IBS, Daejeon 8th June 2017

Implications of decoupling new physics

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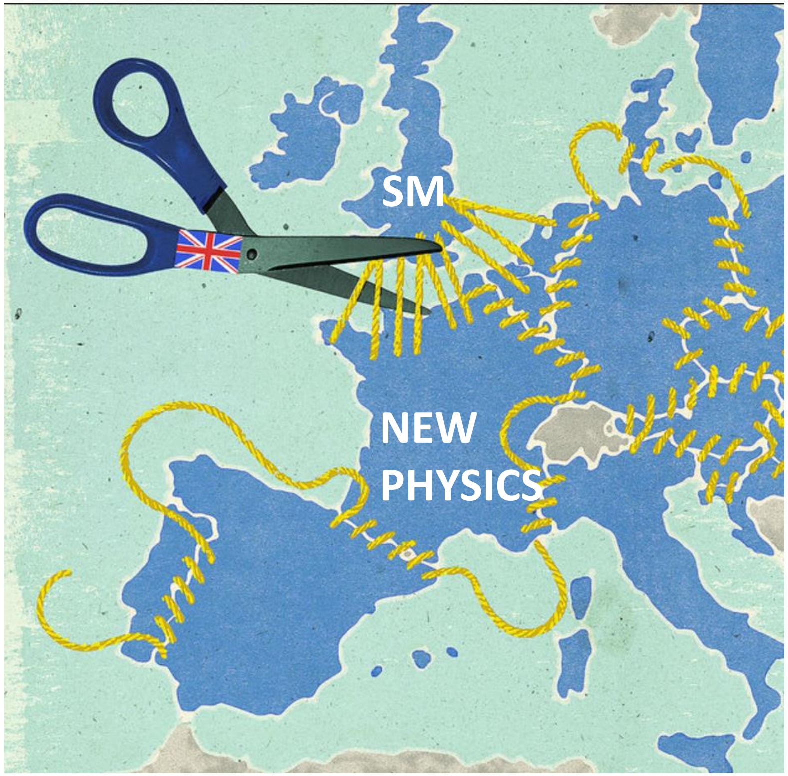
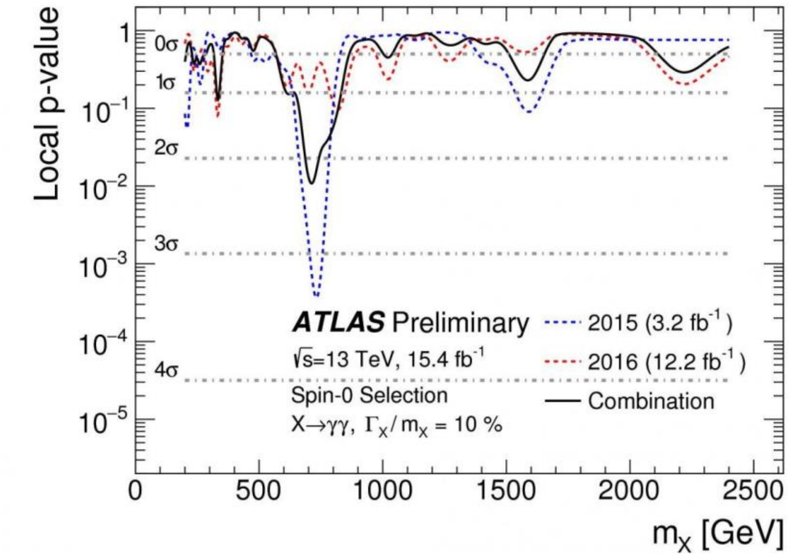
SMEXIT

Implications of decoupling new physics



Introduction

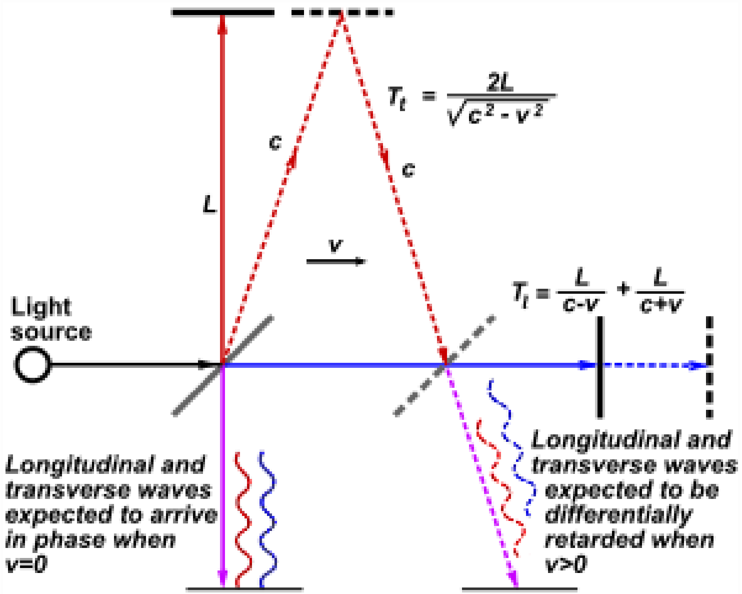
- Last year may have been disappointing for some of us…



- Direct searches for NP still **negative**

Introduction

- But null results may still lead to **deeper understanding**



- *No new physics at the TeV scale could be our "Michelson-Morley" moment*

Introduction

- **Soft** exit from the SM: New physics around the corner
  - Usual *SUSY/compositeness/extra-dimensions*… just a bit more fine-tuned
  - *Neutral naturalness/Twin Higgs*… hidden naturalising sector
- **Hard** exit from the SM: New physics decoupled
  - Accept fine-tuning (c.f. cosmological constant), *SUSY/compositeness/extra-dimensions* resolve other problems at heavier scales
  - *Anthropic landscape*
  - *Cosmological relaxation*
- Phenomenological framework: **SM EFT**

Outline

- **Part I: SM EFT**
  - A phenomenological framework for decoupled new physics
- **Part 2: The Universal One-Loop Effective Action**
  - A new way of matching decoupled new physics to EFTs at one-loop
- **Part 3: Cosmological Relaxation**
  - A new approach to decoupling new physics without fine-tuning

References

SM EFT:

-Dimension-6 operator analysis of the CLIC sensitivity to new physics
John Ellis, Philipp Roloff, Veronica Sanz and **TY**
[arXiv:1701.04804]

-Sensitivities of Prospective Future e+e− Colliders to Decoupled New Physics,
John Ellis and **TY**
JHEP 03 (2016 089 [arXiv:1510.04561]

-Comparing EFT and Exact One-Loop Analyses of Non-Degenerate Stops,
Aleksandra Drozd, **John Ellis, Jeremie Quevillon** and **TY**
JHEP 06 (2015) 028 [arXiv:1504.02409]

-The Effective Standard Model after LHC Run I,
John Ellis, **Veronica Sanz** and **TY**
JHEP 29 (2015) 007 [arXiv:1410.7703]

UOLEA:

-Mixed Heavy-Light Matching in the Universal One-Loop Effective Action,
Sebastian A.R. Ellis, Jeremie Quevillon, TY, and **Zhengkang Zhang**
PLB accepted [arXiv:1604.02445]

-The Universal One-Loop Effective Action,
Aleksandra Drozd, **John Ellis, Jeremie Quevillon** and **TY**,
JHEP 03 (2016) 180 [arXiv:1512.03003]

Cosmological Relaxation:

-A Dynamical Weak Scale From Inflation,
TY
[arXiv:1701.09167]

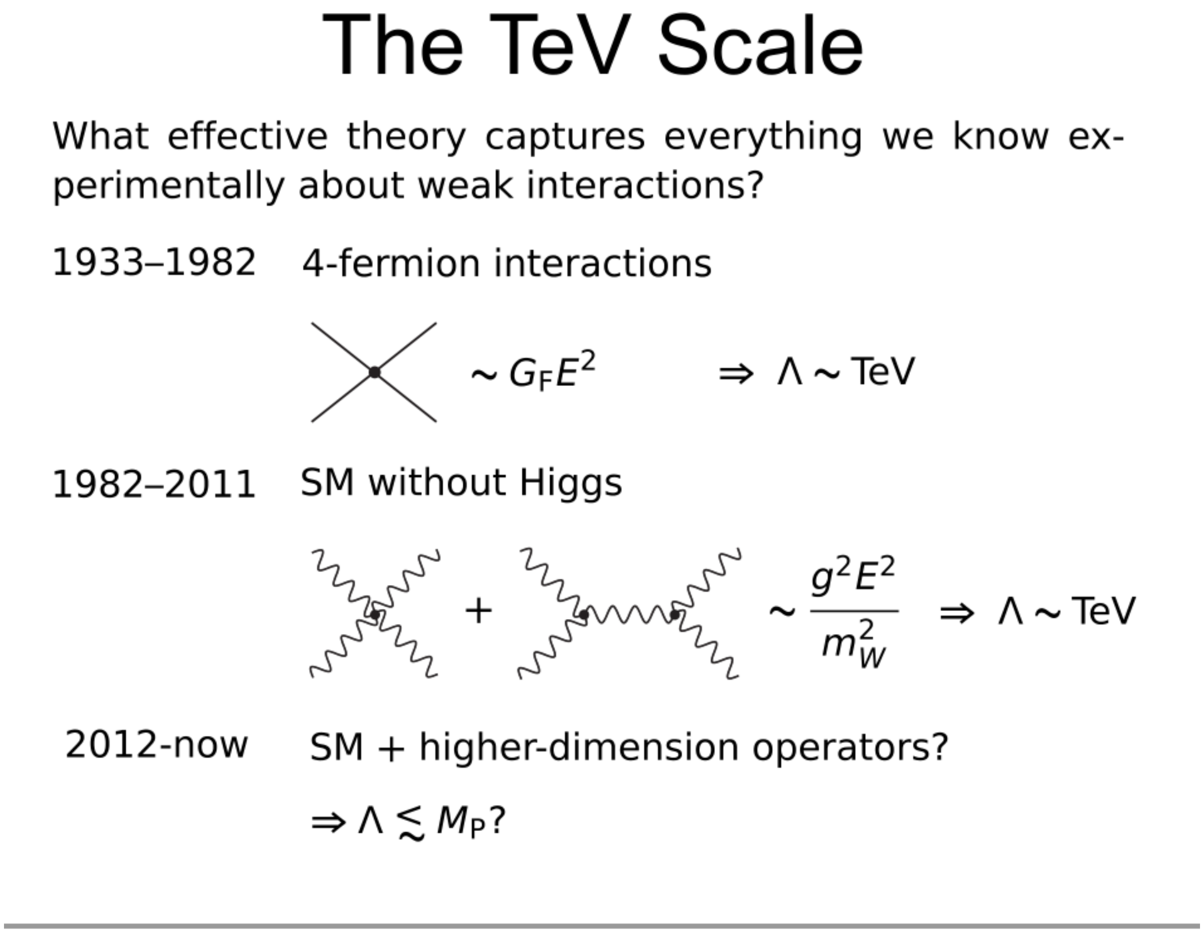
- **Part I: SM EFT**

- Part 2: The Universal One-Loop Effective Action

- Part 3: Cosmological Relaxation

Why SM EFT?

Assuming a SM Higgs and decoupled new physics at higher energies, the SM EFT is the next phenomenological framework



The TeV Scale

What effective theory captures everything we know experimentally about weak interactions?

1933–1982 4-fermion interactions

$\sim G_F E^2 \quad \Rightarrow \Lambda \sim \text{TeV}$

1982–2011 SM without Higgs

$+ \quad \sim \frac{g^2 E^2}{m_W^2} \quad \Rightarrow \Lambda \sim \text{TeV}$

2012-now SM + higher-dimension operators?

$\Rightarrow \Lambda \lesssim M_P$?

Markus Luty PASCOS 2015 slide

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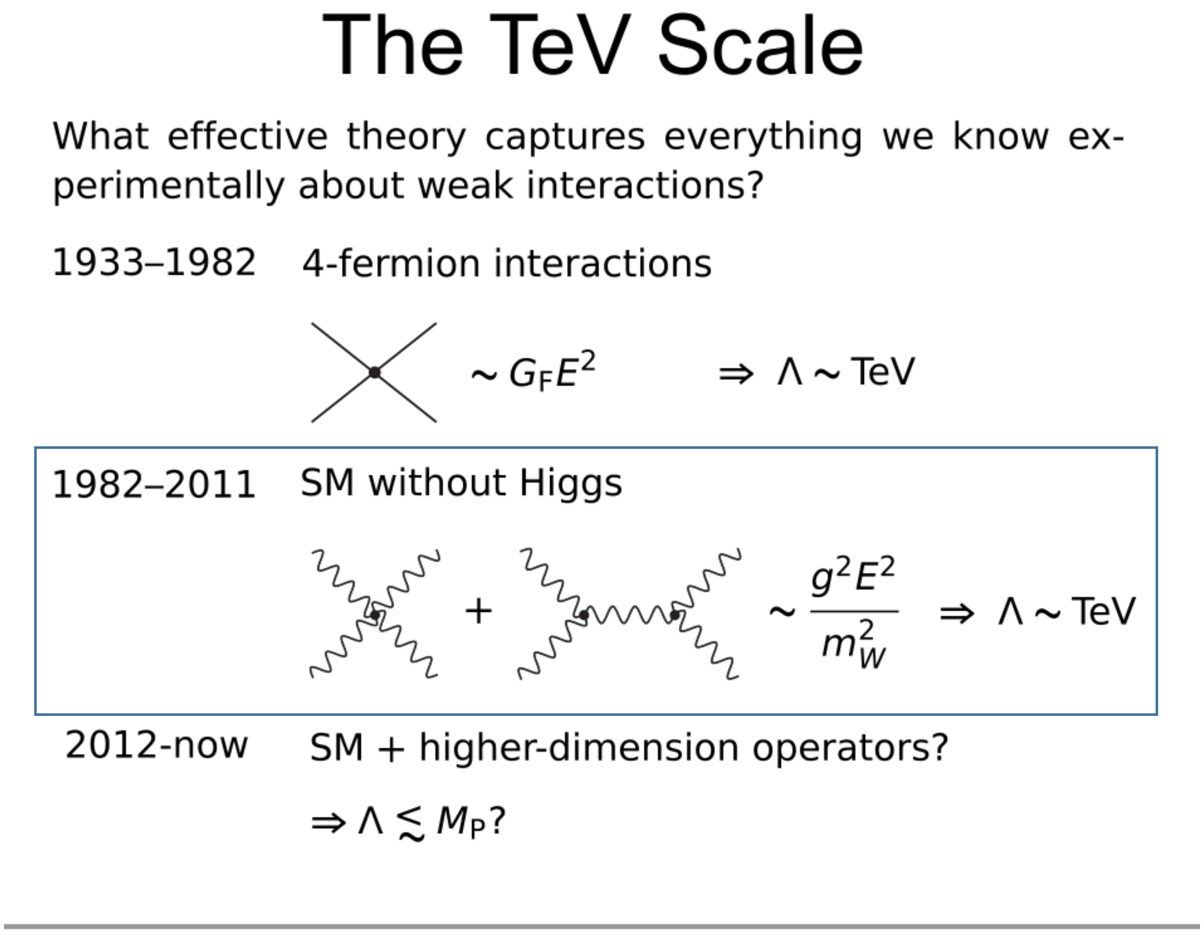

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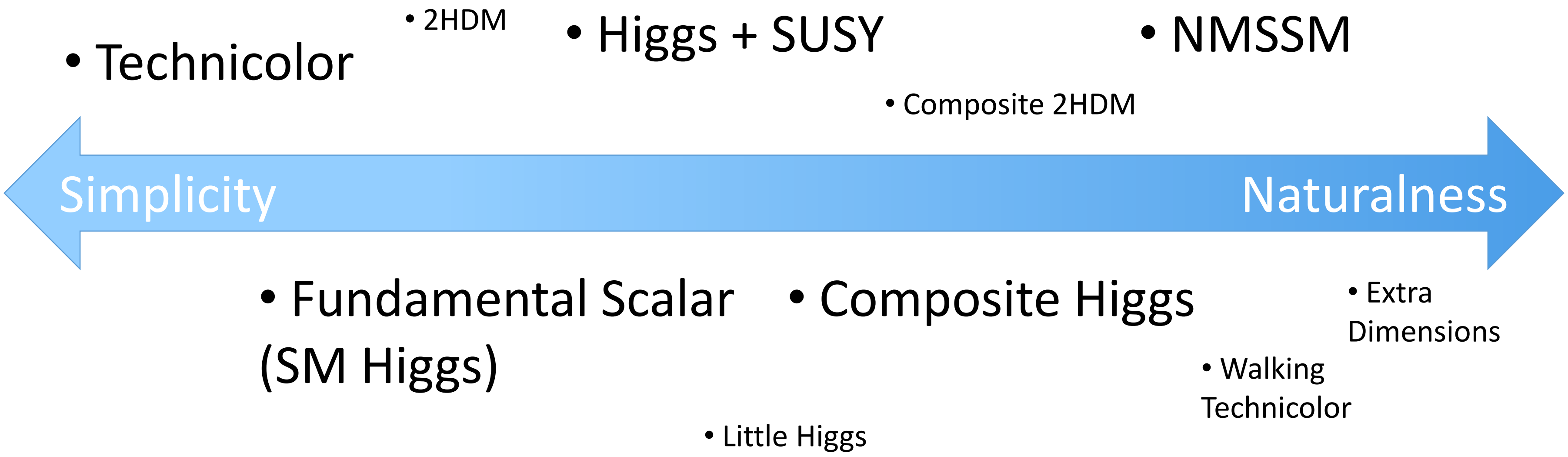
Beyond the Standard Model?

- A priori many ways to break electroweak symmetry!
- But tension between simplicity and naturalness



- Technicolor
- 2HDM
- Higgs + SUSY
- NMSSM
- Composite 2HDM

- Fundamental Scalar (SM Higgs)
- Composite Higgs
- Extra Dimensions
- Walking Technicolor
- Little Higgs

EFT for weak bosons

- 1980s-2012: Discovery of weak bosons -> Non-linear effective Lagrangian for spontaneously-broken global symmetry (breaking mechanism unknown!)
- **Global** symmetry-breaking pattern gives low-energy effective theory regardless of UV mechanism responsible for it

$$SU(2) \times SU(2) \to SU(2)_V \qquad (\rho \equiv M_W / M_Z \cos\theta_w \sim 1)$$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma - m_i \bar{\psi}_L^i \Sigma \psi_R^i + \mathrm{h.c.}$$

$$\Sigma = \exp\left(i \frac{\sigma^a \pi^a}{v} \right)$$

EFT for weak bosons + scalar

- 2012: Non-linear electroweak Lagrangian + general couplings to singlet scalar

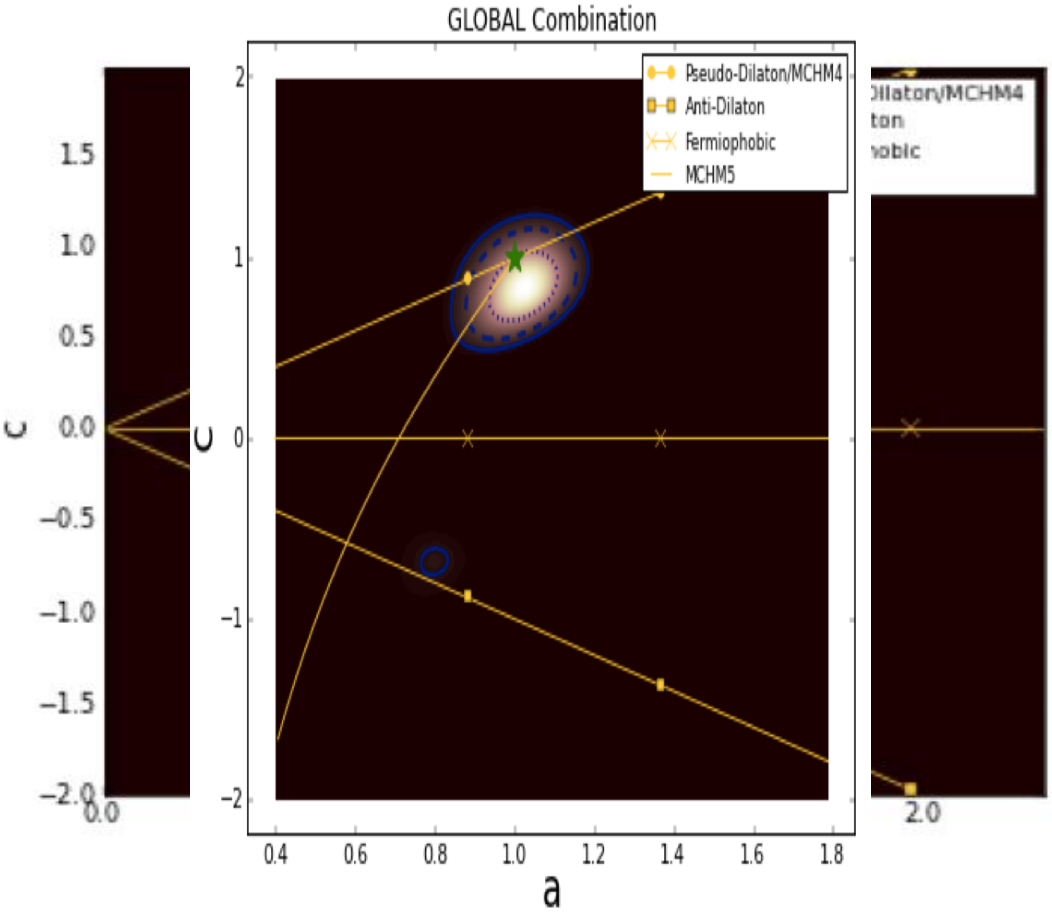
$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \dots\right) - m_i \bar{\psi}^i_L \Sigma \left(1 + c\frac{h}{v} + \dots\right) \psi^i_R + \text{h.c.}$$

$$+ \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}m_h^2 h^2 + d_3 \frac{1}{6}\left(\frac{3m_h^2}{v}\right) h^3 + d_4 \frac{1}{24}\left(\frac{3m_h^2}{v^2}\right) h^4 + \dots \quad ,$$

$$\Sigma = \exp\left(i\frac{\sigma^a \pi^a}{v}\right)$$

Fit experimental data to couplings

- Could have had very different coupling patterns than SM!



March 2012 pre-discovery
July 2012 post-discovery

J. Ellis and T.Y. [arXiv:1207.1693]

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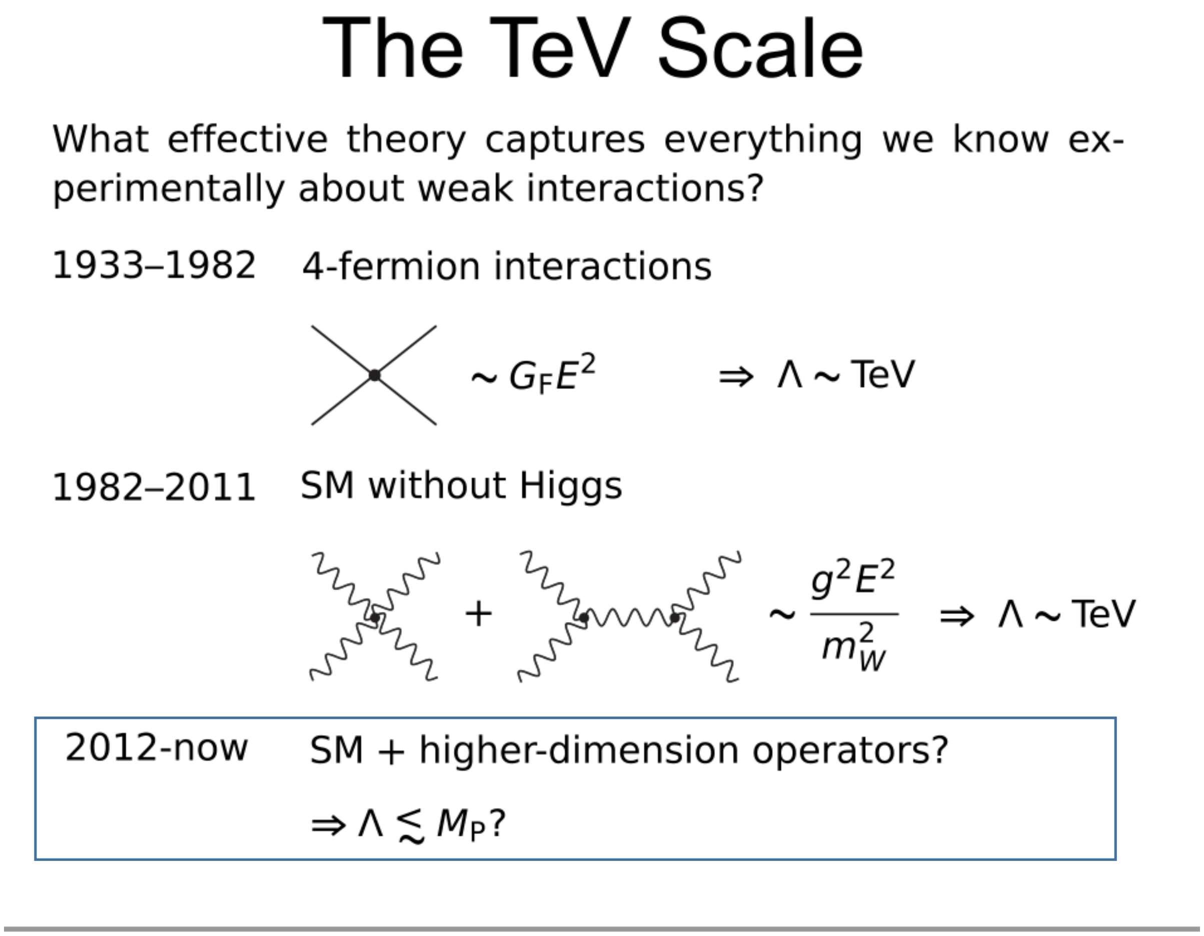

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$\Rightarrow \Lambda \lesssim M_P$?

Markus Luty PASCOS 2015 slide

Dimension-6 Operators

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
q_R^u	$\mathbf{3}$	$\mathbf{1}$	$\frac{2}{3}$
q_R^d	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$
L_L	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
l_R	$\mathbf{1}$	$\mathbf{1}$	-1
ϕ	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$

$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y$$

$$\boxed{\mathcal{L}_{\text{SM}}^{\text{dim-6}} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i}$$

$$\begin{aligned}
\mathcal{L}_m &= \bar{Q}_L i\gamma^\mu D_\mu^L Q_L + \bar{q}_R i\gamma^\mu D_\mu^R q_R + \bar{L}_L i\gamma^\mu D_\mu^L L_L + \bar{l}_R i\gamma^\mu D_\mu^R l_R \\
\mathcal{L}_G &= -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} \\
\mathcal{L}_H &= (D_\mu^L\phi)^\dagger(D^{L\mu}\phi) - V(\phi) \\
\mathcal{L}_Y &= y_d\bar{Q}_L\phi q_R^d + y_u\bar{Q}_L\phi^c q_R^u + y_L\bar{L}_L\phi l_R + \text{h.c.} \qquad ,
\end{aligned}$$

- First classified systematically by Buchmuller and Wyler (Nucl. Phys. B 268 (1986) 621)
- 59 dim-6 CP-even operators in a **non-redundant** basis, assuming minimal flavor structure (Gradkowski et al [arXiv:1008.4884])

$$\begin{aligned}
\mathcal{O}_H &= \tfrac{1}{2}(\partial^\mu|H|^2)^2 \\
\mathcal{O}_T &= \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 \\
\mathcal{O}_6 &= \lambda|H|^6 \\
\mathcal{O}_W &= \tfrac{ig}{2}\left(H^\dagger\sigma^a \overleftrightarrow{D^\mu} H\right) D^\nu W_{\mu\nu}^a \\
\mathcal{O}_B &= \tfrac{ig'}{2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\partial^\nu B_{\mu\nu}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{BB} &= g'^2|H|^2 B_{\mu\nu}B^{\mu\nu} \\
\mathcal{O}_{GG} &= g_s^2|H|^2 G_{\mu\nu}^A G^{A\mu\nu} \\
\mathcal{O}_{HW} &= ig(D^\mu H)^\dagger\sigma^a(D^\nu H)W_{\mu\nu}^a \\
\mathcal{O}_{HB} &= ig'(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu} \\
\mathcal{O}_{3W} &= \tfrac{1}{3!}g\epsilon_{abc}W_\mu^{a\,\nu}W_{\nu\rho}^b W^{c\,\rho\mu}
\end{aligned}$$

Basis adopted from Pomarol and Riva 1308.1426

(SILH basis Giudice et al. hep-ph/0703164)

$\mathcal{O}_{y_u} = y_u\|H\|^2\bar{Q}_L\widetilde{H}u_R + \text{h.c.}$	$\mathcal{O}_{y_d} = y_d\|H\|^2\bar{Q}_L H d_R + \text{h.c.}$	$\mathcal{O}_{y_e} = y_e\|H\|^2\bar{L}_L H e_R + \text{h.c.}$
$\mathcal{O}_R^u = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu u_R)$ $\mathcal{O}_L^q = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)$ $\mathcal{O}_L^{(3)\,q} = (iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L\sigma^a\gamma^\mu Q_L)$	$\mathcal{O}_R^d = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}_R^e = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu e_R)$
$\mathcal{O}_{LL}^{(3)\,ql} = (\bar{Q}_L\sigma^a\gamma_\mu Q_L)(\bar{L}_L\sigma^a\gamma^\mu L_L)$		$\mathcal{O}_{LL}^{(3)\,l} = (\bar{L}_L\sigma^a\gamma^\mu L_L)(\bar{L}_L\sigma^a\gamma_\mu L_L)$

Modifications of EWPO from dim-6 Operators

- (Pseudo-)Observables

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_\ell + 3\Gamma_\nu \qquad R_\ell = \frac{\Gamma_{had}}{\Gamma_\ell} \qquad \sigma_{had} = 12\pi \frac{\Gamma_e \Gamma_{had}}{\hat{m}_Z^2 \Gamma_Z^2} \qquad A_{FB}^f = \frac{3}{4} A_e A_f \qquad M_W = c_W M_Z$$

$$R_q = \frac{\Gamma_q}{\Gamma_{had}}$$

- Depends on

$$\Gamma_f = \frac{\sqrt{2} G_F M_Z^2 \hat{M}_Z}{6\pi}\left[(g_L^f)^2 + (g_R^f)^2\right] \qquad A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}$$

$$g^f = T_f^3 - Q_f s_W^2 \qquad s_W^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} G_F m_Z^2}}$$

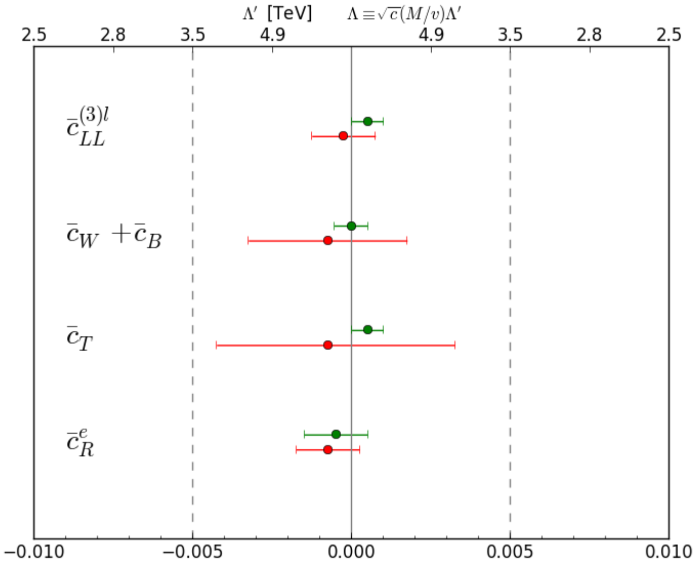
- Dim-6 operators can modify observables directly through Zff couplings contributions or indirectly through redefinitions of input observables

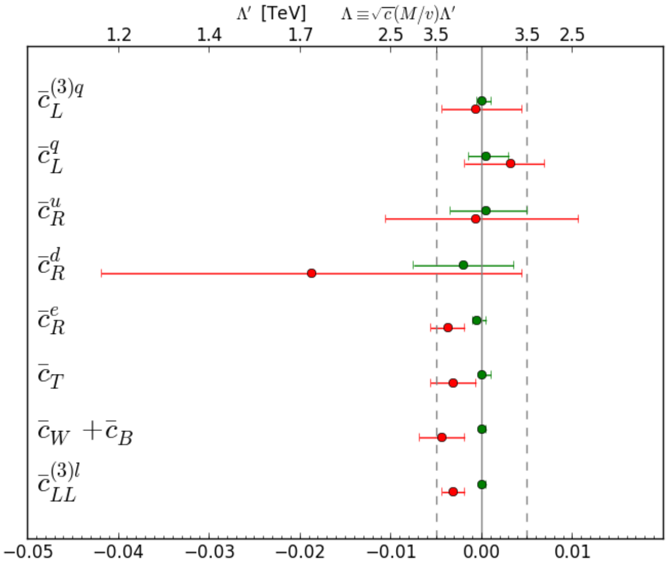
$$m_Z^2 = (m_Z^2)^0 (1 + \Pi_{ZZ}) \qquad G_F = G_F^0 (1 - \Pi_{WW}^0) \qquad \alpha(m_Z) = \alpha^0(m_Z)(1 + \Pi'_{\gamma\gamma})$$

SM EFT Present Constraints

- Marginalized constraints on a complete non-redundant basis of dim-6 operators affecting EWPTs

Ellis, Sanz and T.Y. 1410.7703

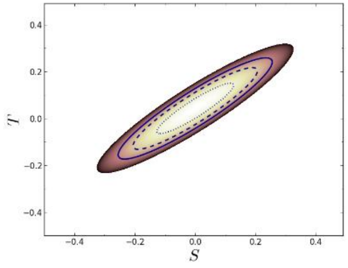




- S,T parameter corresponds to $(c_W + c_B), c_T$ subset

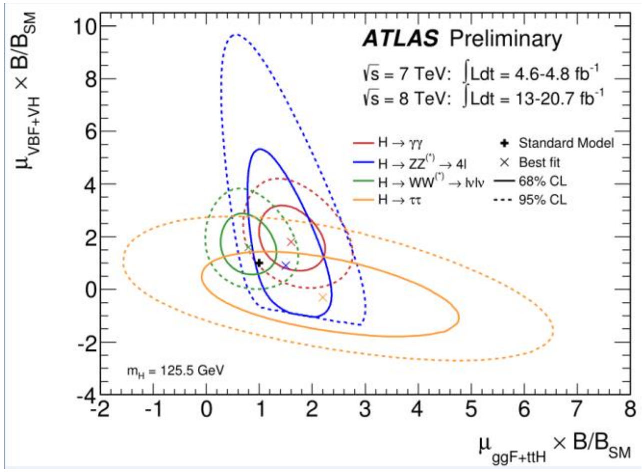
$$S = \frac{4\sin^2\theta_W}{\alpha(m_Z)}(\bar c_W + \bar c_B) \approx 119(\bar c_W + \bar c_B)$$

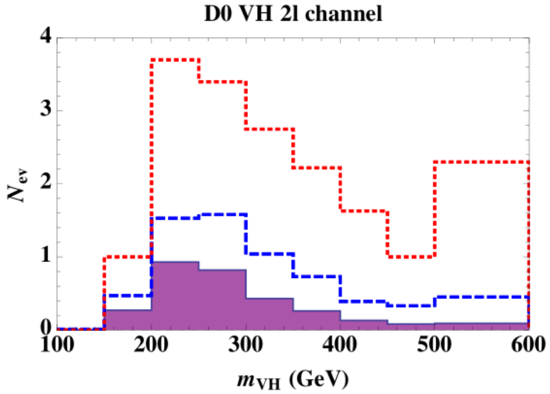
$$T = \frac{1}{\alpha(m_Z)}\bar c_T \approx 129\bar c_T\,.$$

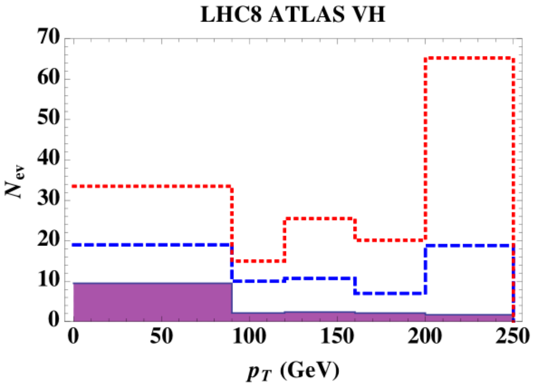


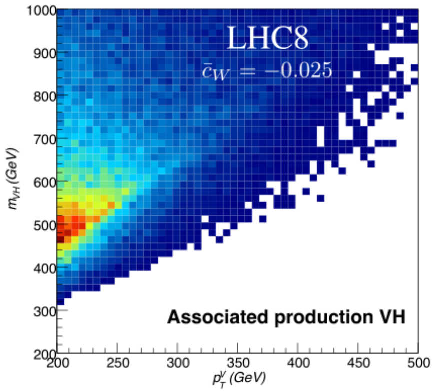
Higgs constraints on dim-6 operators

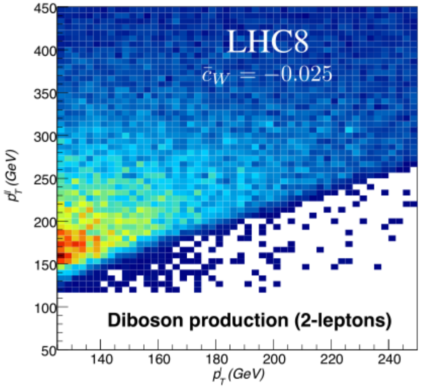
- Operators affect Higgs signal strength measurements, differential distributions







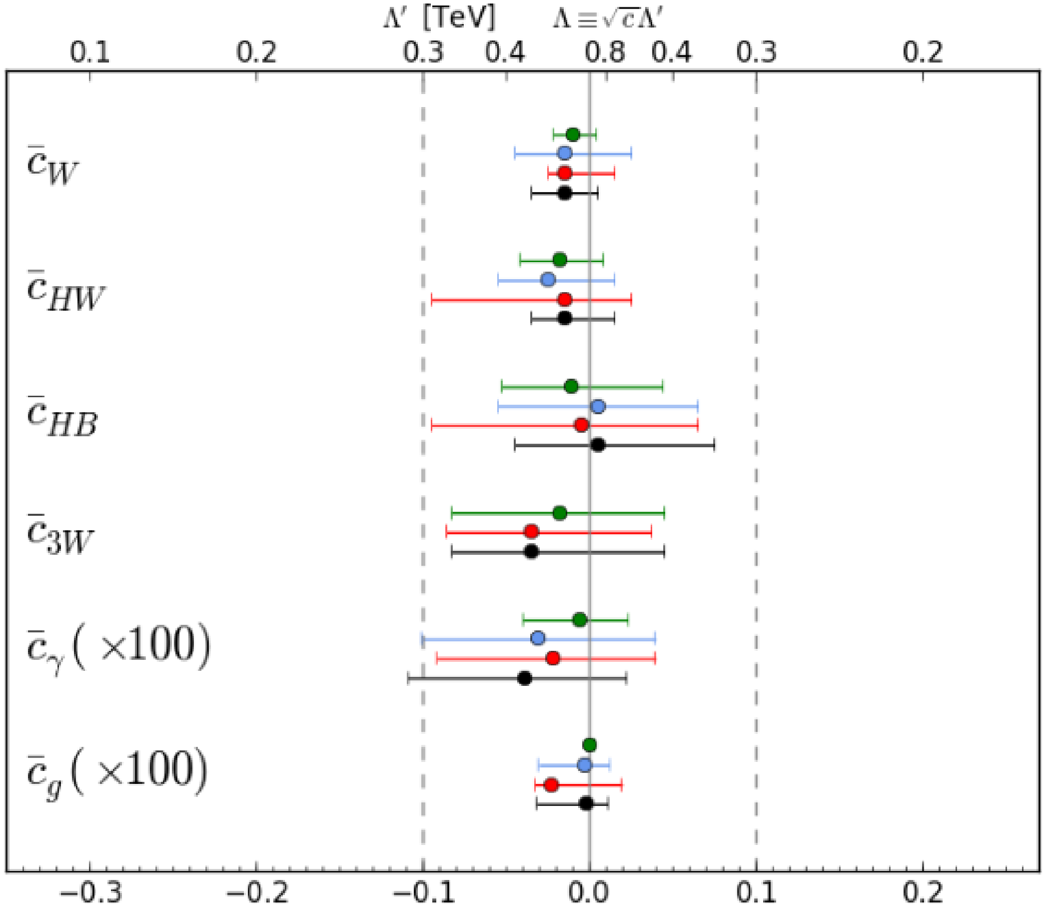




Ellis, Sanz and T.Y. 1410.7703

SM EFT Present Constraints

- Constraints from LHC triple-gauge coupling measurements and Higgs physics

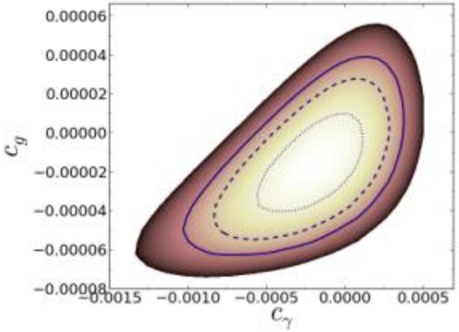


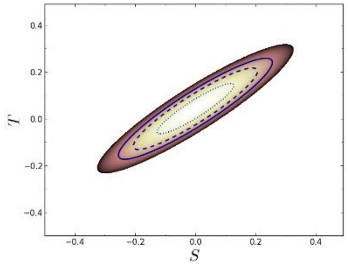
x 10 TeV

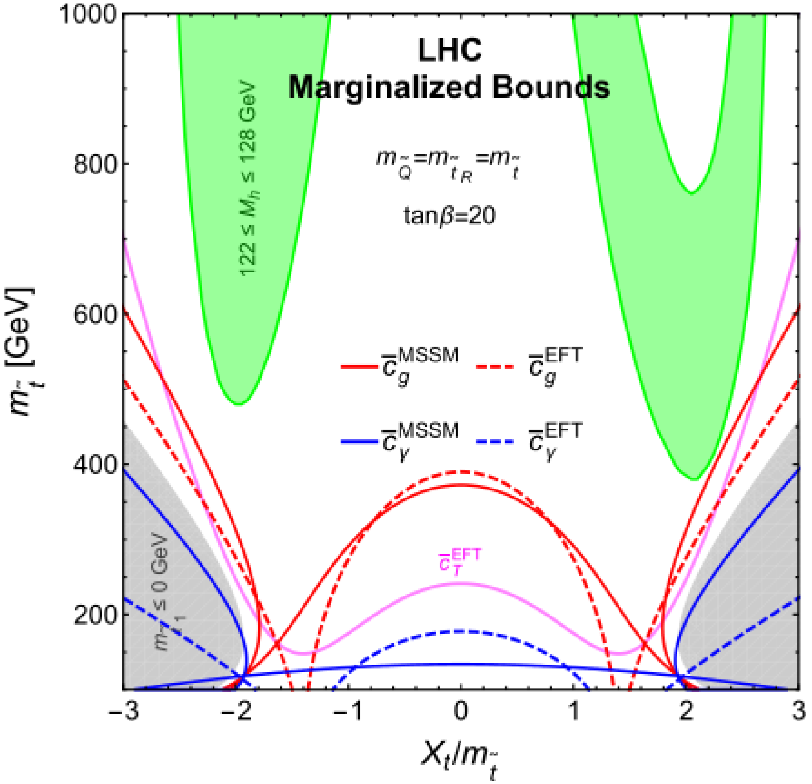
Ellis, Sanz and T.Y. 1410.7703

Translating EFT Constraints to MSSM Stops

Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$, $X_t = 0$
$\bar{c}_g$	LHC	marginalized	$[-4.5, 2.2] \times 10^{-5}$	$\sim$ 410 GeV
		individual	$[-3.0, 2.5] \times 10^{-5}$	$\sim$ 390 GeV
$\bar{c}_\gamma$	LHC	marginalized	$[-6.5, 2.7] \times 10^{-4}$	$\sim$ 215 GeV
		individual	$[-4.0, 2.3] \times 10^{-4}$	$\sim$ 230 GeV
$\bar{c}_T$	LEP	marginalized	$[-10, 10] \times 10^{-4}$	$\sim$ 290 GeV
		individual	$[-5, 5] \times 10^{-4}$	$\sim$ 380 GeV
$\bar{c}_W + \bar{c}_B$	LEP	marginalized	$[-7, 7] \times 10^{-4}$	$\sim$ 185 GeV
		individual	$[-5, 5] \times 10^{-4}$	$\sim$ 195 GeV

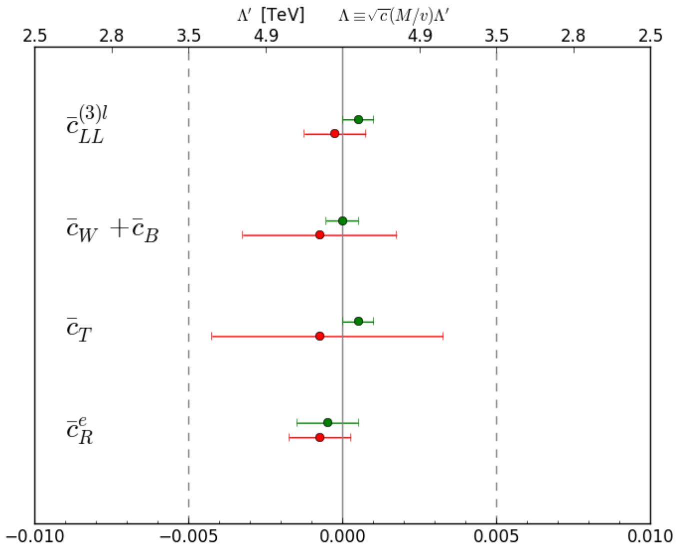






Drozd, Ellis, Quevillon and T.Y.
1504.02409

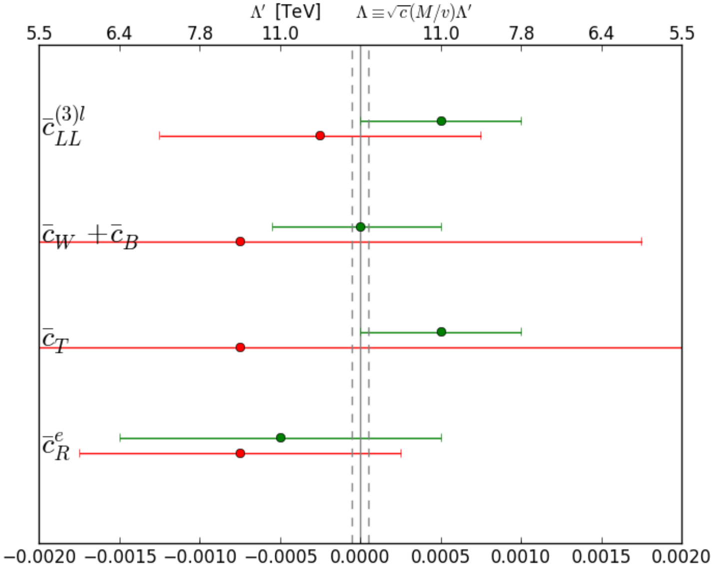
FCC-ee EWPT Constraints



LEP

FCC-ee EWPT Constraints

Tevong You



LEP

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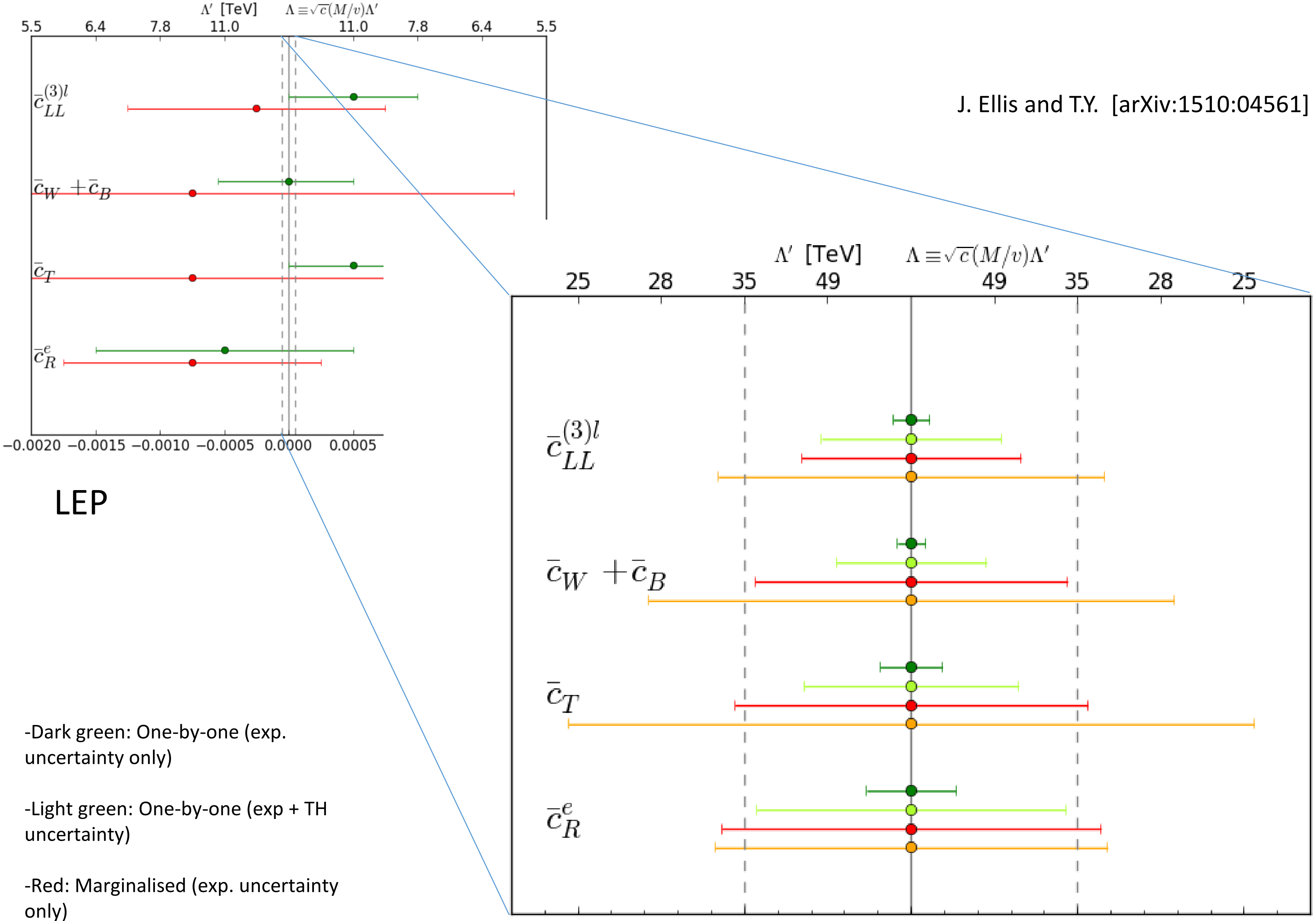
J. Ellis and T.Y. [arXiv:1510:04561]



LEP

FCC-ee

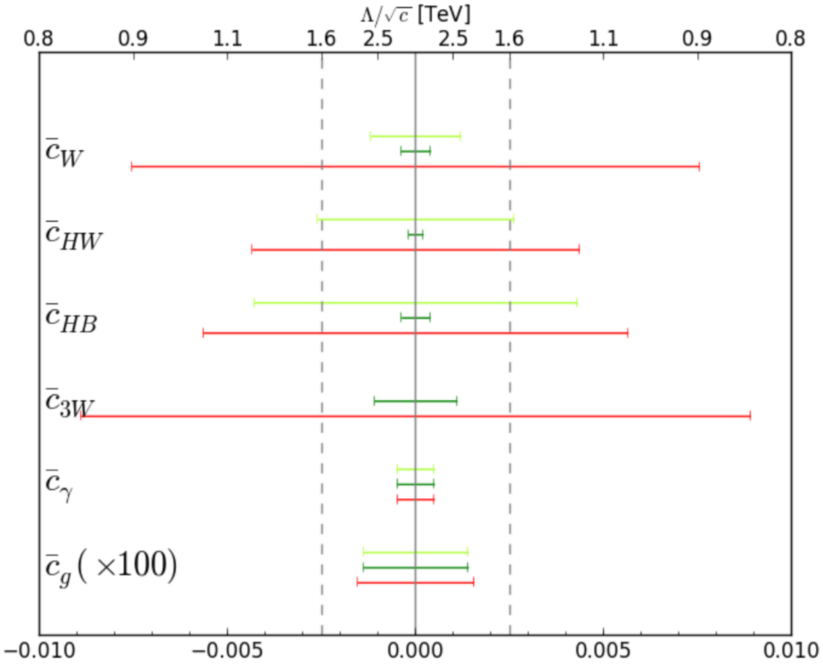
-Dark green: One-by-one (exp. uncertainty only)

-Light green: One-by-one (exp + TH uncertainty)

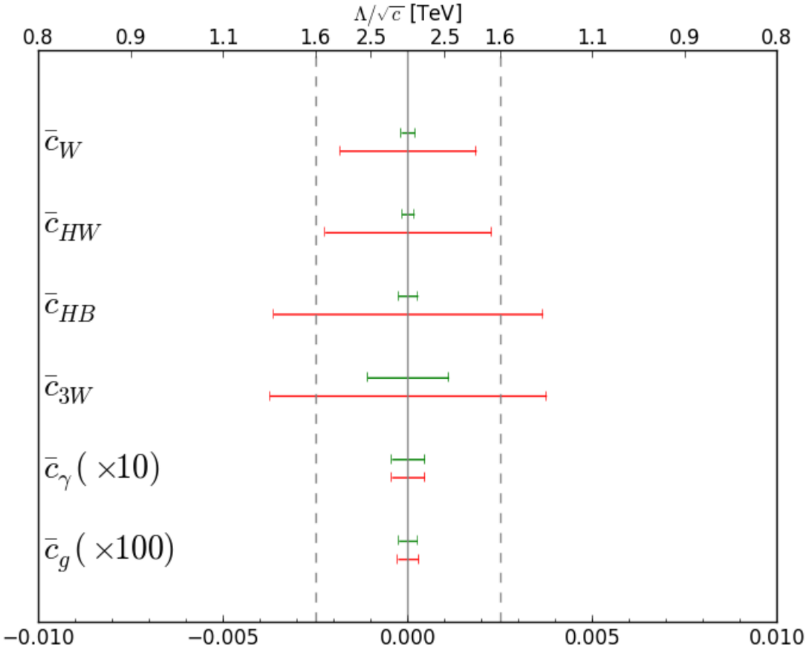
-Red: Marginalised (exp. uncertainty only)

-Orange: Marginalised (exp + TH uncertainty

Future Higgs Constraints



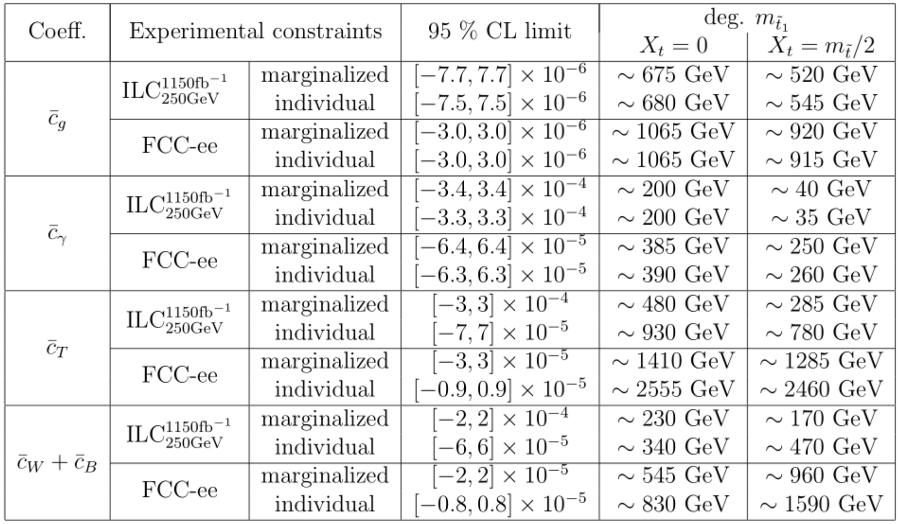
ILC



FCC-ee

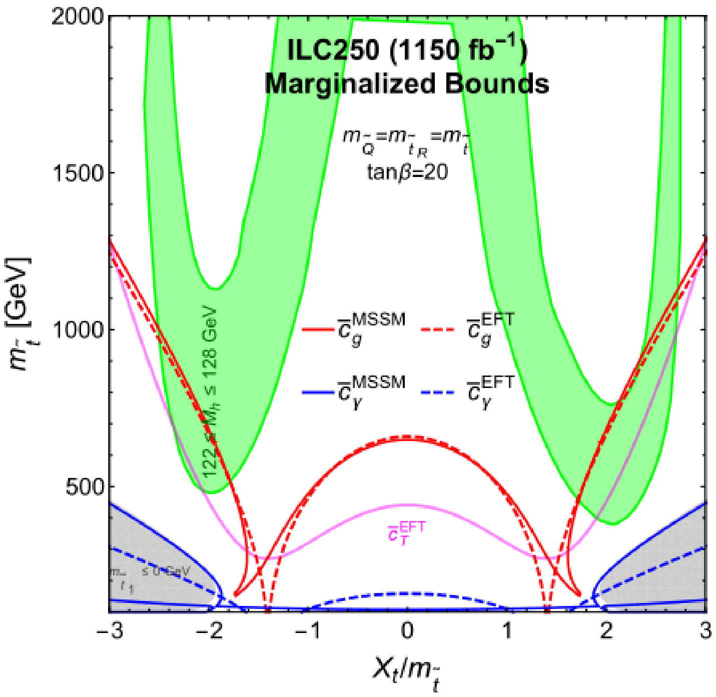
- Similar precision to current EWPT

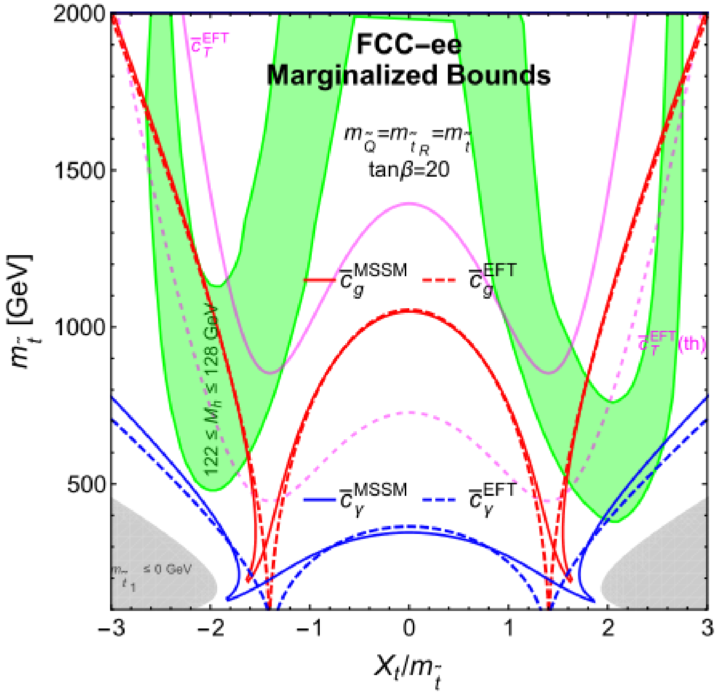
Future Constraints to MSSM Stops



Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$ $X_t = 0$	deg. $m_{\tilde{t}_1}$ $X_t = m_{\tilde{t}}/2$
$\bar{c}_g$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-7.7, 7.7] \times 10^{-6}$	~ 675 GeV	~ 520 GeV
		individual	$[-7.5, 7.5] \times 10^{-6}$	~ 680 GeV	~ 545 GeV
	FCC-ee	marginalized	$[-3.0, 3.0] \times 10^{-6}$	~ 1065 GeV	~ 920 GeV
		individual	$[-3.0, 3.0] \times 10^{-6}$	~ 1065 GeV	~ 915 GeV
$\bar{c}_\gamma$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-3.4, 3.4] \times 10^{-4}$	~ 200 GeV	~ 40 GeV
		individual	$[-3.3, 3.3] \times 10^{-4}$	~ 200 GeV	~ 35 GeV
	FCC-ee	marginalized	$[-6.4, 6.4] \times 10^{-5}$	~ 385 GeV	~ 250 GeV
		individual	$[-6.3, 6.3] \times 10^{-5}$	~ 390 GeV	~ 260 GeV
$\bar{c}_T$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-3, 3] \times 10^{-4}$	~ 480 GeV	~ 285 GeV
		individual	$[-7, 7] \times 10^{-5}$	~ 930 GeV	~ 780 GeV
	FCC-ee	marginalized	$[-3, 3] \times 10^{-5}$	~ 1410 GeV	~ 1285 GeV
		individual	$[-0.9, 0.9] \times 10^{-5}$	~ 2555 GeV	~ 2460 GeV
$\bar{c}_W + \bar{c}_B$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-2, 2] \times 10^{-4}$	~ 230 GeV	~ 170 GeV
		individual	$[-6, 6] \times 10^{-5}$	~ 340 GeV	~ 470 GeV
	FCC-ee	marginalized	$[-2, 2] \times 10^{-5}$	~ 545 GeV	~ 960 GeV
		individual	$[-0.8, 0.8] \times 10^{-5}$	~ 830 GeV	~ 1590 GeV

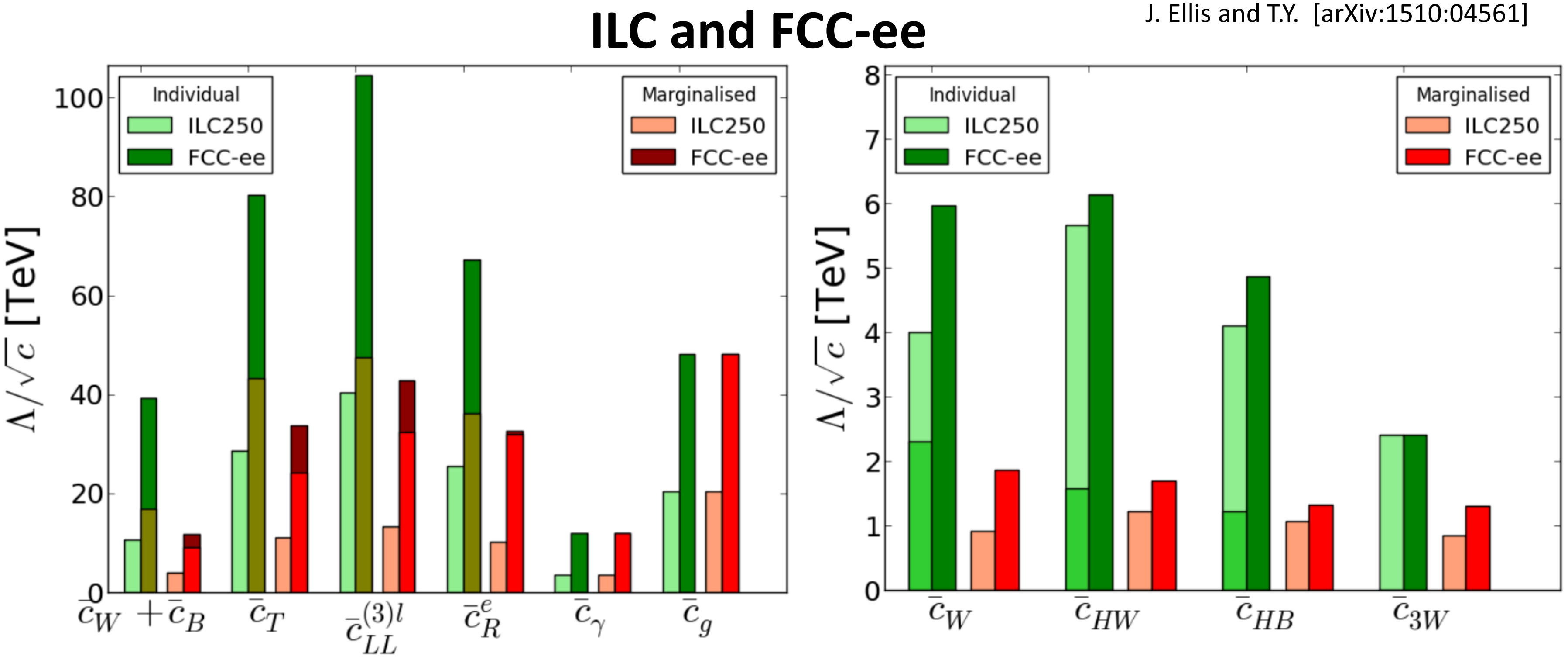
Drozd, Ellis, Quevillon and T.Y.
1504.02409





Future e+e- Constraints

J. Ellis and T.Y. [arXiv:1510:04561]



ILC and FCC-ee

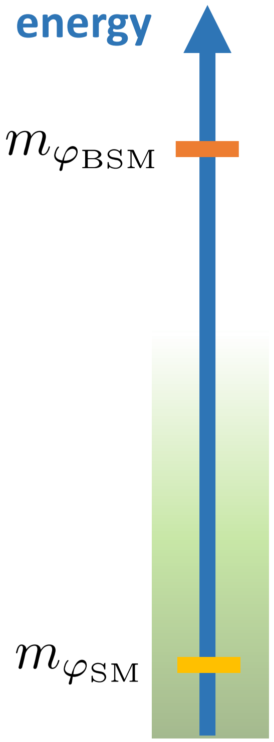
- Future precision sensitive to TeV scale, even for loop-induced operators
- One-loop matching simplified by a **Universal One-Loop Effective Action**

-Henning, Lu & Murayama [arXiv:1412.1837]
-A. Drozd, J. Ellis, J. Quevillon and TY [arXiv:1512.03003]

- Part I: SM EFT
- **Part 2: The Universal One-Loop Effective Action**
- Part 3: Cosmological Relaxation

Introduction

- Matching UV theory onto an EFT Lagrangian:



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

⇩ ***match*** *(UV ←→ EFT dictionary)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

⇩ ***run*** *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\rm exp})$$

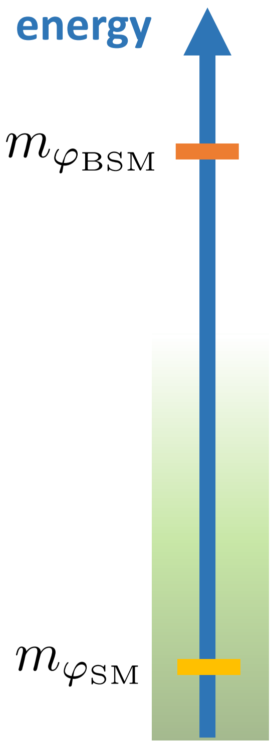
⇩ ***calculate*** *(model-independent)*

observables (cross sections, etc.)

Slide from Z. Zhang

Introduction

- Standard approach is to use Feynman diagrams



$$\mathcal{L}_\mathrm{UV}[\varphi_\mathrm{BSM}, \varphi_\mathrm{SM}]$$

Feynman diagrams

$$\langle \varphi_L \varphi_L \dots \varphi_L \rangle$$

$$\mathcal{L}_\mathrm{EFT}[\varphi_\mathrm{SM}] = \mathcal{L}_\mathrm{SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

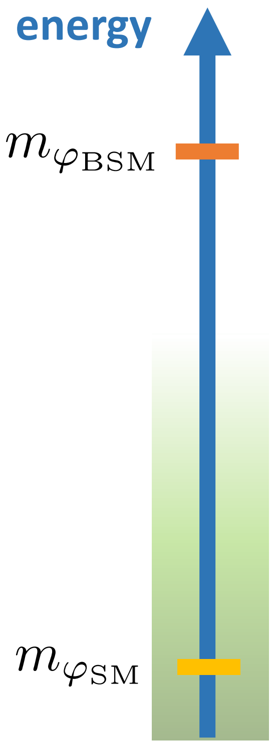
$$\mathcal{L}_\mathrm{EFT}[\varphi_\mathrm{SM}] = \mathcal{L}_\mathrm{SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim E_\mathrm{exp})$$

calculate *(model-independent)*

observables (cross sections, etc.)

Introduction

- Functional method more **elegant** and **direct** way of matching
- But *many* ways of doing this:



$$\mathcal{L}_{\text{UV}}[\varphi_{\text{BSM}}, \varphi_{\text{SM}}]$$

Functional path integral evaluation

$$\mathcal{L}_{\text{EFT}}[\varphi_{\text{SM}}] = \mathcal{L}_{\text{SM}} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\text{EFT}}[\varphi_{\text{SM}}] = \mathcal{L}_{\text{SM}} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\text{exp}})$$

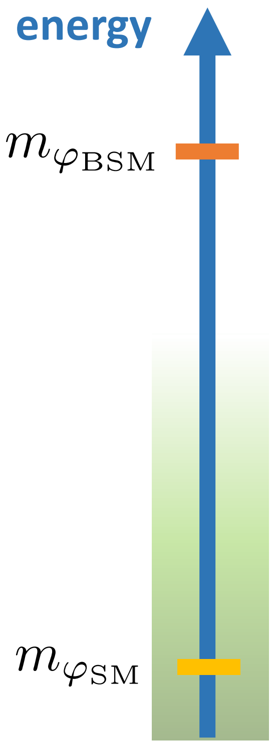
calculate *(model-independent)*

observables (cross sections, etc.)

Introduction

- Functional method more **elegant** and **direct** way of matching
- But *many* ways of doing this:

e.g. Schwinger proper time, **Covariant Derivative Expansion** methods, Various log expansions, heavy-light subtraction procedures, integration by regions, **covariant diagrams**, etc.



$\mathcal{L}_{\mathrm{UV}}[\varphi_{\mathrm{BSM}}$

$$\mathcal{L}_{\mathrm{EFT}}[\varphi_{\mathrm{SM}}] = \mathcal{L}_{\mathrm{SM}} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

***run** (resum large logs)*

$$\mathcal{L}_{\mathrm{EFT}}[\varphi_{\mathrm{SM}}] = \mathcal{L}_{\mathrm{SM}} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim E_{\mathrm{exp}})$$

***calculate** (model-independent)*

observables (cross sections, etc.)

Functional methods: Gaillard-Cheyette CDE

- **Gaillard-Cheyette** *('86, '88)* method of doing Covariant Derivative Expansion reviewed/revived in **HLM** *(Henning, Lu, Murayama, 1412.1837)*

- Evaluate the path integral of the action in the usual way:
  - Expand action around minimum
  - Write Gaussian integral as determinant
  - Write determinant as trace of log in exponent

$$
\begin{aligned}
e^{iS_{\text{eff}}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\
&= \int [D\eta] \underline{e^{i\left(S[\phi,\Phi_c]+\frac{1}{2}\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\eta^2+\mathcal{O}(\eta^3)\right)}} \\
&\approx e^{iS[\phi,\Phi_c]} \underline{\left[\det\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\right)\right]}^{-\frac{1}{2}} \\
&\approx e^{iS[\phi,\Phi_c] \underline{-\frac{1}{2}\text{Tr}\ \ln\left(-\left.\frac{\delta^2 S}{\delta\Phi}\right|_{\Phi=\Phi_c}\right)}} ,
\end{aligned}
$$

- This is common to all functional methods
  - Gaillard-Cheyette also do *momentum shift* before expanding logarithm (see later slide)
  - Also, different methods used for expanding log

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + (\Phi^\dagger \boxed{F(x)} + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - \boxed{U(x)})\Phi + \mathcal{O}(\Phi^3)\,,$$

Model-dependent light fields encapsulated in F and U

$$\left.\begin{aligned}
e^{iS_{\text{eff}}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\
&= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2} \frac{\delta^2 S}{\delta \Phi^2}\Big|_{\Phi=\Phi_c} \eta^2 + \mathcal{O}(\eta^3)\right)} \\
&\approx e^{iS[\phi,\Phi_c]} \left[\det\left(-\frac{\delta^2 S}{\delta \Phi^2}\bigg|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\
&\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}\text{Tr }\ln\left(-\frac{\delta^2 S}{\delta\Phi}\Big|_{\Phi=\Phi_c}\right)}\,,
\end{aligned}\right\}$$

$$\begin{aligned}
S^{\text{eff}}_{\text{1-loop}} &= ic_s \text{Tr }\ln\left(-P^2 + M^2 + \boxed{U}\right) \\
&= ic_s \int d^4x \int \frac{d^4q}{(2\pi)^4} \text{tr }\ln\left(-(P_\mu - q_\mu)^2 + M^2 + \boxed{U}\right)
\end{aligned}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger(P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

Heavy fields can be boson or fermion

$$\left.\begin{aligned} e^{iS_{\rm eff}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\ &= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2}\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\eta^2 + \mathcal{O}(\eta^3)\right)} \\ &\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\ &\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}\text{Tr}\,\ln\left(-\left.\frac{\delta^2 S}{\delta\Phi}\right|_{\Phi=\Phi_c}\right)}\,, \end{aligned}\right\}$$

$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= ic_s \text{Tr}\,\ln\left(-P^2 + M^2 + U\right) \\ &= ic_s \int d^4x \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + U\right) \end{aligned}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

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$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x)) \Phi + \mathcal{O}(\Phi^3)\,,$$

Heavy fields can be boson or fermion

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- Gaillard-Cheyette also do *momentum shift* by inserting $e^{\pm P_\mu \partial/\partial q_\mu}$

$$\begin{aligned} \mathcal{L}^{\text{eff}}_{\text{1-loop}} &= ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln[\underline{e^{P_\mu \partial/\partial q_\mu}}(-(P_\mu - q_\mu)^2 + M^2 + U)\underline{e^{-P_\mu \partial/\partial q_\mu}}] \\ &= ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln[-(\tilde{G}_{\nu\mu} \partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}] \end{aligned}$$

- So covariant derivatives are explicitly in commutators from beginning

$$\underline{\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!} [P_{\alpha_1}, [\ldots [P_{\alpha_n}, G'_{\nu\mu}]]] \frac{\partial^n}{\partial q_{\alpha_1} \ldots q_{\alpha_n}}}$$

$$\underline{\tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!} [P_{\alpha_1}, [\ldots [P_{\alpha_n}, U]]] \frac{\partial^n}{\partial q_{\alpha_1} \ldots q_{\alpha_n}}}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + {\rm h.c.}) + \Phi^\dagger(P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

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$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= ic_s {\rm Tr}\,\ln\left(-P^2 + M^2 + U\right) \\ &= ic_s \int d^4x \int \frac{d^4q}{(2\pi)^4} {\rm tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + U\right) \end{aligned}$$

- Gaillard-Cheyette also do *momentum shift* by inserting $e^{\pm P_\mu \partial/\partial q_\mu}$

$\mathcal{L}^{\rm eff}_{\text{1-loop}} =$ … $e^{-P_\mu\partial/\partial q_\mu}]$

$=$ … $\tilde{U}]$

- So covariant de… …ors from beginning

But **simpler to *avoid* momentum shift!**
Instead, gather result into commutators
after expansion evaluation.

See e.g.
Fuentes-Martin, Portoles, Ruiz-Femenia, 1607.02142;
Z. Zhang, 1610.00710.

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}[P_{\alpha_1},[\ldots[P_{\alpha_n},G_{\nu\mu}]]]\frac{\ldots}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

$$\tilde{U} = \sum_{n=0}\frac{\ldots}{n!}[P_{\alpha_1},[\ldots[P_{\alpha_n},U]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

Functional methods: Heavy-Light loops?

- Linear coupling = tree-level; quadratic coupling = *heavy-only* one-loop

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger \boxed{F(x)} + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - \boxed{U(x)})\Phi + \mathcal{O}(\Phi^3)\,,$$

- What about loops involving both heavy and light fields?
- Naively not accounted for in functional method

See e.g. Bilenky & Santamaria, hep-ph/9310302; Del Aguila, Kunszt, Santiago, 1602.00126.

- Solution: apply background field method to both **heavy** *and* **light** fields

$$\underline{\phi \to \phi_c + \phi'} \quad , \quad \underline{\Phi \to \Phi_c + \Phi'}$$

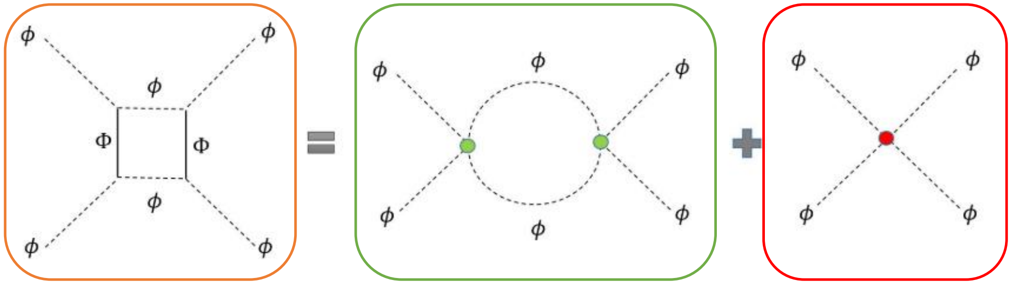
$$\mathcal{L}_{\rm quad} = \frac{1}{2}(\Phi', \phi') \begin{pmatrix} P^2 - M^2 - U_{\Phi\Phi} & -U_{\Phi\phi} \\ -U_{\phi\Phi} & P^2 - m^2 - U_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Phi' \\ \phi' \end{pmatrix}$$

Functional methods: Heavy-Light loops?

- *Just* apply background field method to both heavy and light fields?

$$\phi \to \phi_c + \phi' \quad , \quad \Phi \to \Phi_c + \Phi'$$

- Actually, this gives the one-loop 1PI effective action and **not** $\mathcal{L}_{\rm eff}$
  - Feynman diagram intuition: Heavy-light loops in UV theory match onto both **tree-level-generated** EFT operators *inserted at one-loop*, and **one-loop-generated** EFT operators *inserted at tree-level*



(soft part) *(hard part)*

  - The **former** is not part of $\mathcal{L}_{\rm eff}$, must be subtracted to keep only the **latter**

Functional methods: Heavy-Light subtractions

- Various subtraction procedures proposed

See e.g.
Boggia, Gomez-Ambrosio, Passarino, 1603.03660;
Henning, Lu, Murayama, 1604.01019;
Ellis, Quevillon, TY, Zhang, 1604.02445;
Fuentes-Martin, Portoles, Ruiz-Femenia, 1607.02142.

Universality property also applies to heavy-light case

Integration by regions method avoids subtraction, separates **hard** and **soft** part in integral, greatly simplifies heavy-light treatment

See e.g. Beneke & Smirnov, hep-ph/9711391;
Jantzen, 1111.2589;

- Simplification of evaluating CDE from these developments lead to a **Covariant Diagram** formulation *(Z. Zhang, 1610.00710)*

- But **Universality** of CDE results means evaluation via all these different methods *gives same model-independent expression*

Henning, Lu, Murayama, 1412.1837;
Drozd, J. Ellis, Quevillon, TY, 1512.03003;
S.A.R. Ellis, Quevillon, TY, Z. Zhang; 1705.xxxxx

Universality of the One-Loop Effective Action

- *No need to reinvent the wheel*, every slide up to now can be ignored

- Universality of CDE expansion results first noticed in the **simplified** case of **degenerate mass** for heavy fields *(Henning, Lu, Murayama, 1412.1837)*

- The *general* **Universal One-Loop Effective Action** (UOLEA) subsequently derived without such assumption *(Drozd, J. Ellis, Quevillon, TY, 1512.03003)*

- Extra structures (**heavy-light terms**, “open” covariant derivatives, momentum-shifted-gamma matrices) in CDE expansion not included in initial UOLEA *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1604.02445)*

- Universal **heavy-light** terms now done *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1705.xxxxx)*
- A *complete* UOLEA, including all possible CDE structures, is in sight…

Universality of the One-Loop Effective Action

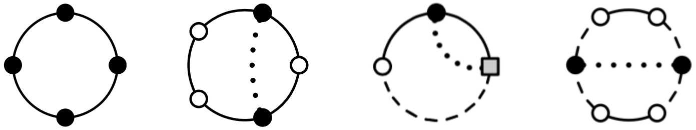
- Neglect these extra structures for now; derivation of universal results e.g. in Gaillard-Cheyette CDE starts from

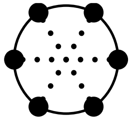
$$\mathcal{L}^{\text{eff}}_{\text{1-loop}} = ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr} \ln[e^{P_\mu \partial/\partial q_\mu}(-(P_\mu - q_\mu)^2 + M^2 + U)e^{-P_\mu \partial/\partial q_\mu}]$$

$$= ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr} \ln[-(\tilde{G}_{\nu\mu} \partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}]$$

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[P_{\alpha_1}, [...[P_{\alpha_n}, G'_{\nu\mu}]]] \frac{\partial^n}{\partial q_{\alpha_1} ... q_{\alpha_n}} \qquad \tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!}[P_{\alpha_1}, [...[P_{\alpha_n}, U]]] \frac{\partial^n}{\partial q_{\alpha_1} ... q_{\alpha_n}}$$

  - (much easier using Covariant Diagrams, see Z. Zhang talk)

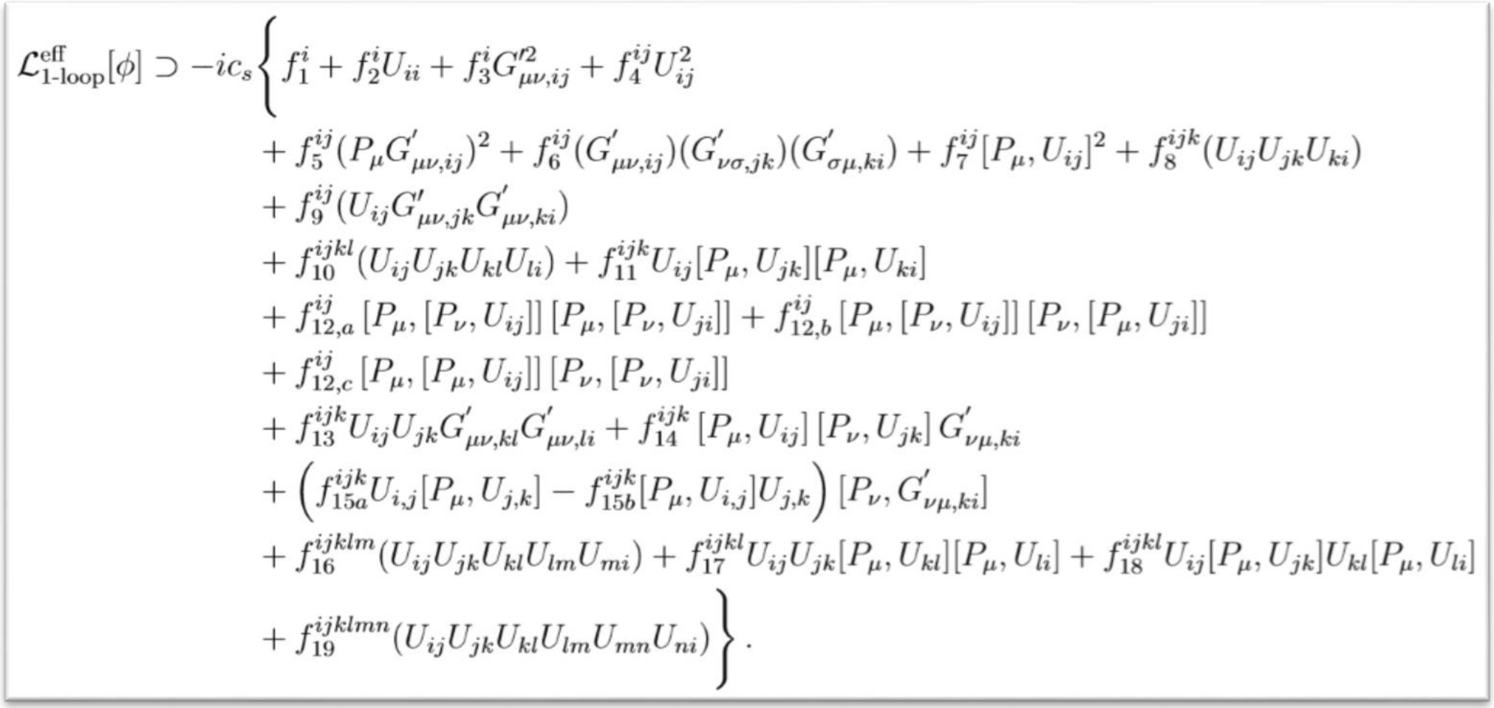




$$= -i \, \frac{1}{6} \mathcal{I}[q^6]^6_i \cdot 2^6 \, \text{tr}(P^\mu P^\nu P^\rho P_\mu P_\nu P_\rho)$$

Universality of the One-Loop Effective Action

- Whatever the method used to obtain it, the resulting UOLEA can be written as



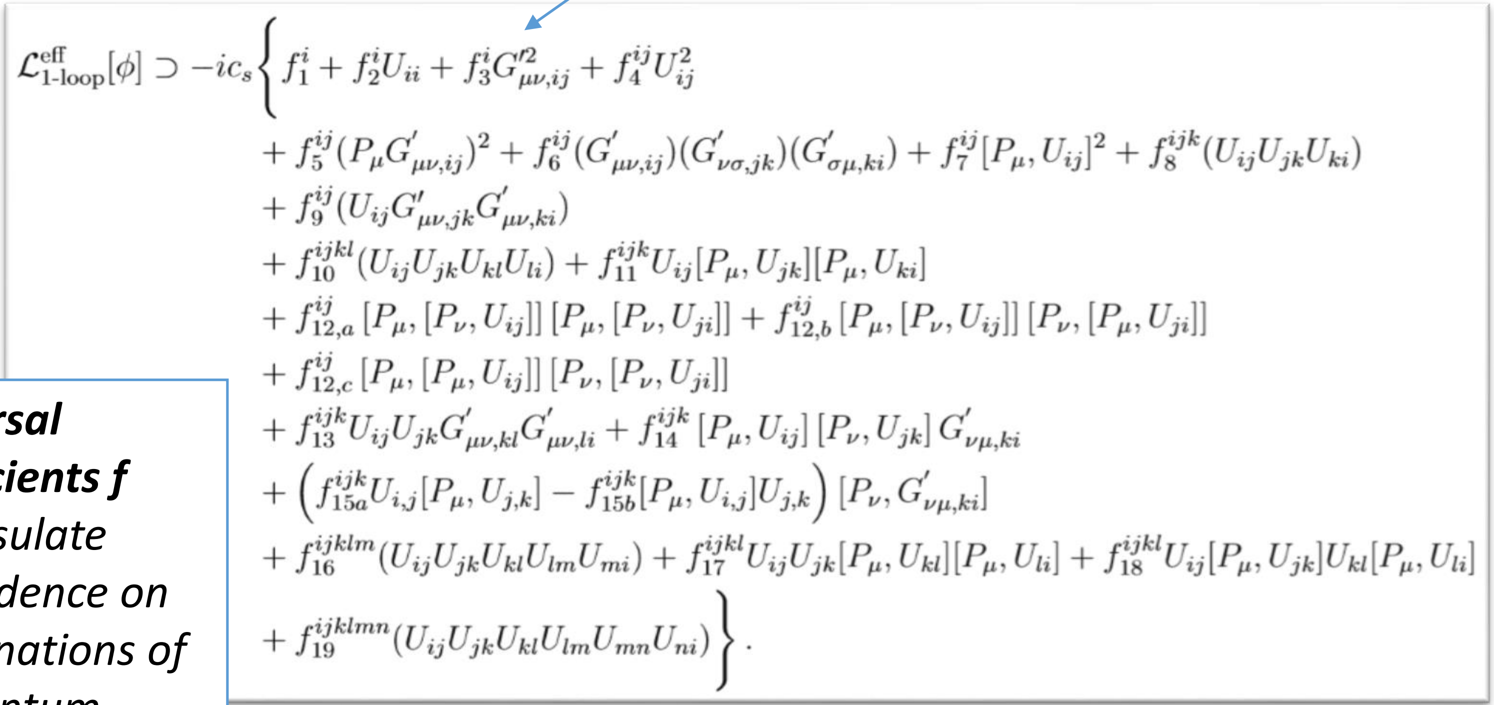
$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U^2_{ij} \\
& + f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2 + f_8^{ijk} (U_{ij} U_{jk} U_{ki}) \\
& + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij} \, [P_\mu, [P_\nu, U_{ij}]] \, [P_\mu, [P_\nu, U_{ji}]] + f_{12,b}^{ij} \, [P_\mu, [P_\nu, U_{ij}]] \, [P_\nu, [P_\mu, U_{ji}]] \\
& + f_{12,c}^{ij} \, [P_\mu, [P_\mu, U_{ij}]] \, [P_\nu, [P_\nu, U_{ji}]] \\
& + f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} \, [P_\mu, U_{ij}] \, [P_\nu, U_{jk}] \, G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] \\
& + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} \, .
\end{aligned}
$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003

Universality of the One-Loop Effective Action

- Whatever the method used to obtain it, the resulting UOLEA can be written as

(f3 universal term calculated by 't Hooft '73)



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& + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu,U_{ij}]^2 + f_8^{ijk}(U_{ij}U_{jk}U_{ki}) \\
& + f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu,U_{jk}][P_\mu,U_{ki}] \\
& + f_{12,a}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\mu,[P_\nu,U_{ji}]] + f_{12,b}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\nu,[P_\mu,U_{ji}]] \\
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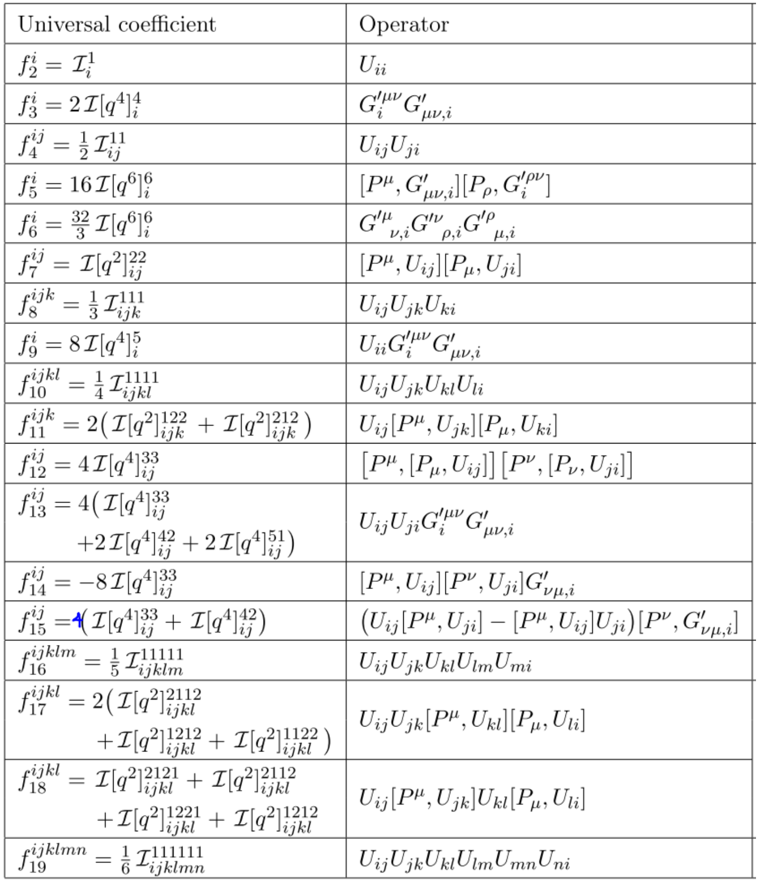
& + f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}\,[P_\mu,U_{ij}]\,[P_\nu,U_{jk}]\,G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk}U_{i,j}[P_\mu,U_{j,k}] - f_{15b}^{ijk}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu,G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] + f_{18}^{ijkl}U_{ij}[P_\mu,U_{jk}]U_{kl}[P_\mu,U_{li}] \\
& + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni})\Big\}\,.
\end{aligned}
$$

Universal coefficients f *encapsulate dependence on combinations of momentum master integrals*

Drozd, J. Ellis, Quevillon, TY, 1512.03003

Universality of the One-Loop Effective Action

- **Universal coefficients** in terms of standard master integrals:



Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33}$ $+2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

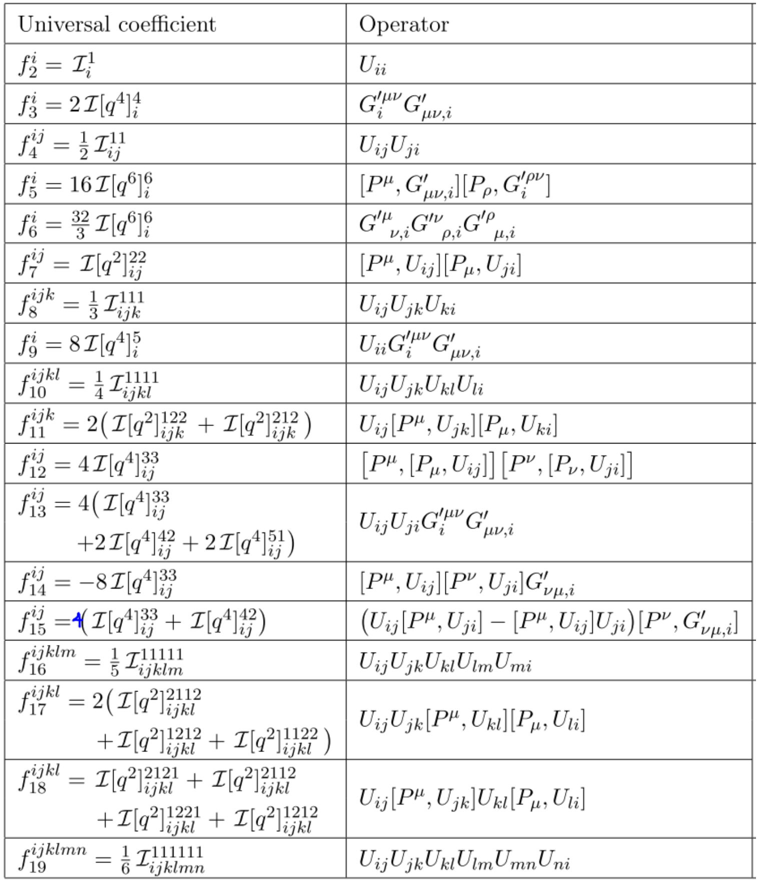
$$\int \frac{d^dq}{(2\pi)^d} \frac{q^{\mu_1}\cdots q^{\mu_{2n_c}}}{(q^2-M_i^2)^{n_i}(q^2-M_j^2)^{n_j}\cdots(q^2)^{n_L}} \equiv g^{\mu_1\cdots\mu_{2n_c}}\,\mathcal{I}[q^{2n_c}]_{ij\ldots0}^{n_in_j\ldots n_L}$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003;

Simplified form by covariant diagram computation shown here from Z. Zhang, 1610.00710.

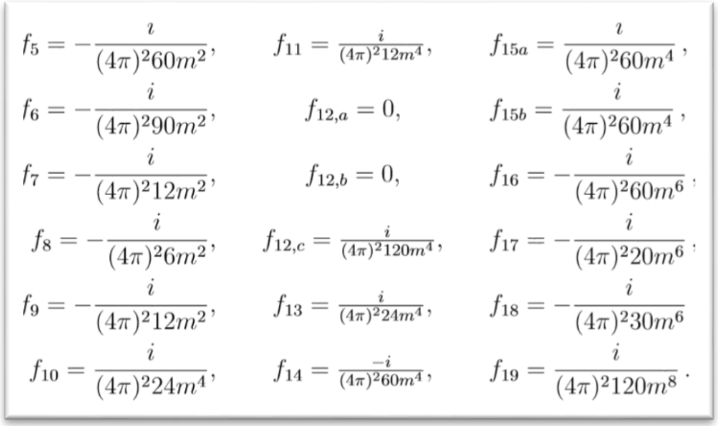
Universality of the One-Loop Effective Action

- **Universal coefficients** in terms of standard master integrals:



Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33}$ $+2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

Degenerate limit *(Henning, Lu, Murayama, 1412.1837)*



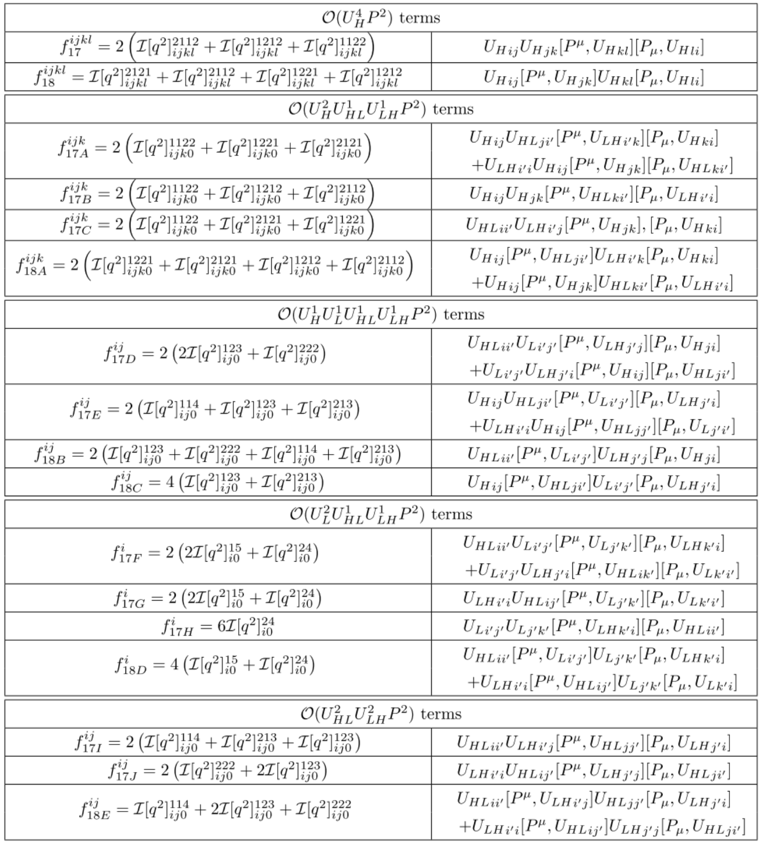
$$
\begin{array}{lll}
f_5 = -\dfrac{i}{(4\pi)^2 60 m^2}, & f_{11} = \frac{i}{(4\pi)^2 12 m^4}, & f_{15a} = \dfrac{i}{(4\pi)^2 60 m^4}, \\
f_6 = -\dfrac{i}{(4\pi)^2 90 m^2}, & f_{12,a} = 0, & f_{15b} = \dfrac{i}{(4\pi)^2 60 m^4}, \\
f_7 = -\dfrac{i}{(4\pi)^2 12 m^2}, & f_{12,b} = 0, & f_{16} = -\dfrac{i}{(4\pi)^2 60 m^6}; \\
f_8 = -\dfrac{i}{(4\pi)^2 6 m^2}, & f_{12,c} = \frac{i}{(4\pi)^2 120 m^4}, & f_{17} = -\dfrac{i}{(4\pi)^2 20 m^6}; \\
f_9 = -\dfrac{i}{(4\pi)^2 12 m^2}, & f_{13} = \frac{i}{(4\pi)^2 24 m^4}, & f_{18} = -\dfrac{i}{(4\pi)^2 30 m^6} \\
f_{10} = \dfrac{i}{(4\pi)^2 24 m^4}, & f_{14} = \frac{-i}{(4\pi)^2 60 m^4}, & f_{19} = \dfrac{i}{(4\pi)^2 120 m^8}.
\end{array}
$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003;

Simplified form by covariant diagram computation shown here from Z. Zhang, 1610.00710.

Universality of the One-Loop Effective Action

- **Heavy-light** extension also done: *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1705.xxxxx)*



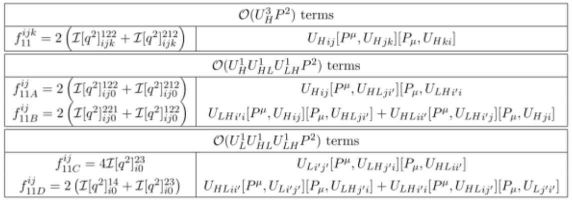
$\mathcal{O}(U_H^4 P^2)$ terms	
$f_{17}^{ijkl} = 2\left(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{Hkl}][P_\mu, U_{Hli}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{Hij}[P^\mu, U_{Hjk}]U_{Hkl}[P_\mu, U_{Hli}]$

$\mathcal{O}(U_H^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17A}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{1221} + \mathcal{I}[q^2]_{ijk0}^{2121}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{LHi'k}][P_\mu, U_{Hki}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{HLki'}]$
$f_{17B}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{1212} + \mathcal{I}[q^2]_{ijk0}^{2112}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{HLki'}][P_\mu, U_{LHi'i}]$
$f_{17C}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{2121} + \mathcal{I}[q^2]_{ijk0}^{1221}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{Hjk}], [P_\mu, U_{Hki}]$
$f_{18A}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1221} + \mathcal{I}[q^2]_{ijk0}^{2121} + \mathcal{I}[q^2]_{ijk0}^{1212} + \mathcal{I}[q^2]_{ijk0}^{2112}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{LHi'k}[P_\mu, U_{Hki}]$ $+U_{Hij}[P^\mu, U_{Hjk}]U_{HLki'}[P_\mu, U_{LHi'i}]$

$\mathcal{O}(U_H^1 U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17D}^{ij} = 2\left(2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{LHj'j}][P_\mu, U_{Hji}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}]$
$f_{17E}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{HLjj'}][P_\mu, U_{Lj'i'}]$
$f_{18B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222} + \mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{LHj'j}[P_\mu, U_{Hji}]$
$f_{18C}^{ij} = 4\left(\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{Li'j'}[P_\mu, U_{LHj'i}]$

$\mathcal{O}(U_L^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17F}^{i} = 2\left(2\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{Lj'k'}][P_\mu, U_{LHk'i}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{HLik'}][P_\mu, U_{Lk'i'}]$
$f_{17G}^{i} = 2\left(2\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{Lj'k'}][P_\mu, U_{Lk'i'}]$
$f_{17H}^{i} = 6\mathcal{I}[q^2]_{i0}^{24}$	$U_{Li'j'}U_{Lj'k'}[P^\mu, U_{LHk'i}][P_\mu, U_{HLii'}]$
$f_{18D}^{i} = 4\left(\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{Lj'k'}[P_\mu, U_{LHk'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{Lj'k'}[P_\mu, U_{Lk'i}]$

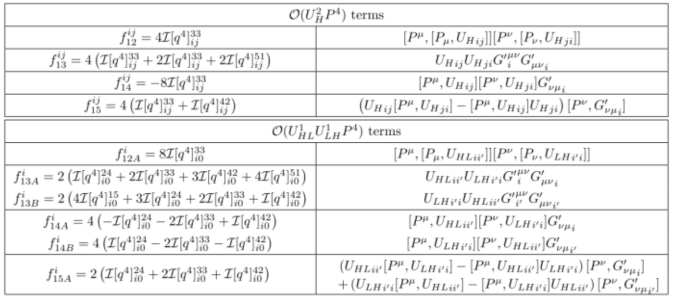
$\mathcal{O}(U_{HL}^2 U_{LH}^2 P^2)$ terms	
$f_{17I}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213} + \mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{HLjj'}][P_\mu, U_{LHj'i}]$
$f_{17J}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{222} + 2\mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{LHj'j}][P_\mu, U_{HLji'}]$
$f_{18E}^{ij} = \mathcal{I}[q^2]_{ij0}^{114} + 2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}$	$U_{HLii'}[P^\mu, U_{LHi'j}]U_{HLjj'}[P_\mu, U_{LHj'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{LHj'j}[P_\mu, U_{HLji'}]$



$\mathcal{O}(U_H^3 P^2)$ terms	
$f_{11}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\right)$	$U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{Hki}]$

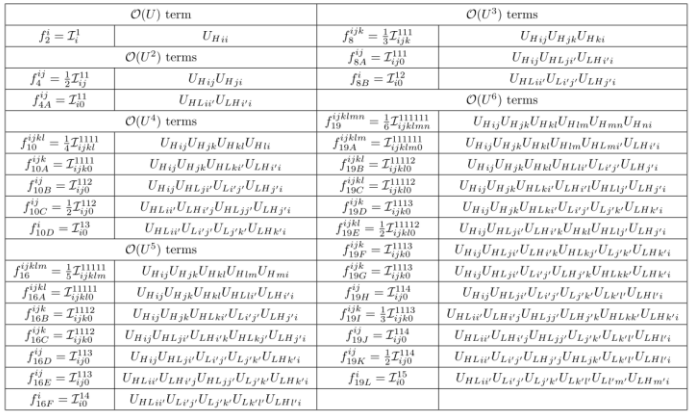
$\mathcal{O}(U_H^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11A}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{122} + \mathcal{I}[q^2]_{ij0}^{212}\right)$ $f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{221} + \mathcal{I}[q^2]_{ij0}^{122}\right)$	$U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{LHi'i}$ $U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11C}^{ij} = 4\mathcal{I}[q^2]_{i0}^{23}$ $f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]_{i0}^{14} + \mathcal{I}[q^2]_{i0}^{23}\right)$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$ $U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

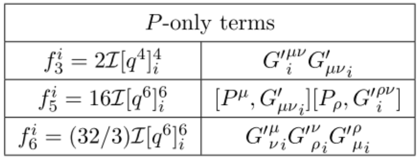


$\mathcal{O}(U_H^2 P^4)$ terms	
$f_{12}^{ij} = 4\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{Hij}]][P^\nu, [P_\nu, U_{Hji}]]$
$f_{13}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{51}\right)$	$U_{Hij}U_{Hji}G'^{\mu\nu}_i G'_{\mu\nu_i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{Hij}][P^\nu, U_{Hji}]G'_{\nu\mu_i}$
$f_{15}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\right)$	$\left(U_{Hij}[P^\mu, U_{Hji}] - [P^\mu, U_{Hij}]U_{Hji}\right)[P^\nu, G'_{\nu\mu_i}]$

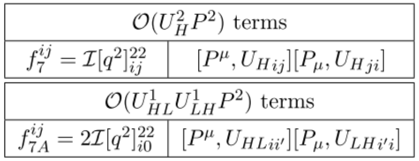
$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^4)$ terms	
$f_{12A}^{i} = 8\mathcal{I}[q^4]_{i0}^{33}$	$[P^\mu, [P_\mu, U_{HLii'}]][P^\nu, [P_\nu, U_{LHi'i}]]$
$f_{13A}^{i} = 2\left(\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + 3\mathcal{I}[q^4]_{i0}^{42} + 4\mathcal{I}[q^4]_{i0}^{51}\right)$ $f_{13B}^{i} = 2\left(4\mathcal{I}[q^4]_{i0}^{15} + 3\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$	$U_{HLii'}U_{LHi'i}G'^{\mu\nu}_i G'_{\mu\nu_i}$ $U_{LHi'i}U_{HLii'}G'^{\mu\nu}_{i'} G'_{\mu\nu_{i'}}$
$f_{14A}^{i} = 4\left(-\mathcal{I}[q^4]_{i0}^{24} - 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$ $f_{14B}^{i} = 4\left(\mathcal{I}[q^4]_{i0}^{24} - 2\mathcal{I}[q^4]_{i0}^{33} - \mathcal{I}[q^4]_{i0}^{42}\right)$	$[P^\mu, U_{HLii'}][P^\nu, U_{LHi'i}]G'_{\nu\mu_i}$ $[P^\mu, U_{LHi'i}][P^\nu, U_{HLii'}]G'_{\nu\mu_{i'}}$
$f_{15A}^{i} = 2\left(\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$	$\left(U_{HLii'}[P^\mu, U_{LHi'i}] - [P^\mu, U_{HLii'}]U_{LHi'i}\right)[P^\nu, G'_{\nu\mu_i}]$ $+\left(U_{LHi'i}[P^\mu, U_{HLii'}] - [P^\mu, U_{LHi'i}]U_{HLii'}\right)[P^\nu, G'_{\nu\mu_{i'}}]$



$\mathcal{O}(U)$ term		$\mathcal{O}(U^3)$ terms	
$f_2^i = \mathcal{I}_i^1$	U_{Hii}	$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{Hij}U_{Hjk}U_{Hki}$
$\mathcal{O}(U^2)$ terms		$f_{8A}^{ij} = \mathcal{I}_{ij0}^{111}$	$U_{Hij}U_{HLji'}U_{LHi'i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{Hij}U_{Hji}$	$f_{8B}^{i} = \mathcal{I}_{i0}^{12}$	$U_{HLii'}U_{Li'j'}U_{LHj'i}$
$f_{4A}^{ij} = \mathcal{I}_{i0}^{11}$	$U_{HLii'}U_{LHi'i}$	$\mathcal{O}(U^6)$ terms	
$\mathcal{O}(U^4)$ terms		$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmn}U_{Hni}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hli}$	$f_{19A}^{ijklm} = \mathcal{I}_{ijklm0}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{HLmi'}U_{LHi'i}$
$f_{10A}^{ijk} = \mathcal{I}_{ijk0}^{1111}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'i}$	$f_{19B}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{Li'j'}U_{LHj'i}$
$f_{10B}^{ij} = \mathcal{I}_{ij0}^{112}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f_{19C}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{112}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f_{19D}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{10D}^{i} = \mathcal{I}_{i0}^{13}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f_{19F}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f_{19G}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16A}^{ijkl} = \mathcal{I}_{ijkl0}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f_{19H}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16B}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$	$f_{19I}^{ijk} = \frac{1}{3}\mathcal{I}_{ijk0}^{1113}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16C}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f_{19J}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16D}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19K}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{16E}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f_{19L}^{i} = \mathcal{I}_{i0}^{15}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f_{16F}^{i} = \mathcal{I}_{i0}^{14}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

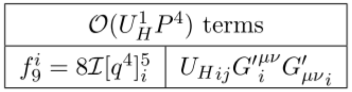


P-only terms	
$f_3^i = 2\mathcal{I}[q^4]_i^4$	$G'^{\mu\nu}_i G'_{\mu\nu_i}$
$f_5^i = 16\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu_i}][P_\rho, G'^{\rho\nu}_i]$
$f_6^i = (32/3)\mathcal{I}[q^6]_i^6$	$G'^{\mu}_{\nu_i} G'^{\nu}_{\rho_i} G'^{\rho}_{\mu_i}$



$\mathcal{O}(U_H^2 P^2)$ terms	
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{7A}^{ij} = 2\mathcal{I}[q^2]_{i0}^{22}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$



$\mathcal{O}(U_H^1 P^4)$ terms	
$f_9^i = 8\mathcal{I}[q^4]_i^5$	$U_{Hij}G'^{\mu\nu}_i G'_{\mu\nu_i}$

Application of the UOLEA: MSSM Stops

- Write UV Lagrangian for heavy multiplet in appropriate form to extract U matrix, mass matrix, and covariant derivative:

$$\boxed{\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \cancel{(\Phi^\dagger F(x) + {\rm h.c.})} + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)},$$

(R-parity)

$$\Phi = (\tilde{Q}\,,\tilde{t}_R^*), \qquad M^2 = \begin{pmatrix} m_{\tilde{Q}}^2 & 0 \\ 0 & m_{\tilde{t}_R}^2 \end{pmatrix} \qquad G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

Application of the UOLEA: MSSM Stops

- Write UV Lagrangian for heavy multiplet in appropriate form to extract U matrix, mass matrix, and covariant derivative:

$$\boxed{\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \boxed{\Phi^\dagger}(\boxed{P^2} - \boxed{M^2} - \boxed{U(x)})\boxed{\Phi} + \mathcal{O}(\Phi^3),}$$

(R-parity)

$$\boxed{\Phi = (\tilde{Q}\,, \tilde{t}_R^*),}$$

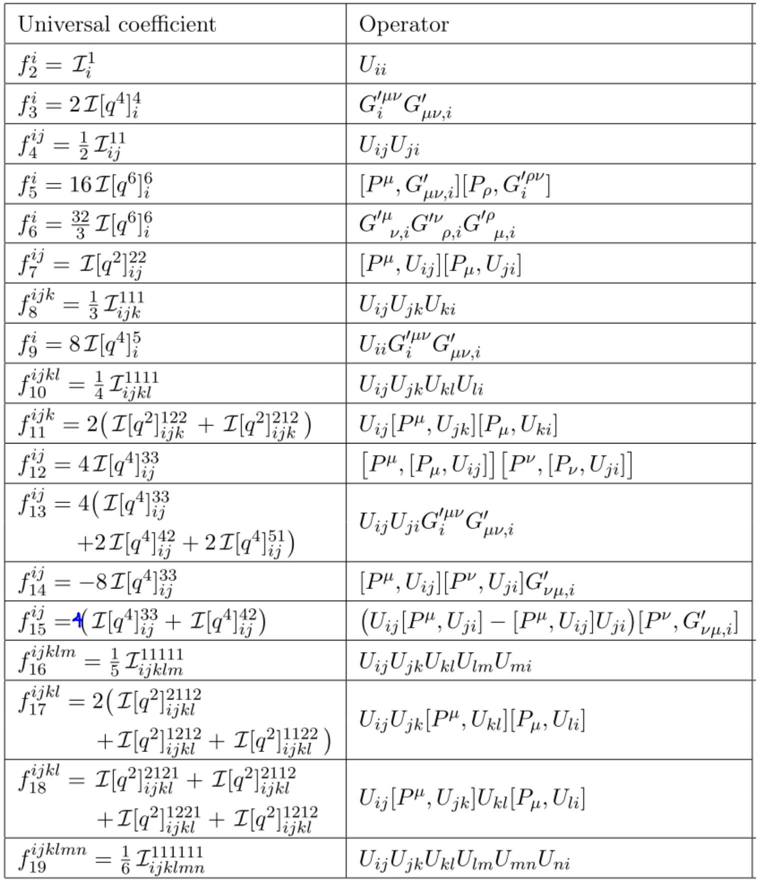
$$\boxed{M^2 = \begin{pmatrix} m_{\tilde{Q}}^2 & 0 \\ 0 & m_{\tilde{t}_R}^2 \end{pmatrix}}$$

$$\boxed{G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}}$$

$$\boxed{U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}}$$

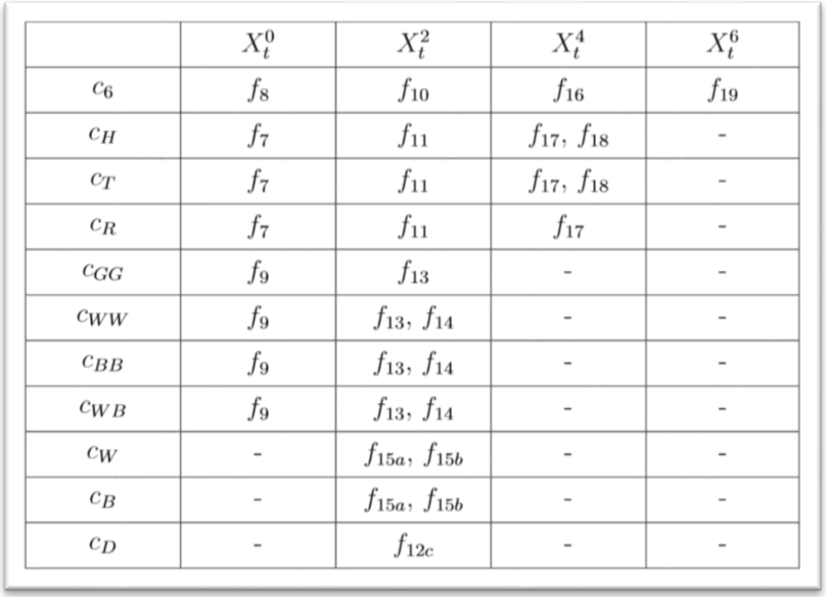
Application of the UOLEA: MSSM Stops

- Pick the relevant operators by counting operator dimensions



Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33}$ $+2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

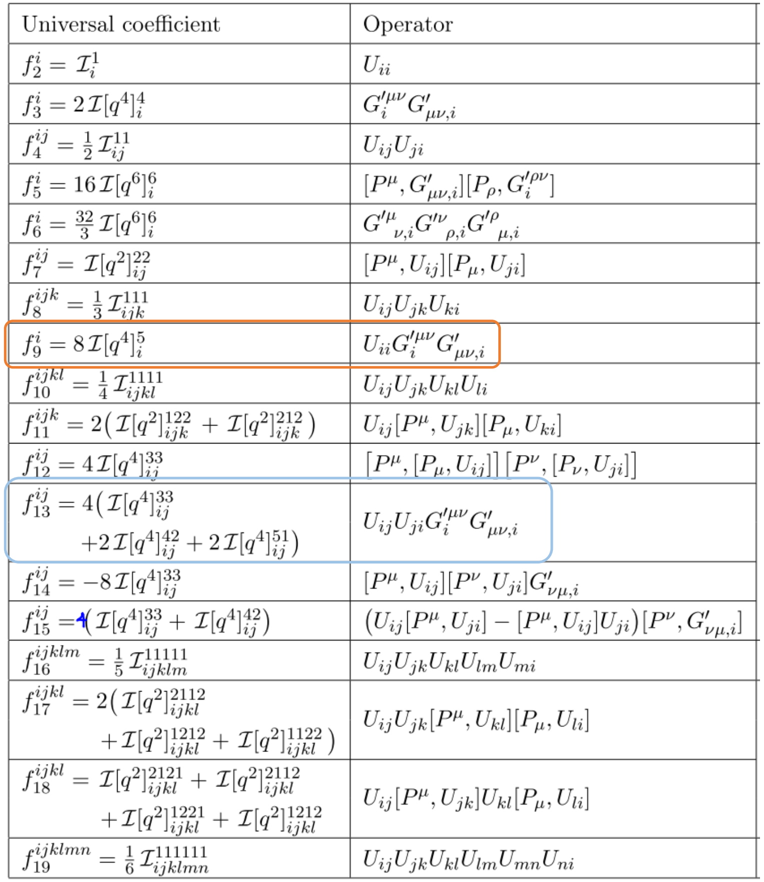
$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\boxed{\tilde{H}\tilde{H}^\dagger} + \frac{1}{2}g_2^2 s_\beta^2 \boxed{H H^\dagger} - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)\boxed{|H|^2} & h_t X_t \boxed{\tilde{H}} \\ h_t X_t \boxed{\tilde{H}^\dagger} & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})\boxed{|H|^2} \end{pmatrix}$$



	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
c_{WB}	f_9	f_{13}, f_{14}	-	-
c_W	-	f_{15a}, f_{15b}	-	-
c_B	-	f_{15a}, f_{15b}	-	-
c_D	-	f_{12c}	-	-

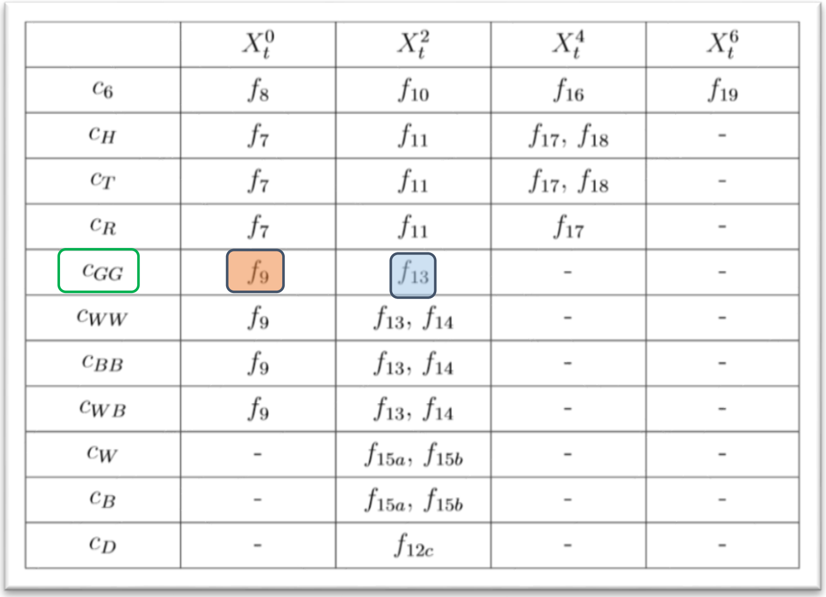
Application of the UOLEA: MSSM Stops

- Example: $\boxed{\mathcal{O}_{GG} \quad = \quad g_s^2 \, |H|^2 \, G^a_{\mu\nu} G^{a,\mu\nu}}$



Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{ij}]]\,[P^\nu, [P_\nu, U_{ji}]]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33}$ $+2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}] G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\boxed{\tilde{H}\tilde{H}^\dagger} + \frac{1}{2}g_2^2 s_\beta^2 \boxed{HH^\dagger} - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)\boxed{|H|^2} & h_t X_t \boxed{\tilde{H}} \\ h_t X_t \boxed{\tilde{H}^\dagger} & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})\boxed{|H|^2} \end{pmatrix}$$

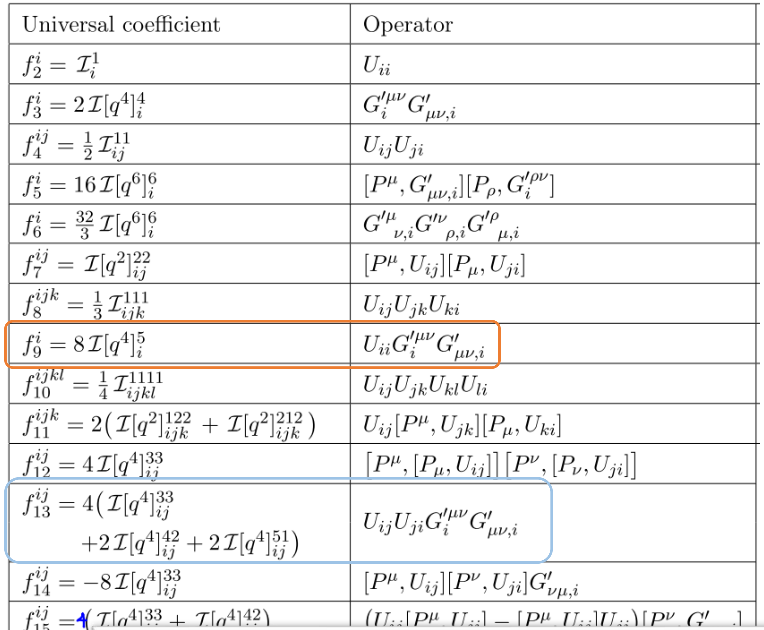


	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
c_{WB}	f_9	f_{13}, f_{14}	-	-
c_W	-	f_{15a}, f_{15b}	-	-
c_B	-	f_{15a}, f_{15b}	-	-
c_D	-	f_{12c}	-	-

Application of the UOLEA: MSSM Stops

- Example:

$$\mathcal{O}_{GG} \;=\; g_s^2\,|H|^2\,G^a_{\mu\nu}G^{a,\mu\nu}$$



Universal coefficient	Operator
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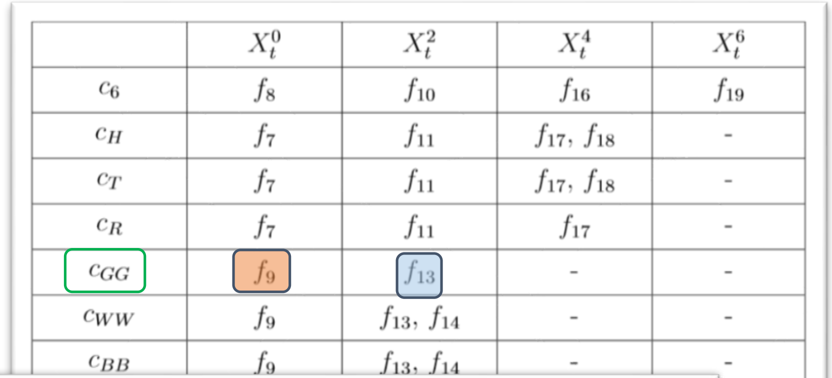
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i}G'^{\nu}{}_{\rho,i}G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4(\mathcal{I}[q^4]^{33} + \mathcal{I}[q^4]^{42})$	
f_{16}^{ijklm}	
f_{17}^{ijkl} =	
f_{18}^{ijkl} =	
f_{19}^{ijklmn}	

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2s_\beta^2 HH^\dagger - \frac{1}{2}(g_1^2Y_{\tilde{Q}}c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_tX_t\tilde{H} \\ h_tX_t\tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2Y_{\tilde{t}_R}c_{2\beta})|H|^2 \end{pmatrix}$$



	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
				-
				-
				-
				-

$$c_{GG} \;=\; \frac{1}{24}\left(\frac{h_t^2 - \frac{1}{6}g_1^2c_{2\beta}}{m_{\tilde{Q}}^2} + \frac{h_t^2 + \frac{1}{3}g_1^2c_{2\beta}}{m_{\tilde{t}_R}^2} - \frac{\bar{X}_t^2}{m_{\tilde{Q}}^2m_{\tilde{t}_R}^2}\right)$$

Application of the UOLEA: MSSM Stops

- Example: $\boxed{\mathcal{O}_{GG} \; = \; g_s^2 \, |H|^2 \, G^a_{\mu\nu} G^{a,\mu\nu}}$

For full results see Drozd, J. Ellis, Quevillon, TY, 1512.03003.

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4(\mathcal{I}[q^4]^{33} + \mathcal{I}[q^4]^{42})$	$(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji})[P^\nu, G'$
f_{16}^{ijklm}	
$f_{17}^{ijkl} =$	
$f_{18}^{ijkl} =$	
f_{19}^{ijklmn}	

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\boxed{\tilde{H}\tilde{H}^\dagger} + \frac{1}{2}g_2^2 s_\beta^2 \boxed{HH^\dagger} - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)\boxed{|H|^2} & h_t X_t \boxed{\tilde{H}} \\ h_t X_t \boxed{\tilde{H}^\dagger} & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})\boxed{|H|^2} \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
$\boxed{c_{GG}}$	$\boxed{f_9}$	$\boxed{f_{13}}$	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
				-
				-
				-
				-

$$\boxed{c_{GG} \;=\; \frac{1}{24}\left(\frac{h_t^2 - \frac{1}{6}g_1^2 c_{2\beta}}{m_{\tilde{Q}}^2} + \frac{h_t^2 + \frac{1}{3}g_1^2 c_{2\beta}}{m_{\tilde{t}_R}^2} - \frac{\bar{X}_t^2}{m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2}\right)}$$

Application of the UOLEA: Real Singlet Scalar

$$\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{1}{2}m^2\Phi^2 - \underline{A|H|^2\Phi} - \frac{1}{2}k|H|^2\Phi^2 - \frac{1}{3!}\mu\Phi^3 - \frac{1}{4!}\lambda_\Phi\Phi^4$$

- U matrix includes heavy-light contributions due to linear coupling

$$\mathcal{U} = \begin{pmatrix} \mathcal{U}_\phi & (\mathcal{U}^\dagger_{H\phi})_{1\times 2} \\ (\mathcal{U}_{H\phi})_{2\times 1} & (\ \mathcal{U}_H\ )_{2\times 2} \end{pmatrix}$$

- Classify possible contributions by counting operator dimensions

$$\mathcal{U}_\phi \equiv -\left.\frac{\delta^2\mathcal{L}}{\delta\phi^2}\right|_{\phi_c} = \kappa|H|^2 + \mu\phi_c + \tfrac{1}{2}\lambda_\phi\phi_c^2 \qquad \supset \mathcal{O}(H^2, \partial^2H^2, H^4, \partial^2H^4, H^6)$$

$$\mathcal{U}_{H\phi} \equiv -\left.\frac{\delta^2\mathcal{L}}{\delta H^\dagger\delta\phi}\right|_{\phi_c} = AH + \kappa H\phi_c \qquad \supset \mathcal{O}(H, H^3, H^5, H\partial^2H^2)$$

$$\mathcal{U}_{\tilde{H}\phi} \equiv -\left.\frac{\delta^2\mathcal{L}}{\delta\tilde{H}^\dagger\delta\phi}\right|_{\phi_c} = A\tilde{H} + \kappa\tilde{H}\phi_c \qquad \supset \mathcal{O}(H, H^3, H^5, H\partial^2H^2)$$

etc…

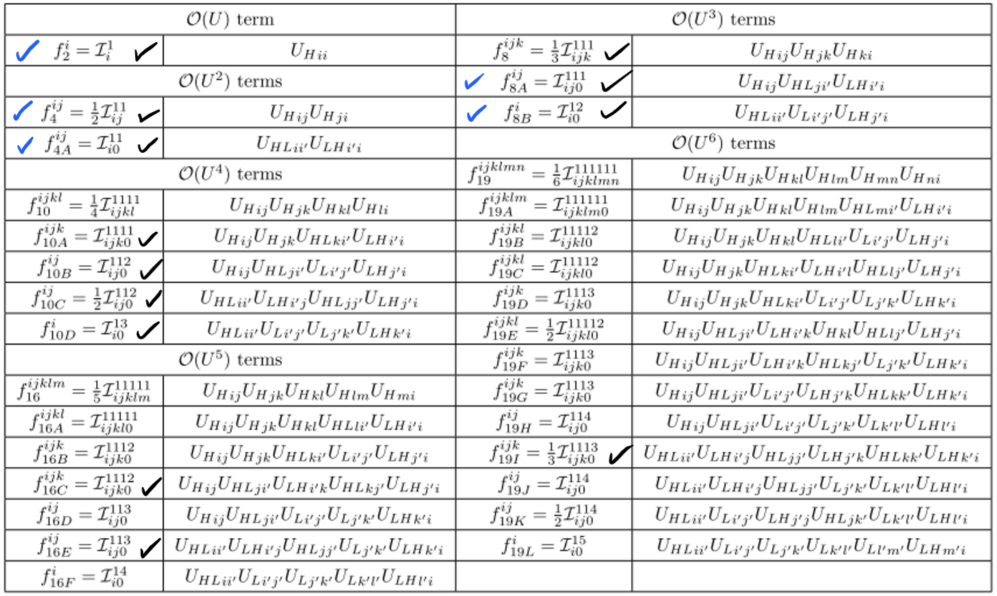
See S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1705.xxxxx

Application of the UOLEA: Real Singlet Scalar

$$\mathcal{O}_6 \quad = \quad |H|^6$$

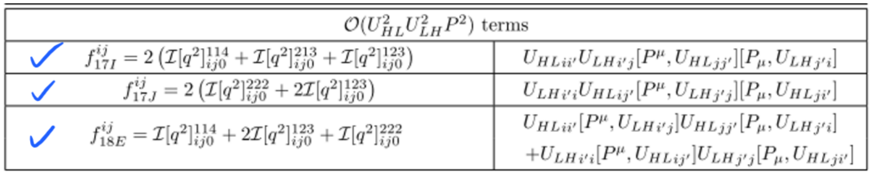
$$\mathcal{O}_H \quad = \quad \tfrac{1}{2}\left(\partial_\mu |H|^2\right)^2 \qquad \mathcal{O}_T \quad = \quad \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 \qquad \mathcal{O}_R \quad = \quad |H|^2 \left|D_\mu H\right|^2$$

- Classify possible contributions by counting operator dimensions

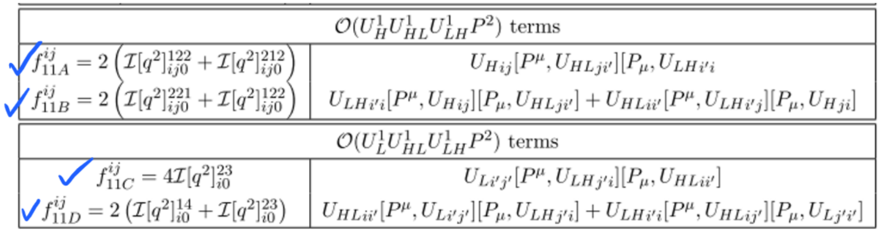


$\mathcal{O}(U)$ term	
✓ $f_2^i = \mathcal{I}_i^1$ ✓	U_{Hii}
$\mathcal{O}(U^2)$ terms	
✓ $f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$ ✓	$U_{Hij}U_{Hji}$
✓ $f_{4A}^{ij} = \mathcal{I}_{i0}^{11}$ ✓	$U_{HLii'}U_{LHi'i}$
$\mathcal{O}(U^4)$ terms	
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hli}$
$f_{10A}^{ijk} = \mathcal{I}_{ijk0}^{1111}$ ✓	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'i}$
$f_{10B}^{ij} = \mathcal{I}_{ij0}^{112}$ ✓	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{112}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$
$f_{10D}^{i} = \mathcal{I}_{i0}^{13}$ ✓	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$\mathcal{O}(U^5)$ terms	
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$
$f_{16A}^{ijkl} = \mathcal{I}_{ijkl0}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$
$f_{16B}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$
$f_{16C}^{ijk} = \mathcal{I}_{ijk0}^{1112}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$
$f_{16D}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{16E}^{ij} = \mathcal{I}_{ij0}^{113}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$
$f_{16F}^{i} = \mathcal{I}_{i0}^{14}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$

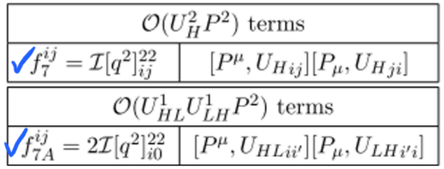
$\mathcal{O}(U^3)$ terms	
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$ ✓	$U_{Hij}U_{Hjk}U_{Hki}$
✓ $f_{8A}^{ij} = \mathcal{I}_{ij0}^{111}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'i}$
✓ $f_{8B}^{i} = \mathcal{I}_{i0}^{12}$ ✓	$U_{HLii'}U_{Li'j'}U_{LHj'i}$
$\mathcal{O}(U^6)$ terms	
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmn}U_{Hni}$
$f_{19A}^{ijklm} = \mathcal{I}_{ijklm0}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{HLmi'}U_{LHi'i}$
$f_{19B}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{Li'j'}U_{LHj'i}$
$f_{19C}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{19D}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$f_{19F}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{19G}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{19H}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{19I}^{ijk} = \frac{1}{3}\mathcal{I}_{ijk0}^{1113}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{19J}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{19K}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{19L}^{i} = \mathcal{I}_{i0}^{15}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$



$\mathcal{O}(U_{HL}^2 U_{LH}^2 P^2)$ terms	
✓ $f_{17I}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213} + \mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{HLjj'}][P_\mu, U_{LHj'i}]$
✓ $f_{17J}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{222} + 2\mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{LHj'j}][P_\mu, U_{HLji'}]$
✓ $f_{18E}^{ij} = \mathcal{I}[q^2]_{ij0}^{114} + 2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}$	$U_{HLii'}[P^\mu, U_{LHi'j}]U_{HLjj'}[P_\mu, U_{LHj'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{LHj'j}[P_\mu, U_{HLji'}]$



$\mathcal{O}(U_H^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{11A}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{122} + \mathcal{I}[q^2]_{ij0}^{212}\right)$	$U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{LHi'i}$
✓ $f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{221} + \mathcal{I}[q^2]_{ij0}^{122}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{11C}^{ij} = 4\mathcal{I}[q^2]_{i0}^{23}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
✓ $f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]_{i0}^{14} + \mathcal{I}[q^2]_{i0}^{23}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$



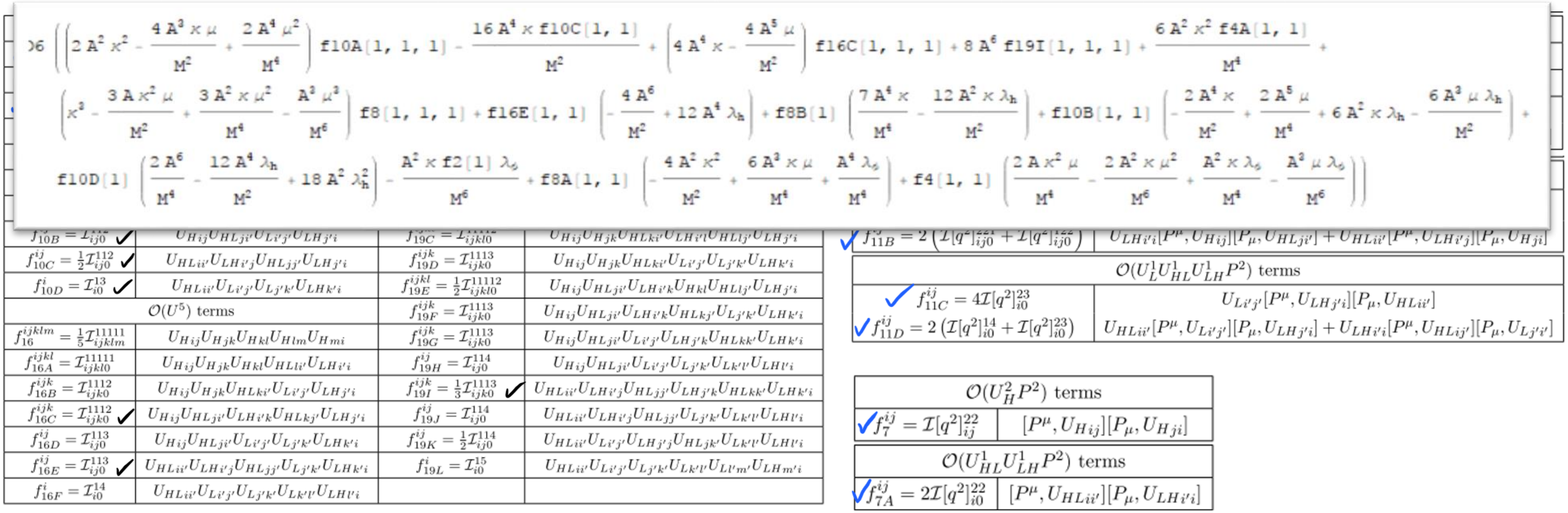
$\mathcal{O}(U_H^2 P^2)$ terms	
✓ $f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{7A}^{ij} = 2\mathcal{I}[q^2]_{i0}^{22}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

Application of the UOLEA: Real Singlet Scalar

$$\mathcal{O}_6 = |H|^6$$

$$\mathcal{O}_H = \tfrac{1}{2}(\partial_\mu |H|^2)^2 \qquad \mathcal{O}_T = \tfrac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2 \qquad \mathcal{O}_R = |H|^2 |D_\mu H|^2$$

- Evaluate sum over each term to get full result for e.g. O6:



$$6\left(\left(2A^2\kappa^2 - \frac{4A^3\kappa\mu}{M^2} + \frac{2A^4\mu^2}{M^4}\right) \mathtt{f10A[1,1,1]} - \frac{16A^4\kappa\,\mathtt{f10C[1,1]}}{M^2} + \left(4A^4\kappa - \frac{4A^5\mu}{M^2}\right)\mathtt{f16C[1,1,1]} + 8A^6\,\mathtt{f19I[1,1,1]} + \frac{6A^2\kappa^2\,\mathtt{f4A[1,1]}}{M^4} + \left(\kappa^3 - \frac{3A\kappa^2\mu}{M^2} + \frac{3A^2\kappa\mu^2}{M^4} - \frac{A^3\mu^3}{M^6}\right)\mathtt{f8[1,1,1]} + \mathtt{f16E[1,1]}\left(-\frac{4A^6}{M^2} + 12A^4\lambda_h\right) + \mathtt{f8B[1]}\left(\frac{7A^4\kappa}{M^4} - \frac{12A^2\kappa\lambda_h}{M^2}\right) + \mathtt{f10B[1,1]}\left(-\frac{2A^4\kappa}{M^2} + \frac{2A^5\mu}{M^4} + 6A^2\kappa\lambda_h - \frac{6A^3\mu\lambda_h}{M^2}\right) + \mathtt{f10D[1]}\left(\frac{2A^6}{M^4} - \frac{12A^4\lambda_h}{M^2} + 18A^2\lambda_h^2\right) - \frac{A^2\kappa\,\mathtt{f2[1]}\,\lambda_s}{M^6} + \mathtt{f8A[1,1]}\left(-\frac{4A^2\kappa^2}{M^2} + \frac{6A^3\kappa\mu}{M^4} + \frac{A^4\lambda_s}{M^4}\right) + \mathtt{f4[1,1]}\left(\frac{2A\kappa^2\mu}{M^4} - \frac{2A^2\kappa\mu^2}{M^6} + \frac{A^2\kappa\lambda_s}{M^4} - \frac{A^3\mu\lambda_s}{M^6}\right)\right)$$

$f^{ij}_{10B} = \mathcal{I}^{112}_{ij0}$ ✓	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f^{ijkl}_{19C} = \mathcal{I}^{11112}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f^{ij}_{10C} = \frac{1}{2}\mathcal{I}^{112}_{ij0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f^{ijk}_{19D} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f^{i}_{10D} = \mathcal{I}^{13}_{i0}$ ✓	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f^{ijkl}_{19E} = \frac{1}{2}\mathcal{I}^{11112}_{ijkl0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f^{ijk}_{19F} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f^{ijklm}_{16} = \frac{1}{5}\mathcal{I}^{11111}_{ijklm}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f^{ijk}_{19G} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f^{ijkl}_{16A} = \mathcal{I}^{11111}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f^{ij}_{19H} = \mathcal{I}^{114}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f^{ijk}_{16B} = \mathcal{I}^{1112}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$	$f^{ijk}_{19I} = \frac{1}{3}\mathcal{I}^{1113}_{ijk0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f^{ijk}_{16C} = \mathcal{I}^{1112}_{ijk0}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f^{ij}_{19J} = \mathcal{I}^{114}_{ij0}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f^{ij}_{16D} = \mathcal{I}^{113}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f^{ij}_{19K} = \frac{1}{2}\mathcal{I}^{114}_{ij0}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f^{ij}_{16E} = \mathcal{I}^{113}_{ij0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f^{i}_{19L} = \mathcal{I}^{15}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f^{i}_{16F} = \mathcal{I}^{14}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

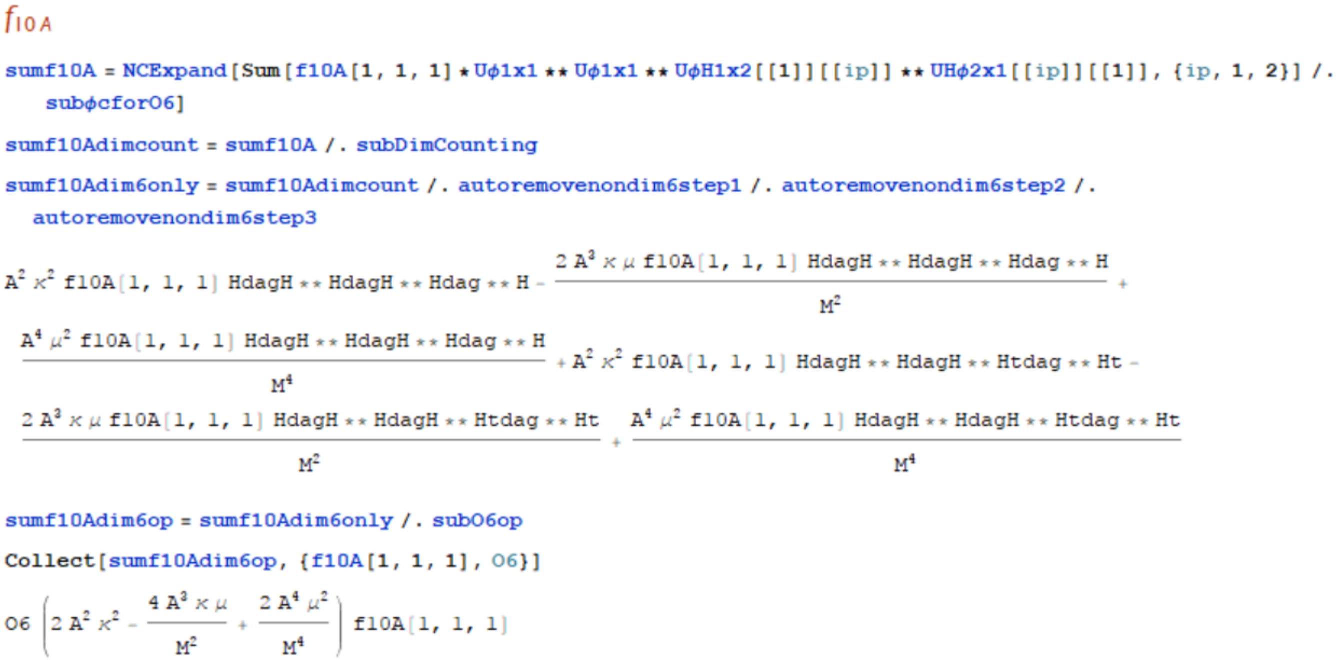
✓ $f^{ij}_{11B} = 2\left(\mathcal{I}[q^2]^{221}_{ij0} + \mathcal{I}[q^2]^{122}_{ij0}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$
$\mathcal{O}(U^1_L U^1_{HL} U^1_{LH} P^2)$ terms	
✓ $f^{ij}_{11C} = 4\mathcal{I}[q^2]^{23}_{i0}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
✓ $f^{ij}_{11D} = 2\left(\mathcal{I}[q^2]^{14}_{i0} + \mathcal{I}[q^2]^{23}_{i0}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

$\mathcal{O}(U^2_H P^2)$ terms	
✓ $f^{ij}_{7} = \mathcal{I}[q^2]^{22}_{ij}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$

$\mathcal{O}(U^1_{HL} U^1_{LH} P^2)$ terms	
✓ $f^{ij}_{7A} = 2\mathcal{I}[q^2]^{22}_{i0}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

Application of the UOLEA: Real Singlet Scalar

- Can (partially) **automate** evaluation of each term, e.g.



f_{10A}

```
sumf10A = NCExpand[Sum[f10A[1, 1, 1] * Uϕ1x1 ** Uϕ1x1 ** UϕH1x2[[1]][[ip]] ** UHϕ2x1[[ip]][[1]], {ip, 1, 2}] /.
   subϕcforO6]

sumf10Adimcount = sumf10A /. subDimCounting

sumf10Adim6only = sumf10Adimcount /. autoremovenondim6step1 /. autoremovenondim6step2 /.
  autoremovenondim6step3
```

$$A^2 \kappa^2 \,\text{f10A}[1,1,1]\ \text{HdagH} ** \text{HdagH} ** \text{Hdag} ** \text{H} - \frac{2 A^3 \kappa \mu \,\text{f10A}[1,1,1]\ \text{HdagH} ** \text{HdagH} ** \text{Hdag} ** \text{H}}{M^2} + \frac{A^4 \mu^2 \,\text{f10A}[1,1,1]\ \text{HdagH} ** \text{HdagH} ** \text{Hdag} ** \text{H}}{M^4} + A^2 \kappa^2 \,\text{f10A}[1,1,1]\ \text{HdagH} ** \text{HdagH} ** \text{Htdag} ** \text{Ht} - \frac{2 A^3 \kappa \mu \,\text{f10A}[1,1,1]\ \text{HdagH} ** \text{HdagH} ** \text{Htdag} ** \text{Ht}}{M^2} + \frac{A^4 \mu^2 \,\text{f10A}[1,1,1]\ \text{HdagH} ** \text{HdagH} ** \text{Htdag} ** \text{Ht}}{M^4}$$

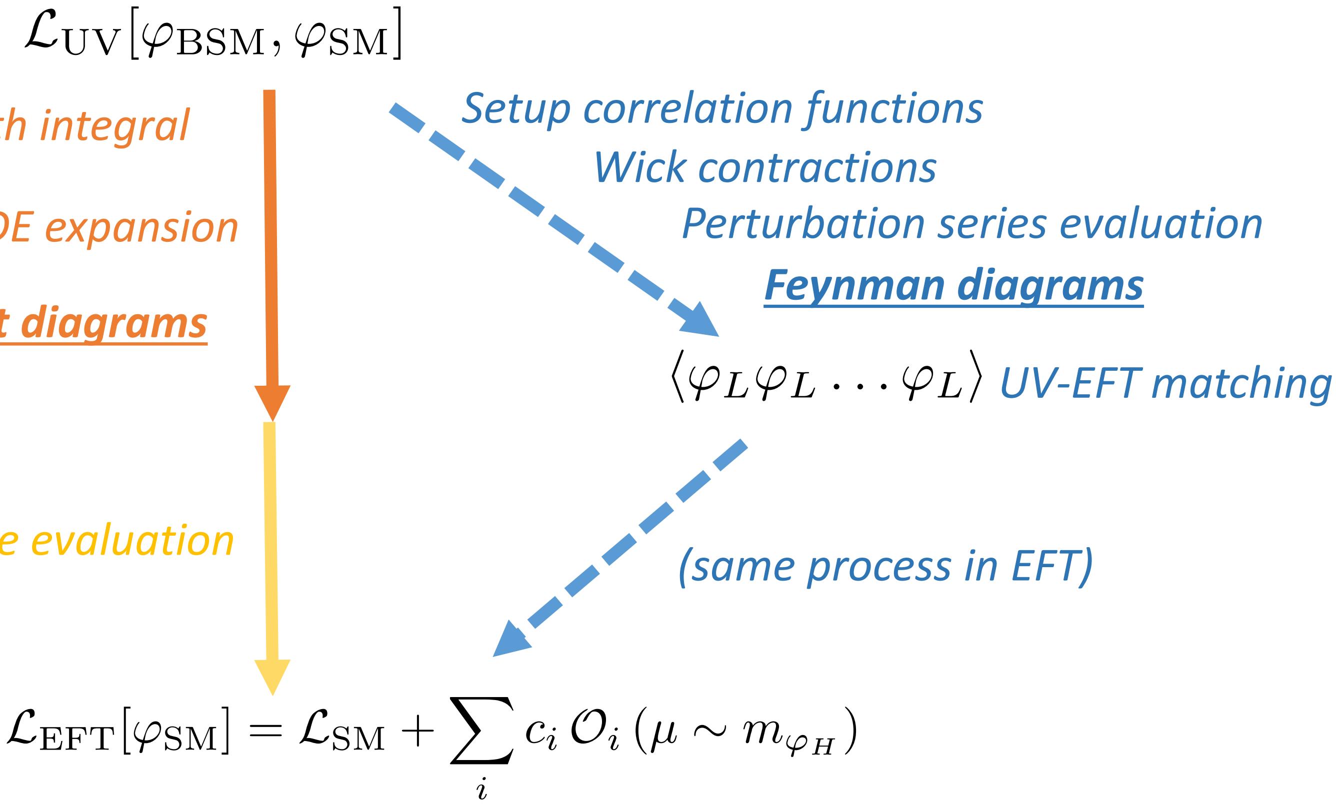
```
sumf10Adim6op = sumf10Adim6only /. subO6op

Collect[sumf10Adim6op, {f10A[1, 1, 1], O6}]
```

$$O6 \left(2 A^2 \kappa^2 - \frac{4 A^3 \kappa \mu}{M^2} + \frac{2 A^4 \mu^2}{M^4}\right) \text{f10A}[1,1,1]$$

- Substitute operator structure relations for desired basis, worked out by hand
  - This example trivial but in general most of the work involved is in this step
  - Possible automation: dictionary of operator relations, or work out algorithm

Take-home message

- When calculating Feynman diagrams we don’t Wick contract and calculate symmetry factors by hand every time
- Similar redundancy in evaluating CDE from the beginning every time we use functional methods for one-loop matching



$$\mathcal{L}_{\text{UV}}[\varphi_{\text{BSM}}, \varphi_{\text{SM}}]$$

Setup path integral

Evaluate CDE expansion

Covariant diagrams

Matrix trace evaluation

Setup correlation functions

Wick contractions

Perturbation series evaluation

Feynman diagrams

$\langle \varphi_L \varphi_L \cdots \varphi_L \rangle$ *UV-EFT matching*

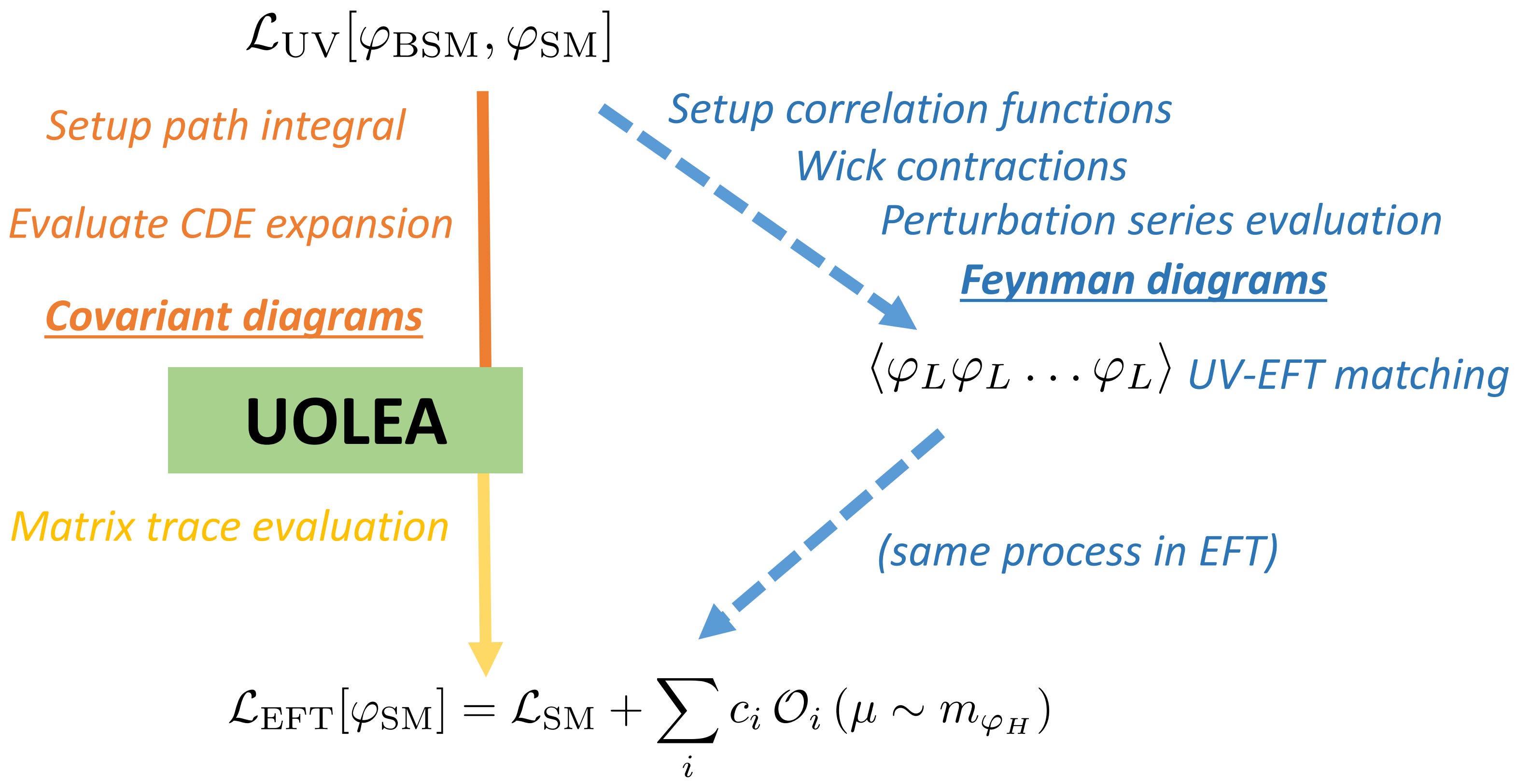
(same process in EFT)

$$\mathcal{L}_{\text{EFT}}[\varphi_{\text{SM}}] = \mathcal{L}_{\text{SM}} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

- Standardise functional one-loop matching procedure…

Take-home message

- When calculating Feynman diagrams we don't Wick contract and calculate symmetry factors by hand every time
- Similar redundancy in evaluating CDE from the beginning every time we use functional methods for one-loop matching



$$\mathcal{L}_{\text{UV}}[\varphi_{\text{BSM}}, \varphi_{\text{SM}}]$$

Setup path integral

Evaluate CDE expansion

Covariant diagrams

UOLEA

Matrix trace evaluation

Setup correlation functions

Wick contractions

Perturbation series evaluation

Feynman diagrams

$\langle \varphi_L \varphi_L \cdots \varphi_L \rangle$ *UV-EFT matching*

(same process in EFT)

$$\mathcal{L}_{\text{EFT}}[\varphi_{\text{SM}}] = \mathcal{L}_{\text{SM}} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

- **Start directly from UOLEA!**

- Part I: SM EFT
- Part 2: The Universal One-Loop Effective Action
- **Part 3: Cosmological Relaxation**

Beyond the Standard Model?

- Hierarchy problem is a real problem: $(m_h)^2_{\text{tree}} + (m_h)^2_{\text{radiative}} = (m_h)^2_{v}$

$$\delta m_\phi^2 \propto m_{\text{heavy}}^2, \quad \delta m_\psi \propto m_\psi \log\left(\frac{m_{\text{heavy}}}{\mu}\right)$$

- Earliest example of an unnatural feature of a fundamental theory:

$$m_{\text{inertial}} = q_{\text{gravity}}$$

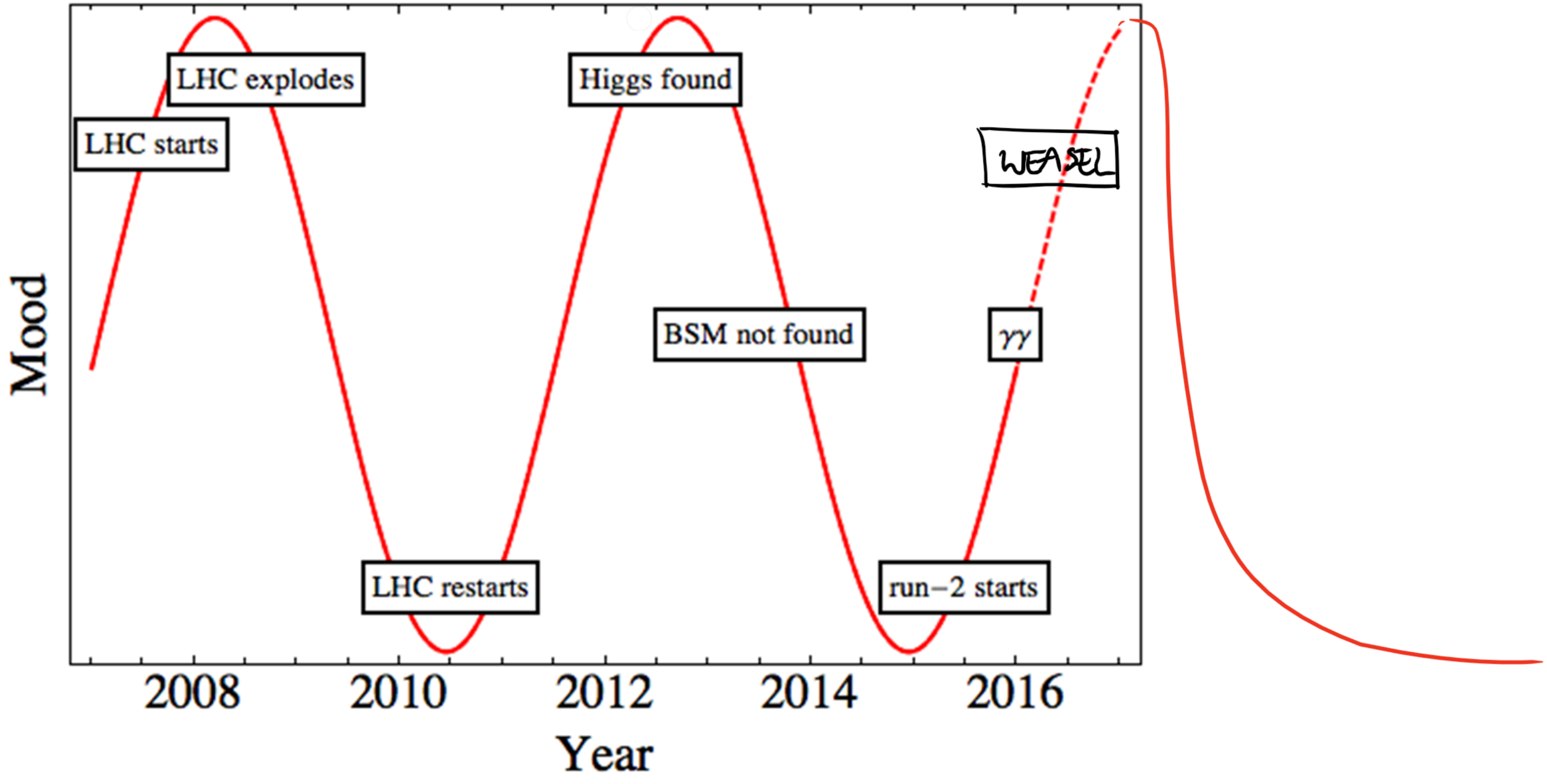
- Classical electromagnetism fine-tuning:

$$(m_e c^2)_{\text{obs}} = (m_e c^2)_{\text{bare}} + \Delta E_{\text{coulomb}}, \qquad \Delta E_{\text{coulomb}} = \frac{e^2}{4\pi\epsilon_0 r_e}$$

- Pions, cut-off also at natural scale

- Higgs? Expect new physics close to weak scale

Beyond the Standard Model?



http://resonaances.blogspot.com.es/2016/01/do-or-die-year.html

- Maybe Nature is trying to tell us we are missing something in the way we think about the hierarchy problem

Cosmological Relaxation

- Natural solution with a high cut-off scale

P. W. Graham, D. E. Kaplan and S. Rajendran, Phys. Rev. Lett. 115 (2015) 22, 221801 [arXiv:1507.07551]

- Originally proposed in the context of cosmological constant

L. F. Abbott, Phys. Lett. B 150 (1985) 427

- Axion-like field *a* with shift symmetry and periodic potential, softly broken

potential barrier height proportional to <h>

$$\mathcal{L} = \frac{1}{32\pi^2}\frac{a}{f}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}G_{\mu\nu}G_{\rho\sigma} \qquad V_{\cos}(a) = \Lambda_G^4 \cos(a/f)$$

Higgs mass at cut-off scale

$$V_{\rm soft}(a) \simeq \underline{(ga - M^2)}|h|^2 + gM^2a + \ldots$$

potential slope gives slow-roll

- Effective Higgs mass scanned by slow-rolling *a* during inflation

- Bonus: Minimal SUSY-breaking mechanism

B. Batell, G. F. Giudice and M. McCullough, JHEP 1512 (2015) 162 [arXiv:1509.00834]

Minimal QCD relaxion model

- e.g. G = colour SU(3)

$$m_\pi^2 f_\pi^2 = (m_u + m_d)\langle\bar{\psi}\psi\rangle$$

$$V_{\cos}(a) \simeq \Lambda_G^3 \langle h\rangle \cos(a/f)$$

$$\Updownarrow$$

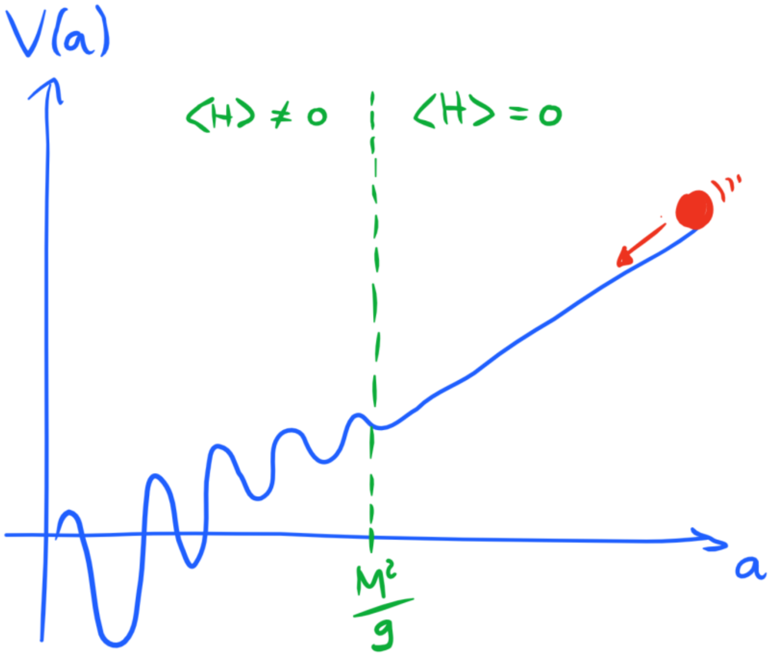
$$V_{\text{soft}}(a) \simeq (ga - M^2)|h|^2 + gM^2 a + \ldots$$

$$M_H^2\big|_{eff} = ga^2 - M^2$$

- Slow-roll scanning stops when barrier height slope = soft-breaking slope

$$\langle h\rangle \sim \underline{g}M^2 f/\Lambda_G^3$$

technically natural
protected parameter



- Strong-CP problem, effective Θ-angle of O(1)

Cosmological Relaxation and Inflation

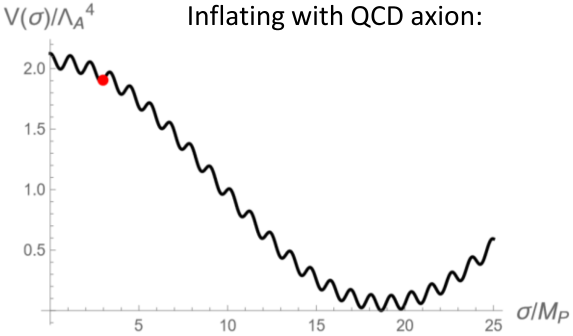
TY, arXiv:1701.09167

- Minimal relaxion setup, no v-dependence of periodic potential barrier

$$\mathcal{L} \supset \left(M^2 - g\phi\right)|h|^2 + gM^2\phi + \ldots + \Lambda_G^4 \cos\left(\frac{\phi}{f_\phi}\right) - \frac{\alpha_D}{f_D}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \, ,$$

- Backreaction of Higgs vacuum expectation ends inflation, e.g.

Inflating with QCD axion:



See e.g. Freese and Liu 0502177

Inflating with electroweak dissipation:

$$\mathcal{L} \supset -\frac{\alpha}{f}\sigma F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + V'_\sigma(\sigma) = -I\frac{\alpha}{f}\left(\frac{H}{\xi}\right)^4 e^{2\pi\xi} \, ,$$

$$\xi \equiv \frac{\alpha}{2f}\frac{\dot{\sigma}}{H}$$

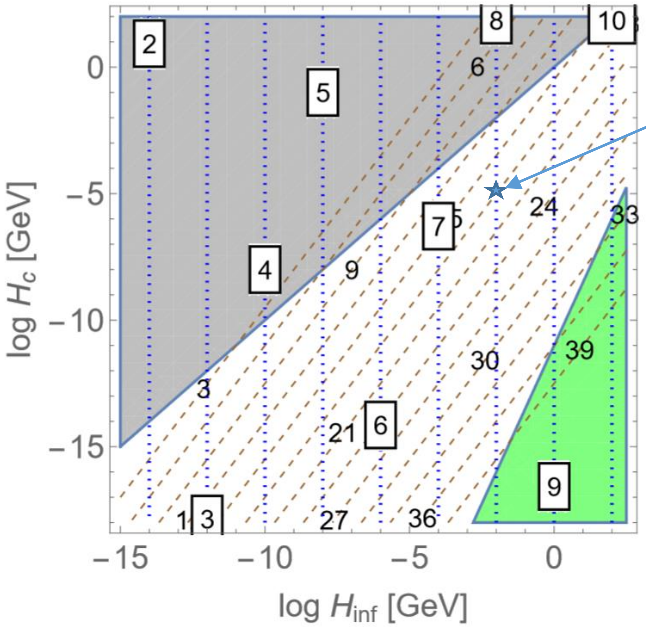
See e.g. Anber and Sorbo 0908.4089

- Decreasing Hubble falls below relaxion dissipation threshold
- This additional friction slows and traps the relaxion near the weak scale

Cosmological Relaxation and Inflation

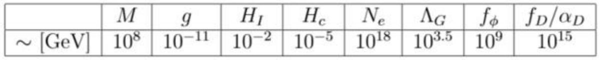
$$\mathcal{L} \supset \left(M^2 - g\phi\right)|h|^2 + gM^2\phi + ... + \Lambda_G^4 \cos\left(\frac{\phi}{f_\phi}\right) - \frac{\alpha_D}{f_D}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \, ,$$

- Parameter space



e.g.

Figure 2: *Parameter space of the relaxion sector determined by the Hubble scale of inflation H_{inf} vs the critical Hubble threshold H_c at which the relaxion is trapped. The upper grey shaded region is excluded because the latter is restricted to $H_c \lesssim H_{inf}$, and the lower green shaded region is when H_c is too small so the relaxion is trapped after rolling past the weak scale. The vertical blue dotted lines labelled by white rectangles denote the log of the maximum cut-off M in GeV. The diagonal brown dashed lines are the log of the number of e-foldings, for the value of g and M that saturate the bounds in Eqs. 4.4 and 4.6.*



	M	g	H_I	H_c	N_e	Λ_G	f_ϕ	f_D/α_D
~ [GeV]	10^8	10^{-11}	10^{-2}	10^{-5}	10^{18}	$10^{3.5}$	10^9	10^{15}

Table 1: *An example of typical parameter values that satisfy all the constraints listed for our relaxion model. M is the effective theory cut-off, g parametrises the explicit shift-symmetry-breaking slope, H_I is the Hubble scale of inflation, H_c the critical Hubble threshold below which the relaxion is trapped, N_e the required e-foldings of inflation during relaxation, Λ_G the trapping barrier height of the relaxion's periodic potential with period $2\pi f_\phi$, and f_D the decay constant of the relaxion's axial gauge field coupling responsible for dissipation into dark gauge bosons.*

Conclusion

- A SM-like Higgs boson and no direct signs of new physics may turn out to be a significant experimental null result
- Decoupled new physics motivates a SM EFT approach to phenomenology
- Future precision may probe even loop-induced operators at the TeV scale
- Universal approach to one-loop matching will become a standard calculational method
- A desert above the weak scale has interesting implications for naturalness and model-building
- Cosmological relaxation mechanism one possible avenue to explore

Backups