Flavor Physics I

Chaehyun Yu

Aug 7 2017, IBS, Korea

Outline

- Flavor Physics and the Standard Model
  - a short history of flavor physics
  - the Standard Model
- Discrete Symmetry and CKM matrix
- Renormalization and muon g-2
- RG and Effective Field Theory
- CP Violation and BSM Flavor Physics

References

- Cheng and Li, Gauge theory of elementary particle physics (1982)
- Ho Kim, Elementary Particles and Their Interactions (2004)
- Peskin and Schroeder, Introduction to Quantum Field Theory (1995)
- Thomas Mannel, Effective Field Theories in Flavour Physics (2004)
- J. Romao, Modern Techniques for One-Loop Calculations (2006)
- A. Buras, Weak Hamiltonican, CP Violation and Rare Decays, hep-ph/9806471
- M. Knecht, The anomalous magnetic moment of the muon: a theoretical introduction, hep-ph/0307239
- K.S. Babu, TASI Lectures on Flavor Physics, arXiv:0910.2948
- S.J. Lee and H. Serodio, A Short Guide to Flavour Physics and CP Violation, arXiv:1504.07549

Flavor originates from Gell-Mann

Murray Gell-Mann

The term *flavor* was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his student at the time, Harald Fritzsch, at a Baskin-Robbins ice-cream store in Pasadena. Just as ice-cream has both color and flavor so do quarks (Fritzsch, 2008).

FPCP 2017

June 5 – 9, Prague
Czech Republic

15th Conference on Flavor Physics and CP Violation

Program at a glance

Monday, June 5, 2017
8:00 Registration start
9:00 Opening
9:30 Charm Physics
11:00 Rare K
12:00 Lattice QCD
14:30 Semileptonic & Leptonic B Decays
17:45 Poster Session
19:00 Closing

Tuesday, June 6, 2017
9:00 HF Production / Spectroscopy
14:00 CPV
18:00 Closing

Wednesday, June 7, 2017
9:00 Neutrino
14:00 Sightseeing Tour of Prague

Thursday, June 8, 2017
9:00 Tau / g-2 / Dark / Muons
12:00 CPV
14:00 Top, Higgs, Energy Frontier BSM Searches
18:00 Closing
19:30 Conference diner

Friday, June 9, 2017
9:00 Neutrino
10:00 Future Experiments
14:30 Summary / Outlook
16:10 Closing

Why is flavor physics so important?

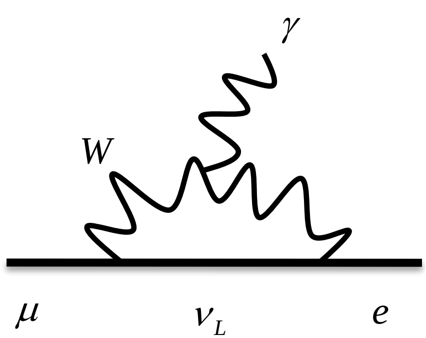
- Unique source for CP violation in the SM

  - but, not enough for explaining the baryon asymmetry of the Universe

- Flavor-changing neutral current (FCNC) is forbidden at the tree level in the SM (GIM mechanism)

- Flavor physics is sensitive to new physics. For example, consider $\mu \to e\gamma$

  The lepton flavor violation generated by massive neutrinos



=0 by GIM $\sum_i U_{ei}U^*_{\mu i} = 0$

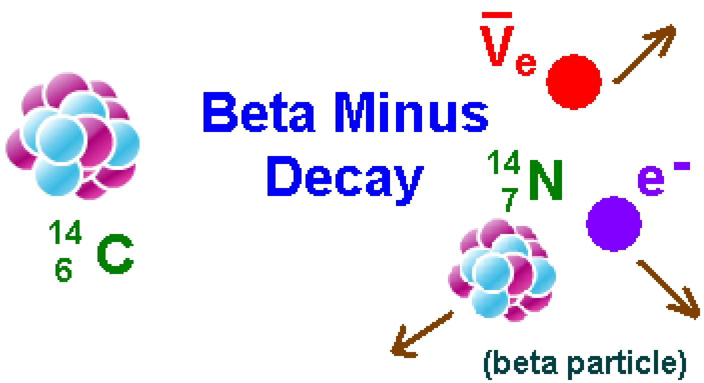
$$\mathrm{Br}(\mu \to e\gamma) \approx \frac{3\alpha}{32\pi}\left|\sum_i U_{ei}U^*_{\mu i}\left(\cancel{-\frac{10}{3}} + \frac{m_i^2}{M_W^2}\right)\right|^2$$

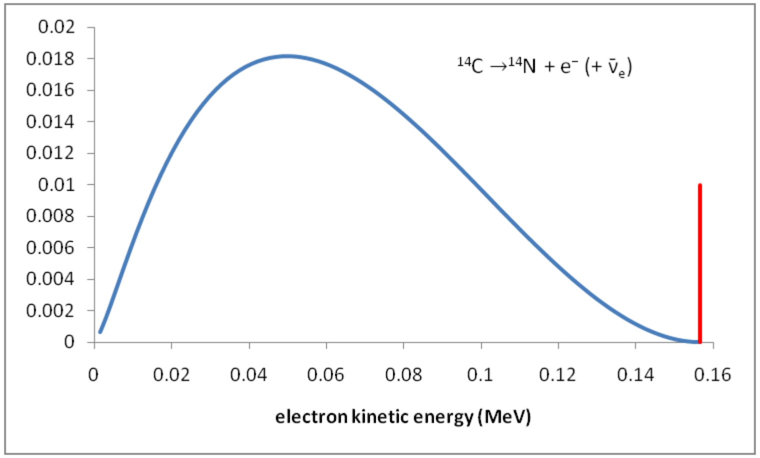
$$= \frac{3\alpha}{32\pi}\left|\sum_{i=1,2} U_{ei}U^*_{\mu i}\frac{\Delta m_{i3}^2}{M_W^2}\right|^2 \sim 10^{-54}$$

Beginning of flavor physics

- The beginning of flavor physics: the discovery of nuclear β decay by Becquerel and Rutherford in the late 19th century

- a mystery: the β ray has a continuous energy spectrum (Chadwick,1914)





- the violation of energy conservation? (by Ellis and Wooster in 1927)

$$^{210}_{83}\,\mathrm{Bi} \longrightarrow {}^{210}_{84}\,\mathrm{Po} \qquad \langle E_\beta \rangle = 350\ \mathrm{keV} \ll E_\beta^{\max} = 1050\ \mathrm{keV}$$

Pauli's postulation

- Pauli postulated a particle that escaped observation to save energy conservation ~ neutrino, the interaction of which had to be so weak that it did not leave any trace in experiments at that time

Dear Radioactive Ladies and Gentlemen,

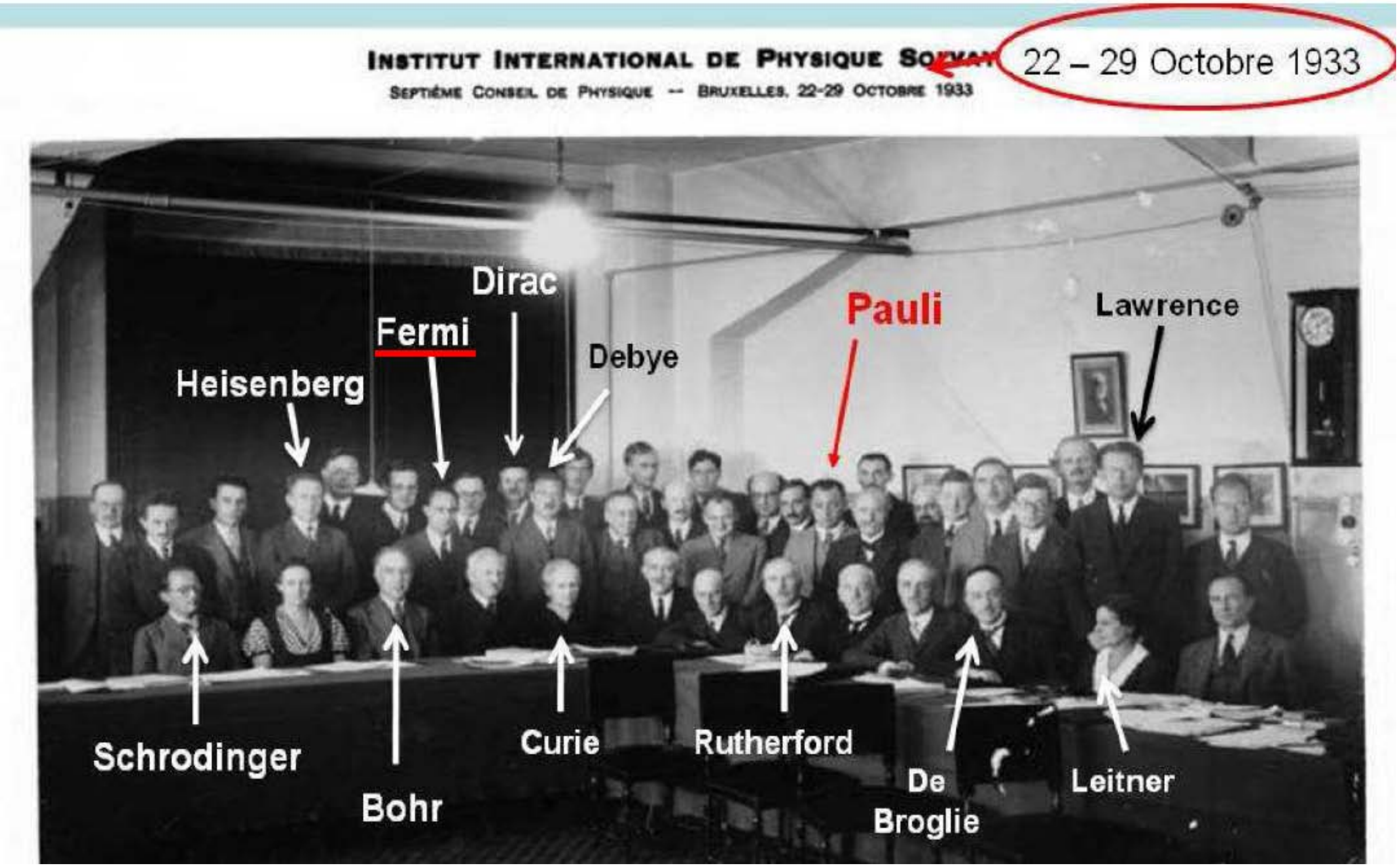
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, because of the "wrong" statistics of the N- and Li-6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" (1) of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin 1/2 and obey the exclusion principle and that further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton mass. - The continuous beta spectrum would then make sense with the assumption that in beta decay, in addition to the electron, a neutron is emitted such that the sum of the energies of neutron and electron is constant.

This proposal is contained in an 'open letter to the group of radioactives at the regional meeting in Tübingen' dated Zürich, 4 December 1930, and written from his office at Gloriastrasse [11]. In this letter, Pauli asks the addressees to examine the question of the experimental detection of such a neutron and gives more details about its probable characteristics. But he also says that he does not yet dare to publish the idea. Indeed, only the two old particles, proton and electron, were accepted at that time. The letter closes by excusing himself not to be able to attend the meeting because of a dance he had to attend in Zürich.

Fermi theory

- Pauli postulated a particle that escaped observation to save energy conservation ~ neutrino, the interaction of which had to be so weak that it did not leave any trace in experiments at that time

Pauli gave a talk on his neutrino proposal in this congress.

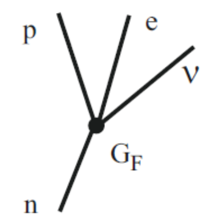


Fermi theory

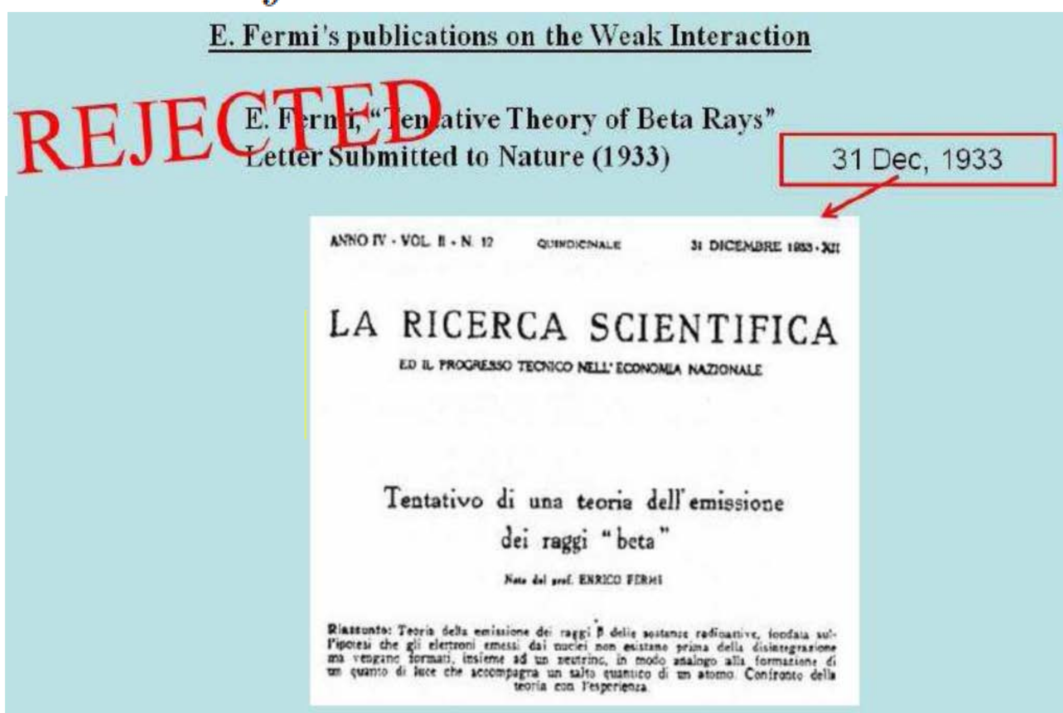
- Pauli postulated a particle that escaped observation to save energy conservation ~ neutrino, the interaction of which had to be so weak that it did not leave any trace in experiments at that time

- Fermi theory for the β decay (1933)

$$n \rightarrow p + e^- + \overline{\nu}_e$$



$$H_{\text{int}} = G \int d^3x \, [\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma_\mu \nu(x)]$$



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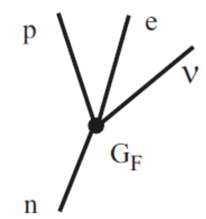


$$H_{\text{int}} = G \int d^3x \, [\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma_\mu \nu(x)]$$

- With more precise data Gamov suggested the generalization of Fermi theory (1936)

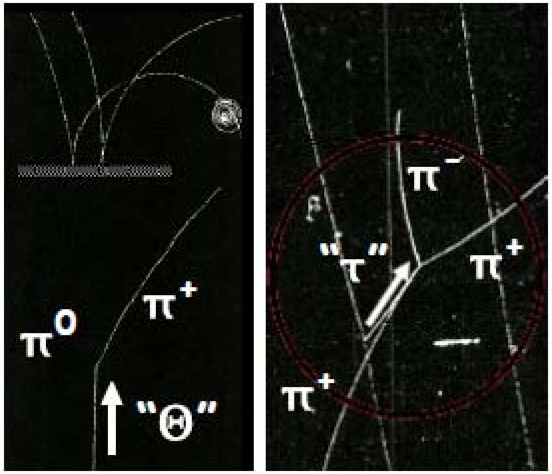
$$H_{\text{int}} = \sum_j \int d^3x \, g_j [\bar{p}(x) M_j n(x)][\bar{e}(x) M_j \nu(x)]$$

$$M_j \otimes M_j = 1 \otimes 1 \, , \, \gamma_5 \otimes \gamma_5 \, , \, \gamma_\mu \otimes \gamma^\mu \, , \, \gamma_\mu\gamma_5 \otimes \gamma^\mu\gamma_5 \, , \, \sigma_{\mu\nu} \otimes \sigma^{\mu\nu}$$

  – Parity and Charge conjugation hold separately

$\Theta-\tau$ puzzle

- Θ and τ decay to final states with different parities, assuming an s-wave decay, but it turned out that they have the same mass and lifetime
  - spin of Θ and τ is known to be s=0
  - spin parity of π: $J^{P}(\pi^{\pm}) = 0^{-}$ $J^{PC}(\pi^{0}) = 0^{-+}$



$$\Theta \rightarrow \pi^{+}\pi^{0}$$

- the orbital angular momentum of $\pi\pi$, $l = 0$

$$P(\pi^{+}\pi^{0}) = (-1)^{2}(-1)^{l} = +1$$

$$\tau \rightarrow \pi^{+}\pi^{+}\pi^{-}$$

- $\boldsymbol{l} + \boldsymbol{L} = \boldsymbol{J} = 0$, which means $l = L$

$$P(\pi^{+}\pi^{+}\pi^{-}) = (-1)^{3}(-1)^{l+L} = -1$$

$$Y_l^m = P_l^m(\cos\theta)e^{im\phi}$$

$$PY_l^m = (-1)^{l+m} \times (-1)^m \times Y_l^m = (-1)^l \times Y_l^m$$

l : the orbital angular momentum of $\pi\pi$

L : the orbital angular momentum of π and $(\pi\pi)$

Parity violation

TD Lee

CN Yang

- C.N. Yang and T.D. Lee (1956) suggested that Θ and τ are identical (K^+) and weak interactions violate parity

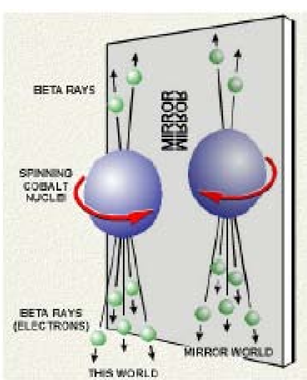
$$H_{int} = -\frac{G_\beta}{\sqrt{2}} \int d^3x \left[\bar{p}(x)\gamma_\mu \left(1 - \frac{g_A}{g_V}\gamma_5\right) n(x) \right] [\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu(x)]$$

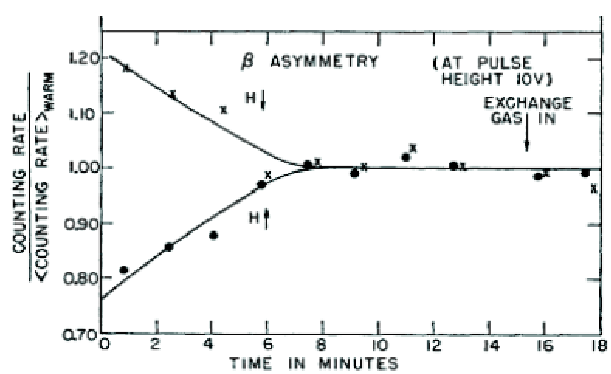
best fit implies $G_\beta = (1.14730 \pm 0.00064) \times 10^{-5}\ \mathrm{GeV}^{-2}$

$$\frac{g_A}{g_V} = 1.255 \pm 0.006 \neq 1$$

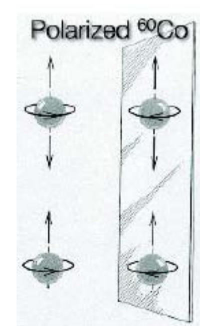
Neither proton nor neutron is an elementary particle

- Soon after the idea, parity violation was experimentally verified by Wu et al. (1956) and Garwin et al. (1957)





CS Wu (1956)



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$$\frac{g_A}{g_V} = 1.255 \pm 0.006 \neq 1$$

Neither proton nor neutron is an elementary particle

- After implementing parity violation, pion, muon, and neutron decays are basically descried by (V-A) current × (V-A) current with the same coupling and universality in the weak interactions is recovered

Strangeness

- In the 1950's, "strangely behaved" particles discovered, starting with the kaons and other strange particles
  - large production cross sections which is typical for strong interaction
  - long lifetimes of order 10^{-10}s which is typical for weak decays
  - always produced in pairs

- Gell-Mann and Nishijima (1954) suggested new quantum number S (strangeness), which is conserved in the production (strong interaction) and not conserved in the decay (weak interaction)

- Universality of weak interaction implies that all couplings in weak interactions must be identical, but

Strangeness-conserving		Strangeness-changing
$\pi \longrightarrow \mu\bar{\nu}$		$K^+ \longrightarrow \pi^+\pi^0$
$\Gamma(\pi \to \mu\bar{\nu})$	$\gg$	$\Gamma(K^+ \to \pi^+\pi^0)$

$$\frac{\Gamma(\pi \to \mu\bar{\nu})}{\Gamma(K^+ \to \pi^+\pi^0)} \sim 20$$

contradicts the concept of universality of weak interactions

Cabibbo angle

- Cabibbo(1964): The total hadronic V-A current should have “unit length”

$$H_\mu = \underbrace{H_\mu^{\Delta S=0}}_{\text{strangeness-conserving}} \cos\Theta + \underbrace{H_\mu^{\Delta S=1}}_{\text{strangeness-changing}} \sin\Theta$$

$$\Rightarrow\quad H_\mu = \bar{u}\gamma_\mu(1-\gamma_5)\left[d\cos\Theta + s\sin\Theta\right]$$

Cabibbo angle
sinΘ≈0.22

$$\frac{\cos^2\Theta}{\sin^2\Theta} \sim 20$$

- Weak interactions becomes universal again with the rotation
- a neutral current with the Cabibbo angle

$$H_{\text{neutral}} = \bar{u}Mu + \left[\bar{d}\cos\Theta + \bar{s}\sin\Theta\right] M' \left[d\cos\Theta + s\sin\Theta\right]$$

- a $\Delta S=\pm1$ case

$$H_{\text{neutral}}^{\Delta S=\pm1} = \cos\Theta\sin\Theta\left[\bar{d}M's + \bar{s}M'd\right]$$

$$\Rightarrow\quad \frac{\Gamma(K^+\to\pi^+\nu\bar{\nu})}{\Gamma(K^+\to\pi^0 e^+\bar{\nu})} \sim \frac{\sin^2\Theta\cos^2\Theta}{\sin^2\Theta} \sim O(1)$$

GIM mechanism

- In experiments it is found that the interaction in the flavor-changing neutral current (FCNC) is strongly suppressed

$$\frac{\Gamma(K^+ \to \pi^+ \nu \bar{\nu})}{\Gamma(K^+ \to \pi^0 e^+ \bar{\nu})} < 10^{-5} \ll 1$$

- this problem was resolved by the GIM (Glashow, Ilipoulos, Maiani, 1970)

  – introduce a new quark (charm) which couples to the "orhtogonal" combination $s \cos \Theta - d \sin \Theta$

$$H_\mu = \bar{u}\gamma_\mu(1-\gamma_5)\,[d\cos\Theta + s\sin\Theta] + \underline{\bar{c}\gamma_\mu(1-\gamma_5)\,[s\cos\Theta - d\sin\Theta]}$$

- The tree-level FCNC is forbidden

$$\begin{aligned} H_{\text{neutral}} &= \bar{u}Mu + \bar{c}Mc \\ &+[\bar{d}\cos\Theta + \bar{s}\sin\Theta]M'[d\cos\Theta + s\sin\Theta] \\ &+[\bar{s}\cos\Theta - \bar{d}\sin\Theta]M'[s\cos\Theta - d\sin\Theta] \\ &= \bar{u}Mu + \bar{c}Mc + \bar{d}M'd + \bar{s}M's \end{aligned}$$

same as the rotation of down-type quark by an orthognal matrix

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

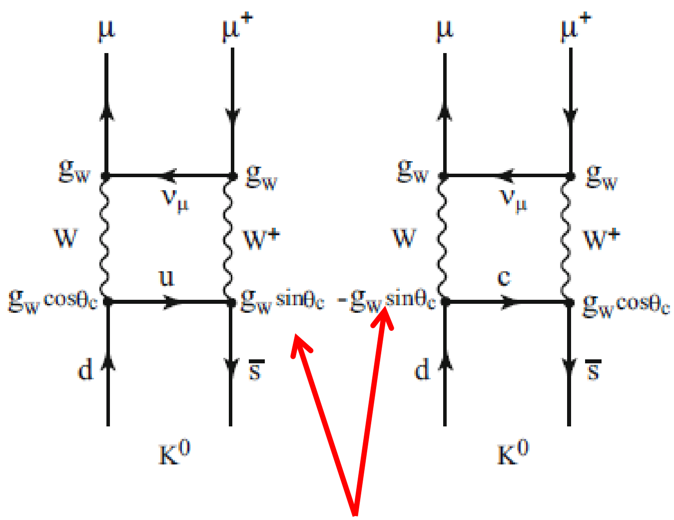
FCNC

- FCNC is non-zero

CC	BR	NC	BR
$K^+ \to \mu^+ \nu_\mu$	$63.54 \pm 0.14\%$	$K^0 \to \mu^- \mu^+$	$6.84 \pm 0.11 \times 10^{-9}$
$K^+ \to \pi^0 e^+ \nu_e$	$5.08 \pm 0.05\%$	$K^+ \to \pi^+ \overline{\nu}_e \nu_e$	$1.5^{+1.3}_{-0.9} \times 10^{-10}$

- Why is FCNC so small?

$K_L^0 \to \mu^+ \mu^-$



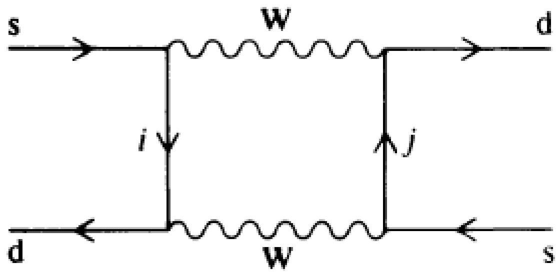
opposite signs

exact cancellation for degenerate quark masses

"GIM suppression"

November revolution

- The mass difference of K_L and K_S predicts the mass of the charm quark (Gaillard and Lee, 1974)



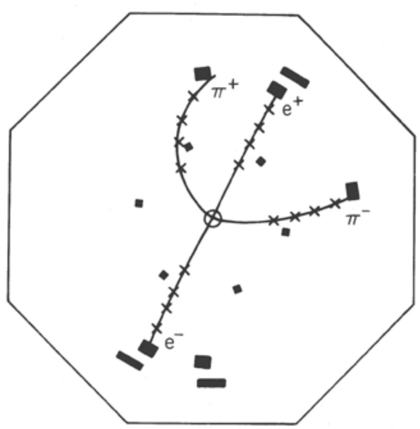
$$\frac{m_L - m_S}{m_K} \simeq \frac{G_F}{\sqrt{2}} f_K{}^2 \frac{\alpha}{4\pi} \epsilon_0 \sin^2\theta_C \cos^2\theta_C$$

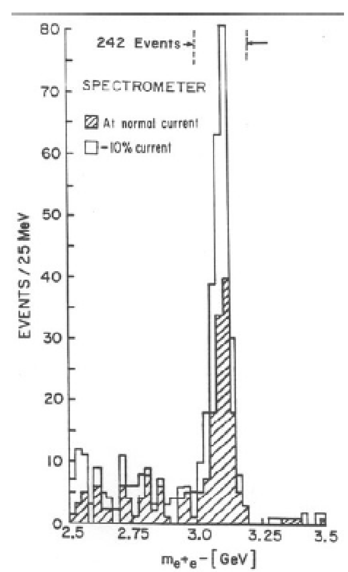
$$\simeq \epsilon_0 \times 5 \times 10^{-12} \ .$$

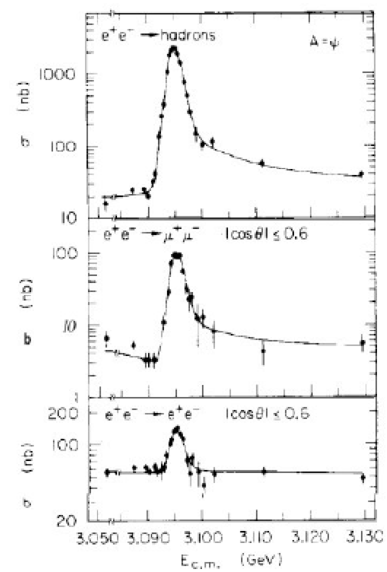
$$m_{\varphi'} \simeq 1.5 \text{ GeV}$$

- in November 1974 the discovery of J/ψ

$$\psi(2S) \to J/\psi\pi^+\pi^-$$







CP violation

- After the discovery of parity violation, it was believed that CP still seems to be a good discrete symmetry of weak interactions, but …

- Cronin and Fitch discovered CP-violating decays of neutral kaons (1964)

$$K^0 = d\bar{s}\,(S=+1) \qquad \overline{K^0} = s\bar{d}\,(S=-1)$$

CP eigenstates $K_1 = \frac{K^0 + \overline{K^0}}{\sqrt{2}}(CP=+) \sim K_S \qquad K_2 = \frac{K^0 - \overline{K^0}}{\sqrt{2}}(CP=-1) \sim K_L$

$$\Rightarrow \pi\pi(CP=+) \qquad \Rightarrow \pi\pi\pi(CP=-)$$

The details of the experiment have been described many times[7,6] and I will not repeat them here. Suffice it to say that, previous to our work, the best limit on the decay of K long to 2 π was from a Soviet group[8] who analyzed 597 decays and found no candidates fitting the 2 π decay. We found 48 events in 24,000 decays, about 1 in 500. What about the original Adair results? If they had been correct we would have seen about 15 times as many 2π events in our hydrogen target data than we did, in fact, observe. The Adair experiment was repeated using a bubble chamber with a .07 cm wall instead of the original 1.6 cm. It was shown[9] that the original effect came, to a large extent, from the walls (no empty target data ??).

Fitch(2008)

After the original CP violation paper

The original CP paper was published in July of 1964.[10] Almost immediately, within a couple of weeks, theoretical papers began to appear. They were in two categories: those that accepted the results as demonstrating CP (and T) violation and those that attempted to explain the phenomena in terms of a new long-range interaction, as Adair *et al* had done earlier.

Fitch(2008)

- Wu and Yang: a phenominological analysis on the basis of CP violation
- Truong: the effect might be completely in the $\Delta I=3/2$ decay amplitude
- Wolfenstein: propose a new $\Delta S=2$ superweak interaction (excluded by the discovery of direct CP violation called ϵ' in 1999)
- Bell, Perring, Bernstein, Cabibbo, Lee: proposed a new particle, a hyperphoton
- Sakharov showed that CP violation combined with baryon non-conservation and nonequilibrium dynamics could account for the matter-antimatter asymmetry in the Universe

Kobayashi-Maskawa mechanism

- CP violation can be generated by complex phase in quark-mixing if three quark doublets exist (Kobayashi, Maskawa, 1972)

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

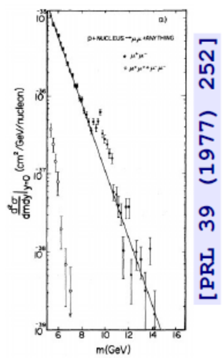
In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

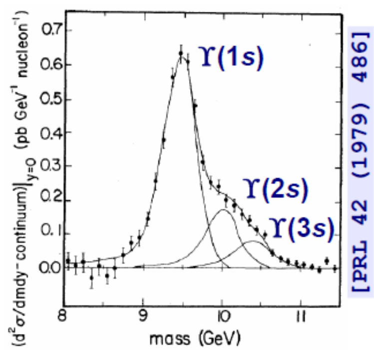
The discovery of third generation

- The tau lepton was discovered at the SLAC by Perl et al. (1975)

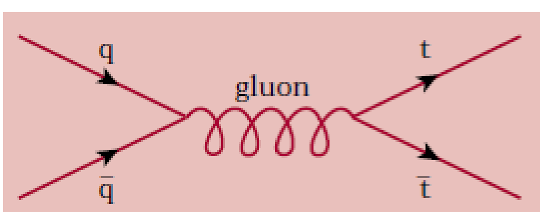
$$e^+ + e^- \rightarrow e^\pm + \mu^\mp + \geqslant 2 \text{ undetected particles}$$

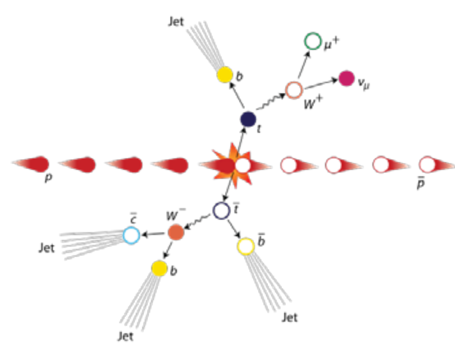
- The discovery of the b quark at the Fermilab (1977): p+Cu→ $\mu^+\mu^-$+X

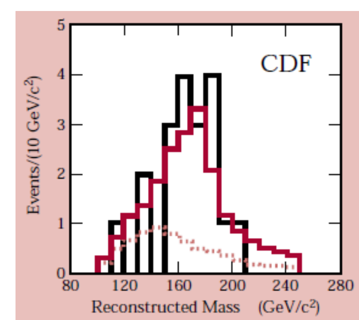




- The top quark was discovered at the Tevatron (1990's)



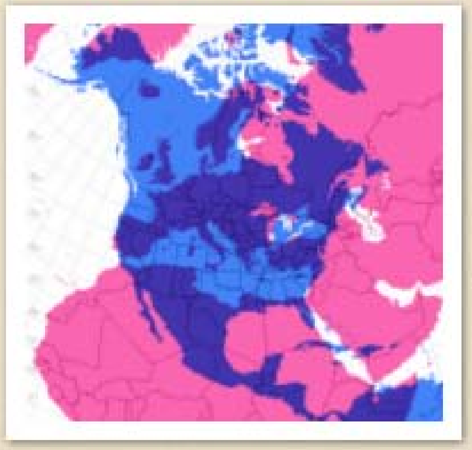




- The tau neutrino discovery at the DONUT in the Fermilab (2000)

Pauli's Other Exclusion Principle

As I am currently stretched between continents, I ponder over the differences between the US and Europe. Apart from the taste of food and the size of humans, there seems to be a fundamental difference at the level of particle physics. Let's have a closer look at the time and place of discoveries of elementary particles:



- Tau neutrino, 2000, Fermilab, United States
- Top quark, 1995, Fermilab, United States
- W and Z bosons, 1983, CERN, Switzerland
- Gluon, 1979, DESY, Germany
- Bottom quark, 1977, Fermilab, United States
- Tau, 1975, SLAC, United States
- Charm quark, 1974, SLAC/Brookhaven, United States
- Up, down, and strange quarks, 1968, SLAC, United States
- Muon neutrino, 1962, Brookhaven, United States
- Electron neutrino, 1956, Los Alamos, United States
- Muon, 1936, Caltech, United States
- Photon, 1905, Patent Office in Bern, Switzerland
- Electron...let's skip that one for simplicity...

+ Higgs boson, 2012, CERN, Switzerland

This can be summarized as Pauli's other exclusion principle:

Fermions are discovered in the US, whereas bosons are discovered in Europe.

To the Standard Model

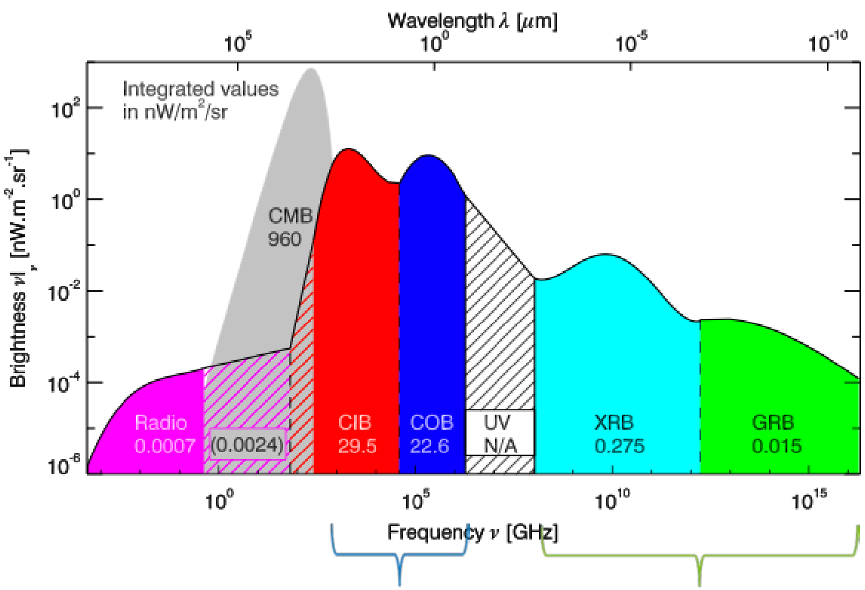
- The Standard Model must include
  - The QED and QCD preserve the flavor symmetry
  - The flavor symmetry can be broken by weak interactions
  - The SM should respect the GIM mechanism
  - At least three generations are required for CP violation

Matter-antimatter asymmetry

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma}$$

$$\begin{aligned} n_B &= \text{number density of baryons} \\ n_{\bar{B}} &= \text{number density of antibaryons} \\ n_\gamma &= \text{number density of photons} \\ &\equiv \frac{\zeta(3)}{\pi^2} g_* T^3, \qquad g_* = 2 \text{ spin polarizations} \end{aligned}$$

$$\boxed{\text{unit phase space} \sim \frac{d^3x d^3p}{(2\pi\hbar)^3}}$$



from galaxies

active galactic nuclii (quasar, etc)

$$\boxed{\begin{aligned} n_\gamma &= g_* \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{(E-\mu)/T} - 1} \\ &\simeq \frac{2 \times 4\pi}{(2\pi)^3} \int \frac{p^2 dp}{e^{p/T} - 1} \\ &= \frac{T^3}{\pi^2} \int_0^\infty \frac{x^2 dx}{e^x - 1} \\ &= \frac{2\zeta(3) T^3}{\pi^2} \end{aligned}}$$

- Most of photons in the Universe are CMB photons (~98%)

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} \mathrm{d}x$$

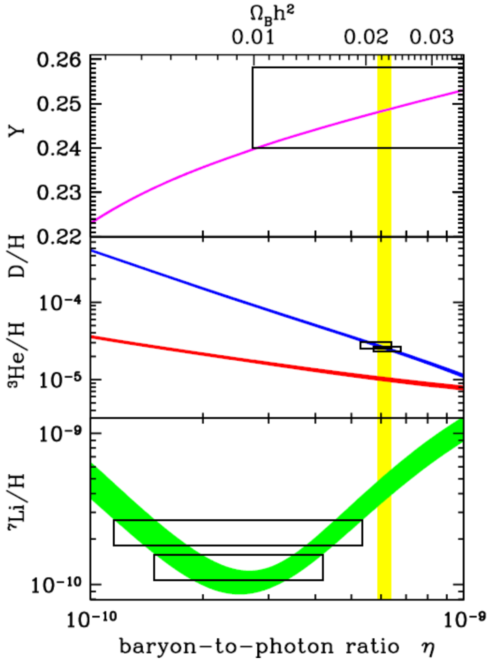
Baryogenesis

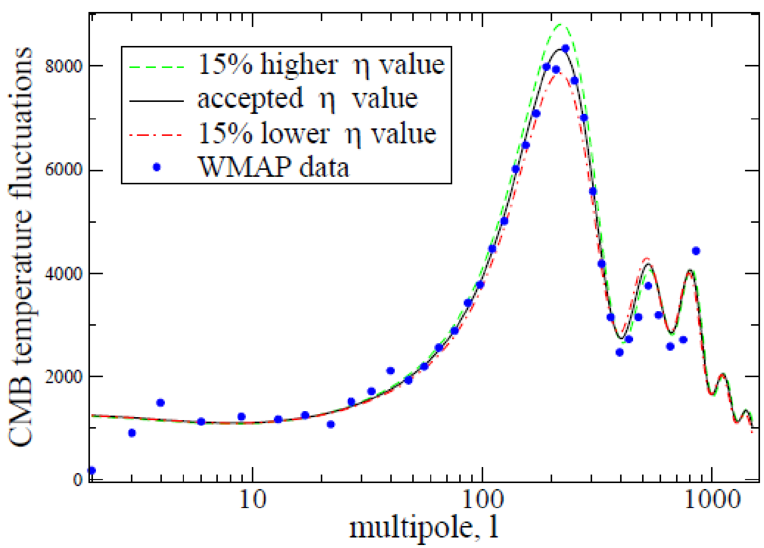
$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma}$$

$$\begin{aligned} n_B &= \text{number density of baryons} \\ n_{\bar{B}} &= \text{number density of antibaryons} \\ n_\gamma &= \text{number density of photons} \\ &\equiv \frac{\zeta(3)}{\pi^2} g_* T^3, \qquad g_* = 2 \text{ spin polarizations} \end{aligned}$$

$$\eta = 10^{-10} \times \begin{cases} 6.28 \pm 0.35 \\ 5.92 \pm 0.56 \end{cases}$$

$$\eta = (6.14 \pm 0.25) \times 10^{-10}$$





Sakharov conditions

- The three necessary Sakharov conditions to explain matter-antimatter asymmetry of the universe
  - B violation
  - C and CP violation
  - Out of thermal equilibrium
- The conservation of baryon number should be violated: obvious
- If C or CP conserved, we get $n_b = n_{\bar{b}}$

$$\begin{array}{ccc} X \to \bar{\nu}_R d & \overset{C}{\leftrightarrow} & \overline{X} \to \nu_R \bar{d} \\ P \updownarrow & & P \updownarrow \\ X \to \bar{\nu}_L d & \overset{C}{\leftrightarrow} & \overline{X} \to \nu_L \bar{d} \end{array}$$

- Under CPT transformation,

$$(CPT)H(CPT)^{-1} = H, \quad (CPT)B(CPT)^{-1} = -B.$$

In thermal equilibrium

the density operator

$$\rho = e^{-\beta \mathcal{H}}$$

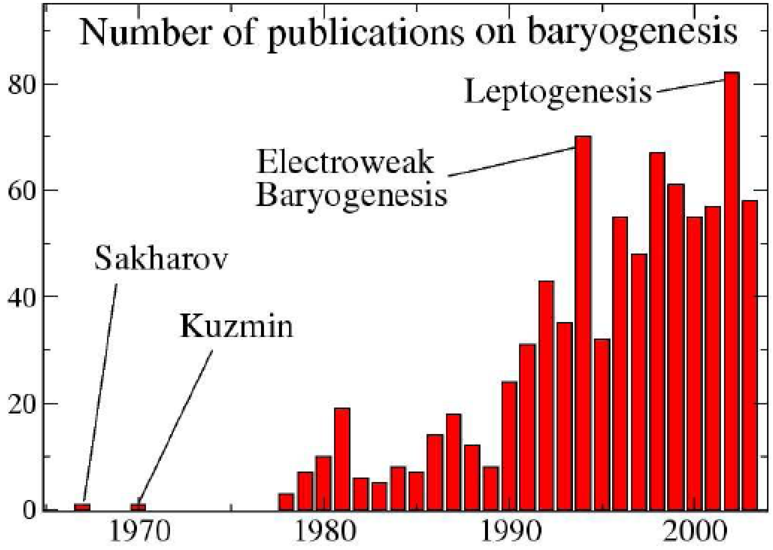
$$\beta = 1/T.$$

$$\langle B \rangle = \mathrm{Tr}\left[e^{-\beta H} B\right]$$

$$\begin{aligned} \langle B \rangle &= \mathrm{Tr}\left[(CPT)e^{-\beta H} B (CPT)^{-1}\right] \\ &= \mathrm{Tr}\left[e^{-\beta H}(CPT)B(CPT)^{-1}\right] = -\langle B \rangle. \end{aligned}$$

Sakharov conditions

- The three necessary Sakharov conditions to explain matter-antimatter asymmetry of the universe
  - B violation
  - C and CP violation
  - Out of thermal equilibrium
- In the early days of big bang cosmology, the baryon asymmetry was considered to be an initial condition, but any baryon asymmetry would be diluted to a negligible during inflation



Baryogenesis in the SM

- The SM satisfies all the three conditions

  – B: violated by the anomaly $\longrightarrow$ very small, but can be large at finite T (sphaleron)
  – C: maximally violated, CP: violated by a complex phase in the CKM matrix
  – Out of thermal equilibrium: electroweak phase transition

- But the CP violation in the SM is too small

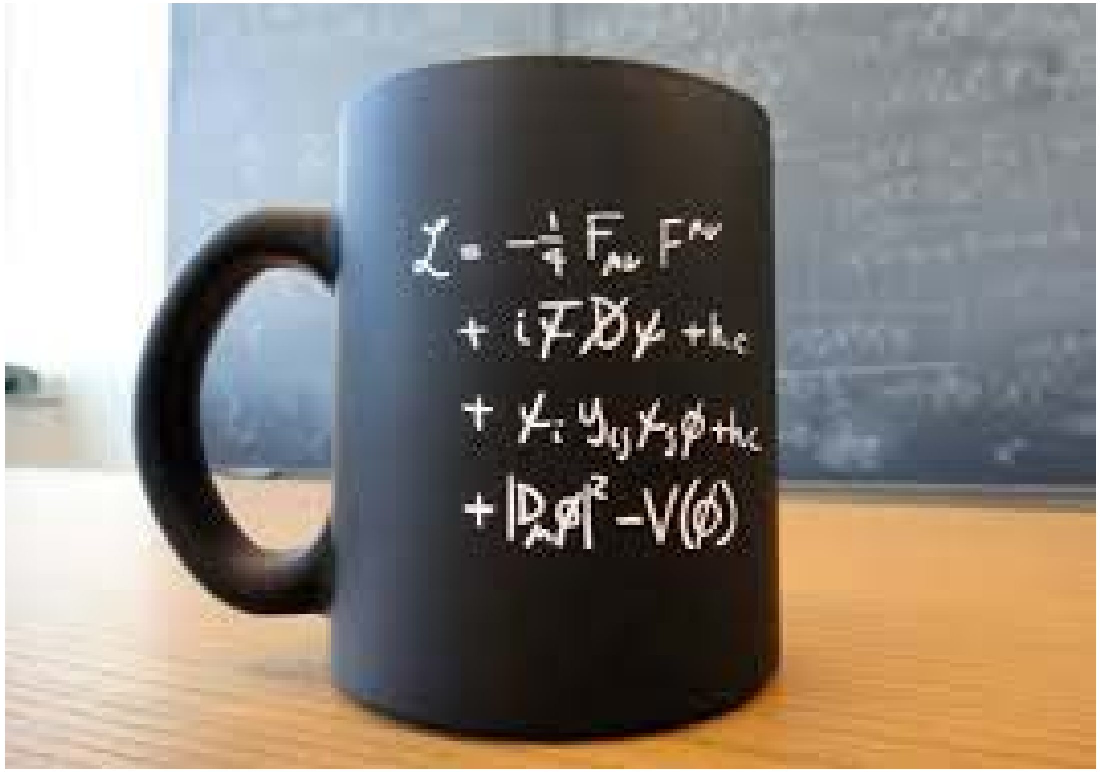
$$
\begin{aligned}
J &= (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\
&\quad (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\, K
\end{aligned}
\qquad
\begin{aligned}
K &= s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta \\
&= \mathrm{Im}\, V_{ii} V_{jj} V_{ij}^* V_{ji}^* \text{ for } i \neq j
\end{aligned}
$$

$$
J = \det\left[m_u^2,\, m_d^2\right] \qquad \frac{J}{(100\ \mathrm{GeV})^{12}} \sim 10^{-20}
$$

- The strong first order phase transition needed to explain the current baryon asymmetry required Higgs mass < 40 GeV

- We need new physics for baryogenesis

Standard Model



Abelian gauge theory

- Consider the Lagrangian for a free-electron field ψ(x)

$$\mathscr{L}_0 = \underbrace{\bar{\psi}(x)(\mathrm{i}\gamma^{\mu}\,\partial_{\mu} - m)\psi(x)}_{\text{Lorentz scalar}}$$

The Euler-Lagrange equation yields the Dirac equation

$$\partial_{\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\phi)}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0$$

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi(x) = 0$$

- Cleary it has a global U(1) symmetry (invariant under a phase change)

$$\psi(x) \to \psi'(x) = \mathrm{e}^{-\mathrm{i}\alpha}\psi(x) \qquad \bar{\psi}(x) \to \bar{\psi}'(x) = \mathrm{e}^{\mathrm{i}\alpha}\bar{\psi}(x)$$

- turn global symmetry into a local symmetry by replacing α by α(x)

$$\begin{aligned}\bar{\psi}(x)\,\partial_{\mu}\psi(x) \to \bar{\psi}'(x)\,\partial_{\mu}\psi'(x) &= \bar{\psi}(x)\,\mathrm{e}^{\mathrm{i}\alpha(x)}\,\partial_{\mu}(\mathrm{e}^{-\mathrm{i}\alpha(x)}\psi(x)) \\ &= \bar{\psi}(x)\,\partial_{\mu}\psi(x)\boxed{-\,\mathrm{i}\bar{\psi}(x)\,\partial_{\mu}\alpha(x)\psi(x)}\end{aligned}$$

The gauge invariance is spoiled

Abelian gauge theory

- A gauge-covariant derivative D_μ with a gauge field $A_\mu(x)$

$$D_\mu \psi = (\partial_\mu + ieA_\mu)\psi$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$$

$$D_\mu \psi(x) \to [D_\mu \psi(x)]' = e^{-i\alpha(x)} D_\mu \psi(x)$$

e is a free parameter

- A gauge-invariant kinetic term of $A_\mu(x)$

$$\mathscr{L}_A = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

The electromagnetic field tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The antisymmetric tensor is related to the covariant derivative

$$\underline{(D_\mu D_\nu - D_\nu D_\mu)}\psi = ieF_{\mu\nu}\psi$$

not derivative

- The gauge-invariant QED Lagrangian

$$\mathscr{L} = \bar{\psi} i\gamma^\mu(\partial_\mu + ieA_\mu)\psi - m\bar{\psi}\psi - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

  – The photon is massless $\cancel{A_\mu A^\mu}$

  – Renormlizability is imposed $\cancel{\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}}$

Lagrangian (density)

- Lorentz invariance (special relativity)

$$\phi, \phi^2, \phi^3, \phi^4, \phi^5 \qquad \partial_\mu \phi \partial^\mu \phi$$

$$\overline{\psi_L} \psi_R + h.c. \qquad \phi \bar{\psi} \psi \qquad \bar{\psi} \gamma^\mu \partial_\mu \psi$$

$$A_\mu A^\mu \qquad F_{\mu\nu} F^{\mu\nu} \qquad \phi F_{\mu\nu} F^{\mu\nu}$$

$$A_\mu \bar{\Psi} \gamma^\mu \Psi \qquad F_{\mu\nu} \bar{\psi} [\gamma^\mu, \gamma^\nu] \psi$$

- renormalizability: restricts Lorentz invariant form up to dimension four
- must be anomaly-free
- invariant under local gauge symmetry (remove redundancy)

  SM: $SU(3)_C \times SU(2)_L \times U(1)_Y$

- Gauge symmetry may be broken spontaneously
- Accidental global symmetry may remain after symmetry breaking

Non-Abelian gauge theory (SU(2))

- The fermion field is an isospin doublet (simplest case)

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad \psi(x) \to \psi'(x) = \exp\left\{\frac{-\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{\theta}}{2}\right\}\psi(x)$$

$\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ The Pauli matrices $\left[\frac{\tau_i}{2}, \frac{\tau_j}{2}\right] = \mathrm{i}\varepsilon_{ijk}\frac{\tau_k}{2} \qquad i, j, k = 1, 2, 3$

$\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ The SU(2) transformation parameters

- The free Lagrangian is invariant under the global SU(2) (θ: space-time indep.)

$$\mathscr{L}_0 = \bar{\psi}(x)(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi(x)$$

- Under the local symmetry transformation

$$\bar{\psi}(x)\, \partial_\mu \psi(x) \to \bar{\psi}'(x)\, \partial_\mu \psi'(x) = \bar{\psi}(x)\, \partial_\mu \psi(x) \boxed{+ \bar{\psi}(x) U^{-1}(\theta)[\partial_\mu U(\theta)]\psi(x)}$$

The gauge invariance is spoiled

Non-Abelian gauge theory (SU(2))

- introduce the covariant derivative D_μ with the vector gauge fields $A_\mu^i(x)$

$$D_\mu\psi = \left(\partial_\mu - ig\frac{\boldsymbol{\tau}\cdot\mathbf{A}_\mu}{2}\right)\psi$$

$$D_\mu\psi \to (D_\mu\psi)' = U(\theta)D_\mu\psi$$

$$\left(\partial_\mu - ig\frac{\boldsymbol{\tau}\cdot\mathbf{A}'_\mu}{2}\right)(U(\theta)\psi) = U(\theta)\left(\partial_\mu - ig\frac{\boldsymbol{\tau}\cdot\mathbf{A}_\mu}{2}\right)\psi$$

$$\left[\partial_\mu U(\theta) - ig\frac{\boldsymbol{\tau}\cdot\mathbf{A}'_\mu}{2}U(\theta)\right]\psi = -igU(\theta)\frac{\boldsymbol{\tau}\cdot\mathbf{A}_\mu}{2}\psi$$

$$\boxed{\frac{\boldsymbol{\tau}\cdot\mathbf{A}'_\mu}{2} = U(\theta)\frac{\boldsymbol{\tau}\cdot\mathbf{A}_\mu}{2}U^{-1}(\theta) - \frac{i}{g}[\partial_\mu U(\theta)]U^{-1}(\theta)}$$

- For an infinitesimal change $\theta(x) \ll 1$, $U(\theta) \cong 1 - i\dfrac{\boldsymbol{\tau}\cdot\boldsymbol{\theta}(x)}{2}$

$$\frac{\boldsymbol{\tau}\cdot\mathbf{A}'_\mu}{2} = \frac{\boldsymbol{\tau}\cdot\mathbf{A}_\mu}{2} - i\theta^j A_\mu^k\left[\frac{\tau_j}{2}, \frac{\tau_k}{2}\right] - \frac{1}{g}\left(\frac{\boldsymbol{\tau}}{2}\cdot\partial_\mu\boldsymbol{\theta}\right)$$

$$\boxed{A_\mu^{i\prime} = A_\mu^i + \varepsilon^{ijk}\theta^j A_\mu^k - \frac{1}{g}\partial_\mu\theta^i}$$

Non-Abelian gauge theory (SU(2))

- The antisymmetric second-rank tensor of the gauge fields

$$(\mathrm{D}_\mu \mathrm{D}_\nu - \mathrm{D}_\nu \mathrm{D}_\mu)\psi \equiv \mathrm{i}g\left(\frac{\tau^i}{2} F^i_{\mu\nu}\right)\psi$$

$$\frac{\boldsymbol{\tau}\cdot\mathbf{F}_{\mu\nu}}{2} = \partial_\mu \frac{\boldsymbol{\tau}\cdot\mathbf{A}_\nu}{2} - \partial_\nu \frac{\boldsymbol{\tau}\cdot\mathbf{A}_\mu}{2} - \mathrm{i}g\left[\frac{\boldsymbol{\tau}\cdot\mathbf{A}_\mu}{2}, \frac{\boldsymbol{\tau}\cdot\mathbf{A}_\nu}{2}\right]$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\varepsilon^{ijk} A^j_\mu A^k_\nu$$

- Now we can construct the SU(2) invariant Lagrangian

$$\mathscr{L} = -\tfrac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \bar{\psi}\mathrm{i}\gamma^\mu \mathrm{D}_\mu \psi - m\bar{\psi}\psi$$

with infinitesimal transformation forms

$$\psi \to \psi' = \psi - \mathrm{i}\frac{\boldsymbol{\tau}\cdot\boldsymbol{\theta}}{2}\psi$$

$$A^i_\mu \to A^{i\prime}_\mu = A^i_\mu + \varepsilon^{ijk}\theta^j A^k_\mu - \frac{1}{g}\partial_\mu \theta^i$$

Generalization to higher groups

- G: some simple Lie group with generators satisfying the algebra

$$[F^a, F^b] = iC^{abc}F^c$$

- ψ: belong some representation with representation matrices T^a

$$[T^a, T^b] = iC^{abc}T^c$$

- The rest is straightforward

$$D_\mu\psi = (\partial_\mu - igT^a A^a_\mu)\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gC^{abc}A^b_\mu A^c_\nu$$

$$(\mathbf{T}\cdot\mathbf{F})_{\mu\nu} = \partial_\mu(\mathbf{T}\cdot\mathbf{A}_\nu) - \partial_\nu(\mathbf{T}\cdot\mathbf{A}_\mu) - ig[\mathbf{T}\cdot\mathbf{A}_\mu, \mathbf{T}\cdot\mathbf{A}_\nu]$$

$$\mathscr{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

- The Lagrangian is invariant under the transformation

$$\psi(x) \to \psi'(x) = U(\mathbf{T}\cdot\boldsymbol{\theta}(x))\psi(x) \equiv U(\theta_x)\psi(x)$$

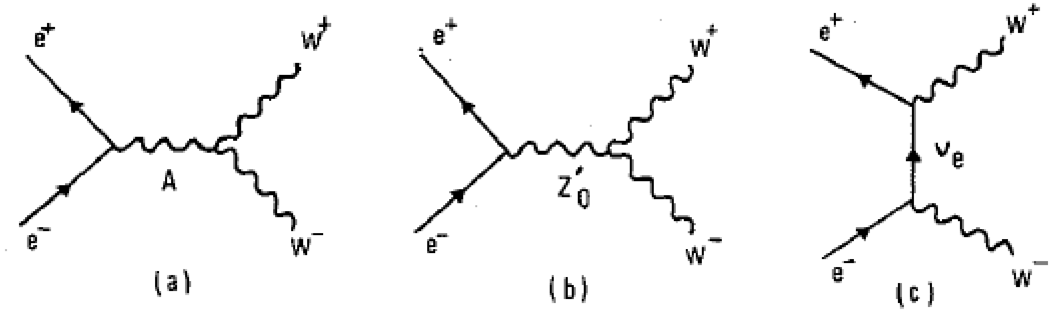
$$\mathbf{T}\cdot\mathbf{A}_\mu(x) \to \mathbf{T}\cdot\mathbf{A}'_\mu(x) = U(\theta_x)\mathbf{T}\cdot\mathbf{A}_\mu U^{-1}(\theta_x) - \frac{i}{g}[\partial_\mu U(\theta_x)]U^{-1}(\theta_x)$$

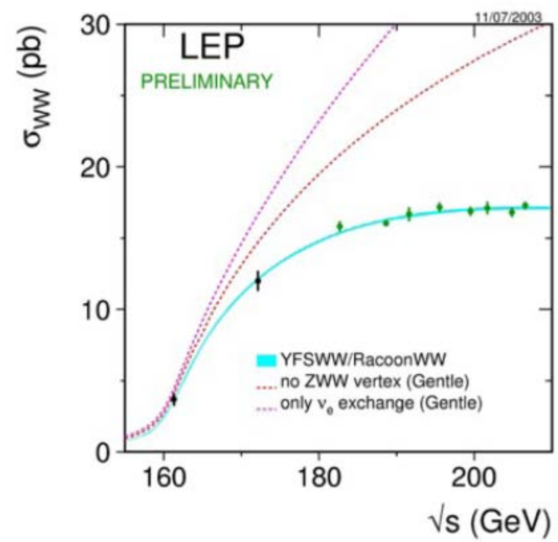
Abelian vs non-Abelian

- non-Abelian contains self interactions

$$-gC^{abc}\,\partial_\mu A^a_\nu A^{b\mu} A^{c\nu} - \frac{g^2}{4} C^{abc} C^{ade} A^b_\mu A^c_\nu A^{d\mu} A^{e\nu}$$

- The self couplings are proved at the LEP II





- In the Abelian case, no restriction in the gauge coupling. I.e. a charge e for one particle, while λe for another particle
- In the non-Abelian case, an arbitrary coupling or rescaling of coupling constant is strictly forbidden. (Why?)

Spontaneous symmetry breaking

- Local $SU(2)_L \times U(1)_Y$ gauge invariance forbids massive gauge bosons

$$\frac{1}{2}m_\gamma^2 A_\mu A^\mu \rightarrow \frac{1}{2}m_\gamma^2 (A_\mu + \frac{1}{e}\partial_\mu \alpha)(A^\mu + \frac{1}{e}\partial^\mu \alpha) \neq \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

- Local $SU(2)_L \times U(1)_Y$ gauge invariance forbids massive fermions

$$-m_f \bar{\psi}\psi = -m_f \left(\bar{\psi}_R + \bar{\psi}_L\right)\left(\psi_L + \psi_R\right) = -m_f \left[\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right]$$

$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\vec{\tau} \cdot \vec{W}_\mu + ig'\tfrac{1}{2}Y B_\mu$$

$$\begin{aligned} \psi_L{}' \rightarrow \psi_L &= e^{i\alpha(x)T + i\beta(x)Y}\psi_L \\ \psi_R{}' \rightarrow \psi_R &= e^{i\beta(x)Y}\psi_R \end{aligned}$$

The mass term is not invariant under this transformation

- How to solve these problems?

Introduce a new field with a specific potential which keeps gauge invariance, but the vacuum is not invariant

Spontaneous breaking of local gauge symmetry

- A new scalar field is a best solution

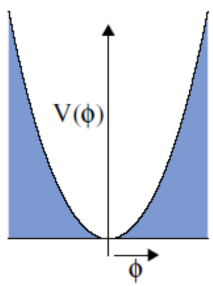
A simple example

- For a real scalar field ϕ with a potential which is symmetric under $\phi \to -\phi$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

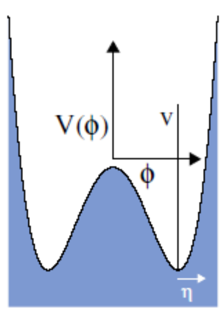
λ must be positive to have a minimum (bounded from below)

- $\mu^2 > 0$: Free particle with additional interactions



$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{free particle, mass } \mu} \quad \underbrace{-\frac{1}{4}\lambda\phi^4}_{\text{interaction}}$$

- $\mu^2 < 0$: Introducing a particle with imaginary mass ?



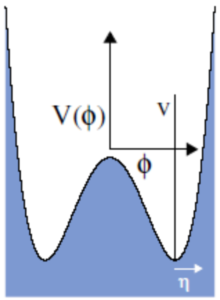
ϕ=0 is not a stable minimum

$$\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} = v$$

true vacuum

A simple example

- expand ϕ around v by $\phi \rightarrow \eta$+v



$$\text{Kinetic term:} \quad \mathcal{L}_{\text{kin}}(\eta) = \frac{1}{2}(\partial_\mu(\eta+v)\partial^\mu(\eta+v))$$

$$= \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) \qquad \text{, since } \partial_\mu v = 0$$

$$\text{Potential term:} \quad \mathrm{V}(\eta) = +\frac{1}{2}\mu^2(\eta+v)^2 + \frac{1}{4}\lambda(\eta+v)^4$$

$$= \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4$$

$$\boxed{\mu^2 = -\lambda v^2}$$

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda v^4$$

self interaction

$$\frac{1}{2}m_\eta^2 = \lambda v^2 \rightarrow m_\eta = \sqrt{2\lambda v^2} \quad \left(= \sqrt{-2\mu^2}\right)$$

Abelian case

- Abelian U(1) gauge theory with a complex scalar ϕ

$$\mathscr{L} = (\mathrm{D}_\mu\phi)^\dagger(\mathrm{D}^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathrm{D}_\mu\phi = (\partial_\mu - \mathrm{i}gA_\mu)\phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- The Lagrangian is invariant under the local gauge transformation

$$\phi(x) \to \phi'(x) = \mathrm{e}^{-\mathrm{i}\alpha(x)}\phi(x)$$

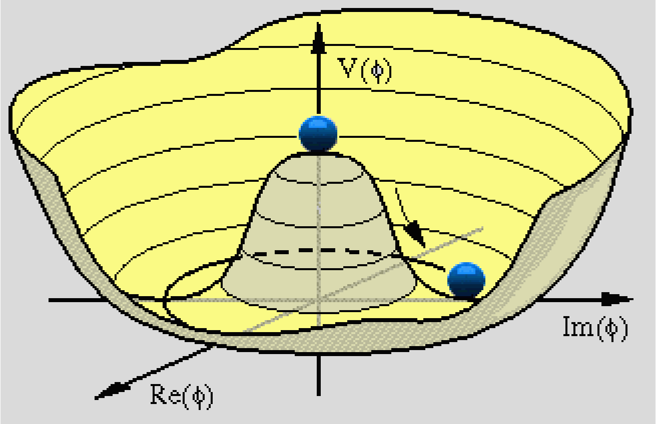
$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{g}\partial_\mu\alpha(x)$$

- The minimum of the potential with $\mu^2<0$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$|\phi| = v/\sqrt{2}$$

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$



Abelian case

- rewrite ϕ in terms of two real fields ϕ_1 and ϕ_2

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

- choose the vacuum

$$\langle 0|\phi_1|0\rangle = v \quad \text{and} \quad \langle 0|\phi_2|0\rangle = 0$$

- expand around the vacuum

$$\phi_1' = \phi_1 - v \quad \text{and} \quad \underline{\phi_2' = \phi_2}$$

massless Goldstone boson

- The covariant derivative term yields

$$\begin{aligned}|D_\mu\phi|^2 &= |(\partial_\mu - igA_\mu)\phi|^2 \\ &= \tfrac{1}{2}(\partial_\mu\phi_1' + gA_\mu\phi_2')^2 + \tfrac{1}{2}(\partial_\mu\phi_2' - gA_\mu\phi_1')^2 \\ &\quad - \underline{gvA^\mu(\partial_\mu\phi_2'} + gA_\mu\phi_1') + \frac{g^2v^2}{2}A^\mu A_\mu .\end{aligned}$$

mixing term

- A_μ acquires a mass $M = gv$

Higgs mechanism for Abelian case

- parametrize ϕ in polar variables and shift the modulus field

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \eta(x)]\exp(\mathrm{i}\xi(x)/v)$$

$$= \frac{1}{\sqrt{2}}[v + \eta(x) + \mathrm{i}\xi(x) + \ldots]$$

For small oscillation, essentially same as previous parametrization

- gauge fixing (unitary gauge)

$$\phi^u(x) = \exp(-\mathrm{i}\xi/v)\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x))$$

$$B_\mu(x) = A_\mu(x) - \frac{1}{gv}\partial_\mu\xi(x)$$

- The Lagrangian is $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1$

$$\mathscr{L}_0 = \frac{1}{2}(\partial_\mu\eta)^2 + \mu^2\eta^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \frac{1}{2}(gv)^2 B_\mu B^\mu$$

$$\mathscr{L}_1 = \frac{1}{2}g^2 B_\mu B^\mu \eta(2v + \eta) - \lambda v^2\eta^3 - \frac{1}{4}\lambda\eta^4$$

	d.o.f
before	$\phi_1, \phi_2, A(\pm)$
after	$\eta, B(\pm, 0)$

ξ : would-be-Goldstone boson

Non-Abelian case

- straightforward to generalize to an SU(2) gauge theory with complex doublet scalar

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \mathscr{L} = (\mathrm{D}_\mu\phi)^\dagger(\mathrm{D}_\mu\phi) - V(\phi) - \tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

$$\mathrm{D}_\mu\phi = \left(\partial_\mu - \mathrm{i}g\frac{\boldsymbol{\tau}}{2}\cdot\mathbf{A}_\mu\right)\phi \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc}A^b_\mu A^c_\nu$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

- The minimum of the potential with $\mu^2<0$

$$\langle\phi^\dagger\phi\rangle_0 = v^2/2 \qquad v = \sqrt{-\frac{\mu^2}{\lambda}}$$

- choose the vacuum and define a new field

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \phi' = \phi - \langle\phi\rangle_0$$

- yields a correct mass term for gauge bosons

$$\frac{1}{4}g^2\langle\phi\rangle_0^\dagger\boldsymbol{\tau}\cdot\mathbf{A}_\mu\boldsymbol{\tau}\cdot\mathbf{A}_\mu\langle\phi\rangle_0 = \frac{1}{2}\left(\frac{gv}{2}\right)^2\mathbf{A}_\mu\mathbf{A}^\mu$$

$$\begin{aligned} &(0 \quad v)\tau_i\tau_j\begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= (0 \quad v)\delta_{ij}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} + (0 \quad v)i\varepsilon_{ijk}\tau_k\begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= v^2\delta_{ij} - iv^2\varepsilon_{ij3} \end{aligned}$$

Higgs mechanism for Non-Abelian case

- parametrize the scalar doublet $\phi(x) = \exp\left\{\mathrm{i}\frac{\boldsymbol{\tau}}{v}\cdot\boldsymbol{\xi}(x)\right\}\begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$
- gauge fixing (unitary gauge)

$$\phi^u(x) = U(x)\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\eta \end{pmatrix}$$

$$\frac{\boldsymbol{\tau}}{2}\cdot\mathbf{B}_\mu = U(x)\frac{\boldsymbol{\tau}}{2}\cdot\mathbf{A}_\mu U^{-1}(x) - \frac{\mathrm{i}}{g}[\partial_\mu U(x)]U^{-1}(x)$$

$$U(x) = \exp\left\{-\mathrm{i}\frac{\boldsymbol{\tau}}{v}\cdot\boldsymbol{\xi}(x)\right\}$$

- The Lagrangian in the unitary gauge

$$\mathscr{L} = (\mathrm{D}_\mu\phi^u)^\dagger(\mathrm{D}^\mu\phi^u) - \frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4 - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}$$

$$\mathrm{D}_\mu\phi^u = \left(\partial_\mu - \mathrm{i}g\frac{\boldsymbol{\tau}}{2}\cdot\mathbf{B}_\mu\right)\phi^u$$

$$G^a_{\mu\nu} = (\partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\varepsilon^{abc}B^b_\mu B^c_\nu)$$

Mass term $$\frac{g^2}{8}(0, v)(\boldsymbol{\tau}\cdot\mathbf{B}_\mu\,\boldsymbol{\tau}\cdot\mathbf{B}^\mu)\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2}\left(\frac{gv}{2}\right)^2\mathbf{B}_\mu\mathbf{B}^\mu$$

	d.o.f
before	$\phi_1(2), \phi_2(2), A^i_\mu(\pm)$
after	$\eta, B^i_\mu(\pm, 0)$

ξ : would-be-Goldstone boson

Unitary gauge

- In the unitary gauge, the Goldstone boson is gauged away and absorbed as the longitudinal polarization state of the vector boson

- The vector boson has gained mass and the Goldstone boson has been eliminated

- All fictitious particles are absent

- But it is not clear that the resultant Lagrangian is renormalizable

- A massive vector boson field, whose propagator for large momentum behaves as $1/M_A^2$ rather than as $1/p^2$ (characteristic of massless gauge boson) and does not lead to an obviously renormalizable theory

$$D_{\mu\nu}(p) = \frac{-g_{\mu\nu} + p_\mu p_\nu / M_A^2}{p^2 - M_A^2 + \mathrm{i}\varepsilon}$$

- The unitarity is obvious, but the renormalizability is not clear

R_ξ gauge

- R_ξ gauge: not manifestly unitary, but explictly renormalizable
- Add the gauge-fixing term (in the Abelian gauge theory)

$$D_\mu\varphi\, D^\mu\varphi^* = \frac{1}{2}\left(\partial_\mu\chi_1 - qA_\mu\chi_2\right)^2 + \frac{1}{2}\left(\partial_\mu\chi_2 + qvA_\mu + qA_\mu\chi_1\right)^2$$

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu - f\right)^2 = -\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 + \frac{1}{\xi}(\partial_\mu A^\mu)\, f - \frac{1}{2\xi}f^2$$

- ξ: a positive real constant which defines the gauge
- f is chosen so as to cancel the mixing term of A and χ_2

$$\xi^{-1}(\partial_\mu A^\mu)\, f + qv\, A^\mu\partial_\mu\chi_2 = 0$$

$$f = \xi\, qv\, \chi_2 \qquad \text{(up to the total derivative)}$$

R_ξ gauge

- With the gauge-fixing term the Lagrangian is

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu\chi_1)^2 + 2\mu^2\chi_1^2\right] + \frac{1}{2}\left[(\partial_\mu\chi_2)^2 - \xi(qv)^2\chi_2^2\right]$$
$$-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}(qv)^2A_\mu^2 + \text{higher-order terms}$$

- Three fields A, χ_1, χ_2 are decoupled from each other up to the mass terms

- χ_1 is identified as the Higgs field (H) while χ_2 is the Goldstone mode

- The propagators of three fields (χ_2,H,A_μ) are

$$\Delta(p, \sqrt{\xi}M_{\mathrm{A}}) = [p^2 - \xi M_{\mathrm{A}}^2]^{-1}$$

$$\Delta(p, M_{\mathrm{H}}) = [p^2 - M_{\mathrm{H}}^2 + \mathrm{i}\varepsilon]^{-1}$$

$$D_{\mu\nu}(p) = \frac{-g_{\mu\nu} + (1-\xi)p_\mu p_\nu/(p^2 - \xi\, M_{\mathrm{A}}^2)}{p^2 - M_{\mathrm{A}}^2 + \mathrm{i}\varepsilon}$$

ξ=1: Feynman gauge

ξ=0: Landau gauge

ξ=∞: unitary gauge

For finite ξ, the propagator of A at large momentum behaves $1/p^2$ like a massless gauge boson: renormalizability is manifest

How to build a Model

- space-time dimensions
  - Minkowski, extra dimensions, super space, …
- specify gauge symmetry

  $U(1), SU(N), SO(N), E_6, \cdots$
- determine matter representation
  - spin 1/2, 3/2
  - fundamental or adjoint representation
  - Dirac or Majorana
- determine the pattern of SSB

  $$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \text{ or } \langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}, \ldots$$

  - or a few copies of doublet ϕ and/or singlets

write down all possible interactions allowed by symmetry

The model might be required to respect
- anomaly free
- unitarity
- Lorentz symmetry
- renormalizability
- naturalness
- unification

Standard Model

- space-time dimensions
  - Minkowski spacetime with a metric $g^{\mu\nu} = (1, -1, -1, -1)$
- specify gauge symmetry

$$SU(3)_C \times SU(2)_L \times U(1)_Y \text{ with gauge couplings } g_s, g, g'$$

- determine matter representation

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3, 2, Y_Q)$$

$$u_R \sim (3, 1, Y_u)$$

$$d_R \sim (3, 1, Y_d)$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2, Y_\ell)$$

$$e_R \sim (1, 1, Y_e)$$

+ 2 copies = 3 generation

hypercharges

$$Y_Q = \frac{1}{6}$$

$$Y_u = \frac{2}{3}$$

$$Y_d = -\frac{1}{3}$$

$$Y_l = -\frac{1}{2}$$

$$Y_e = -1$$

$$Y_\phi = +\frac{1}{2}$$

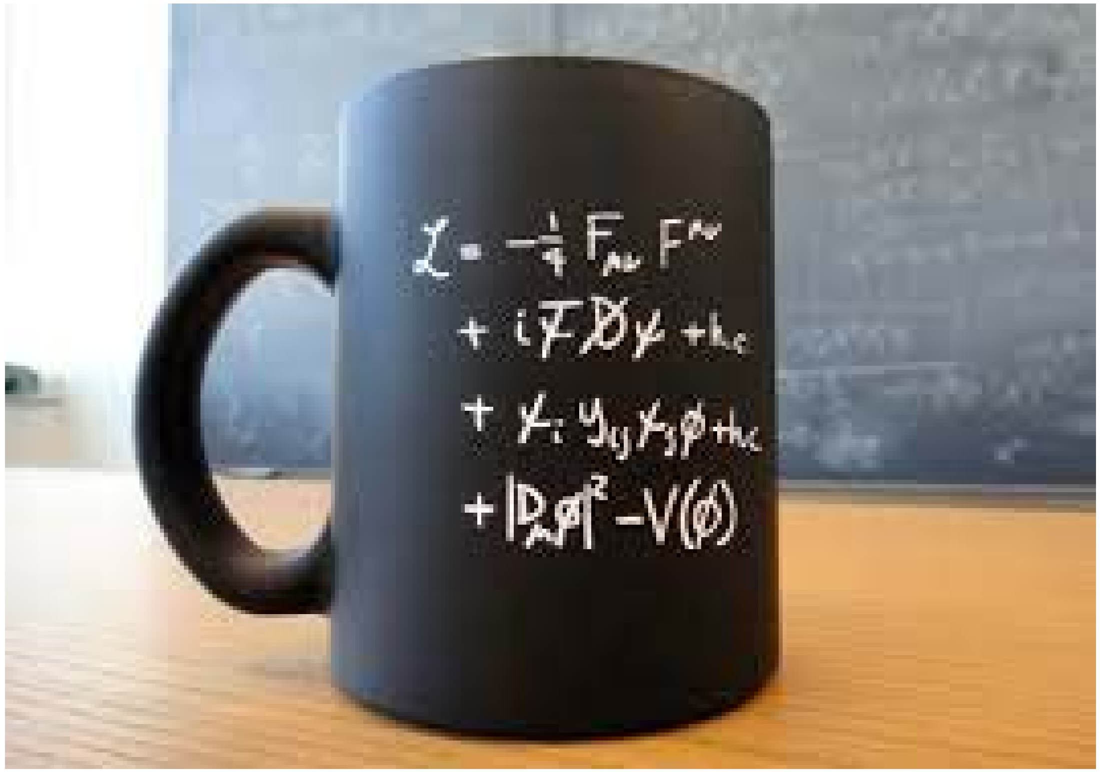
charges

$$Q = T_3 + Y$$

- determine the pattern of SSB

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Standard Model

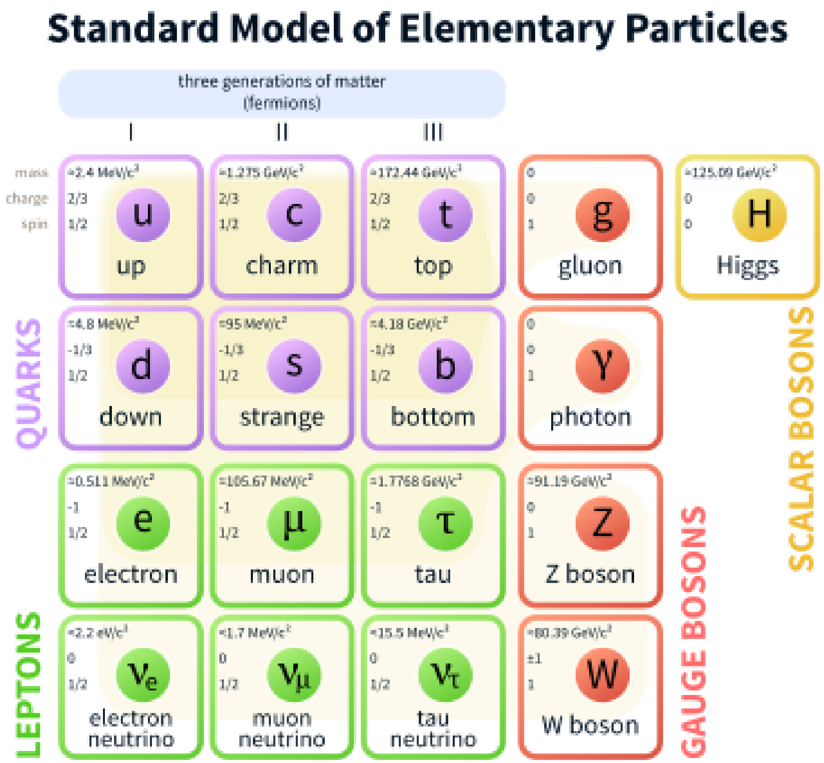


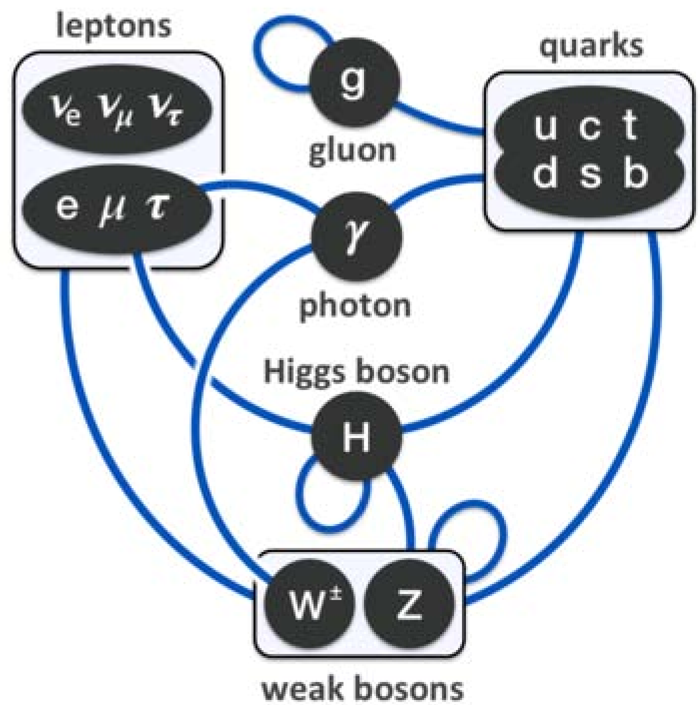
The Standard Model is a chiral theory

$SU(3)_C \times SU(2)_L \times U(1)_Y$ breaks down to $SU(3)_C \times U(1)_{\mathrm{QED}}$

The constructed model must be consistent with experiments

Standard Model





Gauge sector in the SM

- The covariant derivative for $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' B^\mu Y$$

$$L_a = \frac{\lambda_a}{2} : SU(3)_C \text{ generators, } T_b = \frac{\tau_b}{2} : SU(2)_L \text{ generators}$$

$$a = 1, \ldots, 8, \quad b = 1, 2, 3$$

$$\lambda^i = \begin{pmatrix} \tau^i & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Gell-Mann matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli matrices

- The covariant derivatives depend on charges of fields

Higgs

$$D^\mu \phi = \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{2} g' B^\mu \right) \phi$$

left-handed quarks

$$D^\mu Q_{Li} = \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{6} g' B^\mu \right) Q_{Li}$$

right-handed up-type quarks

$$D^\mu U_{Ri} = \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{2i}{3} g' B^\mu \right) U_{Ri}$$

right-handed down-type quarks

$$D^\mu D_{Ri} = \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a - \frac{i}{3} g' B^\mu \right) D_{Ri}$$

left-handed leptons

$$D^\mu L_{Li} = \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b - \frac{i}{2} g' B^\mu \right) L_{Li}$$

right-handed leptons

$$D^\mu E_{Ri} = \left(\partial^\mu - i g' B^\mu \right) E_{Ri}$$

$i = 1, 2, 3$: the generation index

hypercharges

$$Y_Q = \frac{1}{6}, \quad Y_u = \frac{2}{3}, \quad Y_d = -\frac{1}{3}, \quad Y_l = -\frac{1}{2}, \quad Y_e = -1, \quad Y_\phi = +\frac{1}{2}$$

Gauge sector in the SM

- The field strengths for each gauge group

$$\begin{aligned} G^{\mu\nu}_a &= \partial^\mu G^\nu_a - \partial^\nu G^\mu_a - g_s f_{abc} G^\mu_b G^\nu_c \\ W^{\mu\nu}_a &= \partial^\mu W^\nu_a - \partial^\nu W^\mu_a - g \epsilon_{abc} W^\mu_b W^\nu_c \\ B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \end{aligned}$$

$f_{abc}, \varepsilon_{abc}$: the structure constants

- The Lagrangian for the gauge section is

$$\begin{aligned} \mathcal{L}_{\text{gauge}} &= -\frac{1}{4} G^{\mu\nu}_a G_{a\mu\nu} - \frac{1}{4} W^{\mu\nu}_b W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\ &\quad - i\overline{Q_{Li}} \not{D} Q_{Li} - i\overline{U_{Ri}} \not{D} U_{Ri} - i\overline{D_{Ri}} \not{D} D_{Ri} - i\overline{L_{Li}} \not{D} L_{Li} - i\overline{E_{Ri}} \not{D} E_{Ri} \\ &\quad - (D^\mu \phi)^\dagger (D_\mu \phi) \end{aligned}$$

  - 3 gauge couplings
  - Highly symmetric: $SU(3)_c \times SU(2)_L \times U(1)_Y$ local symmetry+global symmetries
  - 3 identical replica of the basic fermion family $\Longrightarrow$ huge flavour-degeneracy

Gauge sector in the SM

- The field strengths for each gauge group

$$
\begin{aligned}
G_a^{\mu\nu} &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu \\
W_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu \\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu
\end{aligned}
$$

$f_{abc}, \varepsilon_{abc}$: the structure constants

- The Lagrangian for the gauge section is

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_b^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\
&\quad - i\overline{Q_{Li}} \not{D} Q_{Li} - i\overline{U_{Ri}} \not{D} U_{Ri} - i\overline{D_{Ri}} \not{D} D_{Ri} - i\overline{L_{Li}} \not{D} L_{Li} - i\overline{E_{Ri}} \not{D} E_{Ri} \\
&\quad - (D^\mu \phi)^\dagger (D_\mu \phi)
\end{aligned}
$$

- global symmetry $G_{\text{global}} = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$

$$
\begin{aligned}
& Q_L^i \to U_Q^{ij} Q_L^j, U_R^i \to U_U^{ij} U_R^j, D_R^i \to U_D^{ij} D_R^j \\
& L_L^i \to U_L^{ij} L_L^j, E_R^i \to U_E^{ij} E_R^j
\end{aligned}
$$

- The global symmetries are broken by Yukawa interactions

Higgs potential in the SM

- The potential for the scalar doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$$\mathcal{L}_\phi^{\text{SM}} = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

2 parameters

- Spontaneous symmetry breaking: $\mu^2<0$ and $\lambda>0$

  4 possible choices for the vacuum $\phi = \begin{pmatrix} \phi_r^+ + i\phi_i^+ \\ \phi_r^0 + i\phi_i^0 \end{pmatrix}$

  We choose $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

- The VEV breaks the SU(2)×U(1) symmetry down to a U(1) subgroup

  → One linear combination T_3+Y annihilates the vacuum state

  $T_3 + Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ for ϕ

- The unbroken subgroup is identified with $U(1)_{EM}$ and its generator Q is identified as

$$Q = T_3 + Y.$$

Vector boson mass

- The relavant Lagrangian is $(D_\mu\langle\phi\rangle)^\dagger(D^\mu\langle\phi\rangle)$

$$D^\mu\langle\phi\rangle = \frac{i}{\sqrt{8}}(gW_a^\mu\tau_a + g'B^\mu)\begin{pmatrix}0\\ v\end{pmatrix} = \frac{i}{\sqrt{8}}\begin{pmatrix} gW_3^\mu + g'B^\mu & g(W_1^\mu - iW_2^\mu) \\ g(W_1^\mu + iW_2^\mu) & -gW_3^\mu + g'B^\mu \end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}$$

- The mass term for the vector bosons

$$\mathcal{L}_{M_V} = \frac{1}{8}(0\ v)\begin{pmatrix} gW_{3\mu} + g'B_\mu & g(W_1 - iW_2)_\mu \\ g(W_1 + iW_2)_\mu & -gW_{3\mu} + g'B_\mu \end{pmatrix}\begin{pmatrix} gW_3^\mu + g'B^\mu & g(W_1 - iW_2)^\mu \\ g(W_1 + iW_2)^\mu & -gW_3^\mu + g'B^\mu \end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}$$

- The Weinberg angle $\tan\theta_W \equiv \frac{g'}{g}$ denotes the rotation of (W_3,B) → (Z,A)

- Four gauge bosons are defined by

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)_\mu, \quad Z_\mu^0 = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu, \quad A_\mu^0 = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu$$

$$\mathcal{L}_{M_V} = \frac{1}{4}g^2v^2W^{+\mu}W_\mu^- + \frac{1}{8}(g^2 + g'^2)v^2Z^{0\mu}Z_\mu^0$$

$$m_W^2 = \frac{1}{4}g^2v^2, \qquad m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2, \qquad m_A^2 = 0.$$

3 massive bosons +
1 massless boson

Questions

- Why doesn't the photon interact with the Higgs boson?

- Show that the charges of $W^{\pm}$ are determined to be ± 1?

- What happens if ϕ^{+} has a vacuum instead of ϕ^{0}?

ρ parameter

- The mass relation $\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2 + g'^2}$
- The ρ parameter at the tree level

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1$$

- The global fit in experimental observables

$$\rho_0 = 1.00037 \pm 0.00023$$

- The ρ parameter provides a stringent bound on new physics models

Fermion gauge interactions

- Cosider the Lagrangian for a electron and neutrino

$$\mathcal{L}_\ell = \overline{\psi}_\mathrm{L}\, \mathrm{i}\gamma^\mu D_\mu^\mathrm{L}\, \psi_\mathrm{L} + \overline{\psi}_\mathrm{R}\, \mathrm{i}\gamma^\mu D_\mu^\mathrm{R}\, \psi_\mathrm{R}$$

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \psi_R = e_R$$

$$\boxed{A_\mu = \tfrac{1}{2}\tau_i A_\mu^i}$$

$$\begin{aligned}\overline{\psi}_\mathrm{L}\mathrm{i}\gamma^\mu D_\mu^\mathrm{L}\, \psi_\mathrm{L} &= \overline{\psi}_\mathrm{L}\mathrm{i}\gamma^\mu(\partial_\mu + \mathrm{i}g A_\mu - \tfrac{\mathrm{i}}{2}g' B_\mu)\psi_\mathrm{L} \\ &= \overline{\psi}_\mathrm{L}\mathrm{i}\gamma^\mu\left[\partial_\mu + \tfrac{\mathrm{i}}{\sqrt{2}}g(W_\mu\tau_+ + W_\mu^\dagger\tau_-) + \tfrac{\mathrm{i}}{2}(gA_{3\mu}\tau_3 - g' B_\mu)\right]\psi_\mathrm{L}\end{aligned}$$

$$\begin{aligned}\overline{\psi}_\mathrm{R}\, \mathrm{i}\gamma^\mu D_\mu^\mathrm{R}\, \psi_\mathrm{R} &= \overline{\psi}_\mathrm{R}\, \mathrm{i}\gamma^\mu \left(\partial_\mu + \mathrm{i}g' \frac{Y_\mathrm{R}}{2} B_\mu\right)\psi_\mathrm{R} \\ &= \overline{e}_\mathrm{R}\mathrm{i}\gamma^\mu\partial_\mu e_\mathrm{R} + g'\, \overline{e}_\mathrm{R}\gamma^\mu e_\mathrm{R}\, B_\mu\, .\end{aligned}$$

$$\boxed{\tau_\pm = \tfrac{1}{2}(\tau_1 \pm \mathrm{i}\tau_2)}$$

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

- The kinetic term

$$\overline{\psi}_\mathrm{L}\mathrm{i}\gamma^\mu\partial_\mu\psi_\mathrm{L} + \overline{\psi}_\mathrm{R}\mathrm{i}\gamma^\mu\partial_\mu\psi_\mathrm{R} = \overline{\nu}_\mathrm{L}\, \mathrm{i}\gamma^\mu\, \partial_\mu\nu_\mathrm{L} + \overline{e}\, \mathrm{i}\gamma^\mu\, \partial_\mu e$$

Charged and neutral currents

- The charged currents

$$\mathcal{L}^{\ell}_{\rm cc} = -\frac{g}{\sqrt{2}}\left[(\overline{\psi}_{\rm L}\gamma^{\mu}\tau_{+}\psi_{\rm L})\,W_{\mu} + (\overline{\psi}_{\rm L}\gamma^{\mu}\tau_{-}\psi_{\rm L})\,W^{\dagger}_{\mu}\right]$$

$$= -\frac{g}{\sqrt{2}}\left(J^{\mu\dagger}W_{\mu} + J^{\mu}W^{\dagger}_{\mu}\right),$$

$$\psi_L = \begin{pmatrix}\nu_L \\ e_L\end{pmatrix}, \psi_R = e_R$$

$$\tau_+ = \begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}$$

$$\tau_- = \begin{pmatrix}0 & 0 \\ 1 & 0\end{pmatrix}$$

$$J_{\mu} = j^1_{\mu} - {\rm i}j^2_{\mu} = \overline{\psi}_{\rm L}\gamma_{\mu}\tau_{-}\psi_{\rm L} = \overline{e}_{\rm L}\gamma_{\mu}\nu_{\rm L}$$

$$J^{\dagger}_{\mu} = j^1_{\mu} + {\rm i}j^2_{\mu} = \overline{\psi}_{\rm L}\gamma_{\mu}\tau_{+}\psi_{\rm L} = \overline{\nu}_{\rm L}\gamma_{\mu}e_{\rm L}$$

- The neutral currents in terms of A and Z

$$\mathcal{L}^{\ell}_{\rm nc} = -e\,j^{\rm em}_{\mu}A^{\mu} - \frac{g}{c_{\rm W}}\,j^{\rm Z}_{\mu}Z^{\mu}$$

$$e = g s_{\rm W} = g' c_{\rm W}$$

$$j^{\rm em}_{\mu} = Q\,\overline{e}\gamma_{\mu}e$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$

$$j^{\rm Z}_{\mu} = \overline{\psi}_{\rm L}\gamma_{\mu}Z_{\rm L}\psi_{\rm L} + \overline{\psi}_{\rm R}\gamma_{\mu}Z_{\rm R}\psi_{\rm R}$$

$$Z_{\rm L} = T_{3\rm L} - Q s^2_{\rm W}\,, \quad Z_{\rm R} = -Q s^2_{\rm W}$$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

$g \sim 0.63$

$g' \sim 0.34$

Yukawa sector in the SM

- The gauge invariant Yukawa interactions are given by

$$\mathcal{L}_{\rm Y}^{\rm SM} = Y_{ij}^{d}\overline{Q_{Li}}\phi D_{Rj} + Y_{ij}^{u}\overline{Q_{Li}}\tilde{\phi}U_{Rj} + Y_{ij}^{e}\overline{L_{Li}}\phi E_{Rj} + \text{h.c.}$$

$$\begin{array}{lcccl} SU(3)_C & \overline{3} & 1 & 3 & \\ SU(2)_L & \overline{2} & 2 & 1 & \\ U(1)_Y & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & = 0 \end{array}$$

- ϕ gives the mass term for the down-type quarks and charged leptons

$$y^d\,\overline{Q_L}\phi d_R = y^d\begin{pmatrix}\overline{u_L} & \overline{d_L}\end{pmatrix}\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}d_R = \frac{y^d v}{\sqrt{2}}\overline{d_L}d_R = m_d\,\overline{d_L}d_R$$

The color index is suppressed

- The up-type quarks can get their mass by $\tilde{\phi} = i\tau_2\phi^{\dagger} = i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}\phi^{0^*}\\ \phi^{-}\end{pmatrix} = \begin{pmatrix}\phi^{0^*}\\ -\phi^{-}\end{pmatrix}$

$$y^u\,\overline{Q_L}\tilde{\phi}u_R = y^u\begin{pmatrix}\overline{u_L} & \overline{d_L}\end{pmatrix}\begin{pmatrix}\frac{v}{\sqrt{2}}\\ 0\end{pmatrix}u_R = \frac{y^u v}{\sqrt{2}}\overline{u_L}u_R = m_u\,\overline{u_L}u_R$$

- One may introduce another doublet ϕ_2 with a hypercharge -1/2 for the mass generation of the up-type quarks (Two Higgs doublet model)

Yukawa sector in the SM

- In general Yukawa couplings are complex

$$\mathcal{L}_{\mathrm{Y}}^{\mathrm{SM}} = Y_{ij}^{d}\overline{Q_{Li}}\phi D_{Rj} + Y_{ij}^{u}\overline{Q_{Li}}\tilde{\phi}U_{Rj} + Y_{ij}^{e}\overline{L_{Li}}\phi E_{Rj} + \mathrm{h.c.}$$

  - Ad hoc
  - necessary to describe data
  - origin of the flavor structure of the model
  - flavor degeneracy is broken

- The mass matrix is not flavor-diagonal

$$(m_d)_{ij} = \frac{y_{ij}^{d} v}{\sqrt{2}},\ (m_u)_{ij} = \frac{y_{ij}^{u} v}{\sqrt{2}},\ (m_e)_{ij} = \frac{y_{ij}^{e} v}{\sqrt{2}}$$

- The mass eigenstates can be obtained by the bi-unitary transformation

Parameters in the (B)SM

- 3 gauge couplings: α, G_F, α_s
- 2 Higgs parameters: μ^2, λ

Flavour parameters

- 6 quark masses: $m_d, m_u, m_s, m_c, m_b, m_t$
- 3 quark mixing angles + 1 phase: λ, A, ρ, η
- 3 charged lepton masses: m_e, m_μ, m_τ
- 3 neutrino masses
- 3 lepton mixing angles + 1 phase + 2 Majorana phases

BSM

+ strong CP