Cosmological stochastic Higgs stabilization

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arXiv:1705.11178

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Outline

1. Introduction

2. Higgs domain wall

3. Vacuum stabilization and inflation

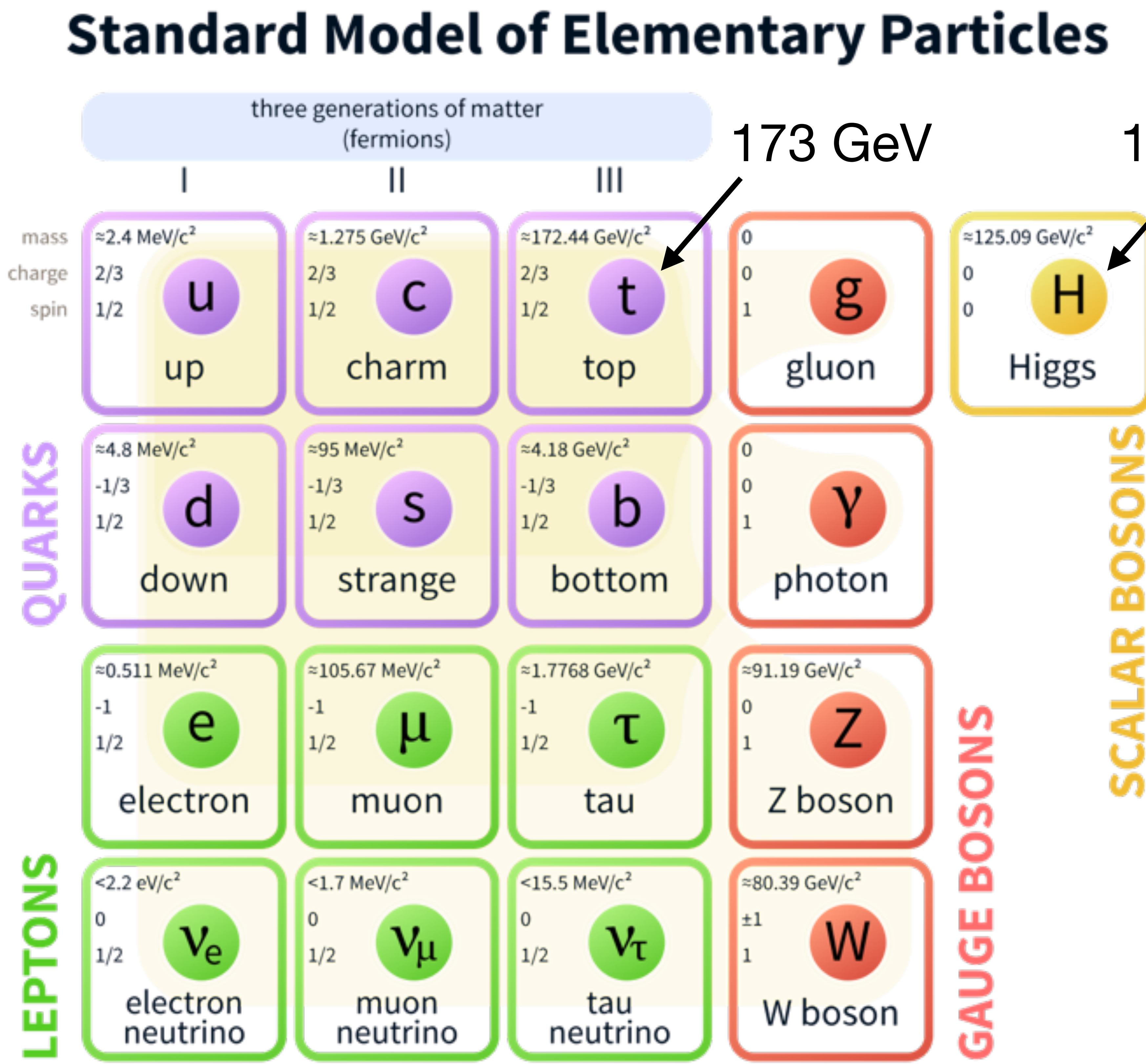
4. Multi-field stochastic dynamics during inflation

5. Summary

1. Introduction

173 GeV

125 GeV

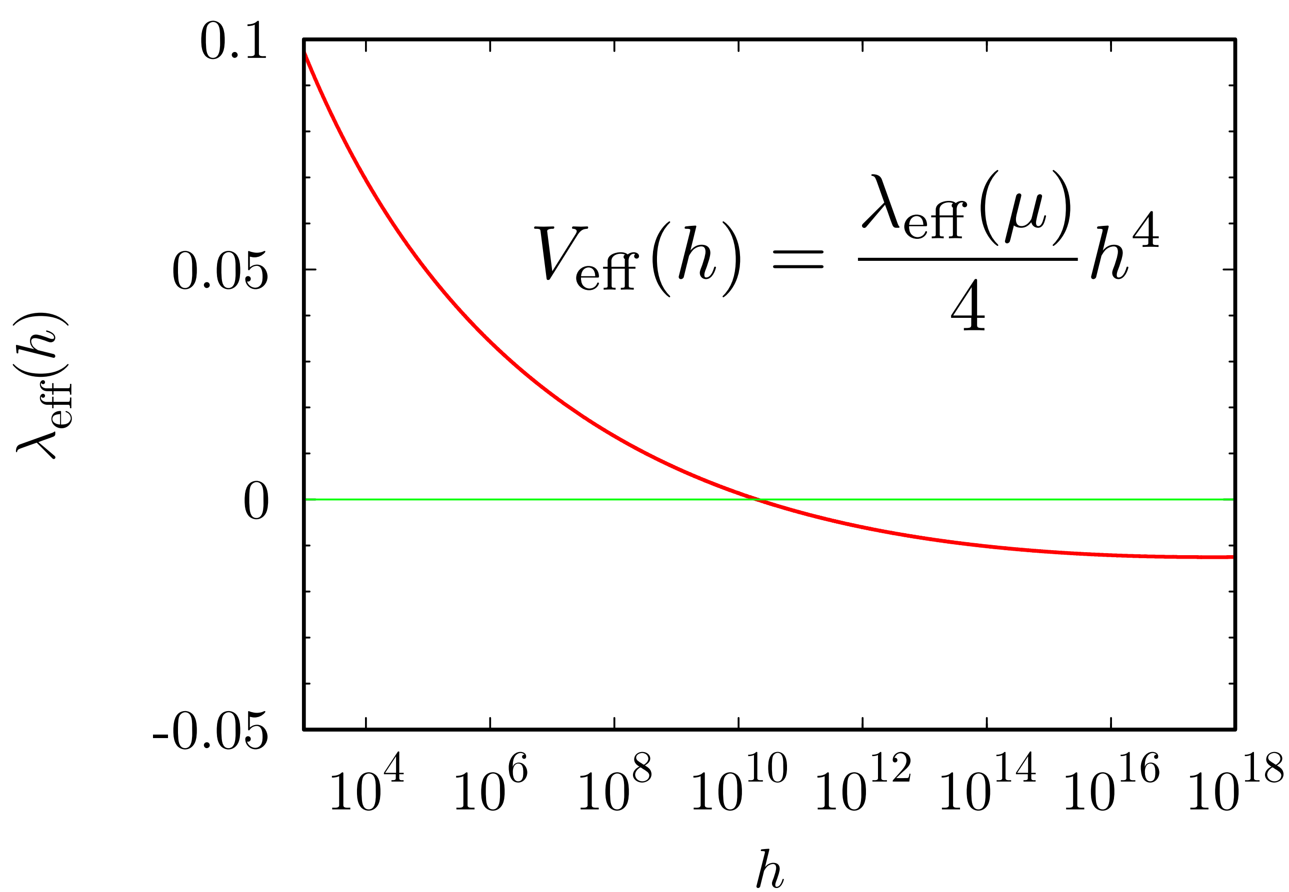


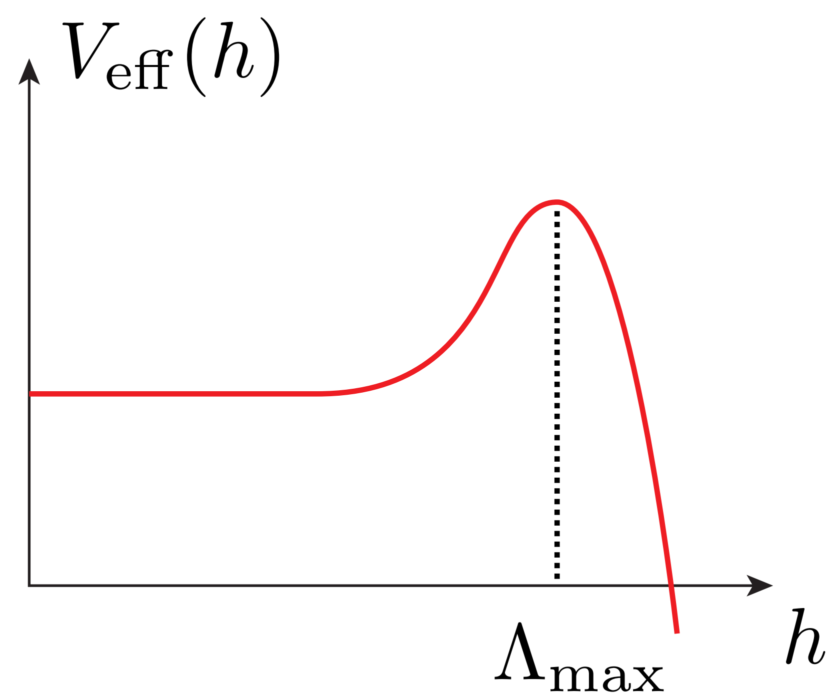
https://en.wikipedia.org/wiki/Standard_Model

Higgs & top masses, Higgs effective potential

$$m_h = 125.09 \pm 0.21 \pm 0.11 \text{ GeV}$$

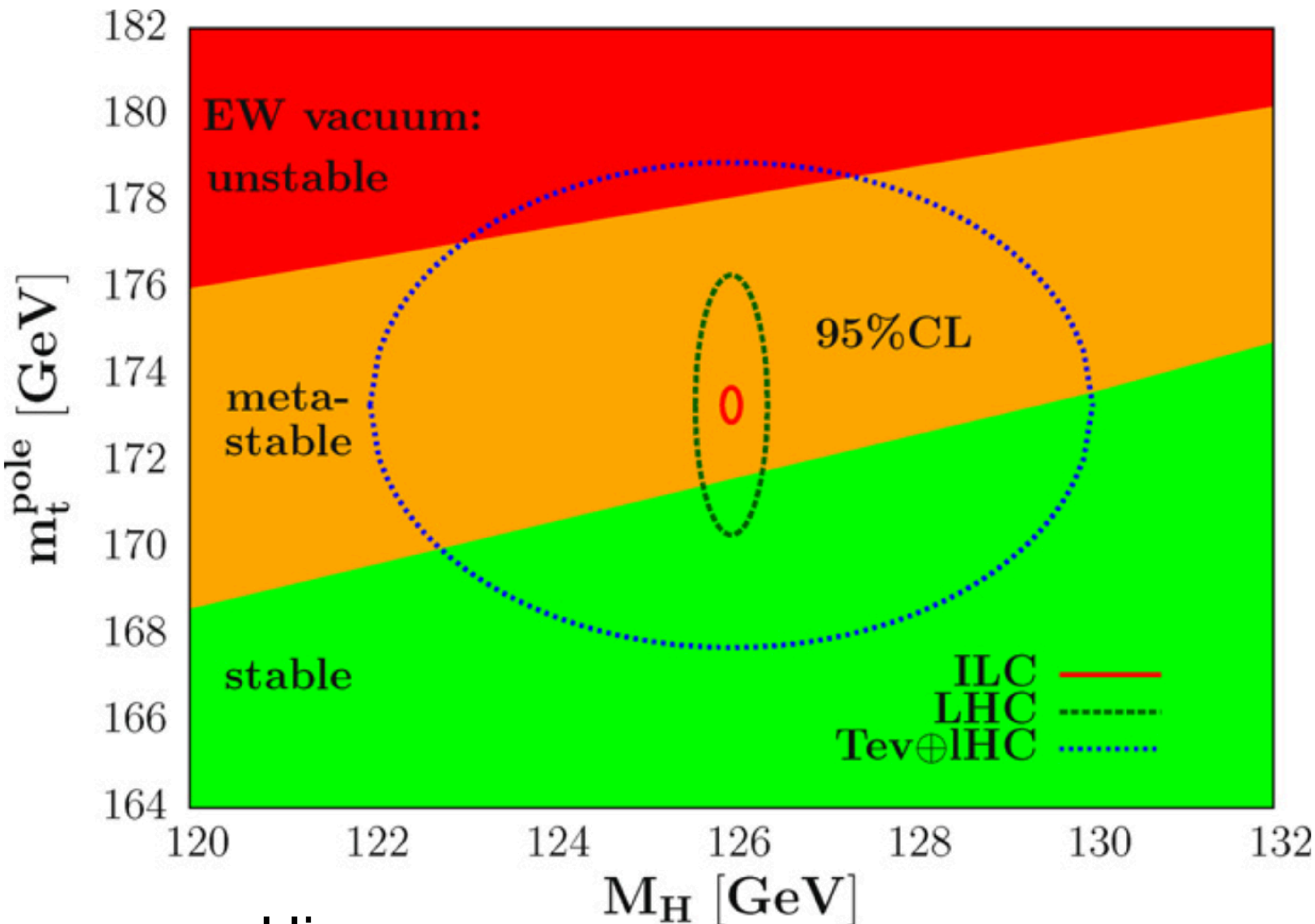
$$m_t = 173.34 \pm 0.76 \pm 0.3 \text{ GeV}$$





$$\Lambda_{\text{max}} \sim 10^{10-11} \text{ GeV}$$

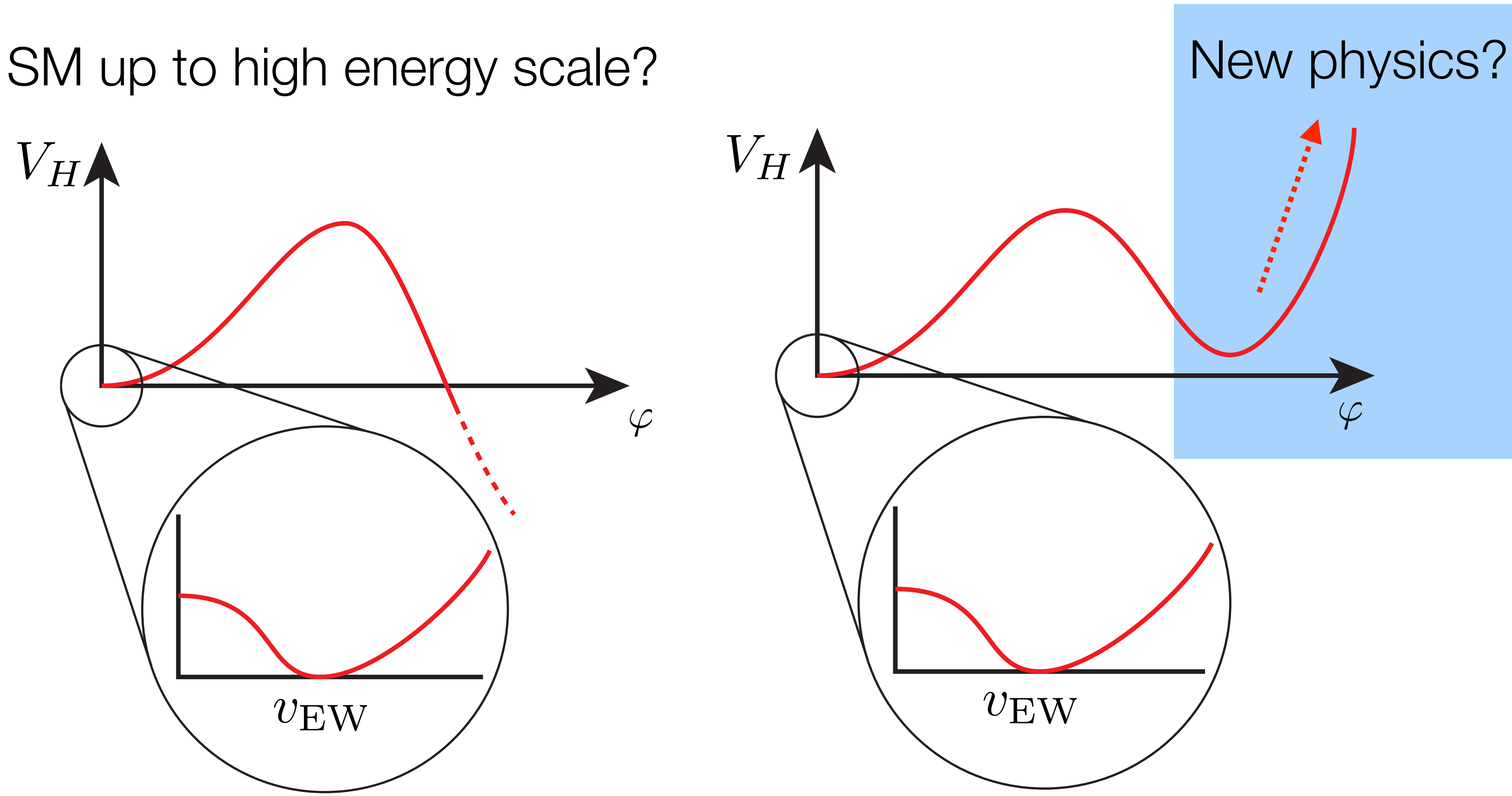
Higgs & top masses, vacuum stability



top quark mass

Higgs mass

Alekhin, Djouadi, Moch, 1207.0980

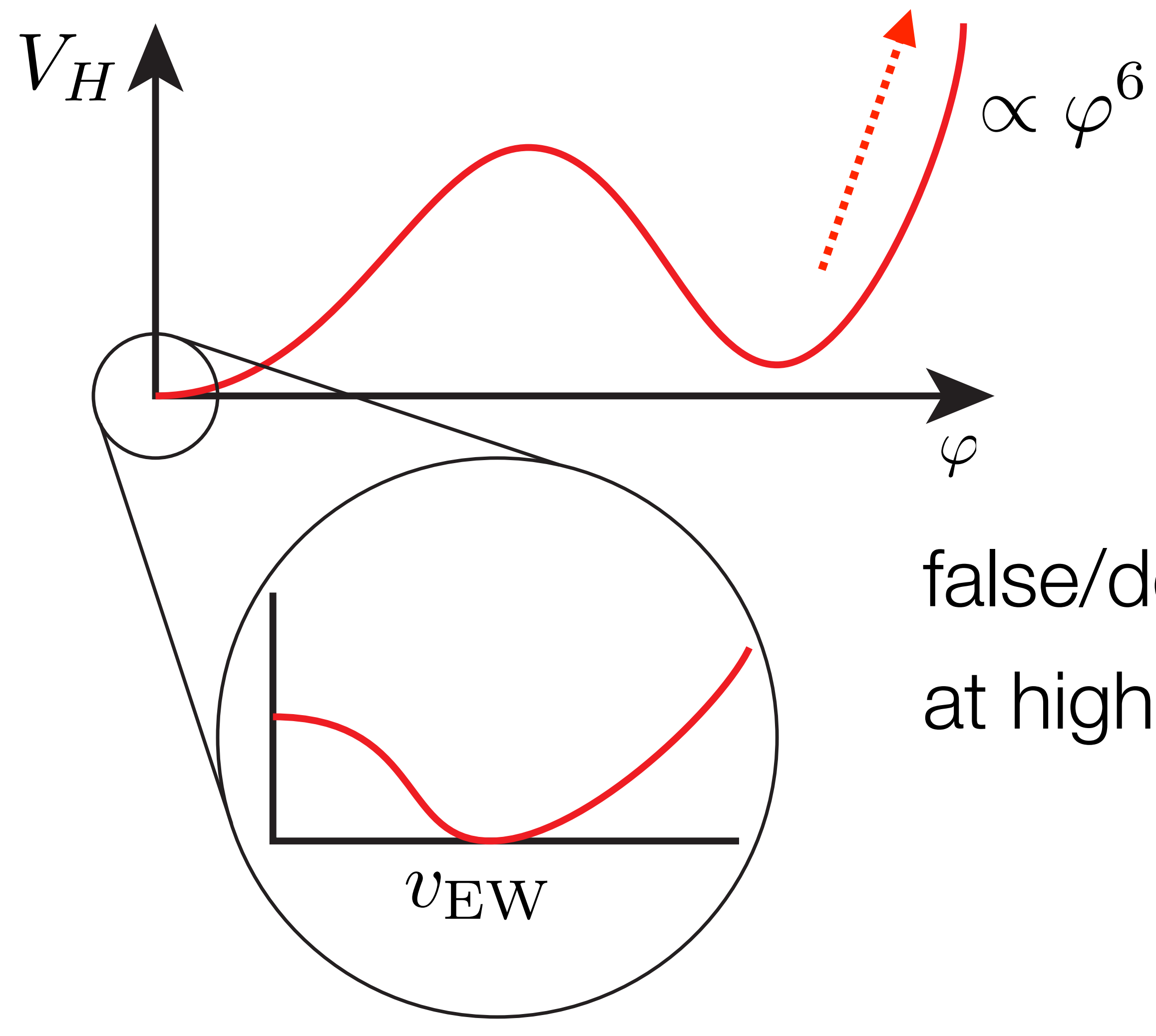

SM up to high energy scale?
V_H
φ
v_{EW}
New physics?
V_H
φ
v_{EW}

2. Higgs domain wall

NK, F. Takahashi, 1502.03725

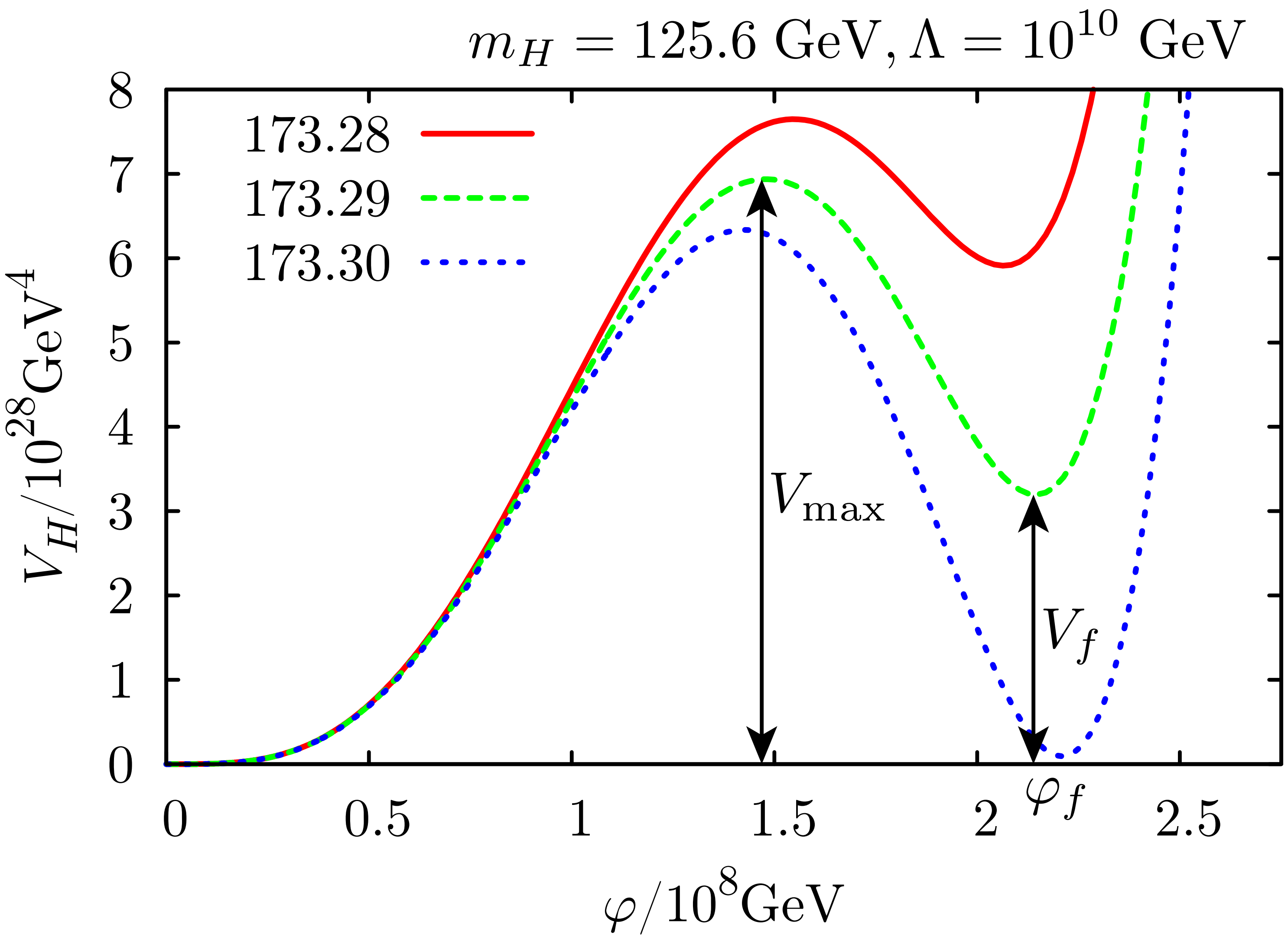
Higgs potential

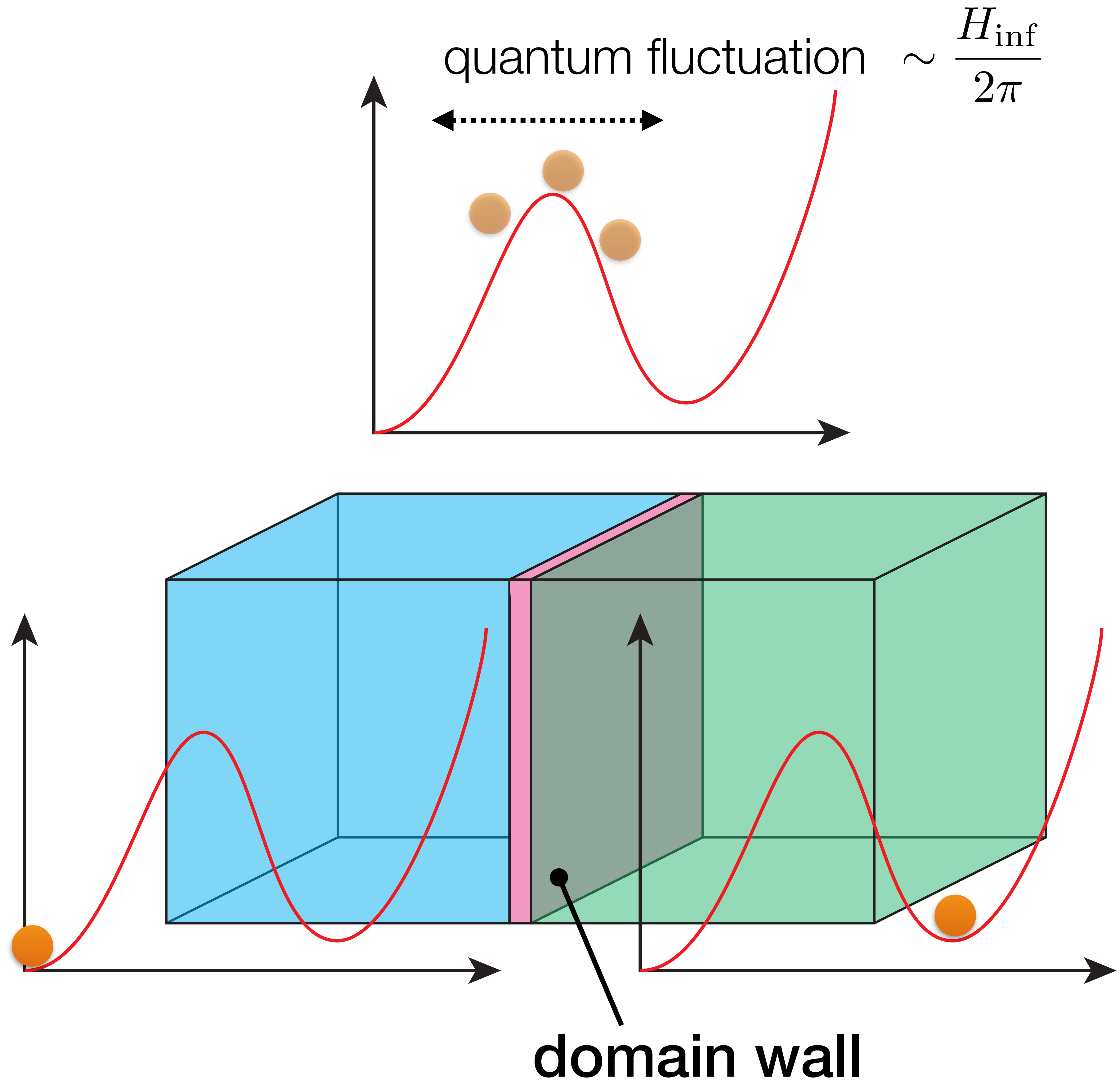
Higgs potential at high energy scales: $V_H = \frac{1}{4}\lambda(\varphi)\varphi^4 \boxed{+\frac{\varphi^6}{\Lambda^2}}$



false/degenerate vacuum
at high energy scale

Higgs potential

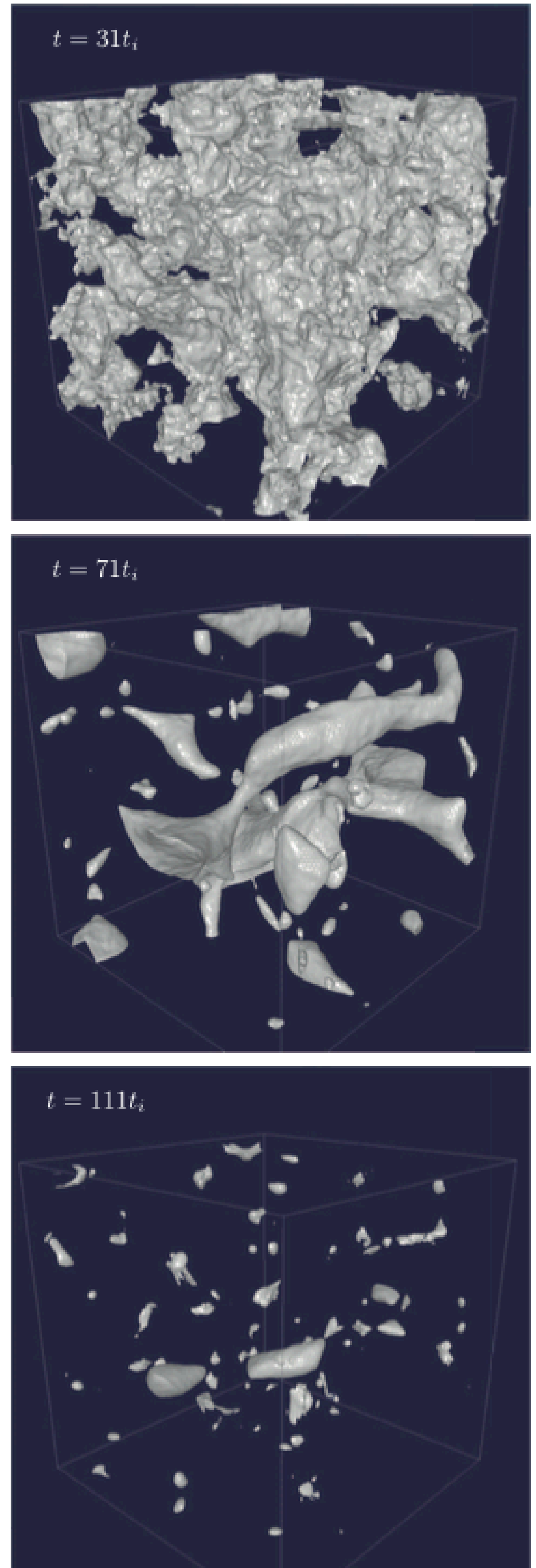


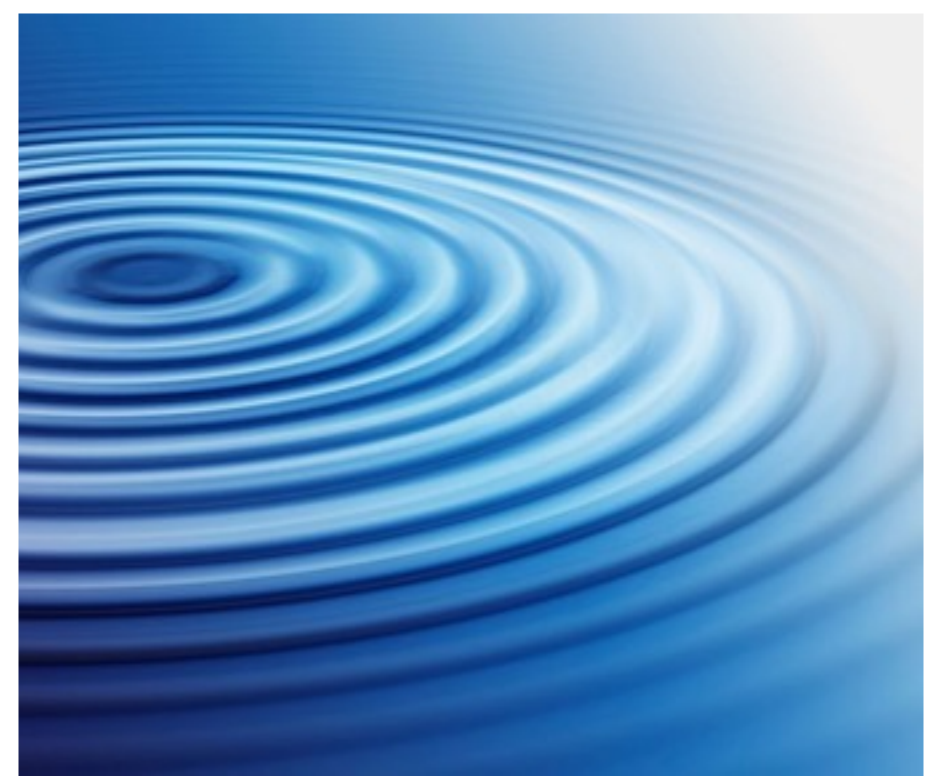

quantum fluctuation $\sim \frac{H_{\rm inf}}{2\pi}$
domain wall

Domain wall collapse

↓ ↓ ↓

Gravitational waves

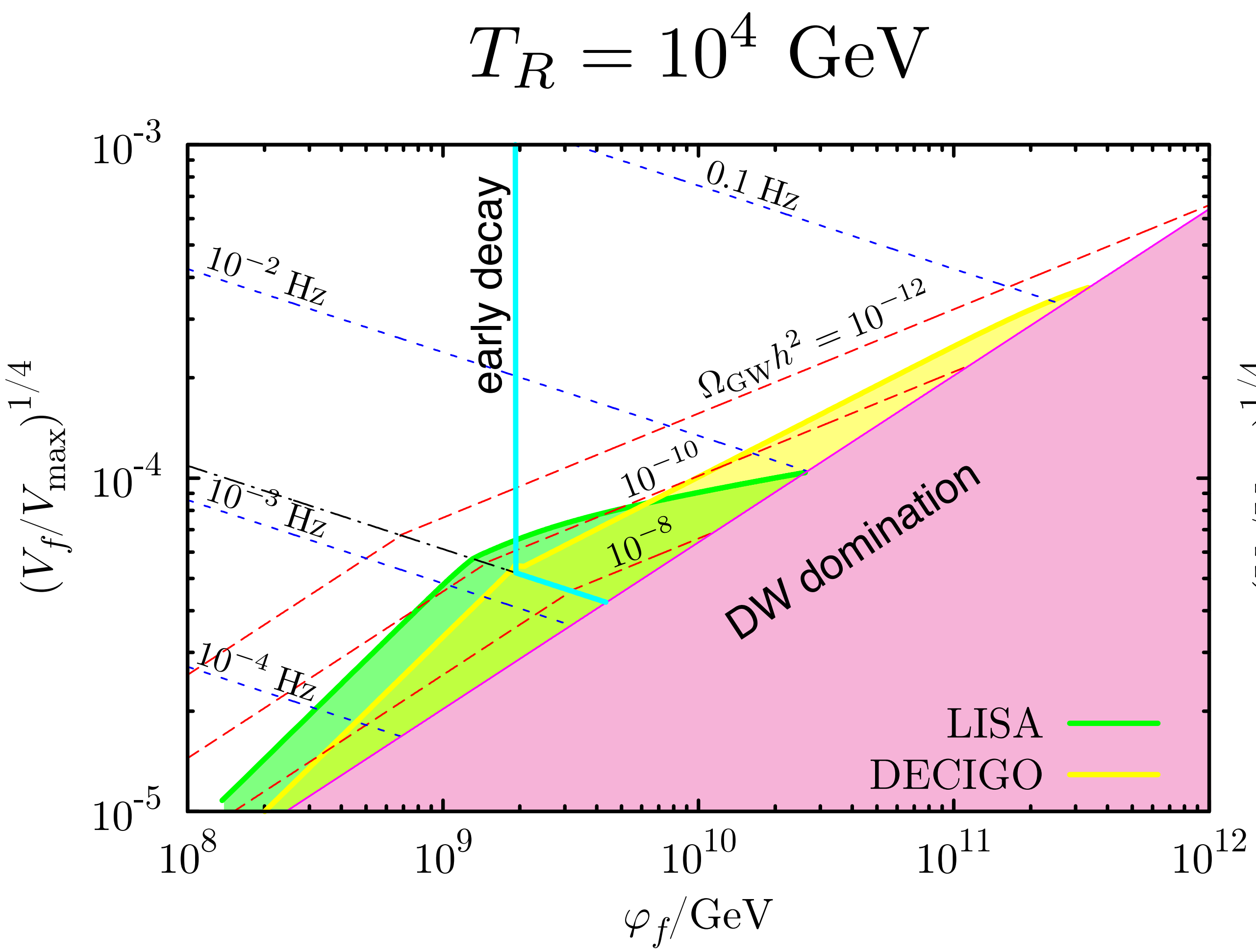




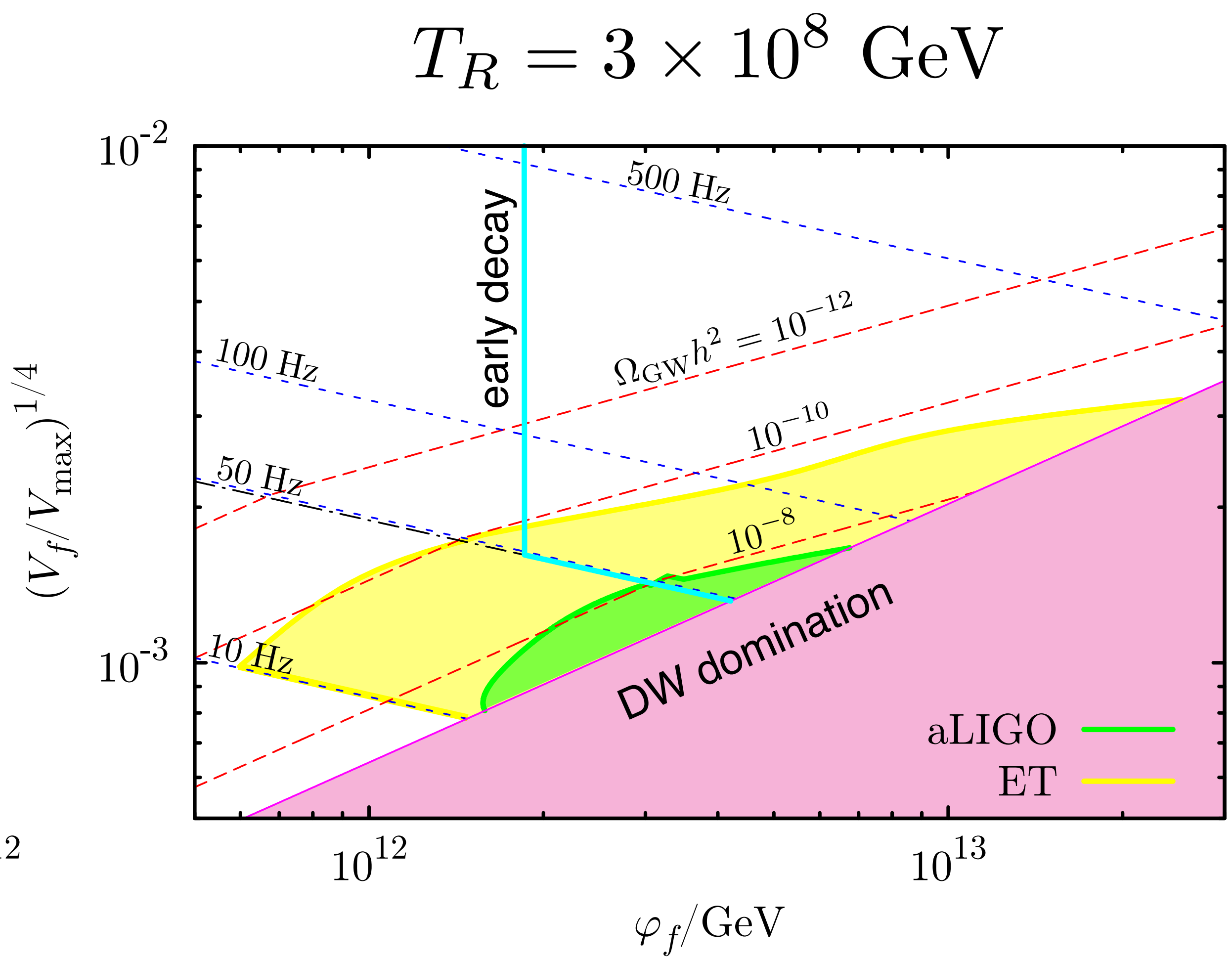
Hiramatsu,Kawasaki,Saikawa (2010)

Detectable region

NK, F. Takahashi, 1502.03725

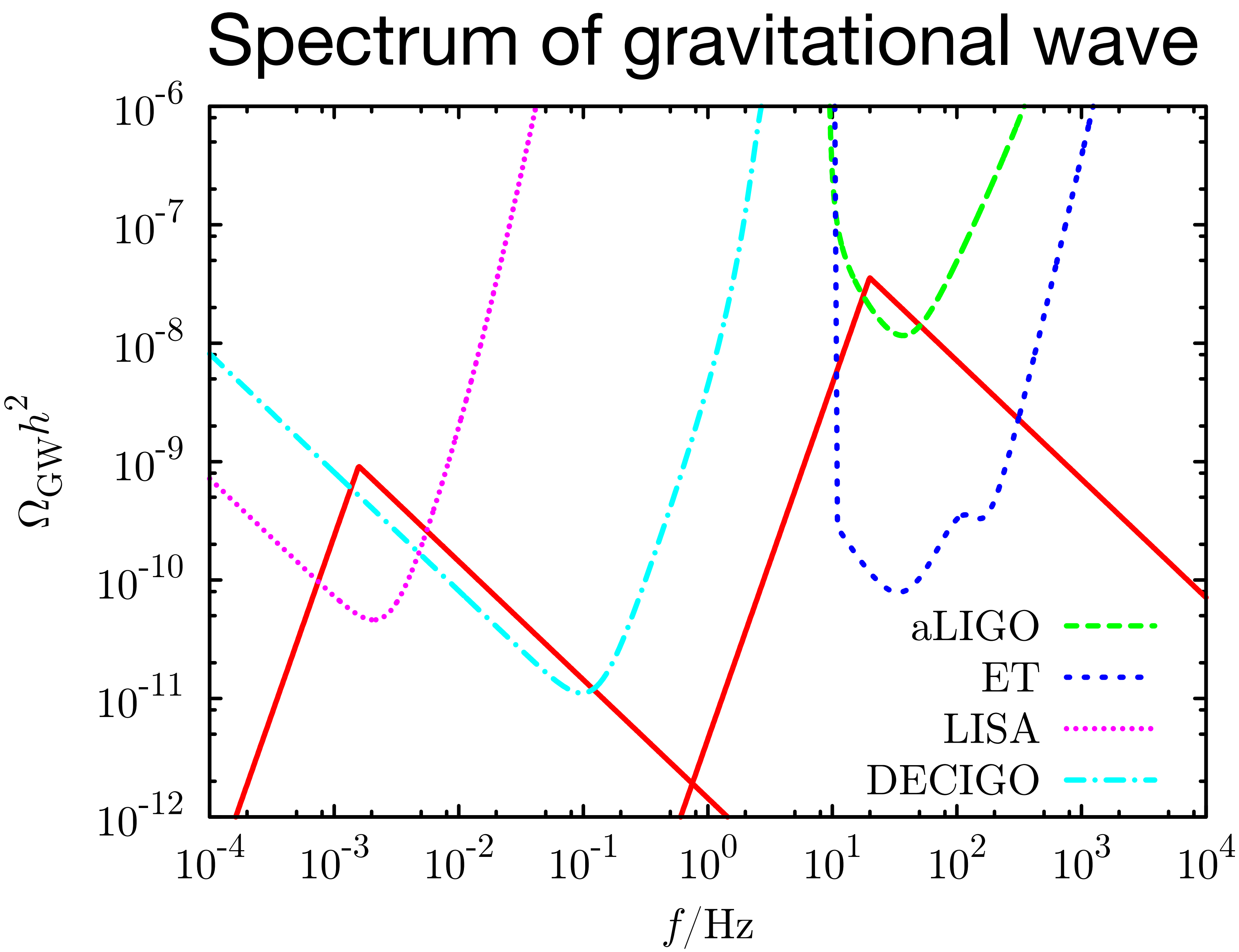


Space-based GW detectors



Ground-based GW detectors

Gravitational waves from Higgs domain wall



left: $\varphi_f = 2 \times 10^9\ \mathrm{GeV},\ (V_f/V_{\rm max})^{1/4} = 5 \times 10^{-5}$

right: $\varphi_f = 2 \times 10^{12}\ \mathrm{GeV},\ (V_f/V_{\rm max})^{1/4} = 10^{-3}$

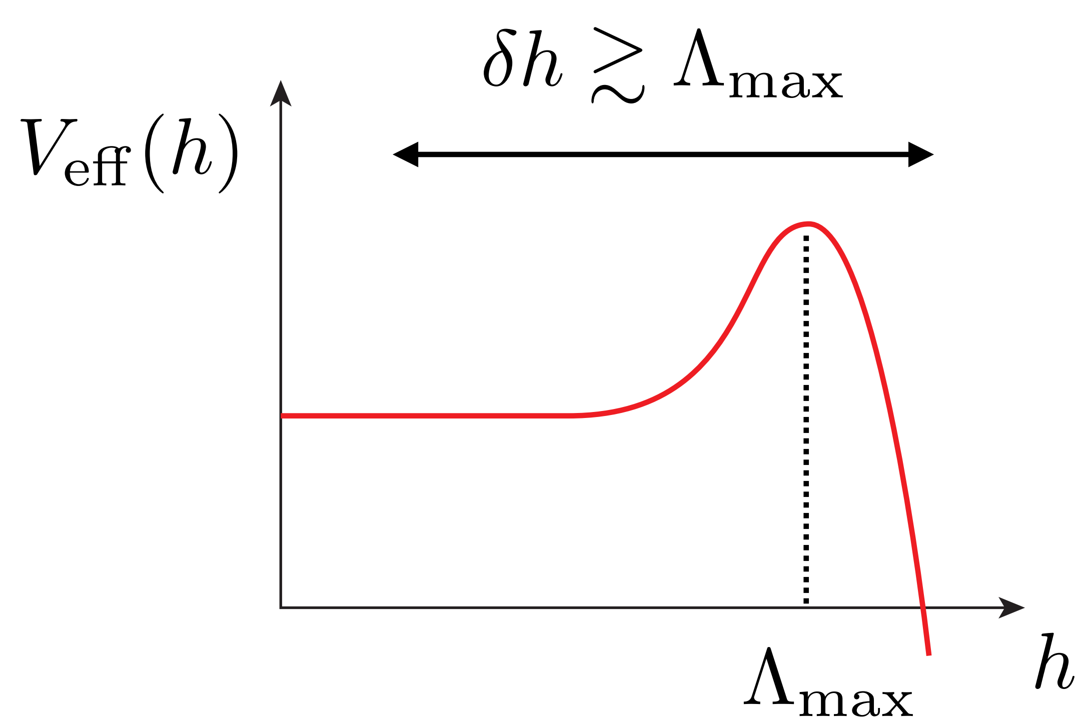
2. Vacuum stabilization and inflation

Inflation and quantum fluctuation

Hubble parameter during inflation

$$H_{\rm inf} \lesssim A_{\mathcal{R}}\pi\sqrt{\frac{r}{2}}M_P \approx 6.73\times 10^{13}\ \mathrm{GeV}\left(\frac{r}{0.07}\right)^{1/2}$$

Quantum fluctuation of Higgs field : $\delta h \sim \dfrac{H_{\rm inf}}{2\pi}$



EW vacuum stabilization during inflation

- Low-scale inflation : $H_{\rm inf} \lesssim 10^{10-11}{\rm GeV}$

- Inflaton — Higgs coupling : $\Delta\mathcal{L} = -\frac{1}{2}\lambda_{\phi h}\phi^2 h^2$

Lebedev, Westphal, 1210.6987

- Non-minimal coupling with gravity : $\Delta\mathcal{L} = -\frac{1}{2}\xi R h^2$

Espinosa, Giudice, Riotto, 0710.2484

$\lambda_{\phi h}\phi^2,\ \xi R \gtrsim H_{\rm inf}^2$ —> Higgs is stabilized during inflation

However, parametric resonance / tachyonic instability after inflation destabilizes the EW vacuum.

Herranen, Markkanen, Nurmi, Rajantie, 1407.3141
Ema, Mukaida, Nakayama, 1602.00483

Fokker-Planck equation

$$\boxed{\frac{\partial P(h,N)}{\partial N} = \frac{\partial}{\partial h}\left[\frac{V'(h)}{3H^2}P(h,N) + \frac{H^2}{8\pi^2}\frac{\partial P(h,N)}{\partial h}\right]}$$

initial condition : $P(h,0) = \delta(h)$

classical motion : $\Delta h_{\rm cl} = \dot{h}\Delta t = -\frac{V'_{\rm eff}(h)}{3H^2}$

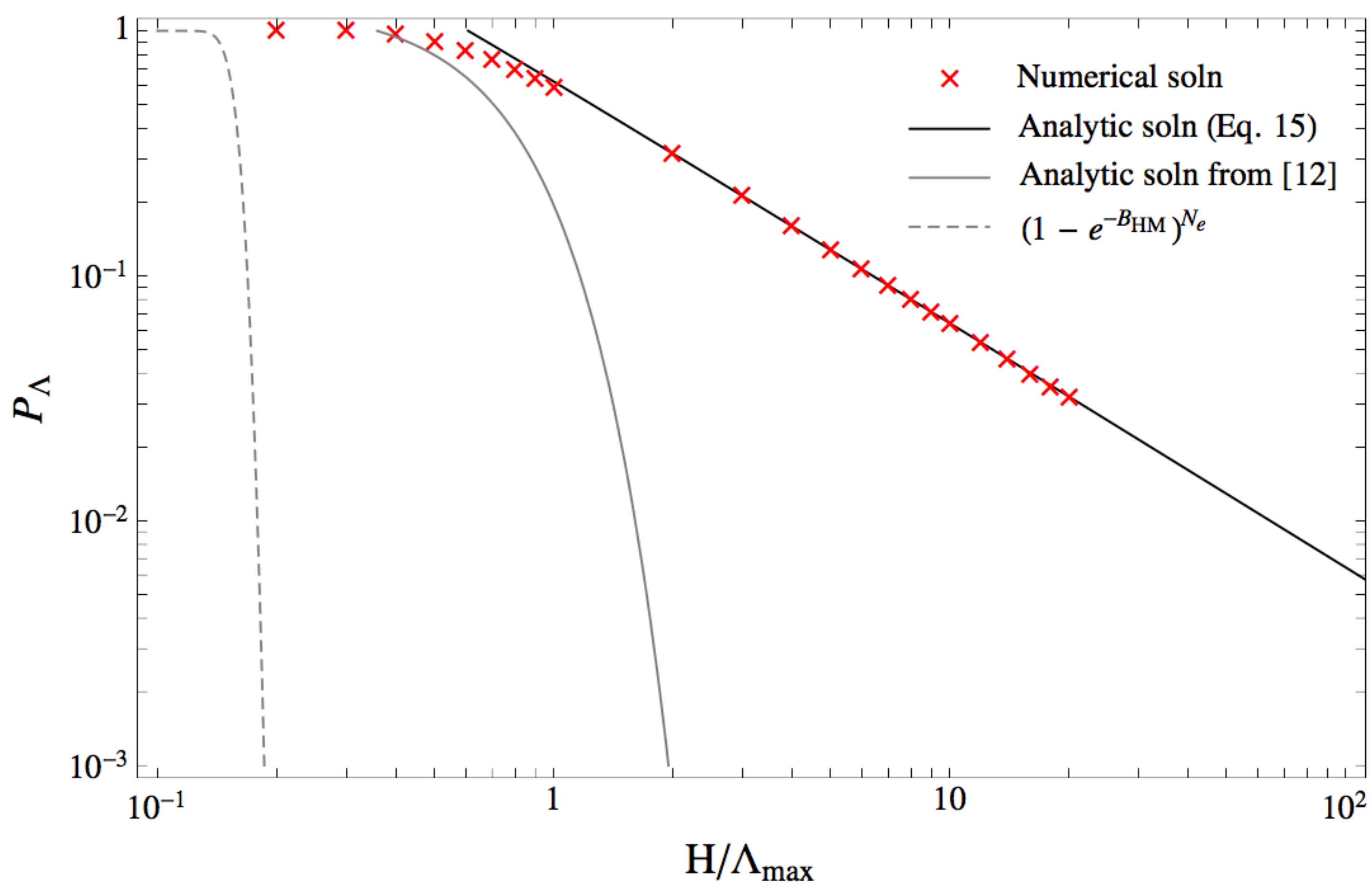
quantum fluctuation : $\Delta h_{\rm qm} = \frac{H}{2\pi}$

$\Big\downarrow$ $P(h,N) = 0$ for $\Delta h_{\rm qm} < \Delta h_{\rm cl}$

boundary condition : $|V'_{\rm eff}(\pm\Lambda_c)| \equiv \frac{3H^2}{2\pi}$

Survival probability

$$P_\Lambda = \int_{-\Lambda_{\rm max}}^{\Lambda_{\rm max}} dh P(h, N)$$



Hook, Kearney, Shakya, Zurek, 1404.5953

3. Multi-field stochastic dynamics during inflation

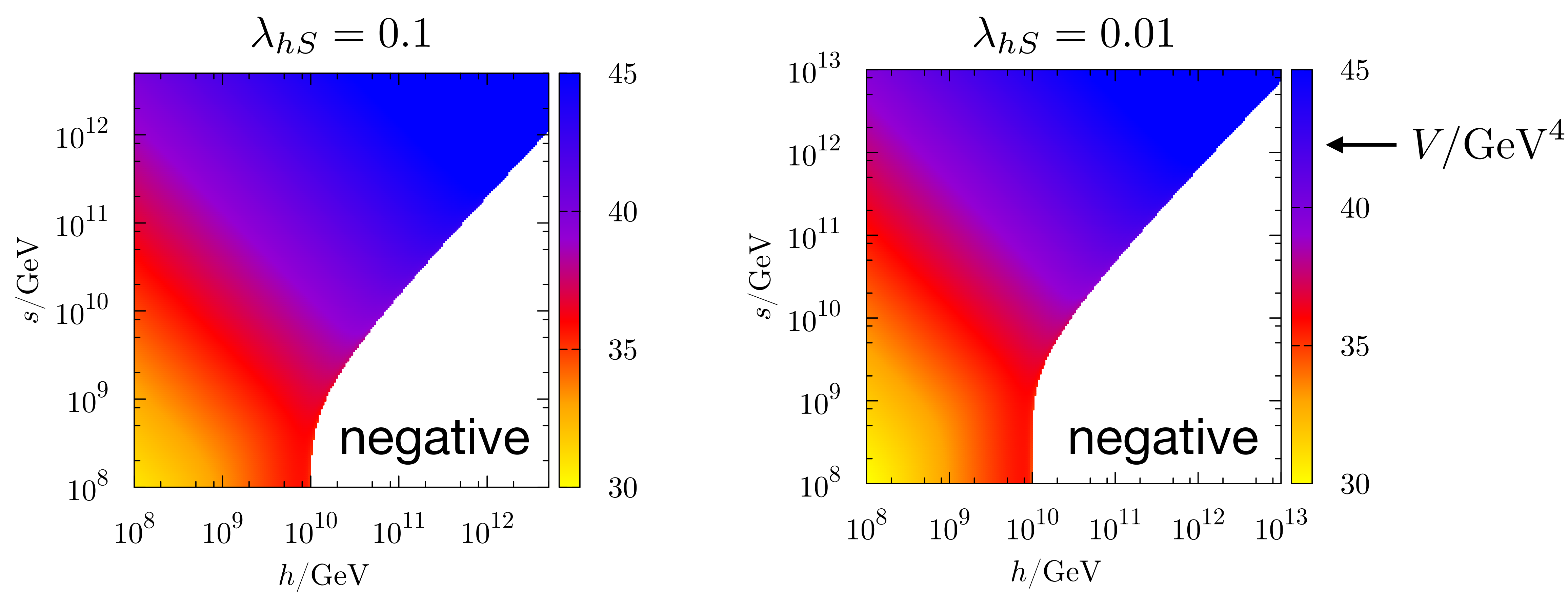
J.O. Gong, NK, 1705.11178

Model : Higgs (h) & singlet spectator field (S)

Potential : $$V(h,S) = \frac{1}{4}\lambda_{\rm eff}(\mu)h^4 + \frac{1}{2}m_S^2 S^2 + \frac{1}{2}\lambda_{hS}h^2S^2 + \frac{1}{4}\lambda_S S^4$$

with $\mu = \sqrt{h^2 + H_{\rm inf}^2}$

Potential



RGEs (up to one loop order)

$$(4\pi)^2 \frac{d\lambda_h}{dt} = 24\lambda_h^2 + 12\lambda y_t^2 - 6y_t^4 - 3\lambda_h(g'^2 + 3g^2) + \frac{3}{8}[2g^4 + (g'^2 + g^2)^2] + \frac{\lambda_{hS}^2}{2}$$

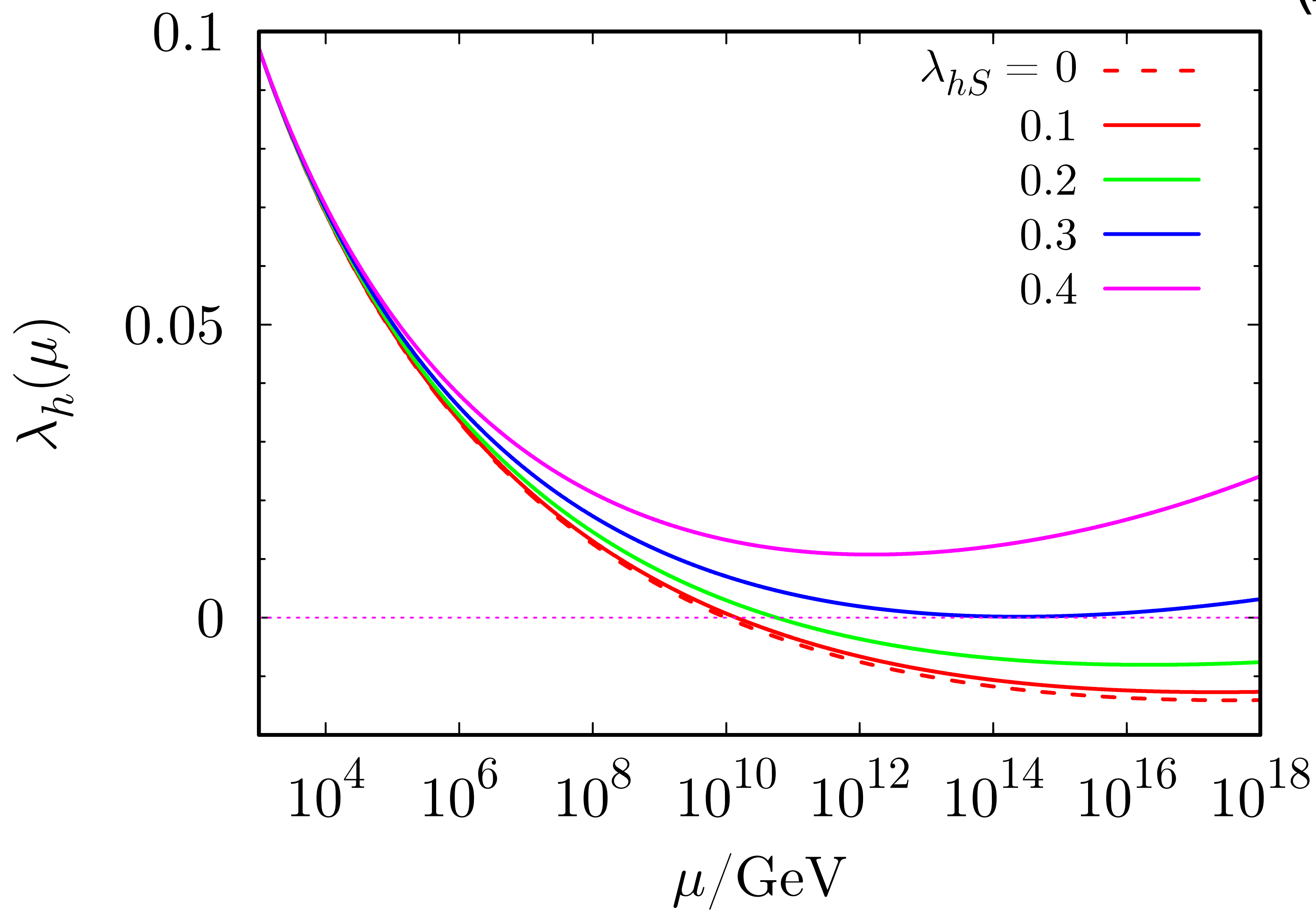
$$(4\pi)^2 \frac{d\lambda_{hS}}{dt} = \lambda_{hS}\left[4\lambda_{hS} + 12\lambda_h + \lambda_S + 6y_t^2 - \frac{3}{2}(g'^2 + 3g^2)\right]$$

$$(4\pi)^2 \frac{d\lambda_S}{dt} = 3\lambda_S + 12\lambda_{hS}$$

with $t = \ln(\mu/M_t)$

Higgs self coupling in the presence of h-S coupling

(2-loop order)



$\lambda_{hS} \gtrsim 0.3$ —> EW vacuum is absolutely stable

Otherwise, the EW vacuum is still metastable…

Multi-field Fokker-Planck equation

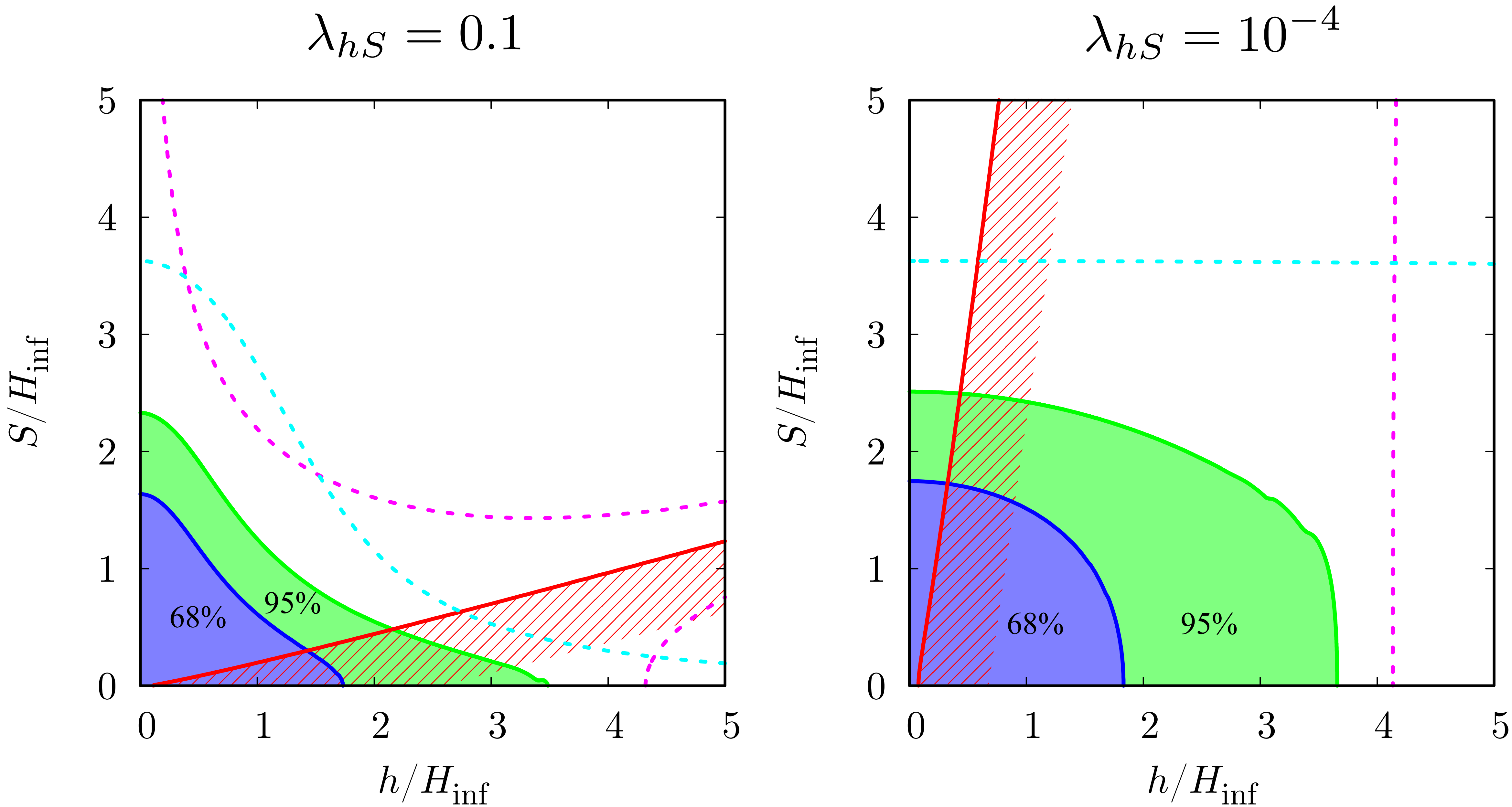
$$\boxed{\frac{\partial P(h,S,N)}{\partial N} = \sum_{\phi=h,S} \frac{\partial}{\partial \phi}\left[\frac{\partial V(h,S)}{\partial \phi}\frac{P(h,S,N)}{3H^2} + \frac{H^2}{8\pi^2}\frac{\partial P(h,S,N)}{\partial \phi}\right]}$$

with initial distribution : $P(h,S,0) = \delta(h)\delta(S)$

+ boundary condition : $P(\pm\Lambda_h,S,N) = P(h,\pm\Lambda_S,N) = 0$

with $\left|\frac{\partial V}{\partial h}\right|_{|h|=\Lambda_h} = \left|\frac{\partial V}{\partial S}\right|_{|S|=\Lambda_S} = \frac{3H_{\rm inf}^3}{2\pi}$

Equal probability contours



$$H_{\rm inf} = 10^{11}\ \text{GeV},\ N = 60,\ \lambda_S = 0.01$$

Analytic (approximate) solution

$$\boxed{\frac{\partial P(h,S,N)}{\partial N} = \sum_{\phi=h,S} \frac{\partial}{\partial \phi}\left[\frac{\partial V(h,S)}{\partial \phi}\frac{P(h,S,N)}{3H^2} + \frac{H^2}{8\pi^2}\frac{\partial P(h,S,N)}{\partial \phi}\right]}$$

separation of variables : $P(h,S,N) = \Psi(N)\Phi(h,S)$

2nd term

$$\Psi(N) \propto e^{-\alpha N} \quad \& \quad \left(\frac{\partial^2}{\partial h^2} + \frac{\partial^2}{\partial S^2}\right)\Phi(h,S) = -\frac{8\pi^2}{H_{\rm inf}^2}\alpha\Phi(h,S)$$

$$P(h,S,N) = \frac{1}{\Lambda_h \Lambda_S}\sum_{n,m\geq 0} e^{-\alpha_{nm}N}\cos\left[\pi\left(n+\frac{1}{2}\right)\frac{h}{\Lambda_h}\right]\left[\pi\left(m+\frac{1}{2}\right)\frac{S}{\Lambda_S}\right]$$

with $\alpha_{nm} = \left(n+\frac{1}{2}\right)^2\frac{H_{\rm inf}^2}{8\Lambda_h^2} + \left(m+\frac{1}{2}\right)^2\frac{H_{\rm inf}^2}{8\Lambda_S^2}$

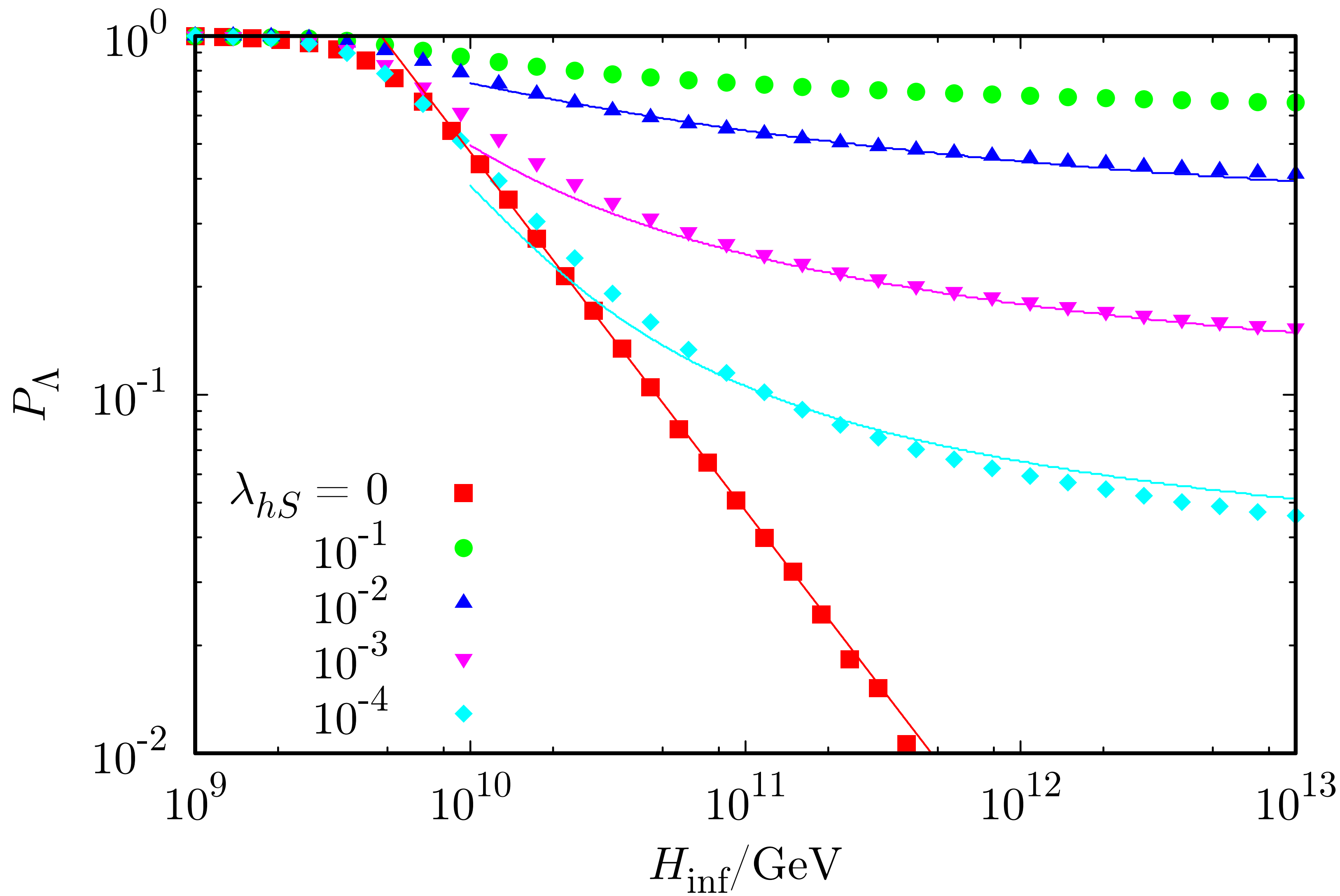
survival probability : $P_\Lambda = \int_{-\Lambda_S}^{\Lambda_S} dS \int_{-\Lambda_{\max}}^{\Lambda_{\max}} dh P(h,S)$

$$\boxed{\Lambda_{\max} = \sqrt{\frac{\lambda_{hS}}{-\partial\lambda_{\rm eff}/4\partial\ln h - \lambda_{\rm eff}}} S \approx 10\sqrt{\lambda_{hS}} S}$$

$$P_\Lambda = \frac{4p}{\pi^2} \sum_{n,m\geq 0} e^{-q\alpha_{nm}N} \frac{-\frac{m+1/2}{n+1/2}\Lambda_h^2 \sin\left[\beta\pi\left(n+\frac{1}{2}\right)\frac{\Lambda_S}{\Lambda_h}\right] + \beta\Lambda_s\Lambda_S}{\beta^2(n+1/2)^2\Lambda_S^2 - (m+1/2)^2\Lambda_h^2}$$

$$\beta = \Lambda_{\max}/S, \quad p, q : \text{ numerical fudge factors}$$

Survival probability just after inflation



$$N = 60,\ \lambda_S = 0.01$$

analytic curve -> $p = 2,\ q = 5$

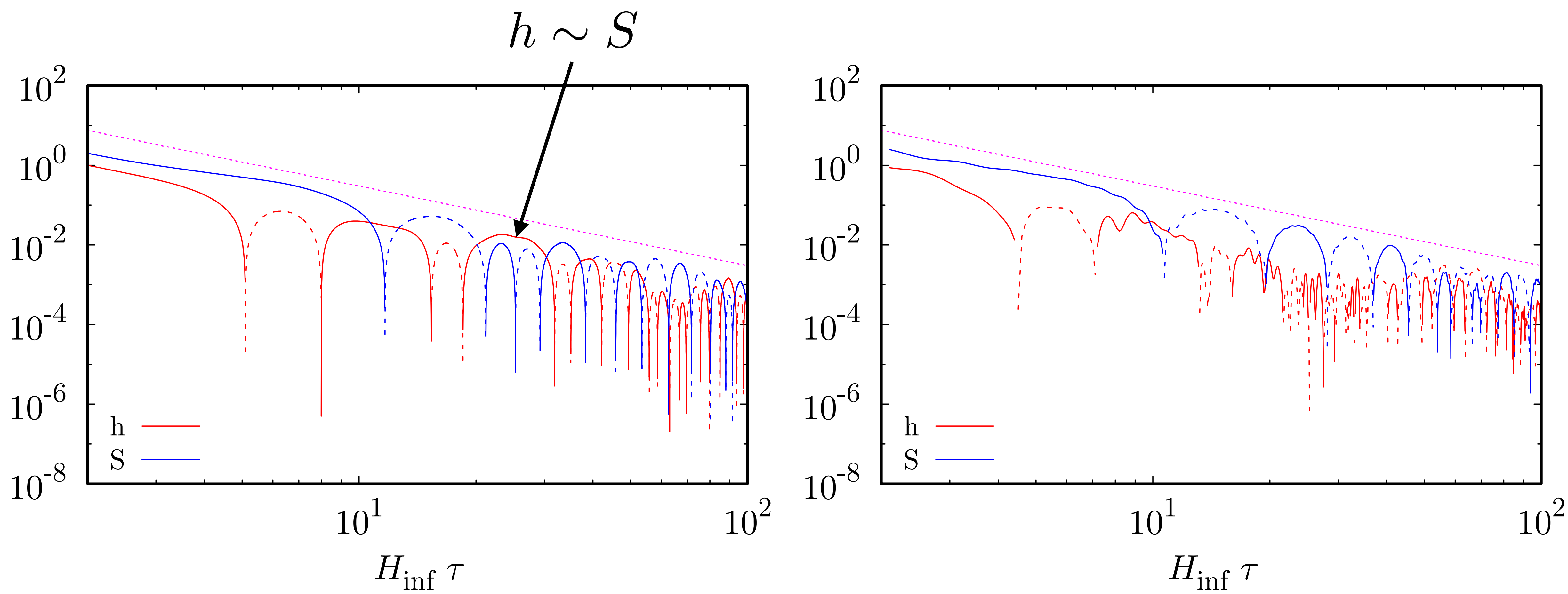
Scalar field dynamics after inflation

$$\ddot{h} + 3H\dot{h} - \frac{\nabla^2 h}{a^2} + \left(\lambda_{\rm eff} + \frac{1}{4}\frac{\partial \lambda_{\rm eff}}{\partial \ln h}\right) h^3 + \lambda_{hS} S^2 h = 0$$

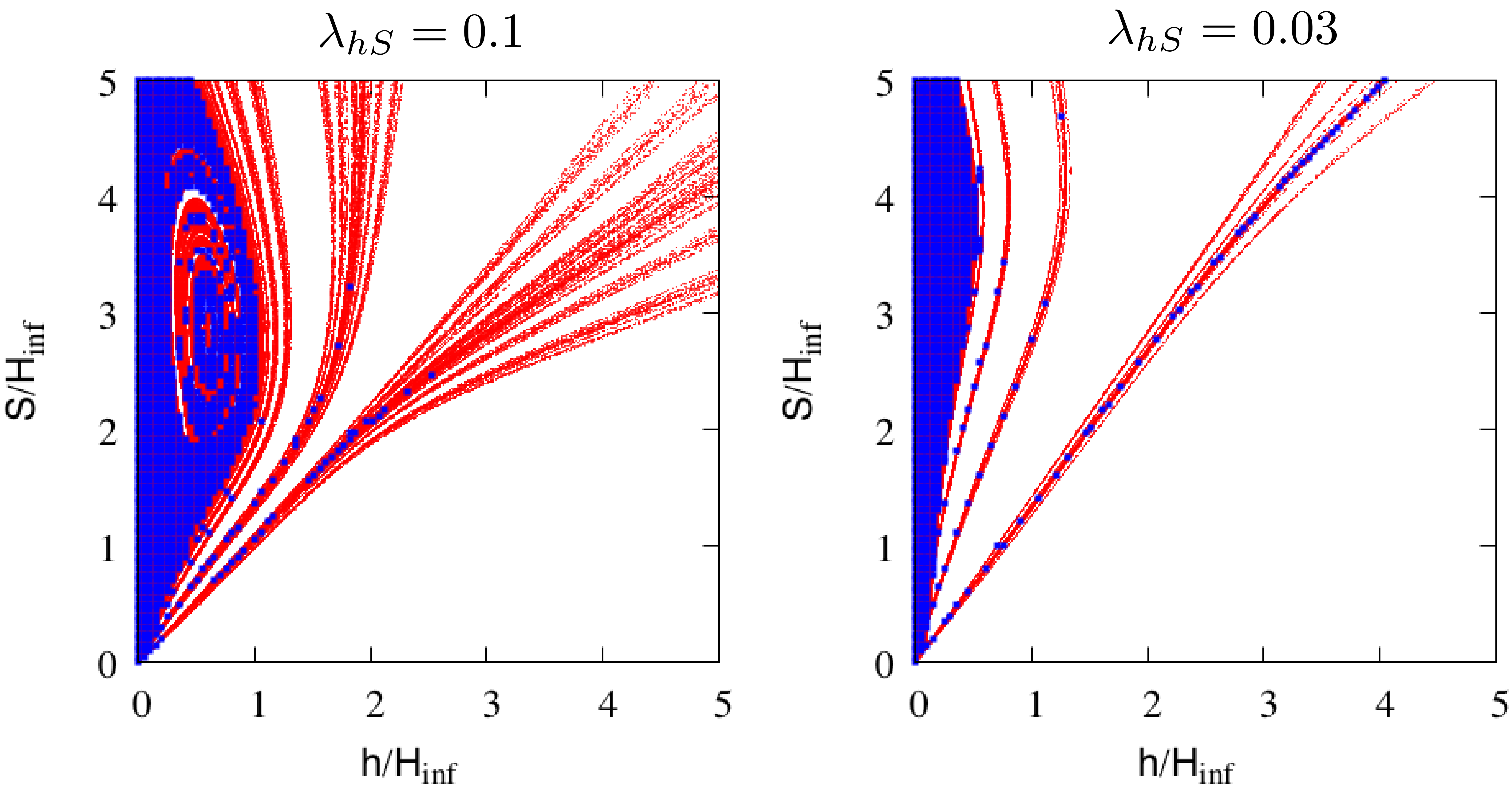
$$\ddot{S} + 3H\dot{S} - \frac{\nabla^2 S}{a^2} + \lambda_S S^3 + \lambda_{hS} h^2 S = 0$$

initial condition : $h_i = h_{\rm inf}, \quad S_i = S_{\rm inf}$

Background evolution

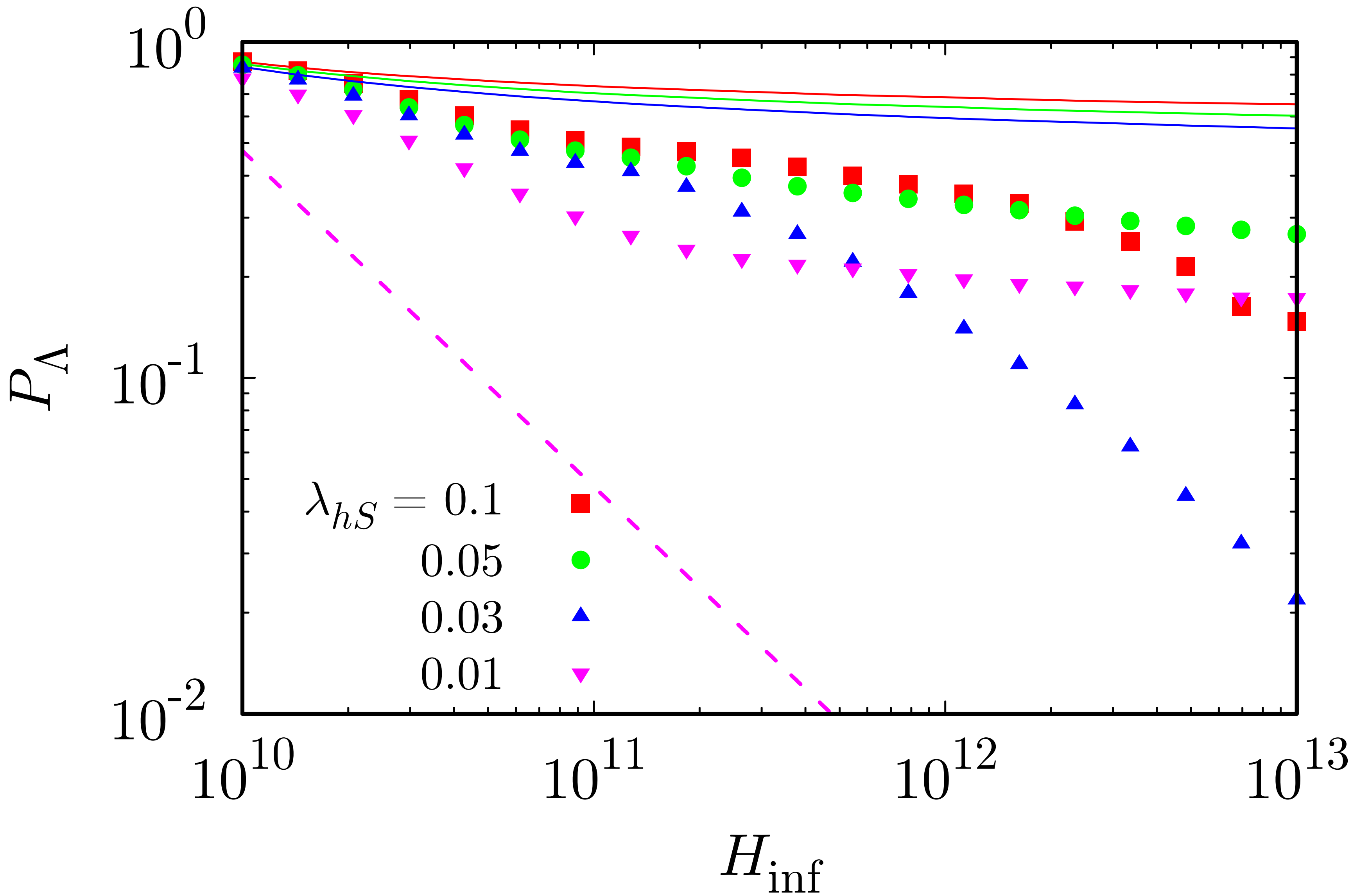


stability/instability chart



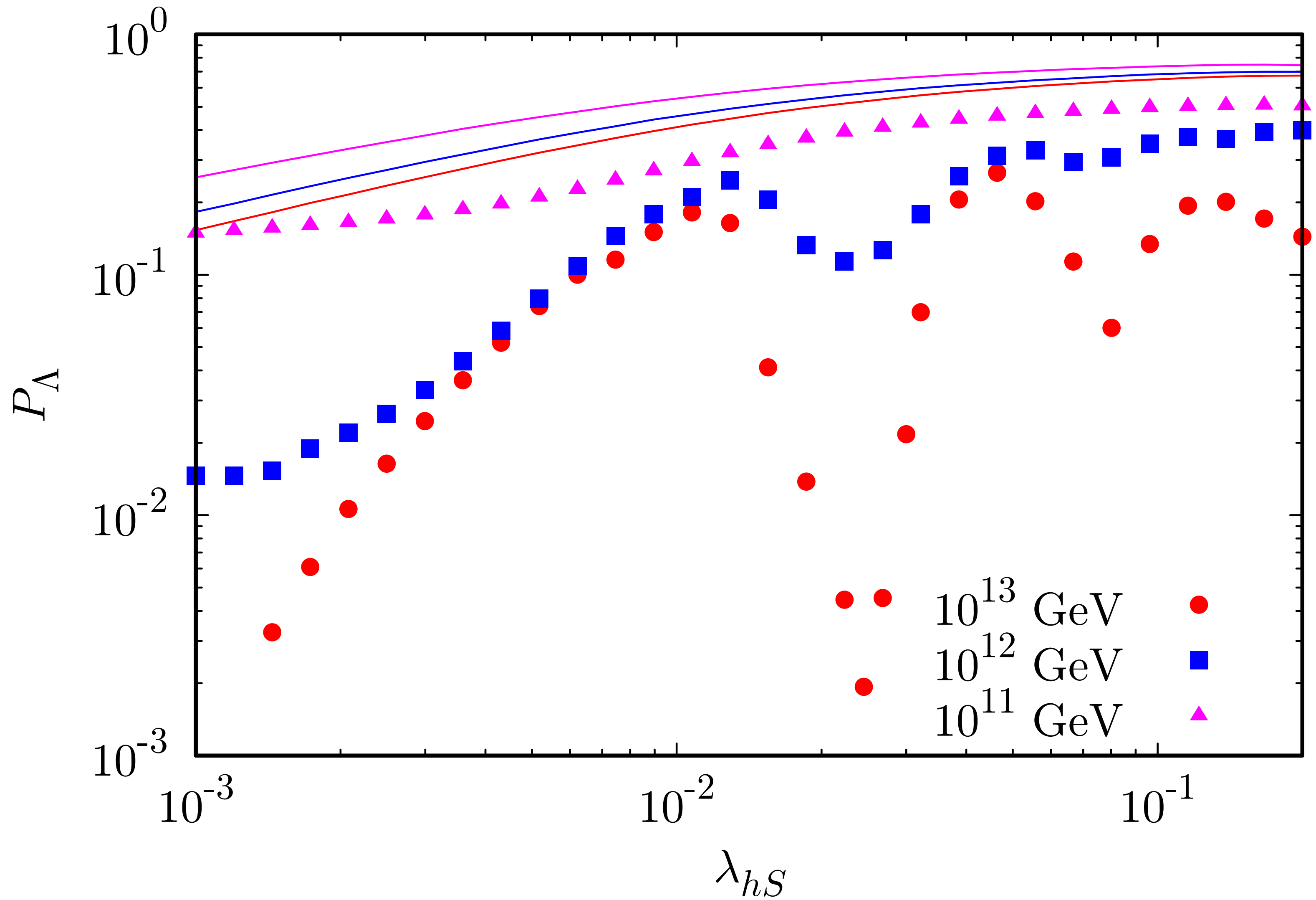
$$H_{\rm inf} = 10^{12}\ \mathrm{GeV},\ \ \lambda_S = 0.01$$

Survival probability after preheating



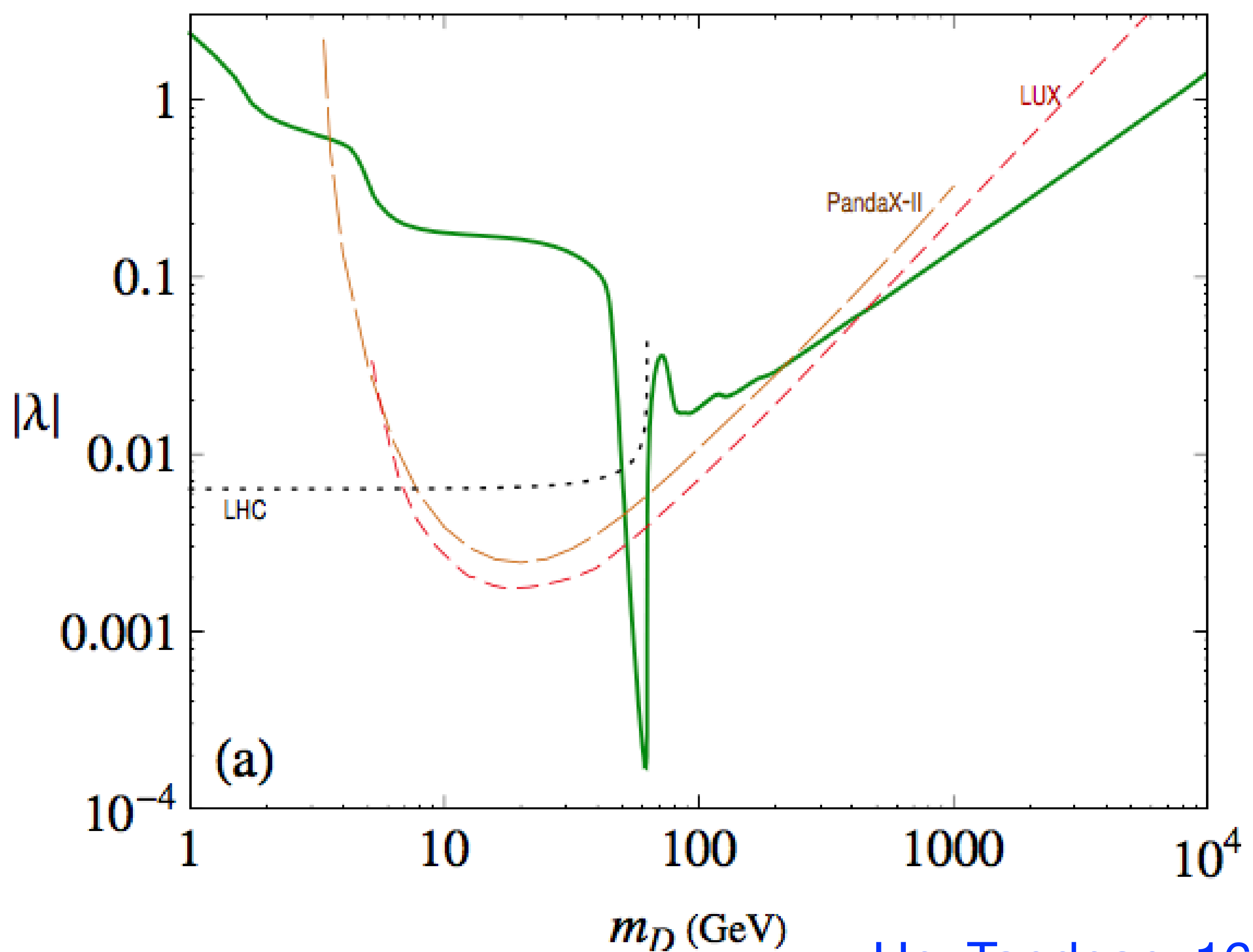
$N = 60, \ \lambda_S = 0.01$

Survival probability after preheating



$$N = 60,\ \lambda_S = 0.01$$

Singlet spectator field as dark matter



He, Tandean, 1609.03551

Summary

- We considered the singlet extension of SM
and solved multi-field stochastic dynamics during inflation
(Fokker-Planck equation)

- Stochastic dynamics of singlet (spectator) field
can largely enhance the survival probability during inflation

- Parametric resonance can occur in our model
but the resonance can be suppressed in a wide parameter range

- The singlet spectator field can be a dark matter
(marginally) consistent with the current constraint.