Flavor Physics II

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Outline

- Flavor Physics and the Standard Model
- Discrete Symmetry and CKM matrix
  - Parity, Charge Conjugation and CPT theorem
  - CKM matrix
- Renormalization and Muon g-2
- RG and Effective Field Theory
- CP Violation and BSM Flavor Physics

Parity

- Parity: $(t, \mathbf{x}) \xrightarrow{\mathrm{P}} (t, -\mathbf{x})$

$$\phi(\mathbf{x}, t) \xrightarrow{\mathrm{P}} \phi^{\mathrm{P}}(\mathbf{x}, t) = \eta_{\mathrm{P}}\, \phi(-\mathbf{x}, t) \qquad \begin{cases} \eta_P = \ \ 1 & \text{for a scalar} \\ \eta_P = -1 & \text{for a pseudoscalar} \end{cases}$$

$$\partial_\mu \phi(\mathbf{x}, t) \xrightarrow{\mathrm{P}} \partial_\mu \phi^{\mathrm{P}}(\mathbf{x}, t) = -(1 - 2\delta_{\mu 0})\eta_{\mathrm{P}}\, \partial_\mu \phi(-\mathbf{x}, t) \qquad \begin{cases} \eta_P = \ \ 1 & \text{for a vector} \\ \eta_P = -1 & \text{for an axial vector} \end{cases}$$

$$A_\mu(\mathbf{x}, t) \xrightarrow{\mathrm{P}} A_\mu^{\mathrm{P}}(\mathbf{x}, t) = -(1 - 2\delta_{\mu 0}) A_\mu(-\mathbf{x}, t)$$

The photon field transforms as a vector (clear from $\partial_\mu \to \partial_\mu - \mathrm{i}A_\mu$)

– The Lagrangian is strictly speaking not invariant under parity, but the space-time integration domain in the action also changes according to the parity transformation

– the equations of motion are invariant

$$S = \int \mathcal{L}(\phi, \partial_\mu \phi)\, d^4x$$

Parity: scalar field

$$\phi(x) = \sum_p \left[a_p \phi_p^{(+)}(x) + b_p^\dagger \phi_{-p}^{(-)}(x) \right]$$

$$\phi_p^{(+)}(x) = C_p e^{i(p\cdot x - Et)} = C_p e^{-ip\cdot x}$$

$$\phi_{-p}^{(-)}(x) = C_p e^{i(-p\cdot x + Et)} = C_p e^{ip\cdot x}$$

- Transformation of scalar fields

$$\mathcal{P}\,\phi(t, x)\,\mathcal{P}^{-1} = \eta_B \phi(t, -x)$$

: $\mathcal{P}$ is a linear operator in the Hilbert space and does not act on c-number quantities

$$\mathcal{P}\,\phi(x)\,\mathcal{P}^{-1} = \sum_p C_p \left[\mathcal{P}\,a_p\,\mathcal{P}^{-1} e^{-i(Et - p\cdot x)} + \mathcal{P}\,b_p^\dagger\,\mathcal{P}^{-1} e^{i(Et - p\cdot x)} \right]$$

$$\begin{aligned}\eta_B\,\phi(t, -x) &= \eta_B \sum_p C_p \left[a_p\, e^{-i(Et + p\cdot x)} + b_p^\dagger\, e^{i(Et + p\cdot x)} \right] \\ &= \eta_B \sum_p C_p \left[a_{-p}\, e^{-i(Et - p\cdot x)} + b_{-p}^\dagger\, e^{i(Et - p\cdot x)} \right]\end{aligned}$$

– we obtain the following property

$$\mathcal{P}\,a_p\,\mathcal{P}^{-1} = \eta_B\, a_{-p} \qquad \mathcal{P}\,a_p^\dagger\,\mathcal{P}^{-1} = \eta_B\, a_{-p}^\dagger$$

$$\mathcal{P}\,b_p\,\mathcal{P}^{-1} = \eta_B\, b_{-p} \qquad \mathcal{P}\,b_p^\dagger\,\mathcal{P}^{-1} = \eta_B\, b_{-p}^\dagger$$

– a one-body state of momentum p

$$\begin{aligned}\mathcal{P}\,|p\rangle &= \mathcal{P}\,a_p^\dagger\,|0\rangle = \mathcal{P} a_p^\dagger \mathcal{P}^{-1}\mathcal{P}\,|0\rangle \\ &= \eta_B a_{-p}^\dagger\,|0\rangle = \eta_B\,|-p\rangle\end{aligned}$$

Parity: Dirac wave function

- Transformation of Dirac wave function ψ(x)
  - Lorentz transformation $x^\mu \to x'^\mu = a^\mu{}_\nu x^\nu$
  - $\psi'(x')$ and $\psi(x)$ are connected by a linear relation

$$\psi'_a(x') = S_{ab}(a)\,\psi_b(x)$$

  - $\psi'(x')$ is also a solution of the Dirac equation

$$(\mathrm{i}\gamma^\mu \partial'_\mu - m)\psi'(x') = 0$$

  - compare it with the original equation

$$(\mathrm{i}\,\partial\!\!\!/ - m)\psi(x) = 0 \quad \Rightarrow \quad (\mathrm{i}S\gamma^\mu S^{-1}\partial_\mu - m)\psi'(x') = 0$$

$$S\gamma^\mu S^{-1}\partial_\mu = \gamma^\mu \partial'_\mu \qquad \partial_\mu = a^\nu{}_\mu \partial'_\nu$$

$$S^{-1}(a)\gamma^\mu S(a) = a^\mu{}_\nu \gamma^\nu$$

  - parity $\mathcal{P}: \boldsymbol{x} \to \boldsymbol{x}' = -\boldsymbol{x}\,, \quad t \to t' = t$

$$a^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$S(a) = \eta_{\mathrm{F}}\,\gamma_0 \qquad \eta_{\mathrm{F}} = \pm 1$$

intrinsic parity

Parity: Dirac field

$$\gamma_0 u(-\boldsymbol{p}, s) = u(\boldsymbol{p}, s)$$
$$\gamma_0 v(-\boldsymbol{p}, s) = -v(\boldsymbol{p}, s)$$

- In analogy with the classical wave function, the Dirac field operators transforms according to

$$\mathcal{P}\,\psi(x)\,\mathcal{P}^{-1} = \eta_{\mathrm{F}}\,\gamma_0\,\psi(t, -\boldsymbol{x})$$

– the Dirac field is expressed as

$$\psi(x) = \sum_{\boldsymbol{p},s} C_{\boldsymbol{p}} \left[b(\boldsymbol{p}, s) u(\boldsymbol{p}, s)\, \mathrm{e}^{-\mathrm{i}p\cdot x} + d^{\dagger}(\boldsymbol{p}, s) v(\boldsymbol{p}, s)\, \mathrm{e}^{\mathrm{i}p\cdot x} \right]$$

$$\Rightarrow \quad \eta_{\mathrm{F}} \gamma_0 \psi(t, -\boldsymbol{x}) = \eta_{\mathrm{F}} \sum_{\boldsymbol{p},s} C_{\boldsymbol{p}} \left[b(-\boldsymbol{p}, s) u(\boldsymbol{p}, s)\, \mathrm{e}^{-\mathrm{i}p\cdot x} - d^{\dagger}(-\boldsymbol{p}, s) v(\boldsymbol{p}, s)\, \mathrm{e}^{\mathrm{i}p\cdot x} \right]$$

$$\mathcal{P}\, b(\boldsymbol{p}, s)\, \mathcal{P}^{-1} = \eta_{\mathrm{F}}\, b(-\boldsymbol{p}, s), \qquad \mathcal{P}\, b^{\dagger}(\boldsymbol{p}, s)\, \mathcal{P}^{-1} = \eta_{\mathrm{F}}\, b^{\dagger}(-\boldsymbol{p}, s),$$

$$\mathcal{P}\, d(\boldsymbol{p}, s)\, \mathcal{P}^{-1} = -\eta_{\mathrm{F}}\, d(-\boldsymbol{p}, s), \qquad \mathcal{P}\, d^{\dagger}(\boldsymbol{p}, s)\, \mathcal{P}^{-1} = -\eta_{\mathrm{F}}\, d^{\dagger}(-\boldsymbol{p}, s)$$

the negative sign in the right-handed side means that the intrinsic parity of the antifermion is opposite in sign to tht of the corresponding fermion

Parity: Dirac field bilinear

- the one-fermion state transforms as

$$b^\dagger(\boldsymbol{p}, s)\ |0\rangle \quad \Rightarrow \quad b^\dagger(-\boldsymbol{p}, s)\ |0\rangle$$

$$d^\dagger(\boldsymbol{p}, s)\ |0\rangle \quad \Rightarrow \quad -d^\dagger(-\boldsymbol{p}, s)\ |0\rangle$$

- for any Dirac field bilinear

$$\mathcal{P}\ \overline{\psi}(x)\Gamma\psi(x)\,\mathcal{P}^{-1} = \overline{\psi}(t, -\boldsymbol{x})\ \gamma_0\Gamma\gamma_0\,\psi(t, -\boldsymbol{x})$$

	$\bar{\psi}\psi$	$i\bar{\psi}\gamma^5\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma^5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$	∂_μ
P	$+1$	-1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$	$(-1)^\mu$

$$(-1)^\mu \equiv 1 \ \text{ for } \mu = 0 \text{ and } (-1)^\mu \equiv -1 \text{ for } \mu = 1, 2, 3$$

– the free Dirac Lagrangian is invariant under P

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

Charge conjugation

- C: a unitary operator that reverses the signs of charges of particles including electric charge, lepton number, flavor number, …

$$\mathcal{C}\;|\boldsymbol{p}, s, Q\rangle = \xi\;|\boldsymbol{p}, s, -Q\rangle$$

ξ is a unimodular phase factor

  – charge conjugation = field conjugation for physical particles

- a complex scalar field

$$\mathcal{C}\,\phi(x)\,\mathcal{C}^{-1} = \xi_{\mathrm{B}}\phi^{\dagger}(x) \qquad \mathcal{C}^{\dagger}\mathcal{C} = 1, \quad |\xi_{\mathrm{B}}|^2 = 1$$

$$\sum_{p} C_{\boldsymbol{p}}[\,\mathcal{C}\,a_{\boldsymbol{p}}\,\mathcal{C}^{-1}\mathrm{e}^{-\mathrm{i}p\cdot x} + \mathcal{C}\,b^{\dagger}_{\boldsymbol{p}}\,\mathcal{C}^{-1}\mathrm{e}^{\mathrm{i}p\cdot x}\,] = \xi_{\mathrm{B}}\sum_{p} C_{\boldsymbol{p}}[\,a^{\dagger}_{\boldsymbol{p}}\mathrm{e}^{\mathrm{i}p\cdot x} + b_{\boldsymbol{p}}\mathrm{e}^{-\mathrm{i}p\cdot x}\,]$$

$$\mathcal{C}\,a_{\boldsymbol{p}}\,\mathcal{C}^{-1} = \xi_{\mathrm{B}}\;b_{\boldsymbol{p}} \qquad \mathcal{C}\,b_{\boldsymbol{p}}\,\mathcal{C}^{-1} = \xi^{*}_{\mathrm{B}}a_{\boldsymbol{p}}$$

  – a one-particle state of momentum p

$$\mathcal{C}a^{\dagger}_{\boldsymbol{p}}\;|0\rangle = \mathcal{C}\,a^{\dagger}_{\boldsymbol{p}}\,\mathcal{C}^{-1}\mathcal{C}\;|0\rangle = \xi^{*}_{\mathrm{B}}b^{\dagger}_{\boldsymbol{p}}\;|0\rangle \qquad \mathcal{C}b^{\dagger}_{\boldsymbol{p}}\;|0\rangle = \mathcal{C}\,b^{\dagger}_{\boldsymbol{p}}\,\mathcal{C}^{-1}\mathcal{C}\;|0\rangle = \xi_{\mathrm{B}}\;a^{\dagger}_{\boldsymbol{p}}\;|0\rangle$$

Charge conjugation

- electromagnetic field
  - the current density for a boson field

$$j^{\mu}(x) = \mathrm{i}\,[\phi^{\dagger}(x)\partial^{\mu}\phi(x) - (\partial^{\mu}\phi^{\dagger}(x))\phi(x)]$$

$$\mathcal{C}\, j_{\mu}(x)\, \mathcal{C}^{-1} = \mathrm{i}\,[\phi\partial_{\mu}\phi^{\dagger} - (\partial_{\mu}\phi)\phi^{\dagger}] = -\, j_{\mu}(x)$$

  - Similarly the electromagnetic field must transform as

$$\mathcal{C}\, A_{\mu}(x)\, \mathcal{C}^{-1} = -A_{\mu}(x)$$

$$\mathcal{C}\, a(\boldsymbol{k}, \lambda)\, \mathcal{C}^{-1} = -a(\boldsymbol{k}, \lambda)$$

$$\mathcal{C}\, |\boldsymbol{k}, \lambda\rangle = -\, |\boldsymbol{k}, \lambda\rangle$$

- The photon is odd under charge conjugation

Charge conjugation

- Dirac field: proportional to its complex conjugate like a boson field

$$\mathcal{C}\,\psi(x)\,\mathcal{C}^{-1} = \xi_F B \psi^*(x), \quad |\xi_F| = 1$$

B is a 4×4 unitary matrix on the spinor representation

$$C = B\gamma_0^*$$

- more convenient to use the following form

$$\overline{\psi}^T = \left((\psi^*)^T \gamma_0\right)^T = (\gamma_0)^T \psi^*$$

$$\mathcal{C}\,\psi(x)\,\mathcal{C}^{-1} = \xi_F C \overline{\psi}^T(x), \quad |\xi_F| = 1 \qquad C^\dagger C = 1$$

Because of the complex conjugate, the charge conjugation operator C depends on the representation of the γ matrix

- The Lagrangian is invariant under charge conjugation

$$\mathcal{C}\,\mathcal{L}_F(x)\,\mathcal{C}^{-1} = \mathcal{L}_F(x)$$

$$\mathcal{L}_F \equiv \mathcal{L}_1 + \mathcal{L}_1^\dagger = \frac{1}{2}\overline{\psi}\left[i\gamma^\mu \overrightarrow{\partial}_\mu - m\right]\psi + \frac{1}{2}\overline{\psi}\left[-i\gamma^\mu \overleftarrow{\partial}_\mu - m\right]\psi$$

- One can derive

$$\mathcal{C}\,\psi^\dagger\,\mathcal{C}^{-1} = \xi_F^* \psi^T \gamma_0^* C^\dagger$$

$$\mathcal{C}\,\mathcal{L}_1\,\mathcal{C}^{-1} = \frac{1}{2}\psi^T \gamma_0^* C^\dagger (i\gamma^0\gamma^\mu\partial_\mu - \gamma_0 m) C \overline{\psi}^T$$

scalar, invariant under transpose in the spinor space

$$= \frac{1}{2}\overline{\psi} C^T \left(-i\gamma^{\mu T}\gamma^{0T}\overleftarrow{\partial}_\mu + \gamma_0^T m\right) C^* \gamma_0 \psi$$

transpose and an additional (-) sign for fermion exchange

Charge conjugation

- require $\mathcal{C}\,\mathcal{L}_1\,\mathcal{C}^{-1} = \mathcal{L}_1^\dagger$ to satisfy $\mathcal{C}\,\mathcal{L}_\mathrm{F}(x)\,\mathcal{C}^{-1} = \mathcal{L}_\mathrm{F}(x)$

$$C^T \gamma^{\mu T} \gamma_0{}^T C^* \gamma_0 = \gamma^\mu$$
$$C^T \gamma_0{}^T C^* \gamma_0 = 1$$

$$\Rightarrow \quad C^\dagger \gamma_\mu C = -\gamma_\mu^\mathrm{T}$$

in arbitrary γ-matrix representation

- in the standard representation of γ, we choose

$$C = \mathrm{i}\gamma^2\gamma^0$$

$$C\bar{u}^\mathrm{T}(\boldsymbol{p}, s) = v(\boldsymbol{p}, s)$$
$$C\bar{v}^\mathrm{T}(\boldsymbol{p}, s) = u(\boldsymbol{p}, s)$$

$$u(\boldsymbol{p}, s) = N \begin{pmatrix} \chi_s \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi_s \end{pmatrix}$$

$$v(\boldsymbol{p}, s) = N' \begin{pmatrix} \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\eta_s \\ \eta_s \end{pmatrix}$$

$$\eta_s = -\mathrm{i}\sigma^2\chi_s$$

- Then one finds

$$\mathcal{C}\,\psi\,\mathcal{C}^{-1} = \sum_{\boldsymbol{p},s} C_{\boldsymbol{p}} \left[\mathcal{C}\,b(\boldsymbol{p}, s)\,\mathcal{C}^{-1} u(\boldsymbol{p}, s)\mathrm{e}^{-\mathrm{i}p\cdot x} + \mathcal{C}\,d^\dagger(\boldsymbol{p}, s)\,\mathcal{C}^{-1} v(\boldsymbol{p}, s)\mathrm{e}^{\mathrm{i}p\cdot x} \right]$$

$$\xi_\mathrm{F} C\overline{\psi}^\mathrm{T} = \xi_\mathrm{F} \sum_{\boldsymbol{p},s} C_{\boldsymbol{p}} \left[b^\dagger(\boldsymbol{p}, s) v(\boldsymbol{p}, s)\,\mathrm{e}^{\mathrm{i}p\cdot x} + d(\boldsymbol{p}, s) u(\boldsymbol{p}, s)\,\mathrm{e}^{-\mathrm{i}p\cdot x} \right]$$

$$\mathcal{C}\,b(\boldsymbol{p}, s)\,\mathcal{C}^{-1} = \xi_\mathrm{F}\,d(\boldsymbol{p}, s) \qquad \mathcal{C}\,d(\boldsymbol{p}, s)\,\mathcal{C}^{-1} = \xi_\mathrm{F}^*\,b(\boldsymbol{p}, s)$$

- Note that in the Weyl representation (Peskin & Schroeder)

$$C\psi(x)C = -i\gamma^2\psi^*(x) = -i\gamma^2(\psi^\dagger)^T = -i\left(\overline{\psi}\gamma^0\gamma^2\right)^T$$

C, P, and T

- charge conjugation for Dirac field bilinear

$$\mathcal{C}\,\psi(x)\,\mathcal{C}^{-1} = \xi C\overline{\psi}^{\mathrm{T}}(x)$$

$$\mathcal{C}\,\overline{\psi}(x)\,\mathcal{C}^{-1} = -\xi^{*}\psi^{\mathrm{T}}(x)C^{\dagger}$$

$$\mathcal{C}\,\overline{\psi}_1(x)\Gamma\psi_2(x)\,\mathcal{C}^{-1} = \xi_1^{*}\xi_2\overline{\psi}_2(x)\,C\Gamma^{\mathrm{T}}C^{\dagger}\,\psi_1(x)$$

- transformation properties

	$\bar\psi\psi$	$i\bar\psi\gamma^5\psi$	$\bar\psi\gamma^\mu\psi$	$\bar\psi\gamma^\mu\gamma^5\psi$	$\bar\psi\sigma^{\mu\nu}\psi$	∂_μ
P	$+1$	-1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$	$(-1)^\mu$
T	$+1$	-1	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu(-1)^\nu$	$-(-1)^\mu$
C	$+1$	$+1$	-1	$+1$	-1	$+1$
CPT	$+1$	$+1$	-1	-1	$+1$	-1

$$(-1)^\mu \equiv 1 \ \text{ for } \mu = 0 \text{ and } (-1)^\mu \equiv -1 \text{ for } \mu = 1,2,3$$

$\mathcal{L}_0 = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi$ is invariant under C, P, and T separately

C, P, and T

Bilinear	$\mathcal{P}$	$\mathcal{T}$	$\mathcal{C}$	$\mathcal{CP}$	$\mathcal{CPT}$
$\overline{\psi}\chi$	$\overline{\psi}\chi$	$\overline{\psi}\chi$	$\overline{\chi}\psi$	$\overline{\chi}\psi$	$\overline{\chi}\psi$
$\overline{\psi}\gamma_5\chi$	$-\overline{\psi}\gamma_5\chi$	$\overline{\psi}\gamma_5\chi$	$\overline{\chi}\gamma_5\psi$	$-\overline{\chi}\gamma_5\psi$	$-\overline{\chi}\gamma_5\psi$
$\overline{\psi}P_{L,R}\chi$	$\overline{\psi}P_{R,L}\chi$	$\overline{\psi}P_{L,R}\chi$	$\overline{\chi}P_{L,R}\psi$	$\overline{\chi}P_{R,L}\psi$	$\overline{\chi}P_{R,L}\psi$
$\overline{\psi}\gamma^\mu\chi$	$\overline{\psi}\gamma_\mu\chi$	$\overline{\psi}\gamma_\mu\chi$	$-\overline{\chi}\gamma^\mu\psi$	$-\overline{\chi}\gamma_\mu\psi$	$-\overline{\chi}\gamma^\mu\psi$
$\overline{\psi}\gamma^\mu\gamma_5\chi$	$-\overline{\psi}\gamma_\mu\gamma_5\chi$	$\overline{\psi}\gamma_\mu\gamma_5\chi$	$\overline{\chi}\gamma^\mu\gamma_5\psi$	$-\overline{\chi}\gamma_\mu\gamma_5\psi$	$-\overline{\chi}\gamma^\mu\gamma_5\psi$
$\overline{\psi}\gamma^\mu P_{L,R}\chi$	$\overline{\psi}\gamma_\mu P_{R,L}\chi$	$\overline{\psi}\gamma_\mu P_{L,R}\chi$	$-\overline{\chi}\gamma^\mu P_{R,L}\psi$	$-\overline{\chi}\gamma_\mu P_{L,R}\psi$	$-\overline{\chi}\gamma^\mu P_{L,R}\psi$
$\overline{\psi}\sigma^{\mu\nu}\chi$	$\overline{\psi}\sigma_{\mu\nu}\chi$	$-\overline{\psi}\sigma_{\mu\nu}\chi$	$-\overline{\chi}\sigma^{\mu\nu}\psi$	$-\overline{\chi}\sigma_{\mu\nu}\psi$	$\overline{\chi}\sigma^{\mu\nu}\psi$

$$(-1)^\mu \equiv 1 \;\text{ for } \mu = 0 \text{ and } (-1)^\mu \equiv -1 \text{ for } \mu = 1, 2, 3$$

$\mathcal{L}_0 = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi$ is invariant under C, P, and T separately

- CP: $e_L^- \leftrightarrow e_R^+$

$$\mathcal{L} \propto V_{ij}\overline{U}_i\gamma^\mu(1-\gamma_5)D_jW_\mu^\dagger + V_{ij}^*\overline{D}_j\gamma^\mu(1-\gamma_5)U_iW_\mu$$

CP conjugation

$$\mathcal{L}_{\rm CP} \propto V_{ij}\overline{D}_j\gamma^\mu(1-\gamma_5)U_iW_\mu + V_{ij}^*\overline{U}_i\gamma^\mu(1-\gamma_5)D_jW_\mu^\dagger$$

If $V_{ij}^* = V_{ij} \rightarrow L = L_{\rm CP}$: i.e. CP conservation

CPT theorem

CPT theorem:

$$(CPT)^{-1}\left(\overline{\Psi}\Psi\right)CPT = +\overline{\Psi}\Psi\ ,$$
$$(CPT)^{-1}\left(\overline{\Psi}i\gamma_5\Psi\right)CPT = +\overline{\Psi}i\gamma_5\Psi\ ,$$
$$(CPT)^{-1}\left(\overline{\Psi}\gamma^\mu\Psi\right)CPT = -\overline{\Psi}\gamma^\mu\Psi\ ,$$
$$(CPT)^{-1}\left(\overline{\Psi}\gamma^\mu\gamma_5\Psi\right)CPT = -\overline{\Psi}\gamma^\mu\gamma_5\Psi\ ,$$

even under CPT

odd

General rule: a fermion bilinear with n vector indices is even (odd) under CPT if n is even (odd); this also applies to derivatives ∂_μ .

Thus any hermitian combination of fields and derivatives that is Lorentz invariant (has no uncontracted Lorentz indices) is even under CPT!

Lagrangian is formed from such terms, $\mathcal{L}(x) \to \mathcal{L}(-x)$ under CPT, and so the action $S = \int d^4x\, \mathcal{L}$ is invariant under CPT.

Lorentz invariance ⟷ CPT

Yukawa matrix

- The third generation was introduced before the discovery of the charm quark by Kobayashi and Maskawa (1972)

- The Yukawa or mass matrices are arbitrary 3×3 complex matrices

$$\mathcal{L}_Y^{\rm SM} = y_{ij}^d \overline{Q_{Li}} \phi d_{Rj} + y_{ij}^u \overline{Q_{Li}} \tilde{\phi} u_{Rj} + y_{ij}^e \overline{L_{Li}} \phi e_{Rj}$$

$$(m_d)_{ij} = \frac{y_{ij}^d v}{\sqrt{2}},\ (m_u)_{ij} = \frac{y_{ij}^u v}{\sqrt{2}},\ (m_e)_{ij} = \frac{y_{ij}^e v}{\sqrt{2}}$$

- No flavor changing coupling for $y_{ij}^f = \delta_{ij}$

- In general, the Yukawa couplings are non-diagonal in the generation basis (flavor basis)

$$y_{ij}^f \neq \delta_{ij}$$

- The mass matrices must be diagonalized to describe the physical process

Diagonalization of mass matrix

- The Yukawa matrices can be diagonalized by bi-unitary transformations

$$y^f = V_L^f y^f_{\rm diag} V_R^{f\dagger} \qquad y^f_{\rm diag} = \begin{pmatrix} y_1^f & & \\ & y_2^f & \\ & & y_3^f \end{pmatrix} \qquad V_L^f, V_R^f : \text{unitary matrices}$$

- The mass matrices for quarks are diagonalized as

$$\bar{u}_L \underbrace{M_u}_{} u_R + \bar{d}_L M_d d_R \quad (V_L^{u\dagger} V_L^u,\ V_R^{u\dagger} V_R^u,\ V_L^{d\dagger} V_L^d,\ V_R^{d\dagger} V_R^d) \Rightarrow \bar{u}'_L M_u^{diag} u'_R + \bar{d}'_L M_d^{diag} d'_R$$

$$u^{(')}_{L,R} = \begin{pmatrix} u^{(')}_{L,R} \\ c^{(')}_{L,R} \\ t^{(')}_{L,R} \end{pmatrix}, \quad d^{(')}_{L,R} = \begin{pmatrix} d^{(')}_{L,R} \\ s^{(')}_{L,R} \\ b^{(')}_{L,R} \end{pmatrix} \qquad M_u^{diag} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}, \quad M_d^{diag} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

- The quark fields are transforms as (field redefinitions)

$$\psi_i \to V_{ij}^f \psi_j \,(\psi = Q_L, U_R, D_R)$$

- Kinetic terms are not changed by field redefinitions by the unitary rotation in the generation basis

$$\mathcal{L}_{\rm kin} \sim -i\overline{\psi_i}\,\not{D}\psi_i$$

Neutral current

- The coupling to neutral gauge bosons γ, Z, g are not modified by the field redefinitions

$$\mathcal{L}_{\text{neutral}} \sim \overline{u}_L^i \delta_{ij} u_L^j \to \overline{u}_L^k (V_L^u)_{ki} \delta_{ij} (V_L^{u\dagger})_{jl} u_L^j = \overline{u}_L^i \delta_{ij} u_L^j$$

  – The FCNC is forbidden at the tree level

- In the SM, all up (or down)-type quarks have the same charges Y and T_3

- This simply forbids the FCNC which might be generated by the field redefinitions because the couplings to neutral gauge bosons depend on Y and T_3

- the couplings to the Higgs boson are aligned to the mass matrix so that there is no FCNC related to the Higgs boson in the SM

- The FCNC process to any gauge boson would be a good signal for probing new physics

CKM matrix

- The couplings to the W bosons are modified as

$$\mathcal{L}_W \sim \overline{u}_L^i \delta_{ij} d_L^j \to \overline{u}_L^k (V_L^u)_{ki} \delta_{ij} (V_L^{d\dagger})_{jl} d_L^l = \overline{u}_L^i (V_{\mathrm{CKM}})_{ij} d_L^j$$

$$V_{\mathrm{CKM}} = V_L^u V_L^{d\dagger}$$

- The CKM matrix is the generalization of the Cabibbo rotation

$$\begin{pmatrix} d'' \\ s'' \\ b'' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

- The CKM matrix is a 3×3 complex matrix and unitary by definition

$$V_{\mathrm{CKM}}^{\dagger} V_{\mathrm{CKM}} = 1$$

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- The mixing of down-type quarks ~ a (historical) convention
  - in the lepton case, the mixing occurs in the up-type components (neutrinos)

Weak phase

- To generate CP violation, we need a complex phase in the Lagrangian
- To be hermitian, all couplings except for the CKM matrix are real
- If the CKM matrix is real, no CP violation

$$\mathcal{L} \propto V_{ij}\overline{U}_i\gamma^\mu(1-\gamma_5)D_jW_\mu^\dagger + V_{ij}^*\overline{D}_j\gamma^\mu(1-\gamma_5)U_iW_\mu$$

$\updownarrow$ CP conjugation

$$\mathcal{L}_{\text{CP}} \propto V_{ij}\overline{D}_j\gamma^\mu(1-\gamma_5)U_iW_\mu + V_{ij}^*\overline{U}_i\gamma^\mu(1-\gamma_5)D_jW_\mu^\dagger$$

If $V_{ij}^* = V_{ij} \rightarrow L = L_{\text{CP}}$: i.e. CP conservation

- If CP violation exists in the framework of the SM, the CKM matrix must be complex
- Because the phase in the CKM matrix is originated from weak interactions, the CKM phase is called as a “weak phase”
- In the BSM, another phase may exist in the new couplings or new vacuum

3 generation is required for CP violation

- An arbitrary N×N complex matrix has $2N^2$ parameters
- Unitary conditions ($VV^\dagger=1$) remove N^2 degree of freedoms

$$\sum_j V_{ij}V_{ji}^* = 1,\ \sum_j V_{ij}V_{jk}^* = 0\ (i \neq k) \Rightarrow N + 2\,{}_N C_2 = N^2$$

- For a real matrix, N directions have ${}_N C_2$ independent (relative) angles

$${}_N C_2 = \frac{N(N-1)}{2} \text{ angles}$$

- The remaining parameters are complex phases

$$N^2 - \frac{N(N-1)}{2} = \frac{N(N+1)}{2} \text{ phases}$$

- Each quark field can change its phase without modifying other terms

$$q_j \to q_j' = e^{i\phi_j} q_j$$

q_R should have an opposite phase to q_L to make the mass term invariant.

N(up-type quarks)+N(down-type quarks)=2N rephasing possible

$$\mathcal{L}_W \to \mathcal{L}_W \sim \overline{u}_L^i e^{i(\delta_i - \delta_j)} (V_{\mathrm{CKM}})_{ij} d_L^j$$

3 generation is required for CP violation

- Among 2N phases, one phase which is called as “overall phase” is not independent
- Therefore (2N-1) phases can be removed by rephasing quark fields
- The number of phases which cannot be removed is N(N+1)/2-(2N-1)=

$$\frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$$

n	$\frac{(n-1)(n-2)}{2}$
1	0
2	0
3	1
4	3

- For CP violation, $N \geq 3$
- In N=3, 3 mixing angles and 1 (weak) phase (the only phase in the SM Langrangian that is necessary for CP violation)
- Rephasing example in N=3

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

the rephasing phase of u quark is multiplied to the first row and can make Vud real $u_L \to e^{i\phi(u)} u_L$

Similarly c quark → Vcd, t quark → Vtd, s quark → Vus, b quark → Vub but rephasing d quark cannot be applied because all elements of the first column are real

Parametrization of CKM matrix

- The orginal CKM matrix in Kobayashi and Maskawa's paper

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}.$$

- The standard CKM matrix which is adopted by PDG

$$V_{\rm CKM} \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = R_{23}(I_{\delta_D} R_{13} I^\dagger_{\delta_D}) R_{12}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$I_{\delta_D} = {\rm diag}(1, 1, e^{i\delta}) \qquad 0 \le \theta_{ij} \le \pi/2\,, \quad 0 \le \delta \le 2\pi$$

$$s_{ij} = \sin\theta_{ij}\,,\ c_{ij} = \cos\theta_{ij}$$

Neutrino and PMNS matrix

- If neutrions are massive, we can construct the Pontecorvo-Maki-Nakagata-Saki (PMNS) matrix similar to the CKM matrix

CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- In the case of Dirac neutrinos, the PMNS matrix are exactly the same as the CKM matrix

- In the case of Majorana neutrinos, there are two more Majorana phases

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Because of an additional mass term $m\,\overline{\nu_L^C}\,\nu_L$, we cannot redefine the phase of neutrino fields

$$\nu_L \to e^{i\phi}\nu_L \Rightarrow m\,\overline{\nu_L^C}\,\nu_L \to e^{i2\phi} m\,\overline{\nu_L^C}\,\nu_L$$

Dirac vs Majorana

- the conventions with respec to charge conjugation and helicity projection

$$\psi^c = C\gamma^0\psi^* = i\gamma^2\psi^*, \qquad \bar{\psi}^c = \psi^T C$$

$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\psi, \qquad \psi_R = \tfrac{1}{2}(1+\gamma_5)\psi$$

$C = i\gamma^2\gamma^0$ $\quad (\gamma^k)^\dagger = -\gamma^k$

$$\overline{\psi^c} = \left(i\gamma_2\psi^*\right)^\dagger \gamma_0 = -i\psi^T\left(\gamma_2\right)^\dagger \gamma_0 = \psi^T C$$

- notation

$$\psi_L^c \equiv (\psi_L)^c = \tfrac{1}{2}(1+\gamma_5)\psi^c = (\psi^c)_R$$

$$(\psi_L)^c = i\gamma_2\frac{1}{2}(1-\gamma_5)\psi^* = \frac{1}{2}(1+\gamma_5)i\gamma_2\psi^*$$

➡ the charge conjugate of the left-handed field acts as a right-handed field

- Dirac mass term

$$\mathscr{L}_D = D(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) = D\bar{\psi}\psi$$

- Majorana mass term

$$\mathscr{L}_{MA} = A(\bar{\psi}_L^c\psi_L + \bar{\psi}_L\psi_L^c) = A\bar{\chi}\chi$$

$$\mathscr{L}_{MB} = B(\bar{\psi}_R^c\psi_R + \bar{\psi}_R\psi_R^c) = B\bar{\omega}\omega$$

➡ Majorana fields must not be charged

$$\chi = \psi_L + \psi_L^c, \qquad \chi^c = \chi \qquad \psi_L = \tfrac{1}{2}(1-\gamma_5)\chi; \qquad \psi_L^c = \tfrac{1}{2}(1+\gamma_5)\chi$$

$$\omega = \psi_R + \psi_R^c, \qquad \omega^c = \omega \qquad \psi_R = \tfrac{1}{2}(1+\gamma_5)\omega; \qquad \psi_R^c = \tfrac{1}{2}(1-\gamma_5)\omega$$

Questions

1. Show that there is no CP violation for N=3 if any two of the quarks are mass degenerate.

2. Explain why there is no CKM-like matrix in the lepton sector of the SM.

3. It is observed that neutrinos oscillate. Can the charged leptons also oscillate?

Wolfenstein parametrization

- More popular parametrization is Wolfenstein parametrization
- V_{CKM} in nature there is hirarchical $\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$

$\lambda = \sin\theta_c \approx 0.22$

$A, \rho, \eta \sim \mathcal{O}(1)$

$$\sum_j V_{ij} V_{ji}^* = 1, \quad \sum_j V_{ij} V_{jk}^* = 0 \ (i \neq k)$$

Cabibbo rotation

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & \\ -\lambda & 1 - \frac{\lambda^2}{2} & \\ & & \end{pmatrix}$$

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Cabibbo rotation

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \\ -\lambda & 1-\frac{\lambda^2}{2} & \\ & & \end{pmatrix}$$

$$V_{cb} \sim 0.06 = O(\lambda^2)$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ & -A\lambda^2 & 1 \end{pmatrix}$$

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Cabibbo rotation

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$$V_{cb} \sim 0.06 = O(\lambda^2)$$

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Wolfenstein parametrization

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Only V_{ub} and V_{td} are complex

- In the W decays, suppressing the common factor and phase space factor

$$\Gamma(W^+ \to \ell^+ \nu_\ell) \propto 1 \qquad \Gamma(W^+ \to u_i \overline{d_j}) \propto 3|V_{ij}|^2 \qquad (i = 1, 2;\ j = 1, 2, 3)$$

- The top quark is omitted and the number of lepton is 3
- The unitarity condition for the CKM matrixc

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

$$\Gamma(W \to \text{hadrons}) \approx 2\Gamma(W \to \text{leptons})$$

Experimentally, $\Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) = 2.09 \pm 0.01$

Wolfenstein parametrization beyond LO

- In the standard parametrization, we define $(\lambda, A, \varrho, \eta)$

$$s_{12} = \lambda\,, \qquad s_{23} = A\lambda^2\,, \qquad s_{13}e^{-i\delta} = A\lambda^3(\varrho - i\eta)$$

$$\varrho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta, \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta$$

- The higher order terms are obtained as

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6)$$

$$V_{us} = \lambda + \mathcal{O}(\lambda^7)$$

$$V_{ub} = A\lambda^3(\varrho - i\eta)$$

$$V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\varrho + i\eta)] + \mathcal{O}(\lambda^7)$$

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) + \mathcal{O}(\lambda^6)$$

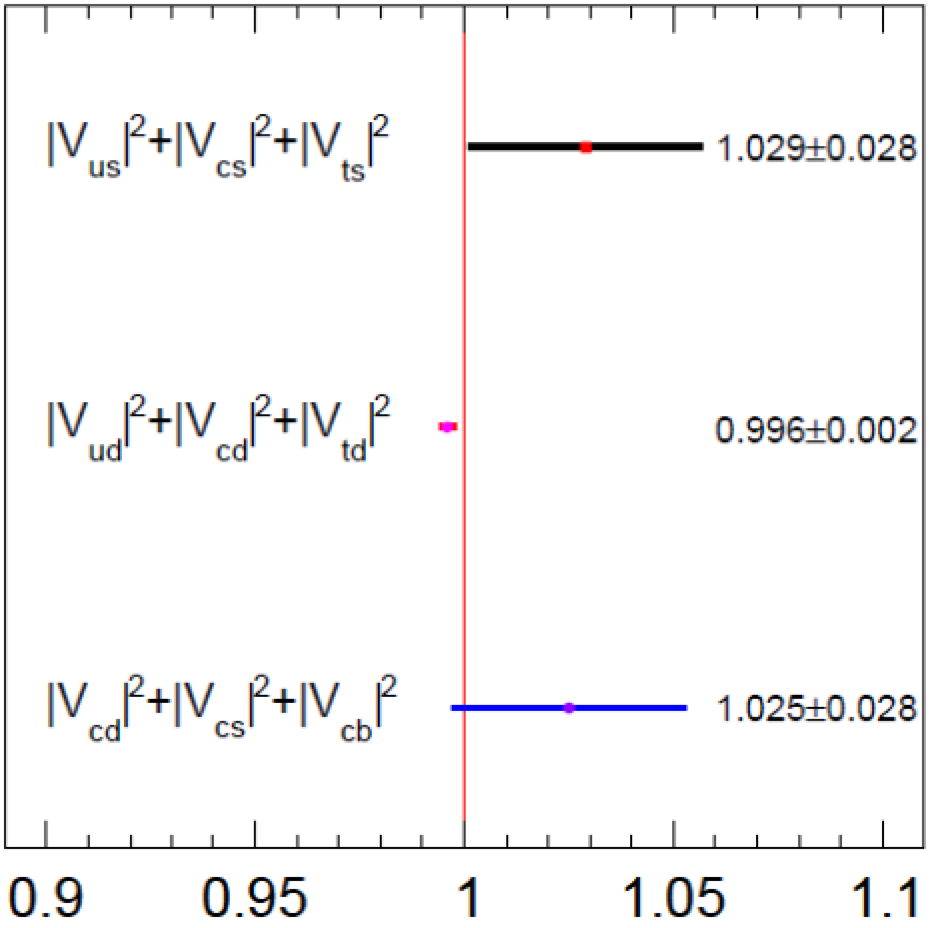
$$V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^8)$$

$$V_{td} = A\lambda^3\left[1 - (\varrho + i\eta)(1 - \frac{1}{2}\lambda^2)\right] + \mathcal{O}(\lambda^7)$$

$$V_{ts} = -A\lambda^2 + \frac{1}{2}A(1 - 2\varrho)\lambda^4 - i\eta A\lambda^4 + \mathcal{O}(\lambda^6)$$

$$V_{tb} = 1 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6)$$

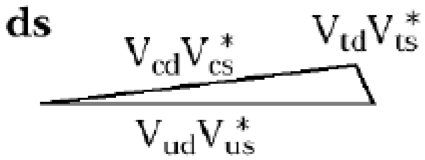
Unitarity check



Hierarchy in Unitary relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

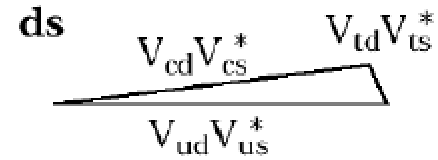
$$\underset{\mathcal{O}(\lambda)}{V_{ud}\,V_{us}^*} \quad + \quad \underset{\mathcal{O}(\lambda)}{V_{cd}\,V_{cs}^*} \quad + \quad \underset{\mathcal{O}(\lambda^5)}{V_{td}\,V_{ts}^*} \quad = 0$$



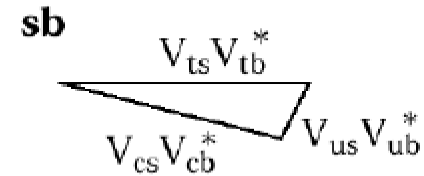
Hierarchy in Unitary relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\underset{\mathcal{O}(\lambda)}{V_{ud}\,V_{us}^{*}} + \underset{\mathcal{O}(\lambda)}{V_{cd}\,V_{cs}^{*}} + \underset{\mathcal{O}(\lambda^5)}{V_{td}\,V_{ts}^{*}} = 0$$



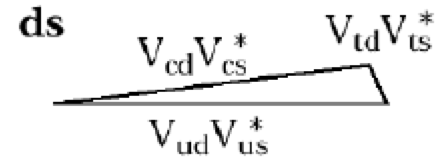
$$\underset{\mathcal{O}(\lambda^4)}{V_{us}\,V_{ub}^{*}} + \underset{\mathcal{O}(\lambda^2)}{V_{cs}\,V_{cb}^{*}} + \underset{\mathcal{O}(\lambda^2)}{V_{ts}\,V_{tb}^{*}} = 0$$



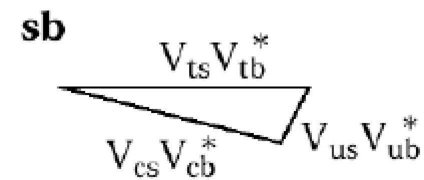
Hierarchy in Unitary relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

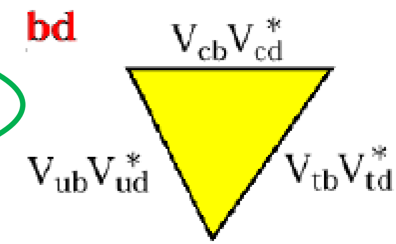
$$\underset{\mathcal{O}(\lambda)}{V_{ud}V_{us}^*} + \underset{\mathcal{O}(\lambda)}{V_{cd}V_{cs}^*} + \underset{\mathcal{O}(\lambda^5)}{V_{td}V_{ts}^*} = 0$$



$$\underset{\mathcal{O}(\lambda^4)}{V_{us}V_{ub}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{cs}V_{cb}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{ts}V_{tb}^*} = 0$$



$$\underset{\mathcal{O}(\lambda^3)}{V_{ud}V_{ub}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{cd}V_{cb}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{td}V_{tb}^*} = 0$$



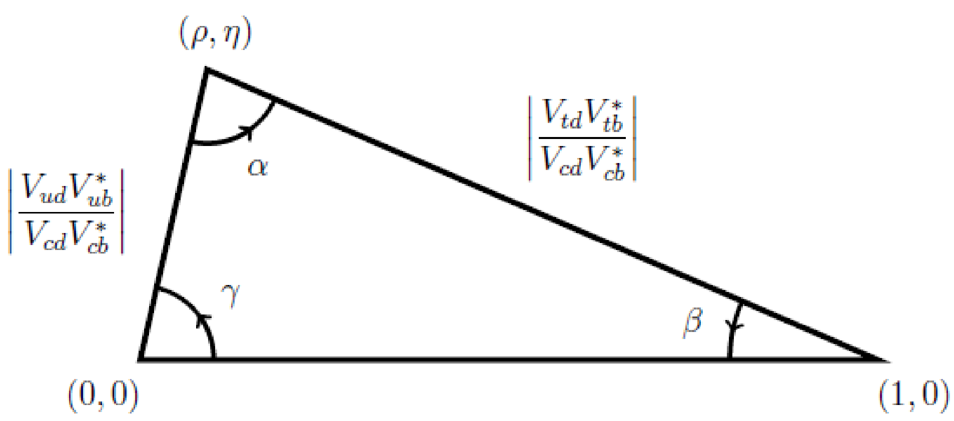
The CKM triangle

Unitarity triangle

$$\underset{\mathcal{O}(\lambda^3)}{V_{ud}V_{ub}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{cd}V_{cb}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{td}V_{tb}^*} = 0$$

$$A\lambda^3[(\rho + i\eta) + (-1) + (1 - \rho - i\eta)] = 0$$

- Resize the sides by $V_{cd}V_{cb}^*$

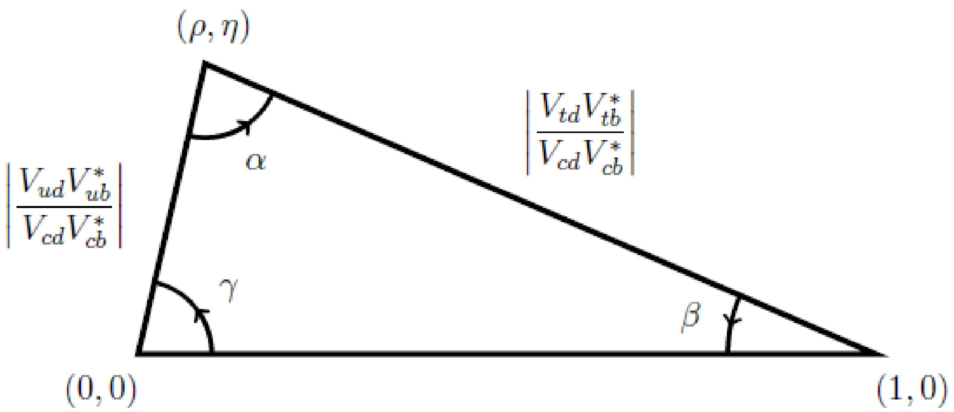


$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

Jarlskog invariant



$$\text{(area)}=\left(\frac{1}{2}\text{(base)}\times\text{(height)}\right)\times\text{(scale factor)}=\frac{1}{2}\text{Im}\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)\times\left|V_{cd}V_{cb}^*\right|^2$$

$$=\frac{1}{2}\text{Im}\left(-V_{ud}V_{ub}^*V_{cd}^*V_{cb}\right)=\frac{1}{2}\text{Im}\left(c_{12}c_{13}s_{13}e^{i\delta}(s_{12}c_{23}+c_{12}s_{23}s_{13}e^{-i\delta})s_{23}c_{13}\right)$$

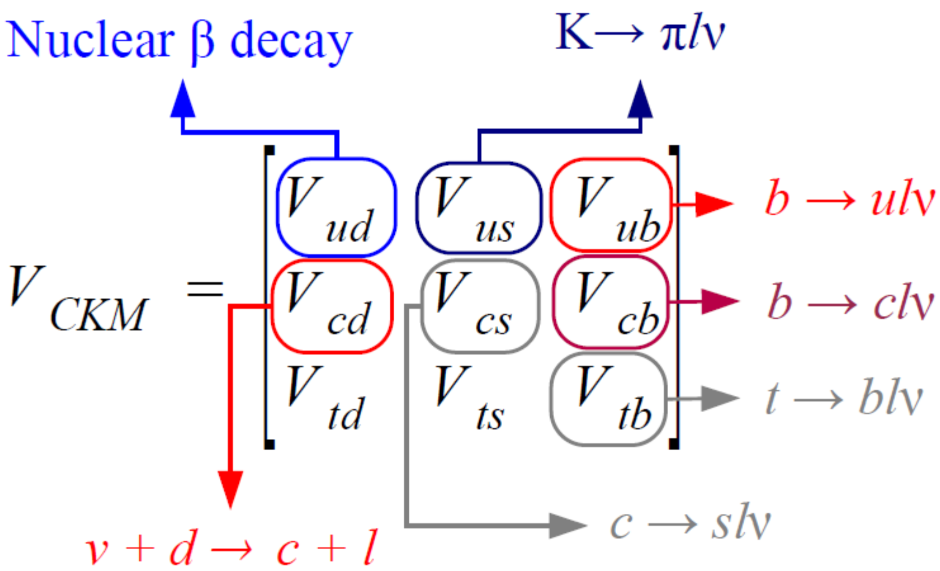
$$=\frac{1}{2}c_{12}c_{23}c_{13}^2s_{12}s_{23}s_{13}\sin\delta$$

- basis-independent quantity that identifies CP violation = Jarlskog invariant

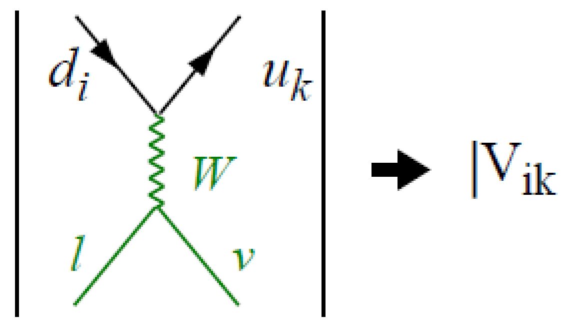
$$\text{Im}\left[V_{ij}V_{kl}V_{il}^*V_{kj}^*\right]=J\sum_{n,m}\epsilon_{ikn}\epsilon_{jlm}\quad\text{(no sum in } i, j, k, l)$$

$$J=c_{12}c_{23}c_{13}^2s_{12}s_{23}s_{13}\sin\delta\sim\lambda^6A^2\eta\quad\text{(in the SM)}$$

Determination of CKM matrix elements



Once we assume unitarity, the CKM matrix can be completely determined using only exp. info from processes mediated by tree-level c.c. amplitudes



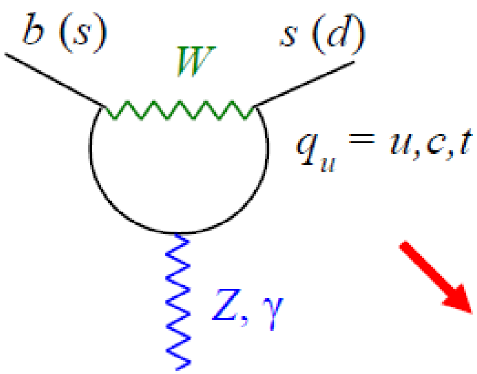
Excellent determination (error ~ 0.5 %)
Very good determination (error ~ 0.1%)
Good determination (error ~ 2 %)
Sizable error (5-15 %)
Not competitive with unitarity constraints

Determination of CKM matrix elements

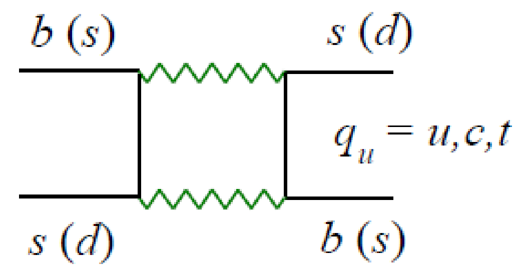
The only CKM elements we cannot access via tree-level processes are V_{ts} & V_{td}

Loop-induced amplitudes:

$\Delta F = 1$ FCNC



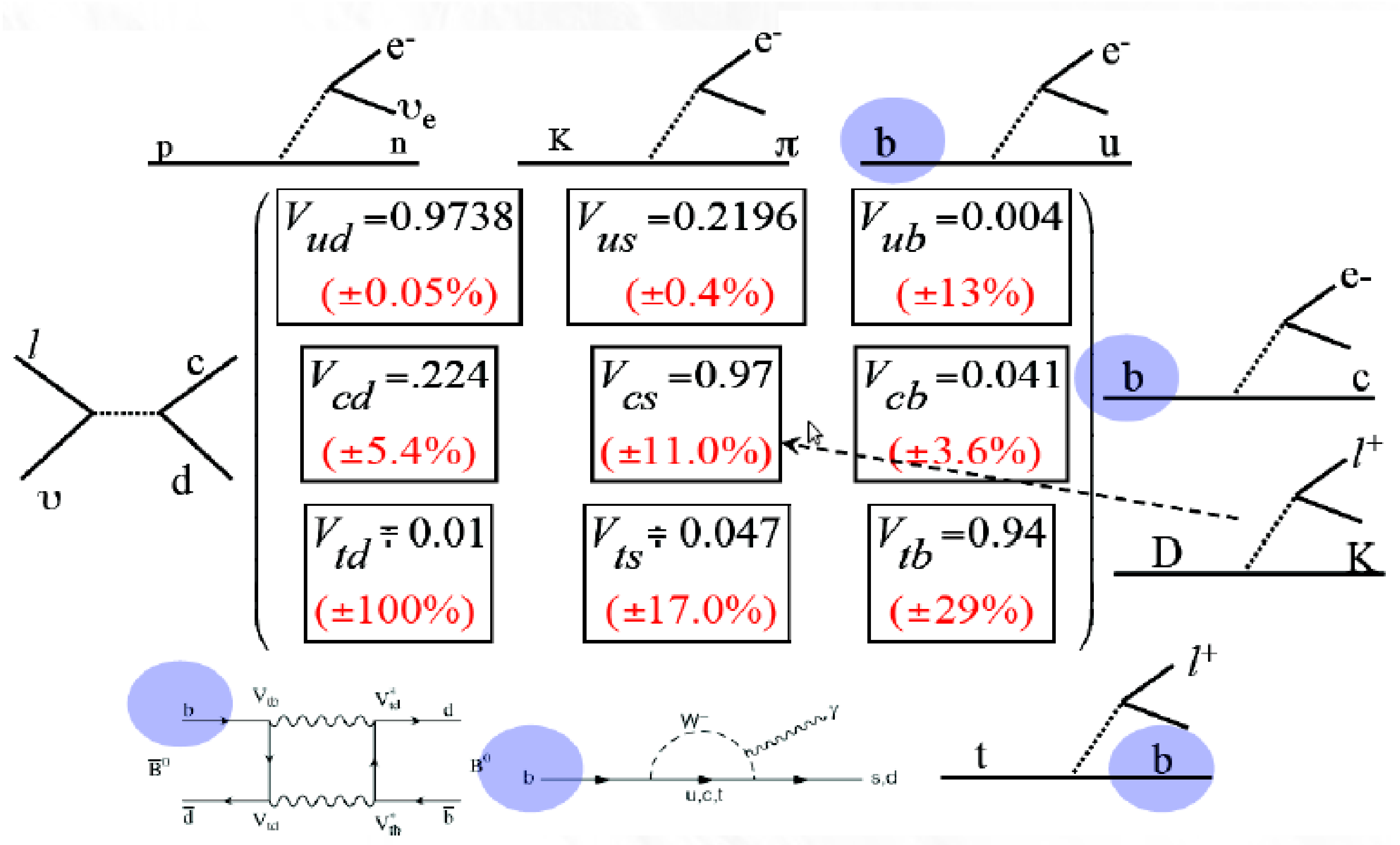
$\Delta F = 2$ (neutral-meson mixing)



GIM mechanism

[large top-quark contribution: $A \sim m_t^2\, V_{tq}^{*} V_{tb}$]

Determination of CKM matrix elements



The Unitarity Triangle

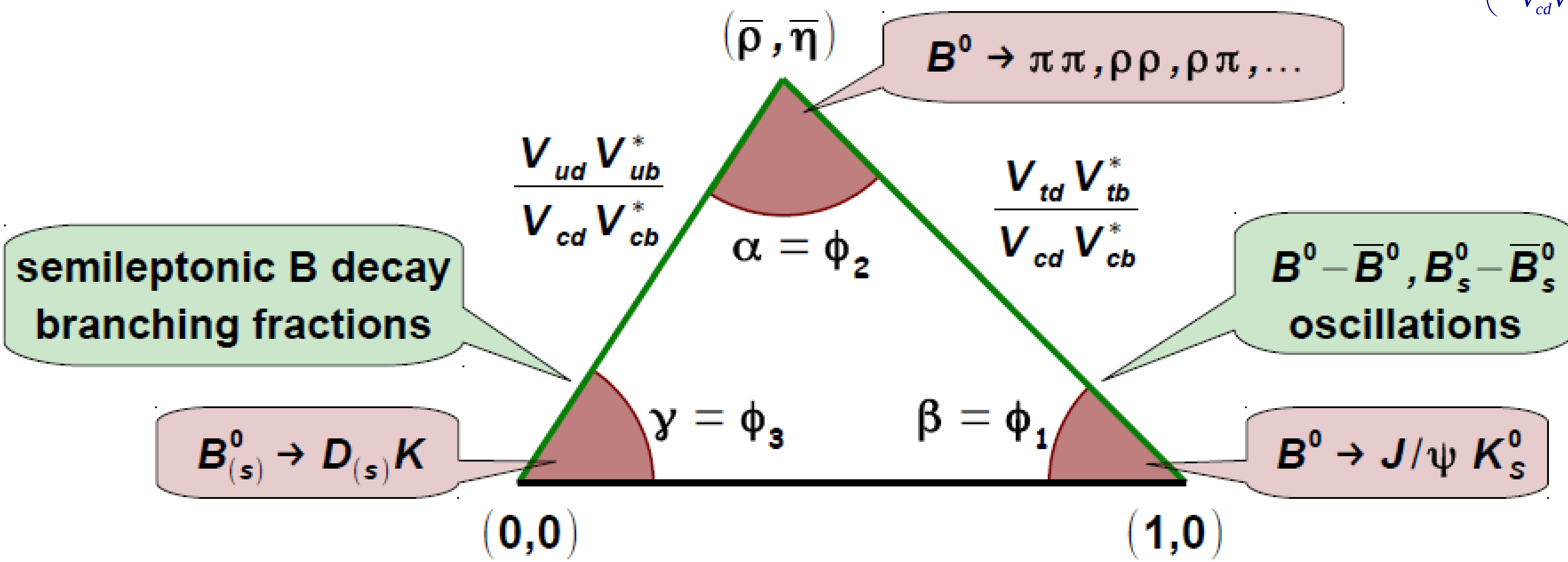
$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \Rightarrow 1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

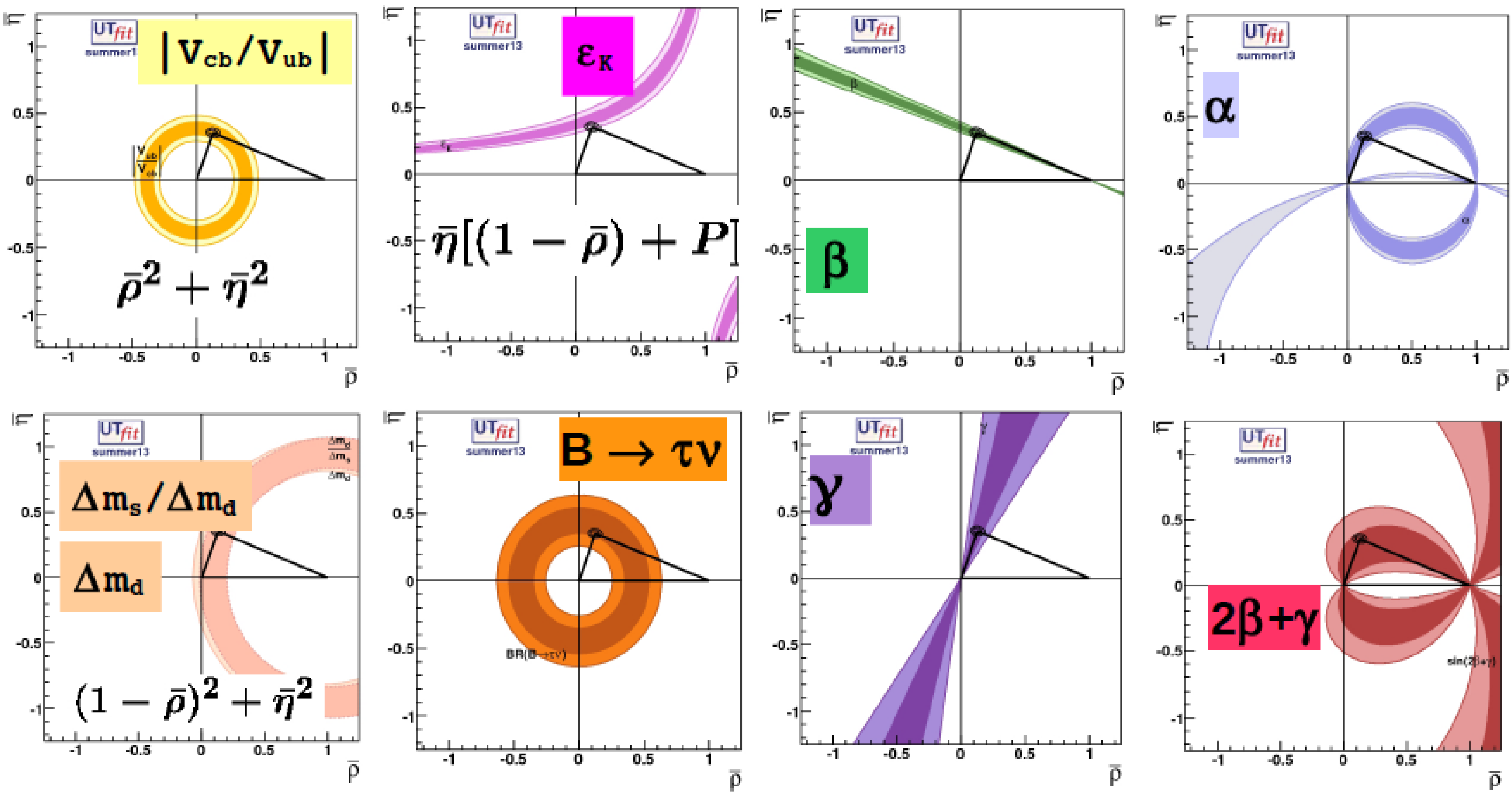
$A\lambda^3(\rho - i\eta)$ $\quad -A\lambda^3$ $\quad A\lambda^3(1-\rho-i\eta)$



- measure the lengths of the two sides: *CP* conserving quantities
- measure all three angles: *CP* violating quantities (angles = phases !)
- many observables $\to$ overconstraint determination of triangle

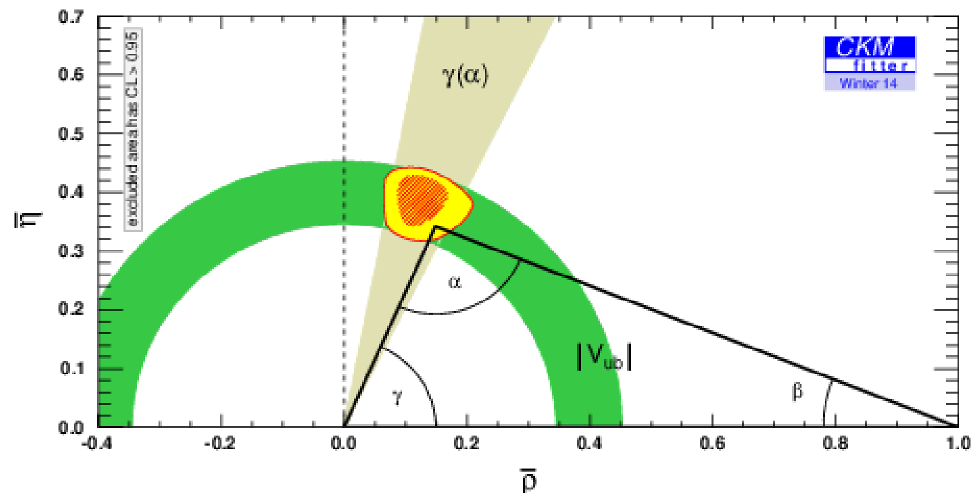
consistency check of Standard Model !

Unitarity Triangle analysis in the SM

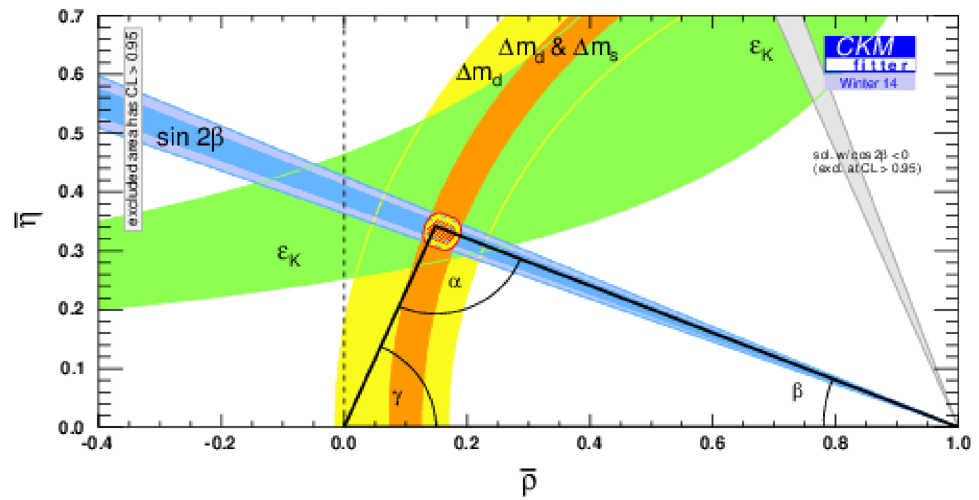


The Unitarity Triangle : Tree vs Loop

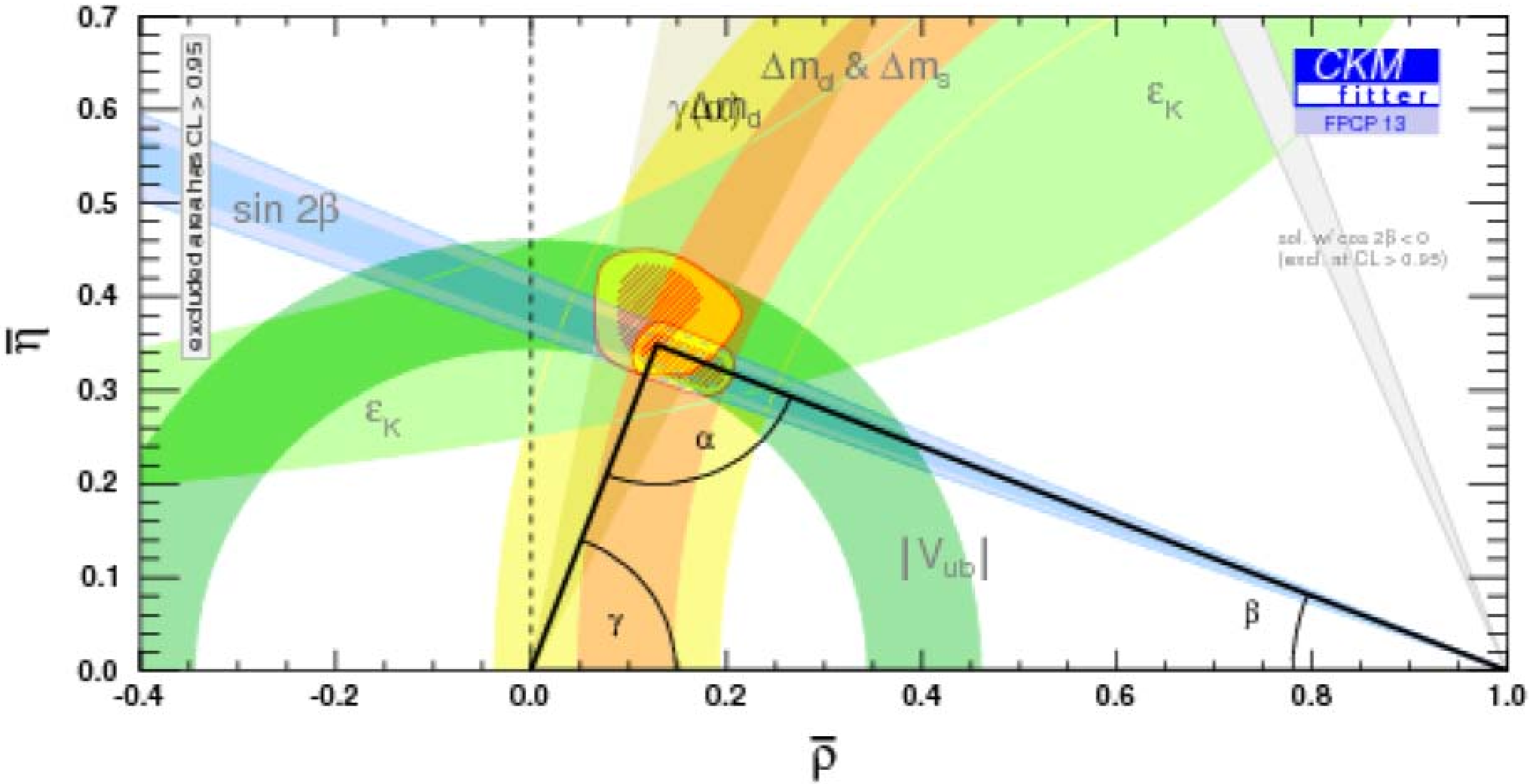
Unitarity triangle from "tree observables"



Unitarity triangle from "loop observables"

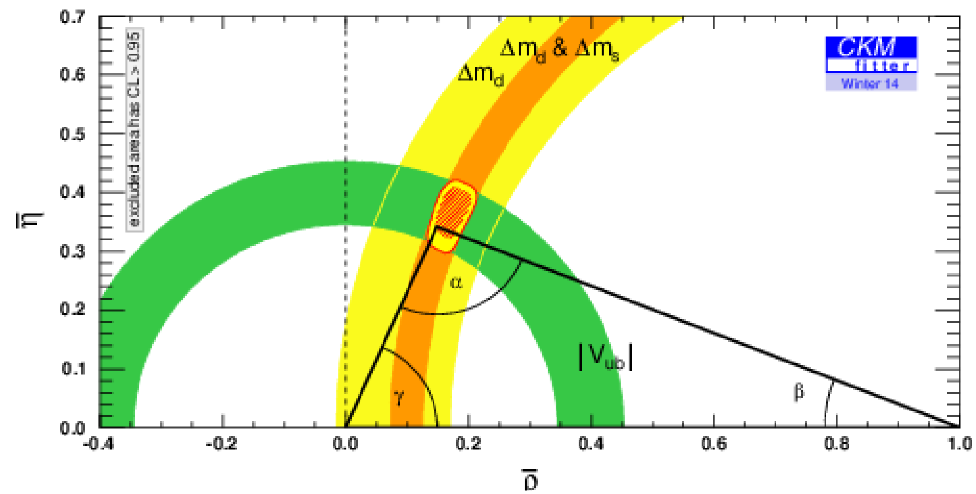


The Unitarity Triangle : Tree vs Loop

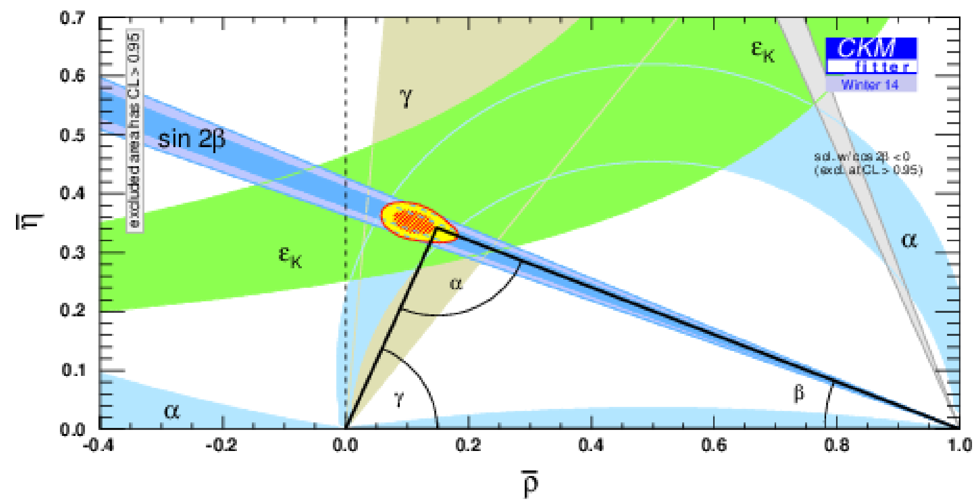


The Unitarity Triangle : CP vs ~~CP~~

Unitarity triangle from "CP conserving observables"

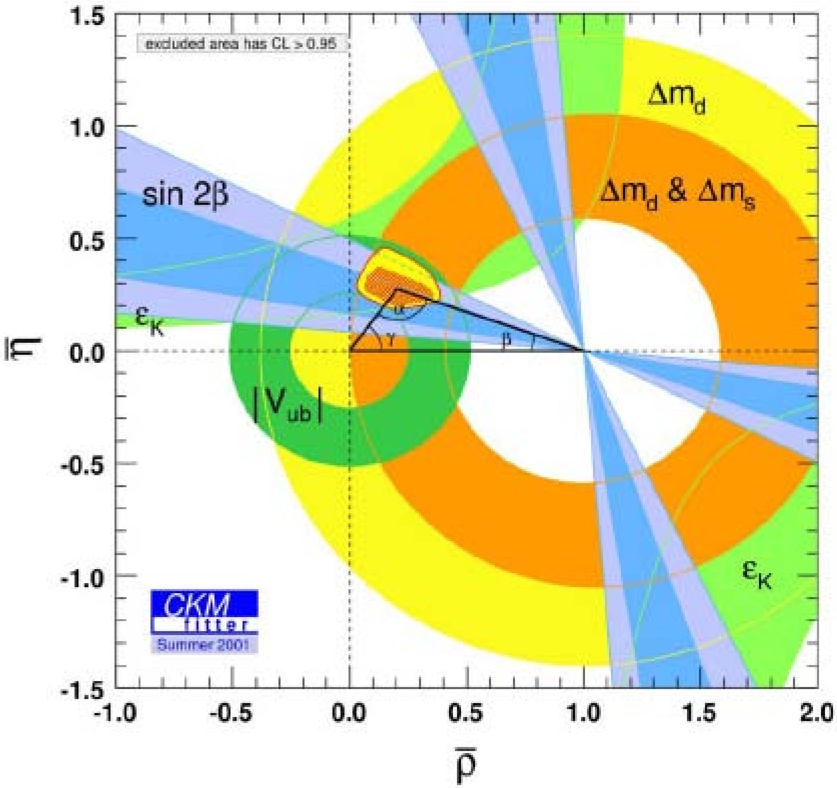


Unitarity triangle from "CP violating observables"

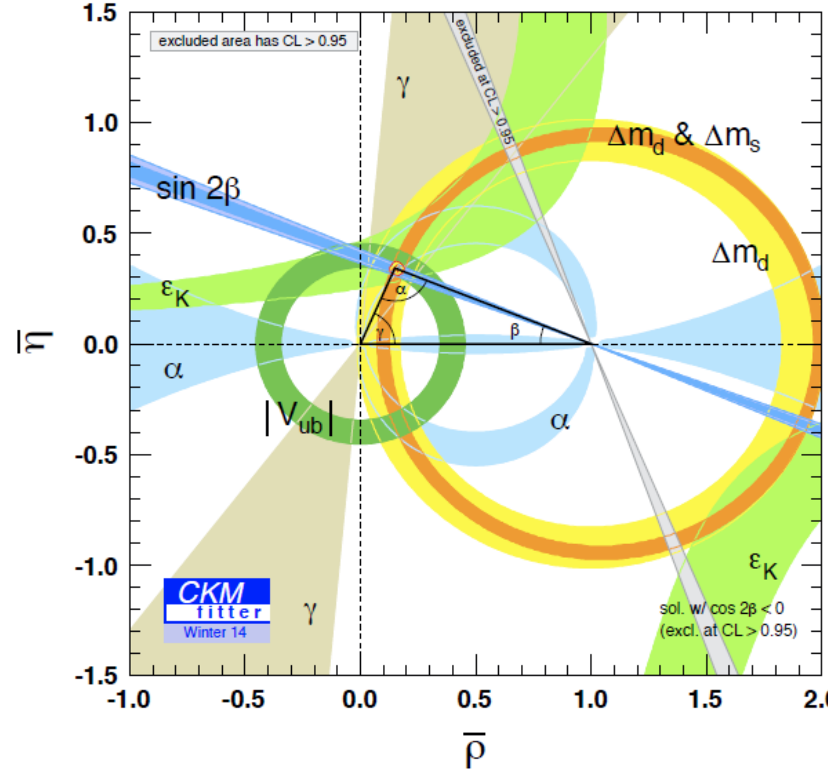


The Unitarity Triangle: 2001 vs 2014

Summer 2001



Winter 2014



$$\alpha + \beta + \gamma = (175.2 \pm 9.3)^o$$

	Direct	CKM fit
$\alpha =$	$\left(88.8^{+4.5}_{-4.3}\right)^o$	$\left(93.6^{+3.2}_{-2.9}\right)^o$
$\beta =$	$\left(21.5^{+0.8}_{-0.7}\right)^o$	$\left(25.38^{+0.80}_{-1.57}\right)^o$
$\gamma =$	$\left(70^{+7.7}_{-9.0}\right)^o$	$\left(66.4^{+1.2}_{-3.3}\right)^o$
$-2\beta_s =$	$+0.00 \pm 0.07$	$-0.0363^{+0.0014}_{-0.0012}$

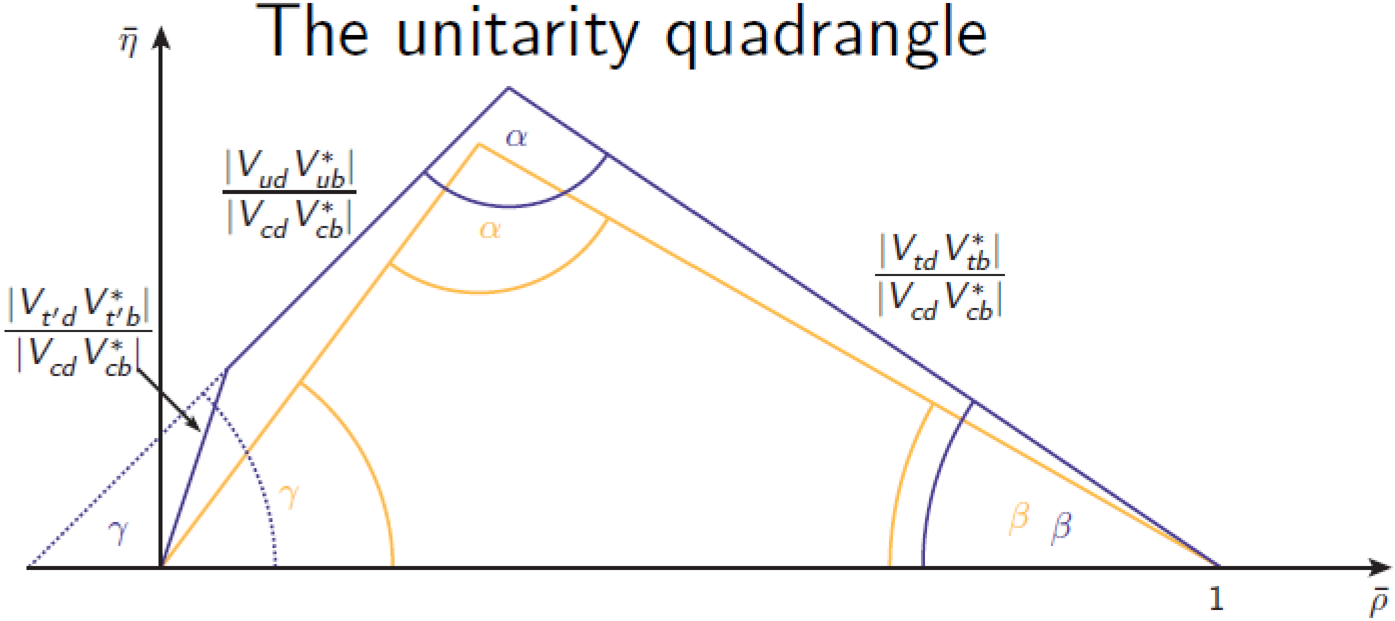
well with the CKM picture at O(10%) level

4th generation and CKM matrix

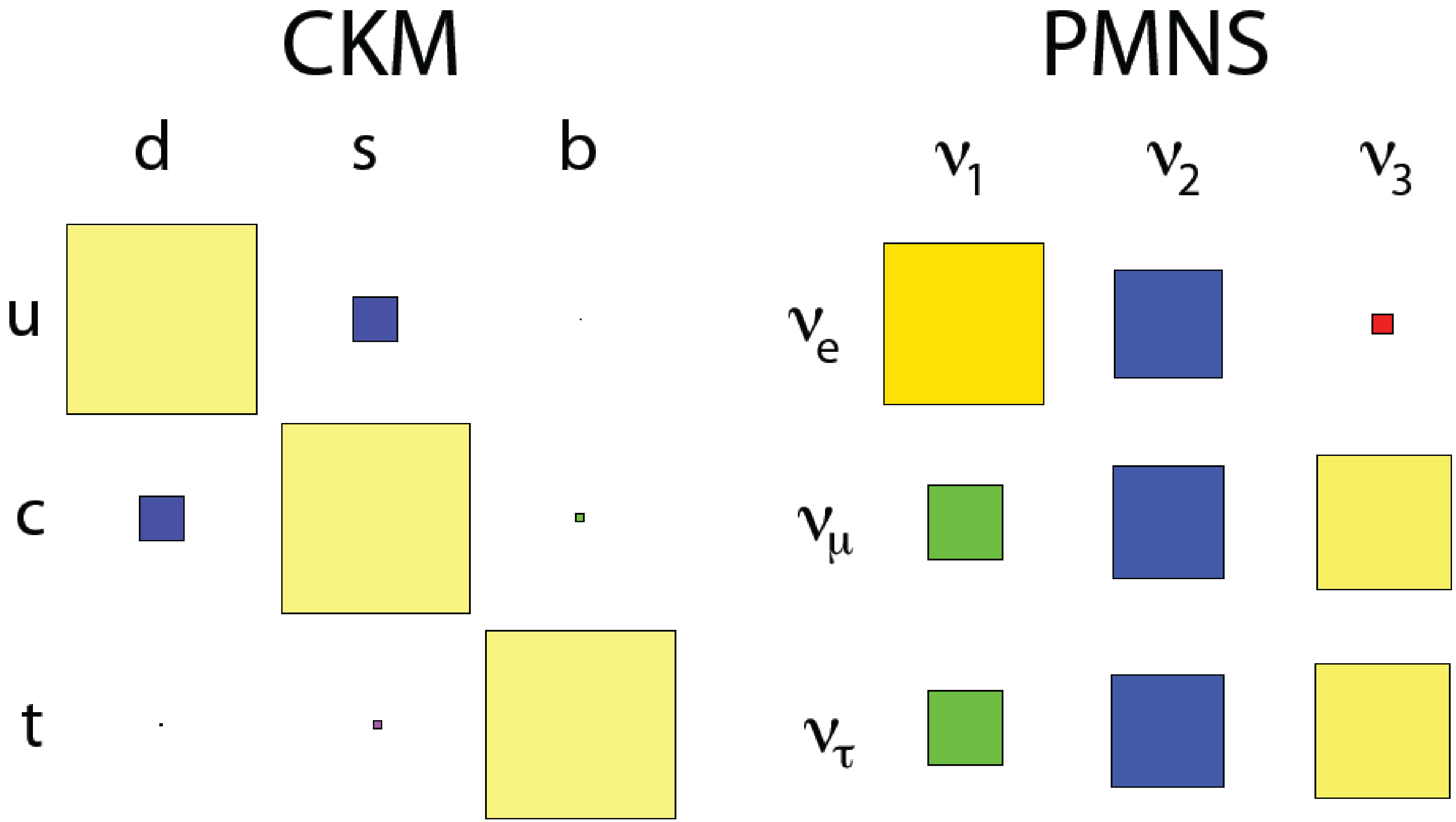
The 4 × 4 CKM matrix

$$
V^{\mathrm{CKM4}} = \begin{pmatrix}
c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\
\begin{array}{l} -c_{23}c_{24}s_{12} \\ -c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} &
\begin{array}{l} c_{12}c_{23}c_{24} \\ -c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} &
\begin{array}{l} c_{13}c_{24}s_{23} \\ -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} \end{array} &
c_{14}s_{24}e^{-i\delta_{24}} \\
\begin{array}{l} -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} \\ +c_{34}s_{12}s_{23} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{l} -c_{12}c_{34}s_{23} \\ -c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} \\ -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} \\ +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{l} c_{13}c_{23}c_{34} \\ -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} \end{array} &
c_{14}c_{24}s_{34} \\
\begin{array}{l} -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} \\ -s_{12}s_{23}s_{34} \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{l} -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} \\ +c_{12}s_{23}s_{34} \\ -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{l} -c_{13}c_{23}s_{34} \\ -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} \\ -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} \end{array} &
c_{14}c_{24}c_{34}
\end{pmatrix}
$$

The unitarity quadrangle



CKM vs PMNS: flavor problems



- Why are the CKM elements so hierarchical and diagonal?
- Why is the PMNS matrix so different from the CKM matrix?