Radiative Kähler moduli stabilization

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Introduction

The standard model of particle physics

Gauge group : SU(3) × SU(2) × U(1)

Matter content :

	spin1/2	$SU(3)_c,\ SU(2)_L,\ U(1)_Y$
quarks (×3 families)	$Q^i\!=\!(u_L,\ d_L)^i$	(3, 2, 1/6)
	u_R^i	($\bar{3}$, 2, -2/3)
	d_R^i	($\bar{3}$, 1, 1/3)
leptons (×3 families)	$L^i\ =\!(\nu,\ e_L)^i$	(1, 2, -1/2)
	e_R^i	(1, 1, 1)
	spin0	
Higgs	$H\!=\!(H^+,\ H^0)$	(1, 2, -1/2)

	spin1	$SU(3)_c,SU(2)_L,U(1)_Y$
gluon	g	(8, 1, 0)
W bosons	$W^\pm\ \ W^0$	(1, 3, 0)
B boson	B^0	(1, 1, 0)

Introduction

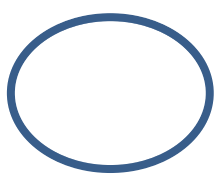
Problem:

No gravitational interaction in the standard model

String theory

A good candidate for the unified theory of the gauge and gravitational interactions

Closed string



→Graviton

Dp-brane

Ramond-Ramond field: C_{p+1}

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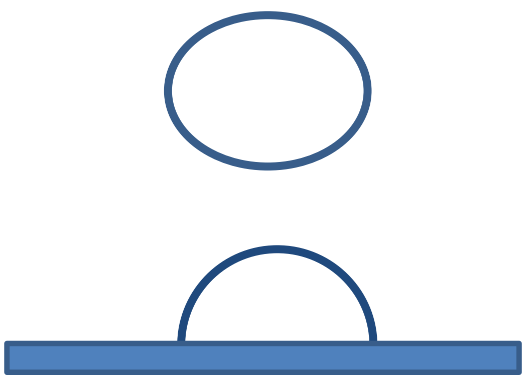
String theory

A good candidate for the unified theory of the gauge and gravitational interactions

Closed string

Open string



Dp-brane

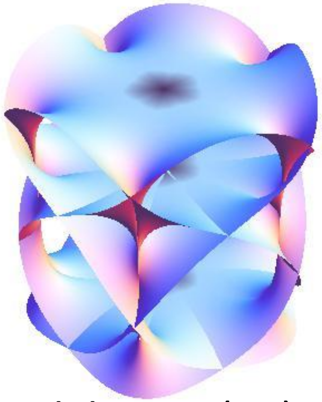
→Graviton

Ramond-Ramond field: C_{p+1}

→Gauge fields

(Perturbative) superstring theory requires the extra 6 dimension.

10＝4＋6



6D Calabi-Yau (CY)Manifold

$R_{ij} = 0$

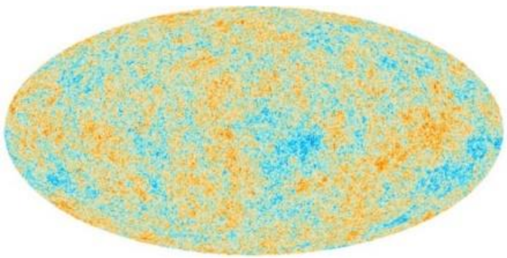
〇The geometric parameters of extra 6D dimensional space
→4D scalar fields (called moduli)

〇Unless they are stabilized, it will lead to unobserved fifth forces.

〇Stabilization of the extra dimensional space
→ <u>Moduli stabilization (creating a moduli potential)</u>

Moduli are ubiquitous in string compactifications

○ Good candidate of inflaton



○ Supersymmetry breaking

○ Moduli cosmology

Moduli interact with matter fields gravitationally.

→Such long-lived particles affect the cosmology of the early Universe

(e.g., dark matter abundance, baryon asymmetry,…)

○ Ultralight axions

In the LARGE volume scenario,
(Type IIB string on Calabi-Yau (CY) manifold)

$$V_{\mathrm{axion}} \sim e^{-\mathcal{V}^{2/3}} \cos\left(\frac{a}{f}\right)$$

Axion mass is suppressed by the LARGE CY volume $\langle \mathcal{V} \rangle \gg l_s^6$

l_s: string length

Such an ultralight axion is a target of astrophysical and cosmological observations.

From such phenomenological points of view,
it is quite important to discuss the moduli dynamics.

Outline

○ Introduction

○ **Moduli stabilization in type IIB string**

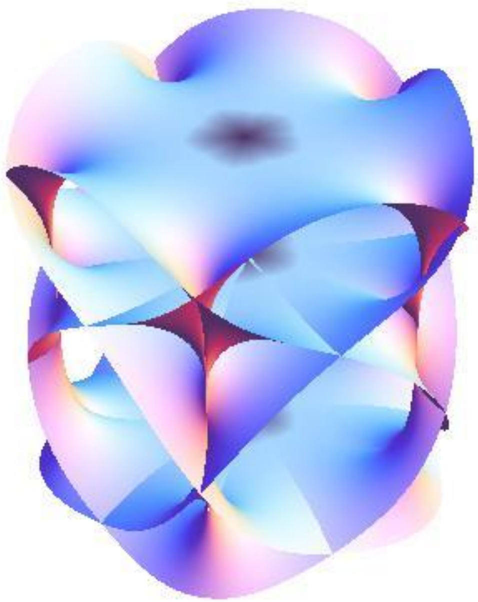
(i) KKLT scenario
(ii) LARGE volume scenario (LVS)

○ **Radiative Kähler moduli stabilization**

○ **Conclusion**

Moduli fields (4D massless scalar fields) :

Closed string moduli



i) Dilaton : S

$\langle \mathrm{Re}(S) \rangle = g_s^{-1}$
g_s: string coupling

ii) Kähler moduli : T
Size of the internal cycles

iii) Complex structure moduli : U
Shape

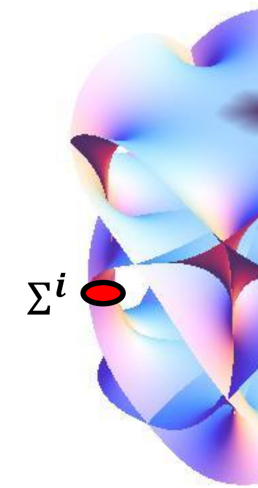
○ String axions

Imaginary parts of moduli fields

e.g.

Integrating the 4-form C_4 over the internal 4-cycles Σ^{i} of internal manifold,

$$\sigma^i(x) = \int_{\Sigma^i} C_4$$



Kähler moduli : $T^i = \tau^i + i\sigma^i, \quad (i = 1,2,\cdots,h^{1,1})$

τ^i: The volume of 4-cycles

A lot of axions originating from higher dimensional form fields

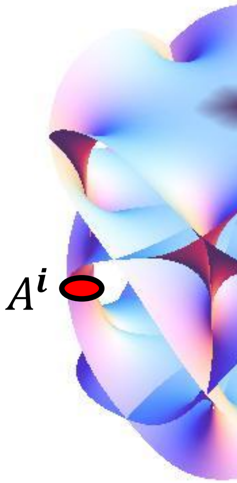
○ String axions

Imaginary parts of moduli fields

e.g.

Integrating the hol.3-form Ω over the internal 3-cycles A^{i} of internal manifold,

$$U^i(x) = \int_{A^i} \Omega$$



Complex structure moduli

$$i = 1,2,\cdots,h^{2,1}$$

$h^{2,1}$: # of three-cycles of CY

Flux compactification

Flux compactification is useful to stabilize the moduli fields.

Let us consider higher-dimensional Maxwell's theory on $R^{1,3} \times M$,

$$\int_{R^{1,3}\times M} F_p \wedge * F_p$$

When there exists a magnetic flux F_p in a cycle Σ_p of M

$$\int_{\Sigma_p} F_p = n \in Z$$

It generates a potential depending on the metric of extra dimension
→ Generating moduli potential

Flux compactification in type IIB string on Calabi-Yau(CY)

Type IIB string on $R^{1,3} \times \mathrm{CY}$,

$$\int_{R^{1,3}\times\mathrm{CY}} G_3 \wedge * \, G_3$$

$G_3 = F_3 - iSH_3$: three-form

The fluxes on the three-cycle of CY (Σ_3) generate the moduli potential,

$$\int_{\Sigma_3} F_3 \qquad \int_{\Sigma_3'} \mathrm{H}_3$$

We can stabilize the dilaton S and all the complex structure moduli U^i $(i = 1,2,\ldots,h^{2,1})$.

[Giddings-Kachru-Polchinski ’02]

$h^{2,1}$: # of three-cycles of CY

Flux compactification in type IIB string on Calabi-Yau(CY)

After the Kaluza-Klein reductions,
type IIB string on CY orientifold is described by 4D N=1 supergravity action.

Scalar potential (in units of $M_{\rm pl} = 2.4 \times 10^{18}{\rm GeV}$):

$$V = e^K \left(\sum_{I,J} K^{I\bar{J}} D_I W D_{\bar{J}} W - 3|W|^2 \right)$$

$$D_I = \partial_I + K_I$$
$$K_I = \partial_I K$$

The flux-induced superpotential

[Gukov-Vafa-Witten '00]

$$W = W_{\rm flux}(S, U)$$

leads to the stabilization of S and U at the supersymmetric minimum.

$$D_S W = D_U W = 0$$

The remaining problem of string compactification is how to stabilize the Kähler moduli.

○<u>KKLT scenario</u>

[Kachru-Kallosh-Linde-Trivedi ’03]

Non-perturbative effects

○<u>LARGE volume scenario</u> [Balasubramanian-Berglund-Conlon-Quevedo ’05]

Stringy correction(α'-corrections)+Non-perturbative effects

○<u>Radiative Kahler moduli stabilization</u> [Kobayashi-Omoto-Otsuka-Tatsuishi ’17]

Stringy correction(α'-corrections)+ radiative corrections from sparticles

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○ **Conclusion**

①KKLT scenario

[Kachru-Kallosh-Linde-Trivedi '03]

○Non-perturbative effects in the superpotential

$$K = -2\ln(\mathcal{V})$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}$$

$\mathcal{V}(T)$: CY volume

$W_0 = \langle W_{\rm flux} \rangle$

$$V_F = e^K \left(K^{T_i \bar{T}_j} D_{T_i} W D_{\bar{T}_j} \bar{W} - 3|W|^2 \right)$$

Stabilization of Kähler moduli:

$$D_{T_i} W = \partial_{T_i} W + (\partial_{T_i} K) W = 0$$

$$\mathrm{Re}(T_i) \sim \frac{|\ln(W_0)|}{a_i} \gg 1$$

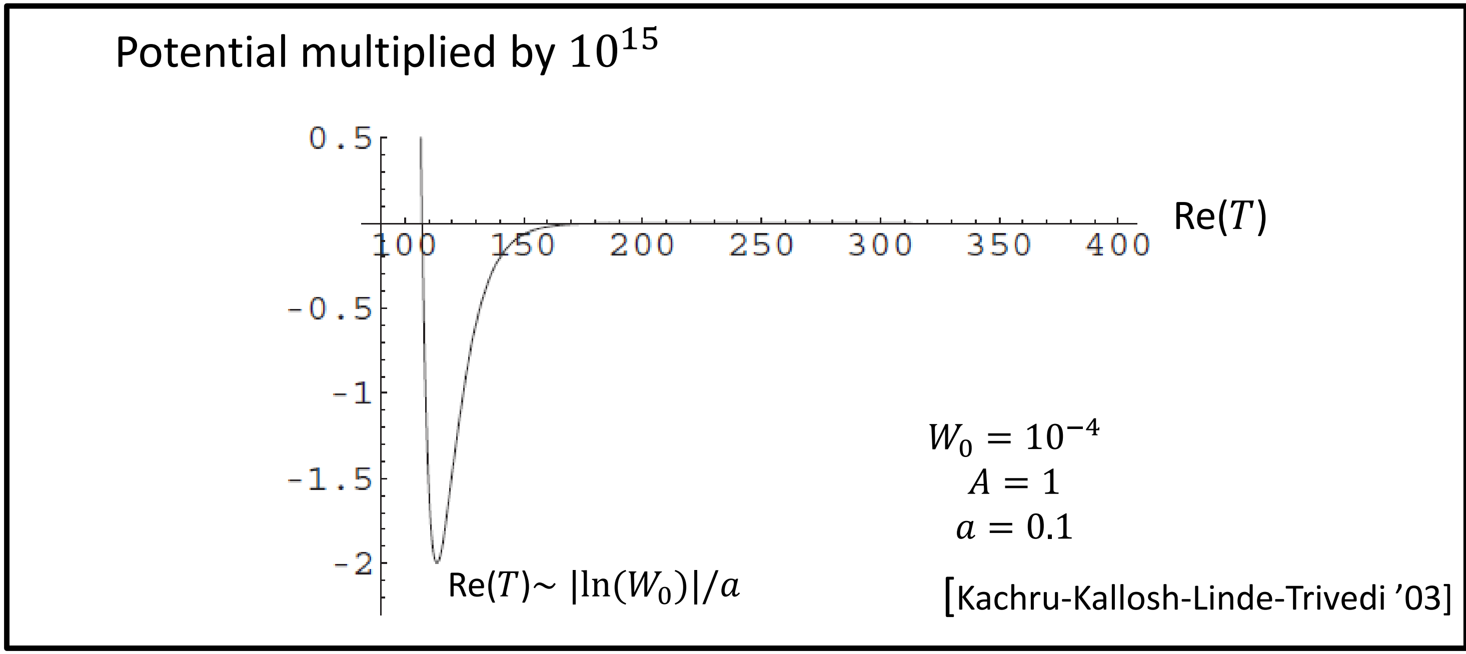
However, we require the tuning of W_0 to achieve $D_{T_i} W = 0$.

①KKLT scenario

When the number of Kahler modulus is $h^{1,1} = 1$,

$$K = -3\ln(T + \bar{T})$$

$$W = W_0 + Ae^{-aT}$$



However, we require the tuning of W_0 to achieve Re(T)$\gg 1$.

②LARGE volume scenario

[Balasubramanian-Berglund-Conlon-Quevedo '05]

○The inclusion of stringy α'-corrections to the Kähler potential
○Non-perturbative effects in the superpotential

$$K = -2\ln(\mathcal{V} + \xi)$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}$$

$\mathcal{V}(T)$: CY volume

$W_0 = \langle W_{\rm flux} \rangle$

The stringy α'-corrections (from 10D R^4 term)

[Becker-Becker-Haack-Louis '02]

$$\xi \simeq -2.4 \times 10^{-3} \chi\, g_s^{-3/2}$$

χ : Euler number of CY

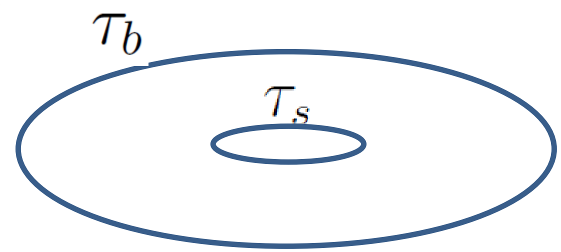
g_s : String coupling

②LARGE volume scenario

[Balasubramanian-Berglund-Conlon-Quevedo '05]

For Calabi-Yau $P^4_{[1,1,1,6,9]}$ with two Kähler moduli,

Volume of CY : $\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}$



$$K = -2\ln(\mathcal{V} + \xi)$$

$$W = W_0 + A_s e^{-a_s T_s}$$

$$\tau_s = \mathrm{Re}(T_s)$$

The scalar potential:

$$V_F = \frac{a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{3\xi |W|^2}{4\mathcal{V}^3} + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right)$$

The two Kähler moduli are stabilized at

$$\langle \mathcal{V} \rangle \sim |W_0| e^{a_s\tau_s} \qquad \langle \tau_s \rangle \sim \xi^{2/3}$$

②LARGE volume scenario

[Balasubramanian-Berglund-Conlon-Quevedo '05]

The two Kähler moduli are stabilized at

$$\langle \mathcal{V} \rangle \sim |W_0| e^{a_s \tau_s} \qquad \langle \tau_s \rangle \sim \xi^{2/3}$$

○CY volume is extremely large $\langle \mathcal{V} \rangle \sim |W_0| e^{a_s \tau_s} \gg 1$

○We require the positive ξ

$$\xi \simeq -2.4 \times 10^{-3} \chi \, g_s^{-3/2} > 0$$

Euler number of CY should be negative, namely

$$\chi = 2(h^{1,1} - h^{2,1}) < 0 \leftrightarrow \quad h^{2,1} > h^{1,1}$$

<u># of complex structure moduli > # of Kahler moduli</u>

②LARGE volume scenario

[Balasubramanian-Berglund-Conlon-Quevedo ’05]

〇Ultralight axion :

Non-perturbative effects

$$W_{\rm non} = A_b e^{-a_b T_b}$$

$$T_b = \tau_b + i\sigma_b$$

generate the potential of axion associated with volume modulus.

$$V_{\rm axion} \sim e^{-\mathcal{V}^{2/3}} \cos(a_b \sigma_b)$$

$$\langle \mathcal{V} \rangle \sim |W_0| e^{a_s \tau_s} \gg 1$$

LARGE volume leads to the tiny mass of axion.

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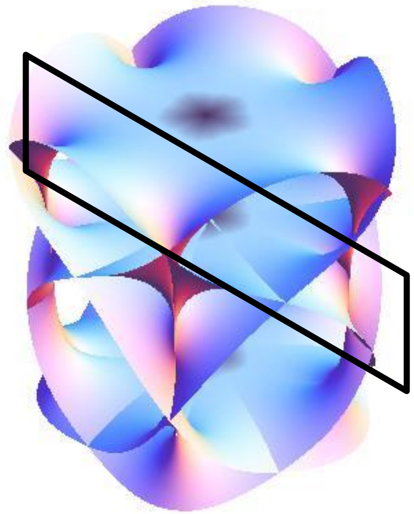
Radiative Kähler moduli stabilization

[Kobayashi-Omoto-Otsuka-Tatsuishi '17]

○The inclusion of matter fields Q_a living on D7-branes



D7-branes wrapping T_i

$i \in \{1,2,\cdots,h^{1,1}\}$

$$K_{\rm matter} = Z^{(i)}_{a\bar{a}}|Q_a|^2$$

Kähler metric: $Z^{(i)}_{a\bar{a}} = (T_i + \bar{T}_i)^{-n_i^a}$

Also, the gaugino fields localized on D7-branes

Gauge kinetic function : $f = \dfrac{T_i}{2\pi}$

○The inclusion of matter fields Q_a living on D7-branes

$$K = -2\ln(\mathcal{V}+\xi) + Z^{(i)}_{a\bar{a}}|Q_a|^2$$

$$W = W_0$$

$$W_0 = \langle W_{\rm flux}\rangle$$

Scalar potential : $V_{\alpha'} \simeq \dfrac{3\xi|W|^2}{4\mathcal{V}^3}$

Volume modulus $\mathcal{V}$ runs away to $\mathcal{V}\to\infty$

Moduli F-term

$$F^{T_i} = -e^K K^{T_i\bar{T}_j} K_{\bar{T}_j}\bar{W}_0 \neq 0$$

generates the soft masses

$$m_a^2 \simeq V_{\alpha'} + (1-n_i^a)m_{3/2}^2$$

$$M_f \simeq m_{3/2}$$

$$m_{3/2} \simeq W_0/\mathcal{V}$$

Sparticle masses depend on the CY volume,

$$m_a^2 \simeq V_{\alpha'} + (1 - n_i^a) m_{3/2}^2$$

$$M_f \simeq m_{3/2}$$

$$m_{3/2} \simeq W_0 / \mathcal{V}$$

→Radiative corrections through the Coleman-Weinberg (CW) potential

$$V_{\rm CW} = \frac{1}{32\pi^2} \int^{\Lambda^2} dk^2 k^2 {\rm STr} \ln(k^2 + M^2)$$

$$\simeq \frac{c_1}{32\pi^2} \Lambda^2 m_{3/2}^2 + \mathcal{O}(m_{3/2}^4)$$

$$c_1 \simeq c_{\rm b} - 2c_{\rm f} + 4$$

We fix the cutoff scale $\Lambda = m_{\rm KK}$ as the mass of Kaluza-Klein mode.

$$m_{\rm KK} = \frac{\sqrt{\pi}}{\mathcal{V}^{2/3}} M_{\rm Pl}$$

Scalar potential

○ α'-corrections and radiative corrections

$$V_F = V_{\alpha'} + V_{\rm CW}$$

In units of $\Lambda = m_{\rm KK} = 1$

$$M_{\rm Pl} = \frac{\mathcal{V}^{2/3}}{\sqrt{\pi}} \qquad W_0 \equiv \hat{W}_0 M_{\rm Pl}^3$$

$$V_F \simeq \frac{|\hat{W}_0|^2}{\pi^2}\left[\frac{3\xi}{4\mathcal{V}^{1/3}} + \frac{c_1}{32\pi^2}\frac{\pi}{\mathcal{V}^{2/3}}\right]$$

Volume modulus is perturbatively stabilized at

$$\langle \mathcal{V} \rangle \simeq 18664\left(-\frac{c_1/\xi}{10^3}\right)^3$$

From the positivity of $\langle \partial_{\mathcal{V}} \partial_{\mathcal{V}} V_F \rangle$ $c_1 > 0$ and $\xi < 0$

Volume modulus is perturbatively stabilized at

$$\langle \mathcal{V} \rangle \simeq 18664 \left(-\frac{c_1/\xi}{10^3} \right)^3$$

$c_1 > 0$ and $\xi < 0$

However, the potential energy becomes negative at this minimum,

$$V_F < 0$$

To achieve tiny cosmological constant,
we introduce the anti-D3 branes

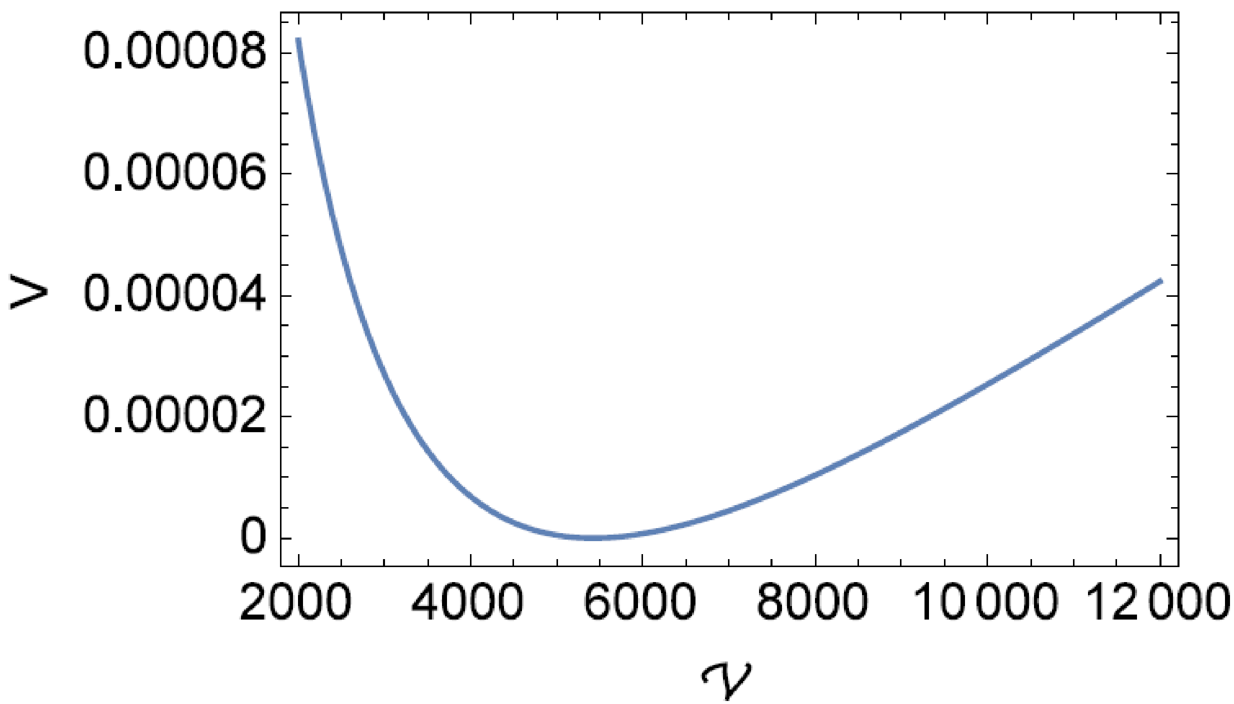
$$V_{\rm up} = \frac{\epsilon}{\mathcal{V}^2} M_{\rm Pl}^4$$

We have checked that such an uplifting term does not change the structure of moduli stabilization scenario.

○The scalar potential in units of $\Lambda = m_{\mathrm{KK}}$

$e^{\langle K(S,U)\rangle/2}|\hat{W}_0| = 1$ $g_s = 0.1$ $\xi = -0.1$

$n_i^a = 0,$ $c_b = 120$ $c_f = 12.$



○Mass scales of typical modes

Volume modulus

$$m_\tau \simeq 3.5 \times 10^{-3}\, |\hat{W}_0| \left(-\frac{10^3}{c_1/\xi}\right)^{3/2} (-\xi)^{1/2} m_{\mathrm{KK}}$$

Gravitino

$$m_{3/2} \simeq 3.2 \times 10^{-2} \left(-\frac{10^3}{c_1/\xi}\right) |\hat{W}_0| m_{\mathrm{KK}}$$

Dilaton and Complex structure moduli

$$m_{U,S} = N m_{3/2}$$

KK mode

$$m_{\mathrm{KK}} = 1$$

Stringy mode

$$m_{\mathrm{st}} = 2.4 \left(\frac{g_s}{10^{-1}}\right)^{1/4} \left(-\frac{10^3}{c_1/\xi}\right)^{1/2} m_{\mathrm{KK}}$$

Planck scale

$$M_{\mathrm{Pl}} = 176 \left(-\frac{c_1/\xi}{10^3}\right)^2 m_{\mathrm{KK}}$$

$$\boxed{m_\tau\,, m_{3/2} < m_U, m_S < m_{\mathrm{KK}} < m_{\mathrm{st}} < M_{\mathrm{Pl}}}$$

○Mass scales of typical modes

$$e^{\langle K(S,U)\rangle/2}|\hat{W}_0| = 1 \qquad g_s = 0.1 \qquad \xi = -0.1$$

$$c_1 \simeq c_{\rm b} - 2c_{\rm f} + 4 \qquad c_b^{(\rm MSSM)} = 52 \qquad c_f^{(\rm MSSM)} = 12 \qquad \mathcal{V} \simeq \kappa(T+\bar{T})^{3/2}$$

Scale	$c_1 = 50$	$c_1 = 100$	$c_1 = 1000$
$\mathcal{V}$	691	5530	5.5×10^6
τ	$39\kappa^{-2/3}$	$156\kappa^{-2/3}$	$1.6 \times 10^4\kappa^{-2/3}$
m_τ[GeV]	1.7×10^{14}	1.5×10^{13}	4.8×10^9
$m_{3/2}$[GeV]	3.5×10^{15}	4.4×10^{14}	4.4×10^{11}
$m_{U,S}$[GeV]	$3.5N \times 10^{15}$	$4.4N \times 10^{14}$	$4.4N \times 10^{11}$
$m_{\rm KK}$[GeV]	5.5×10^{16}	1.4×10^{16}	1.4×10^{14}
$m_{\rm st}$[GeV]	9.1×10^{16}	3.2×10^{16}	1.0×10^{15}
$M_{\rm Pl}$[GeV]	2.4×10^{18}	2.4×10^{18}	2.4×10^{18}

Radiative Kähler moduli stabilization

○ The inclusion of α'-corrections and radiative corrections

→ Stabilization of CY volume without tuning flux-induced superpotential

$$\langle \mathcal{V} \rangle \simeq 18664 \left(-\frac{c_1/\xi}{10^3} \right)^3$$

○ Large number of sparticle contributions

$$c_1 \simeq c_{\rm b} - 2c_{\rm f} + 4 = 10^{2-3}$$

○ Positive Euler number of CY

$$\xi \simeq -2.4 \times 10^{-3} \chi \, g_s^{-3/2} < 0$$

$$\chi = 2(h^{1,1} - h^{2,1}) > 0 \leftrightarrow \quad h^{1,1} > h^{2,1}$$

<u># of complex structure moduli < # of Kähler moduli</u>

Comment on the stabilization of other Kähler Moduli

(I) Non-perturbative effects

Similar to the LVS,

$$K = -2\ln(\mathcal{V}+\xi)$$

$$W = W_0 + A_s e^{-a_s T_s}$$

$$V_F = \underbrace{\frac{a_s^2|A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s|A_s W|\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{3\xi|W|^2}{4\mathcal{V}^3}}_{V_{\rm LVS}} + \underbrace{\frac{c_1}{32\pi^2}\Lambda^2 m_{3/2}^2}_{V_{\rm CW}}$$

Volume modulus and another modulus can be stabilized at

$$a_s\tau_s \sim \ln(\mathcal{V}) \qquad \mathcal{V} \simeq 18664\left(-\frac{c_1/\hat{\xi}}{10^3}\right)^3$$

$$\hat{\xi} \equiv \xi - \frac{4}{3}\frac{(\tau_s)^2}{(-\kappa_{ssi}t^i)}$$

Comment on the stabilization of other Kähler Moduli

(II) D-term stabilization

Anomalous U(1)s on hidden $D7_i$-branes (or fractional D3-branes)

$$V_D^{D7} = \sum_i \left(q_{T_i}^{(D7_i)} \partial_{T_i} K - q_m |\phi_m|^2 \right)^2$$

generate the moduli potential induced by the Fayet-Iliopoulos term.

Anomalous U(1) gauge bosons eat the linear combination of string axions.

For vanishing matter fields,
some moduli correspond to the bow-up modes (brane at singularities).

Ultralight axion

○Scalar potential is a function of CY volume.
→The axion associated with CY volume remains massless.

Axion potential is generated by the non-perturbative effects,

$$W = W_0 + e^{-\frac{2\pi}{n}T}$$

where CY volume is approximated as $\mathcal{V} \simeq (T+\bar{T})^{3/2}$

Axion mass and its decay constant

$$\frac{m_\theta}{m_{3/2}} \simeq \mathcal{V} e^{-\frac{\pi}{2n}\mathcal{V}^{2/3}} \qquad f_\theta \simeq \frac{\sqrt{6}n}{2\pi}\frac{M_{\rm Pl}}{\mathcal{V}^{2/3}}$$

Scale	$n=1$	$n=3$	$n=5$	$n=7$	$n=9$
$m_\theta/m_{3/2}$	2.3×10^{-209}	7.4×10^{-68}	9.7×10^{-40}	9.1×10^{-28}	3.7×10^{-21}
f_θ[GeV]	3×10^{15}	9×10^{15}	1.5×10^{16}	2×10^{16}	2.7×10^{16}

Kähler moduli stabilization

○KKLT scenario

[Kachru-Kallosh-Linde-Trivedi ’03]

Non-perturbative effects

Tuning : $|W_{\rm flux}| \ll M_{\rm Pl}^3$

○LARGE volume scenario

[Balasubramanian-Berglund-Conlon-Quevedo ’05]

Stringy corrections +Non-perturbative effects

No tuning : $|W_{\rm flux}| \sim M_{\rm Pl}^3$ Euler number of CY is negative

○Radiative Kähler moduli stabilization

[Kobayashi-Omoto-Otsuka-Tatsuishi ’17]

Stringy corrections + radiative corrections from sparticles

No tuning : $|W_{\rm flux}| \sim M_{\rm Pl}^3$ Euler number of CY is positive

Conclusion

○ Leading α'-and radiative corrections

→ **Stabilization of overall Kähler modulus**

if the Euler number of CY is positive

○ Other Kahler moduli can be stabilized by
D-terms and/or non-perturbative effects

○ Prediction of Ultralight axion

Discussion

○ Cosmology and phenomenology of the ultralight axion

○ Explicit moduli stabilization in a detailed setup