



Coherent elastic neutrino-nucleus scattering and Nonstandard neutrino Interactions

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with D. Marfatia, based on work in

Phys. Lett. B 775, 54 (2017)

[arXiv:1708.04255]

IBS CTPU, Daejeon, Republic of Korea, 11/27/2017

Outline

- COHERENT experiment
- Nonstandard Interactions
- COHERENT constraints on NSI
- Summary

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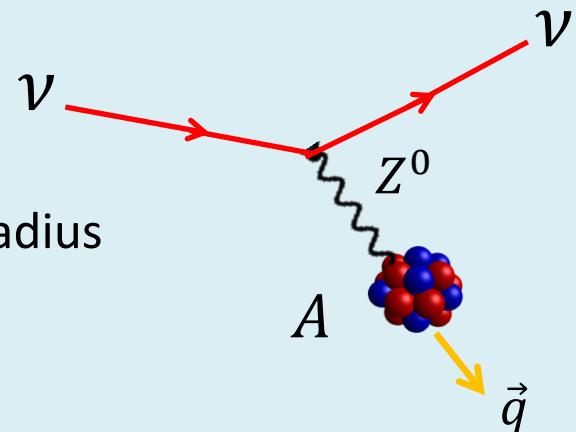
Coherent Elastic neutrino-Nucleus Scattering (CEvNS)

Moment transfer $\longrightarrow Q \lesssim 1/R \longleftarrow$ Nuclear radius

Can be Satisfied for $E_\nu < 50\text{MeV}$

$$\text{Nuclear recoil energy } E_r \leq \frac{2E_\nu^2}{M+2E_\nu} \sim 50 \text{ keV}$$

DM direct detection experiments \longrightarrow detection thresholds of 10 keV



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1 MARCH 1974

Coherent effects of a weak neutral current

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(Received 15 October 1973; revised manuscript received 19 November 1973)

If there is a weak neutral current, then the elastic scattering process $\nu + A \rightarrow \nu + A$ should have a sharp coherent forward peak just as $e + A \rightarrow e + A$ does. Experiments to observe this peak can give important information on the isospin structure of the neutral current. The experiments are very difficult, although the estimated cross sections (about 10^{-38} cm^2 on carbon) are favorable. The coherent cross sections (in contrast to incoherent) are almost energy-independent. Therefore, energies as low as 100 MeV may be suitable. Quasi-coherent nuclear excitation processes $\nu + A \rightarrow \nu + A^*$ provide possible tests of the conservation of the weak neutral current. Because of strong coherent effects at very low energies, the nuclear elastic scattering process may be important in inhibiting cooling by neutrino emission in stellar collapse and neutron stars.

CEvNS in the SM

$$\mathcal{L}_{NC}^{SM} = -\frac{G_F}{\sqrt{2}} \sum_{\substack{q=u,d \\ \alpha=e,\mu,\tau}} [\bar{\nu}_\alpha \gamma^\mu (1 - \gamma^5) \nu_\alpha] (g_L^q [\bar{q} \gamma_\mu (1 - \gamma^5) q] + g_R^q [\bar{q} \gamma_\mu (1 + \gamma^5) q])$$

$$\frac{d\sigma}{dE_r} = \frac{G_F^2 M}{2\pi} F^2(Q^2) \left[(G_V + G_A)^2 + (G_V - G_A)^2 \left(1 - \frac{E_r}{E_\nu}\right)^2 - (G_V^2 - G_A^2) \frac{M E_r}{E_\nu^2} \right]$$

$$G_V = g_p^V Z + g_n^V N$$

$F(Q^2)$: Nuclear form factor $Q^2 = 2ME_r$

$$G_A = g_p^A (Z_+ - Z_-) + g_n^A (N_+ - N_-)$$

→ Small for most nuclei

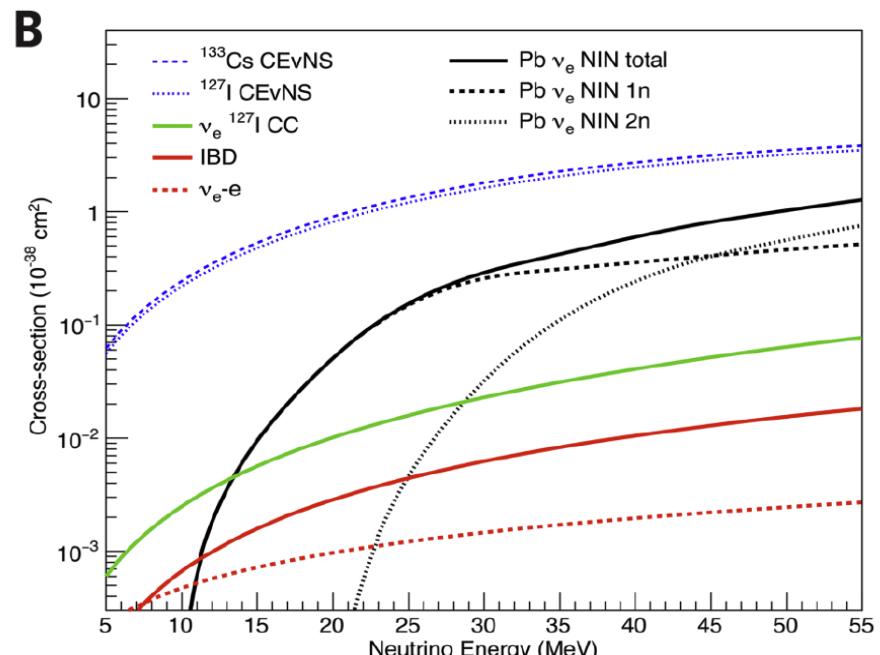
For $E_r \ll E_\nu$

$$\frac{d\sigma_\alpha}{dE_r} = \frac{G_F^2}{2\pi} Q_\alpha^2 F^2(2ME_r) M \left(2 - \frac{ME_r}{E_\nu^2}\right)$$

SM nuclear Charge $Q_{\alpha,SM}^2 = (Zg_p^V + Ng_n^V)^2$

$$g_p^V = \frac{1}{2} - 2 \sin^2 \theta_W \sim 0.04$$

$$g_n^V = -\frac{1}{2} \quad \rightarrow \quad \sigma_{SM} \propto N^2$$



COHERENT experiment

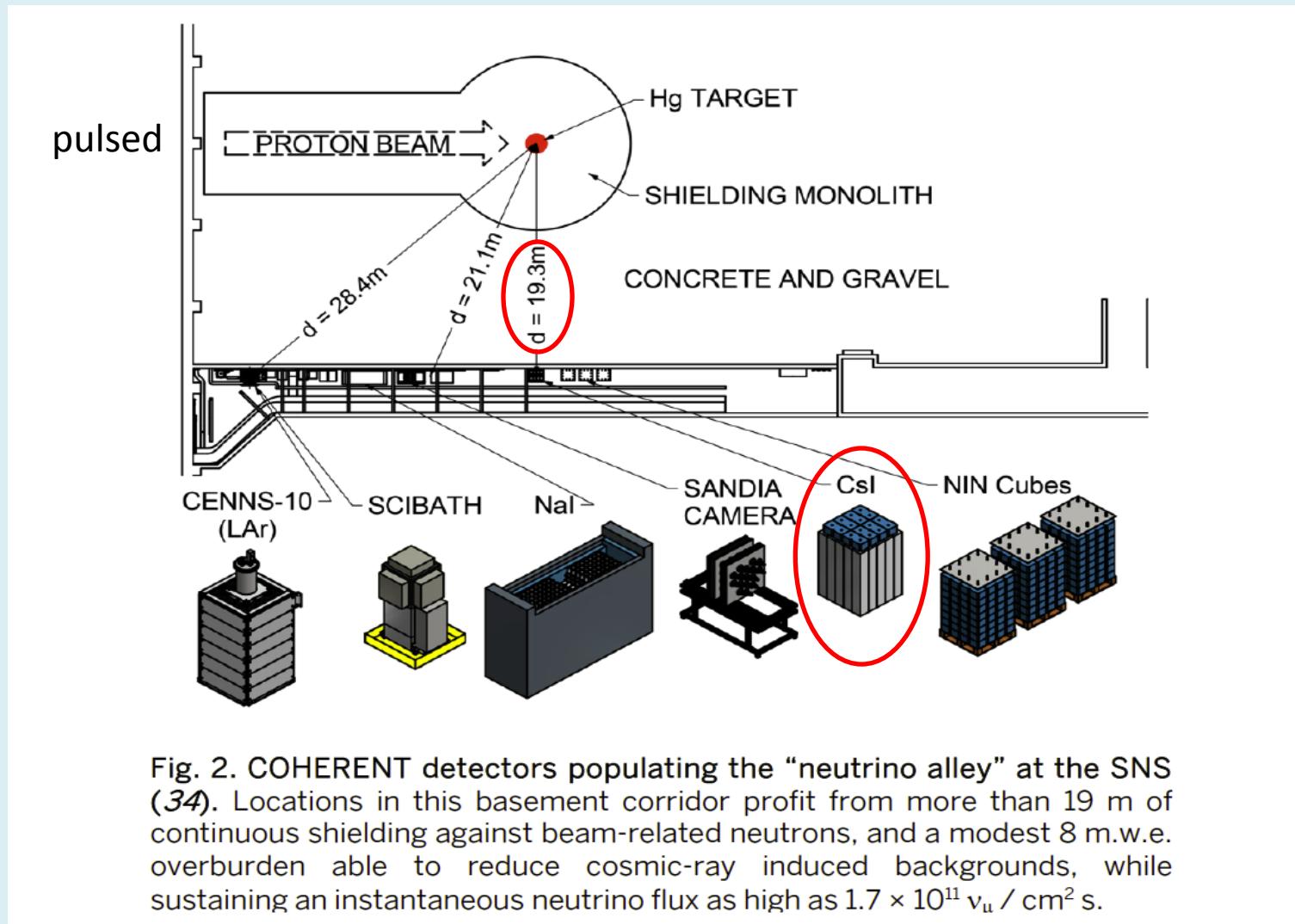


Fig. 2. COHERENT detectors populating the “neutrino alley” at the SNS (34). Locations in this basement corridor profit from more than 19 m of continuous shielding against beam-related neutrons, and a modest 8 m.w.e. overburden able to reduce cosmic-ray induced backgrounds, while sustaining an instantaneous neutrino flux as high as $1.7 \times 10^{11} \nu_{\text{u}} / \text{cm}^2 \text{s}$.

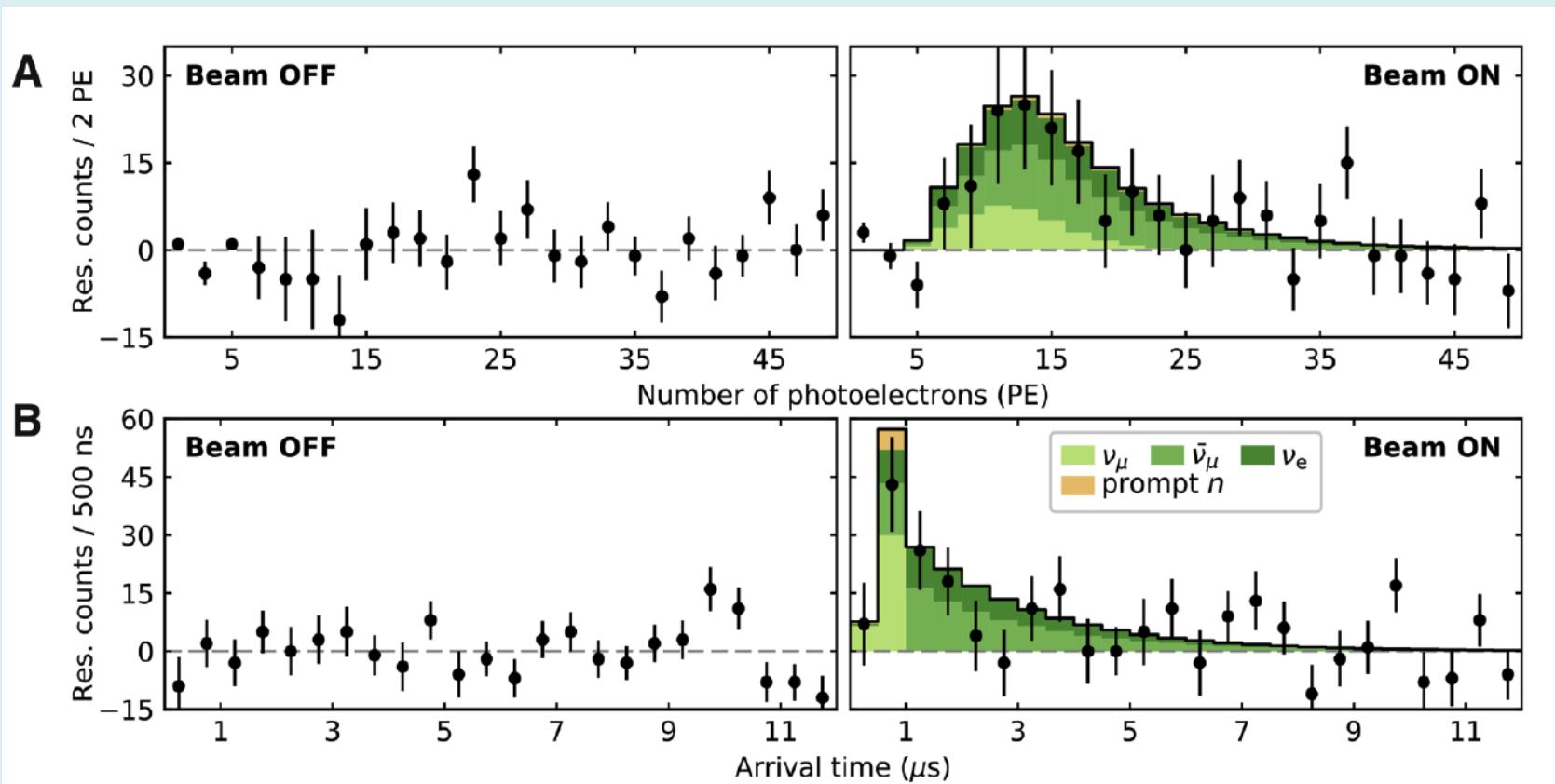


COHERENT data

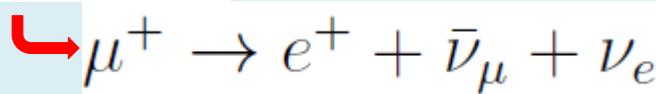
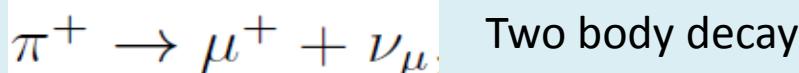
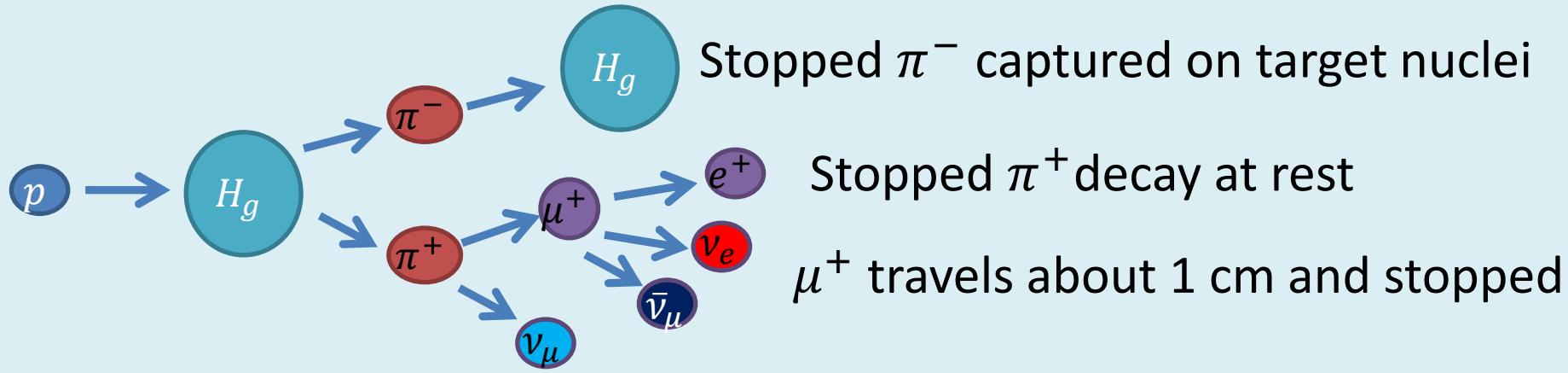
134 ± 22 events observed

173 ± 48 events predicted in the SM

6.7σ CL evidence for CEvNS



Neutrino Flux

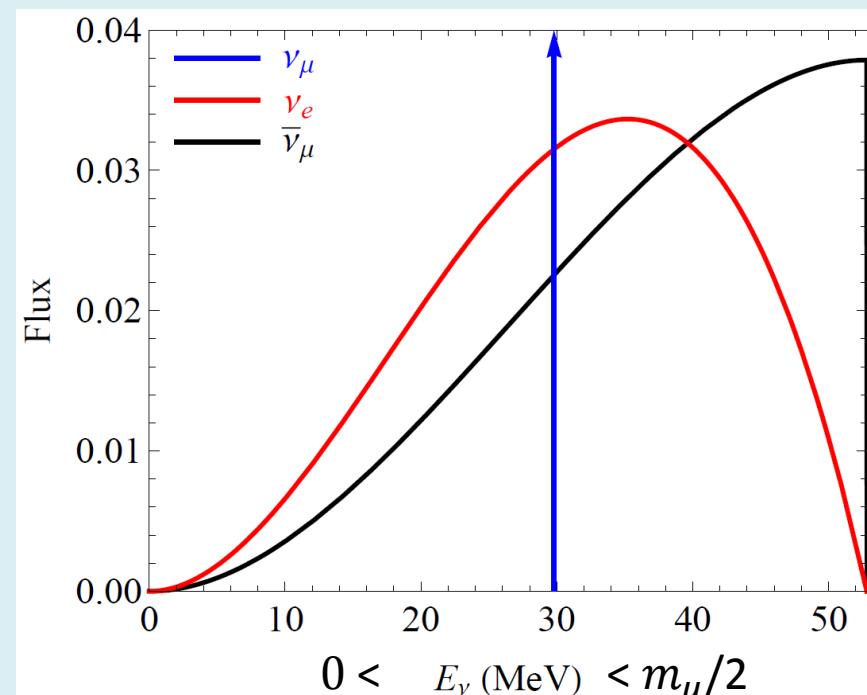


Three body decay

$$\phi_{\nu_\mu}(E_\nu) = \delta \left(E_\nu - \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \right),$$

$$\phi_{\bar{\nu}_\mu}(E_\nu) = \frac{64E_\nu^2}{m_\mu^3} \left(\frac{3}{4} - \frac{E_\nu}{m_\mu} \right),$$

$$\phi_{\nu_e}(E_\nu) = \frac{192E_\nu^2}{m_\mu^3} \left(\frac{1}{2} - \frac{E_\nu}{m_\mu} \right),$$



$$N_{\alpha}^i = \frac{r N_{\text{POT}}}{4\pi L^2} \times \frac{2m_{\text{det}}}{M_{\text{CsI}}} N_A \times \int dn_{\text{PE}} f(n_{\text{PE}}) \frac{dE_r}{dn_{\text{PE}}} \int dE_{\nu} \phi_{\alpha}(E_{\nu}) \frac{d\sigma_{\alpha}}{dE_r}(E_{\nu}, E_r)$$

$r = 0.08$ is the number of neutrinos per flavor for each proton on target

$N_{\text{POT}} = 1.76 \times 10^{23}$ is the total number of protons delivered to the target

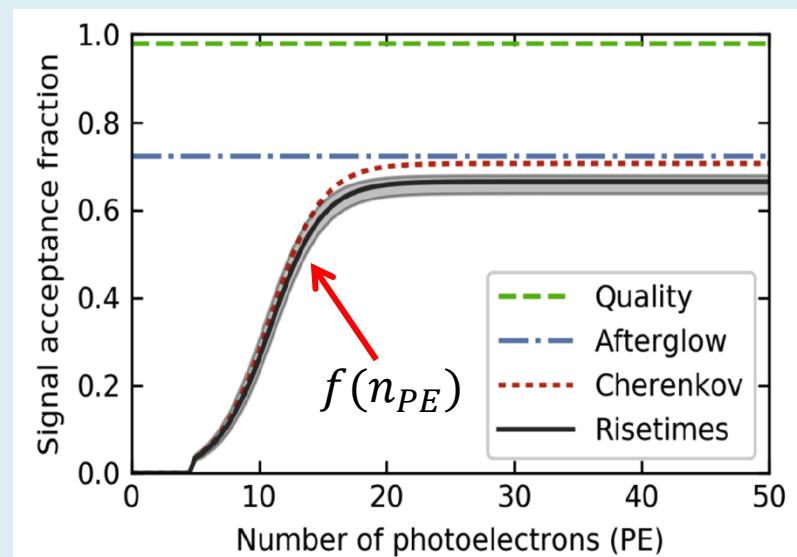
$L = 19.3 \text{ m}$ is the distance between the source and the CsI detector

$m_{\text{det}} = 14.6 \text{ kg}$ is the mass of detector, M_{CsI} is the molar mass of CsI

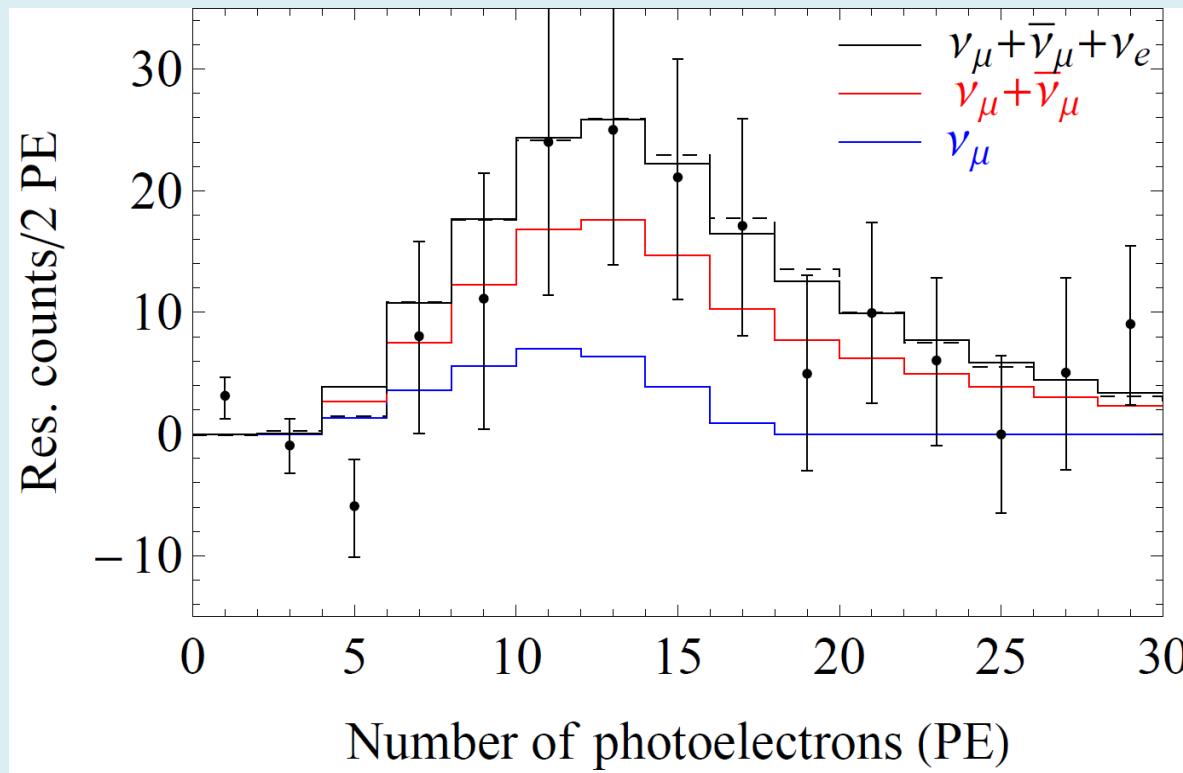
“Approximately 1.17 photoelectrons are expected per keV of cesium or iodine nuclear recoil energy”

$$n_{\text{PE}} = 1.17 \left(\frac{E_r}{\text{keV}} \right)$$

COHERENT Collaboration, Science (2017)



SM spectrum



Sterile neutrinos: Anderson et al. 2012; Dutta et al. 2015; Kosmas et al. 2017

Neutrinos magnetic moment: Dodd et al. 1991; Scholberg 2005; Kosmas et al. 2015

Light dark matter: deNiverville et al. 2015

Nonstandard neutrino interactions: Barranco et al. 2005, 2007; Scholberg 2005; Dutta et al. 2015; Lindner et al. 2016; Dent et al. 2016; Coloma et al. 2017; Shoemaker 2017

see also COHERENT Collaboration, Science (2017), Coloma et al. 1708.02899

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Matter NSI



$$\mathcal{L}_{NC}^{SM} = -\frac{G_F}{\sqrt{2}} \sum_{\substack{q=u,d \\ \alpha=e,\mu,\tau}} [\bar{\nu}_\alpha \gamma^\mu (1 - \gamma^5) \nu_\alpha] (g_L^q [\bar{q} \gamma_\mu (1 - \gamma^5) q] + g_R^q [\bar{q} \gamma_\mu (1 + \gamma^5) q])$$

$$\mathcal{L}_{NC}^{NSI} = -\frac{G_F}{\sqrt{2}} \sum_{\substack{q=u,d \\ \alpha,\beta=e,\mu,\tau}} [\bar{\nu}_\alpha \gamma^\mu (1 - \gamma^5) \nu_\beta] (\varepsilon_{\alpha\beta}^{qL} [\bar{q} \gamma_\mu (1 - \gamma^5) q] + \varepsilon_{\alpha\beta}^{qR} [\bar{q} \gamma_\mu (1 + \gamma^5) q])$$

Wolfenstein, Phys. Rev. D 17, 2369 (1978)

Modification of matter potential via the MSW mechanism

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + A \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

SM CC

Effective parameters

$$\varepsilon_{\alpha\beta} \equiv \sum_{f,C} \varepsilon_{\alpha\beta}^{fC} \frac{N_f}{N_e}$$

$$A \equiv 2\sqrt{2}G_F N_e E$$

On earth $N_u = N_d = 3N_e$

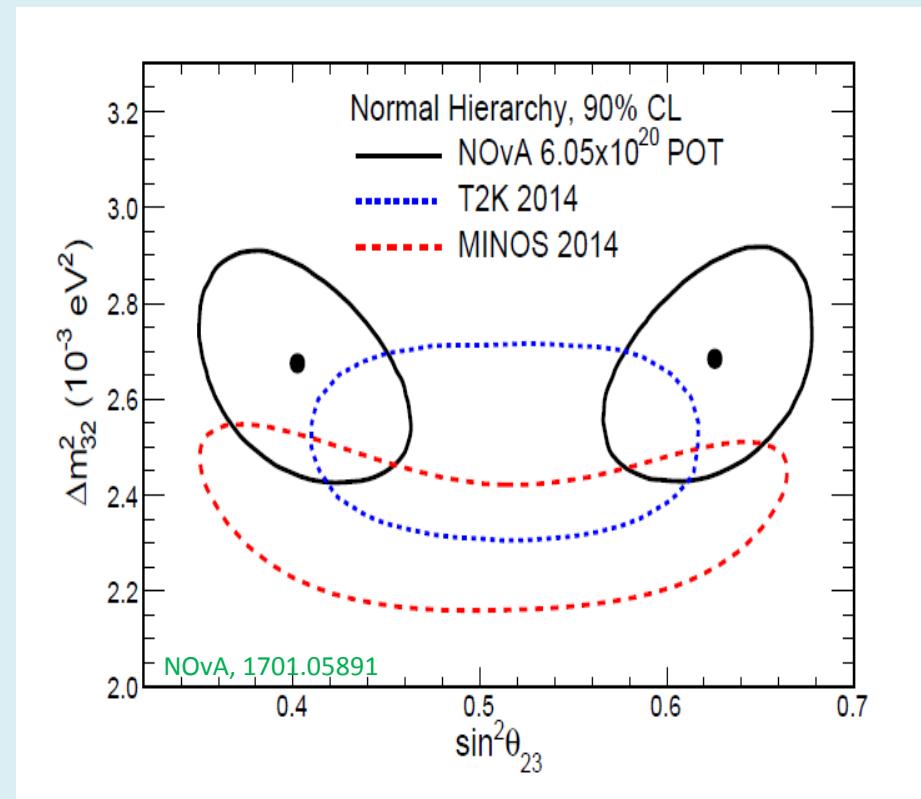
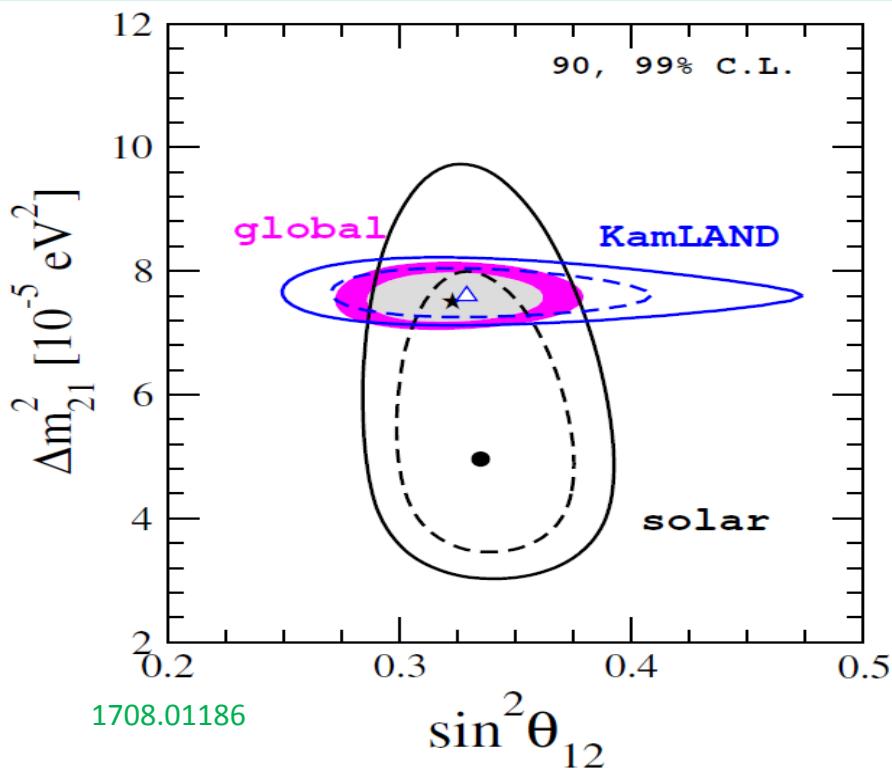
$$\epsilon_{\alpha\beta}^{fV} \equiv \epsilon_{\alpha\beta}^{fL} + \epsilon_{\alpha\beta}^{fR}$$

JL, D. Marfatia, K. Whisnant,
Phys. Lett. B 767, 350 (2017)

NOvA: 810 km, $E \sim 2$ GeV

T2K: 295 km, $E \sim 0.6$ GeV

$$\epsilon_{\tau\tau} = 0.6 \text{ and } \epsilon_{e\tau} = 1.2$$



Solar neutrinos produced at the center of the Sun, and travel large distance through the Earth during the night
KamLAND: $\langle L \rangle \sim 180$ km



$$\epsilon_{ee}^u = \epsilon_{ee}^d \sim 0.1$$

JL, D. Marfatia, K. Whisnant,
Phys. Lett. B 771, 247(2017)

Global Analysis

Gonzalez-Garcia, Maltoni [1307.3092]

		90% CL		3 σ	
Param.	best-fit	LMA	LMA \oplus LMA-D	LMA	LMA \oplus LMA-D
$\varepsilon_{ee}^u - \varepsilon_{\mu\mu}^u$	+0.298	[+0.00, +0.51]	\oplus [-1.19, -0.81]	[-0.09, +0.71]	\oplus [-1.40, -0.68]
$\varepsilon_{\tau\tau}^u - \varepsilon_{\mu\mu}^u$	+0.001	[-0.01, +0.03]	[-0.03, +0.03]	[-0.03, +0.20]	[-0.19, +0.20]
$\varepsilon_{e\mu}^u$	-0.021	[-0.09, +0.04]	[-0.09, +0.10]	[-0.16, +0.11]	[-0.16, +0.17]
$\varepsilon_{e\tau}^u$	+0.021	[-0.14, +0.14]	[-0.15, +0.14]	[-0.40, +0.30]	[-0.40, +0.40]
$\varepsilon_{\mu\tau}^u$	-0.001	[-0.01, +0.01]	[-0.01, +0.01]	[-0.03, +0.03]	[-0.03, +0.03]
ε_D^u	-0.140	[-0.24, -0.01]	\oplus [+0.40, +0.58]	[-0.34, +0.04]	\oplus [+0.34, +0.67]
ε_N^u	-0.030	[-0.14, +0.13]	[-0.15, +0.13]	[-0.29, +0.21]	[-0.29, +0.21]
$\varepsilon_{ee}^d - \varepsilon_{\mu\mu}^d$	+0.310	[+0.02, +0.51]	\oplus [-1.17, -1.03]	[-0.10, +0.71]	\oplus [-1.44, -0.87]
$\varepsilon_{\tau\tau}^d - \varepsilon_{\mu\mu}^d$	+0.001	[-0.01, +0.03]	[-0.01, +0.03]	[-0.03, +0.19]	[-0.16, +0.19]
$\varepsilon_{e\mu}^d$	-0.023	[-0.09, +0.04]	[-0.09, +0.08]	[-0.16, +0.11]	[-0.16, +0.17]
$\varepsilon_{e\tau}^d$	+0.023	[-0.13, +0.14]	[-0.13, +0.14]	[-0.38, +0.29]	[-0.38, +0.35]
$\varepsilon_{\mu\tau}^d$	-0.001	[-0.01, +0.01]	[-0.01, +0.01]	[-0.03, +0.03]	[-0.03, +0.03]
ε_D^d	-0.145	[-0.25, -0.02]	\oplus [+0.49, +0.57]	[-0.34, +0.05]	\oplus [+0.42, +0.70]
ε_N^d	-0.036	[-0.14, +0.12]	[-0.14, +0.12]	[-0.28, +0.21]	[-0.28, +0.21]

Future experiments

JL, D. Marfatia, K. Whisnant, JHEP 1701, 071 (2017)

DUNE: 1300 km, $E \sim 3$ GeV

T2HK: 295 km, $E \sim 0.6$ GeV

T2HKK-1.5: 295 km, $E \sim 0.6$ GeV
1100 km, $E \sim 0.8$ GeV

T2HKK-2.5: 295 km, $E \sim 0.6$ GeV
1100 km, $E \sim 0.6$ GeV

Generalized MH

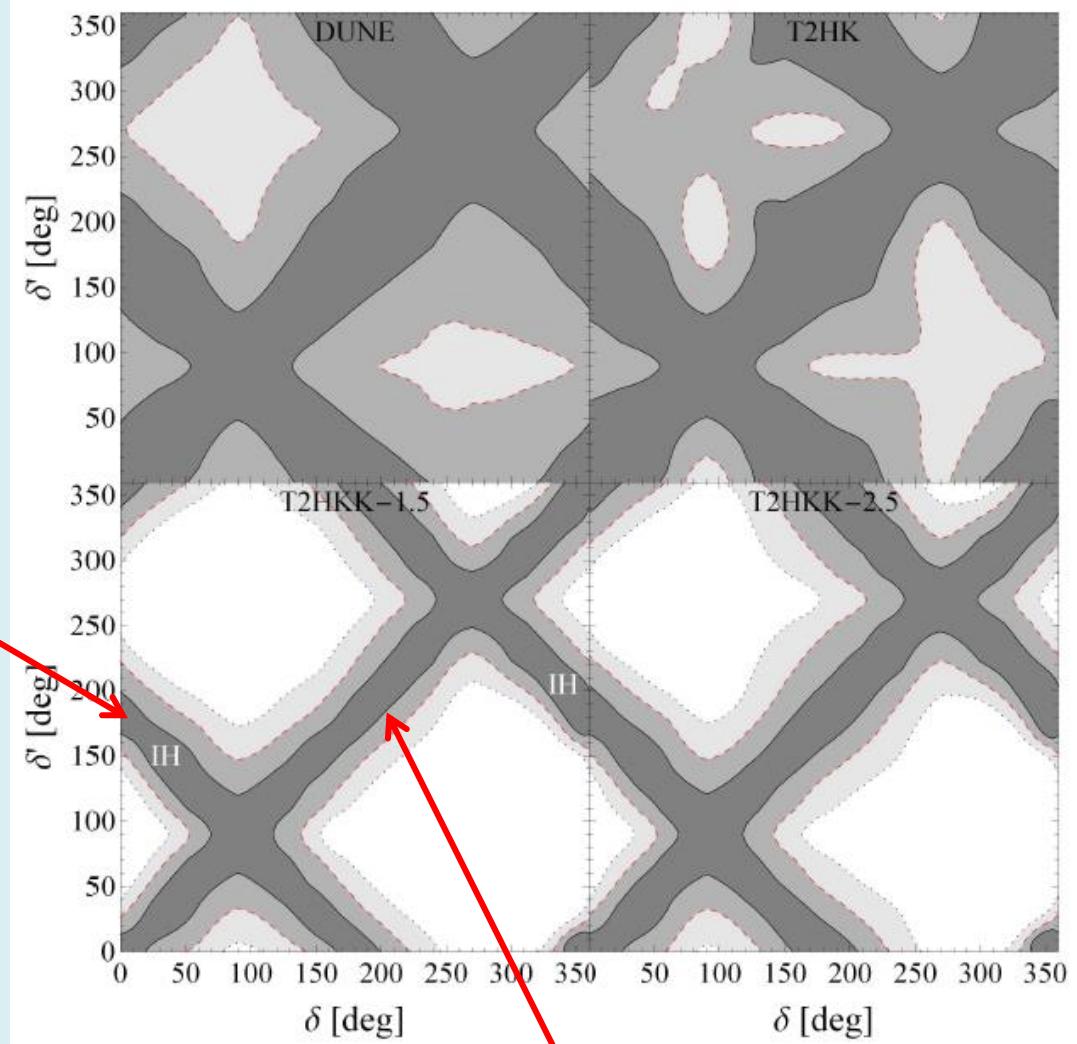
$$\Delta m_{31}^2 \rightarrow -\Delta m_{31}^2$$
$$\sin \theta_{12} \leftrightarrow \cos \theta_{12}$$
$$\delta \rightarrow \pi - \delta$$

$$\epsilon_{ee} \rightarrow -\epsilon_{ee} - 2,$$
$$\epsilon_{\alpha\beta} \rightarrow -\epsilon_{\alpha\beta}^* \quad (\alpha\beta \neq ee)$$

→ $H \rightarrow -H^*$

Same oscillation probabilities

Coloma, Schwetz [1604.05772]

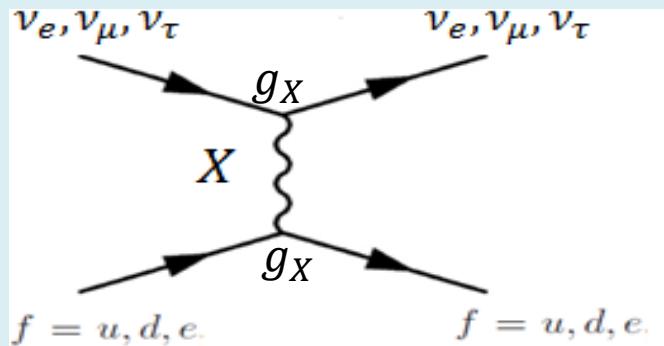


$\delta' = \delta$ holds when $\epsilon = 0$

Large NSI Models

Above EW scale, CLFV impose very tight constraints: $\varepsilon < \mathcal{O}(10^{-3})$

Below EW scale, large NSI is generated with a light gauge boson X



$$\text{propagator} \propto \frac{g_X^2}{q^2 - m_X^2}$$

Farzan, Phys. Lett. B748, 311 (2015)

Farzan and Shoemaker, JHEP 07 033 (2016)

Farzan and Heeck, Phys. Rev. D94, 053010 (2016)

Coherent forward scattering

$$q^2 \rightarrow 0$$

$$\varepsilon \propto \frac{g_X^2 m_W^2}{m_X^2}$$

$$g_X \sim 10^{-5}, m_X \sim 10 \text{ MeV}, \varepsilon \sim \mathcal{O}(1)$$

NuTeV and CHARM: $q^2 \geq 10 \text{ GeV}$

For $q^2 \gg m_X^2$, sensitivity to NSI is suppressed by $\frac{g_X^2}{q^2} m_X^2$

COHERENT?

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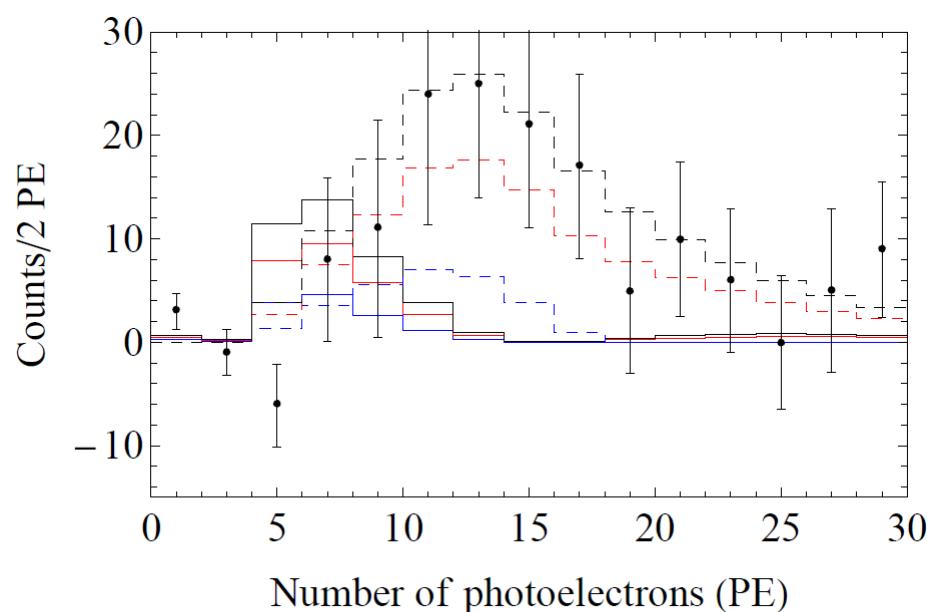
Light mediator

A purely vector mediator

$$\mathcal{L}_{\text{NSI}} = -g (\bar{\nu} \gamma^\rho \nu + \bar{\mu} \gamma^\rho \mu + \bar{u} \gamma^\rho u + \bar{d} \gamma^\rho d) Z'_\rho$$

Modified effective nuclear charge

$$Q_{\alpha, \text{NSI}}^2 = \left[Z \left(g_p^V + \frac{3g^2}{2\sqrt{2}G_F(Q^2 + M_{Z'}^2)} \right) + N \left(g_n^V + \frac{3g^2}{2\sqrt{2}G_F(Q^2 + M_{Z'}^2)} \right) \right]^2$$



Moment transfer

$$Q^2 = 2ME_r$$

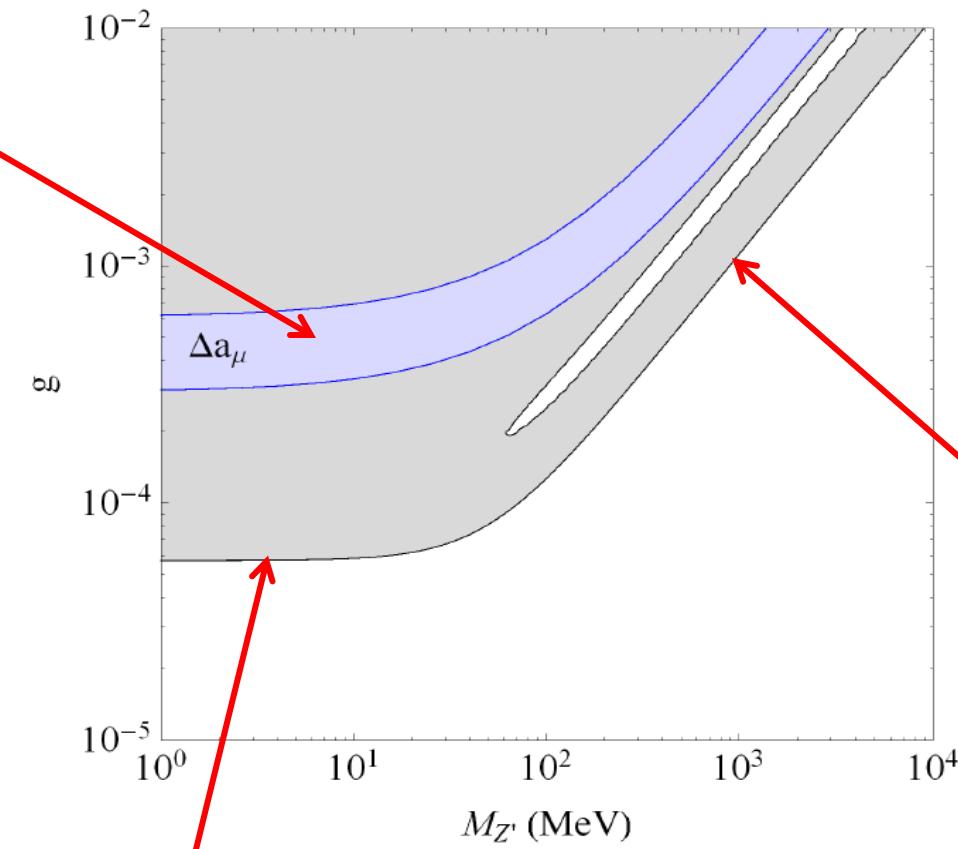
$$M_{Z'} = 10 \text{ MeV} \text{ and } g = 10^{-4}$$

$$\chi^2 = \sum_i \left[\frac{N_{\text{exp}}^i - N_{\text{NSI}}^i(1 + \alpha)}{\sigma_{\text{stat}}^i} \right]^2 + \left(\frac{\alpha}{\sigma_\alpha} \right)^2$$

12 bins, $6 \leq \text{PE} < 30$

$$\sigma_\alpha = 0.28$$

Explains
 Δa_μ at 2σ



Propagator

$$\text{NSI} \propto \frac{g^2}{2ME_r + M_{Z'}^2}$$

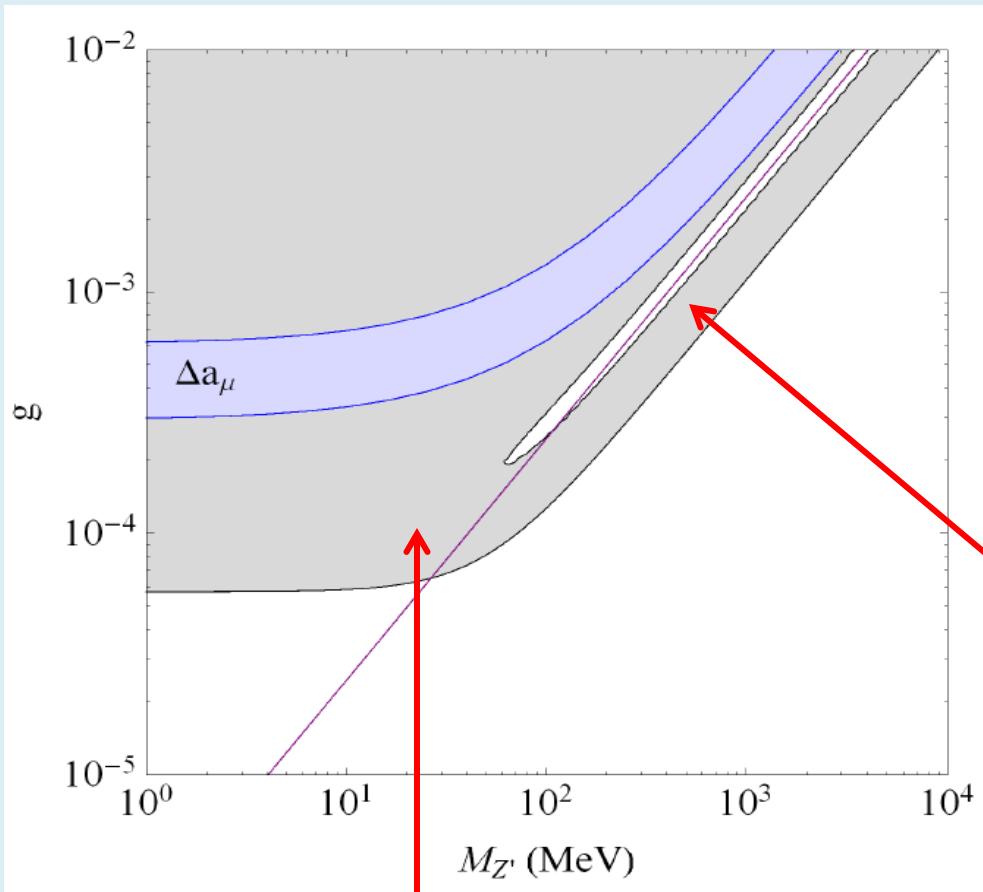
For heavy mediators, i.e., $M_{Z'} \gg \sqrt{2ME_r}$, the limit depends on $\frac{g^2}{M_{Z'}^2}$ and can constrain NSI matter effect

For very light mediators, i.e., $M_{Z'} < 50$ MeV, the limit is only sensitive to g , while the NSI matter effect is sensitive to $\frac{g^2}{M_{Z'}^2}$, hence the COHERENT constraint does not apply to matter NSI induced by a very light mediator.

$$\chi^2 = \sum_i \left[\frac{N_{\text{exp}}^i - N_{\text{NSI}}^i(1 + \alpha)}{\sigma_{\text{stat}}^i} \right]^2 + \left(\frac{\alpha}{\sigma_\alpha} \right)^2$$

12 bins, $6 \leq \text{PE} < 30$

$$\sigma_\alpha = 0.28$$



$$Q_{\alpha, \text{NSI}} = -Q_{\alpha, \text{SM}}$$

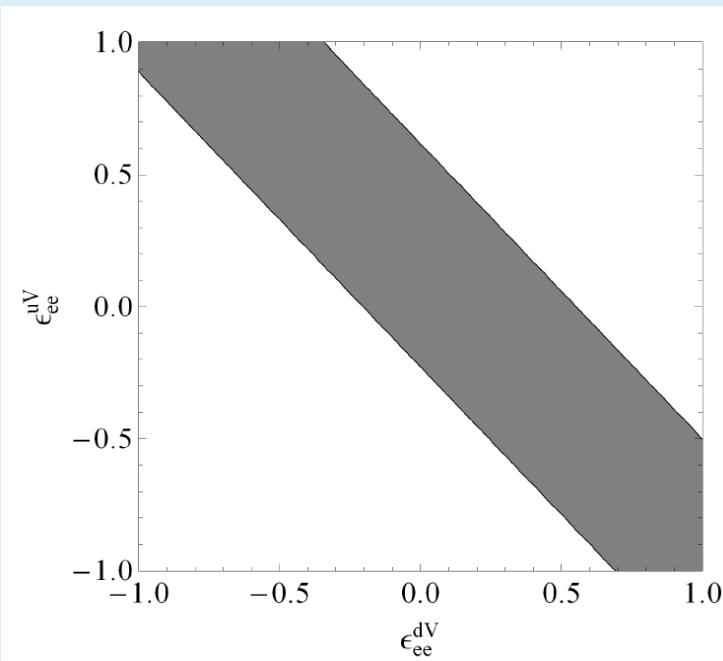
$$\frac{g^2}{M_{Z'}^2} = -\frac{4\sqrt{2}(Zg_p^V + Ng_n^V)}{3(Z+N)}G_F$$

For very light mediator, modification of the spectral shapes breaks the degeneracy.

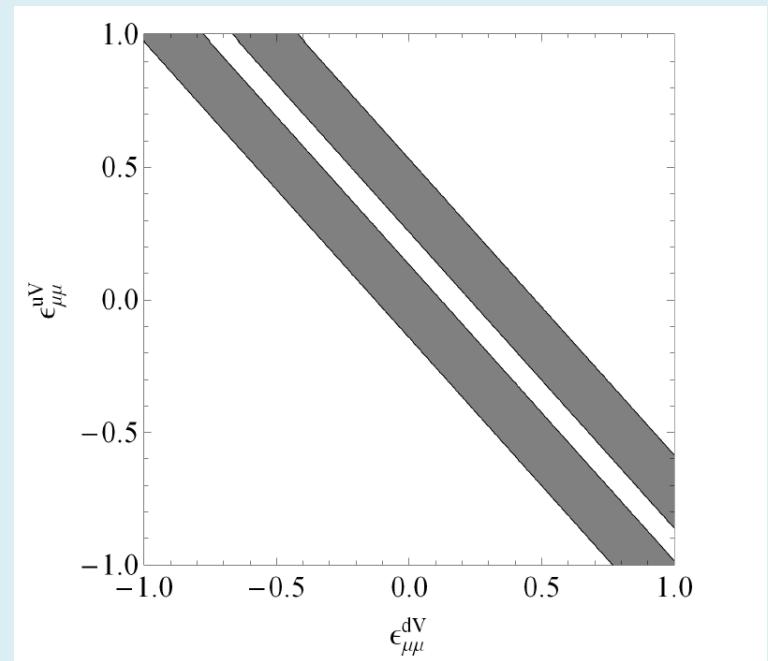
Heavy mediator

$$Q_\alpha^2 = [Z(g_p^V + 2\epsilon_{\alpha\alpha}^{uV} + \epsilon_{\alpha\alpha}^{dV}) + N(g_n^V + \epsilon_{\alpha\alpha}^{uV} + 2\epsilon_{\alpha\alpha}^{dV})]^2 + \sum_{\beta \neq \alpha} [Z(2\epsilon_{\alpha\beta}^{uV} + \epsilon_{\alpha\beta}^{dV}) + N(\epsilon_{\alpha\beta}^{uV} + 2\epsilon_{\alpha\beta}^{dV})]^2.$$

(a) $\epsilon_{ee}^{uV} \neq 0, \epsilon_{ee}^{dV} \neq 0.$



(b) $\epsilon_{\mu\mu}^{uV} \neq 0, \epsilon_{\mu\mu}^{dV} \neq 0.$



$$Zg_p^V + Ng_n^V = \pm [Z(g_p^V + 2\epsilon_{\alpha\alpha}^{uV} + \epsilon_{\alpha\alpha}^{dV}) + N(g_n^V + \epsilon_{\alpha\alpha}^{uV} + 2\epsilon_{\alpha\alpha}^{dV})]$$



Two Linear bands

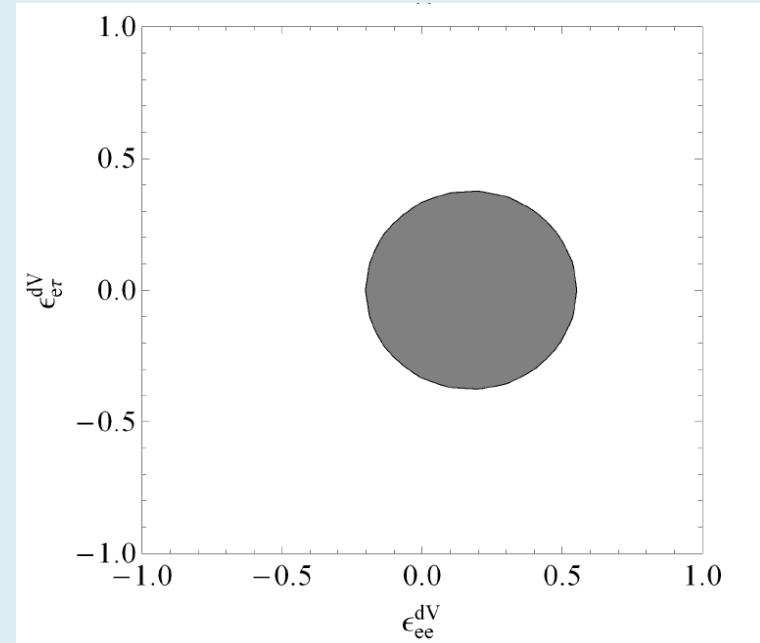
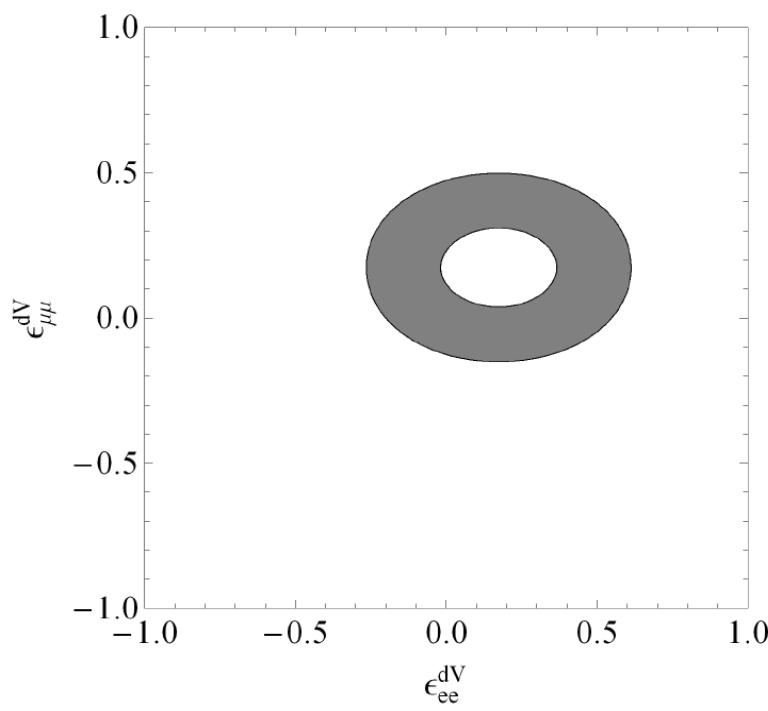
Cancellation between the NSI coupling to up and down quarks permit large values of fundamental NSI parameters

Heavy mediator

$$Q_\alpha^2 = \left[Z(g_p^V + 2\epsilon_{\alpha\alpha}^{uV} + \epsilon_{\alpha\alpha}^{dV}) + N(g_n^V + \epsilon_{\alpha\alpha}^{uV} + 2\epsilon_{\alpha\alpha}^{dV}) \right]^2 + \sum_{\beta \neq \alpha} \left[Z(2\epsilon_{\alpha\beta}^{uV} + \epsilon_{\alpha\beta}^{dV}) + N(\epsilon_{\alpha\beta}^{uV} + 2\epsilon_{\alpha\beta}^{dV}) \right]^2.$$

(c) $\epsilon_{ee}^{dV} \neq 0, \epsilon_{\mu\mu}^{dV} \neq 0.$

(d) $\epsilon_{ee}^{dV} \neq 0, \epsilon_{e\tau}^{dV} \neq 0.$



$$(Zg_p^V + Ng_n^V)^2 = \left[Z(g_p^V + \epsilon_{ee}^{dV}) + N(g_n^V + 2\epsilon_{ee}^{dV}) \right]^2 + \left[Z\epsilon_{e\tau}^{dV} + 2N\epsilon_{e\tau}^{dV} \right]^2$$

Effective NSI parameters

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + A \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

Effective parameters

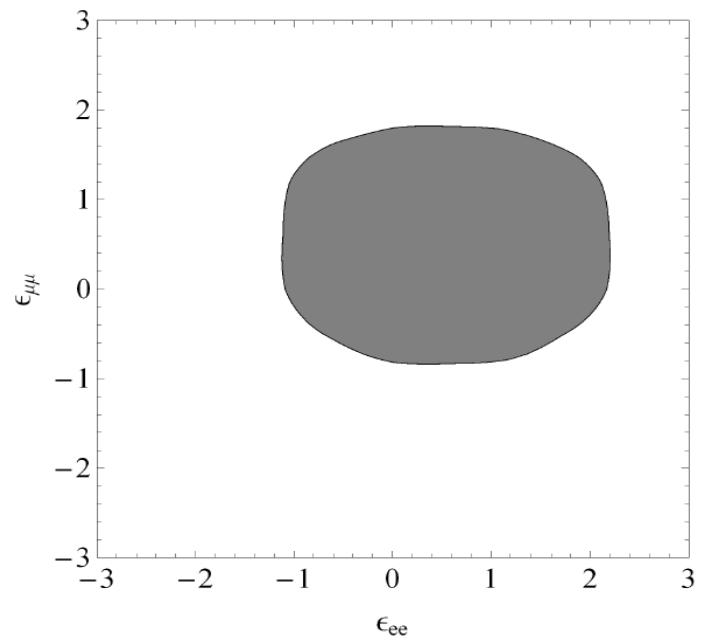
On earth $N_u = N_d = 3N_e$

$$\epsilon_{\alpha\beta} \equiv \sum_f \epsilon_{\alpha\beta}^{fV} \frac{N_f}{N_e}$$

$$\epsilon_{\alpha\alpha} \approx 3(\epsilon_{\alpha\alpha}^{uV} + \epsilon_{\alpha\alpha}^{dV})$$

$$-0.95 \leq \epsilon_{ee} \leq 1.95, \quad -0.66 \leq \epsilon_{\mu\mu} \leq 1.57.$$

For heavy mediators $M_Z' > 50$ MeV
 $\epsilon_{ee} = -2$ is excluded at 90% CL



Summary

Parameter degeneracies due to the NSI parameters strongly affect sensitivities of future neutrino oscillation experiments.

COHERENT constraints on NSI depend on the mediator mass.

- For a **lighter** mediator, COHERENT data only constrain the mediator coupling, and large NSI can still be generated by a very light mediator.
- For a **heavier** mediator, the COHERENT constraints are weakened by degeneracies between different combinations of fundamental NSI parameters. However, the data can place meaningful constraints on the effective NSI parameters in Earth matter for mediator $M_Z > 50$ MeV.

Thank you!

Backup slides

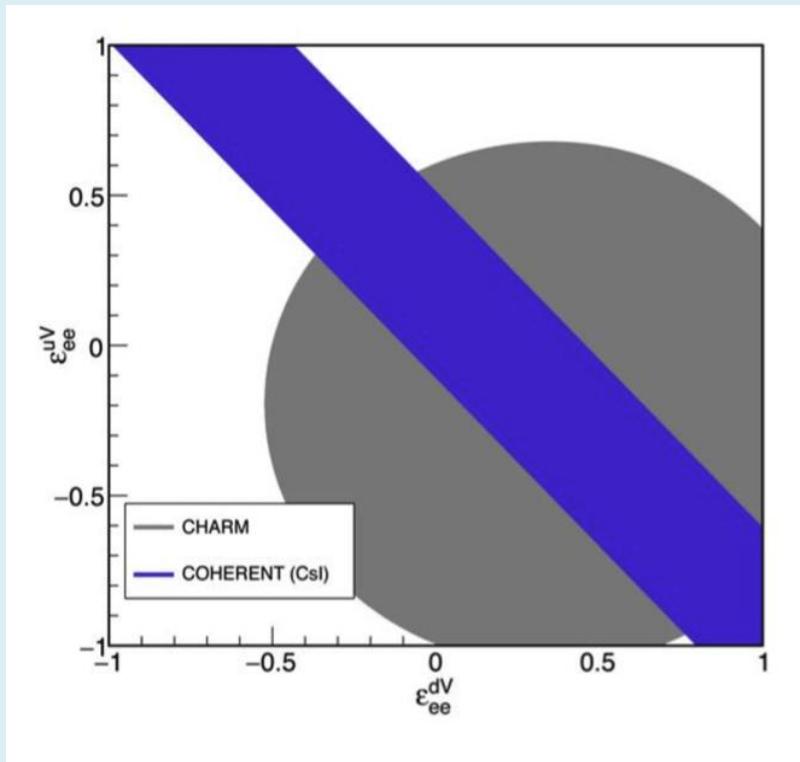
Next generation long baseline experiments

JL, D. Marfatia, K. Whisnant, JHEP 1701, 071 (2017)

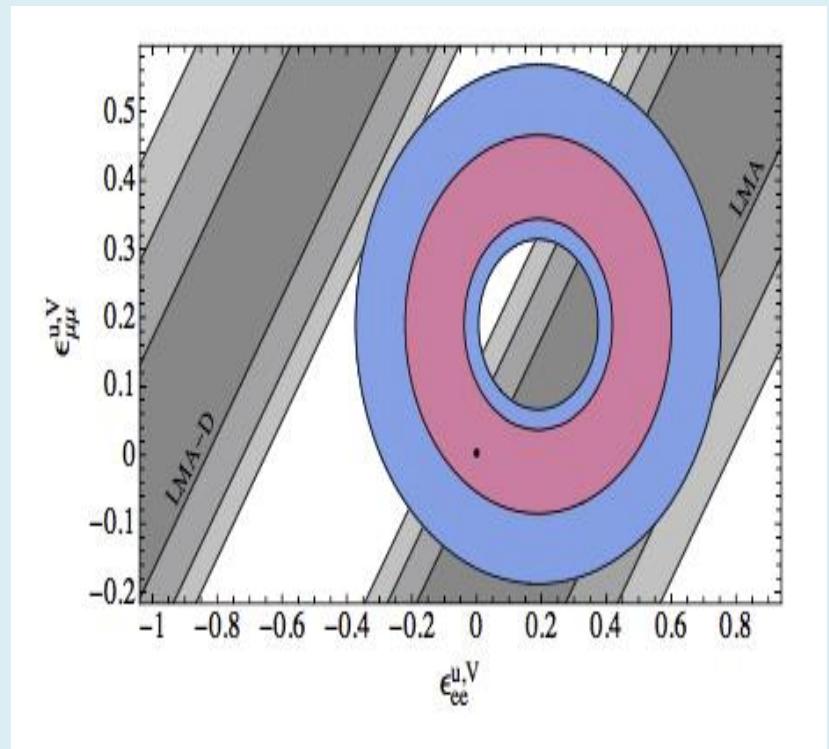
Experiment	$\frac{L(\text{km})}{E_{\text{peak}}(\text{GeV})}$	$\nu + \bar{\nu}$ Exposure (kt·MW· 10^7 s)	Signal norm. uncertainty	Background norm. uncertainty
DUNE (LAr)	$\frac{1300}{3.0}$	264 + 264 (80 GeV protons, 1.07 MW power, 1.47×10^{21} POT/yr, 40 kt fiducial mass, 3.5+3.5 yr)	app: 2.0% dis: 5.0%	app: 5-20% dis: 5-20%
T2HK (WC)	$\frac{295}{0.6}$	864.5 + 2593.5 (30 GeV protons, 1.3 MW power, 2.7×10^{21} POT/yr, 0.19 Mt each tank, 1.5+4.5 yr with 1 tank, 1+3 yr with 2 tanks)	app: 2.5% dis: 2.5%	app: 5% dis: 20%
T2HKK-1.5 (WC)	$\frac{295}{0.6} + \frac{1100}{0.8}$	1235 + 3705 (30 GeV protons, 1.3 MW power, 2.7×10^{21} POT/yr, 0.19 Mt each tank, 2.5+7.5 yr with 1 tank at KD and HK)	app: 2.5% dis: 2.5%	app: 5% dis: 20%
T2HKK-2.5 (WC)	$\frac{295}{0.6} + \frac{1100}{0.6}$			

For DUNE, 1 yr = 1.76×10^7 s; for HyperK, 1 yr = 1.0×10^7 s.

Other results

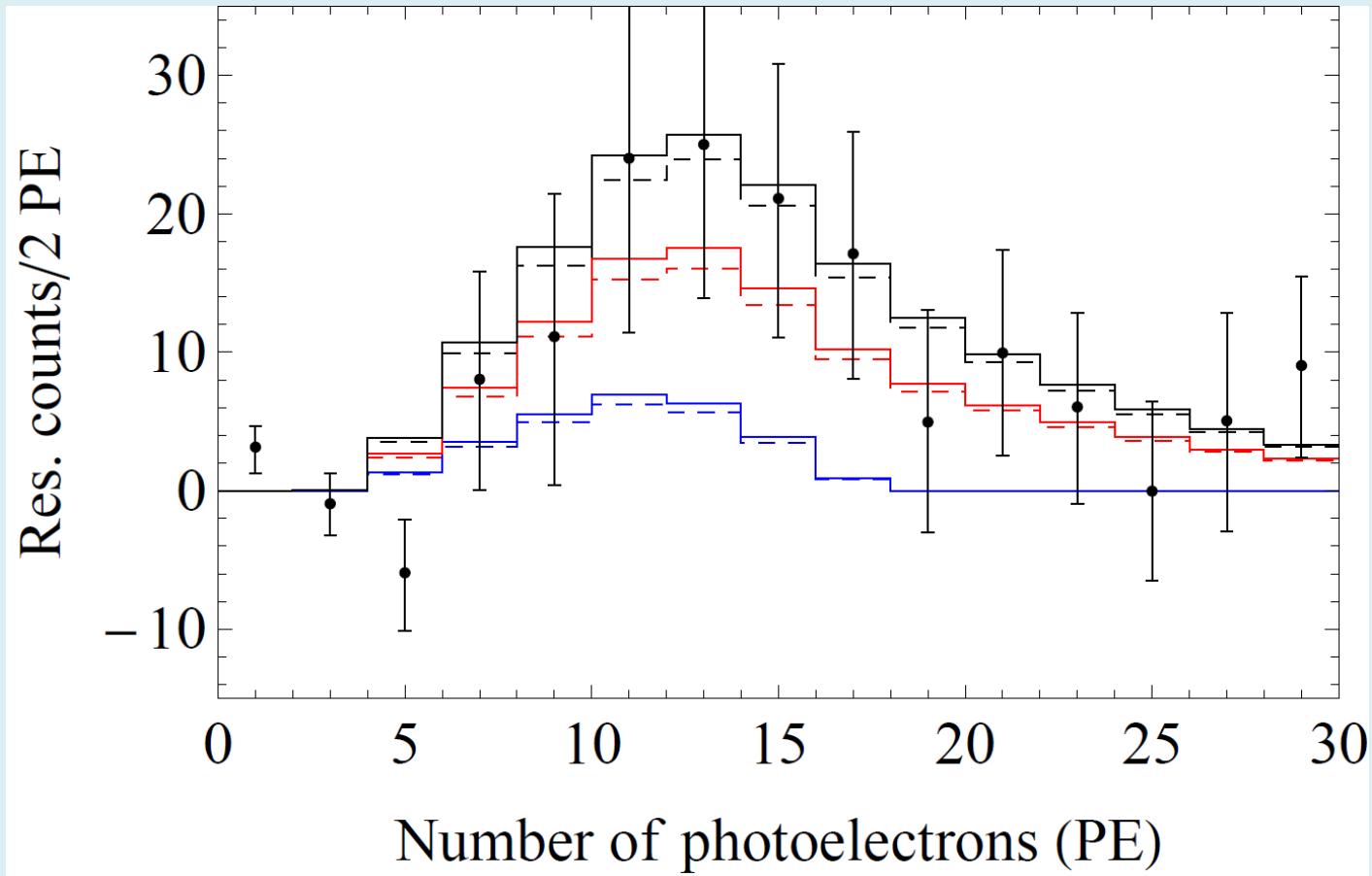


COHERENT Collaboration, Science (2017)



Coloma et al. 1708.02899

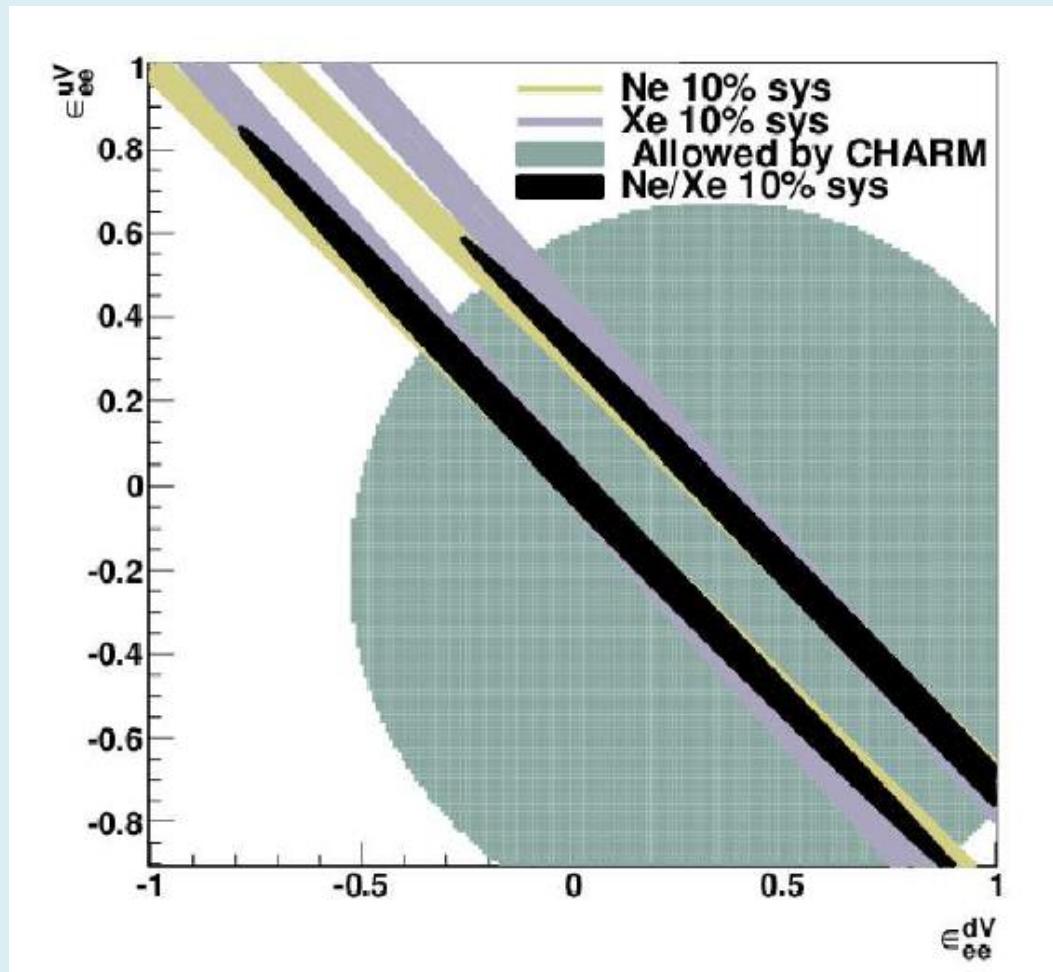
Sterile neutrinos



$$\Delta m_{41}^2 = 1.75 \text{ eV}^2, |U_{e4}| = 0.164, |U_{\mu 4}| = 0.119$$

Collin et al. Phys. Rev. Lett. 117, 221801 (2016)

Multiple elements



Scholberg, Phys. Rev. D 73, 033005 (2006)