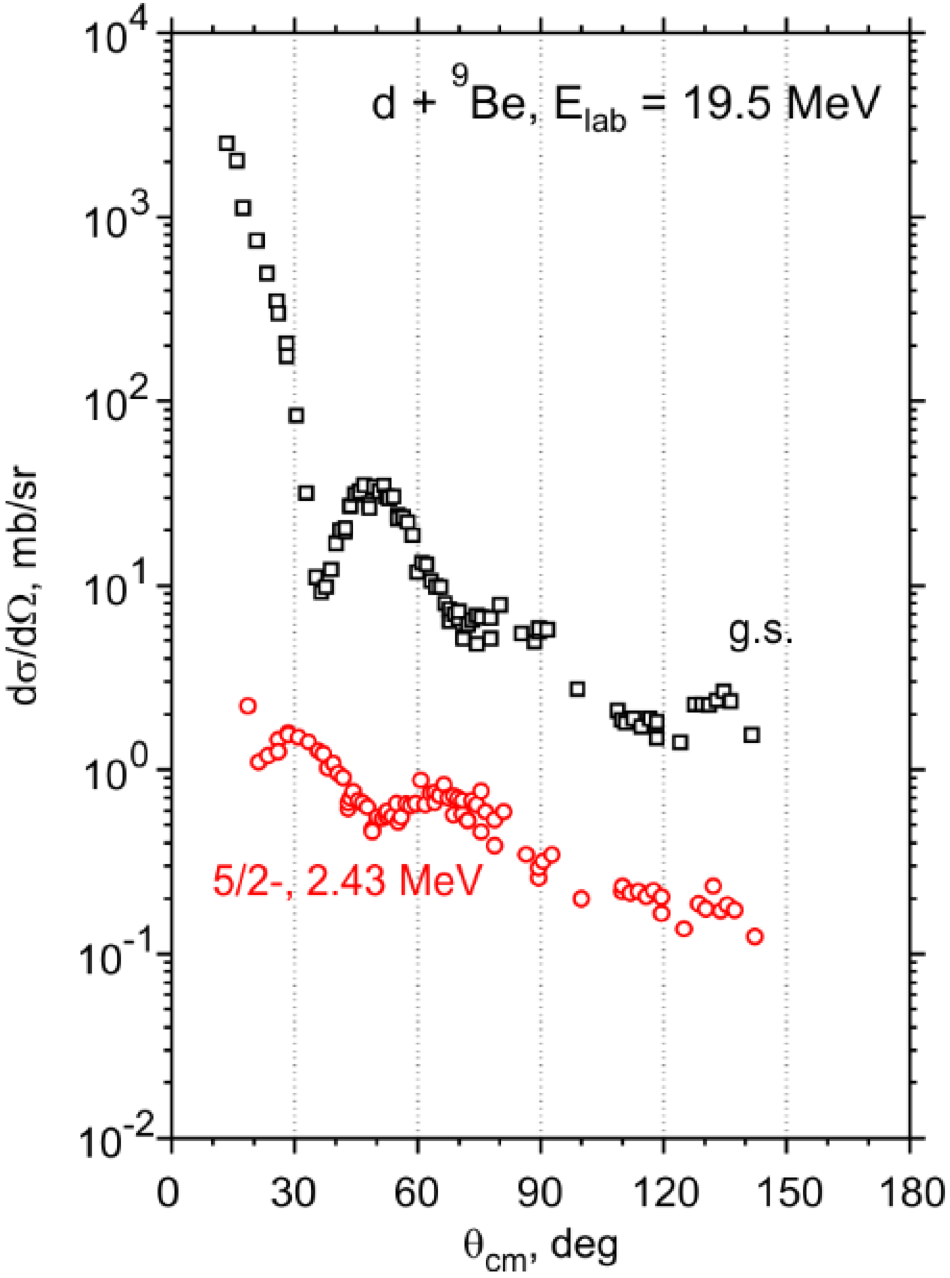
http://nrv.jinr.ru/denikin/data/d9Be.txt

1. Get optical potential
2. Run computational codes (NRV+ FRESCO)
3. Define OMP for CC
4. Define ^{9}Be deformation parameter

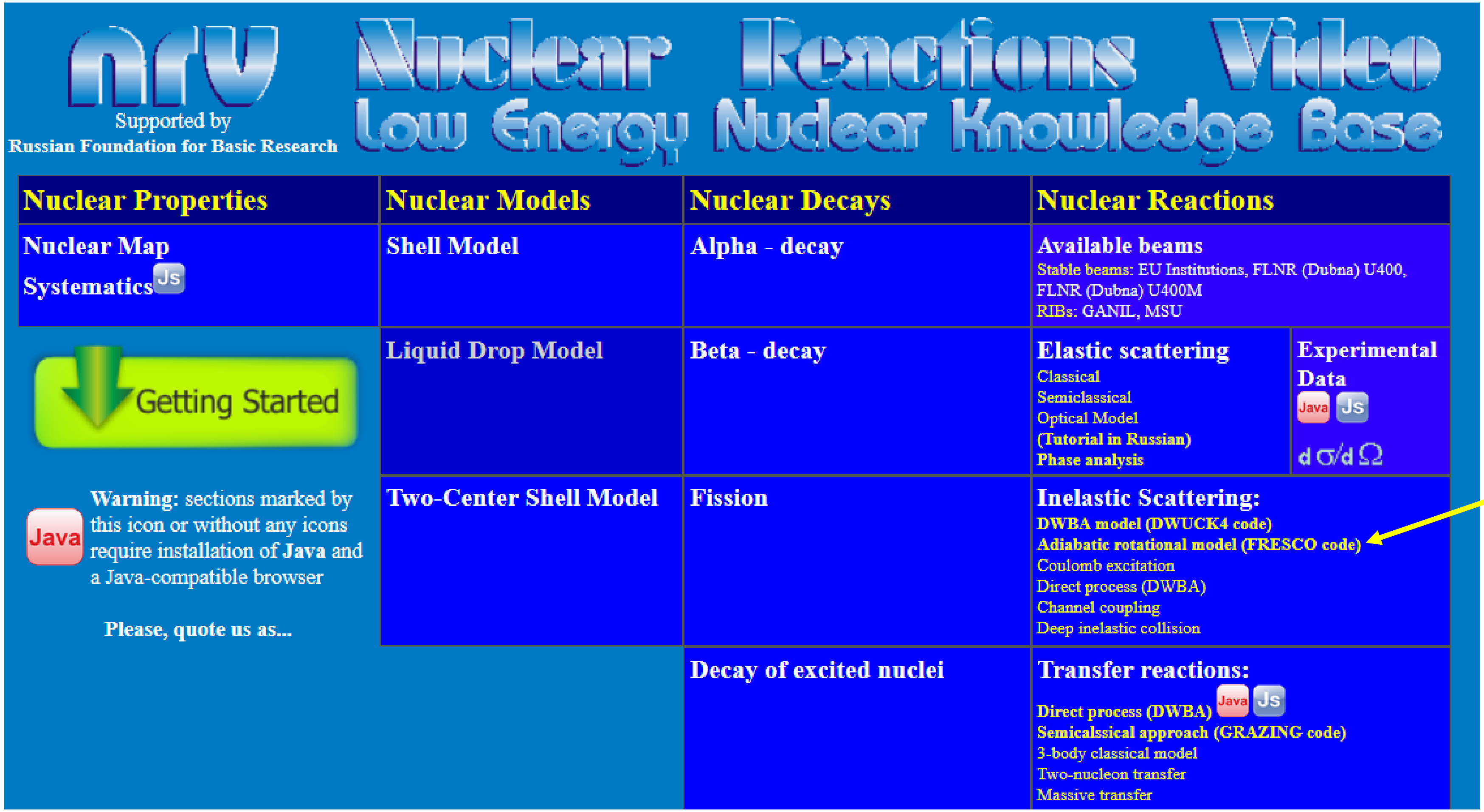


^{9}Be (3/2-)

Rotational band with K = 3/2



COUPLED CHANNELS IN NRV



Based on FRESCO by I.J. Thompson

Click here!

Set-up your reaction

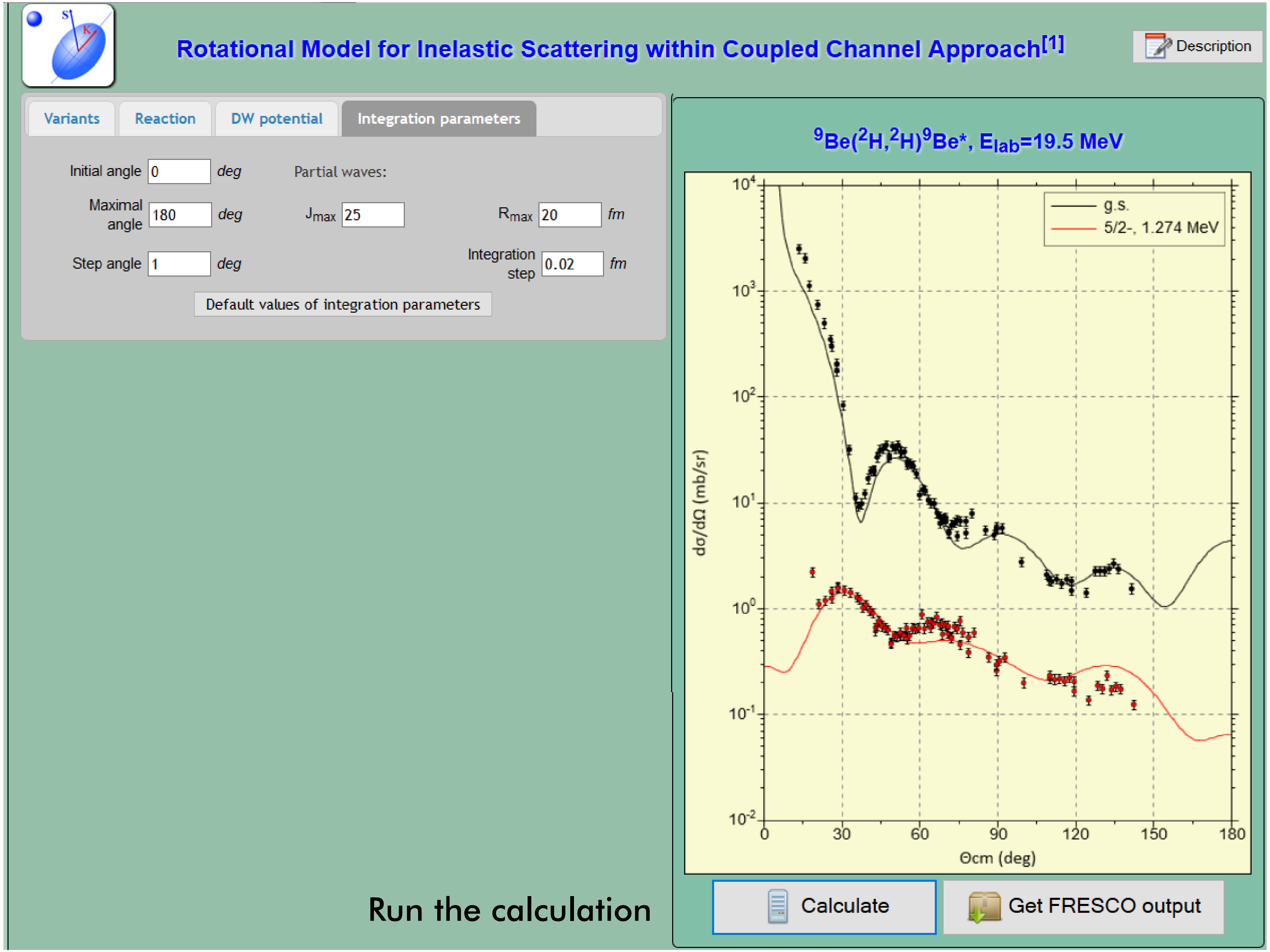
Set-up integration parameters

Set-up excited states details

Choose the exp. data

Set-up interaction potential

Prepare the exp. data

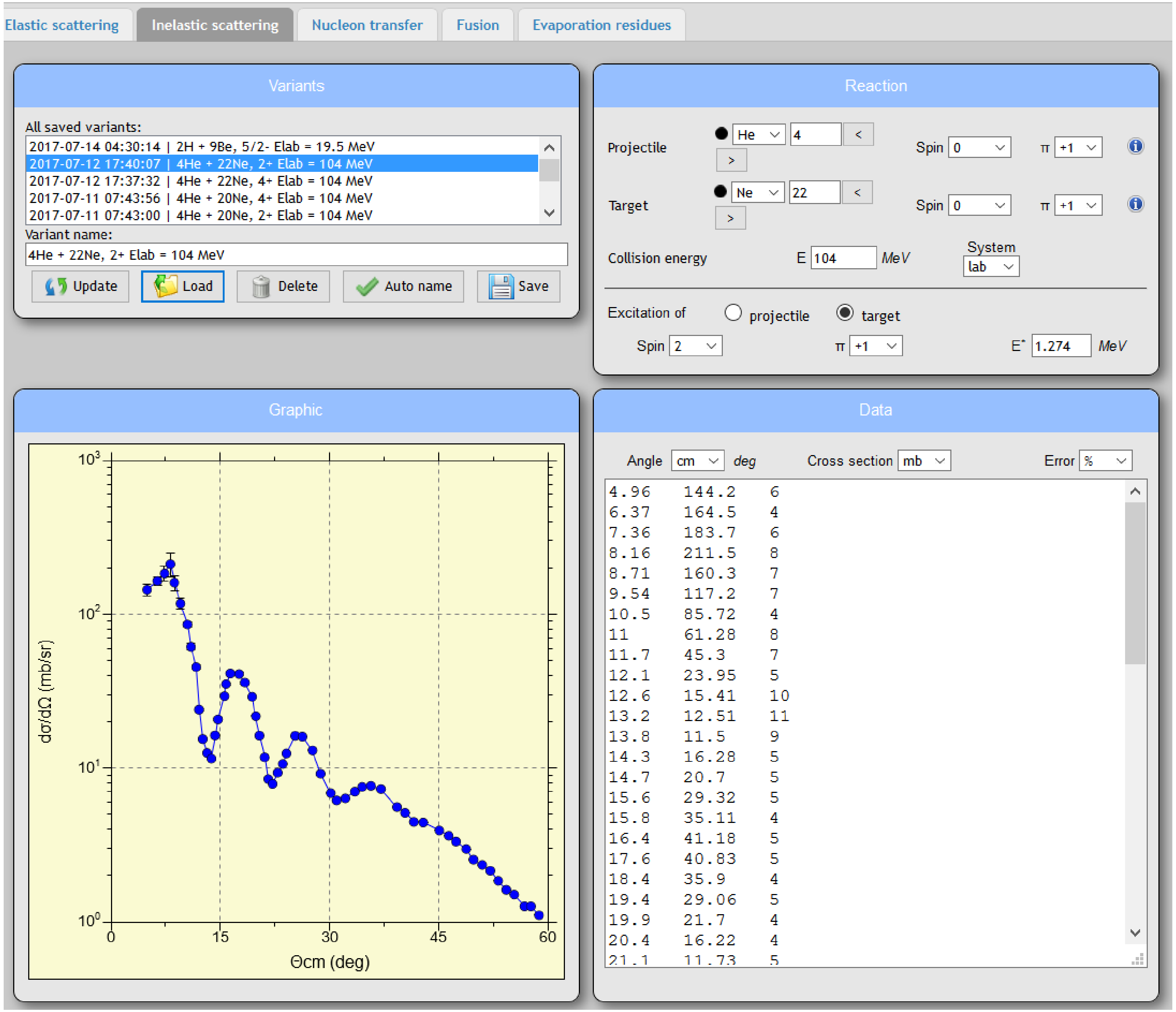


Run the calculation

List of predefined and stored data

Navigation buttons

Illustration plot



Set-up your reaction

Cross section Data (only in 3 columns)

USEFUL LINKS

1. NRV Knowledge base on low energy nuclear physics http://nrv.jinr.ru/nrv/
2. National Nuclear Data Center (Brookhaven) http://www.nndc.bnl.gov/index.jsp
3. Nuclear Data Services https://www-nds.iaea.org/
4. Centre for Photonuclear Experiments Data http://cdfe.sinp.msu.ru/index.en.html
5. JAEA Nuclear Data Center http://wwwndc.jaea.go.jp/
6. TUNL Nuclear Data Evaluation http://www.tunl.duke.edu/nucldata/index.shtml
7. NACRE II: European Compilation of Reactions Rates for Astrophysics http://www.astro.ulb.ac.be/nacreii/

CLASSICAL AND NEW TEXTBOOKS

1. G.R. Satchler, Direct Nuclear Reactions
2. M.L. Goldberger, K.M. Watson, Collision theory
3. P. Froebrich, R. Lindermeier, Theory of Nuclear Reactions
4. N.K. Glendenning, Direct Nuclear reactions
5. R.G. Newton, Scattering theory of waves and particles
6. I.J. Thompson, F.M. Nunes, Nuclear Reactions for Astrophysics
7. K. Langanke, J.A. Maruhn, S.E. Koonin, Computational Nuclear Physics
8. R. Bass, Nuclear Reactions with Heavy Ions
9. H. Schieck, Nuclear Reactions
10. …