Halo-independent determination of the unmodulated WIMP signal in DAMA and other model-independent approaches in the analysis of Dark Matter direct detection data

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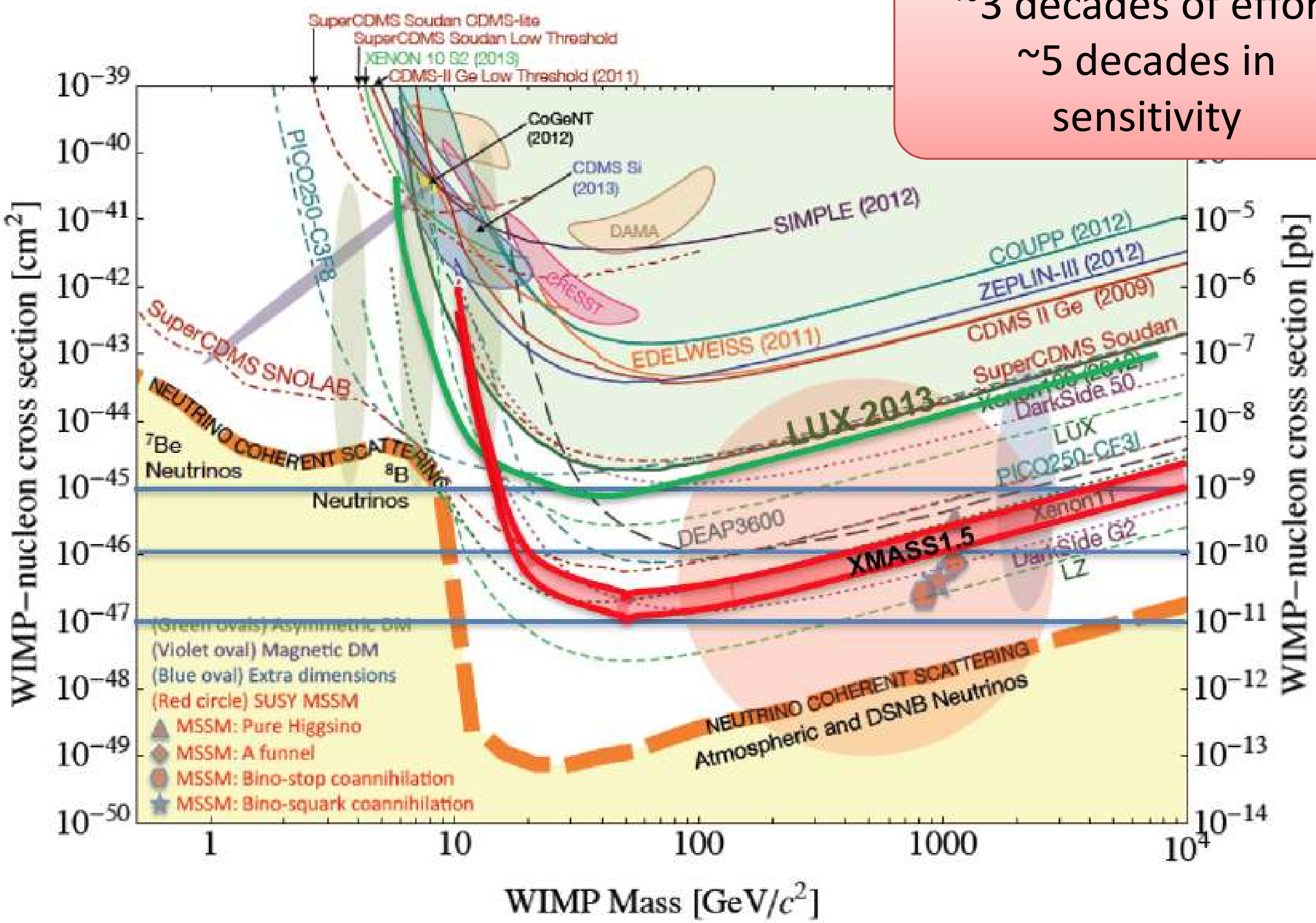
Plan of the talk:

- Introduction
- Sources of uncertainty in direct detection and generalizations
- Are constraints robust? A few counter-examples (DAMA and more)
- Late development: halo-independent determination of modulation fraction
- The path from direct detection to relic abundance in effective modes

WIMP direct searches: spin-independent interaction+Maxwellian distribution

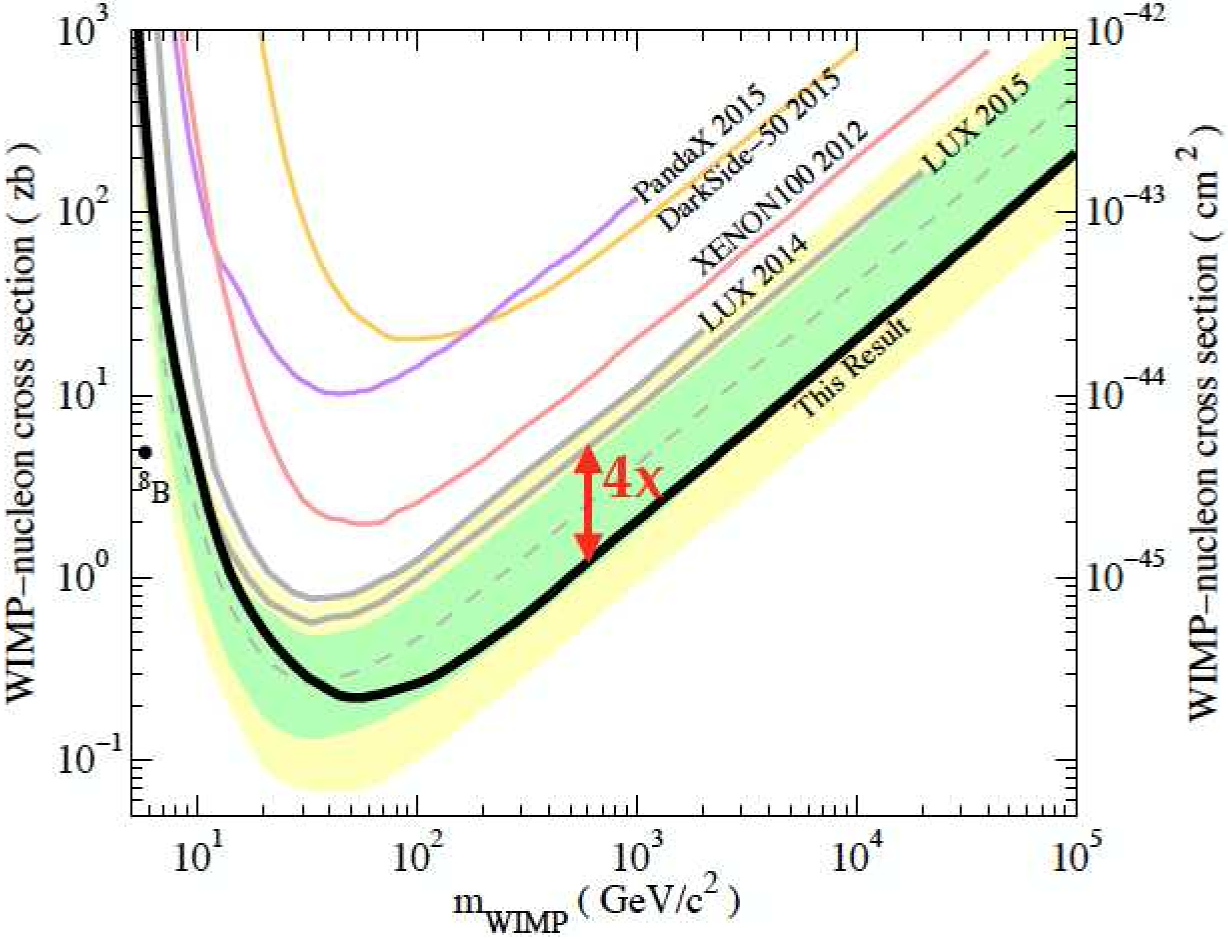
~3 decades of effort
~5 decades in sensitivity



Will the race discover DM before eventually reaching the irreducible background of solar and atmospheric neutrinos???

(from Y. Suzuki talk @IDM 2016, July 2016)

LUX 2016 (332 live days)



(A. Manalaysay, IDM 2016)

N.B.: theoretical predictions for the WIMP direct detection rate depend on two main ingredients:

1) a scaling law for the cross section, in order to compare experiments using different targets

   Traditionally spin-independent cross section (proportional to (atomic mass number)^2) or spin-dependent cross section (proportional to the product $\boldsymbol{S}_{WIMP} \cdot \boldsymbol{S}_{nucleus}$) is assumed

2) a model for the velocity distribution of WIMPs

   Traditionally a Maxwellian distribution is assumed

WIMP differential detection rate

$$\frac{dR}{dE_R} = N_T \frac{\rho_\chi}{m_\chi} \int_{vmin}^{v_{max}} d\vec{v} f(\vec{v}) |\vec{v}| \frac{d\sigma(\vec{v}, E_R)}{dE_R}$$

E_R=nuclear energy
N_T=# of nuclear targets
v=WIMP velocity in the Earth's rest frame

Astrophysics

- ρ_χ=WIMP local density
- f(v)= WIMP velocity distribution function

Particle and nuclear physics

- $\frac{d\sigma(\vec{v}, E_R)}{dE_R}$ =WIMP-nucleus elastic cross section

$$\frac{d\sigma(\vec{v}, E_R)}{dE_R} = \left(\frac{d\sigma(\vec{v}, E_R)}{dE_R}\right)_{\text{coherent}} + \left(\frac{d\sigma(\vec{v}, E_R)}{dE_R}\right)_{\text{spin-dependent}}$$

usually dominates, α (atomic number)^2

N.B.: dependence on galactic model contained in function:

$$\mathcal{I}(v_{min}) \equiv \int_{v_{min}} \frac{f(v)}{v} d^3\vec{v}$$

f(v) usually assumed to be at Maxwellian at rest in the Galactic system (possibility of *corotation* can be also considered):

$$f_G(\vec{v}_G) = \left(\frac{3}{2\pi v_{rms}^2}\right)^{\frac{3}{2}} e^{-\frac{3v_G^2}{2v_{rms}^2}} d^3\vec{v}_G$$

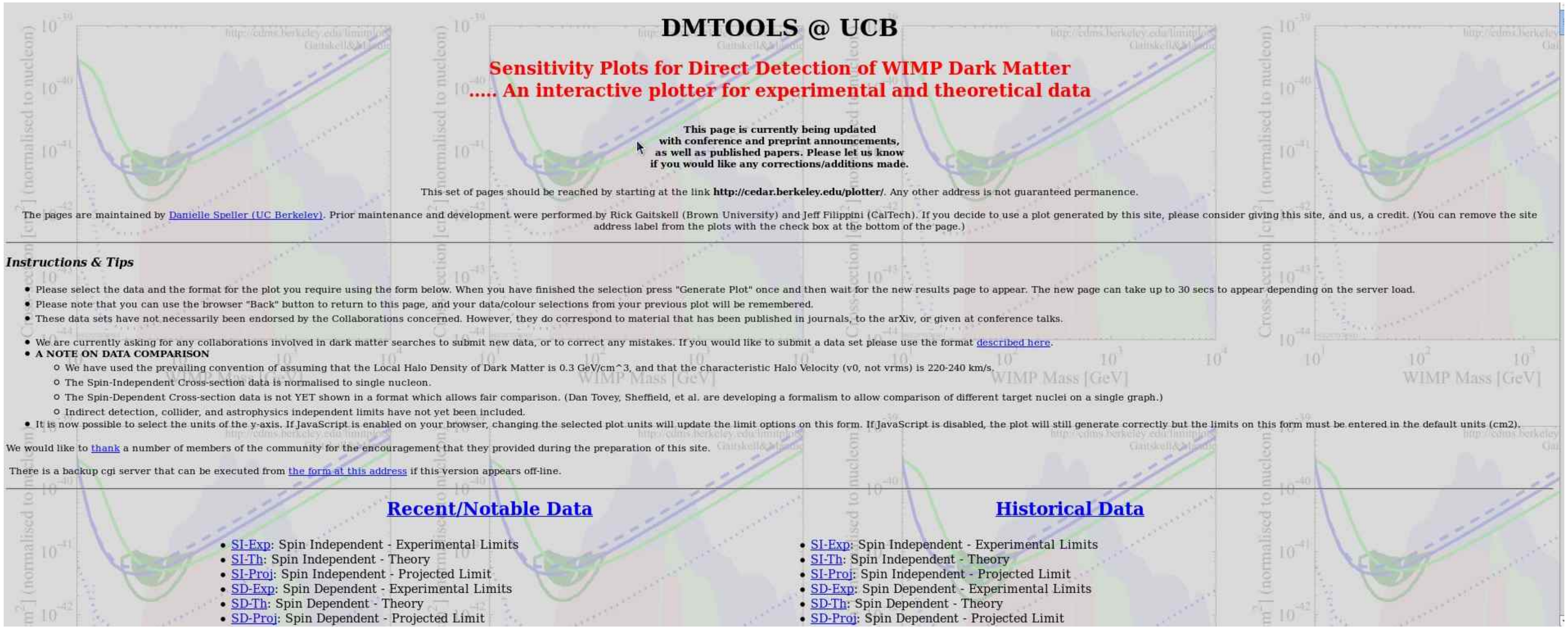
$$\vec{v}_G = \vec{w} + \vec{v}$$

WIMP velocity in Galactic reference frame

Earth velocity in Galactic reference frame

WIMP velocity in Earth reference frame

Getting an updated mass-cross section plot has never been easier!



DMTOOLS @ UCB

Sensitivity Plots for Direct Detection of WIMP Dark Matter
..... An interactive plotter for experimental and theoretical data

This page is currently being updated with conference and preprint announcements, as well as published papers. Please let us know if you would like any corrections/additions made.

This set of pages should be reached by starting at the link **http://cedar.berkeley.edu/plotter/**. Any other address is not guaranteed permanence.

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Instructions & Tips

- Please select the data and the format for the plot you require using the form below. When you have finished the selection press "Generate Plot" once and then wait for the new results page to appear. The new page can take up to 30 secs to appear depending on the server load.
- Please note that you can use the browser "Back" button to return to this page, and your data/colour selections from your previous plot will be remembered.
- These data sets have not necessarily been endorsed by the Collaborations concerned. However, they do correspond to material that has been published in journals, to the arXiv, or given at conference talks.
- We are currently asking for any collaborations involved in dark matter searches to submit new data, or to correct any mistakes. If you would like to submit a data set please use the format described here.
- **A NOTE ON DATA COMPARISON**
  - We have used the prevailing convention of assuming that the Local Halo Density of Dark Matter is 0.3 GeV/cm^3, and that the characteristic Halo Velocity (v0, not vrms) is 220-240 km/s.
  - The Spin-Independent Cross-section data is normalised to single nucleon.
  - The Spin-Dependent Cross-section data is not YET shown in a format which allows fair comparison. (Dan Tovey, Sheffield, et al. are developing a formalism to allow comparison of different target nuclei on a single graph.)
  - Indirect detection, collider, and astrophysics independent limits have not yet been included.
- It is now possible to select the units of the y-axis. If JavaScript is enabled on your browser, changing the selected plot units will update the limit options on this form. If JavaScript is disabled, the plot will still generate correctly but the limits on this form must be entered in the default units (cm2).

We would like to thank a number of members of the community for the encouragement that they provided during the preparation of this site.

There is a backup cgi server that can be executed from the form at this address if this version appears off-line.

Recent/Notable Data

- SI-Exp: Spin Independent - Experimental Limits
- SI-Th: Spin Independent - Theory
- SI-Proj: Spin Independent - Projected Limit
- SD-Exp: Spin Dependent - Experimental Limits
- SD-Th: Spin Dependent - Theory
- SD-Proj: Spin Dependent - Projected Limit

Historical Data

- SI-Exp: Spin Independent - Experimental Limits
- SI-Th: Spin Independent - Theory
- SI-Proj: Spin Independent - Projected Limit
- SD-Exp: Spin Dependent - Experimental Limits
- SD-Th: Spin Dependent - Theory
- SD-Proj: Spin Dependent - Projected Limit

(http://cedar.berkeley.edu/plotter/)

…at least for the most common assumptions: spin-independent, spin-dependent interaction+ Maxwellian

Indeed, spin-independent and spin-dependent cross sections are predicted for the neutralino in supersymmetry and numerical simulations of galaxy formation support the choice of a Maxwellian for the velocity distributions.

However a bottom-up approach would also be desireable,especially if no hints come from high-energy physics about the fundamental properties of the WIMP particle. Indeed two questions arise:

- what is the most general class of scaling laws for a WIMP-nucleus cross section?
- the detailed merger history of the Milky Way is not known, allowing for the possibility of the presence of sizeable non–thermal components for which the density, direction and speed of WIMPs are hard to predict, *especially in the high velocity tail of the distribution*: do we need to assume a Maxwellian velocity distribution?

Recently both aspects have been addressed

Compatibility among different experiments (ex. DAMA/Libra vs. CoGeNT) can be verified without assuming any model for the halo

Write expected WIMP rate as:

$$\frac{dR}{dE_R} = \frac{\rho_\chi \sigma_n}{2m_\chi \mu_{n\chi}^2} \frac{C_T}{f_n^2} F^2(E_R)\epsilon(E_R)g(v_{\min}, t)$$

$F^2(E_R)$ is the form factor, and the function:

$$g(v_{\min}, t) = \int_{v_{\min}}^{\infty} \frac{f_{\text{local}}(\vec{v}, t)}{v} d^3v$$

contains all the dependence on the halo model with:

$$v_{\min} = \sqrt{\frac{m_N E_R}{2\mu^2}}$$

So there is a one-to-one correspondence between the recoil energy E_R and v_{min}

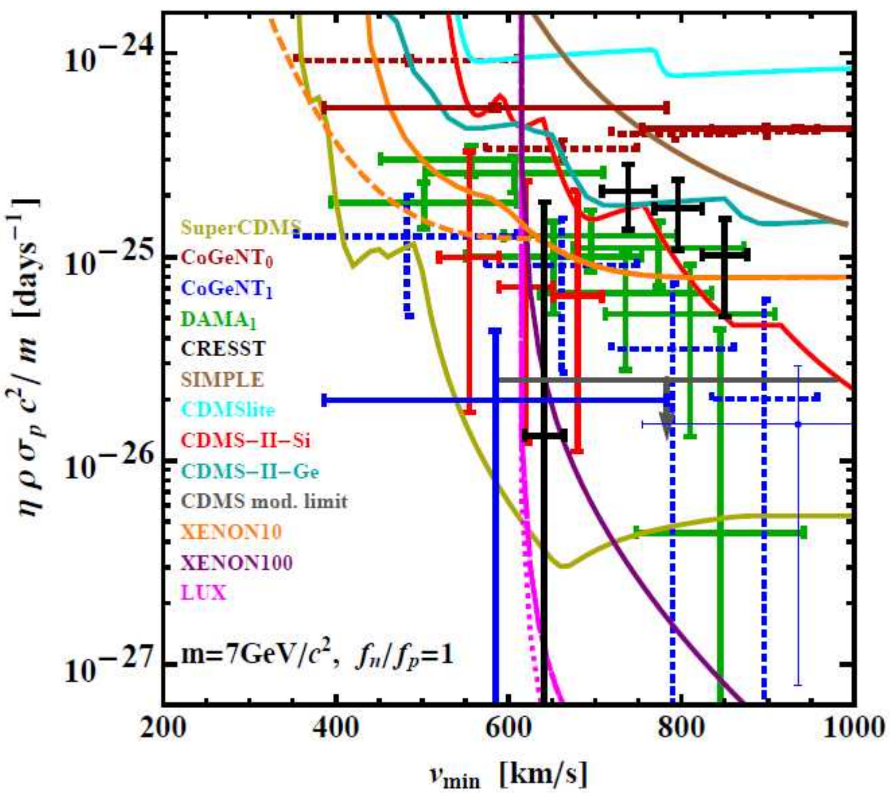
→ map the event rate expected in different experiments into the same intervals in v_{min} (P.J. Fox, J. Liu, N. Weiner, PRD83,103514 (2011))

In this way the dependence on the galactic model cancels out in the ratio of the expected count rates of the two experiments because <u>they depend on the same integrals of $f_{local}(v)$</u>

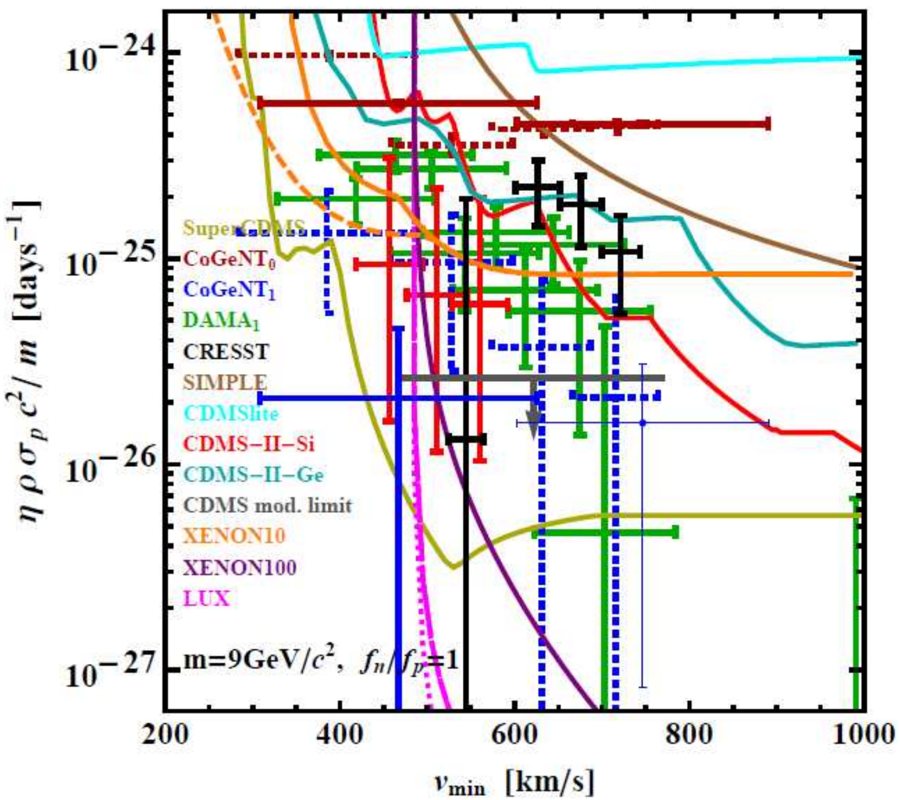
halo-independent analysis for elastic scattering

Del Nobile, Gelmini, Gondolo, Huh, arXiv:1405.5582

m_{WIMP}=7 GeV



m_{WIMP}=9 GeV



$$R^{\rm SI}_{[E'_1,E'_2]}(t) = \int_0^\infty {\rm d}v_{\rm min}\, \tilde\eta(v_{\rm min},t)\, \mathcal{R}^{\rm SI}_{[E'_1,E'_2]}(v_{\rm min})$$

$$\tilde\eta(v_{\rm min},t) \equiv \frac{\rho\sigma_p}{m}\int_{v\geqslant v_{\rm min}} {\rm d}^3v\, \frac{f(\mathbf{v},t)}{v}$$

$$\tilde\eta(v_{\rm min},t) \simeq \tilde\eta^0(v_{\rm min}) + \tilde\eta^1(v_{\rm min})\cos[\omega(t-t_0)]$$

N.B. : only halo dependence factorized. Results depend on assumptions on other quantities such as quenching factors, L_{eff}, Q_y etc.

$$\tilde{\eta}(v_{min}, t) \equiv \frac{\rho}{m_{WIMP}} \sigma_0 \eta(v_{min}, t)$$

- Annual modulation

Experimental data fits (DAMA, CoGeNT, KIMS) assume a sinusoidal behaviour:

$$\tilde{\eta}(v_{\rm min}, t) \simeq \tilde{\eta}^0(v_{\rm min}) + \tilde{\eta}^1(v_{\rm min}) \cos[\omega(t - t_0)]$$

The usual "halo-independent" approach to analyze yearly modulation data: factorize a modulated halo function $\tilde{\eta}_1$ with the only constraint $\tilde{\eta}_1 < \tilde{\eta}_0$.

(In the case of a Maxwellian typically $\tilde{\eta}_1 / \tilde{\eta}_0 \leq 0.07$)

Standard lore: cannot predict $\tilde{\eta}_1 / \tilde{\eta}_0$ without a model for the velocity distribution. Is it really so? More on that later

Summarizing, the minimal requirements for halo functions $\eta_{0,1}$ are:

$$\tilde{\eta}_0(v_{\min,2}) \leq \tilde{\eta}_0(v_{\min,1}) \quad \text{if } v_{\min,2} > v_{\min,1}$$ (decreasing function)

$$\tilde{\eta}_1 \leq \tilde{\eta}_0 \quad \text{at the same } v_{\min}$$ (modulated part<100%)

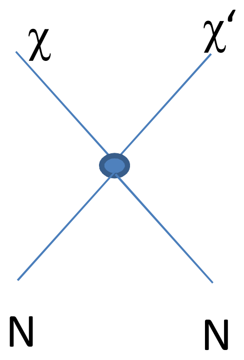
$$\tilde{\eta}_0(v_{\min} \geq v_{\rm esc}) = 0.$$ (no bound WIMPs<escape velocity)

Inelastic Dark Matter

D. Tucker-Smith and N.Weiner, Phys.Rev.D 64, 043502 (2001), hep-ph/0101138

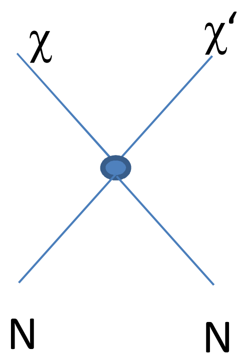
Two mass eigenstates χ and χ' very close in mass: $m_\chi - m_{\chi'} \equiv \delta$ with $\chi + N \to \chi + N$ forbidden

"Endothermic "scattering ($\delta>0$)



Kinetic energy needed to "overcome" step → rate no longer exponentially decaying with energy, maximum at finite energy E_*

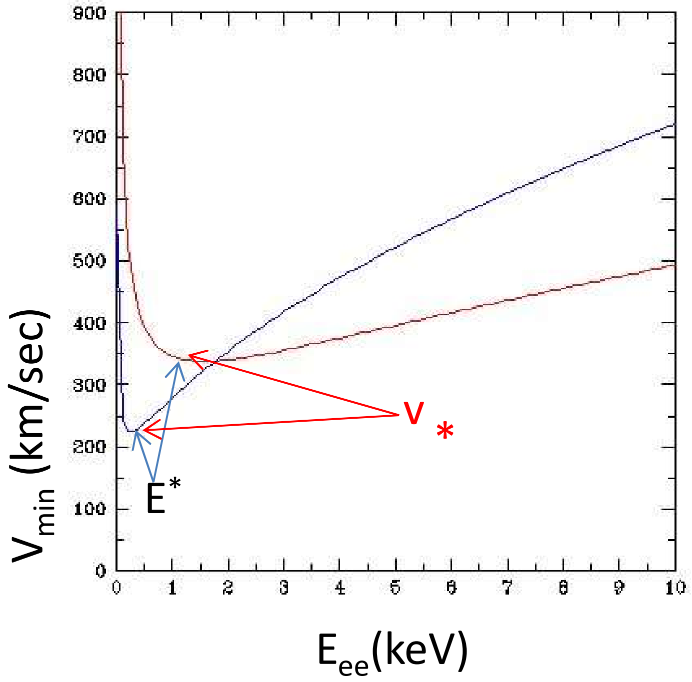
"Exothermic" scattering ($\delta<0$)



χ is metastable, δ energy deposited independently on initial kinetic energy (even for WIMPs at rest)

Inelastic DM and the halo-independent approach: recoil energy E_{ee} is no longer monotonically growing with v_{min} (energy E^* corresponds to minimal v_{min})

$$v_{min} = \frac{1}{\sqrt{2 m_N E_R}}\left(\frac{m_N E_R}{\mu} + \delta\right) = a\sqrt{E_r} + \frac{b}{\sqrt{E_R}}$$



N.B. for δ>0 WIMPs need a minimal absolute incoming speed v_* to upscatter to the heavier state → vanishing rate if $v_* > v_{esc}$ (escape velocity)

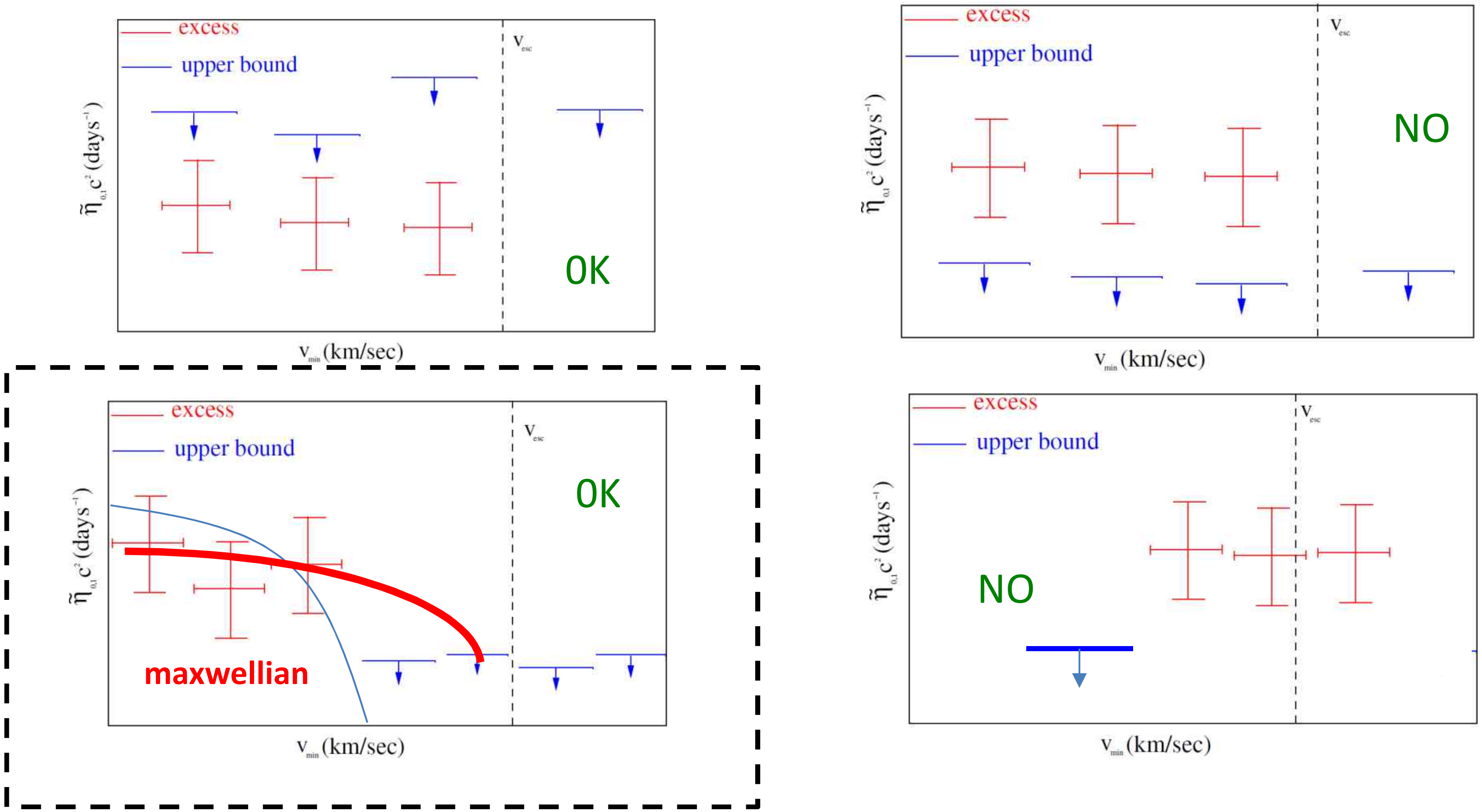
Need to rebin the data in such a way that the relation between v_{min} and E_R is invertible in each bin (easy: just ensure that for all target nuclei E^* corresponds to one of the bin boundaries)

S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)

comparison among different experiments for Inelastic DM

if conflicting experimental results can be mapped into non-overlapping ranges of v_{min} and if the v_{min} range of the constraint is at higher values compared to the excess (while that of the signal remains below v_{esc}) the tension between the two results can be eliminated by an appropriate choice of the $\eta_{0,1}$ functions

Four cases:



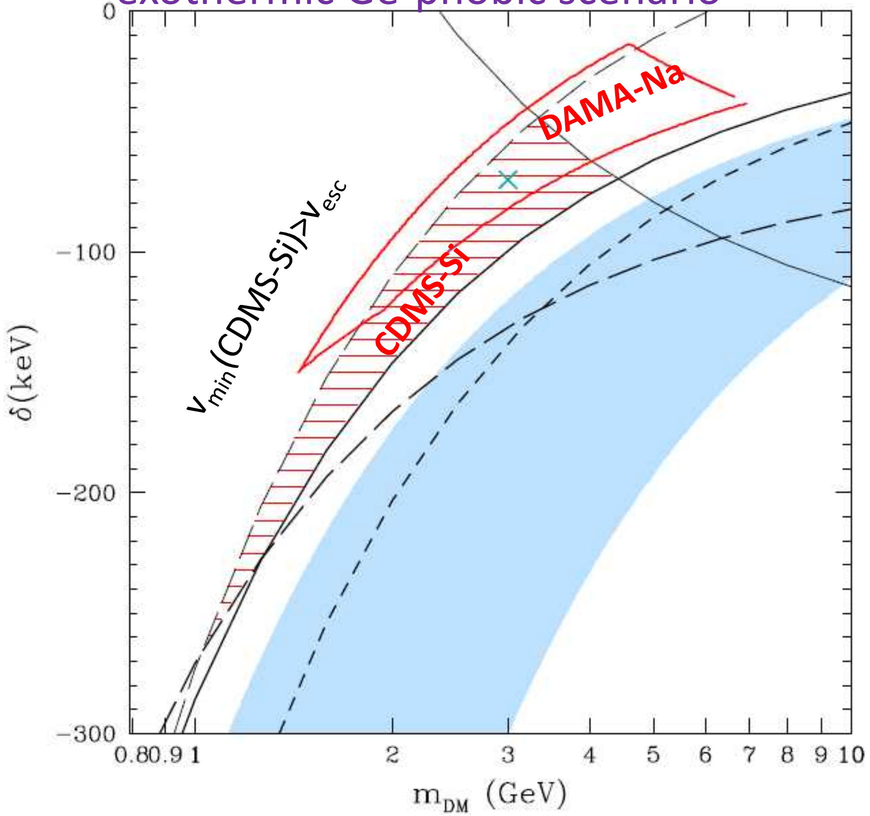
N.B: the effect of inelastic scattering (δ≠0) only implies a "horizontal shift" of η estimations (up to negligible effects) → pick appropriate m_{DM}, δ combination to shift-away the bounds without shifting away the signal!

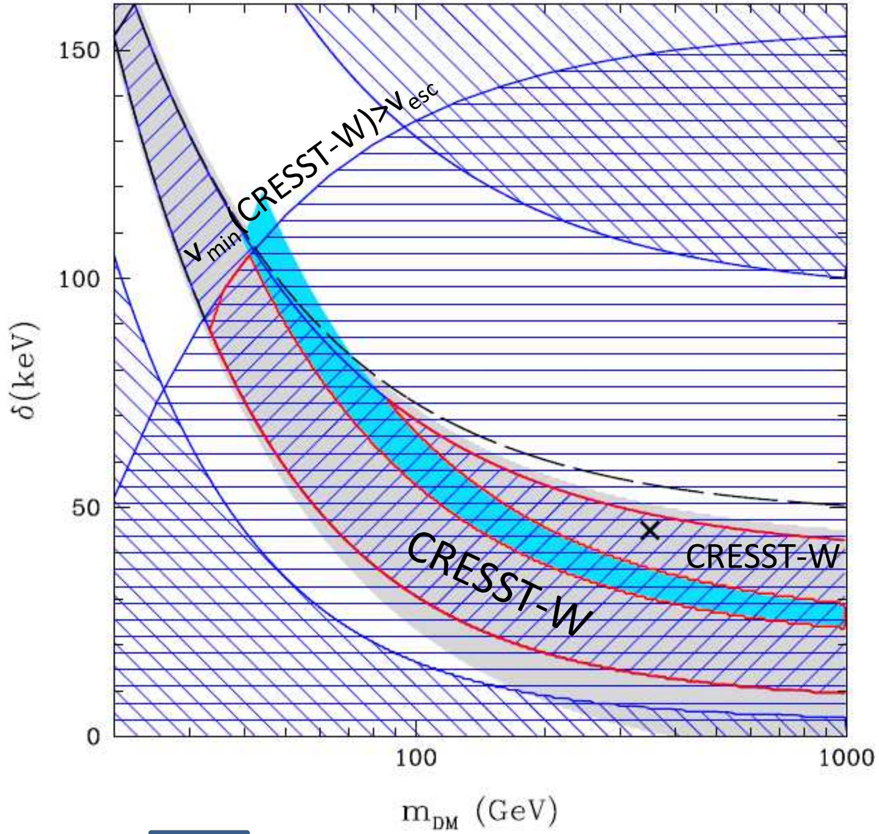
S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)

Halo-independent analysis of inelastic Dark Matter

Kinematic conditions for v_{min}(bounds)>v_{min}(signals) and v_{min}(signals)<v_{esc}

"exothermic Ge-phobic scenario"



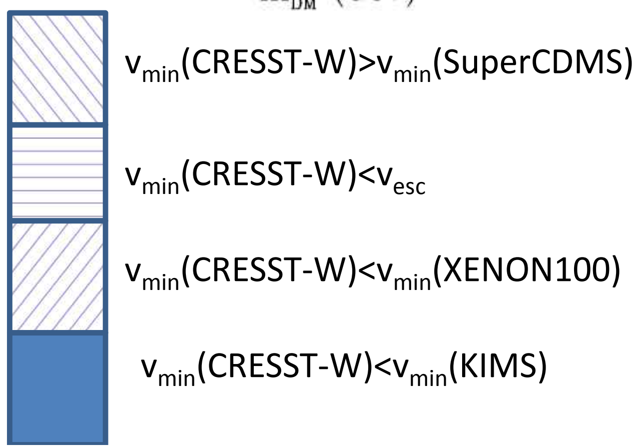


N.B. only kinematics involved (valid for different scaling laws)

At higher masses <u>upper bound</u> of ROI is constraining

In LUX, XENON100→XENON100 more constraining than LUX due to <u>lower</u> light yield

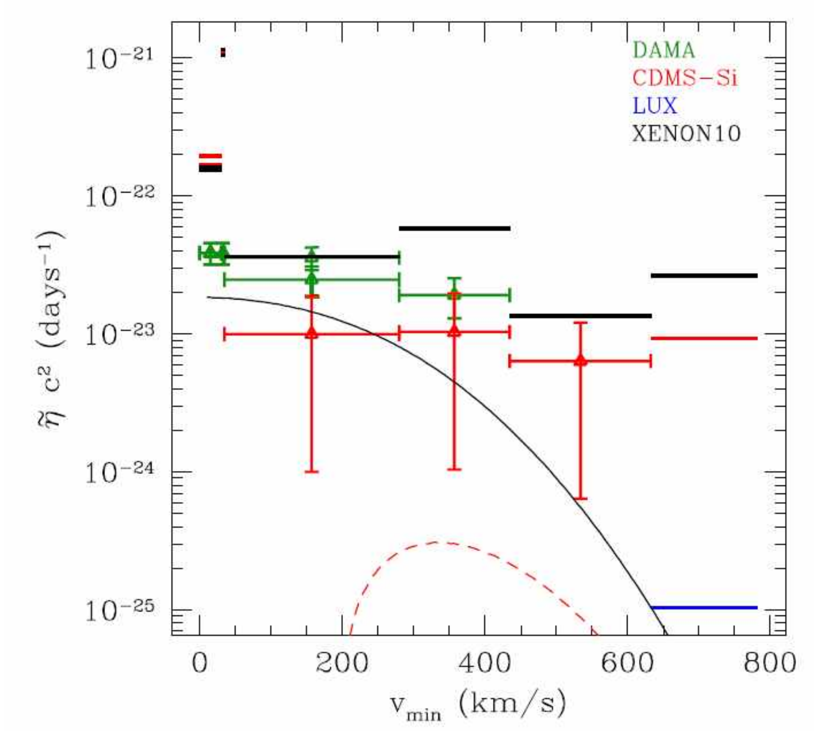
S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)



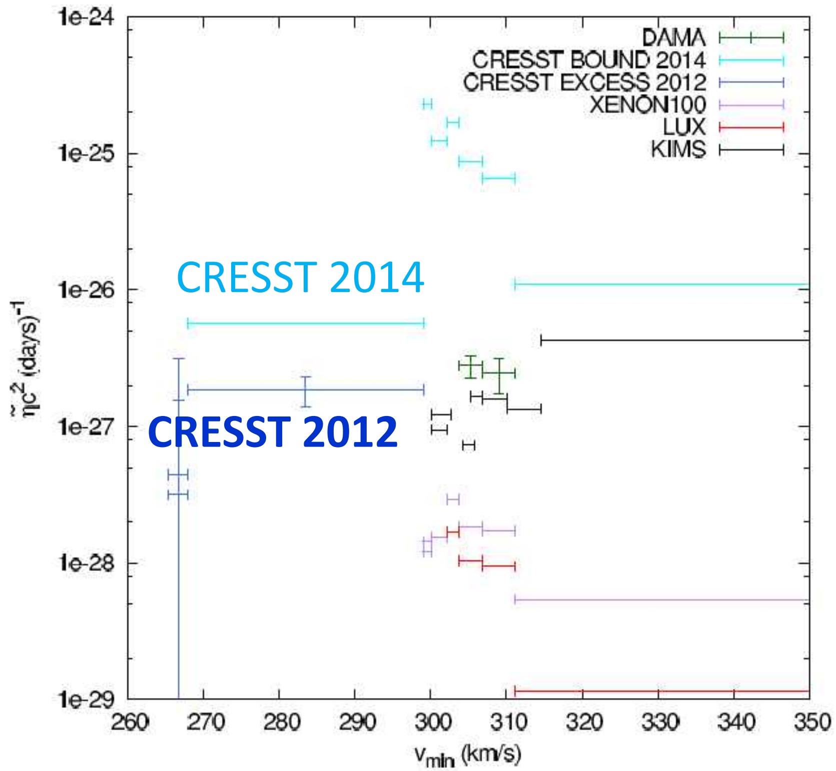
Halo-independent analysis of inelastic Dark Matter

"Agnostic" approach about velocity integral: a constraint does not affect values of v_{min} below its covered range, i.e. if $v_{min}(bound) > v_{min}(signal)$

m_{DM} =3 GeV, δ=-70 keV, f_n/f_p =-0.79



m_{DM} =350 GeV, δ=45 keV, f_n/f_p =1



- DAMA and CDMS-Si can be separately OK with bounds, but are always in tension between themselves
- Assuming standard Maxwellian more tension arises
- high-mass CRESST solution not affected by recent reanalysis due to low statistics

S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)

isospin violation (more properly: isovector interaction)

$$R = \sigma_p \sum_i \eta_i \frac{\mu_{A_i}^2}{\mu_p^2} I_{A_i} \left[Z + (A_i - Z) f_n / f_p \right]^2$$

(spin-independent cross section, same for other interactions)

sum over isotopes

Cancellation between f_p (WIMP-proton coupling) and f_n (WIMP-nucleon coupling) when $f_n/f_p \sim -Z/(A-Z) \rightarrow$ can suppress the scattering cross section ona specific target (i.e. $f_n/f_p \sim -0.79$ for Germanium)

Minimal "degrading factors", i.e. maximal factors by which the reciprocal scaling law between two elements can be reduced (limited by multiple isotopes, one choice of f_n/f_p ratio cannot fit all)

Element	Xe	Ge	Si	Ca	W	Ne	C
Xe (54, *)	1.00	8.79	149.55	138.21	10.91	34.31	387.66
Ge (32, *)	22.43	1.00	68.35	63.14	130.45	15.53	176.47
Si (14, *)	172.27	30.77	1.00	1.06	757.44	1.06	2.67
Ca (20, *)	173.60	31.53	1.17	1.00	782.49	1.10	2.81
W (74, *)	2.98	13.88	177.46	166.15	1.00	41.64	466.75
Ne (10, *)	163.65	28.91	4.39	4.09	726.09	1.00	11.52
C (6, *)	176.35	32.13	1.07	1.02	789.59	1.12	1.00
I (53, 127)	1.94	5.51	127.04	118.35	20.68	28.92	326.95
Cs (55, 133)	1.16	7.15	139.65	127.61	12.32	31.88	355.27
O (8, 16)	178.49	32.13	1.08	1.03	789.90	1.13	1.01
Na (11, 23)	101.68	13.77	8.45	8.33	481.03	2.27	22.68
Ar (18, 36)	178.49	32.13	1.08	1.03	789.90	1.13	1.01
F (9, 19)	89.39	10.88	12.44	11.90	425.93	3.05	33.47

(J.L.Feng, J.Kumar, D.Marfatia and D.Sanford, Phys.Lett.B703, 124 (2011), 1102.4331)

On the most general WIMP-nucleus cross section
(i.e. beyond “spin-dependent” and “spin”independent”)

Most general approach: consider ALL possible NR couplings, including those depending on velocity and momentum

$$\mathcal{H} = \sum_i \left(c_i^0 + c_i^1 \tau_3\right) \mathcal{O}_i$$

τ_3=nuclear isospin operator, i.e.

$$c_i^{\rm p} = (c_i^0 + c_i^1)/2 \quad \text{(proton)}$$

$$c_i^{\rm n} = (c_i^0 - c_i^1)/2 \quad \text{(neutron)}$$

(if $c_i^p = c_i^n \rightarrow c_i^1=0$)

N.R. operators O_i guaranteed to be Hermitian if built out of the following four 3-vectors:

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^\perp, \quad \vec{S}_\chi, \quad \vec{S}_N$$

with:

$$\left.\begin{aligned} \vec{v}^\perp &= \vec{v} + \frac{\vec{q}}{2\mu_N} \\ \vec{v} &\equiv \vec{v}_{\chi,\rm in} - \vec{v}_{N,\rm in} \end{aligned}\right\} \Rightarrow \vec{v}^\perp \cdot \vec{q} = 0$$

$$\mathcal{O}_1 = 1_\chi 1_N,$$
$$\mathcal{O}_2 = (v^\perp)^2,$$
$$\mathcal{O}_3 = i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right),$$
$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N,$$
$$\mathcal{O}_5 = i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right),$$
$$\mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right)$$
$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp,$$
$$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp,$$
$$\mathcal{O}_9 = i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right),$$
$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N},$$
$$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}.$$

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

Additional operators that do not arise for traditional spin≤1 mediators:

$$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp),$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right),$$

$$\mathcal{O}_{14} = i\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)(\vec{S}_N \cdot \vec{v}^\perp),$$

$$\mathcal{O}_{15} = -\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left[(\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right]$$

$$\mathcal{O}_{16} = -\left[(\vec{S}_\chi \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right]\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right)$$

Connection to relativistic effective theory:

j	$\mathcal{L}^j_{\rm int}$	Nonrelativistic reduction	$\sum_i c_i \mathcal{O}_i$	P/T
1	$\bar\chi\chi\bar N N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
2	$i\bar\chi\chi\bar N\gamma^5 N$	$i\frac{\vec q}{m_N}\cdot\vec S_N$	$\mathcal{O}_{10}$	O/O
3	$i\bar\chi\gamma^5\chi\bar N N$	$-i\frac{\vec q}{m_\chi}\cdot\vec S_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$	O/O
4	$\bar\chi\gamma^5\chi\bar N\gamma^5 N$	$-\frac{\vec q}{m_\chi}\cdot\vec S_\chi\frac{\vec q}{m_N}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$	E/E
5	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
6	$\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q^{\,2}}{2m_N m_M}1_\chi 1_N + 2(\frac{\vec q}{m_\chi}\times\vec S_\chi + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$\frac{\vec q^{\,2}}{2m_N m_M}\mathcal{O}_1 - 2\frac{m_N}{m_M}\mathcal{O}_3 + 2\frac{m_N^2}{m_M m_\chi}(\frac{\vec q^{\,2}}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
7	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu\gamma^5 N$	$-2\vec S_N\cdot\vec v^\perp + \frac{2}{m_\chi}i\vec S_\chi\cdot(\vec S_N\times\vec q)$	$-2\mathcal{O}_7 + 2\frac{m_N}{m_\chi}\mathcal{O}_9$	O/E
8	$i\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec q}{m_M}\cdot\vec S_N$	$2\frac{m_N}{m_M}\mathcal{O}_{10}$	O/O
9	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma_\mu N$	$-\frac{\vec q^{\,2}}{2m_\chi m_M}1_\chi 1_N - 2(\frac{\vec q}{m_N}\times\vec S_N + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)$	$-\frac{\vec q^{\,2}}{2m_\chi m_M}\mathcal{O}_1 + \frac{2m_N}{m_M}\mathcal{O}_5 - 2\frac{m_N}{m_M}(\frac{\vec q^{\,2}}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
10	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$4(\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_4 - \frac{m_N^2}{m_M^2}\mathcal{O}_6)$	E/E
11	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma^\mu\gamma^5 N$	$4i(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
12	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-[i\frac{\vec q^{\,2}}{m_\chi m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)]\frac{\vec q}{m_M}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_{10} - 4\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_{12} - 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
13	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma_\mu N$	$2\vec v^\perp\cdot\vec S_\chi + 2i\vec S_\chi\cdot(\vec S_N\times\frac{\vec q}{m_N})$	$2\mathcal{O}_8 + 2\mathcal{O}_9$	O/E
14	$\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4i\vec S_\chi\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
15	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma^\mu\gamma^5 N$	$-4\vec S_\chi\cdot\vec S_N$	$-4\mathcal{O}_4$	E/E
16	$i\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i\vec v^\perp\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_{13}$	E/O
17	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu N$	$2i\frac{\vec q}{m_M}\cdot\vec S_\chi$	$2\frac{m_N}{m_M}\mathcal{O}_{11}$	O/O
18	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q}{m_M}\cdot\vec S_\chi[i\frac{\vec q^{\,2}}{m_N m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_N)]$	$\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
19	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu\gamma^5 N$	$-4i\frac{\vec q}{m_M}\cdot\vec S_\chi\vec v_\perp\cdot\vec S_N$	$-4\frac{m_N}{m_M}\mathcal{O}_{14}$	E/O
20	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4\frac{\vec q}{m_M}\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N^2}{m_M^2}\mathcal{O}_6$	E/E

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In the expected rate WIMP physics (encoded in the R functions that depend on the c_i couplings) and the nuclear physics (contained in 8 (6+2) response functions W factorize in a simple way:

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}E_R} = \sum_T \frac{\mathrm{d}\mathcal{R}_T}{\mathrm{d}E_R} \equiv \sum_T \xi_T \frac{\rho_\chi}{2\pi m_\chi} \int_{v>v_{\mathrm{min}}(q)} \frac{f(\vec{v}+\vec{v}_e(t))}{v} P_{\mathrm{tot}}(v^2,q^2)\, d^3v$$

$$\begin{aligned}
P_{\mathrm{tot}}(v^2,q^2) &= \frac{4\pi}{2j_N+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \Bigg[R_M^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_M^{\tau\tau'}(y) \\
&+ R_{\Sigma''}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Sigma''}^{\tau\tau'}(y) + R_{\Sigma'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Sigma'}^{\tau\tau'}(y) \Bigg] \\
&+ \frac{q^2}{m_N^2} \Bigg[R_{\Phi''}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Phi''}^{\tau\tau'}(y) + R_{\Phi'' M}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Phi'' M}^{\tau\tau'}(y) \\
&+ R_{\tilde{\Phi}'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\tilde{\Phi}'}^{\tau\tau'}(y) + R_{\Delta}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Delta}^{\tau\tau'}(y) \\
&+ R_{\Delta\Sigma'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Delta\Sigma'}^{\tau\tau'}(y) \Bigg] \Bigg\},
\end{aligned}$$

N.B.: besides usual spin-independent and spin-dependent terms new contributions arise, with explicit dependences on the transferred momentum q and the WIMP incoming velocity

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
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WIMPs response funtions

$$R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right]$$

$$R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2}$$

$$R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2}$$

$$R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2}$$

$$R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right]$$

$$R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right]$$

$$R_{\Delta}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2}$$

$$R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}.$$

general form:

$$R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'}\frac{(v_T^{\perp})^2}{c^2} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'}\frac{v_T^2 - v_{min}^2}{c^2}$$

Nuclear response functions

Assuming one-body dark matter-nucleon interactions, the Hamiltonian density for dark matter-nucleus interactions is:

$$\begin{aligned}
\mathcal{H}_{ET}(\vec{x}) &= \sum_{i=1}^{A} l_0(i)\ \delta(\vec{x}-\vec{x}_i) + \sum_{i=1}^{A} l_0^A(i)\ \frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i \cdot \vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i) + \delta(\vec{x}-\vec{x}_i)\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right] \\
&+ \sum_{i=1}^{A} \vec{l}_5(i)\cdot\vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i) + \sum_{i=1}^{A} \vec{l}_M(i)\cdot\frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i\delta(\vec{x}-\vec{x}_i) + \delta(\vec{x}-\vec{x}_i)\frac{1}{i}\overrightarrow{\nabla}_i\right] \\
&+ \sum_{i=1}^{A} \vec{l}_E(i)\cdot\frac{1}{2M}\left[\overleftarrow{\nabla}_i \times \vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i) + \delta(\vec{x}-\vec{x}_i)\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right]
\end{aligned}$$

So the WIMP-nucleus Hamiltonian has the general form:

$$\int d\vec{x}\ e^{-i\vec{q}\cdot\vec{x}}\ \left[l_0\langle J_iM_i|\hat{\rho}(\vec{x})|J_iM_i\rangle - \vec{l}\cdot\langle J_iM_i|\hat{\vec{j}}(\vec{x})|J_iM_i\rangle\right]$$

With:

$$e^{i\vec{q}\cdot\vec{x}_i} = \sum_{J=0}^{\infty}\sqrt{4\pi}\ [J]\ i^J j_J(qx_i)Y_{J0}(\Omega_{x_i})$$

$$\hat{e}_\lambda e^{i\vec{q}\cdot\vec{x}_i} = \begin{cases} \displaystyle\sum_{J=0}^{\infty}\sqrt{4\pi}\ [J]\ i^{J-1}\frac{\vec{\nabla}_i}{q}j_J(qx_i)Y_{J0}(\Omega_{x_i}), & \lambda = 0 \\ \displaystyle\sum_{J\geq 1}^{\infty}\sqrt{2\pi}\ [J]\ i^{J-2}\left[\lambda j_J(qx_i)\vec{Y}_{JJ1}^{\lambda}(\Omega_{x_i}) + \frac{\vec{\nabla}_i}{q}\times j_J(qx_i)\vec{Y}_{JJ1}^{\lambda}(\Omega_{x_i})\right], & \lambda = \pm 1 \end{cases}$$

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which depends on the expectations of six distinct nuclear response functions, defined as:

$$
\begin{aligned}
&M_{JM}(q\vec{x}) \\
&\Delta_{JM}(q\vec{x}) \equiv \vec{M}^M_{JJ}(q\vec{x}) \cdot \frac{1}{q}\vec{\nabla} \\
&\Sigma'_{JM}(q\vec{x}) \equiv -i\left\{\frac{1}{q}\vec{\nabla} \times \vec{M}^M_{JJ}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{-\sqrt{J}\,\vec{M}^M_{JJ+1}(q\vec{x}) + \sqrt{J+1}\,\vec{M}^M_{JJ-1}(q\vec{x})\right\} \cdot \vec{\sigma} \\
&\Sigma''_{JM}(q\vec{x}) \equiv \left\{\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{\sqrt{J+1}\,\vec{M}^M_{JJ+1}(q\vec{x}) + \sqrt{J}\,\vec{M}^M_{JJ-1}(q\vec{x})\right\} \cdot \vec{\sigma} \\
&\tilde{\Phi}'_{JM}(q\vec{x}) \equiv \left(\frac{1}{q}\vec{\nabla} \times \vec{M}^M_{JJ}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right) + \frac{1}{2}\vec{M}^M_{JJ}(q\vec{x}) \cdot \vec{\sigma} \\
&\Phi''_{JM}(q\vec{x}) \equiv i\left(\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right)
\end{aligned}
$$

with $M_{JM} = j_J Y_{JM}$ Bessel spherical harmonics and $\vec{M}^M_{JL} = j_J \vec{Y}_{JM}$ vector spherical harmonics.

•**M**= vector-charge (scalar, **usual spin-independent part**, non-vanishing for all nuclei)

• **Φ''**=vector-longitudinal, related to spin-orpit coupling σ·l (also spin-independent, non-vanishing for all nuclei)

•**Σ'** and **Σ''** = associated to longitudinal and transverse components of nuclear spin, **their sum is the usual spin-dependent interaction**, require nuclear spin j>0

•**Δ**=associated to the orbital angular momentum operator l, also requires j>0

•**Φ̃'**= related to a vector-longitudinal operator that transforms as a tensor under rotations, requires j>1/2

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Squaring the ampitude get the following nuclear response functions:

$$W_O^{\tau\tau'}(y) \equiv \sum_{J=0,2,\ldots}^{\infty} \langle j_N || O_{J;\tau}(q) || j_N \rangle \langle j_N || O_{J;\tau'}(q) || j_N \rangle \text{ for } O = M, \Phi'',$$

$$W_O^{\tau\tau'}(y) \equiv \sum_{J=1,3,\ldots}^{\infty} \langle j_N || O_{J;\tau}(q) || j_N \rangle \langle j_N || O_{J;\tau'}(q) || j_N \rangle \text{ for } O = \Sigma'', \Sigma', \Delta,$$

$$W_{\tilde{\Phi}'}^{\tau\tau'}(y) = \sum_{J=2,4,\ldots}^{\infty} \langle j_N || \tilde{\Phi}'_{J;\tau}(q) || j_N \rangle \langle j_N || \tilde{\Phi}'_{J;\tau'}(q) || j_N \rangle,$$

$$W_{\Phi'' M}^{\tau\tau'}(y) = \sum_{J=0,2,\ldots}^{\infty} \langle j_N || \Phi''_{J;\tau}(q) || j_N \rangle \langle j_N || M_{J;\tau'}(q) || j_N \rangle,$$

(interference terms)

$$W_{\Delta\Sigma'}^{\tau\tau'}(y) = \sum_{J=1,3,\ldots}^{\infty} \langle j_N || \Delta_{J;\tau}(q) || j_N \rangle \langle j_N || \Sigma'_{J;\tau'}(q) || j_N \rangle.$$

These 8 (6+2 interferences) W nuclear response functions have been calculated for most nuclei using a numerical (truncated) harmonic potential shell model (Fitzpatrick et al., JCAP 1302 1302(2013), Catena and Schwabe, JCAP 1504 no. 04, 042 (2015)) with oscillator parameter:

$$b[\text{fm}] = \sqrt{41.467/(45A^{-1/3} - 25A^{-2/3})} \qquad y = (qb/2)^2$$

One of the most popular scenarios for WIMP-nucleus scattering is a spin-dependent interaction where the WIMP particle is a χ fermion (either Dirac or Majorana) that recoils through its coupling to the spin of nucleons N=p,n:

$$\mathcal{L}_{int} \propto \vec{S}_\chi \cdot \vec{S}_N = c^p \vec{S}_\chi \cdot \vec{S}_p + c^n \vec{S}_\chi \cdot \vec{S}_n$$

(for instance, predicted by supersymmetry when the WIMP is a neutralino that couples to quarks via Z-boson or squark exchange)

A few facts of life:

Nuclear spin is mostly carried by odd-numbered nucleons. Even-even isotopes carry no spin.

• the DAMA effect is measured with Sodium Iodide. Both Na and I have spin **carried by an unpaired proton**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{23}Na	3/2	11	12	100 %
^{127}I	5/2	53	74	100 %

Germanium experiments carry only a very small amount of ^{73}Ge, the only isotope with spin, **carried by an unpaired neutron**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{73}Ge	9/2	32	41	7.7 %

Xenon experiment contain two isotopes with spin, **both carried mostly by an unpaired neutron**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{129}Xe	½	54	75	26%
^{131}Xe	3/2	54	77	21%

→several authors have considered the possibility that $c_n \ll c_p$: in this case the WIMP particle is seen by DAMA but does not scatter on xenon and germanium detectors

However another class of Dark Matter experiments (superheated droplet detector and bubble chambers) **all use nuclear targets with an unpaired proton**:

Experiment	Target	Type	Energy thresholds (keV)	Exposition (kg day)
SIMPLE	$C_2Cl\,F_5$	superheated droplets	7.8	6.71
COUPP	$C\,F_3\,I$	bubble chamber	7.8, 11, 15.5	55.8, 70, 311.7
PICASSO	$C_3\,F_8$	bubble chamber	1.7, 2.9, 4.1, 5.8, 6.9, 16.3, 39, 55	114
PICO-2L	$C_3\,F_8$	bubble chamber	3.2, 4.4, 6.1, 8.1	74.8, 16.8, 82.2, 37.8

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{19}F	1/2	9	10	100
^{35}Cl	3/2	17	18	75.77 %
^{37}Cl	3/2	17	20	24.23 %
^{127}I	5/2	53	74	100

These experiments are sensitive to c_p , so for $c_n \ll c_p$ spin-dependent scatterings on Fluorine have been shown to lead to tension with the DAMA (C. Amole et al., (PICO Coll.) PLB711, 153(2012), E. Del Nobile, G.B. Gelmini, A. Georgescu and J.H. Huh, 1502.07682)

N.B. All only sensitive to the energy threshold, which for bubble and droplets nucleation is controlled by the pressure of the liquid

Correspondence between WIMP and non-relativistic EFT nuclear response function

coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$
	velocity-independent	velocity-dependent		velocity-independent	velocity-dependent

(in parenthesis the explicit dependence on q)

$$\mathcal{H} = \sum_i \left(c_i^0 + c_i^1 \tau_3\right) \mathcal{O}_i$$

$$R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{(v_T^{\perp})^2}{c^2} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{v_T^2 - v_{min}^2}{c^2}$$

Relativistic couplings leading in their non-relativistic limits **to the most general spin-dependent** terms:

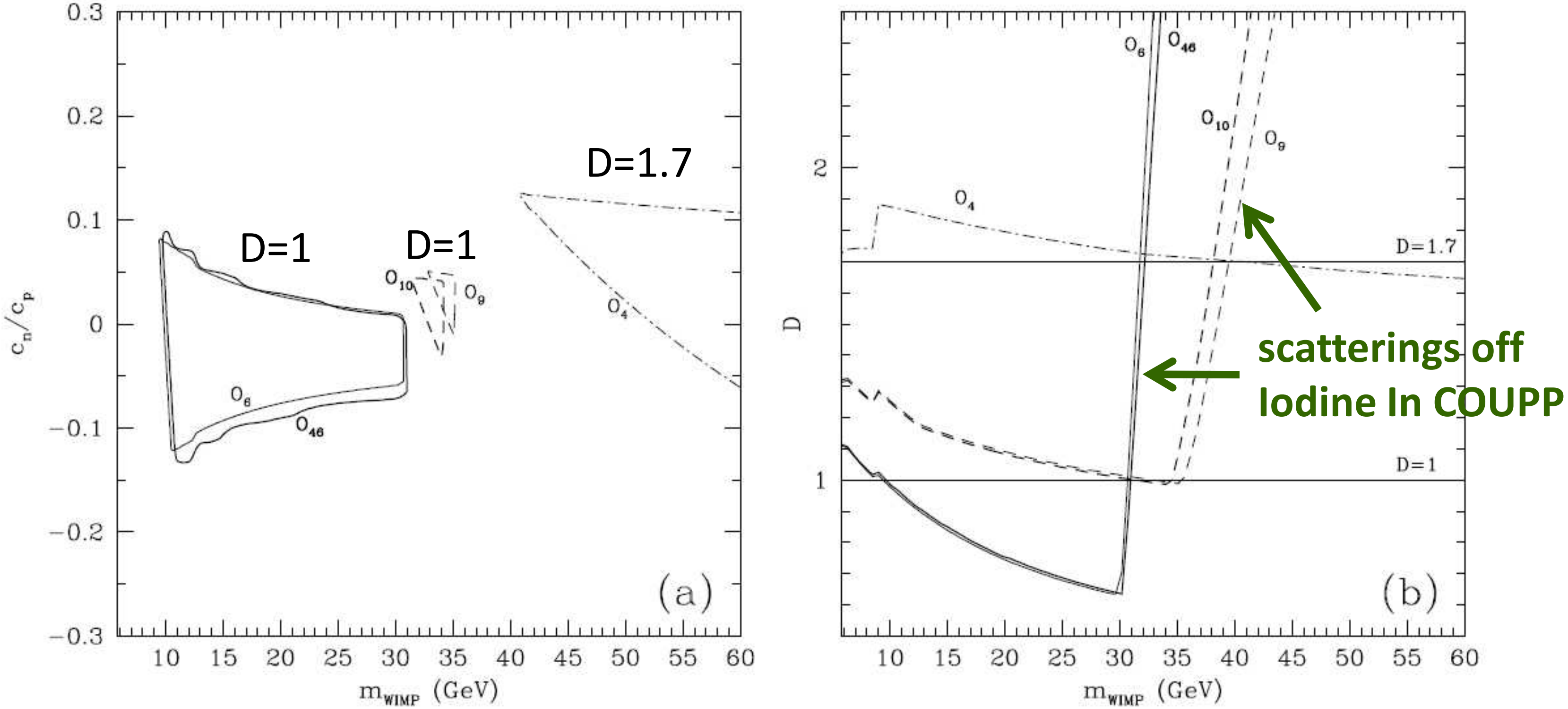
	Relativistic EFT	Nonrelativistic limit	$\sum_i \mathcal{O}_i$	cross section scaling
1	$\bar{\chi}\gamma^{\mu}\gamma^{5}\chi\bar{N}\gamma_{\mu}\gamma^{5}N$	$-4\vec{S}_{\chi}\cdot\vec{S}_N$	$-4\mathcal{O}_4$	$W^{\tau\tau'}_{\Sigma''}(q^2)+W^{\tau\tau'}_{\Sigma'}(q^2)$
2	$2\bar{\chi}\gamma^{\mu}\chi\bar{N}\gamma_{\mu}\gamma^{5}N+$ $+\bar{\chi}\gamma^{\mu}\gamma_5\chi\bar{N}i\sigma_{\mu\nu}\frac{q^{\nu}}{m_{WIMP}}N$	$-4\vec{S}_N\cdot\vec{v}^{\perp}_T$	$-4\mathcal{O}_7$	$(v^{\perp}_T)^2W^{\tau\tau'}_{\Sigma'}(q^2)$
3	$2\bar{\chi}\gamma^{\mu}\chi\bar{N}\gamma_{\mu}\gamma^{5}N+$ $-\bar{\chi}i\sigma_{\mu\nu}\frac{q^{\nu}}{m_{WIMP}}\chi\bar{N}\gamma^{\mu}\gamma_5N$	$-4\vec{S}_N\cdot\vec{v}^{\perp}_T$	$-4\mathcal{O}_7$	$(v^{\perp}_T)^2W^{\tau\tau'}_{\Sigma'}(q^2)$
4	$\bar{\chi}\gamma^{\mu}\chi\bar{N}\gamma_{\mu}\gamma^{5}N$	$-2\vec{S}_N\cdot\vec{v}^{\perp}_T+$ $+\frac{2}{m_{WIMP}}i\vec{S}_{\chi}\cdot\left(\vec{S}_N\times\vec{q}\right)$	$-2\mathcal{O}_7+2\frac{m_N}{m_{WIMP}}\mathcal{O}_9$ $\simeq 2\frac{m_N}{m_{WIMP}}\mathcal{O}_9$	$\simeq q^2W^{\tau\tau'}_{\Sigma'}(q^2)$
5	$\bar{\chi}i\sigma_{\mu\nu}\frac{q^{\nu}}{m_M}\chi\bar{N}\gamma^{\mu}\gamma_5N$	$4i(\frac{\vec{q}}{m_M}\times\vec{S}_{\chi})\cdot\vec{S}_N$	$4\frac{m_N}{m_M}\mathcal{O}_9$	$q^2W^{\tau\tau'}_{\Sigma'}(q^2)$
6	$\bar{\chi}\gamma^{\mu}\gamma_5\chi\bar{N}i\sigma_{\mu\nu}\frac{q^{\nu}}{m_M}N$	$4i\vec{S}_{\chi}\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_9$	$q^2W^{\tau\tau'}_{\Sigma'}(q^2)$
7	$i\bar{\chi}\chi\bar{N}\gamma^{5}N$	$i\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$\mathcal{O}_{10}$	$q^2W^{\tau\tau'}_{\Sigma''}(q^2)$
8	$i\bar{\chi}i\sigma_{\mu\nu}\frac{q^{\nu}}{m_M}\gamma_5\chi\bar{N}\gamma_{\mu}\gamma_5N$	$-4i(\frac{\vec{q}}{m_N}\cdot\vec{S}_{\chi})(\vec{v}^{\perp}_T\cdot\vec{S}_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_{14}$	$(v^{\perp}_T)^2q^2W^{\tau\tau'}_{\Sigma'}(q^2)$
9	$\bar{\chi}\gamma_5\chi\bar{N}\gamma^{5}N$	$-\frac{\vec{q}}{m_{WIMP}}\cdot\vec{S}_{\chi}\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$-\frac{m_N}{m_{WIMP}}\mathcal{O}_6$	$q^4W^{\tau\tau'}_{\Sigma''}(q^2)$
10	$\bar{\chi}i\sigma^{\mu\alpha}\frac{q_{\alpha}}{m_M}\gamma_5\chi\bar{N}i\sigma_{\mu\beta}\frac{q^{\beta}}{m_M}\gamma_5N$	$4\frac{\vec{q}}{m_M}\cdot\vec{S}_{\chi}\frac{\vec{q}}{m_M}\cdot\vec{S}_N$	$4\frac{m_N^2}{m_M^2}\mathcal{O}_6$	$q^4W^{\tau\tau'}_{\Sigma''}(q^2)$
11	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_{\nu}}{m_M}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^{\alpha}}{m_M}N$	$4\left(\frac{\vec{q}}{m_M}\times\vec{S}_{\chi}\right)\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_N\right)$	$4\left(\frac{q^2}{m_M^2}\mathcal{O}_4-\frac{m_N^2}{m_M^2}\mathcal{O}_6\right)$	$q^4W^{\tau\tau'}_{\Sigma'}(q^2)$

- the resulting scaling laws include the most general velocity and momentum dependences allowed by Galilean invariance through the product $(v_T^{\perp})^{2n}$ $(q^2)^m$ (n=0,1; m=0,1,2)

S.Scopel, J.H.Yoon and K.Yoon, JCAP 1507(2015)041

Numerical results

S.Scopel, J.H.Yoon and K.Yoon, JCAP 1507(2015)041



•If D<1 all constraints are verified

•Possible for O_6,O_{46} (q^4 momentum dependence) and to a lesser extent for O_9,O_{10} (q^2 momentum dependence), no compatibility for O_4 (usual spin-dependent interaction, no q dependence)

• as long as scatterings off Fluorine (and/or Chlorine) dominate in bubble chambers and droplets detectors momentum transfers q=sqrt($m_{nucleus}$ E) have a smaller values compared to Sodium , due to the lighter target mass and to the lower energy threshold of the former→ reduced sensitivity to DAMA for $(q^2)^n$, n=1,2

• for m_{WIMP}>30 GeV scatterings off Iodine in COUPP are kinematically accessible with much larger values of momentum transfer q →steep rise in compatibility factor when n=1,2

S.Scopel and K.Yoon, JCAP 1602(2016)050

An alternative way to evade Fluorine constraints for a WIMP with spin-dependent coupling to protons: inelastic scattering (proton-philic Spin-dependent IDM, pSIDM)

$$v_{\min} = \frac{1}{\sqrt{2 m_N E_R}} \left| \frac{m_N E_R}{\mu_{\chi N}} + \delta \right|$$

$v_{min} > v^*_{min}$

$$v^*_{\min} = \sqrt{\frac{2\delta}{\mu_{\chi N}}}$$

$A_{sodium}=23$ $A_{Fluorine}=19$

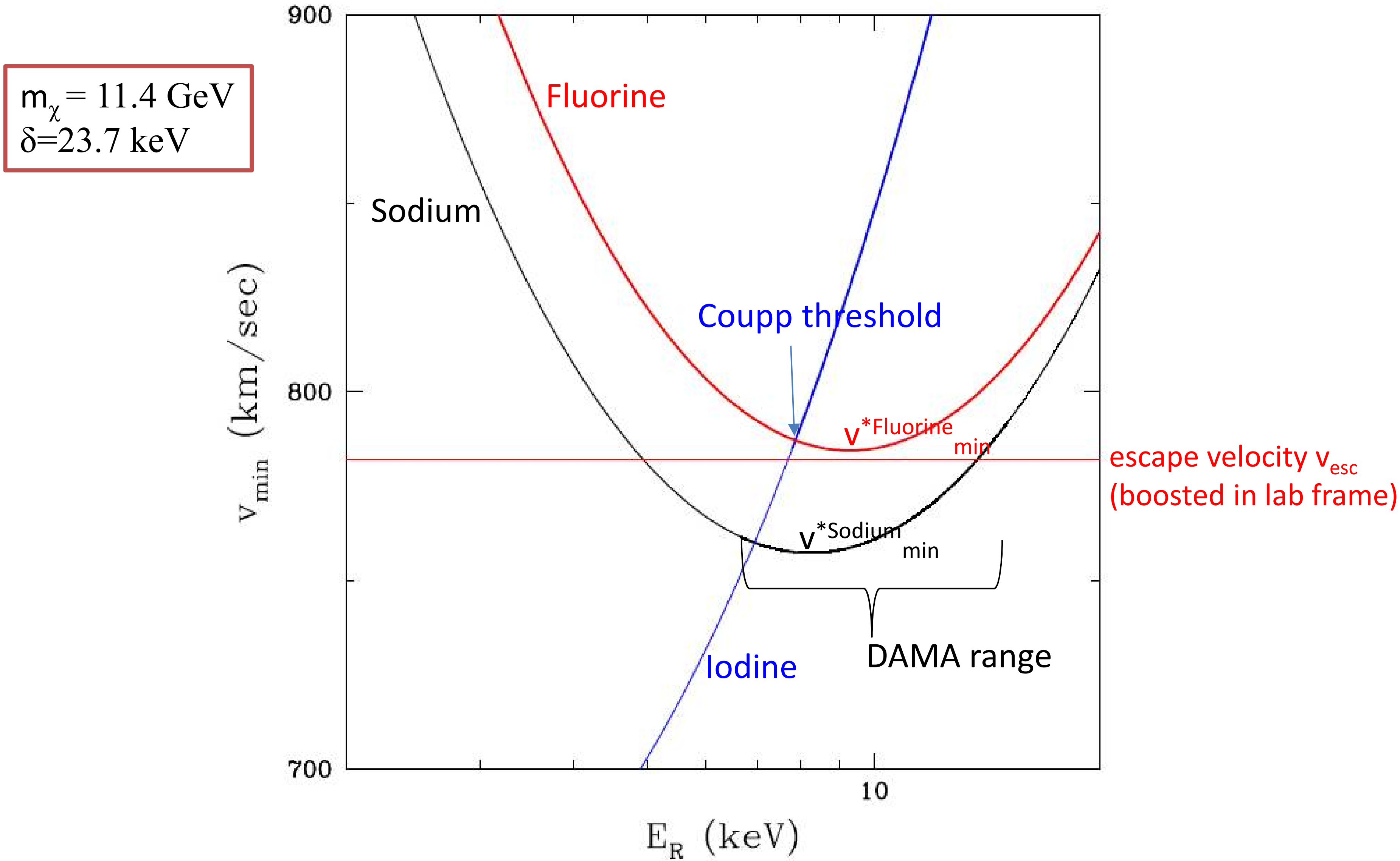
$m_{Sodium} > m_{Fluorine} \rightarrow \mu_{\chi N}^{Sodium} > \mu_{\chi N}^{Sodium}$

$\rightarrow v^{*Sodium}_{min} < v^{*Fluorine}_{min}$

what if $v^{*Sodium}_{min} < v_{esc} < v^{*Fluorine}_{min}$?

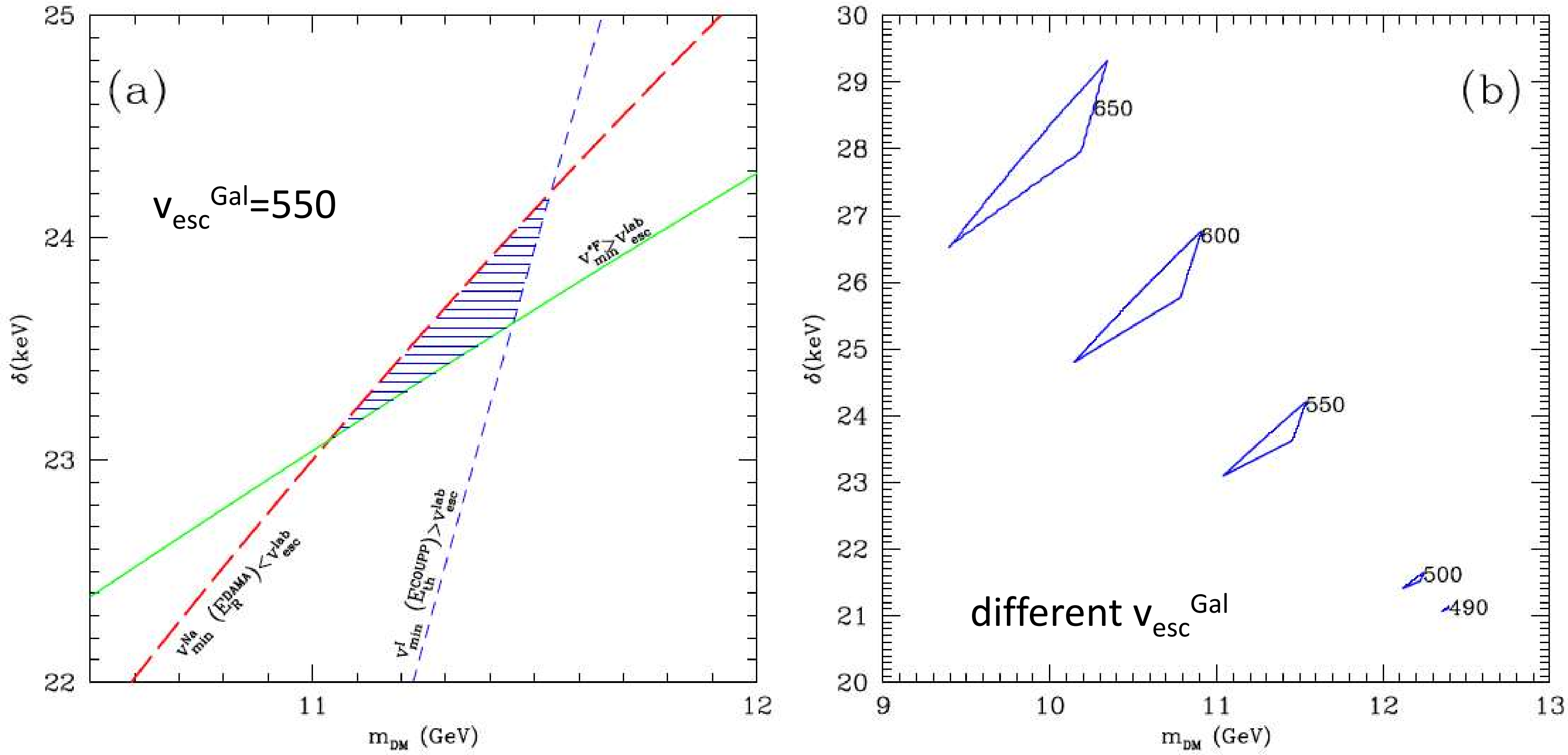
(N.B. v_{esc} in lab frame)

S.Scopel and K.Yoon, JCAP 1602(2016)050



depending on m_χ and δ, can drive Fluorine (and Iodine in COUPP) beyond v_{esc} while Sodium remains below → no constraint on DAMA from droplet detectors and bubble chambers

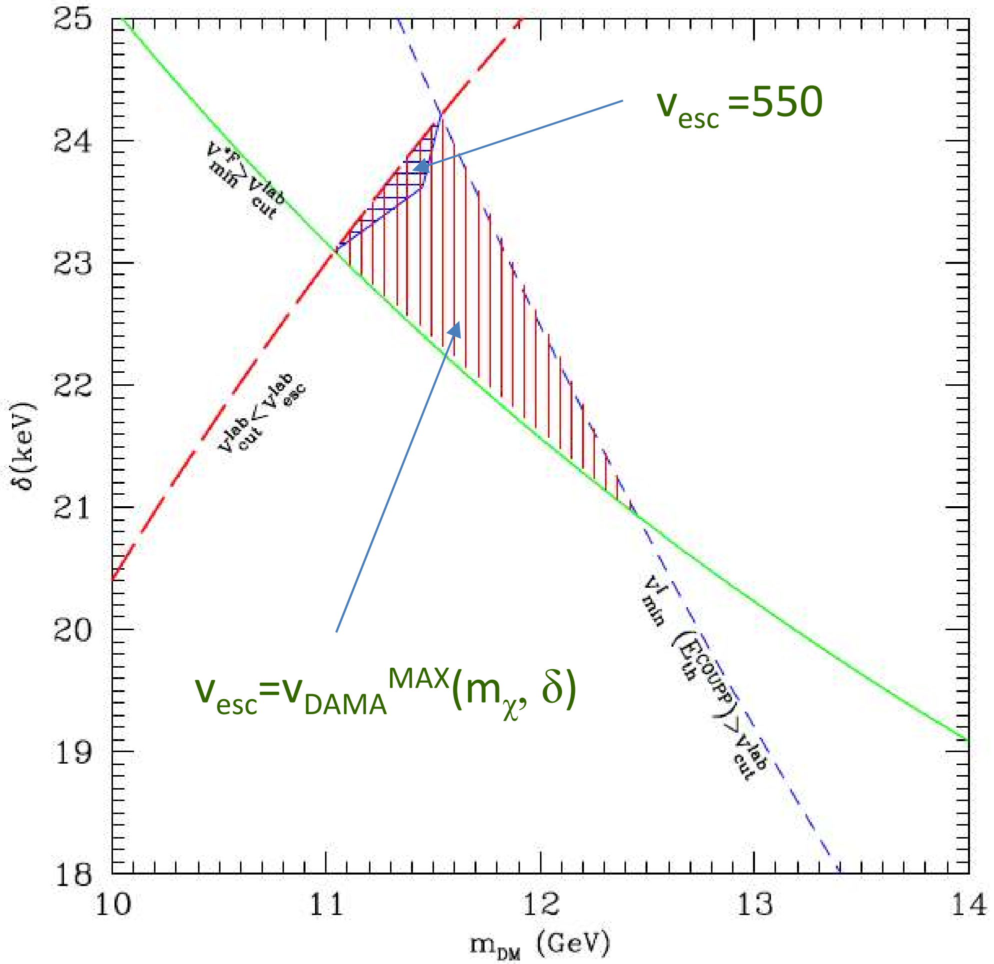
S.Scopel and K.Yoon, JCAP 1602(2016)050



very tuned region. but this is just kinematics

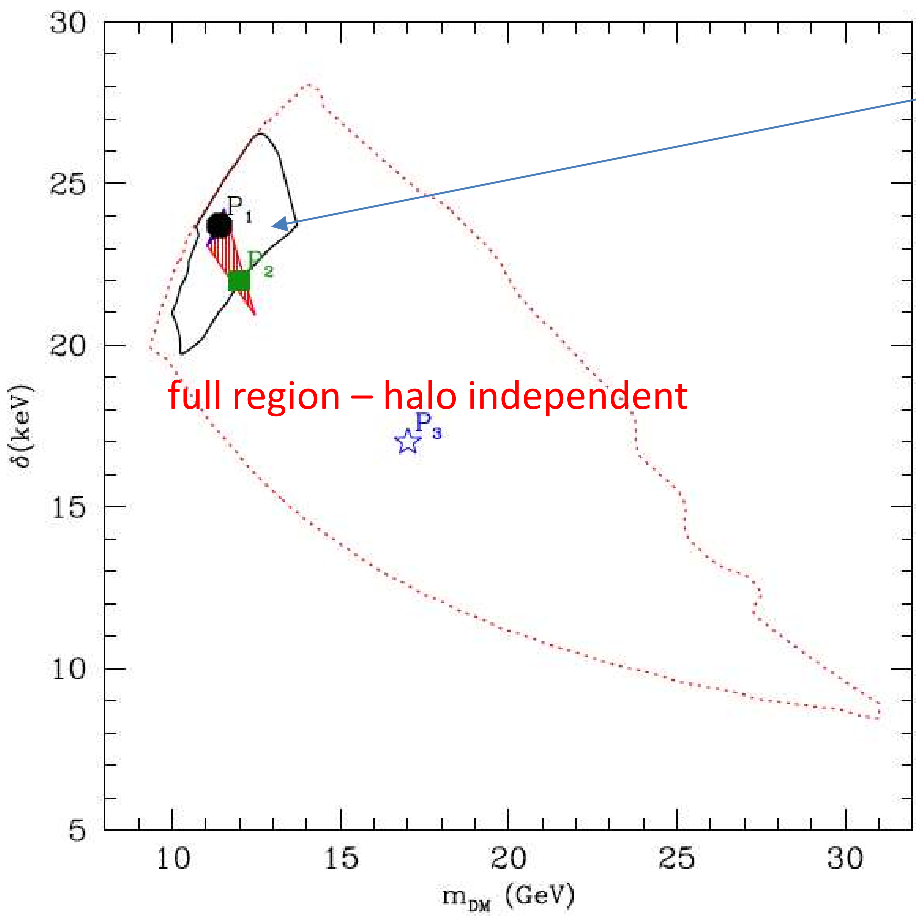
S.Scopel and K.Yoon, JCAP 1602(2016)050

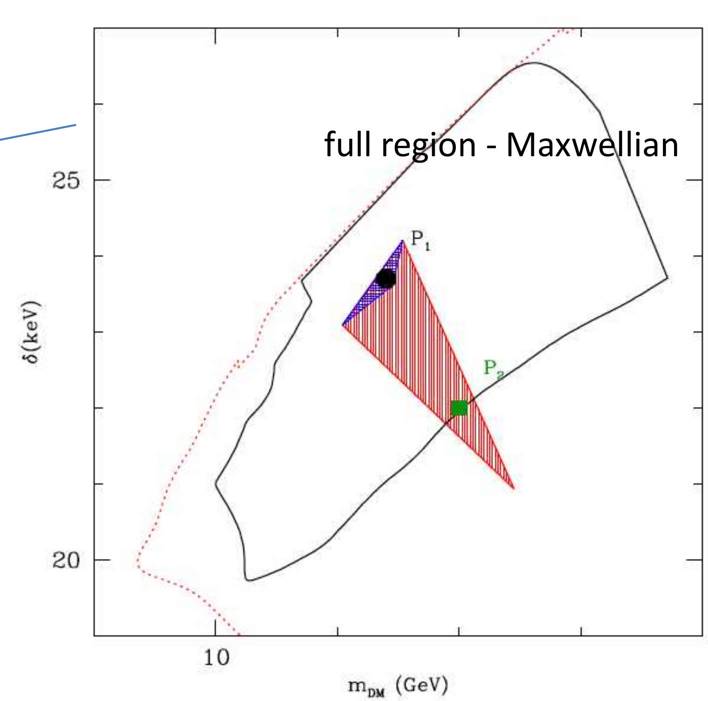
taking $v_{esc}=v_{DAMA}^{MAX}(m_\chi, \delta)$ the kinematic region enlarges considerably



S.Scopel and K.Yoon, JCAP 1602(2016)050

when including also the dynamics (through a full calculation of the compatibility factor) the two regions (Maxwellian and halo-independent) enlarge even more



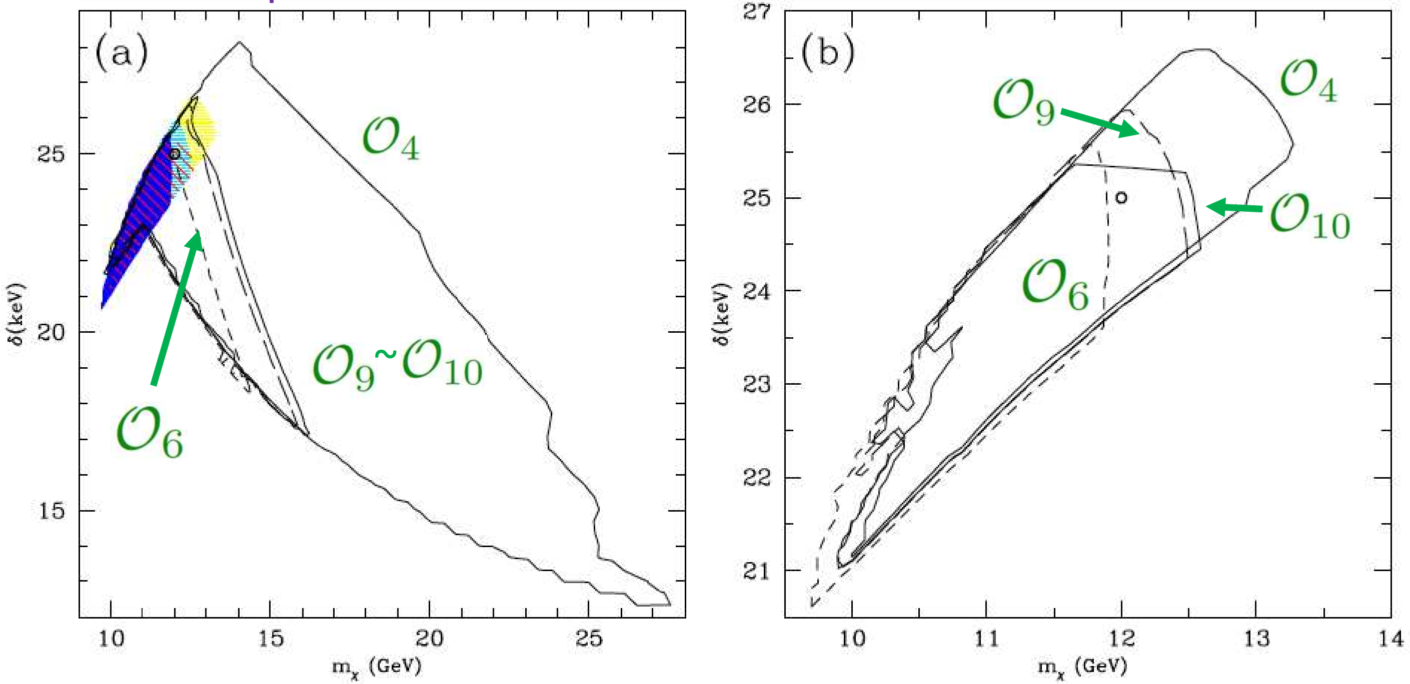


pSIDM: extension to most general spin-dependent couplings allowed by non-relativistic EFT

	Relativistic EFT	Non-relativistic limit	$\sum_i \mathcal{O}_i$	cross section scaling
$\mathcal{O}_4^{AA}$	$\bar\chi_1\gamma^\mu\gamma^5\chi_2\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}$+h.c.	$-4\vec{S}_\chi\cdot\vec{S}_{\mathcal{N}}$	-4$\mathcal{O}_4$	$W^{\tau\tau'}_{\Sigma''}(q^2)+W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_9^{VA}$	$\bar\chi_1\gamma^\mu\chi_2\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}$+h.c.	$+\frac{2}{m_{WIMP}}i\vec{S}_\chi\cdot\left(\vec{S}_{\mathcal{N}}\times\vec{q}\right)$	$\simeq 2\frac{m_{\mathcal{N}}}{m_{WIMP}}\mathcal{O}_9$	$\simeq q^2W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_9^{TA}$	$\bar\chi_1 i\sigma_{\mu\nu}\frac{q^\nu}{m_M}\chi_2\bar{\mathcal{N}}\gamma^\mu\gamma_5\mathcal{N}$+h.c.	$4i(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot\vec{S}_{\mathcal{N}}$	$4\frac{m_{\mathcal{N}}}{m_M}\mathcal{O}_9$	$q^2W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_9^{AT}$	$\bar\chi_1\gamma^\mu\gamma_5\chi_2\bar{\mathcal{N}}i\sigma_{\mu\nu}\frac{q^\nu}{m_M}\mathcal{N}$+h.c.	$4i\vec{S}_\chi\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_{\mathcal{N}})$	$-4\frac{m_{\mathcal{N}}}{m_M}\mathcal{O}_9$	$q^2W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_{10}^{SP}$	$i\bar\chi_1\chi_2\bar{\mathcal{N}}\gamma^5\mathcal{N}$+h.c.	$i\frac{\vec{q}}{m_{\mathcal{N}}}\cdot\vec{S}_{\mathcal{N}}$+h.c.	$\mathcal{O}_{10}$	$q^2W^{\tau\tau'}_{\Sigma''}(q^2)$
$\mathcal{O}_6^{PP}$	$\bar\chi_1\gamma_5\chi_2\bar{\mathcal{N}}\gamma^5\mathcal{N}$+h.c.	$-\frac{\vec{q}}{m_{WIMP}}\cdot\vec{S}_\chi\frac{\vec{q}}{m_{\mathcal{N}}}\cdot\vec{S}_{\mathcal{N}}$	$-\frac{m_{\mathcal{N}}}{m_{WIMP}}\mathcal{O}_6$	$q^4W^{\tau\tau'}_{\Sigma''}(q^2)$
$\mathcal{O}_6^{T'T'}$	$\bar\chi_1 i\sigma^{\mu\alpha}\frac{q_\alpha}{m_M}\gamma_5\chi_2\bar{\mathcal{N}}i\sigma_{\mu\beta}\frac{q^\beta}{m_M}\gamma_5\mathcal{N}$+h.c.	$4\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\frac{\vec{q}}{m_M}\cdot\vec{S}_{\mathcal{N}}$	$4\frac{m_{\mathcal{N}}^2}{m_M^2}\mathcal{O}_6$	$q^4W^{\tau\tau'}_{\Sigma''}(q^2)$

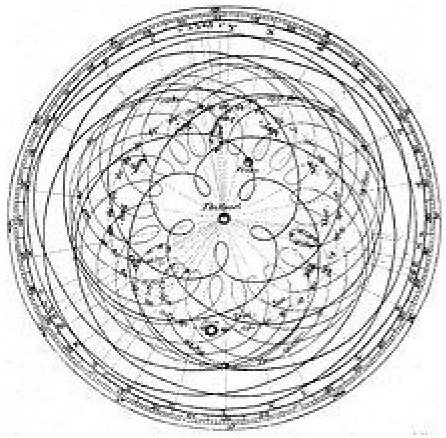
Halo-independent

Maxwellian



Momentum dependence leads to smaller allowed regions

S. Scopel and H. Yu, JCAP 1704 (2017) no.04, 031

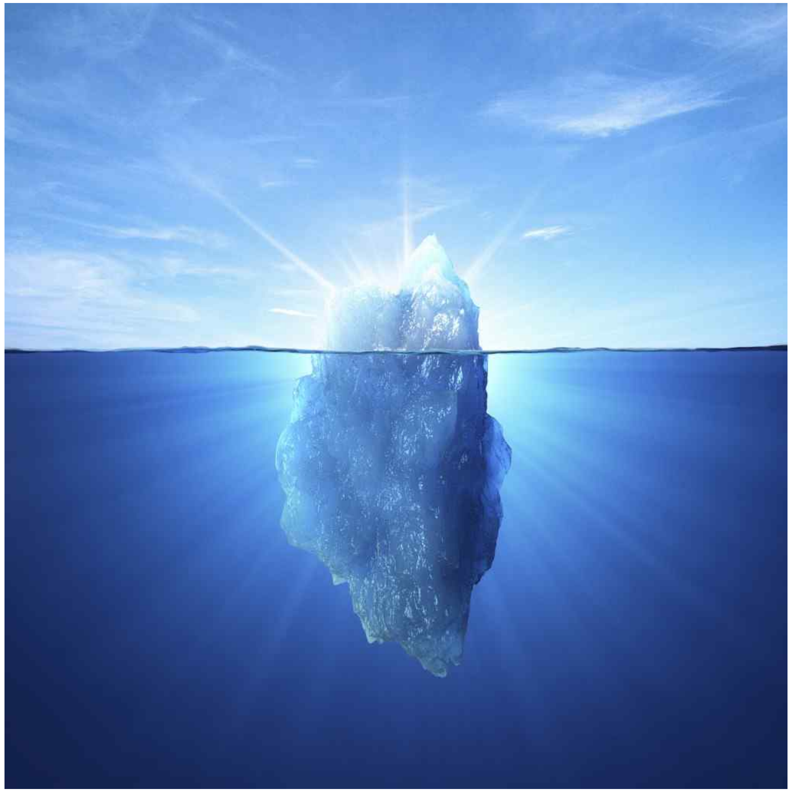


Several epicycles added to the usual scenario:

- Halo-independent
- Non-standard coupling
- Inelastic scattering
- Isospin violation
- ….

- Indeed, combining a halo-independent approach and/or a non-standard coupling (other than SI or SD) and/or inelastic scattering (different kinematics) and/or isospin violation compatibility among any of the "excesses" and constraints from null experiments can be achieved (S.S. and K.H. Yoon, JCAP 1602 (2016) no.02, 050; S.S.,K.H. Yoon and J.H. Yoon, JCAP 1507 (2015) no.07, 041; S.S. and J. H. Yoon, Phys.Rev. D91 (2015) no.1, 015019; S.S. and K.H. Yoon, JCAP 1408 (2014) 060)
- "Proofs of concept"

The bottom line:
Based on very well motivated theoretical assumptions we got used to a very simple WIMP direct detection parameter space (i.e. mass vs. SI sigma exclusion plots for isothermal sphere). However in principle it may be much larger: are we just starting now to scratch its surface?



Using data to study the model-independent halo functions η_0 and η_1 and extract the cross section σ: the stream approach

S.S. P. Gondolo, arXiv:1703.08942

The expected rate in bin of *observed* energy E' is given by:

$$N(t)_{[E'_1,E'_2]} = \int_{E'_1}^{E'_2} \frac{dR}{dE'}(E',t)\, dE'$$

$$v_{min}(E_R) = \left(\frac{m_T E_R}{2\mu_T^2}\right)^{\frac{1}{2}}$$ (elastic scattering)

$$\frac{dR}{dE'}(E',t) = \epsilon(E') \int_0^\infty \frac{dR}{dE_{ee}}(E_{ee},t)\mathcal{G}_T(E',E_{ee})\, dE_{ee} \qquad E_{ee} = Q(E_R)E_R$$

$$\frac{dR}{dE_R}(E_R,t) = \sum_T N_T \frac{\rho}{m_{WIMP}} \int_{v_{min}[E_R]} d^3v f(\vec{v},t) v \frac{d\sigma}{dE_R}$$

$$\frac{d\sigma}{dE_R} = \frac{\sigma_0 F(E_R)}{E_R^{max}} = \frac{\sigma_0 F(E_R)}{2\mu_T/m_N v^2}$$

m_T=nuclear mass of target T

μ_T=WIMP-nucleus reduced mass

$F(E_R)$=nuclear form factor (for finite size nucleus)

$Q(E_R)$=quenching factor=fraction of energy deposited in detected channel such as ionization or scintillation

$\mathcal{G}_T(E',E_{ee})$ =energy resolution of the detector

ε(E')=detector acceptance

N_T= number of targets of isotope T

Combining everything together :

contains all uncertainties

$$N_{[E'_1,E'_2]}(t) = \int_0^\infty \mathcal{R}_{[E'_1,E'_2]}(v_{min})\tilde{\eta}(v_{min},t)d\,v_{min}$$

$$\tilde{\eta}(v_{min},t) \equiv \frac{\rho}{m_{WIMP}}\sigma_0\eta(v_{min},t)$$

(σ_0=point-like WIMP-nucleus cross section)

E. Del Nobile, G. Gelmini, P. Gondolo and J.H. Huh, JCAP 1310 (2013) 048

N.B. the previous derivation requires no explicit velocity dependence in the cross section. Can do better!

An apparently innocuous trick: take out the velocity integral, and write the expected number of events as:

$$N(t)_{[E_1',E_2']} = \int_0^\infty \mathcal{H}_{[E_1',E_2']}(v)\, f(v,t)\, dv$$

where the response function contains all the dependences on the cross section and the experimental quantities. By setting:

$$f(v,t) \equiv -v\frac{\partial \eta(v,t)}{\partial v}$$

integrating by parts and incorporating as usual the point-like cross section and the local density in the definition of the halo function leads to:

$$N_{[E_1',E_2']}(v)(t) = \int_0^\infty \mathcal{R}_{[E_1',E_2']}(v)\, \tilde{\eta}(v,t)\, dv$$

This expression looks pretty much the same as the previous one (with $v_{min} \rightarrow v$) but *is valid in principle for any velocity dependence in the cross section.*

The two different response functions are related by:

$$\mathcal{R}_{[E_1',E_2']}(v) = \frac{\partial}{\partial v}\left[v\mathcal{H}_{[E_1',E_2']}(v)\right], \qquad \mathcal{H}_{[E_1',E_2']}(v) = \frac{1}{v}\int_0^v \mathcal{R}_{[E_1',E_2']}(v')\, dv'$$

Several attempts in the literature to determine the velocity distribution from the data, sampling f(v) in a finite number of velocity bins $N_{streams}$ and using a maximum-likelihood method

(B. Feldstein, F. Kahlhoefer, JCAP 1408 (2014) 065; JCAP 1412 (2014) 12, 052. Alejandro Ibarra, Andreas Rappelt, arXiv:1703.09168)

In this case $f(v) \to \lambda_i$ with $\Sigma_i \lambda_i=1$ with $i=1,N_{streams}$ and the likelihood $L(S^k)$ depends on the direct detection theoretical predictions S^k with $k=1,N_{exp}$ and S^k in the form:

$$S^k = \sum_{i=1}^{N} \lambda_i H_i^k$$

with H_i^k fixed constants determined by experimental properties (response functions).

The problem of this approach is that in general $N_{streams} \ll N_{exp}$ (actually, the idea it to take $N_{streams} \to \infty$) so fixing N_{exp} experimental values leaves a large degeneracy in the λ's.

A different approach: given N_{exp} experimental observations and $N_{streams}$ streams look for the **maximal range** of any other observable of the form $\Sigma_i \lambda_i W_i$ using the lambdas as *nuisance* parameters (P. Gondolo, S. Scopel arXiv:1703.08942).

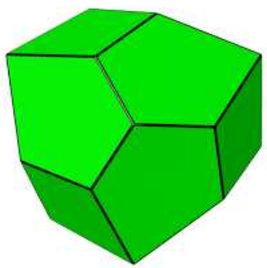
Consider the case of a finite number of streams $N_{Streams}$

P. Gondolo, S. Scopel arXiv:1703.08942

Fixing N_{exp} observations with $N_{streams}$ unknown λ's corresponds to solving the linear system:

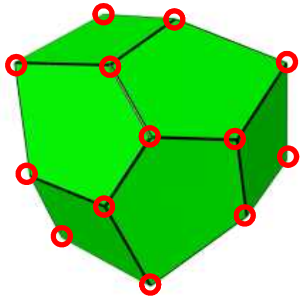
$$\left\{ \begin{aligned} \sum_{i=1}^{N_{streams}} H_i^1 \lambda_i &= N_{exp}^1 \\ &\cdots \\ \sum_{i=1}^{N_{streams}} H_i^{N_{exp}} \lambda_i &= N_{exp}^{N_{exp}} \\ \sum_{i=1} \lambda_i &= 1 \\ \lambda_i > 0 & \end{aligned} \right.$$

The system is unconstrained ($N_{exp} < N_{streams}$) so it singles out a polyhedron (i.e. the subset of a hyperplane when $0 < \lambda_i < 1$) of dimension $N_{streams} - N_{exp} - 1$ in the $N_{streams}$ -dimensional space of the λ's

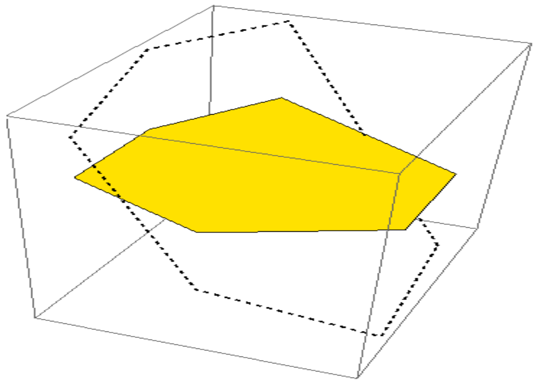


P. Gondolo, S. Scopel arXiv:1703.08942

The vertices of the polyhedron are found by fixing to zero $N_{streams}$ - N_{exp} -1 of the λ's and solving the remaining system of N_{exp} +1 unknowns and N_{exp} +1 constraints



An intuitive theorem: given a linear function $W=\Sigma_i \lambda_i W_i$ defined on a polyhedron its maximum and minimum must correspond to one of the vertices



So to find the extrema of W need only to check the vertices – in practice this means to always consider N_{exp} +1 λ's at a time (setting the other λ's to zero)

The problem is N_{exp} +1 dimensional, not $N_{streams}$ dimensional, and since $N_{exp} \ll N_{streams}$ this implies a huge simplification!

This theorem is also valid in the space of continuous functions:

- given the $N + 1$ known functions $g^i(x)$ $(i = 1, ..., N)$ and $h(x)$ and the unknown function $f(x)$, all defined in the same domain, the N constraints:

$$I_g^i = \int_0^\infty g^i(x)\, f(x)\, dx,\ i = 1, ...N,$$

imply that the extreme values of the integral:

$$I_h = \int_0^\infty h(x)\, f(x)\, dx,$$

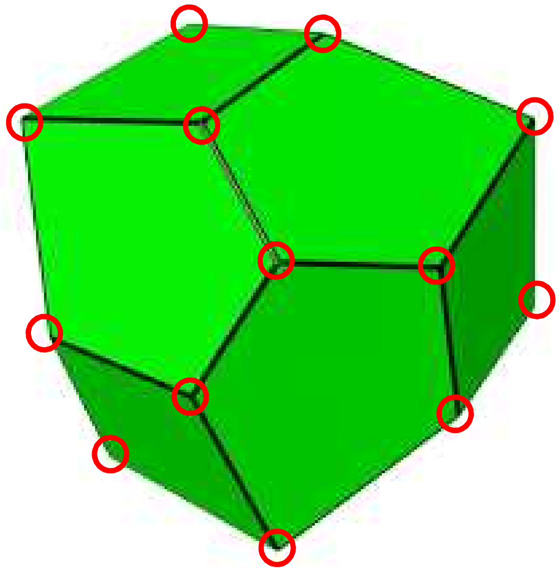
are obtained by expressing the unknown function $f(x)$ in terms of the N parametrizations:

$$f_n(x) = \sum_{j=1}^{n} \lambda_j\, \delta(x - x_j),\ n = 1, ..., N,$$

with $\sum_{i=1}^{n} \lambda_i$=1 and n=1,..,N.

I. Pinelis, *"On the extreme points of moments sets"*, Math. Meth. Oper. Res. 83 (2016) 325–349, [arXiv:1204.0249; H. P. Mulholland and P. Rogers, *"Representation theorems for distribution functions"*, Proc. of London Math. Society s3-8(2) (1958) 177–223.

P. Gondolo, S. Scopel arXiv:1703.08942



In the continuum case the theorem reduces an extremization problem in infinite dimensions (the moment set) into an extremization problem in a finite number of dimensions (the space of extreme distributions, which has dimension at most (1+d)N', where N=N+1 is the number of moment conditions and d is the dimensionality of the velocity space).

P. Gondolo, S. Scopel arXiv:1703.08942

In practice, this means that, at fixed n, the maximal range of the I_g integral is swept by the λ_j , x_j parameters that satisfy the n constraints with $f_n(x)$ given by the superposition of n streams, i.e. the system of n + 1 linear equations:

$$\sum_{j=1}^{n} \lambda_j g^i(x - x_j) = I_f^i, \quad i = 1, n$$

$$\sum_{k=1}^{n} \lambda_k = 1, \; \lambda_k > 0.$$

The full range of I_g is then obtained by combining the N intervals at fixed n.

Direct application to the analysis of direct detection data: given n experimental measurements any other quantity of the form

$$A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$$

can be bracketed for *any* f(v).

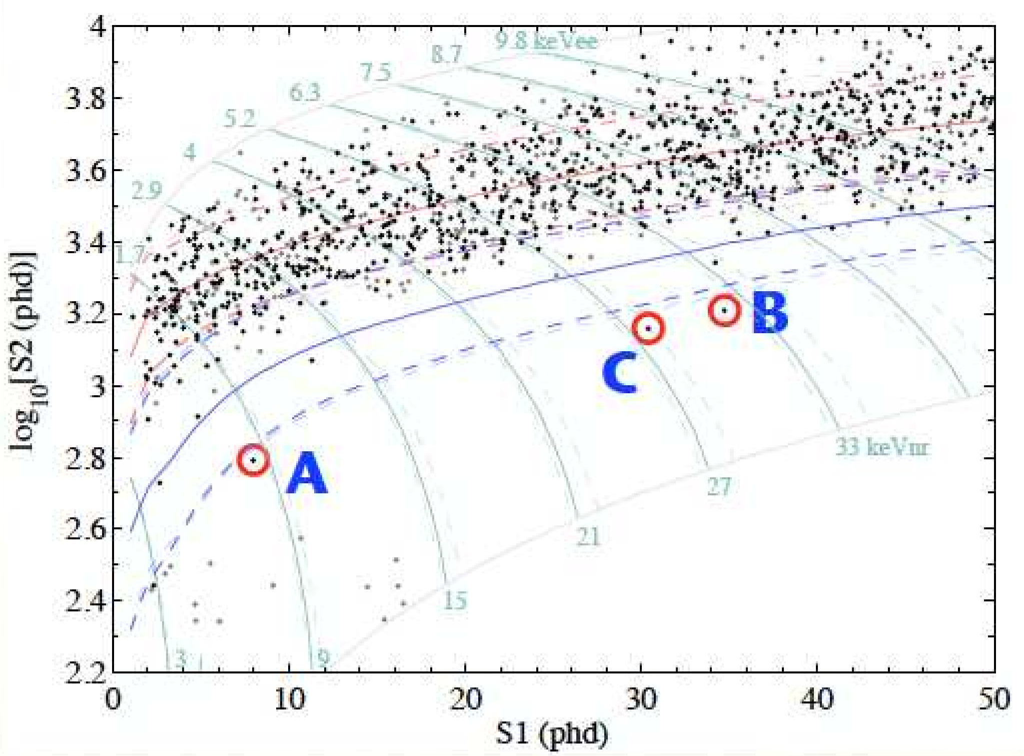
Recap:

- Given n independent direct detection measurements a parameterization of the velocity distribution in terms of n streams, combined with analogous parameterizations for n-1,n-2,…1 brackets *any* observable of the form $A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$ where only A(v) is known.

Where do we get from here?

WIMP-search data

- After salting, events outside the ER band were scrutinized again.
- Two populations of rare pathological events were identified, that had contributed three particularly dangerous events.



From A. Manalaysay talk at IDM2016

Three events surviving unblinding in LUX, excluded by modified pos-unblinding cuts. Let's just assume they were not (**CAVEAT: not for real, just playing with them!**)

Problem #1: only tree events in the $0<S_1<50$ phe window, which binning should we use? (N.B. S_1 in phe=photo-electons can be converted to the recoil energy).

Solution: can use the data to construct the *extended* likelihood function, which does not need binning:

$$\frac{L}{2} = N_{tot} - \sum_{k=1}^{N} \ln \left(\frac{dR}{dS_1} \right)_k$$

with N=3 events and where where both N_{TOT} and dR/dS_1 are given by the sum of a background + a signal contributions:

$$\left(\frac{dR}{dS_1} \right)_k = \left(\frac{dR_s}{dS_1} \right)_k + \left(\frac{dR_b}{dS_1} \right)_k$$

$$N_{tot} = N_{tot,s} + N_{tot,b}$$

$$N_{tot,s} = \int_{S_1^{min}}^{S_1^{max}} \frac{dR_s}{dS_1} dS_1, \quad N_{tot,b} = \int_{S_1^{min}}^{S_1^{max}} \frac{dR_b}{dS_1} dS_1$$

N.B. need an estimation of the background. In LUX the background estimation is 1.5 events in the full range $0.5\ PE<S_1<50\ PE$

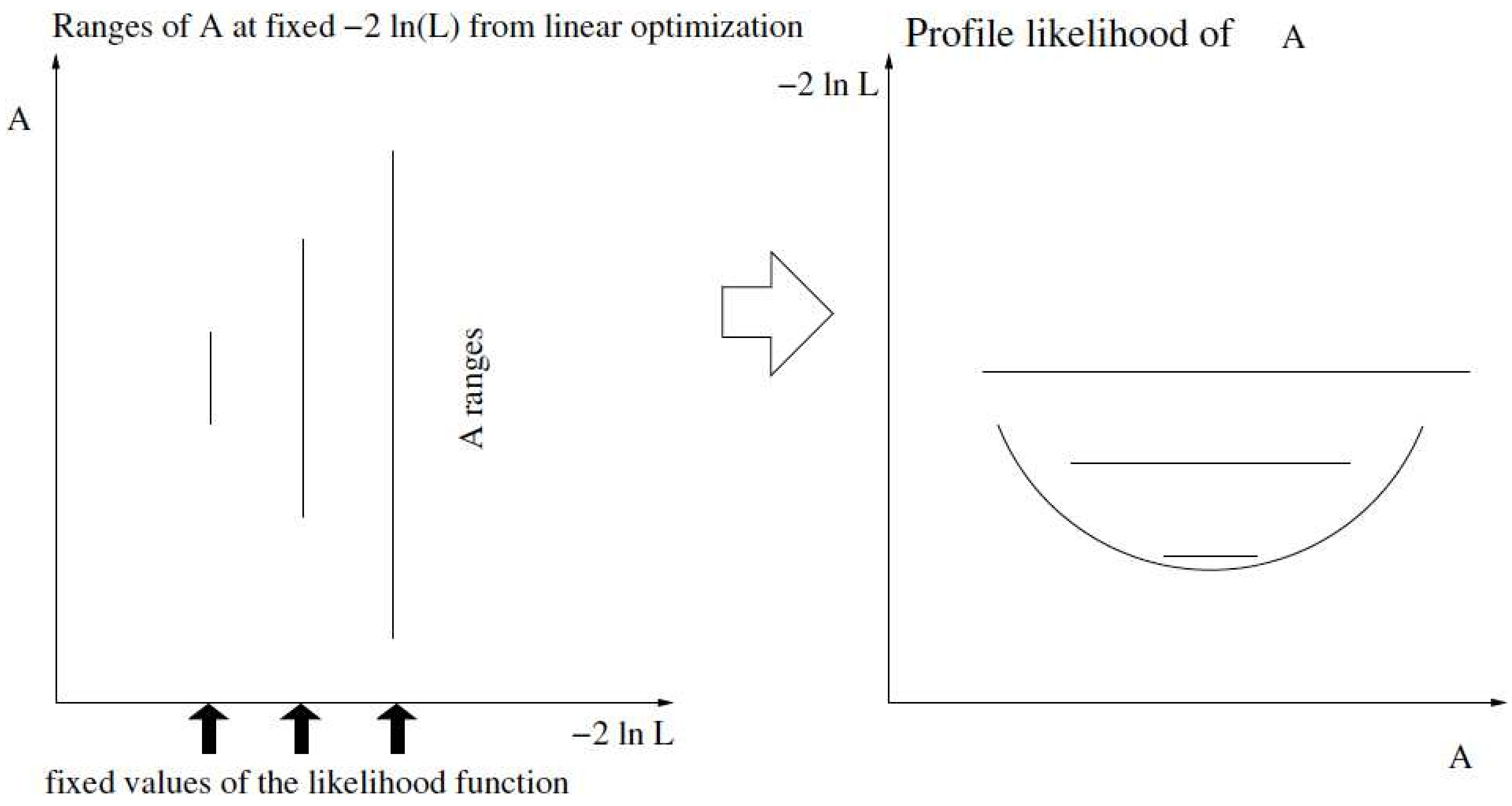
The signal part can be expressed in terms of integrals of f(v) times some response function only dependent on experimental quantities:

$$N_{tot,s} = \int_0^\infty \mathcal{H}_{[S_1^{min},S_1^{max}]}(v) f(v)\, dv,$$

$$\left(\frac{dR_s}{dS_1}\right)_k = \int_0^\infty \mathcal{H}_{S_1^k}(v) f(v)\, dv,$$

According to the previous theorem for any choice of the four quantities N_{TOT} and $(dR/dS_1)_k$ any other quantity of the form $A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$ can be bracketed for any f(v). Actually, fixing N_{TOT} and $(dR/dS_1)_k$ fixes L, *so this is also true for a fixed value of L*

If we can bracket the full range of A at fixed L by turning the plot 90 degrees we can get the profile-likelihood of A



The n-sigma range of A is obtained by taking the points with $L-L_{min}<n^2$

Can choose A as any quantity which can be expressed as an integral of f(v) times a response function. For instance, take an average of the halo function η in some range of v:

$$\tilde{\eta}(v_{min}) = \frac{\rho}{m_\chi}\sigma \int_{v_{min}}^{\infty} f(v)\, dv$$

$$< \tilde{\eta}(v_{min}) >_{[v_{min,1}, v_{min,2}]} = \frac{1}{v_{min,2} - v_{min,1}} \int_{v_{min,1}}^{v_{min,2}} \tilde{\eta}(v_{min})\, dv_{min}$$

Indeed, this average can be expressed as:

$$< \tilde{\eta}(v_{min}) >_{[v_{min,1}, v_{min,2}]} = \boxed{\int_0^{\infty} \mathcal{H}_\eta^{[v_{min,1}, v_{min,2}]}(v) f(v)\, dv}$$

with:

$$\mathcal{H}_\eta^{[v_{min,1}, v_{min,2}]}(v) = \frac{\rho}{m_\chi}\sigma \begin{cases} 0 & \text{if } v < v_{min,1} \\ \frac{1}{v}\frac{v - v_{min,1}}{v_{min,2} - v_{min,1}} & \text{if } v_{min,1} \le v \le v_{min,2} \\ \frac{1}{v} & \text{if } v > v_{min,2}. \end{cases}$$

Problem #2: how do we sample the parameter space with N=1,2,3,4 streams?
According to the theorem the full range of $<\tilde{\eta}(v_{min})>_{[v_{min,1},v_{min,2}]}$ is spanned by using:

$$<\tilde{\eta}(v_{min})>_{[v_{min,1},v_{min,2}]}=\sum_{k=1}^{m}\lambda_k\mathcal{H}_{\eta}^{[v_{min,1},v_{min,2}]}(v_k),\quad m=1,2,...,N+1$$

Need to do that numerically.

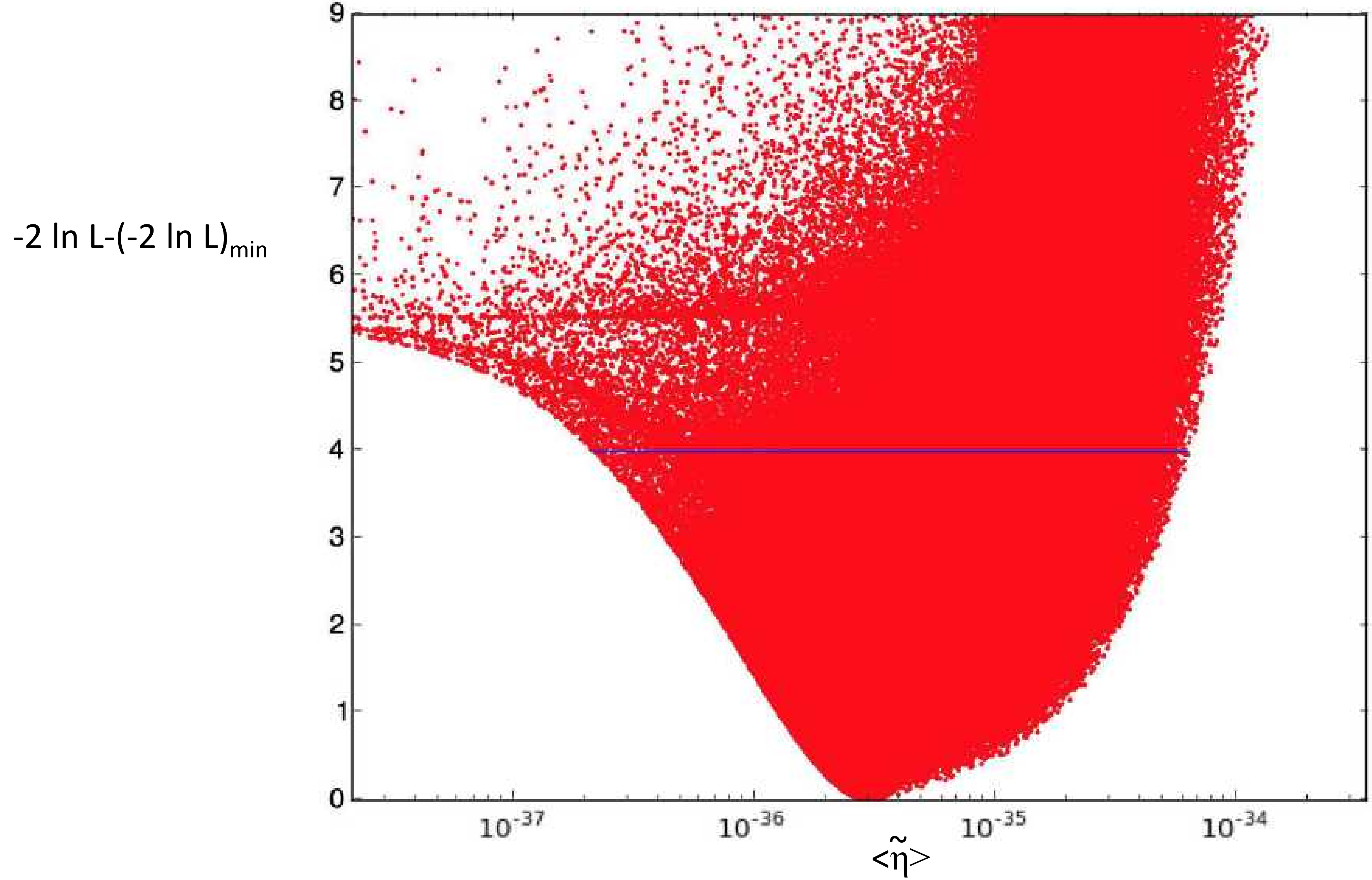
Suitable for a Markov Chain sampling. Two advantages:

- the sampling is driven by the Likelihood itself, don't waste time in low-probability regions
- Perfect for profiling, highest density of points where -2 ln L is minimal

Can use a Markov–Chain Montecarlo code* to generate large sets {v} of v_k velocities and {λ} of λ_k coefficients for $1 \le k \le m$ and $1 \le m \le N+1=4$ to calculate both -2 ln L and $<\tilde{\eta}(v_{min})>$

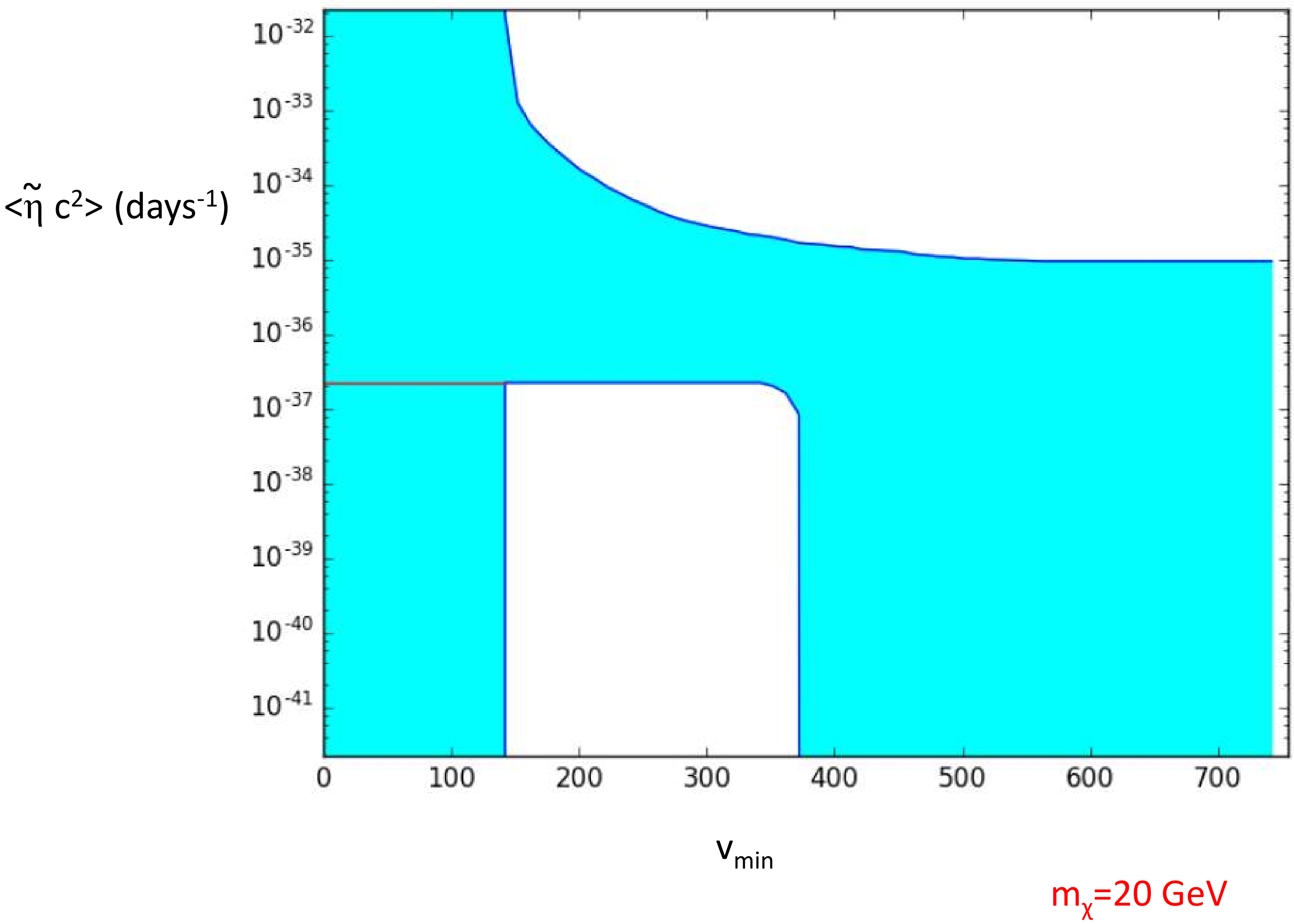
*emcee, D. Foreman-Mackey, D. W. Hogg, D. Lang, and J. Goodman, emcee: The mcmc hammer, Publications of the Astronomical Society of the Pacific 125 (2013), no. 925 306.

Example: 2-sigma interval for <η> in the range 200 km/s<v_{min}<250 km/s and zero background

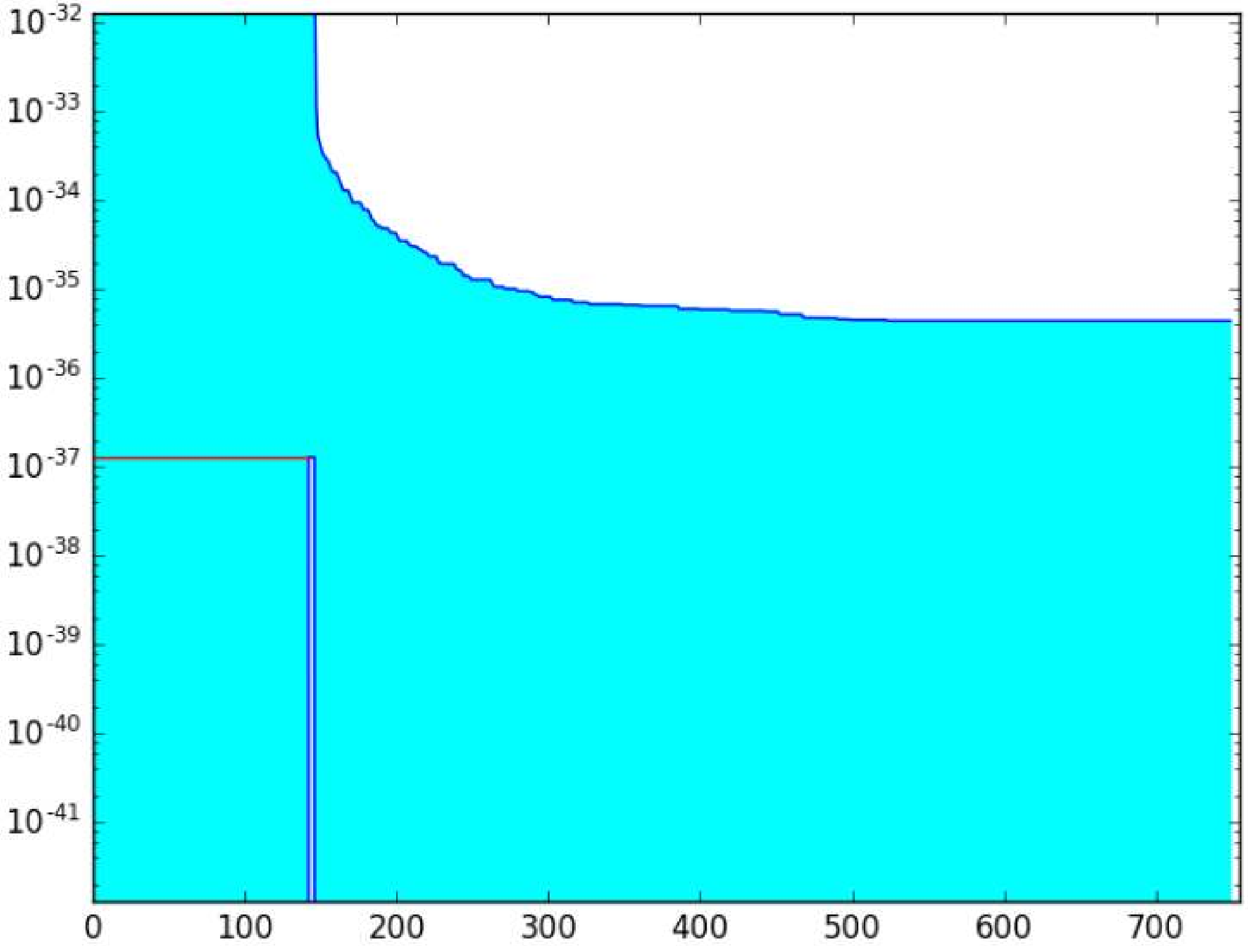


$5x10^6$ points using 250 independent walkers and a Metropolis-Hastings sampler

Repeat the same exercise ad different velocity ranges:



On the other hand, assuming a flat background of 1.5 events, 3 observed events have a much smaller significance (less than 1 sigma, only upper bound for the halo function):



$\langle\tilde{\eta}\, c^2\rangle$ (days^{-1})

v_{min}

m_χ=20 GeV

N.B. the new physics is contained in the cross section σ, which is a normalizing factor in the response function:

$$\tilde{\eta}(v_{min}) = \frac{\rho}{m_\chi}\sigma \int_{v_{min}}^{\infty} f(v)\, dv$$

Since:

$$< \tilde{\eta}(v_{min}) >_{[v_{min,1},v_{min,2}]} = \sum_{k=1}^{m} \lambda_k \mathcal{H}_{\eta}^{[v_{min,1},v_{min,2}]}(v_k)$$

a convenient way is to normalize the streams to σ:

$$\sigma = \sum_k \lambda_k$$

What kind of info can we get on σ? It depends…

From direct detection data to suppression scale (simple halo-independent recipe)

Once $\tilde{\eta}$ is fixed by experiment need f(v) to get info on the cross section and the suppression scale Λ

$$\tilde{\eta}(v_{min}) \equiv \frac{\rho_\chi}{m_\chi}\sigma_0 \eta(v_{min})$$

M*aximize η* and *minimize* cross section taking:

$$f(\vec{v}) = \delta(v_s - v_{min})$$

(v_s = maximal value of the v_{min} range corresponding to the signal)

$$\Rightarrow \quad \tilde{\eta}^{max}(v_{min}) = \tilde{\eta}^{fit}\theta(v_s - v_{min})$$

N.B. corresponds to fitting the experimental etas to a constant value, works only if this is compatible to data

Then use:

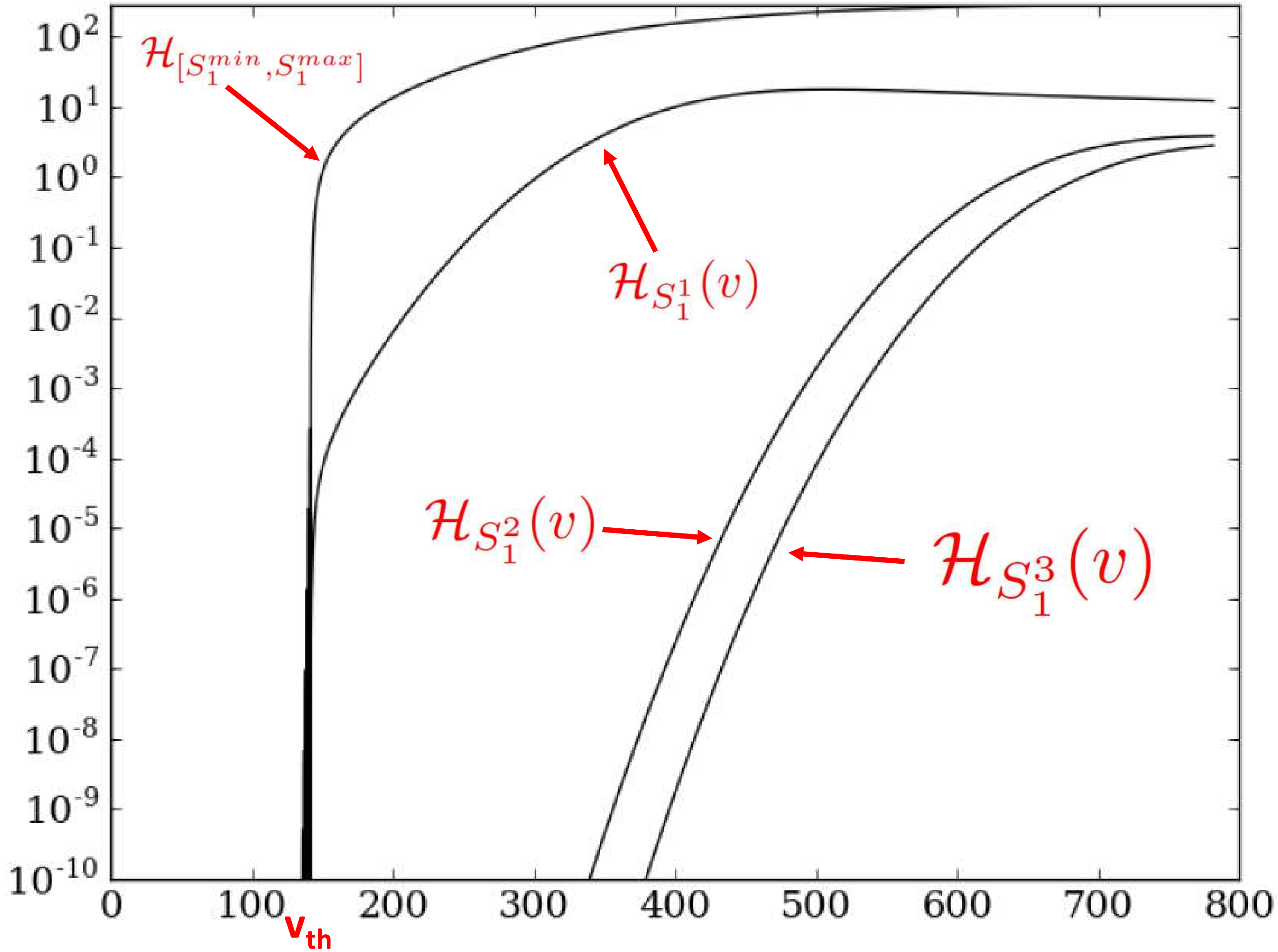
$$\tilde{\eta}^{fit} = \frac{\rho_\chi}{m_\chi}\sigma\frac{1}{v_s}$$

N.B.: only sensitive to the product density times cross section

If the DM density ρ_χ is a fraction ς of the total amount measured in the neighborhood of the Sun ρ_{loc} , a direct detection experiment can only get estimates on $\varsigma\sigma$ rather then σ

Profile likelihood of σ?
(rather, $\varsigma\sigma$)

The four response functions in -2 ln L, corresponding to $N_{tot,s}$ and $(dR/dS_1)_k$ with S_k=(7.9, 30.40, 34.7) PE



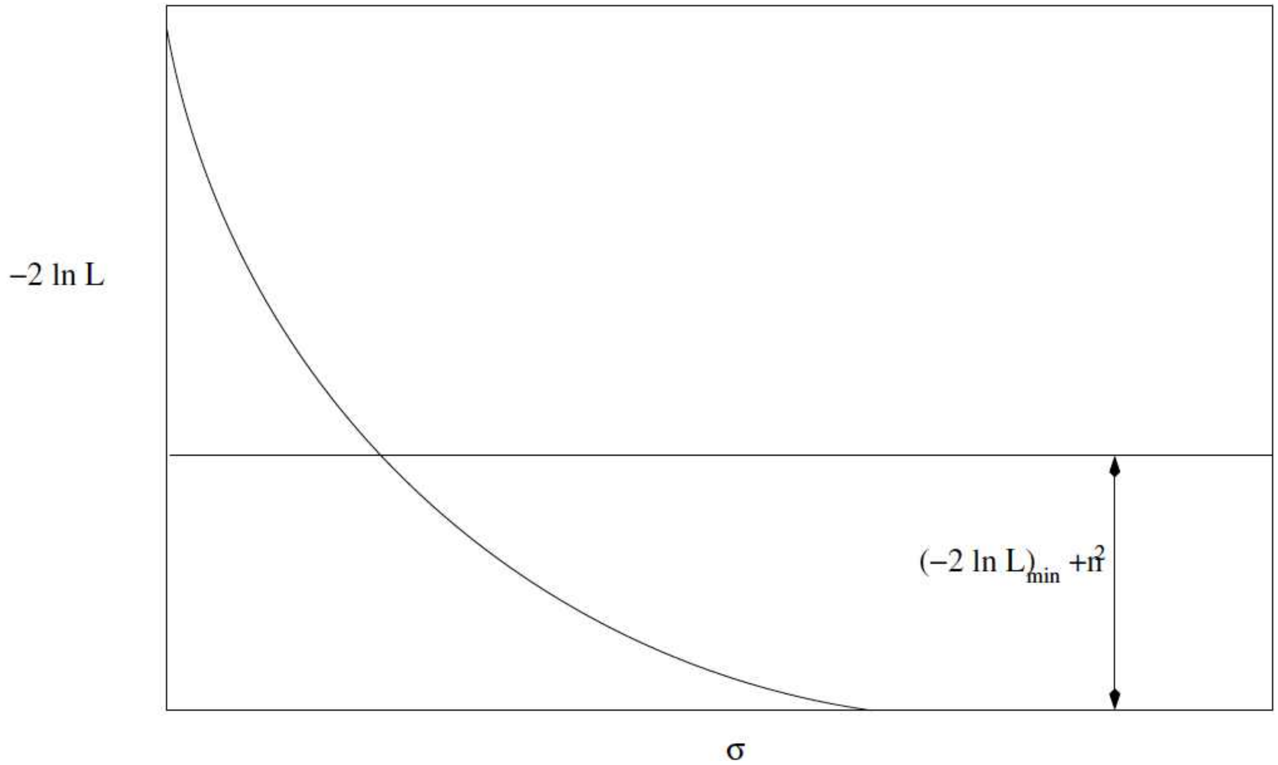
All response functions vanish for $v \rightarrow v_{th}$ (low-energy threshold - extended by energy resolution)

m_χ=20 GeV

This means that the likelihood is degenerate when:

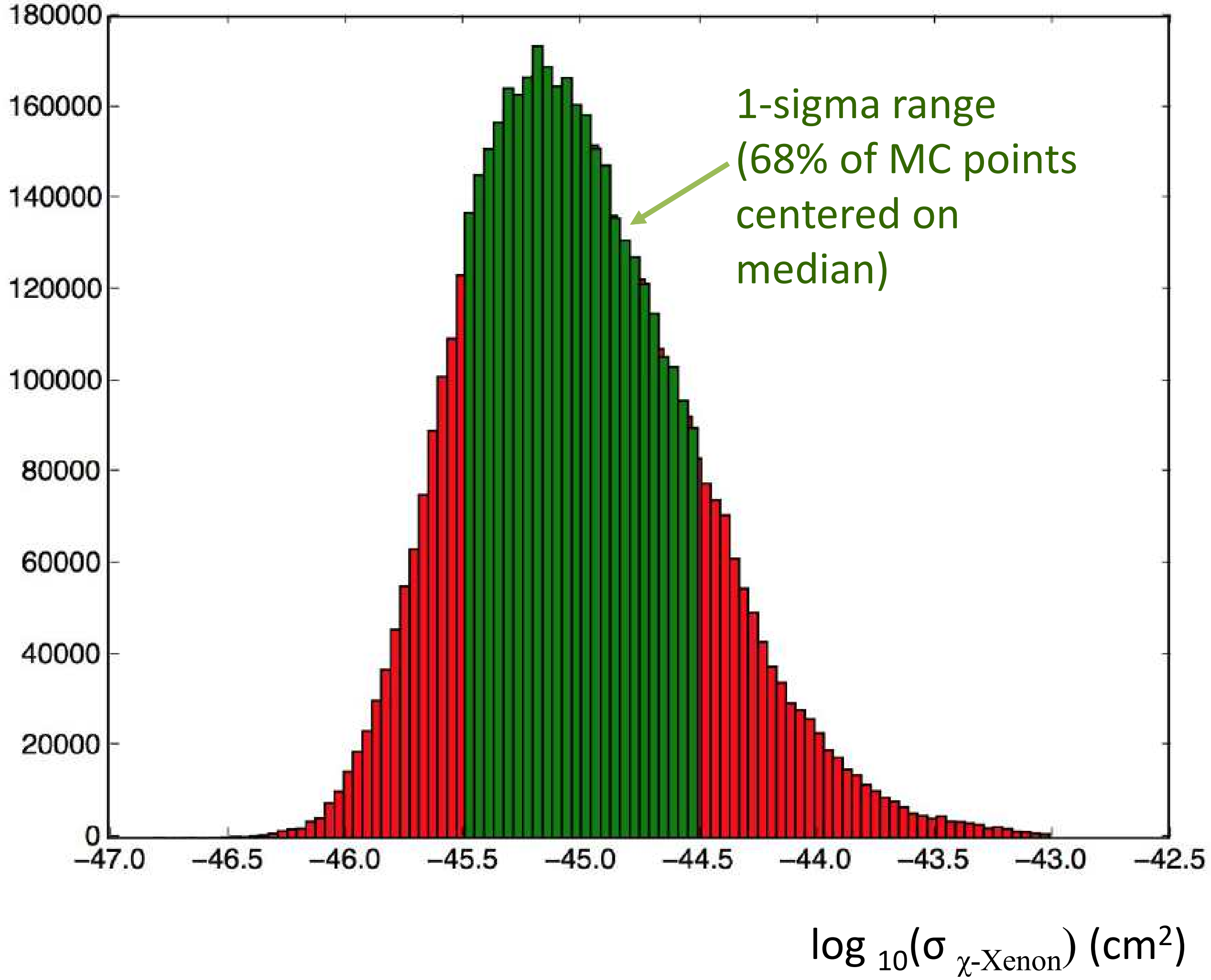
$$\lim_{v_k \to v_{th}} \mathcal{H}_i(v_k) = 0, \ \sigma \to \infty \text{ at fixed } \lambda_k \mathcal{H}_i(v_k)$$

→ profiling of σ at n sigma can only yields a lower bound



Lower boun on σ → upper bound on NP scale

Actually can also get an *interval* on σ through Bayes theorem, but need to assume a *prior distribution* on the parameters (namely, on the v_k's):



(flat prior in range $0<v_k<v_{esc}$=782 km/sec)

N.B. -2 ln L flat directions with $\sigma \rightarrow \infty$ for $v_k \rightarrow v_{th}$, small volume diluted in flat prior range

Halo-independent yearly-modulated fractions

Due to the rotation of the Earth around the Sun the signal in a direct detection experiment depends on time. Assuming that the only time dependence is due to the boost from the Galactic to the Lab rest frame:

$$S(t)_{[E_1',E_2']} = \int \mathcal{H}_{[E_1',E_2']}(v)\, f(v,t)\, dv =$$

$$S(t)_{0,[E_1',E_2']} + S_{m,[E_1',E_2']} \cos\left[\frac{2\pi}{365\,\mathrm{days}}(t-t_0)\right] =$$

$$\int \mathcal{R}_{[E_1',E_2']}(v)\tilde{\eta}(v,t)\, dv =$$

$$\int \mathcal{R}_{[E_1',E_2']}(v) \left\{ \tilde{\eta}_0(v) + \tilde{\eta}_1(v) \cos\left[\frac{2\pi}{365\,\mathrm{days}}(t-t_0)\right]\right\} dv$$

Standard lore: need to know explicitly f(v) to get the modulated fraction $\tilde{\eta}_1(v)/\tilde{\eta}_0(v)$ (ex: <10% for a Maxwellian)

P. Gondolo, S. Scopel arXiv:1703.08942

The problem: how to estimate the time dependence (yearly modulation) of the signal from the time dependence of an unknown quantity [f(v)]?

Very simple solution: a change of variable!

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$$f(\vec{v},t) = f_{gal}\left(\vec{u} = \vec{v} + \vec{v}_{\odot} + \vec{v}_{\oplus}(t)\right)$$

Lab rest frame

Galaxy rest frame

velocity of the Sun

velocity of the Earth

Change integration variable from v (lab frame) to u (Galactic frame):

$$S_{[E_1',E_2']}(t) = \int \mathcal{H}_{[E_1',E_2']}\left(\vec{u} - \vec{v}_{\odot} - \vec{v}_{\oplus}(t)\right) f_{gal}(\vec{u})\, d^3u$$

N.B. : the time dependence is now is only in the response function

The unmodulated and modulated parts are obtained via a Fourier time analysis:

$$S_{0,[E_1',E_2']} = \frac{1}{T}\int_0^T dt, S_{[E_1',E_2']}(t)$$

$$S_{m,[E_1',E_2']} = \frac{1}{T}\int_0^T dt, \cos\left[\frac{2\pi}{365}(t-t_0)\right] S_{[E_1',E_2']}(t)$$

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Assuming isotropic response (this is usually the case) this implies:

$$S_{0,[E'_1,E'_2]} = \int \mathcal{H}_{0,[E'_1,E'_2]}(u)\, f_{gal}(u)\, du$$

$$S_{m,[E'_1,E'_2]} = \int \mathcal{H}_{m,[E'_1,E'_2]}(u)\, f_{gal}(u)\, du$$

$$u \equiv |\vec{u}|$$

with:

$$\mathcal{H}_{0,\mathrm{gal}}(u) = \frac{1}{4\pi} \int d\Omega_u \, \frac{1}{T} \int_0^T dt\, \mathcal{H}\left(|\vec{u} - \vec{v}|\right)$$

$$\mathcal{H}_{m,\mathrm{gal}}(u) = \frac{1}{4\pi} \int d\Omega_u \, \frac{1}{T} \int_0^T dt \cos\left[\frac{2\pi}{365}(t - t_0)\right] \mathcal{H}\left(|\vec{u} - \vec{v}|\right)$$

N.B. The modulated amplitude depends on the cosine transform of the response function, which is completely known→ modulation as a property of the detector

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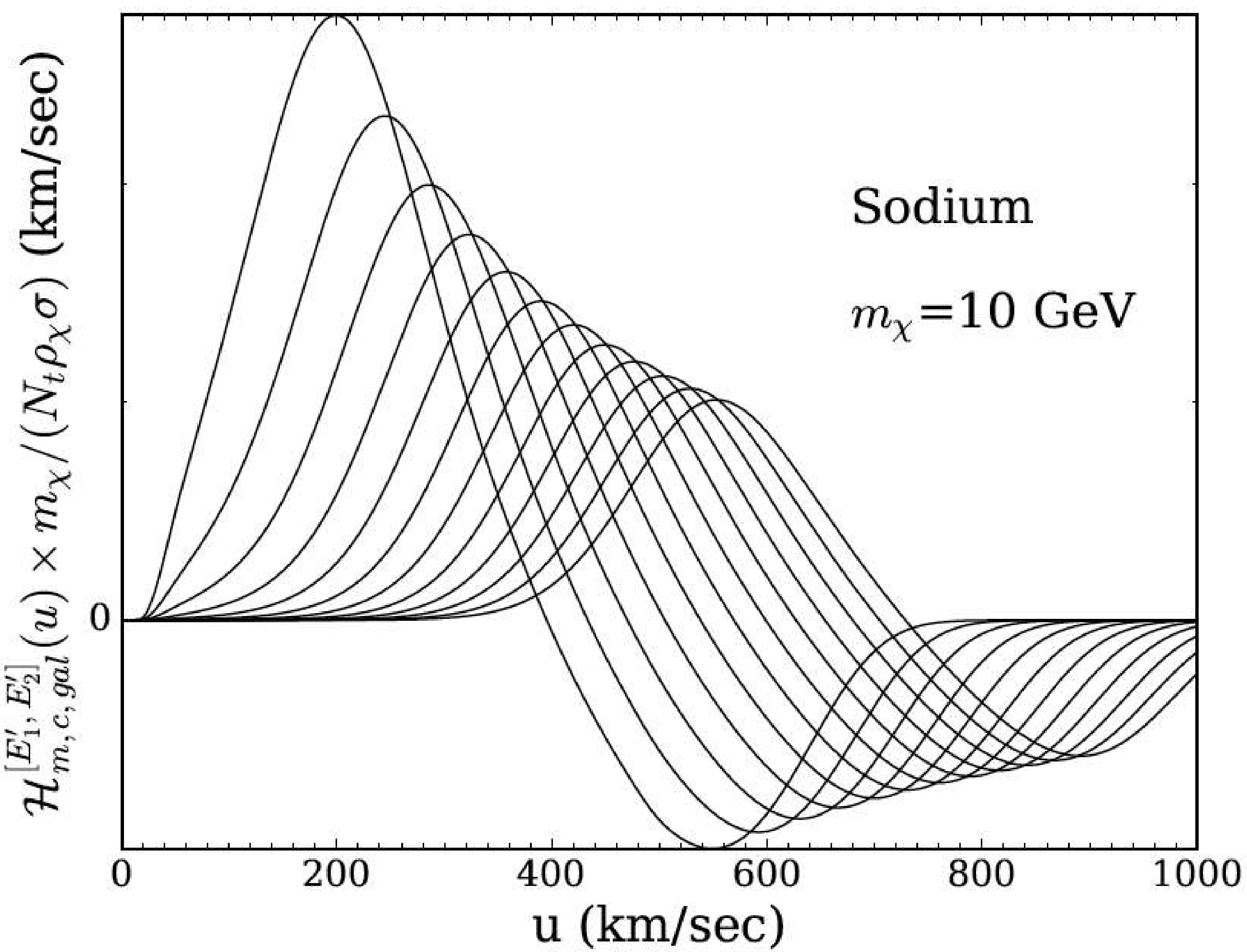
Modulated response functions in DAMA (galactic rest frame)



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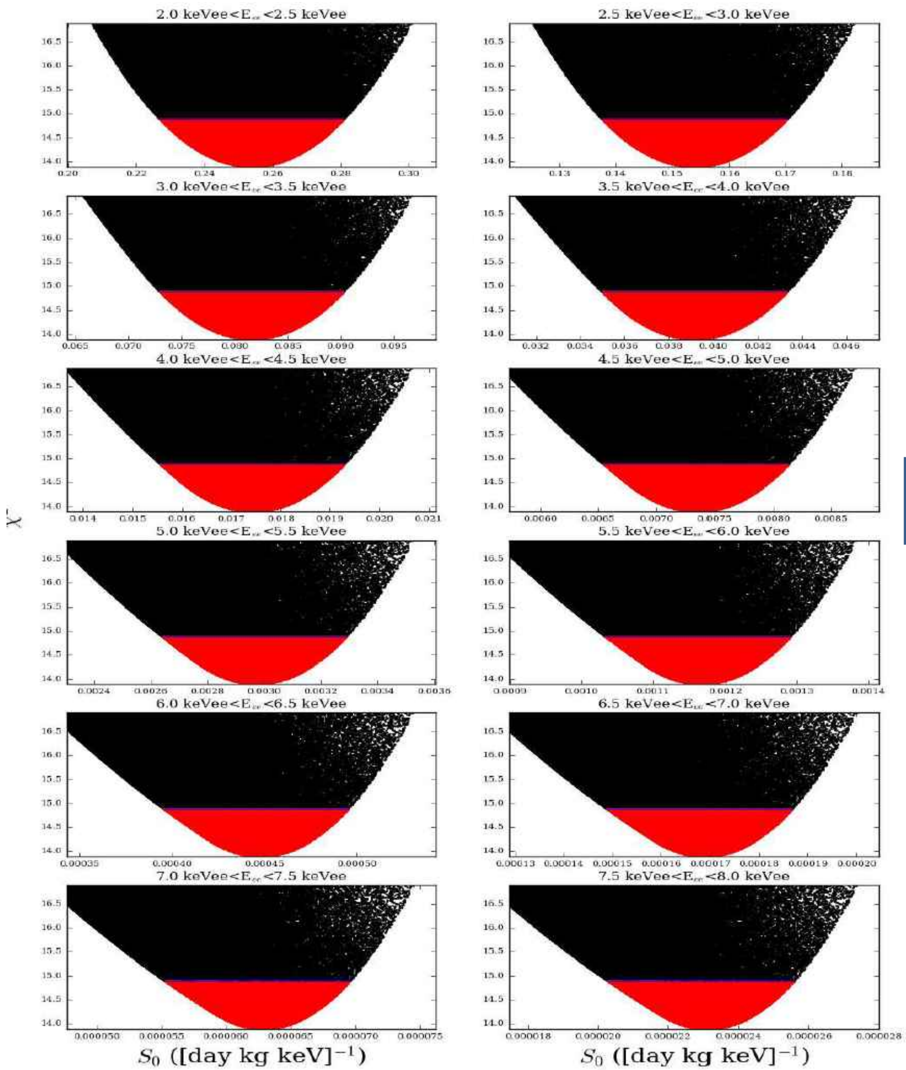
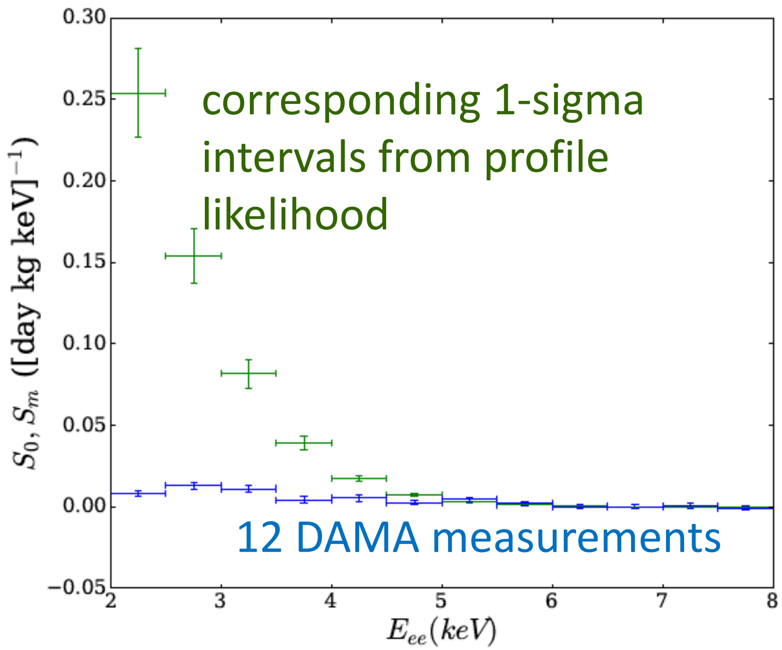
S_0 and S_m are both given by the integral of a know response function times *the same* unknown f(u) → use theorem on extreme distributions to profile out the unmodulated amplitudes in DAMA starting from measured modulated amplitudes

$$-2\ln\mathcal{L} = \sum_{k=1}^{12}\left(\frac{S_m^k - S_{m,exp}^k}{\sigma_k}\right)^2$$

$$f_{gal}(u) = \sum_{k=1}^{N}\lambda_k\delta(u-u_k), \quad N = 1, 12$$

N.B.: The velocity distribution is a *nuisance* parameter, maneageable because we need it only on the boundary, where the dimensionality is reduced





m_χ=5 GeV

5×10^6 points Markov chain, 250 independent walkers Metropolis-Hastings sampler

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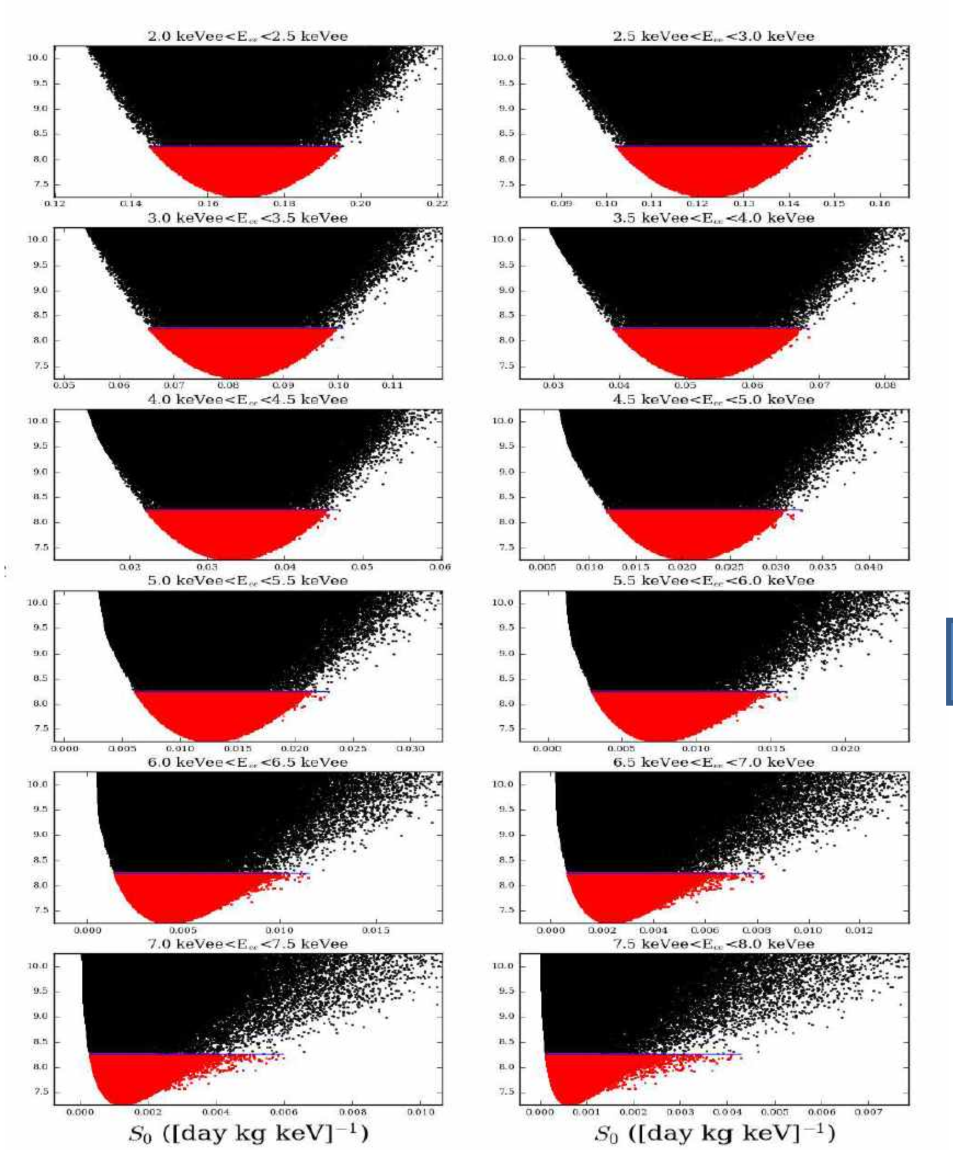

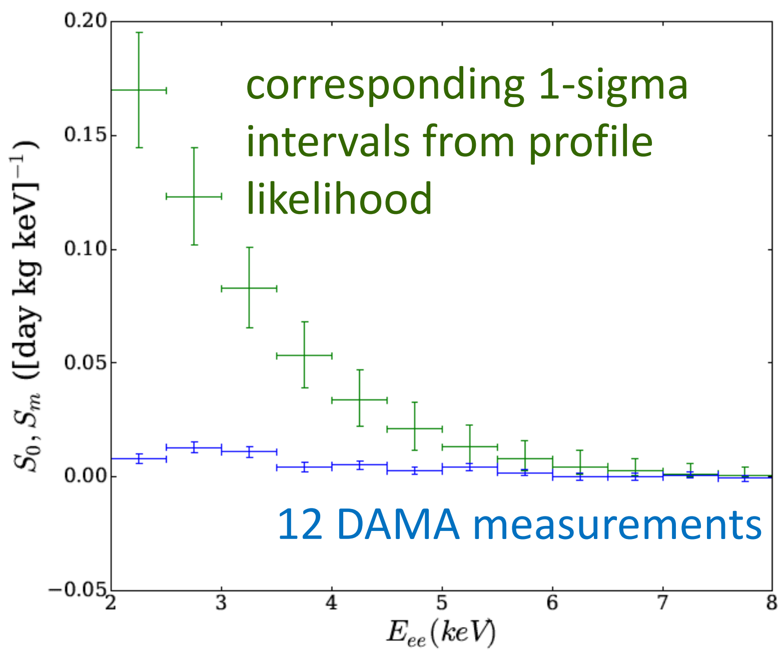

m_χ=10 GeV

5x10^6 points Markov chain, 250 independent walkers Metropolis-Hastings sampler

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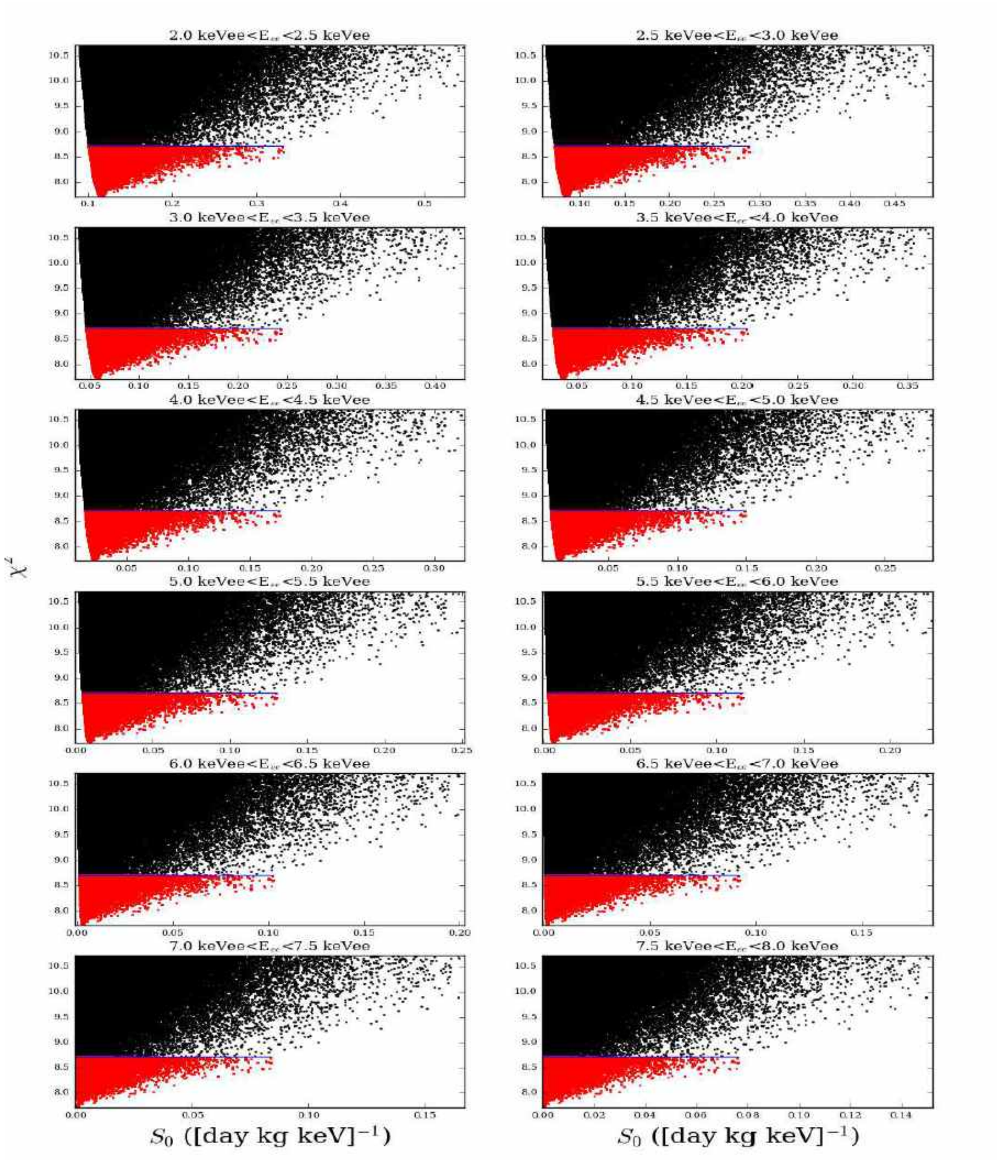

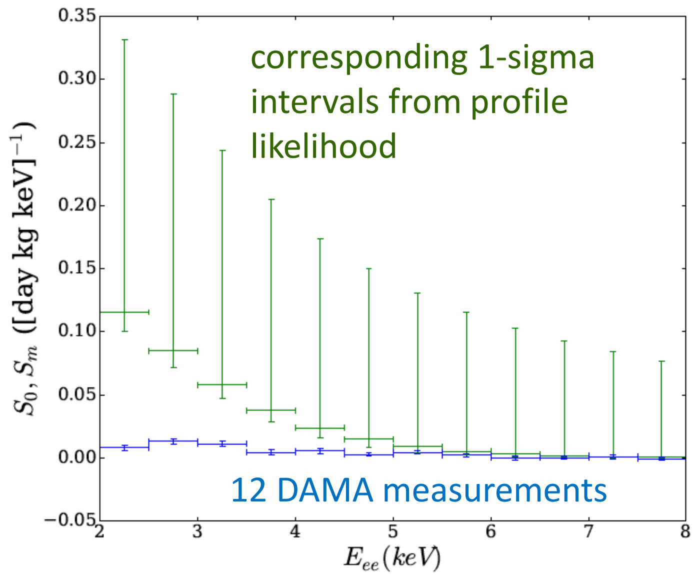

m_χ=15 GeV

$5\text{x}10^6$ points Markov chain, 250 independent walkers Metropolis-Hastings sampler

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Disentangled from the background for
any isotropic velocity distribution

E_i [keVee]	$S_{m,i}$	$S_{0,i}$ $m_\chi = 5$ GeV	$S_{0,i}$ $m_\chi = 10$ GeV	$S_{0,i}$ $m_\chi = 15$ GeV	B_i
2.0–2.5	0.016(39)	$0.25^{+0.027}_{-0.027}$	$0.17^{+0.026}_{-0.025}$	$0.12^{+0.22}_{-0.016}$	1.029
2.5–3.0	0.026(44)	$0.15^{+0.016}_{-0.016}$	$0.12^{+0.022}_{-0.021}$	$0.085^{+0.20}_{-0.013}$	1.228
3.0–3.5	0.022(44)	$0.082^{+0.0086}_{-0.0087}$	$0.083^{+0.018}_{-0.017}$	$0.058^{+0.19}_{-0.011}$	1.294
3.5–4.0	0.0084(40)	$0.039^{+0.0041}_{-0.0042}$	$0.053^{+0.015}_{-0.014}$	$0.038^{+0.17}_{-0.0094}$	1.140
4.0–4.5	0.011(36)	$0.017^{+0.0018}_{-0.0019}$	$0.034^{+0.013}_{-0.012}$	$0.024^{+0.15}_{-0.0078}$	0.956
4.5–5.0	0.0054(32)	$0.0074^{+0.00078}_{-0.00081}$	$0.021^{+0.012}_{-0.0093}$	$0.015^{+0.13}_{-0.0062}$	0.853
5.0–5.5	0.0089(32)	$0.003^{+0.00031}_{-0.00033}$	$0.013^{+0.0098}_{-0.0070}$	$0.0089^{+0.12}_{-0.0046}$	0.868
5.5–6.0	0.0039(31)	$0.0012^{+0.00012}_{-0.00013}$	$0.0078^{+0.0082}_{-0.0049}$	$0.0052^{+0.11}_{-0.0032}$	0.853
6.0–6.5	0.00018(31)	$4.5^{+0.47}_{-0.52} \times 10^{-4}$	$0.0045^{+0.0069}_{-0.0031}$	$0.003^{+0.10}_{-0.002}$	0.868
6.5–7.0	0.00018(28)	$1.7^{+0.18}_{-0.2} \times 10^{-4}$	$0.0025^{+0.0057}_{-0.0019}$	$0.0016^{+0.091}_{-0.0012}$	0.860
7.0–7.5	0.0015(28)	$6.3^{+0.66}_{-0.76} \times 10^{-5}$	$0.0013^{+0.0047}_{-0.0011}$	$8.3^{+830}_{-6.5} \times 10^{-4}$	0.860
7.5–8.0	−0.0013(29)	$2.3^{+0.24}_{-0.28} \times 10^{-5}$	$6.8^{+36}_{-5.6} \times 10^{-4}$	$4.2^{+760}_{-3.4} \times 10^{-4}$	0.890

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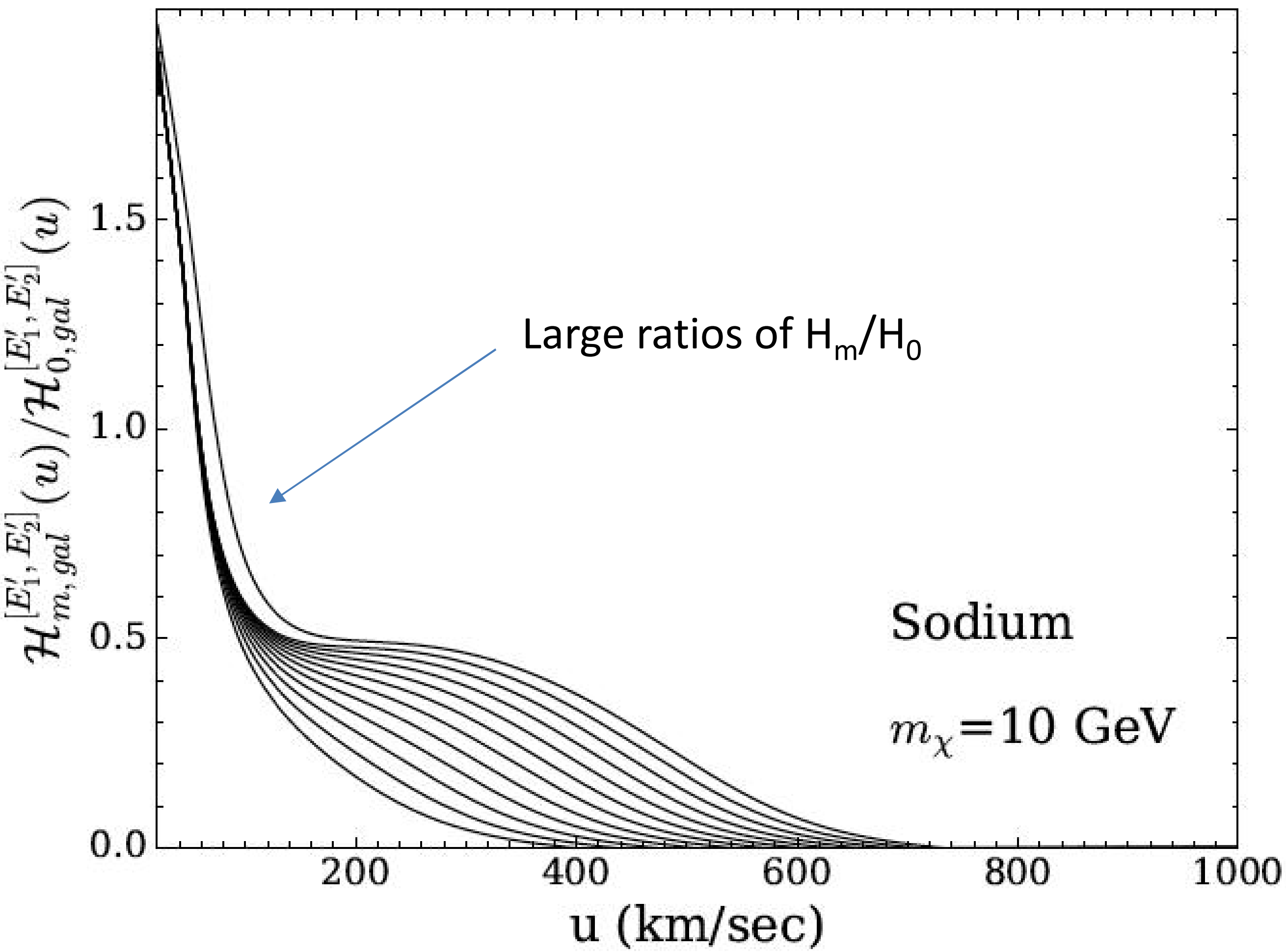
1-sigma ranges for modulation fractions:
m_χ=5: 0.03<S_m/S_0<0.13; m_χ=10: 0.05<S_m/S_0<0.13; m_χ=20: 0.07<S_m/S_0<0.19

N.B. Non-isotropic distributions can easily predict larger modulation fractions (up to 100 %).
However, also the space of isotropic f(u) contains large modulation solutions, which however are disfavored by the data



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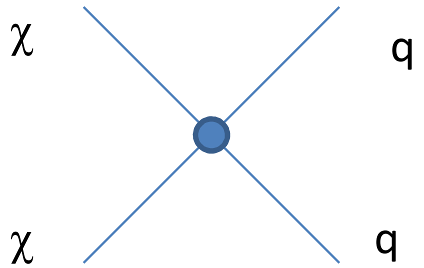
A few comments:

- Cannot adopt this procedure to get a best-likelihood determination of f(v) - the streams parameterization of f(v) is only valid on the boundaries
- Mathematically and statistically sound procedure, with many potential applications.
- Straightforward generalization to non-isotropic velocity distribution – no conceptual problems, but numerically more challenging.

From direct detection to thermal relic abundance

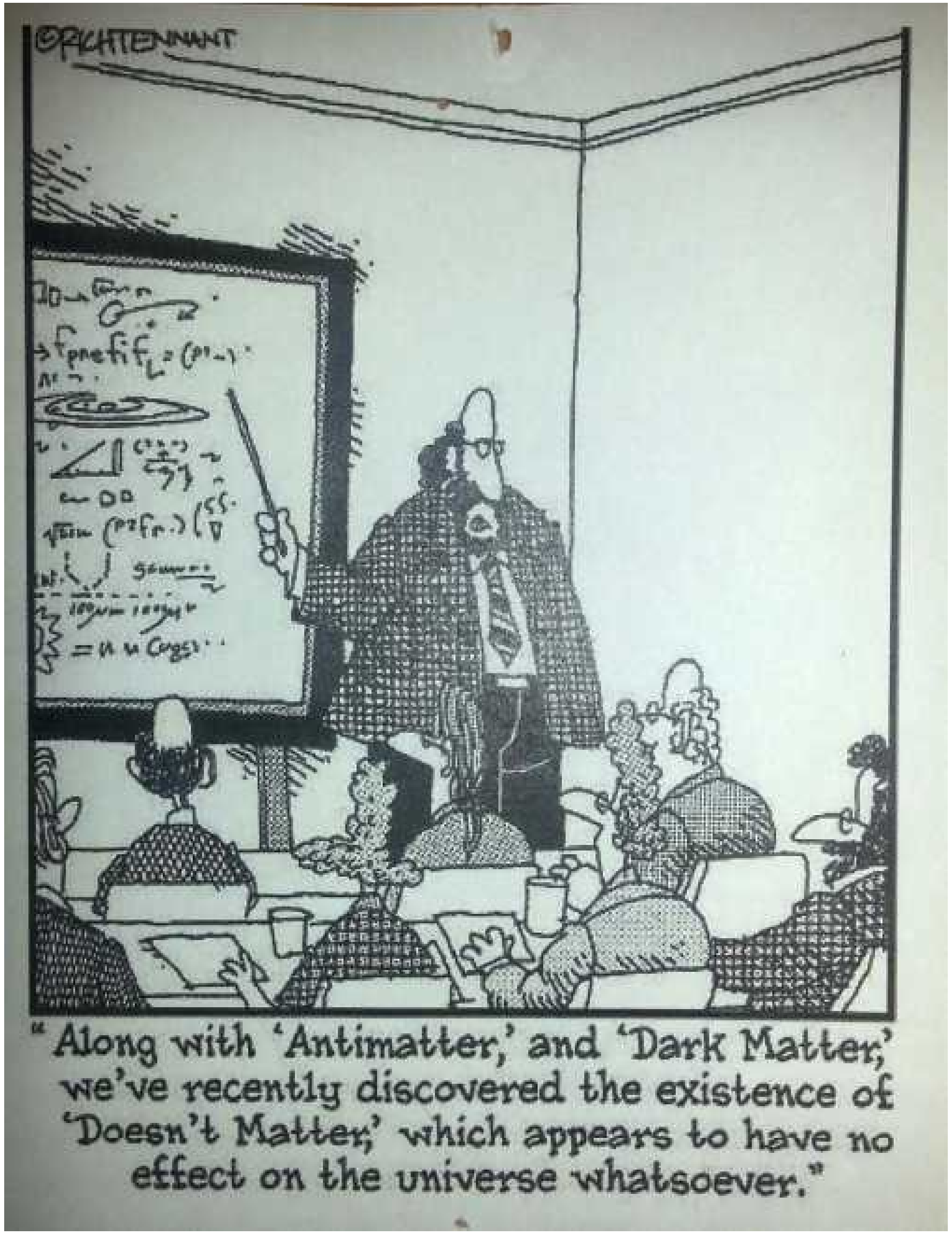
If the same current dominates the EFT up to the annihilation scale at freeze out (E_{CM}~2 m_{χ}) ,can calculate the WIMP thermal relic abundance



$$\Omega h^2 \simeq \frac{< \sigma v >_0}{< \sigma v >}$$

$$< \sigma v >_0 \simeq 2 \times 10^{-9}\, \mathrm{GeV}^{-2}$$

$$\Rightarrow \quad < \sigma v > = \frac{1}{\Lambda^4} \sum_q |c_q|^2 < \tilde{\sigma v} >_q$$



How to handle subdominant DM candidates?

Actually, one should rescale the local DM density ρ_χ used to extract Λ:

$$\rho_\chi \rightarrow \xi \rho_\chi$$

with the rescaling factor:

$$\xi \equiv \frac{\Omega_\chi h^2}{(\Omega h^2)_{obs}} \simeq \frac{\Lambda^4}{\sum_q |c_q|^2 \frac{<\tilde{\sigma v}>_q}{<\sigma v>_0}}$$

In this case a direct detection experiment measures the combination $\varsigma\sigma_{\mathrm{ref,p}}$ with:

$$\sigma_{ref,p} \simeq \frac{\mu^2}{\pi} \frac{c_p^2}{\Lambda^4}$$

$$\xi\sigma_{ref,p} = \frac{\mu^2}{\pi} \frac{c_p^2}{\sum_q |c_q|^2 \frac{<\tilde{\sigma v}>_q}{<\sigma v>_0}} = \frac{\mu^2}{\pi} \frac{c_p^2}{\tilde{\Lambda}^4} = \frac{\mu^2}{\pi} \frac{1}{\tilde{\Lambda}_p^4}$$

<u>dependence on Λ cancels out</u>

Expected effective scale:

$$\tilde{\Lambda}_{p,th} = \left(\frac{\sum_q c_q^2 < \tilde{\sigma v} >_q}{c_p^2 < \tilde{\sigma v} >_0} \right)^{\frac{1}{4}} = \left(\frac{\sum_q r_q^2 < \tilde{\sigma v} >_q}{(c_p/c_u)^2 < \tilde{\sigma v} >_0} \right)^{\frac{1}{4}} = \tilde{\Lambda}_{p,\Omega_0}$$

$\Lambda_{p,\Omega}$ is just the Λ_p fixed by the observed by the relic abundance, and would be m̃easured also ĩf $\Omega h^2 << (\Omega h^2)_{obs}$; $r_q = c_q/c_u$

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A direct detection experiment is bound to measure the corresponding “effective scale” :

$$\tilde{\Lambda}_{p,exp} = \frac{\Lambda}{\xi^{1/4} c_p^{1/2}} = \left(\frac{\mu_{\chi\mathcal{N}}^2}{\pi(\xi\sigma_{ref})_{exp}} \right)^{\frac{1}{4}}$$

If the velocity distribution is known a given experimental excess fixes $\varsigma\sigma$ and so $\tilde{\Lambda}_{p,exp}$, which for a given effective model (i.e. a given m_χ and set of r_q‘s) is bound to be equal to $\tilde{\Lambda}_{p,\Omega_0}$ irrespective to the actual value of Ω

The bottom line: a DM experiment is bound to measure an effective scale corresponding to $\Omega=\Omega_0$ even if $\Omega<\Omega_0$

$$\tilde{\Lambda}_{p,exp} = \tilde{\Lambda}_{p,\Omega_0}$$

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On the other hand, if f(v) is not fixed, can get a lower bound on $\varsigma\sigma$ and so an upper bound on Λ_p

$$\tilde{\Lambda}^{max}_{p,exp} > \tilde{\Lambda}_{p,\Omega_0} > \tilde{\Lambda}^{min}_{p,\Omega_0}$$

Minimized with respect to the free parameters of the EFT (r_q's)

N.B. consistency test , not just a requirement that $\Omega \leq \Omega_0$ when the parameters of the EFT are fixed by direct detection

Actually, the acceptable values of $\Lambda_{p,exp}$ are those for which $\Omega \geq \Omega_0$ when $\varsigma=1$ ("overclosure" is fine, "underclosure" is not)

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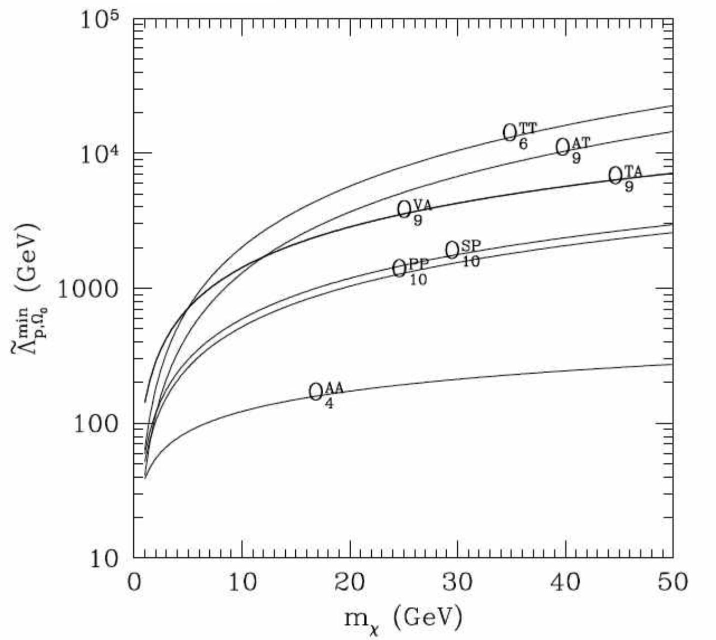
pSIDM generalized to spin-dependent EFT models with momentum dependence

	Relativistic EFT	Non-relativistic limit	$\sum_i \mathcal{O}_i$	cross section scaling
$\mathcal{O}_4^{AA}$	$\bar\chi_1\gamma^\mu\gamma^5\chi_2\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}$+h.c.	$-4\vec{S}_\chi\cdot\vec{S}_\mathcal{N}$	$-4\mathcal{O}_4$	$W^{\tau\tau'}_{\Sigma''}(q^2)+W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_9^{VA}$	$\bar\chi_1\gamma^\mu\chi_2\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}$+h.c.	$+\frac{2}{m_{WIMP}}i\vec{S}_\chi\cdot\left(\vec{S}_\mathcal{N}\times\vec{q}\right)$	$\simeq 2\frac{m_\mathcal{N}}{m_{WIMP}}\mathcal{O}_9$	$\simeq q^2 W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_9^{TA}$	$\bar\chi_1 i\sigma_{\mu\nu}\frac{q^\nu}{m_M}\chi_2\bar{\mathcal{N}}\gamma^\mu\gamma_5\mathcal{N}$+h.c.	$4i(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot\vec{S}_\mathcal{N}$	$4\frac{m_\mathcal{N}}{m_M}\mathcal{O}_9$	$q^2 W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_9^{AT}$	$\bar\chi_1\gamma^\mu\gamma_5\chi_2\bar{\mathcal{N}}i\sigma_{\mu\nu}\frac{q^\nu}{m_M}\mathcal{N}$+h.c.	$4i\vec{S}_\chi\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_\mathcal{N})$	$-4\frac{m_\mathcal{N}}{m_M}\mathcal{O}_9$	$q^2 W^{\tau\tau'}_{\Sigma'}(q^2)$
$\mathcal{O}_{10}^{SP}$	$i\bar\chi_1\chi_2\bar{\mathcal{N}}\gamma^5\mathcal{N}$+h.c.	$i\frac{\vec{q}}{m_\mathcal{N}}\cdot\vec{S}_\mathcal{N}$+h.c.	$\mathcal{O}_{10}$	$q^2 W^{\tau\tau'}_{\Sigma''}(q^2)$
$\mathcal{O}_6^{PP}$	$\bar\chi_1\gamma_5\chi_2\bar{\mathcal{N}}\gamma^5\mathcal{N}$+h.c.	$-\frac{\vec{q}}{m_{WIMP}}\cdot\vec{S}_\chi\frac{\vec{q}}{m_\mathcal{N}}\cdot\vec{S}_\mathcal{N}$	$-\frac{m_\mathcal{N}}{m_{WIMP}}\mathcal{O}_6$	$q^4 W^{\tau\tau'}_{\Sigma''}(q^2)$
$\mathcal{O}_6^{T'T'}$	$\bar\chi_1 i\sigma^{\mu\alpha}\frac{q_\alpha}{m_M}\gamma_5\chi_2\bar{\mathcal{N}}i\sigma_{\mu\beta}\frac{q^\beta}{m_M}\gamma_5\mathcal{N}$+h.c.	$4\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\frac{\vec{q}}{m_M}\cdot\vec{S}_\mathcal{N}$	$4\frac{m_\mathcal{N}^2}{m_M^2}\mathcal{O}_6$	$q^4 W^{\tau\tau'}_{\Sigma''}(q^2)$

$\tilde\Lambda_{p,exp}$ (maximal value in allowed param. Space)

Non–relativistic operator	Halo–independent	Maxwellian
$\mathcal{O}_4$	163	20.4
$\mathcal{O}_6$	23.0	2.79
$\mathcal{O}_9$	27.5	3.27
$\mathcal{O}_{10}$	3.01	0.32

$\tilde\Lambda_{p,\Omega_0}$ (minimum value)



Only the standard spin-dependent interaction passes the consistency test

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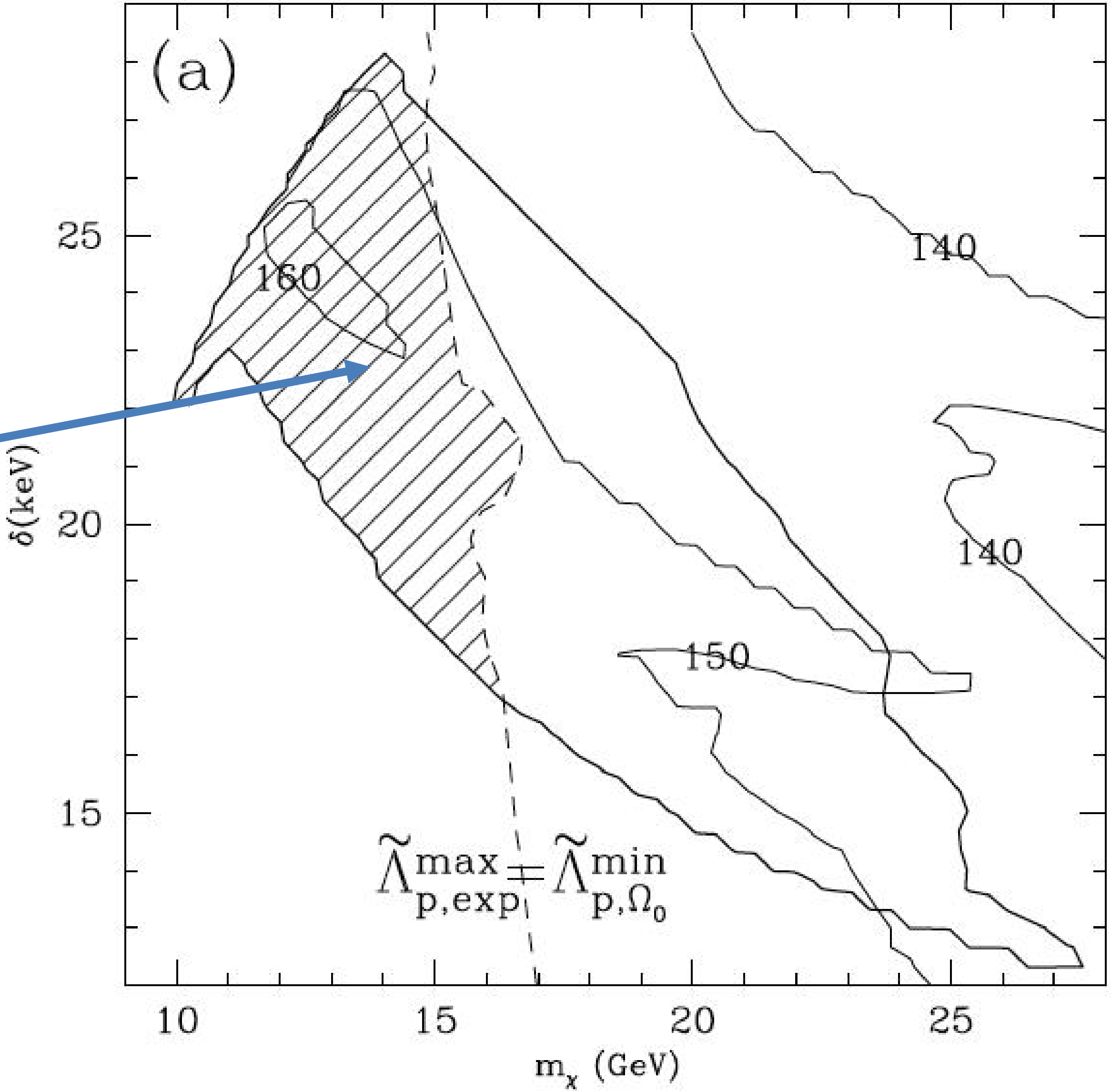
pSIDM, standard spin coupling



DAMA+thermal relic

$(\tilde{\Lambda}^{max}_{p,exp} \geq \tilde{\Lambda}^{min}_{p,\Omega_0})$

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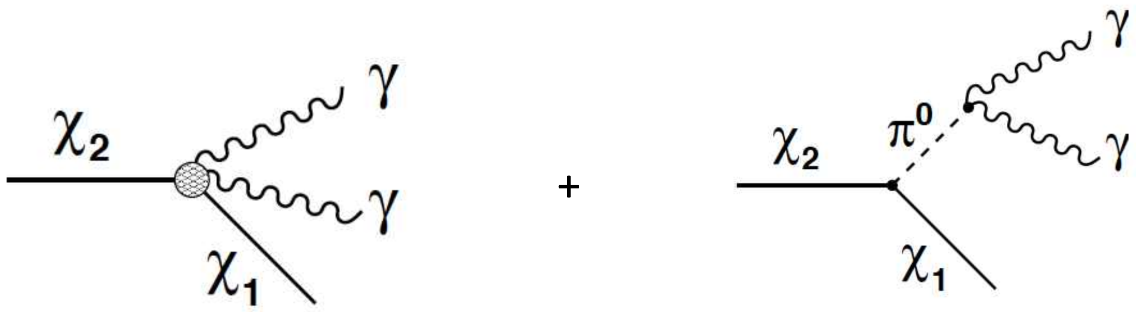
One complication in the pSIDM scenario: χ_2 states decay to χ_1's too slowly if only the effective operator that drives direct detection is present



$$\Gamma_{\gamma\gamma} = 7.2 \times 10^{-56} \left(\frac{\delta}{10\ \text{keV}} \right)^9 \left(\frac{10\ \text{GeV}}{\tilde{\Lambda}} \right)^4 \text{GeV}$$

Age of the Universe: $1.5\times10^{-42}\ \text{GeV}^{-1}$

Downscatterings of χ_2 states are excluded by bubble chambers and droplet detectors, must assume some additional decay mechanism to get rid of them!

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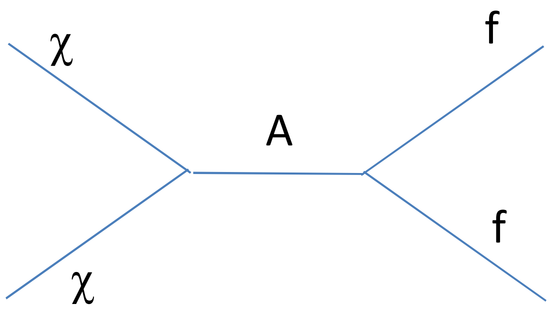
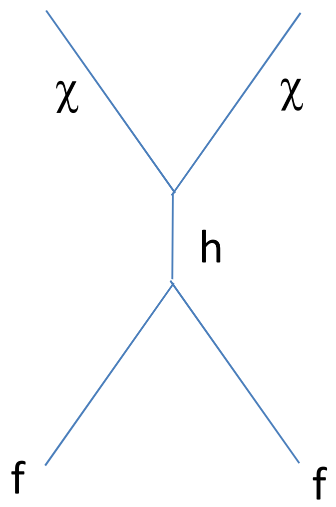
CAVEAT! The counter-example of Higgs-doublet mediation (ex: SUSY)

Several neutral mass eigenstates present (scalars h,H, pseudoscalars A)



In the NR limit a fermion-antifermion (or two-Majorana fermion) state has parity -1→ pseudoscalar Higgs boson A exchange dominates in s-wave annihilation



In direct detection pseudoscalar exchange implies velocity suppression → scalar Higgs boson h dominates

Typically $m_h \neq m_A$ and the correlation between Ωh^2 and $\sigma_{ref,p}$ is lost

Conclusions

• an explanation of the DAMA modulation result (or of other, less statistically significant "excesses") in terms of a WIMP signal is incompatible with the constraints published by other Dark Matter direct detection experiments only if direct-detection data are analyzed with ALL the following assumptions:

1) **spin-dependent** or **isoscalar spin-independent** cross section
2) **Maxwellian** velocity distribution in our Galaxy
3) WIMP **elastic** scattering

All these assumptions are reasonable if for instance the WIMP is a susy neutralino and if the DM particles in our Galaxy are fully thermalized.

•However, without any hint from the LHC about the underlying fundamental physics and without a detailed knowledge of the merger history of our Galaxy it appears safer to adopt a bottom-up layman approach. This includes:

1) using non-relativistic effective theory which introduces new response functions with explicit dependence on the transferred momentum and the WIMP incoming velocity
2) factorizing the halo-function dependence
3) allowing for inelastic scattering
4) allowing for isovector couplings

• In this way a much wider parameter space opens up.

• First explorations show that indeed compatibility between excesses and constraints can be achieved → **full correlation with indirect signals and relic abundance needs still to be worked out**

• **"Proofs of concept" (but if by chance you have a nice model for pSIDM (spin-dependent Inelastic Dark Matter that couples only to protons) it works fine for DAMA)**

- New methods using *representation theorems for distribution functions* allow to get intervals on unknown quantities for the most general halo function (i.e. trea=ng f(v) as a nuisance parameter) → for instance, in this way it is possible to get info on the average rate knowing only the modulated fractions
- Given a signal, strictly speaking halo-independent methods without *any* assumption on the velocity distribution f(v) can only yield a lower bound on the interac=on cross sec=on → an upper bound on σ requires some prior assumptions on the f(v)
- Direct detection is sensitive to the product of local density times cross sec=on → insensi=ve to the cut -off scale of the effective theory