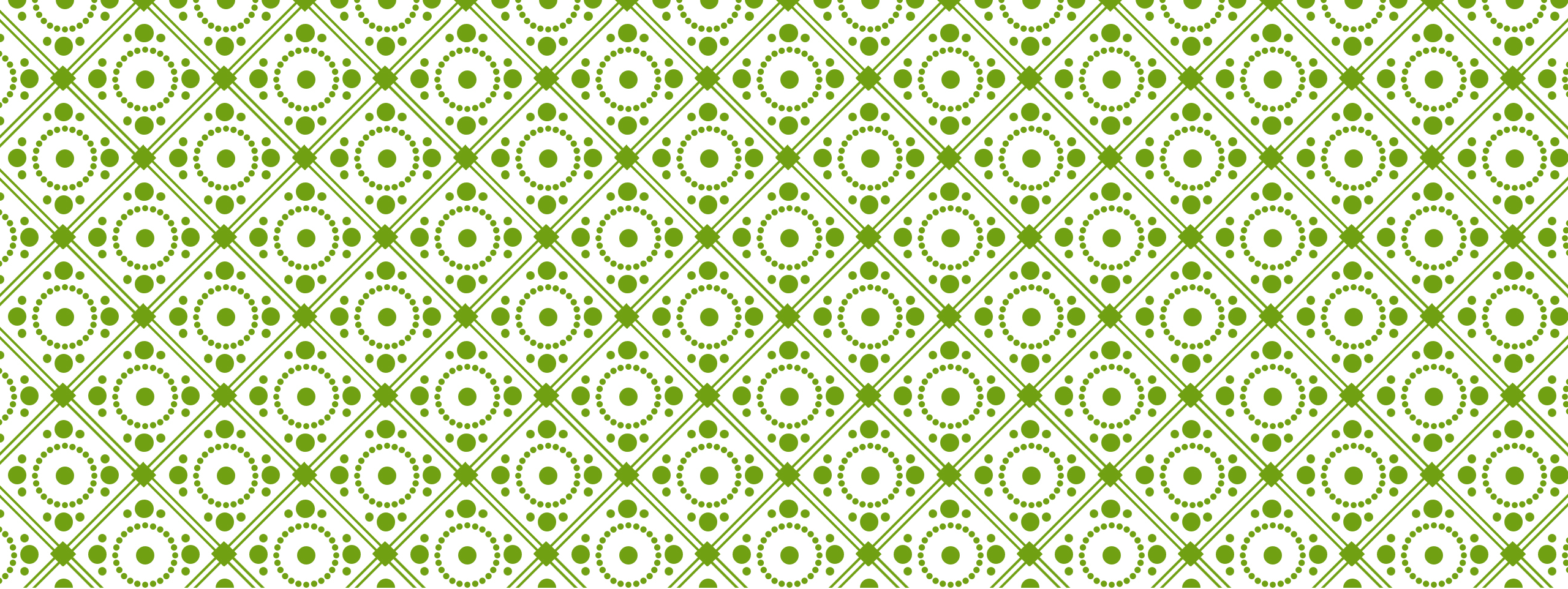




NUCLEAR REACTIONS

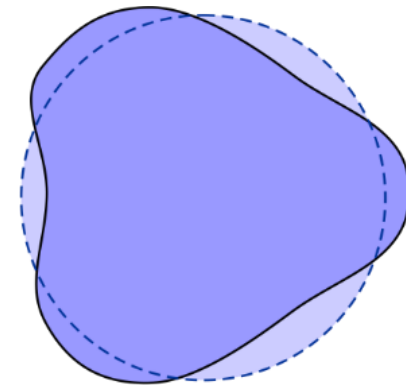
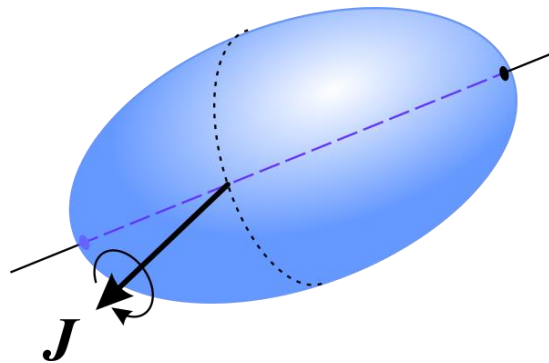
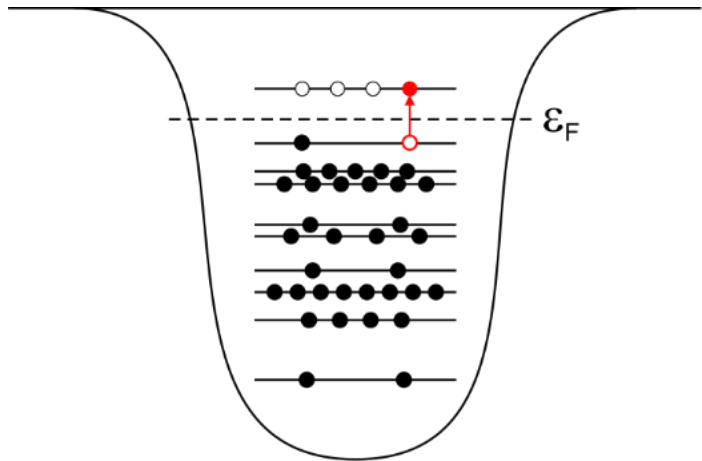
Direct nuclear reactions with RIB (part 3)

Andrea Denikin, PhD
JINR, Dubna University

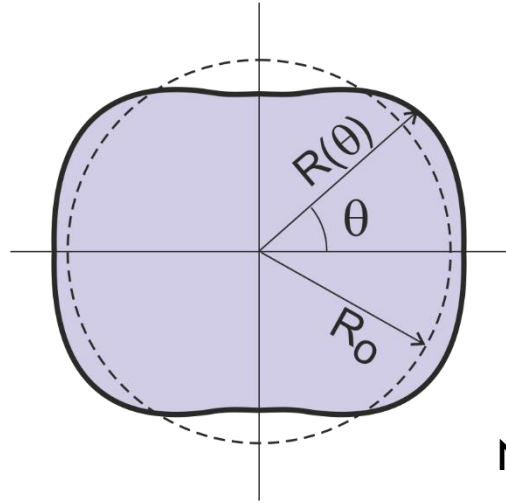
INELASTIC NUCLEAR SCATTERING

Types of inelastic nuclear excitation:

1. Excitation of one-particle-states.
2. Excitation of collective state: surface vibration or/and rotation



NUCLEAR SHAPE



Nuclear shape parametrization

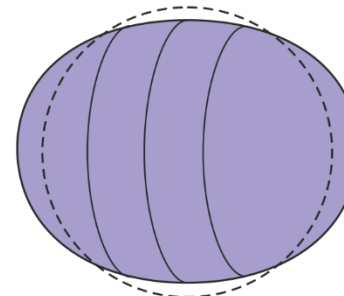
$$R(\Omega) = R_0 \left[1 + \sum_{\lambda \geq 2, \mu} \beta_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right] = R_0 + \delta R(\vec{\beta}, \Omega)$$

Static deformation:
multipole electric moment

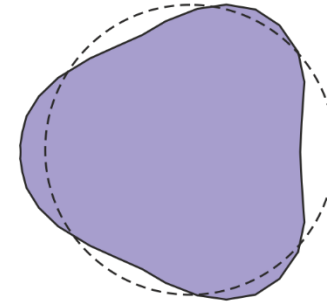
$$Q_\lambda = \frac{3ZR_0^\lambda}{\sqrt{\pi(2\lambda+1)}} \beta_\lambda^{g.s.}$$

Dynamical deformation:
zero vibration amplitude

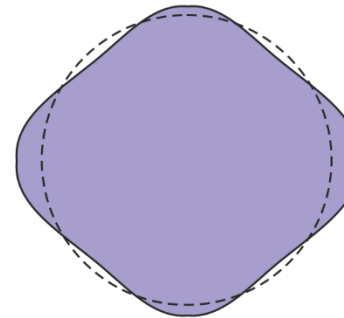
$$\langle \beta_\lambda^0 \rangle = \frac{4\pi}{3ZR_0^\lambda} \left(\frac{B(E\lambda)}{e^2} \right)^{1/2}$$



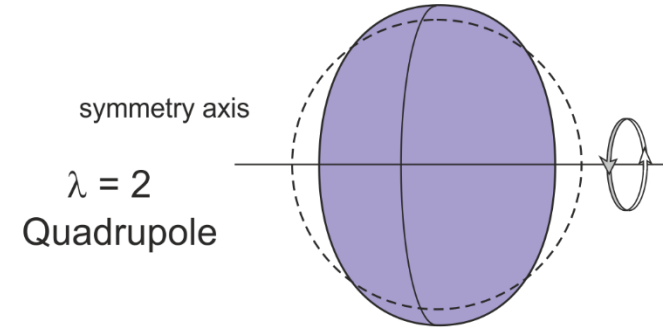
$$\beta_2 = 0.3$$



$$\beta_3 = 0.3$$

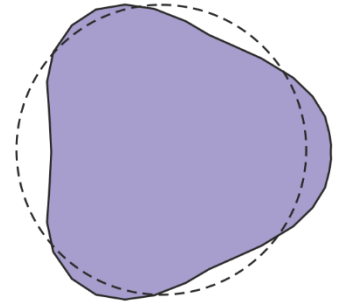


$$\beta_4 = 0.2$$

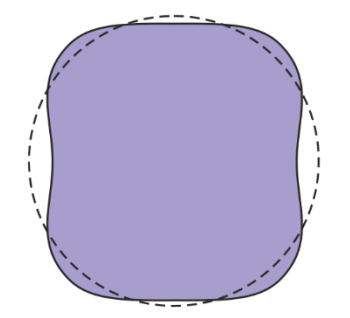


symmetry axis
 $\lambda = 2$
Quadrupole

$$\beta_2 = -0.3$$



$$\beta_3 = -0.3$$

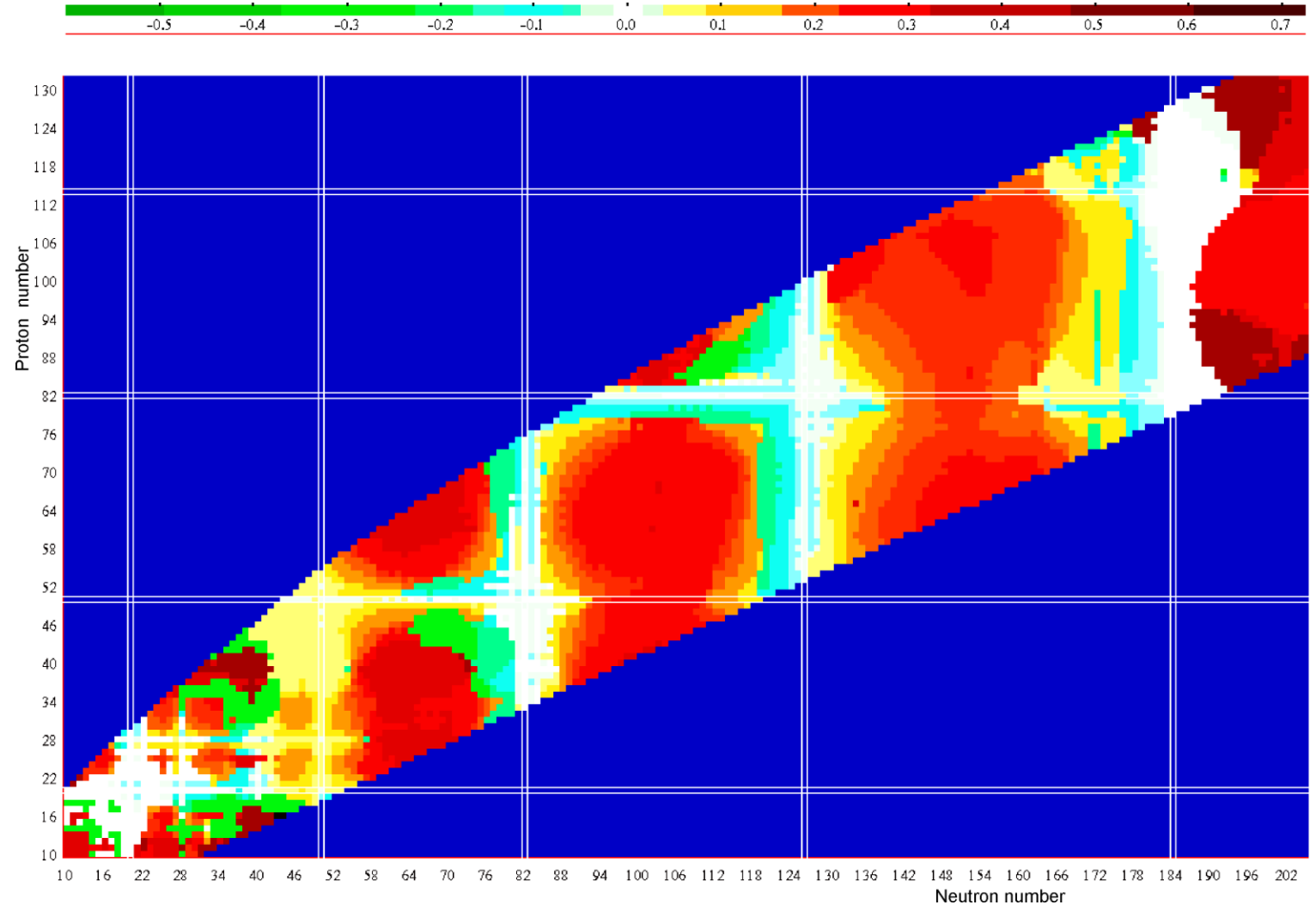


$$\beta_4 = -0.2$$

$\lambda = 4$
Hexadecapole

NUCLEAR SHAPE: DEFORMATION

P. Moller et al., 1995

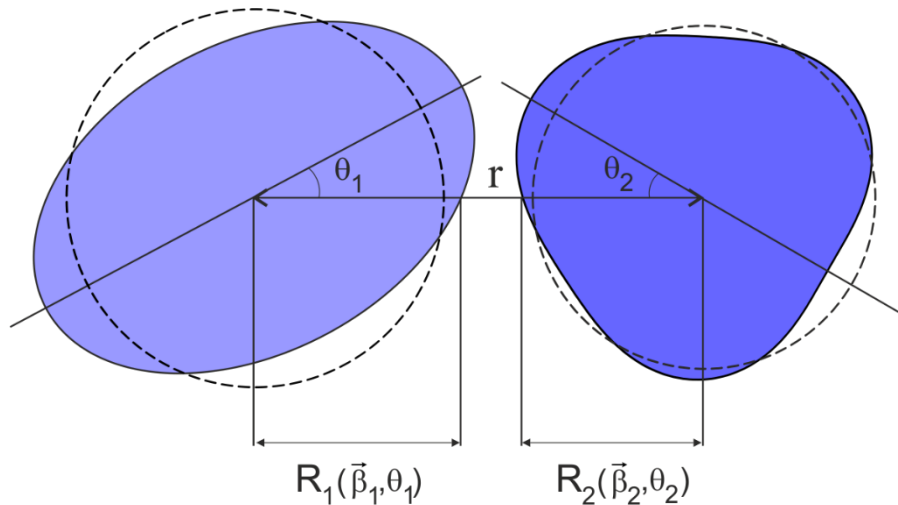


DWBA FOR INELASTIC SCATTERING

Reaction: $a + A \rightarrow a + A^*$

Differential cross section:

$$\frac{d\sigma_{fi}}{d\Omega}(\theta) = \frac{1}{(2\pi)^2} \frac{\mu^2}{\hbar^4} \frac{k_f}{k_i} |T_{fi}(\mathbf{k}_f, \mathbf{k}_i)|^2$$



DWBA amplitude:

$$T_{fi}^{DW}(\mathbf{k}_f, \mathbf{k}_i) \approx \left\langle \Psi_{\mathbf{k}_f}^{(-)}(\mathbf{R}_f) \varphi_A^{j'm'}(\mathbf{r}_f) \left| \Delta V \right| \varphi_A^{jm}(\mathbf{r}_i) \Psi_{\mathbf{k}_i}^{(+)}(\mathbf{R}_i) \right\rangle$$

Nucleus-nucleus interaction potential:

$$U(r) = U_C(r) + U_N(r)$$

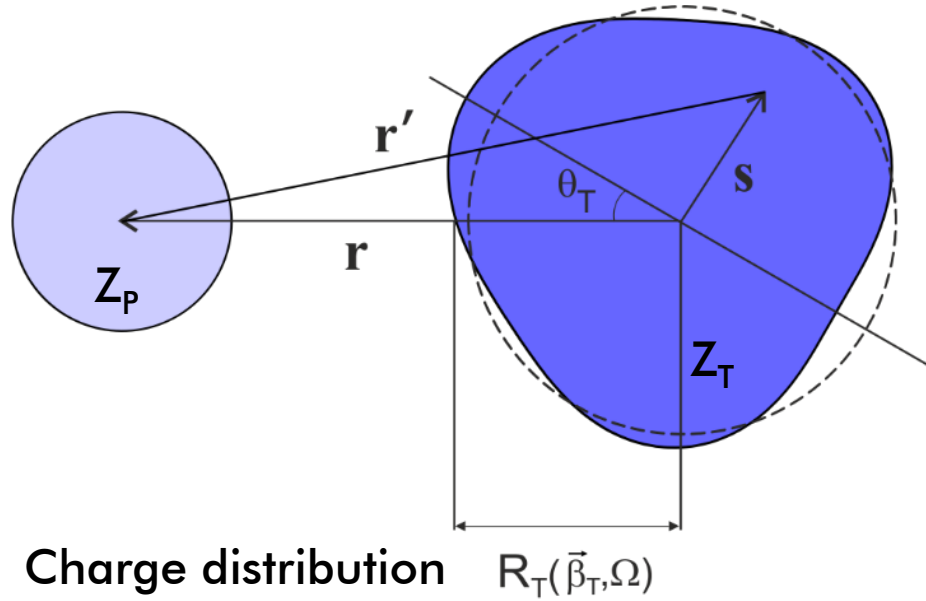
$$U_N(r) = \frac{V_0}{1 + \exp \frac{r - \sum_i R_i(\beta_i, \theta_i)}{a}} = U_0(r) - \delta R \frac{dU}{dr} + \dots$$

$$U_N(r) \approx U_0(r) - R_0 \frac{dU_0}{dr} \sum_{\lambda\mu}^a \beta_{\lambda\mu} Y_{\lambda\mu}(\Omega) + \dots$$

Optical potential

Distortion potential, leading to inelastic excitation

DEFORMED COULOMB POTENTIAL



$$U_C(r, \vec{\beta}_2) = Z_P e \int d\Omega_s \int_0^{R_T(\vec{\beta}_T, \Omega_s)} s^2 ds \frac{\rho_0^C}{|\mathbf{r} - \mathbf{s}|}$$

Use partial decomposition of the function

$$\frac{1}{|\mathbf{r} - \mathbf{s}|} = \sum_{lm} \frac{4\pi}{2l+1} \frac{r_{<}^l}{r_{>}^{l+1}} Y_{lm}^*(\Omega_s) Y_{lm}(\Omega)$$

And perform the integration one obtains:

$$U_C(r, \vec{\beta}_2) = Z_P Z_T e^2 \left\{ \frac{1}{2R_C} \left[3 - \frac{r^2}{R_C^2} \right], \quad r < R_C \right. \\ \left. \frac{1}{r}, \quad r > R_C \right\} + \frac{3Z_P Z_T e^2}{R_C} \sum_{lm} \frac{\beta_l^C Y_{lm}(\Omega)}{2l+1} \left\{ \frac{r^l}{R_C^l}, \quad r < R_C \right. \\ \left. \frac{R_C^{l+1}}{r^{l+1}}, \quad r > R_C \right\}$$

Coulomb part of the optical potential

Coulomb addition to the distortion potential

VIBRATIONAL EXCITATIONS

Vibration: harmonic oscillator

$$\sum_{\lambda\mu} \left[\frac{B_\lambda}{2} |\dot{\beta}_{\lambda\mu}|^2 + \frac{C_\lambda}{2} \beta_{\lambda\mu}^2 \right] |n j m\rangle = \sum_{\lambda\mu} \hbar \omega_\lambda \left(n + \frac{1}{2} \right) |n j m\rangle$$

Coupling potential:

$$\Delta V(\mathbf{r}, \beta) = \Delta V_C(\mathbf{r}, \beta) + \Delta V_N(\mathbf{r}, \beta) = \sum_{\lambda\mu} v_\lambda^C(r) + v_\lambda^N(r) \beta_{\lambda\mu} Y_{\lambda\mu}(\Omega)$$

Transition matrix element:

$$T_{fi} \sim \langle n' j' m' | \Delta V | n j m \rangle = \sum_{\lambda} \mathbb{C} \langle n' j' || \beta_\lambda || n j \rangle v_\lambda^C(r) + v_\lambda^N(r)$$

$$\langle 1 j || \beta_\lambda || 0 0 \rangle = (-)^j \delta_{j\lambda} \sqrt{2j+1} \frac{4\pi}{3ZeR_C^j} \sqrt{B(E\lambda, 0 \rightarrow 1)},$$

$$B(E\lambda, 0 \rightarrow 1) = \left(\frac{3}{4\pi} ZeR_C^\lambda \right)^2 \frac{\hbar}{2B_\lambda \omega_\lambda}, \omega_\lambda = \sqrt{\frac{C_\lambda}{B_\lambda}}$$

here $B(E\lambda)$ is the reduced electric multipole transition probability, which determines the strength of γ -transitions of multipolarity λ between two nuclear states.

$$\frac{d\sigma}{d\Omega}(\lambda \leftarrow 0) = \left(\frac{m}{2\pi\hbar^2} \right)^2 \frac{k'}{k} \frac{\beta_{0\lambda}^2 R_0^2}{2\lambda+1} \sum_{\mu} \left| \int \chi_{k'}^{(-)*}(\mathbf{r}) \left(v_\lambda^C(r) - \frac{dU}{dr} \right) Y_{\lambda\mu}(\Omega) \chi_k^{(+)}(\mathbf{r}) d\mathbf{r} \right|^2$$

EXAMPLE

1808.8 keV 6+	=====	0.2 ps
1791.5 keV 6+	=====	20.8 ps
1550.9 keV 3+	=====	0.56 ps
1314.7 keV 4+	=====	7.4 ps
696.6 keV 2+	=====	2.97 ps
0 keV 0+	////	2.29E+15 y

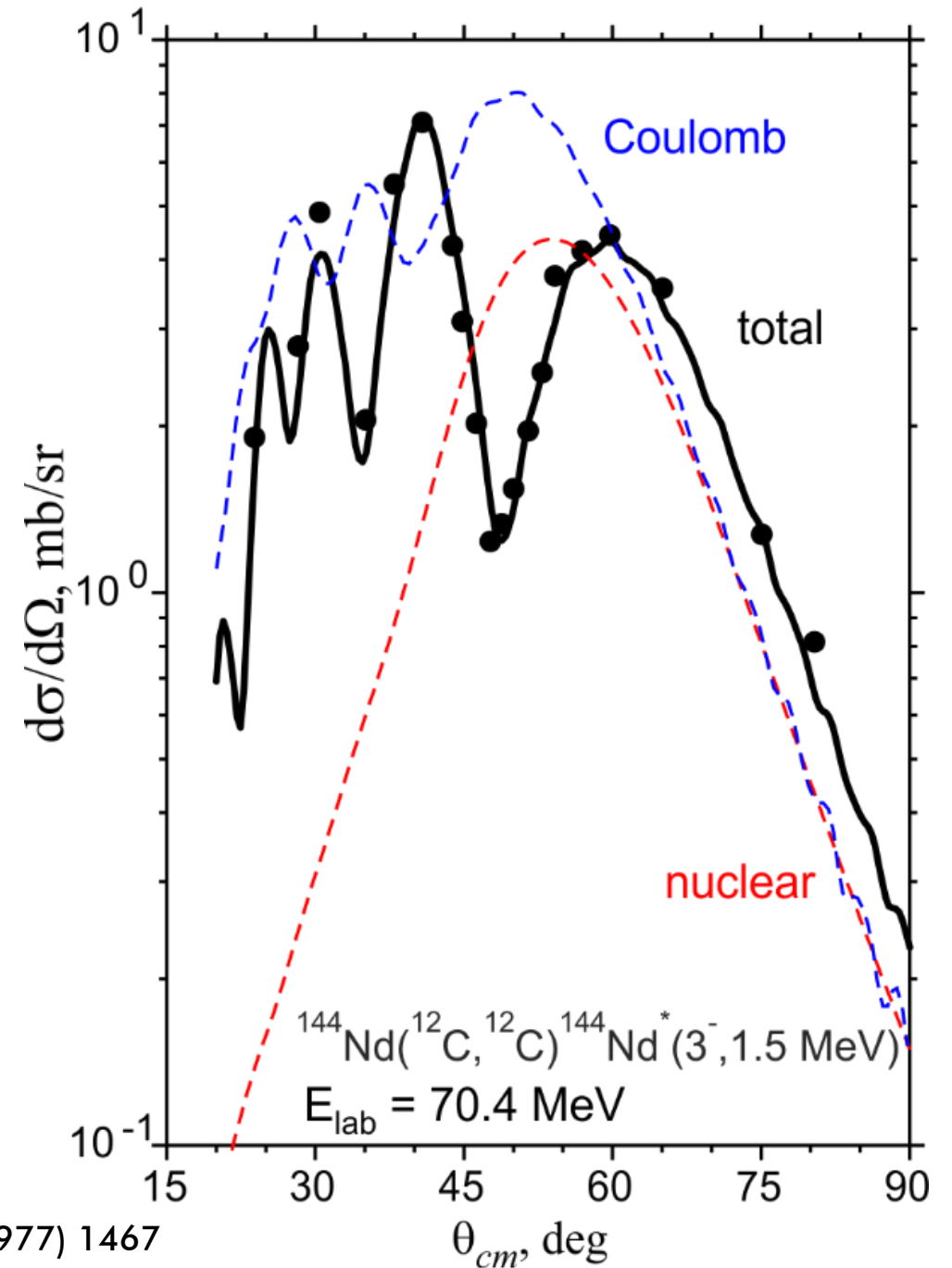
^{144}Nd

Available information:

- ☐ No static deformation
- ☐ $B(E3) = 0.263 \text{ e}^2\text{b}^3$
- ☐ $\beta_{03} = 0.114$

Extracted parameters:

- ☐ $\beta_{03, \text{Nucl}} = 0.1$
- ☐ $\beta_{03, \text{Coul}} = 0.115$



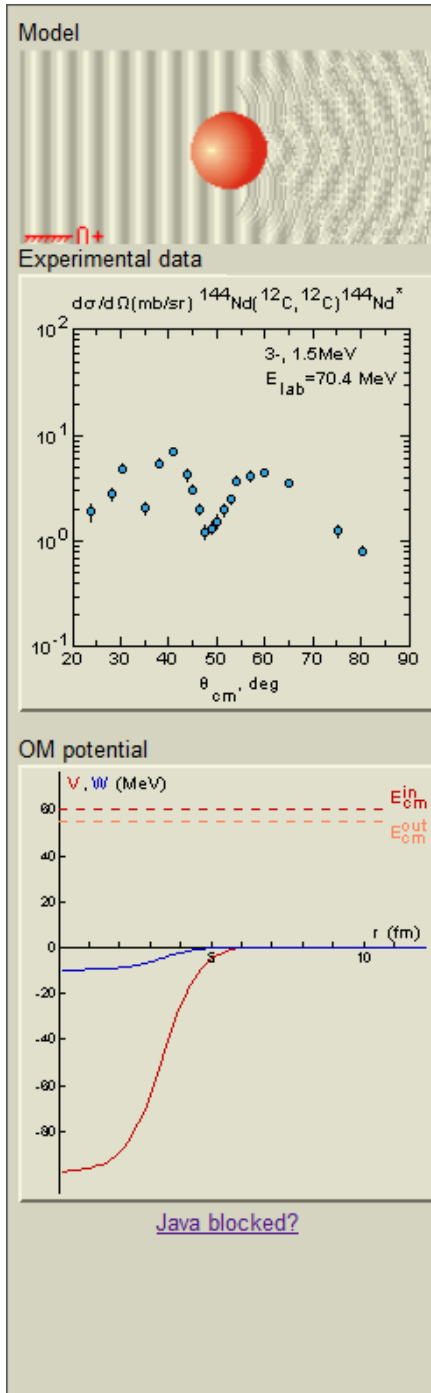
NRV DWBA FOR INELASTIC SCATTERING

nrV Supported by Russian Foundation for Basic Research

Nuclear Reactions Video Low Energy Nuclear Knowledge Base

Nuclear Properties	Nuclear Models	Nuclear Decays	Nuclear Reactions
Nuclear Map Systematics JS  Warning: sections marked by this icon or without any icons require installation of Java and a Java-compatible browser Please, quote us as...	Shell Model Liquid Drop Model Two-Center Shell Model	Alpha - decay Beta - decay Fission Decay of excited nuclei	Available beams Stable beams: EU Institutions, FLNR (Dubna) U400, FLNR (Dubna) U400M RIBs: GANIL, MSU Elastic scattering Classical Semiclassical Optical Model (Tutorial in Russian) Phase analysis Experimental Data Java JS $d\sigma/d\Omega$ Inelastic Scattering: DWBA model (DWUCK4 code) Adiabatic rotational model (FRESCO code) Coulomb excitation Direct process (DWBA) Channel coupling Deep inelastic collision Transfer reactions: Java JS Direct process (DWBA) Semiclassical approach (GRAZING code) 3-body classical model Two-nucleon transfer Massive transfer

Click here!



Inelastic scattering of nuclear particles (one-step excitation of collective state)

Model: DWBA (based on DWUCK4, P.Kunz) NRV Description OMP Compilations

Reaction Sample Open Save

Projectile ■ C 12 < > r₀ 1.2 fm R 2.747 fm

Target ■ Nd 144 < > r₀ 1.2 fm R 6.29 fm

Energy 70.4 MeV ☒ lab ☐ cm ☐ E/A

Inelastic Excitation of ☐ projectile or ☒ target

ground state J(π) 0(+) ▼

excited state J(π) 3(-) ▼ E_{exc} 1.51053 MeV Exp. levels

transferred momenta j_{tr} 3 ħ l_{tr, str} j, 0 ħ

Normalization: β_I 0.1 β_{Coul} 0.115 ☒ Coulomb excitation Unit normalization

Transition Form-Factor (taken as derivative of entrance channel OMP) ☒ include ☐ exclude imaginary part

Experimental data Prepare No data

OM potential in entrance channel

W. S. Volume ▼ V₀^{vol} -17.15 MeV r₀^{vol} 1.292 fm a^{vol} 0.691 fm

V₀^{sur} MeV r₀^{sur} fm a^{sur} fm

Proximity: b fm r₀^{coul} 1.2 fm Folding params

Absorptive potential

W. S. Volume ▼ W₀^{vol} -12.97 MeV r₀^{vol} 1.334 fm a^{vol} 0.428 fm

W₀^{sur} MeV r₀^{sur} fm a^{sur} fm

Spin-orbit interaction

Spin ☒ 0 ☐ 1/2 V₀ 0.9 MeV W₀ 0.0 MeV r₀ 0.96 fm a 0.444 fm

OM potential in exit channel the same

W. S. Volume ▼ V₀^{vol} -17.15 MeV r₀^{vol} 1.292 fm a^{vol} 0.691 fm

V₀^{sur} MeV r₀^{sur} fm a^{sur} fm

Proximity: b fm r₀^{coul} 1.2 fm Folding params

Absorptive potential

W. S. Volume ▼ W₀^{vol} -12.97 MeV r₀^{vol} 1.334 fm a^{vol} 0.428 fm

W₀^{sur} MeV r₀^{sur} fm a^{sur} fm

Spin-orbit interaction

Spin ☒ 0 ☐ 1/2 V₀ 0.9 MeV W₀ 0.0 MeV r₀ 0.96 fm a 0.444 fm

Integration parameters Default values of integration parameters

Initial angle 20 deg. Partial waves: L max 130

Maximal angle 90 deg. Rmax 30.0 fm

Step 0.5 deg. Integration step 0.10 fm

Calculate

Set your reaction

Set properties of excited state

Set exp. data

Set OMP for in-channel

Set OMP for out-channel

Set integration params

Entrance channel OMP

Coulomb $r_0(R)$, fm 1.20 (9.04)	Real part			Imaginary part		
	V_0 , MeV	$r_0(R)$, fm	a , fm	W_0 , MeV	$r_0(R)$, fm	a , fm
Volume	-17.15	1.29 (9.73)	0.69	-12.97	1.33 (10.05)	0.43
Surface						
Spin-Orbit						
Proximity						

Exit channel OMP

Coulomb $r_0(R)$, fm 1.20 (9.04)	Real part			Imaginary part		
	V_0 , MeV	$r_0(R)$, fm	a , fm	W_0 , MeV	$r_0(R)$, fm	a , fm
Volume	-17.15	1.29 (9.73)	0.69	-12.97	1.33 (10.05)	0.43
Surface						
Spin-Orbit						
Proximity						

Reaction parameters

Entrance channel	Exit channel
$E_{lab} = 70.40$ MeV	$E_{lab} = 68.76$ MeV
$E_{cm} = 64.98$ MeV	$E_{cm} = 63.47$ MeV
$k = 5.868$ fm $^{-1}$	$k = 5.799$ fm $^{-1}$
$\eta = 23.402$	$\eta = 23.679$

Inelastic excitation of ^{144}Nd

$J^{\pi}_{g.s.} \rightarrow J^{\pi}$	$0^{+} \rightarrow 3^{-}$
l_{tr}, l_{tr}, s_{tr}	3, 3, 0 $\hbar$
E_{exc}	1.51 MeV
β_l	0.100
β_{Coul}	0.115

Integration parameters

R_{max}	30.0 fm
dr	0.10 fm
L_{max}	130 $\hbar$

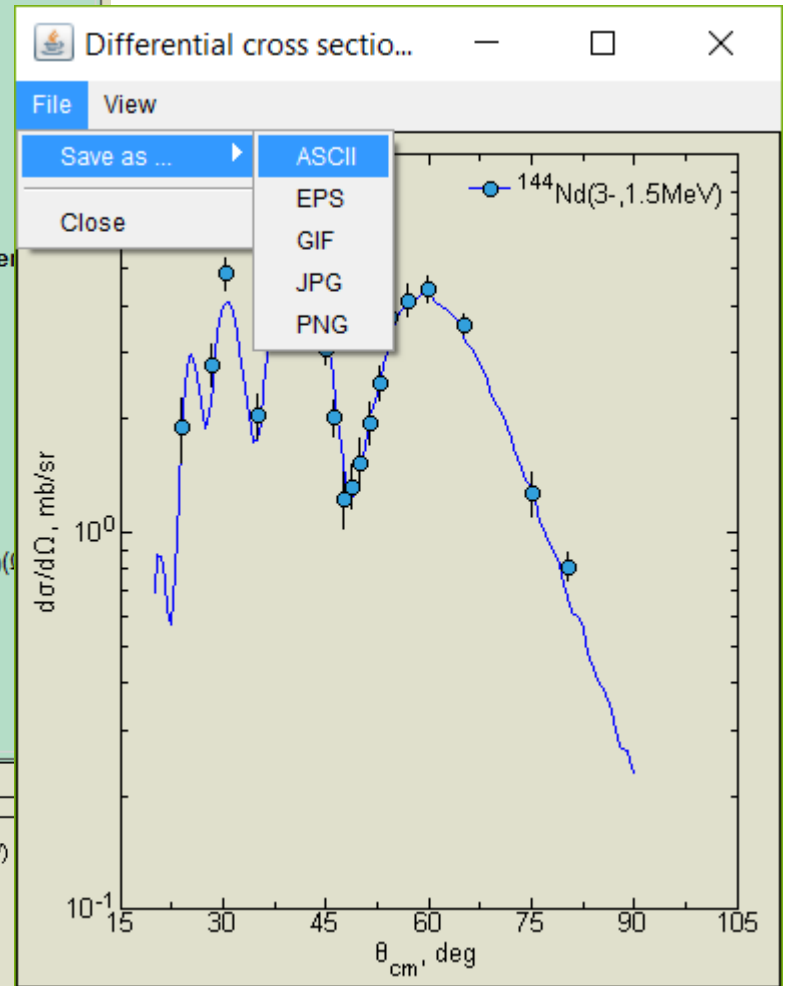
Transition Form Factor

$$\left(-\beta_l R_V \left| \frac{dV}{dr} \right| - i \beta_l R_W \left| \frac{dW}{dr} \right| + \beta_{Coul} \frac{3 Z_1 Z_2 e^2 R_C^l}{(2l+1) r^{l+1}} \right) Y_{10}(\theta)$$

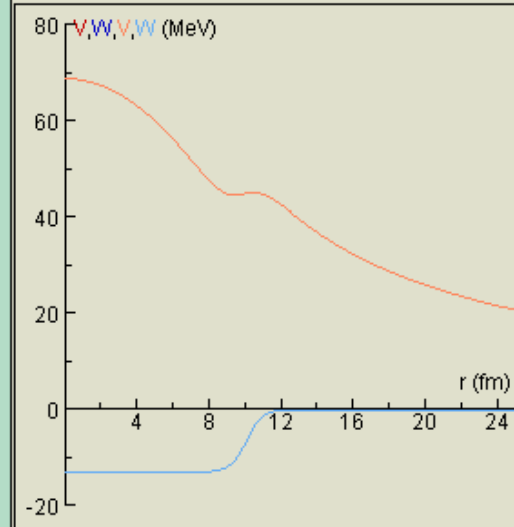
$$\beta_l R_V^{(vol)} = 1.292 \beta_l A_{Nd}^{1/3} = 0.67 \text{ fm}$$

$$\beta_l R_W^{(vol)} = 1.334 \beta_l A_{Nd}^{1/3} = 0.69 \text{ fm}$$

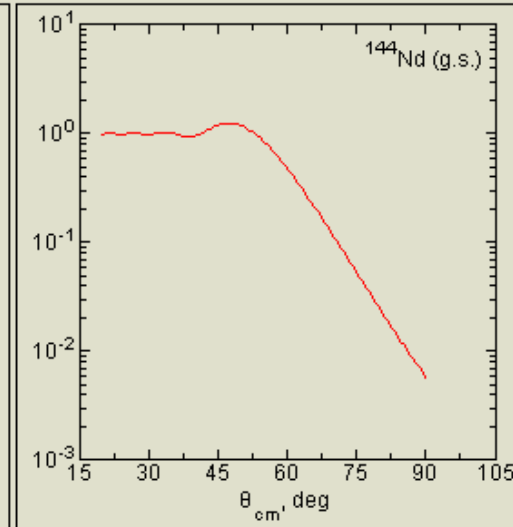
$$\beta_{Coul} R_{Coul} = 1.2 \beta_{Coul} A_{Nd}^{1/3} = 0.72 \text{ fm}$$



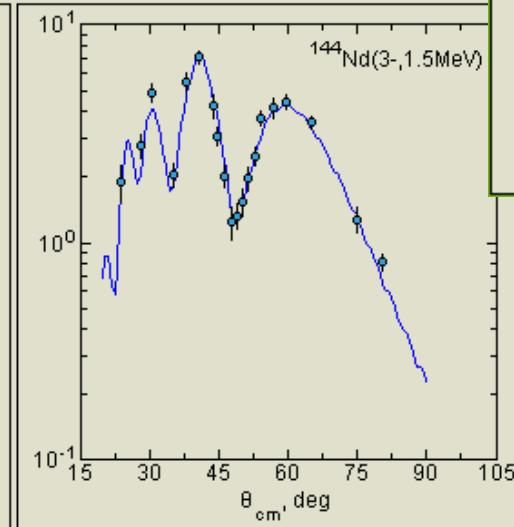
Optical Potentials



Elastic scattering



Inelastic scattering



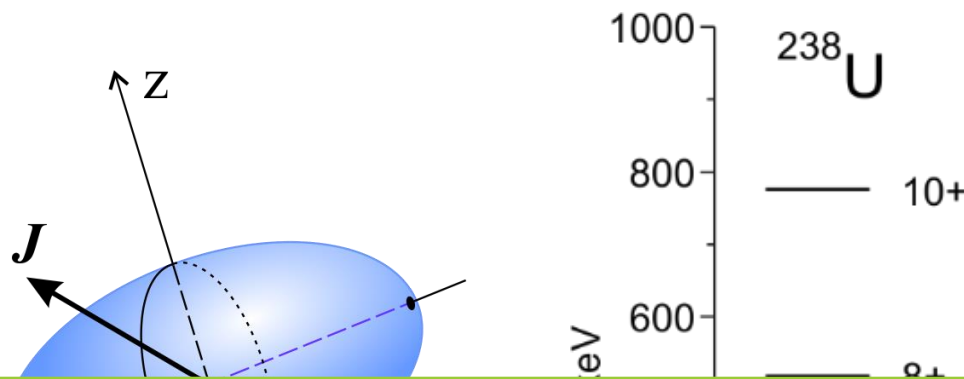
Click on a plot to process it

Get your results and process the data

ROTATIONAL EXCITATIONS

Rotation: quantum axially symmetric rotator

Schrodinger equation $\frac{\hbar^2}{2\mathfrak{I}} \mathbf{I}^2 |JM\rangle = \frac{\hbar^2}{2\mathfrak{I}} J(J+1) |JM\rangle$



$J\pi$	0+	2+	4+	6+
E_j	0	$6\hbar^2/2\mathfrak{I}$	$20\hbar^2/2\mathfrak{I}$	$42\hbar^2/2\mathfrak{I}$
E_j/E_2	0	1	3.33	7
$E_j(^{238}\text{U}), \text{keV}$	0	44.9	148.4	307.2
$E_j/E_2(^{238}\text{U})$	0	1	3.30	6.84

Moment of inertia:

$$\begin{aligned} \text{1} \quad \mathfrak{I}_{rig} &= \int r^2 \rho(r, \theta, \phi) dV = \\ &= \rho_0 \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\phi \int_0^{R(\theta, \phi)} r^4 dr \approx \frac{2}{5} m_N r_0^2 A^{5/3} (1 + 0.31\beta_2) = \\ &= 0.0138 A^{5/3} (1 + 0.31\beta_2) \left[\frac{\hbar^2}{\text{MeV}} \right] \\ \mathfrak{I}_{rig}(^{238}\text{U}) &= 148 \frac{\hbar^2}{\text{MeV}} \\ \text{2} \quad \mathfrak{I}(^{238}\text{U}) &= 3 \frac{\hbar^2}{E_{2+}} = 66.8 \frac{\hbar^2}{\text{MeV}} \end{aligned}$$

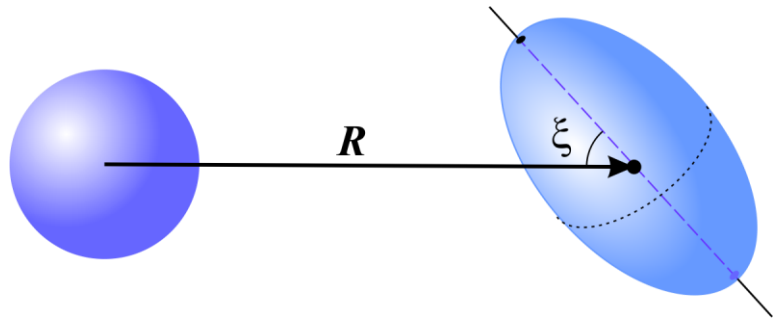
$$\beta_2(^{238}\text{U}) = 0.24$$

Nucleus is not a rigid body!

$$\text{3} \quad \mathfrak{I}_{LD}(^{238}\text{U}) = \frac{45}{16\pi} \mathfrak{I}_{rig} \beta^2 = 7.6 \frac{\hbar^2}{\text{MeV}}$$

Nucleus does not rotate like irrotational flow!

COUPLED CHANNELS



Hamiltonian describes two-nucleus system where one nuclei has internal structure

$$\hat{\mathbf{H}} = \hat{t}_R + \hat{h}_\xi + \hat{V}(R, \xi)$$

$$\hat{\mathbf{H}}\Psi(\mathbf{R}, \xi) = E\Psi(\mathbf{R}, \xi)$$

$$\hat{h}_\xi \varphi_i(\xi) = \varepsilon_i \varphi_i(\xi) \quad \text{here wave functions } \varphi_i(\xi) \text{ form the basis}$$

$$\Psi(\mathbf{R}, \xi) = \sum_i \psi_i(\mathbf{R}) \varphi_i(\xi) \quad \text{decomposition of the total wave function over the basis } \varphi_i(\xi)$$

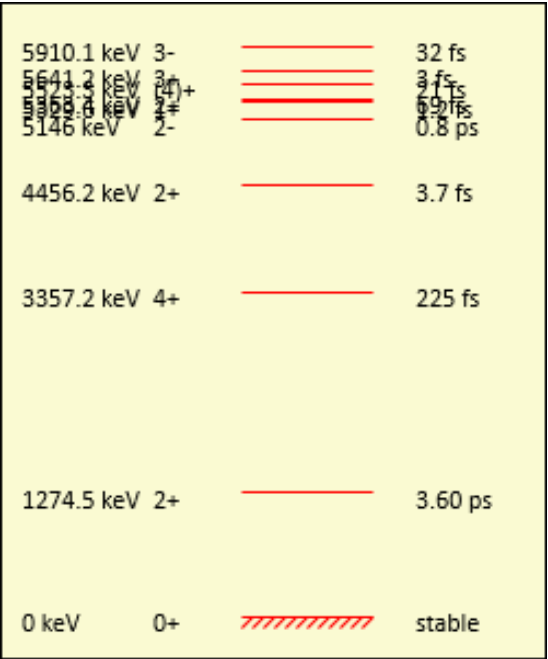
Set of coupled equations:

$$\hat{t}_R + \varepsilon_i \psi_i(\mathbf{R}) + \sum_j \hat{V}_{ij}(\mathbf{R}) \psi_j(\mathbf{R}) = E \psi_i(\mathbf{R}), \quad \psi_i(\mathbf{R}) \rightarrow \delta_{i1} e^{ik_1 R} + f_{i1}(\theta) \frac{e^{ik_i R}}{R}, \quad i = 1, 2, \dots$$

$$\hat{V}_{ij}(\mathbf{R}) = \int \varphi_i^*(\xi) \hat{V}(\mathbf{R}, \xi) \varphi_j(\xi) d\xi \quad \text{matrix elements that couple different reaction channels}$$

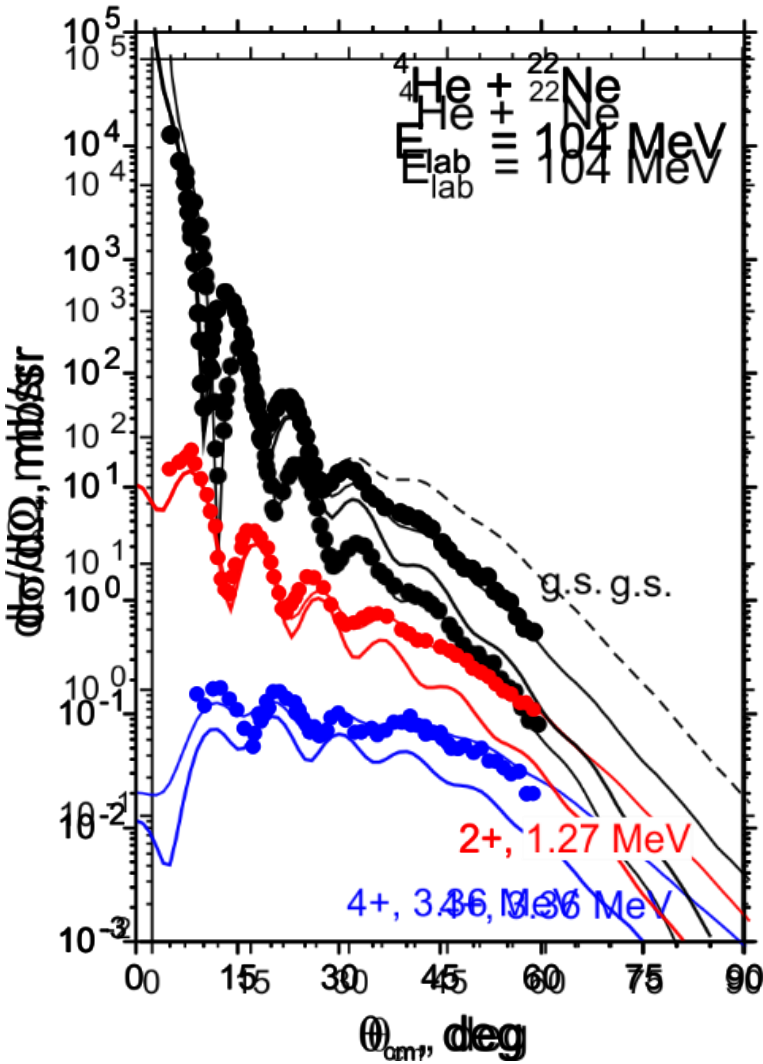
DWBA transition form-factor

EXAMPLE: $^4\text{He} + ^{20}\text{Ne}$ (Coupled channels)



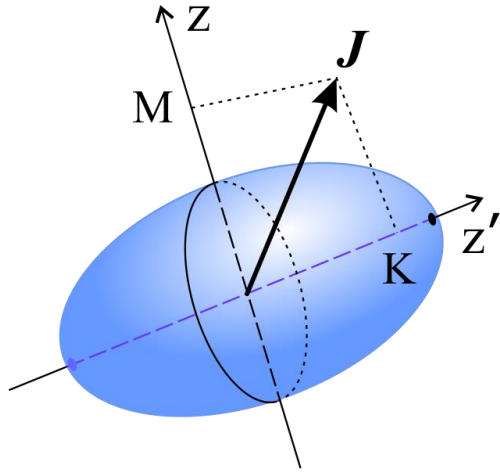
^{22}Ne

P. Moller: $\beta_2 = 0.384$, $\beta_4 = 0.09$



OMP	OM / CC
V_0 , MeV	112.72
r_v , fm	0.789
a_v , fm	0.76
W_0 , MeV	20.83 / 15.7
r_w , fm	1.09
a_w , fm	0.56

ROTATIONAL EXCITATIONS



For nuclei with odd number of either protons or neutrons two contributions to the total angular momentum J can be distinguished

- ❑ the collective contribution from the rotation of even-even core
- ❑ 2 the single-particle contribution from the valence single nucleon.

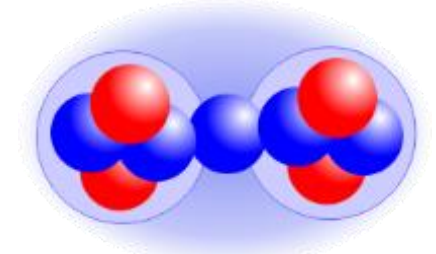
$|JMK\rangle$ one takes into account also projection K onto the internal symmetry axis ("strong coupling limit" & Nilsson model)

Number of rotational bands equals to the number of different projection K . Both, even and odd spins are allowed within each band.

The smallest value of J for the band is K .

The lowest energy state (with spin) is called the band head.

The rotational energies are given by

$$E = E_0 + \frac{\hbar^2}{2\mathfrak{I}_K} J(J+1) - K^2$$


${}^9\text{Be} (3/2^-)$

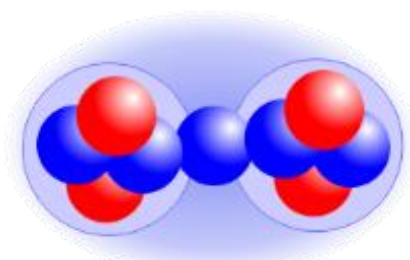
PRACTICES:

Apply Optical model and Coupled channel approach to describe data for inelastic scattering reaction $d(19.5 \text{ MeV}) + {}^9\text{Be}$

Elastic and inelastic data:

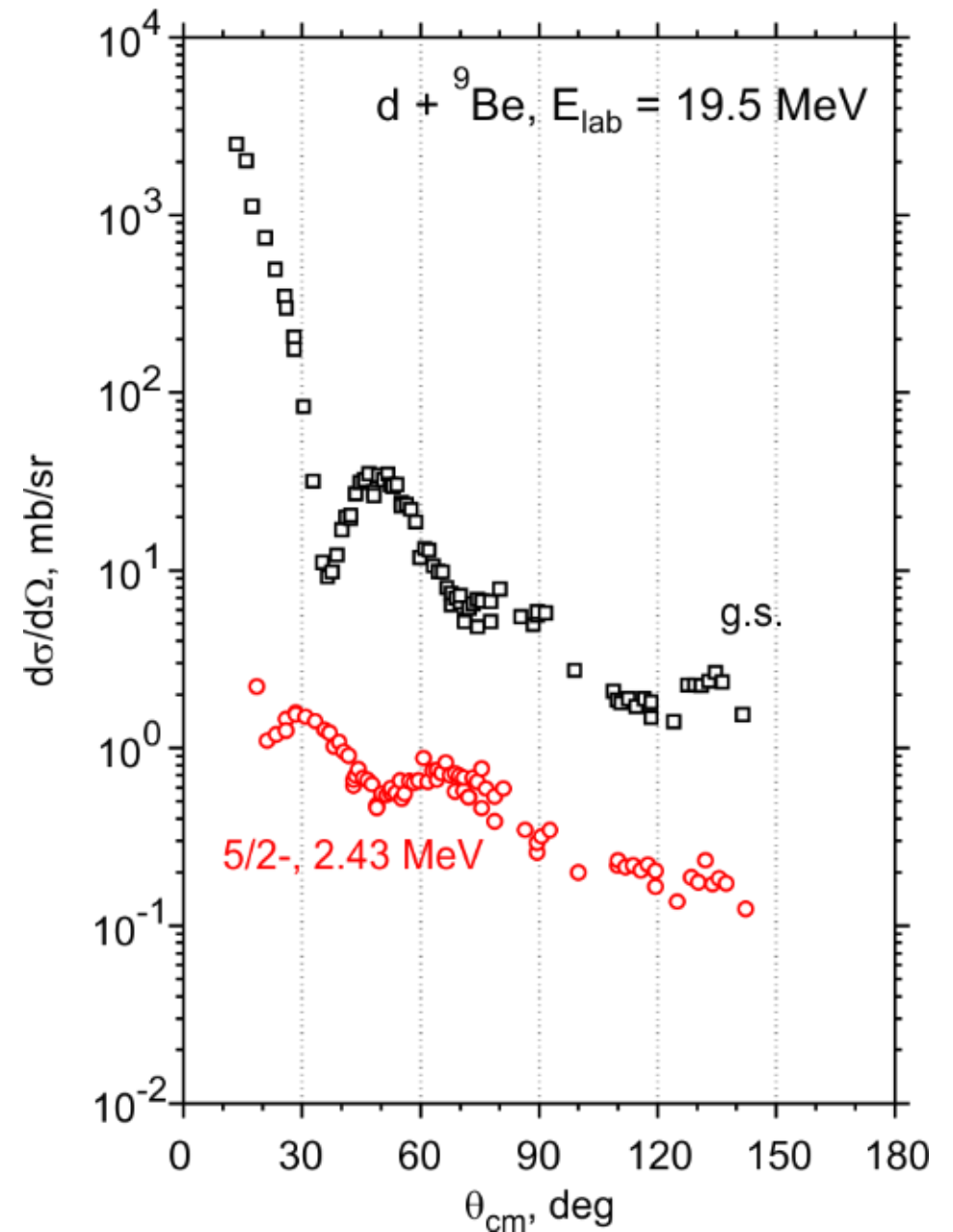
<http://nrv.jinr.ru/denikin/data/d9Be.txt>

1. Get optical potential
2. Run computational codes (NRV+ FRESCO)
3. Define OMP for CC
4. Define ${}^9\text{Be}$ deformation parameter



${}^9\text{Be} (3/2^-)$

Rotational band with $K = 3/2$



COUPLED CHANNELS IN NRV



Nuclear Reactions Video

Low Energy Nuclear Knowledge Base

Supported by
Russian Foundation for Basic Research

Nuclear Properties	Nuclear Models	Nuclear Decays	Nuclear Reactions
Nuclear Map Systematics 	Shell Model	Alpha - decay	Available beams Stable beams: EU Institutions, FLNR (Dubna) U400, FLNR (Dubna) U400M RIBs: GANIL, MSU
 Warning: sections marked by this icon or without any icons require installation of Java and a Java-compatible browser Please, quote us as...	Liquid Drop Model	Beta - decay	Elastic scattering Classical Semiclassical Optical Model (Tutorial in Russian) Phase analysis
	Two-Center Shell Model	Fission	Inelastic Scattering: DWBA model (DWUCK4 code) Adiabatic rotational model (FRESCO code) Coulomb excitation Direct process (DWBA) Channel coupling Deep inelastic collision
		Decay of excited nuclei	Transfer reactions: Direct process (DWBA)   Semiclassical approach (GRAZING code) 3-body classical model Two-nucleon transfer Massive transfer



Getting Started

Experimental Data

$d\sigma/d\Omega$

Based on FRESCO by I.J. Thompson

Click here!

Set-up your reaction

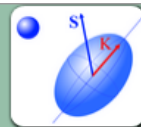
Set-up integration
parameters

Set-up excited
states details

Choose the exp. data

Set-up interaction
potential

Prepare the exp. data



Rotational Model for Inelastic Scattering within Coupled Channel Approach^[1]

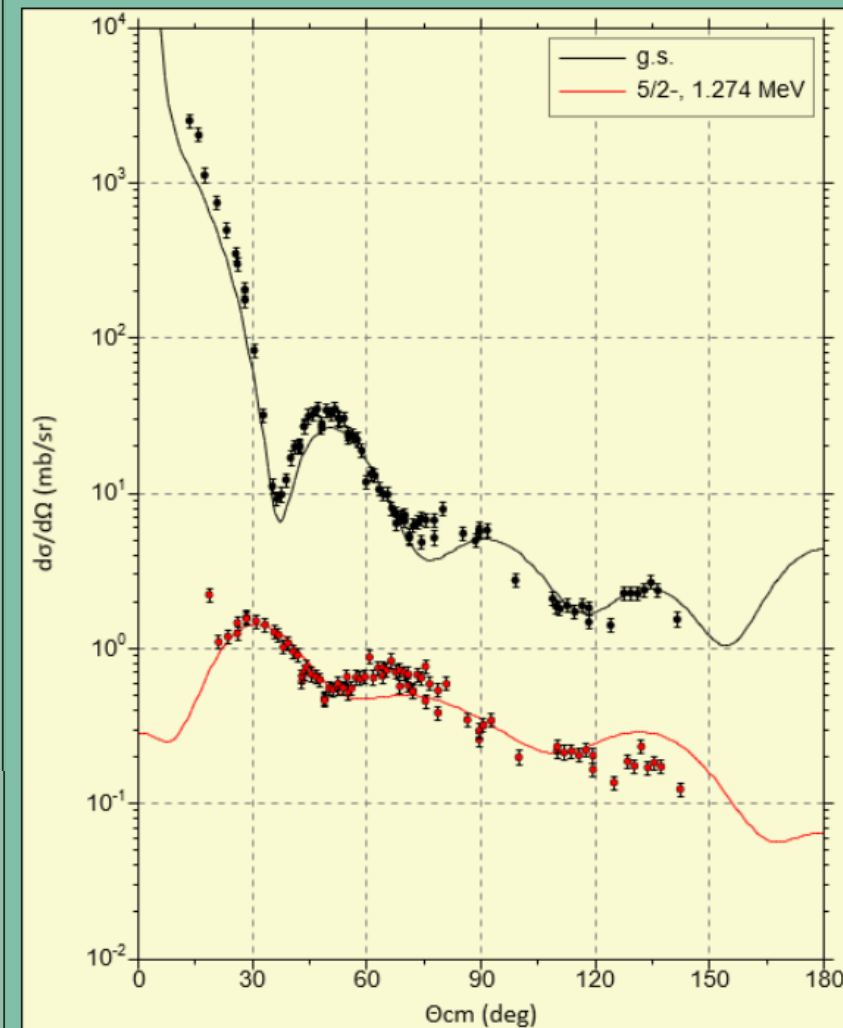
Description

Variants
Reaction
DW potential
Integration parameters

Initial angle deg
Maximal angle deg
Step angle deg
Partial waves:
J_{max}
R_{max} fm
Integration step fm

Default values of integration parameters

$^9\text{Be}(^2\text{H}, ^2\text{H})^9\text{Be}^*$, $E_{\text{lab}}=19.5$ MeV



Run the calculation

Calculate

Get FRESKO output

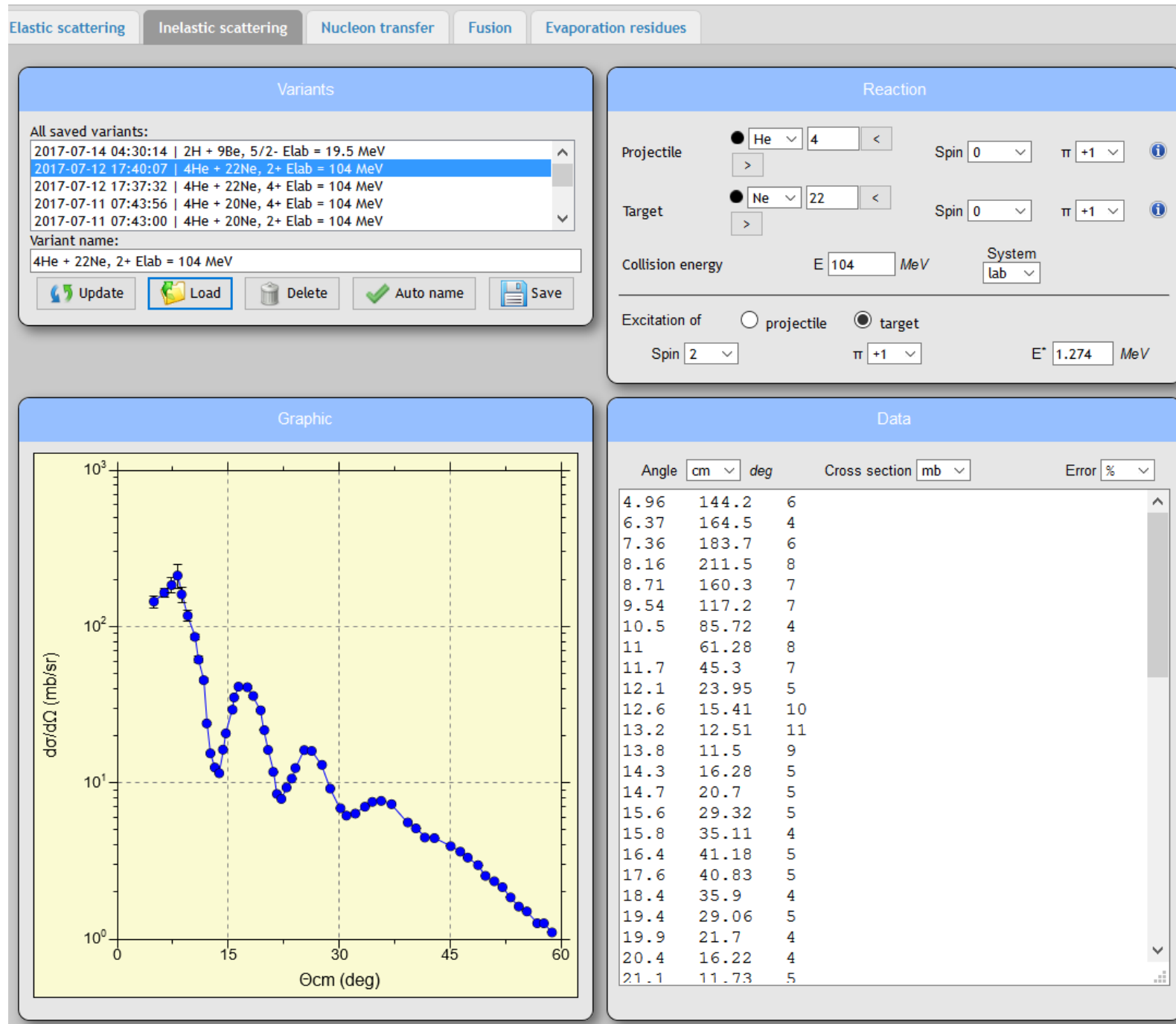
List of
predefined
and stored
data

Navigation
buttons

Illustration
plot

Set-up your
reaction

Cross section
Data (only in 3
columns)



USEFUL LINKS

1. NRV Knowledge base on low energy nuclear physics <http://nrv.jinr.ru/nrv/>
2. National Nuclear Data Center (Brookhaven) <http://www.nndc.bnl.gov/index.jsp>
3. Nuclear Data Services <https://www-nds.iaea.org/>
4. Centre for Photonuclear Experiments Data <http://cdfc.sinp.msu.ru/index.en.html>
5. JAEA Nuclear Data Center <http://wwwndc.jaea.go.jp/>
6. TUNL Nuclear Data Evaluation <http://www.tunl.duke.edu/nucldata/index.shtml>
7. NACRE II: European Compilation of Reactions Rates for Astrophysics
<http://www.astro.ulb.ac.be/nacreii/>

CLASSICAL AND NEW TEXTBOOKS

1. G.R. Satchler, Direct Nuclear Reactions
2. M.L. Goldberger, K.M. Watson, Collision theory
3. P. Froebrich, R. Lindermeier, Theory of Nuclear Reactions
4. N.K. Glendenning, Direct Nuclear reactions
5. R.G. Newton, Scattering theory of waves and particles
6. I.J. Thompson, F.M. Nunes, Nuclear Reactions for Astrophysics
7. K. Langanke, J.A. Maruhn, S.E. Koonin, Computational Nuclear Physics
8. R. Bass, Nuclear Reactions with Heavy Ions
9. H. Schieck, Nuclear Reactions
10. ...