NUCLEAR REACTIONS
Direct nuclear reactions with RIB (part 3)
Andrea Denikin, PhD
JINR, Dubna University
INELASTIC NUCLEAR SCATTERING

Types of inelastic nuclear excitation:
1. Excitation of one-particle-states.
2. Excitation of collective state: surface vibration or/and rotation.
NUCLEAR SHAPE

Nuclear shape parametrization

\[ R(\Theta) = R_0 \left[ 1 + \sum_{\lambda \geq 2, \mu} \beta_{\lambda \mu} Y_{\lambda \mu}(\theta, \phi) \right] = R_0 + \delta R(\vec{\beta}, \Theta) \]

Static deformation:
- multipole electric moment
  \[ Q_\lambda = \frac{3Z R_0^\lambda}{\sqrt{\pi(2\lambda + 1)}} \beta_{\lambda}^{q.s.} \]

Dynamical deformation:
- zero vibration amplitude
  \[ \langle \beta_{\lambda}^0 \rangle = \frac{4\pi}{3Z R_0^\lambda} \left( \frac{B(E\lambda)}{e^2} \right)^{1/2} \]

\( \lambda = 2 \) Quadrupole
\( \beta_2 = 0.3 \)

\( \lambda = 3 \) Octopole
\( \beta_3 = 0.3 \)

\( \lambda = 4 \) Hexadecapole
\( \beta_4 = 0.2 \)

\( \beta_2 = -0.3 \)
\( \beta_3 = -0.3 \)
\( \beta_4 = -0.2 \)
NUCLEAR SHAPE: DEFORMATION

P. Moller et al., 1995
DWBA FOR INELASTIC SCATTERING

Reaction: $a + A \rightarrow a + A^*$

Differential cross section:

$$\frac{d\sigma}{d\Omega}_{fi}(\theta) = \frac{1}{(2\pi)^2} \frac{\mu^2}{\hbar^4} k_f \frac{1}{k_i} |T_{fi}(k_f, k_i)|^2$$

DWBA amplitude:

$$T_{fi}^{DW}(k_f, k_i) \approx \left\langle \Psi^{(-)}_{k_f}(R_f) \varphi_{A_i}^{jm} (r_f) \right| \Delta V \left| \varphi_{A_i}^{jm} (r_i) \Psi^{(+)}_{k_i}(R_i) \right\rangle$$

Nucleus-nucleus interaction potential:

$$U(r) = U_C(r) + U_N(r)$$

$$U_N(r) = \frac{V_0}{1 + \exp\left( -r \sum_i R_i(\beta_i, \theta_i) \right)} = U_0(r) - \delta R \frac{dU}{dr} + \ldots$$

Optical potential

Distortion potential, leading to inelastic excitation
DEFORMED COULOMB POTENTIAL

Charge distribution

\[ U_C(r, \vec{\beta}_2) = Z_p Z_T e^2 \begin{cases} 
\frac{1}{2R_C} \left[ 3 - \frac{r^2}{R_C^2} \right], & r < R_C \\
\frac{1}{r}, & r > R_C 
\end{cases} \]

Coulomb part of the optical potential

\[ U_C(r, \vec{\beta}_2) = Z_p e^2 \int d\Omega_s \int_0^{\beta_{l,m}} s^2 ds \frac{\rho_0^C}{|r-s|} \]

Use partial decomposition of the function

\[ \frac{1}{|r-s|} = \sum_{l,m} \frac{4\pi}{2l+1} \frac{r^l}{r^{l+1}} Y^{*}_{lm}(\Omega_s) Y_{lm}(\Omega) \]

And perform the integration one obtains:

\[ U_C(r, \vec{\beta}_2) = Z_p Z_T e^2 \left\{ \begin{array}{ll}
\frac{1}{2R_C} \left[ 3 - \frac{r^2}{R_C^2} \right], & r < R_C \\
\frac{1}{r}, & r > R_C 
\end{array} \right\} + \frac{3Z_p Z_T e^2}{R_C} \sum_{l,m} \frac{\beta_{l,m}^C}{2l+1} \left\{ \begin{array}{ll}
\frac{r^l}{R_C^{l+1}}, & r < R_C \\
\frac{r^{l+1}}{R_C^l}, & r > R_C 
\end{array} \right\} \]

Coulomb addition to the distortion potential
VIBRATIONAL EXCITATIONS

Vibration: harmonic oscillator

\[
\sum_{\lambda \mu} \left[ \frac{B_\lambda}{2} \beta_{\lambda \mu}^2 + \frac{C_\lambda}{2} \beta_{\lambda \mu}^2 \right] n_{jm} = \sum_{\lambda \mu} \hbar \omega_\lambda \left( n + \frac{1}{2} \right) n_{jm}
\]

Coupling potential:

\[
\Delta V(r, \beta) = \Delta V_C(r, \beta) + \Delta V_N(r, \beta) = \sum_{\lambda \mu} v^C_\lambda(r) + v^N_\lambda(r) \beta_{\lambda \mu} Y_\lambda(\Omega)
\]

Transition matrix element:

\[
T_{ji} \sim \langle n'j'm' | \Delta V | njm \rangle = \sum_{\lambda} C \langle n'j' | \beta_\lambda | nj \rangle (v^C_\lambda(r) + v^N_\lambda(r))
\]

\[
\langle 1j | \beta_\lambda | 00 \rangle = (-)^j \delta_{j\lambda} \sqrt{2j+1} \frac{4\pi}{3ZeR_C^3} \sqrt{B(E\lambda, 0 \rightarrow 1)}, \quad B(E\lambda, 0 \rightarrow 1) = \left( \frac{3}{4\pi} \frac{ZeR_C^3}{B_\lambda \omega_\lambda} \right)^2 \frac{\hbar}{2B_\lambda \omega_\lambda}, \omega_\lambda = \sqrt{\frac{C_\lambda}{B_\lambda}}
\]

where \( B(E\lambda) \) is the reduced electric multipole transition probability, which determines the strength of \( \gamma \)-transitions of multypolarity \( \lambda \) between two nuclear states.

\[
\frac{d\sigma}{d\Omega} (\lambda \rightarrow 0) = \left( \frac{m}{2\pi \hbar^2} \right)^2 \frac{k'}{k} \frac{\beta_\lambda R_0^2}{2\lambda + 1} \sum_{\mu} \left| \int \chi^{(-)^\mu}_{k'}(r) \left( v^C_\lambda(r) - \frac{dU}{dr} \right) Y_\lambda(\Omega) \chi^{(+)}_k(r) \, dr \right|^2
\]
Available information:
- No static deformation
- $B(E3) = 0.263\ e^2b^3$
- $\beta_{03} = 0.114$

Extracted parameters:
- $\beta_{03,\text{Nucl}} = 0.1$
- $\beta_{03,\text{Coul}} = 0.115$
NRV DWBA FOR INELASTIC SCATTERING

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<td>Two-Center Shell Model</td>
<td>Fission</td>
<td>Classical, Semi-classical, Optical Model (Tutorial in Russian)</td>
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<td>Inelastic Scattering:</td>
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<td>Deep inelastic collision</td>
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<td>Inelastic transfer</td>
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Click here!
### Inelastic scattering of nuclear particles (one-step excitation of collective state)

**Model:** DWBA (based on DWUCK4, PKunz)

<table>
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<tr>
<th>Reaction</th>
<th>Sample</th>
<th>Open</th>
<th>Save</th>
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</thead>
<tbody>
<tr>
<td><strong>Projectile</strong></td>
<td>C</td>
<td>12</td>
<td>&lt; &gt;</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td>Nd</td>
<td>144</td>
<td>&lt; &gt;</td>
</tr>
<tr>
<td><strong>Energy</strong></td>
<td>70.4 MeV</td>
<td>lab</td>
<td>cm</td>
</tr>
</tbody>
</table>

**Inelastic Excitation of**
- ground state: J(m) = 0(-) or target
- excited state: J(m) = 3(+)

**Transition Form-Factor**
- taken as derivative of entrance channel OMP
- include or exclude imaginary part

<table>
<thead>
<tr>
<th>Momenta</th>
<th>V0</th>
<th>W0</th>
<th>V0</th>
<th>W0</th>
<th>V0</th>
<th>W0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vol</td>
<td>-17.15</td>
<td>MeV</td>
<td>1.292</td>
<td>fm</td>
<td>a Vol</td>
<td>0.691</td>
</tr>
<tr>
<td>Sur</td>
<td>-12.97</td>
<td>MeV</td>
<td>1.334</td>
<td>fm</td>
<td>a Sur</td>
<td>0.428</td>
</tr>
</tbody>
</table>

**Absorptive potential**
- W.S. Volume

<table>
<thead>
<tr>
<th>Proximity</th>
<th>b Vol</th>
<th>fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vol</td>
<td>1.2</td>
<td>fm</td>
</tr>
<tr>
<td>Sur</td>
<td>1.2</td>
<td>fm</td>
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</tbody>
</table>

**Spin-orbit interaction**

<table>
<thead>
<tr>
<th>Spin</th>
<th>V0</th>
<th>MeV</th>
<th>W0</th>
<th>MeV</th>
<th>r0</th>
<th>fm</th>
<th>a</th>
<th>fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9</td>
<td>MeV</td>
<td>0.0</td>
<td>MeV</td>
<td>0.06</td>
<td>fm</td>
<td>a</td>
<td>0.444</td>
</tr>
<tr>
<td>1/2</td>
<td></td>
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</tbody>
</table>

**OM potential in exit channel**
- the same

<table>
<thead>
<tr>
<th>Proximity</th>
<th>b Vol</th>
<th>fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sur</td>
<td>1.2</td>
<td>fm</td>
</tr>
</tbody>
</table>

***Calculates***

- Initial angle: 20 deg
- Maximal angle: 90 deg
- Step: 0.5 deg

**Integration parameters**

<table>
<thead>
<tr>
<th>Partial waves</th>
<th>L max</th>
<th>R max</th>
<th>Integration step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>130</td>
<td>30.0</td>
<td>0.10 fm</td>
</tr>
</tbody>
</table>

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The 2nd RISP Intensive Program on «Rare Isotope Physics», 2017

Set your reaction
Set properties of excited state
Set exp. data
Set OMP for in-channel
Set OMP for out-channel
Set integration params
The 2nd RISP Intensive Program on «Rare Isotope Physics», 2017

Get your results and process the data

### Entrance channel OMP

<table>
<thead>
<tr>
<th>Coulomb $t_0(R)$, fm</th>
<th>Real part</th>
<th>Imaginary part</th>
</tr>
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<tr>
<td>$V_0$, MeV</td>
<td>$r_0(R)$, fm</td>
<td>$a$, fm</td>
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<tr>
<td>-17.15</td>
<td>1.29 (0.73)</td>
<td>0.69</td>
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### Exit channel OMP

<table>
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<td>1.29 (0.73)</td>
<td>0.69</td>
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### Optical Potentials

### Elastic scattering

### Inelastic scattering

![Graph of Elastic and Inelastic Scattering](image.png)

Transition Form Factor

$$\left( \beta(R) \frac{dV}{dr} \int - \beta(R) \frac{dW}{dr} \int \frac{2Z_1 Z_2 e^2 R_c}{(2l+1) l^{-1}} \right) Y_0(0)$$

- $\beta(R/v_0) = 1.292 \beta(Nd)^{1/3}$
- $\beta(R/v_0) = 1.334 \beta(Nd)^{1/3}$
**ROTATIONAL EXCITATIONS**

Rotation: quantum axially symmetric rotator

Schrodinger equation

\[
\frac{\hbar^2}{2\mathcal{I}} \hat{r}^2 |JM\rangle = \frac{\hbar^2}{2\mathcal{I}} J(J+1) |JM\rangle
\]

Moment of inertia:

\[
\mathcal{I}_{\text{rig}}(^{238}\text{U}) = \int r^2 \rho(r, \theta, \phi) dV = \rho_0 \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\phi \int_0^{\mathcal{I}} r^4 dr \approx \frac{2}{5} m_N r_0^2 A^{5/3} (1 + 0.31 \beta_2) = 0.0138 A^{5/3} (1 + 0.31 \beta_2) \left(\frac{\hbar^2}{\text{MeV}}\right)
\]

\[
\mathcal{I}_{\text{rig}}(^{238}\text{U}) = 148 \left(\frac{\hbar^2}{\text{MeV}}\right)
\]

\[
\mathcal{I}(^{238}\text{U}) = 3 \left(\frac{\hbar^2}{E_{2^+}^{238}\text{U}}}\right) = 66.8 \left(\frac{\hbar^2}{\text{MeV}}\right)
\]

\[
\beta_2(^{238}\text{U}) = 0.24
\]

\[
\mathcal{I}_{LD}(^{238}\text{U}) = \frac{45}{16\pi} \mathcal{I}_{\text{rig}} \beta^2 = 7.6 \left(\frac{\hbar^2}{\text{MeV}}\right)
\]

Nucleus is not a rigid body!

Nucleus does not rotate like irrotational flow!

<table>
<thead>
<tr>
<th>$J\pi$</th>
<th>0+</th>
<th>2+</th>
<th>4+</th>
<th>6+</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_j$</td>
<td>0</td>
<td>6$h^2/2\mathcal{I}$</td>
<td>20$h^2/2\mathcal{I}$</td>
<td>42$h^2/2\mathcal{I}$</td>
</tr>
<tr>
<td>$E_j/E_2$</td>
<td>0</td>
<td>1</td>
<td>3.33</td>
<td>7</td>
</tr>
<tr>
<td>$E_j(^{238}\text{U})$, keV</td>
<td>0</td>
<td>44.9</td>
<td>148.4</td>
<td>307.2</td>
</tr>
<tr>
<td>$E_j/E_2(^{238}\text{U})$</td>
<td>0</td>
<td>1</td>
<td>3.30</td>
<td>6.84</td>
</tr>
</tbody>
</table>
COUPLED CHANNELS

Hamiltonian describes two-nucleus system where one nuclei has internal structure

\[ \hat{H} = \hat{t}_R + \hat{h}_\xi + \hat{V}(R, \xi) \]

\[ \hat{H}\Psi(R, \xi) = E\Psi(R, \xi) \]

\[ \hat{h}_\xi \varphi_i(\xi) = \varepsilon_i \varphi_i(\xi) \]

Here wave functions \( \varphi_i(\xi) \) form the basis

\[ \Psi(R, \xi) = \sum_i \psi_i(R) \varphi_i(\xi) \]

decomposition of the total wave function over the basis \( \varphi_i(\xi) \)

Set of coupled equations:

\[ \hat{t}_R + \varepsilon_i \psi_i(R) + \sum_j \hat{V}_{ij}(R) \psi_j(R) = E\psi_i(R), \quad \psi_i(R) \rightarrow \delta_{i1} e^{ik_1R} + f_{i1}(\theta) \frac{e^{ik_1R}}{R}, \quad i = 1, 2, \ldots \]

\[ \hat{V}_{ij}(R) = \int \varphi_i^*(\xi) \hat{V}(R, \xi) \varphi_j(\xi) d\xi \]

matrix elements that couple different reaction channels

DWBA transition form-factor
EXAMPLE: $^4\text{He} + ^{20}\text{Ne}$ (Coupled channels)

$^{22}\text{Ne}$

P. Moller: $\beta_2 = 0.384$, $\beta_4 = 0.09$

| $V_0$, MeV | 112.72 |
| $r_V$, fm  | 0.789  |
| $a_V$, fm  | 0.76   |
| $W_0$, MeV | 20.83 / 15.7 |
| $r_W$, fm  | 1.09   |
| $a_W$, fm  | 0.56   |
ROTATIONAL EXCITATIONS

For nuclei with odd number of either protons or neutrons two contributions to the total angular momentum $J$ can be distinguished:
- the collective contribution from the rotation of even-even core
- 2 the single-particle contribution from the valence single nucleon.

One takes into account also projection $K$ onto the internal symmetry axis ("strong coupling limit" & Nilsson model).

Number of rotational bands equals to the number of different projection $K$. Both, even and odd spins are allowed within each band.

The smallest value of $J$ for the band is $K$.
The lowest energy state (with spin $J$) is called the band head.

The rotational energies are given by:

$$E = E_0 + \frac{\hbar^2}{2\mathcal{I}_K} J(J + 1) - K^2$$

$^9\text{Be} (3/2^-)$
PRACTICES:

Apply Optical model and Coupled channel approach to describe data for inelastic scattering reaction $d(19.5\text{ MeV}) + ^9\text{Be}$

Elastic and inelastic data:
http://nrv.jinr.ru/denikin/data/d9Be.txt

1. Get optical potential
2. Run computational codes (NRV+ FRESCO)
3. Define OMP for CC
4. Define $^9\text{Be}$ deformation parameter

Rotational band with $K = 3/2$
### COUPLED CHANNELS IN NRV

**Supported by**
Russian Foundation for Basic Research

**Based on FRESCO by I.J. Thompson**

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<td>Two-nucleon transfer</td>
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**Click here!**
Set-up your reaction
Set-up integration parameters
Set-up excited states details

Choose the exp. data
Set-up interaction potential
Prepare the exp. data

Run the calculation
List of predefined and stored data

Navigation buttons

Illustration plot

Set-up your reaction

Cross section Data (only in 3 columns)
USEFUL LINKS

3. Nuclear Data Services [https://www-nds.iaea.org/](https://www-nds.iaea.org/)
5. JAEA Nuclear Data Center [http://wwwndc.jaea.go.jp/](http://wwwndc.jaea.go.jp/)
CLASSICAL AND NEW TEXTBOOKS

1. G.R. Satchler, Direct Nuclear Reactions
3. P. Froebrich, R. Lindermeier, Theory of Nuclear Reactions
4. N.K. Glendenning, Direct Nuclear reactions
5. R.G. Newton, Scattering theory of waves and particles
6. I.J. Thompson, F.M. Nunes, Nuclear Reactions for Astrophysics
7. K. Langanke, J.A. Maruhn, S.E. Koonin, Computational Nuclear Physics
8. R. Bass, Nuclear Reactions with Heavy Ions
9. H. Schieck, Nuclear Reactions
10. ...