Non-perturbative analysis of the spectrum of meson resonances in an ultraviolet-complete composite-Higgs model

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based on arXiV: 1610.09293 [Phys.Rev. D95 (2017)]
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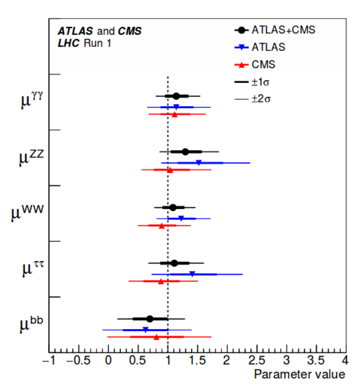
A SM-like Higgs boson

Higgs mass

SM-like Higgs boson discovered at LHC with mass $m_h = 125.09 \pm 0.24$ GeV.
[ATLAS & CMS combination (1503.07589)]
$\Longrightarrow$ No unknown parameters in the SM which becomes predictive.

Higgs couplings



[ATLAS & CMS combination (1606.02266)]

- So far no significant deviations compared to the SM prediction
  $\Rightarrow$ SM is a successful theory up to the EW scale
- precision will continue to increase
  $\Rightarrow$ Any deviations will be the sign of new physics beyond the SM (BSM)

The need of BSM physics

The SM is a successful theory up to the EW scale but he has some shortcomings
$\Rightarrow$ Several hints point towards the necessity to introduce BSM physics

Observational facts

- Dark matter
- Baryon asymmetry
- Neutrinos masses
- ...

Theoretical puzzles

- Hierarchy problem
- Huge Hierarchy between SM fermion masses
- Gauge coupling unification
- ...

No answer to these observational and theoretical issues in the SM
$\Rightarrow$ One necessarily need to introduce BSM physics
$\Rightarrow$ The SM is an <u>effective model</u> valid up to the EW scale

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- New strong dynamics condensates at scale Λ and spontaneously breaks a global symmetry G into H

⇒ Higgs is naturally light as a pNGB leaving in the coset G/H

QCD: $G/H = SU(3)_L \times SU(3)_R/SU(3)_V \rightarrow 8$ pNGBs $(\pi^{\pm}, \pi^0, K^{\pm}, K^0\overline{K}^0, \eta)$

Gauging of SM symmetry

- SM gauge symmetry embedded inside unbroken group H

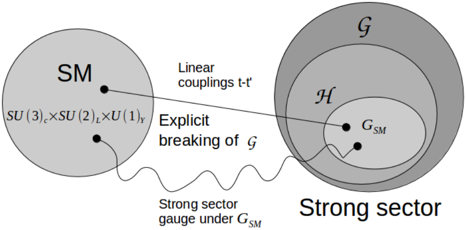
⇒ pNGBs charged under G_{SM} (4 associated to Higgs doublet)

⇒ Gauging explicitly breaks G but can not destabilise Higgs potential and induce EWSB

Partial compositeness

- Potential (and mass) for Higgs generated from another explicit breaking

⇒ Linear couplings between SM fermions and composite spin 1/2 resonances



Underlying dynamics

Barring extra space-time dimensions
⇒ Simplest, well-understood, explicit realization provided by gauge theory of fermions that confines at the multi-TeV scale Λ

Full gauge theory (hypergluons, hyperfermions as d.o.f) hard to study below Λ because of its non-perturbative nature ⇒ Effective models are useful

- Chiral Lagrangians: dictated only by global symmetries

$$\mathcal{L}_{\chi PT} = \frac{F_G^2}{4}\langle (D_\mu U)^\dagger D^\mu U\rangle \qquad U = \exp(2iG^{\hat{A}}T^{\hat{A}}/F_G)\Sigma_\epsilon$$

⇒ Little information on the details of the strong dynamics
⇒ Not sure that an UV completion exists

- 4-fermion interactions (gauge bosons froze-out)

$$\mathcal{L}_{NJL} = (\overline{\Psi}\Gamma^\alpha\Psi)(\overline{\Psi}\Gamma_\alpha\Psi)$$

⇒ Definite UV completion and underlying gauge symmetry respected
⇒ Possible to make calculation of non-perturbative quantities with Nambu Jona-Lasinio (NJL) techniques [Nambu and Jona-Lasinio '61]

The framework

Λ

gauge theory

(fundamental fermions, techni-gluons)

$$\mathcal{L}_{\text{gauge}} = \overline{\Psi} i \not{D} \Psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Exact results on the strong dynamics exist

Intermediate scale effective models

(fermionic bilinears)

$$\mathcal{L}_{\text{NJL}} = \overline{\Psi} i \not{\partial} \Psi + \sum_{\alpha} \left(\overline{\Psi} \Gamma^{\alpha} \Psi \right) \left(\overline{\Psi} \Gamma_{\alpha} \Psi \right)$$

NJL techniques (Large N), ...

f

Low scale effective models

(Goldstone bosons, ...)

$$\mathcal{L}_{\chi PT} = \frac{f^2}{2} \partial_{\mu} \Pi \partial^{\mu} \Pi + \cdots$$

Chiral perturbation theory

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UV completions: EW sector (Higgs as pNGB) + coloured sector (top partners)

Basic requirements for an UV completion

- Higgs as a composite Nambu-Goldstone boson leaving in coset G/H
- Custodial symmetry: $H \supset SU(2)_L \times SU(2)_R$
- No fundamental scalars: gauge theories with fermions

Minimal UV models classified in [Ferretti, Karateev, '14]

Minimal model: $SU(4)/Sp(4) \cong SO(6)/SO(5)$

- $SU(4)/Sp(4) \Rightarrow$ only 5 NGBs: Higgs doublet + singlet η
- 4 Weyl fermions $\psi \Rightarrow SU(4)$ global symmetry
- $Sp(4) \Rightarrow \psi$ belong to a pseudo-real hypercolour representation:
  the fundamental of $Sp(2N)$ [Barnard et al, '13]

			Colour	Flavour	
		Lorentz	$Sp(2N)$	$SU(4)$	$Sp(4)$
Hypercolour fermions	ψ_i^a	$(1/2, 0)$	$\Box_i$	4^a	4
	$\overline{\psi}_{ai} \equiv \psi^{\dagger}_{aj}\Omega_{ji}$	$(0, 1/2)$	$\Box_i$	$\overline{4}_a$	4^*
Spin-zero bilinears	$M^{ab} \sim (\psi^a \psi^b)$	$(0, 0)$	1	6^{ab}	$5 + 1$
	$\overline{M}_{ab} \sim (\overline{\psi}_a \overline{\psi}_b)$	$(0, 0)$	1	$\overline{6}_{ab}$	$5 + 1$
Spin-one bilinears	$a^\mu \sim (\overline{\psi}_a \overline{\sigma}^\mu \psi^a)$	$(1/2, 1/2)$	1	1	1
	$(V^\mu, A^\mu)^b_a \sim (\overline{\psi}_a \overline{\sigma}^\mu \psi^b)$	$(1/2, 1/2)$	1	15^a_b	$10 + 5$

Hypercolour-invariant fermionic bilinears have the quantum numbers of the meson resonances

Lightest composite meson resonances

<u>Scalars:</u> $\sigma + S^{\hat{A}} \sim 1 + 5$ <u>Pseudo-scalars:</u> $\eta' + G^{\hat{A}} \sim 1 + 5$

<u>Vectors:</u> $V^A_\mu \sim 10$ <u>Axial-vector:</u> $a_\mu + A^{\hat{A}}_\mu \sim 1 + 5$

- The model is a vector-like gauge theory: all fermions ψ can be made massive ($m_\psi \psi\psi$), while preserving the gauge hypercolour symmetry $G_c = Sp(2N)$

Three cases in vector-like theories: [Peskin, '80]

- $G = SU(N_f)_L \times SU(N_f)_R$ and $H_m = SU(N_f)_V$ (complex rep. of $\mathcal{G}$)
- $G = SU(2N_f)$ and $H_m = SO(2N_f)$ (real rep.)
  $H_m = Sp(2N_f)$ (pseudo-real rep.)

- Vafa-Witten theorem: The flavour subgroup H of G preserved by m_ψ can not be spontaneously broken $\Rightarrow$ If $SU(4)$ broken, it is broken down to $Sp(4)$

- 't Hooft anomaly matching:
Any global UV anomaly (generated by the hyperfermions ψ) must be matched in the IR, either by massless spin-1/2 baryons or Goldstone boson

ψ's can not form baryons because they are in pseudo-real hypercolour irreps
$\Rightarrow$ $SU(4)$ **unavoidably** spontaneously broken

$$d^{AB\hat{C}} = 2Tr[\{T^A, T^B\}T^{\hat{C}}]$$

$SU(4)$ broken ($T^{\hat{A}}$) and unbroken (T^A) generators combine in non-zero anomaly coefficients $\Rightarrow$ Global anomalies

$$(\psi^a \psi^b) \equiv \psi_i^a \Omega_{ij} \psi_j^b$$

The unique invariant tensor of $Sp(2N)$ is two-index antisymmetric
$\Rightarrow$ $SU(4)$-flavour contraction also antisymmetric
($4 \times 4 = 6_A + 10_S$)

Nambu-Jona Lasinio approximation of strong dynamics: 'froze out' hypergluons inducing 4-fermion interactions

Scalar 4-fermion operators relevant for the spontaneous breaking:

$$\mathcal{L}^{\psi}_{scal} = \frac{\kappa_A}{2N}(\psi^a\psi^b)(\overline{\psi}_a\,\overline{\psi}_b) + \frac{\kappa_B}{8N}\left[\epsilon_{abcd}(\psi^a\psi^b)(\psi^c\psi^d) + h.c.\right]$$

- $\kappa_{A,B} \sim 1/\Lambda^2$ real, dimensionful couplings
- κ_A controls spontaneous symmetry breaking $SU(4) \to Sp(4)$
- κ_B explicitly breaks the anomalous $U(1)$ symmetry

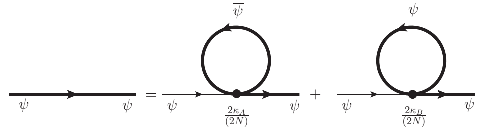
- Introducing auxiliary field M^{ab} whose equation of motion is $M^{ab} = -\frac{(\kappa_A + \kappa_B)}{(2N)}(\psi^a\psi^b)$
- Compute effective potential $V_{eff}(M^{ab})$ induced by fermion loops and minimise
- Minimum is non-zero above a critical coupling κ_A which depends on κ_B $(\kappa_B/\kappa_A < 1)$
  $\Rightarrow M_\psi \neq 0$ and $SU(4) \to Sp(4)$ [Barnard et al, '13]

$$\langle M^{ab}\rangle = \frac{M_\psi}{2}\Sigma_0 = \frac{M_\psi}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

Lagrangian can be rewritten in the 'physical' channels, corresponding to definite $Sp(4)$ representations using $SU(4)$ Fierz identities:

$$\begin{aligned}\mathcal{L}^{\psi}_{scal} &= 2\frac{\kappa_A}{(2N)}\left[\left(\psi\Sigma_0 T^0_\psi\psi\right)\left(\overline{\psi}T^0_\psi\Sigma_0\overline{\psi}\right)+\left(\psi\Sigma_0 T^{\hat{A}}\psi\right)\left(\overline{\psi}T^{\hat{A}}\Sigma_0\overline{\psi}\right)\right]\\ &+ \frac{\kappa_B}{(2N)}\left[\left(\psi\Sigma_0 T^0_\psi\psi\right)\left(\psi\Sigma_0 T^0_\psi\psi\right)-\left(\psi\Sigma_0 T^{\hat{A}}\psi\right)\left(\psi\Sigma_0 T^{\hat{A}}\psi\right)+h.c.\right]\end{aligned}$$

Schwinger Dyson equation determines dynamical fermion mass M_ψ



$$M_\psi = 4(\kappa_A+\kappa_B)M_\psi\tilde{A}_0(M^2_\psi)$$

Self-consistence implicitly ressums all diagrams leading in 1/N

$$\xi \equiv \frac{\Lambda^2(\kappa_A+\kappa_B)}{4\pi^2} = \left[1-\frac{M^2_\psi}{\Lambda^2}\ln\left(\frac{\Lambda^2+M^2_\psi}{M^2_\psi}\right)\right]^{-1}$$

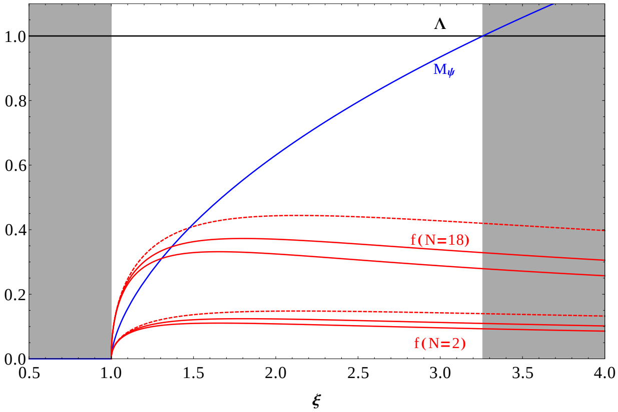
critical coupling $\quad 1<\xi\lesssim 3.25 \quad$ maximal coupling

- Non trivial solution $M_\psi \neq 0$ ($SU(4)$ spontaneously broken) exists only if $\xi > 1$
- Consistent resummation: $0 < M_\psi/\Lambda \lesssim 1$

$$\langle vac|\mathcal{J}_\mu^{\hat{A}}(0)|G^{\hat{B}}(p)\rangle = ip_\mu \frac{f}{\sqrt{2}}\delta^{\hat{A}\hat{B}}$$

EW precision observables receive order v^2/f^2 corrections $\Rightarrow f \gtrsim 0.5 - 1$ TeV

$$\frac{f^2}{2} = \lim_{q^2\to 0}[-q^2\overline{\Pi}_A(q^2)] = \frac{\tilde{\Pi}_A(0)}{1+2\kappa_D\tilde{\Pi}_A(0)/N}, \qquad \tilde{\Pi}_A(0) = -2(2N)M_\psi^2\tilde{B}_0(0,M_\psi^2)$$



- f residue of the Goldstone boson pole in the resummed transverse axial correlator
- f sets the scale of the composite sector
- $f \propto 1/\sqrt{N}$
- f can be as small as $\Lambda/10$ ($\Lambda \equiv$ NJL cutoff)

$\Rightarrow$ possibly large hierarchy

Vector and axial-vector 4-fermion interactions

Vector and axial vector resonances associated to spin 1 bilinears

$$\mathcal{L}^{\psi}_{vect} = \frac{\kappa'_C}{2N}\left(\overline{\psi}_a\overline{\sigma}^{\mu}\psi^a\right)\left(\overline{\psi}_b\overline{\sigma}_{\mu}\psi^b\right) + \frac{\kappa'_D}{2N}\left(\overline{\psi}_a\overline{\sigma}^{\mu}\psi^b\right)\left(\overline{\psi}_b\overline{\sigma}_{\mu}\psi^a\right)$$

Lagrangian can be rewritten in the 'physical' channels, corresponding to definite $Sp(4)$ representations using $SU(4)$ Fierz identities

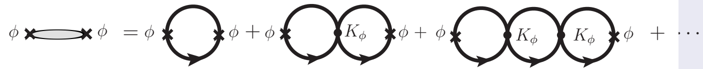
$$\mathcal{L}^{\psi}_{vect} = \frac{\kappa_C}{2N}\left(\overline{\psi}\, T^0_{\psi}\, \overline{\sigma}^{\mu}\psi\right)^2 + \frac{\kappa_D}{2N}\left(\overline{\psi}T^{A}\overline{\sigma}^{\mu}\psi\right)^2 + \frac{\kappa_D}{2N}\left(\overline{\psi}\, T^{\hat{A}}\,\overline{\sigma}^{\mu}\psi\right)^2$$

$\Rightarrow$ Non-tachyonic masses obtained for $\kappa_{C,D} > 0$
(consistent with current-current hypothesis)
$\Rightarrow$ Spin 1 operators with couplings $\kappa_{C,D}$ (vector $10_{Sp(4)}$, axial-vector $(1+5)_{Sp(4)}$)

Additional spin 1 resonances associated to $(\psi^a\sigma^{\mu\nu}\psi^b) \sim 10_{Sp(4)}$ do not appear at the level of four-fermion interactions because of Lorentz and/or $SU(4)$ invariance.

Bethe-Salpether equation

Resummation (geometrical series) of an infinite number of constituent fermion loops at leading order in $1/N$ $\Rightarrow$ Two-point correlators develop a pole



The pole defines the meson mass M_ϕ

$$\overline{\Pi}_\phi(q^2) = \frac{\tilde{\Pi}_\phi(q^2)}{1 - 2K_\phi \tilde{\Pi}_\phi(q^2)} \quad \longrightarrow \quad 1 - 2K_\phi \tilde{\Pi}_\phi(q^2 = M_\phi^2) = 0$$

ϕ	K_ϕ	$\tilde{\Pi}_\phi(q^2)$
$G^{\hat{A}}$	$2(\kappa_A + \kappa_B)/(2N)$	$\tilde{\Pi}_P(q^2) = (2N)\left[\tilde{A}_0(M_\psi^2) - \frac{q^2}{2}\tilde{B}_0(q^2, M_\psi^2)\right]$
η'	$2(\kappa_A - \kappa_B)/(2N)$	
$S^{\hat{A}}$	$2(\kappa_A - \kappa_B)/(2N)$	$\tilde{\Pi}_S(q^2) = (2N)\left[\tilde{A}_0(M_\psi^2) - \frac{1}{2}(q^2 - 4M_\psi^2)\tilde{B}_0(q^2, M_\psi^2)\right]$
σ	$2(\kappa_A + \kappa_B)/(2N)$	

and similarly for the spin one channels V and A

No confinement in the NJL $\Rightarrow$ Prescription for the unphysical imaginary parts

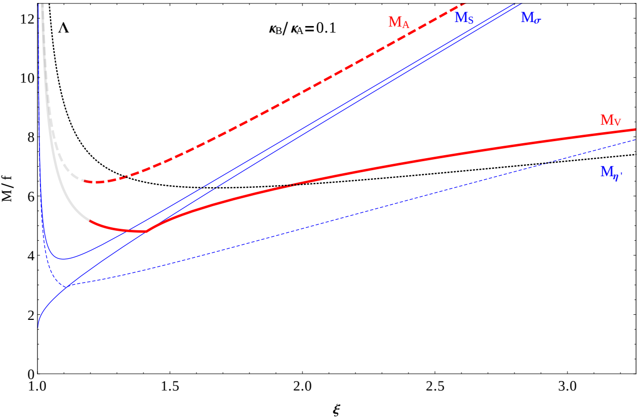
$$1 - 2K_\phi \tilde{\Pi}_\phi(q^2) = c_0^\phi(q^2) + c_1^\phi(q^2) q^2 \quad \longrightarrow \quad M_\phi^2 = Re\left[-\frac{c_0^\phi(M_\phi^2)}{c_1^\phi(M_\phi^2)} \right]$$

$K_\phi \equiv$ four-fermion couplings $\qquad \tilde{\Pi}_\phi(q^2) \equiv$ Polarisation amplitudes

- Inserting the gap-equation, one recovers consistently the Goldstone pole: $\quad M_G = 0$
- Singlet pseudo-scalar proportional to $U(1)$ anomaly and mixes with axial vector: $$M_{\eta'}^2 = -\frac{\kappa_B}{\kappa_A^2 - \kappa_B^2} \frac{[1 - 2K_a \tilde{\Pi}_A^L(M_{\eta'}^2)]}{\tilde{B}_0(M_{\eta'}^2, M_\psi^2)}$$
- Scalars proportional to the mass gap M_ψ: $$M_\sigma^2 = 4M_\psi^2, \qquad M_S^2 = 4M_\psi^2 + M_{\eta'}^2 \frac{\tilde{B}_0(M_{\eta'}^2, M_\psi^2)}{\tilde{B}_0(M_S^2, M_\psi^2)} \simeq M_\sigma^2 + M_{\eta'}^2$$
- Vector heavy even for vanishing mass gap: $$M_V^2 = \frac{-3}{4\kappa_D \tilde{B}_0(M_V^2, M_\psi^2)} + 2M_\psi^2 \frac{\tilde{B}_0(0, M_\psi^2)}{\tilde{B}_0(M_V^2, M_\psi^2)} - 2M_\psi^2$$
- Axial-vector generally the heaviest: $$M_A^2 = \frac{-3}{4\kappa_D \tilde{B}_0(M_A^2, M_\psi^2)} + 2M_\psi^2 \frac{\tilde{B}_0(0, M_\psi^2)}{\tilde{B}_0(M_V A2, M_\psi^2)} + 4M_\psi^2 \simeq M_V^2 + 6M_\psi^2$$

Current-current hypothesis

- Large-N relation among 4-fermion operators dominated by single hypergluon exchange $\rightarrow \kappa_A = \kappa_C = \kappa_D$ $\qquad (M_a = M_A)$



- $M_\phi / f \sim 1/\sqrt{N}$ ($N = 4$ here)
- Free parameters:
  $\xi = \Lambda^2(\kappa_A + \kappa_B)/(4\pi^2)$
  κ_B/κ_A
- EW splitting neglected
  (e.g. $\mathbf{5}_{Sp(4)} = \mathbf{2}_{\pm 1/2}\mathbf{1}_0$)
  $\Rightarrow$ Full $Sp(4)$ multiplets
- Consistently recover NGBs: $M_G = 0$

EW meson masses in units of f ($f \gtrsim 0.5 - 1$ TeV)

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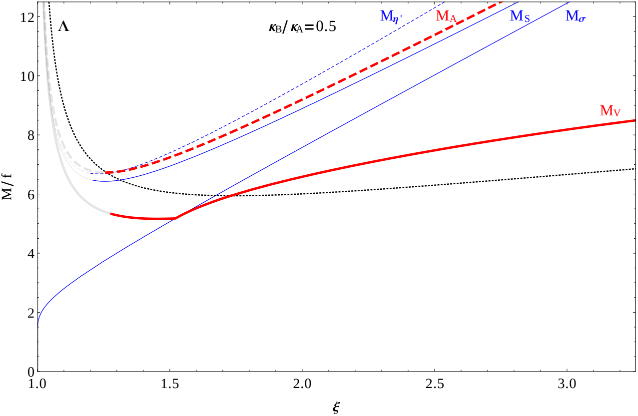

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Four-fermions operators couplings may be related
$\Rightarrow$ Prediction of relative strength between the various physical channels (works well in QCD)

- Start from $Sp(2N)$ current-current operators: encode UV dynamics in 'ladder' approximation, that holds when N is (moderately) large
- Use Fierz transformations to generate various operators

$$\mathcal{L}_{UV} = g_{HC}\mathcal{J}_{\psi}^{\mu I}\mathcal{G}_{\mu I} \qquad \mathcal{J}_{\psi}^{\mu I} = \psi\left(\Omega T^{I}\right)\sigma^{\mu}\overline{\psi}$$

Assume that confining strong dynamics can be described (1^{rst} approximation) by exchange of one hypergluon which acquired a dynamical mass
$\Rightarrow$ 'Ladder' approximation strong dynamics generates $Sp(2N)$ current-current operators

$$\mathcal{L}_{eff} = \frac{\kappa_{UV}}{2N}\mathcal{J}_{\psi}^{\mu I}\mathcal{J}_{\psi\mu}^{I} \qquad \kappa_{UV}/(2N) \sim g_{HC}^{2}/\Lambda^{2} \quad (g_{HC} \sim 1/\sqrt{2N})$$

Lorentz and SU(N) for the fundamental (flavour) Fierz transformations are very well-known but not $Sp(2N)$ that we derived

Four-fermions operators couplings may be related
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- Start from $Sp(2N)$ current-current operators: encode UV dynamics in 'ladder' approximation, that holds when N is (moderately) large
- Use Fierz transformations to generate various operators

$Sp(2N)$ Fierz matrix for the fundamental representation:

$$\begin{pmatrix} (\Omega T^0)_{ij}(\Omega T^0)_{kl} \\ \sum_{I}(\Omega T^I)_{ij}(\Omega T^I)_{kl} \\ \sum_{\hat{I}}(\Omega T^{\hat{I}})_{ij}(\Omega T^{\hat{I}})_{kl} \end{pmatrix} = \begin{pmatrix} \frac{1}{2N} & \frac{1}{2N} & \frac{1}{2N} \\ \frac{2N+1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{(2N+1)(N-1)}{2N} & \frac{N-1}{2N} & -\frac{N+1}{2N} \end{pmatrix} \begin{pmatrix} (\Omega T^0)_{il}(\Omega T^0)_{kj} \\ \sum_{I}(\Omega T^I)_{il}(\Omega T^I)_{kj} \\ \sum_{\hat{I}}(\Omega T^{\hat{I}})_{il}(\Omega T^{\hat{I}})_{kj} \end{pmatrix},$$

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The partial compositeness paradigm

A new explicit breaking source is needed to:

- Destabilise Higgs potential and induce EWSB
- Generate SM fermion masses and couplings with composite Higgs

$\Rightarrow$ Two main possibilities: linear or bilinear couplings between SM fermions and operators of the strong sector

(potential generated mainly by the heaviest SM particle ie top quark)

Bilinear coupling

Exchange of massive states at scale Λ_{UV} generates: $\Delta\mathcal{L} = \lambda_\Psi \overline{\Psi}_{SM}\Psi_{SM}\mathcal{O}$

$\mathcal{O} = \overline{\Psi}_{HC}\Psi_{HC}$

SM fermions masses supressed by at least (Λ/Λ_{UV})

$\Rightarrow$ Too small mass for top quark, except if $\Lambda \simeq 4\pi f \gg v$ but large fine-tunning

Linear coupling (partial compositeness)

SM fermions may mix with Spin1/2 composite resonances:

$\Delta\mathcal{L} = \lambda_\Psi \overline{\Psi}_{SM}\mathcal{O} + h.c.$

$\mathcal{O} = \overline{\Psi}_{HC}\Psi_{HC}\Psi_{HC}$

$\Rightarrow$ A priori "possible" to obtain large enough top quark mass with proper anomalous dimension

$\Rightarrow$ Need to introduce new constituent coloured fermions X^f that can form spin-1/2 baryons

Introduce new constituent coloured fermions X^f that can form spin-1/2 baryons mixing with SM top quark
$\Rightarrow$ Need to go beyond $Sp(2N)$ fundamental representation

Coloured fundamental fermions

- VL embedding of $SU(3)_c$ inside coloured sector implies 6 Weyl fermions X^f
$\Rightarrow$ $SU(6) \to SO(6) \supset SU(3)_c$

- Real representation: 2-index traceless antisymmetric

$$X^f_{ij} = -X^f_{ji} \sim \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array} \qquad X^f_{ij}\Omega_{ji} = 0$$

- $N \geqslant 2$ as $d(\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}) = (2N+1)(N-1)$

$\Rightarrow$ minimal case: $Sp(2) \cong SU(2)$ (EW sector alone) $\to$ lattice results available
$Sp(4) \cong SO(5)$ (EW+ coloured sector)

	Lorentz	$Sp(2N)$	$SU(6)$	$SO(6)$
X^f_{ij}	$(1/2,0)$	$\Box_{ij}$	6^f	6
$\overline{X}_{fij} \equiv \Omega_{ik} X^\dagger_{fkl} \Omega_{lj}$	$(0,1/2)$	$\Box_{ij}$	$\overline{6}_f$	6
$M_c^{fg} \sim (X^f X^g)$	$(0,0)$	1	21^{fg}	$20'+1$
$\overline{M}_{cfg} \sim (\overline{X}_f \overline{X}_g)$	$(0,0)$	1	$\overline{21}_{fg}$	$20'+1$
$a^\mu_X \sim (\overline{X}^f \overline{\sigma}^\mu X_f)$	$(1/2,1/2)$	1	1	1
$(V^\mu_c, A^\mu_c)^g_f \sim (\overline{X}_f \overline{\sigma}^\mu X^g)$	$(1/2,1/2)$	1	35^f_g	$15+20'$

Spin-zero coloured mesons

Spin-one coloured mesons

Lightest composite coloured meson resonances

Scalars: $\sigma_X + S_c^{\hat{F}} \sim 1 + 20'$

Vectors: $V_c^{\mu F} \sim 15$

Pseudo-scalars: $\eta_X + G_c^{\hat{F}} \sim 1 + 20'$

Axial-vector: $a^\mu_c + A_c^{\mu \hat{F}} \sim 1 + 20'$

$$20'_{SO(6)} = (8 + 6 + \overline{6})_{SU(3)_c} \qquad 15_{SO(6)} = (1 + 8 + 3 + \overline{3})_{SU(3)_c}$$

pheno coloured scalars [Cacciapaglia et al, '15]

Lot of changes appears when theory includes both EW and coloured sectors

- Important to consider global fermion numbers $U(1)_\psi$ and $U(1)_X$
- Currents $\mathcal{J}^0_{\mu\psi,X}$ both anomalous w.r.t $Sp(2N)$ (like $U(1)_A$ in QCD)
- However, one linear combination is anomaly free and thus conserved:

$$\mathcal{J}^0_\mu = \mathcal{J}^0_{\mu X} - 3(N-1)\mathcal{J}^0_{\mu\psi}$$

$\Rightarrow$ New Goldstone boson η_0 appears while η' receive a mass from the anomaly

Construct the minimal operator that preserves all exact symmetries but explicitly breaks the anomalous $U(1)$ (generalisation of κ_B-term)

- EW sector: $Sp(2N)$ anomaly breaks $U(1)_\psi \to \mathcal{O}_\psi = -\frac{1}{4}\epsilon_{abcd}(\psi^a\psi^b)(\psi^c\psi^d)$
- Colour sector: anomaly breaks $U(1)_X \to \mathcal{O}_X = -\frac{1}{6!}\epsilon_{f_1\cdots f_6}\epsilon_{g_1\cdots g_6}(X^{f_1}X^{g_1})\cdots(X^{f_6}X^{g_6})$
- Full theory preserves $U(1)_{X-3(N-1)\psi}$: $\to \mathcal{L}_{\psi X} = A_{\psi X}\frac{\mathcal{O}_\psi}{(2N)^2}\left[\frac{\mathcal{O}_X}{[(2N+1)(N-1)]^6}\right]^{(N-1)}$

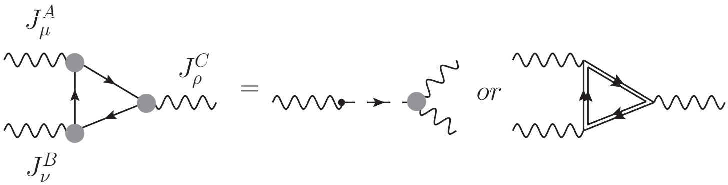
After spontaneous breaking $\mathcal{L}_{\psi X}$ generates effective 4-fermion operators ψ^4, X^4 and $\psi^2 X^2$

Trilinear baryons: $\Psi^{abf} = (\psi^a \psi^b X^f)$, $\Psi^{ab}_f = (\psi^a \psi^b \overline{X}_f)$ $\Psi^{af}_b = (\psi^a \overline{\psi}_b X^f)$
$\Psi^{fgh} = (X^f X^g X^h)$, $\Psi^{fg}_h = (X^f X^g \overline{X}_h)$

Anomaly matching condition:

$$\sum_{i=\psi,X} n_i A(r_i) = \sum_{i=baryon} n_{i'} A(r_i), \qquad 2\,Tr[T_r^{\hat{A}}\{T_r^B, T_r^C\}] = A(r) d^{\hat{A}BC}$$

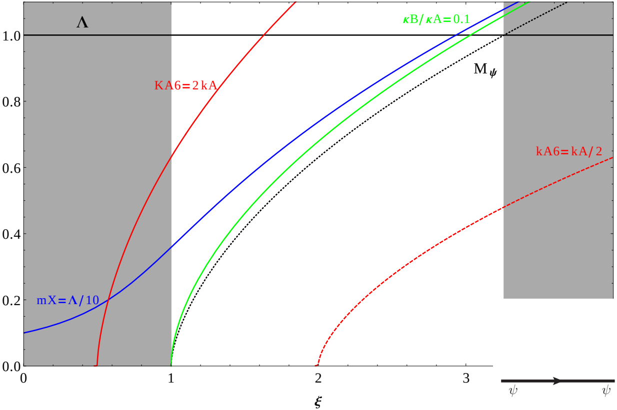
- $SU(4)^3$: Matching impossible for $N \neq 8n \Rightarrow SU(4)$ breaks to $Sp(4)$ and one expects non-zero condensate $\langle \psi\psi \rangle \neq 0$
- $SU(6)^3$: Matching always possible $\Rightarrow SU(6)$ may not break to $SO(6)$ and the condensate $\langle XX \rangle$ may vanish or not
- $SU(4)^2 \times U(1), SU(6)^2 \times U(1), U(1)^3$: $U(1)$ most likely broken by $\langle \psi\psi \rangle$



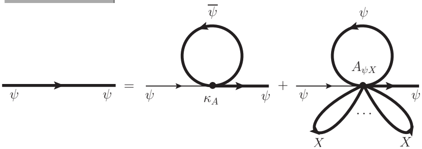
Two coupled mass gap equations:

$$\begin{cases} M_\psi = 4\left[\kappa_A + \kappa_B(M_X^2)\right] M_\psi \tilde{A}_0(M_\psi^2) \\ M_X = 4\left[\kappa_{A6} + \kappa_{B6}(M_\psi^2, M_X^2)\right] M_X \tilde{A}_0(M_X^2) + m_X \end{cases}$$

$$\begin{cases} \kappa_B = \kappa_{B6} = 0 \\ \kappa_A = \kappa_{A6}, m_X = 0 \end{cases}$$
$$\Rightarrow M_\psi = M_X$$

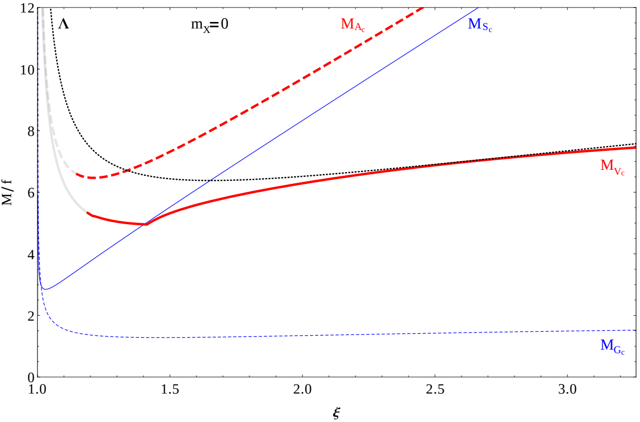


- Coloured sector window [between critical coupling ($M_X = 0$) and maximal coupling ($M_X = \Lambda$)] shifts respect to the EW sector window
- $m_X \neq 0$: No critical coupling as $M_X \geqslant m_X$



Current-current hypothesis

The ratio EW masses/ coloured masses strongly depends on the ratio κ_{A6}/κ_A
Unfortunately the large-N approximation does not determine this ratio uniquely (but still determines $\kappa_{A6} = \kappa_{C6} = \kappa_{D6}$)
$\Rightarrow$ Choose $\kappa_A = \kappa_{A6}$



- $M_\phi/f \sim 1/\sqrt{N}$ ($N = 4$, $\kappa_B/\kappa_A = 1/100$ here)
- Goldstone bosons receive a mass from gluon loops that evade bounds for $f \gtrsim 1$ TeV
- Goldstone (and coloured resonances) mass increase with f

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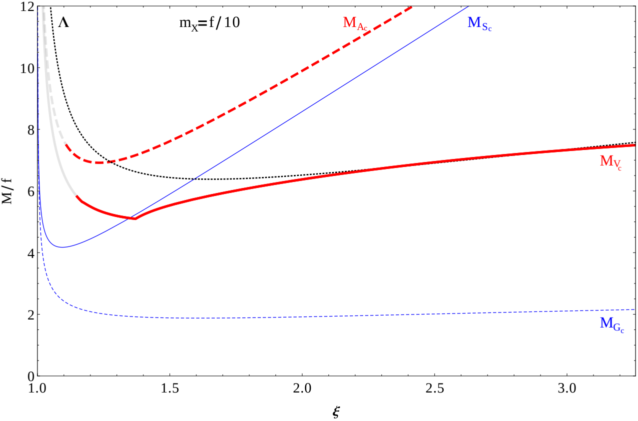

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Gauging explictly breaks G and induce radiative mass to NGBs:

$$\Delta M^2_{G_{\hat{A}}} = -\frac{3}{4\pi}\frac{1}{F_G^2}\frac{g^2_{\mathcal{W}}}{4\pi}\times\int_0^\infty dQ^2\, Q^2\, \Pi_{V\text{-}A}(-Q^2)\times\left[\sum_{\hat{B}}\left(f^{\hat{A}W\hat{B}}\right)^2 - \sum_{B}\left(f^{\hat{A}\hat{W}B}\right)^2\right]$$

$f^{abc} = 2iTr(T^a[T^b, T^c])$
$T^{\mathcal{W}} = T^W + T^{\hat{W}}$ gauged generators, $T^{W,\hat{W}}$ linear combination of $T^{A,\hat{A}}$

As $G_{SM} \subset H \to f^{\hat{A}\hat{W}B} = 0 (T^{\hat{W}} = 0) \Rightarrow$ Always positive contributionin CHMs that can not break EW symmetry

Coloured pNGBs masses

Coloured pNGBs receive mass from gluon loops:

$$\underline{\text{Octet}} : \Delta M^2_{O_c} = -\frac{3}{4\pi}\frac{1}{F^2_{G_c}}\int_0^\infty dQ^2\, Q^2\, \Pi^X_{V\text{-}A}(-Q^2)\times\frac{3}{4\pi}g_s^2$$

$$\underline{\text{Sextet}} : \Delta M^2_{S_c} = -\frac{3}{4\pi}\frac{1}{F^2_{G_c}}\int_0^\infty dQ^2\, Q^2\, \Pi^X_{V\text{-}A}(-Q^2)\times\frac{1}{4\pi}\left(\frac{10}{3}g_s^2 + \frac{16}{9}g'^2\right)$$

$\Rightarrow$ Enough to comply with direct searches even for $f = 1$ TeV
(and even for $m_X = 0$ contarry to the common expectation)

The $Sp(4)$ singlet mesons σ, η', a^μ may mix with the $SO(6)$ singlet ones σ_c, η'_c, a^μ_c (all SM singlets)

If on ignores mixing: $\begin{cases} M_\sigma = 2M_\psi \\ M_{\sigma_c} = 2M_X \end{cases}$ $\begin{cases} M^2_{\eta'} \sim \kappa_B \\ M^2_{\eta'_c} \sim (\kappa_{B6}, m_X) \end{cases}$ $\begin{cases} M_a = M_A \\ M_{a_c} = M_{A_c} \end{cases}$

▶ Axial-vectors: $Sp(2N)$ current-current operators do not induce singlet-singlet mixing operators

⇒ Axial singlet mixing is subleading in $1/N$

▶ (Pseudo-)scalars: Anomalous operator $A_{\psi X}$ induces a coupling $\psi^2 X^2$ of the same order as the couplings ψ^4, X^4

⇒ The mixing is a leading effect for (pseudo-)scalars

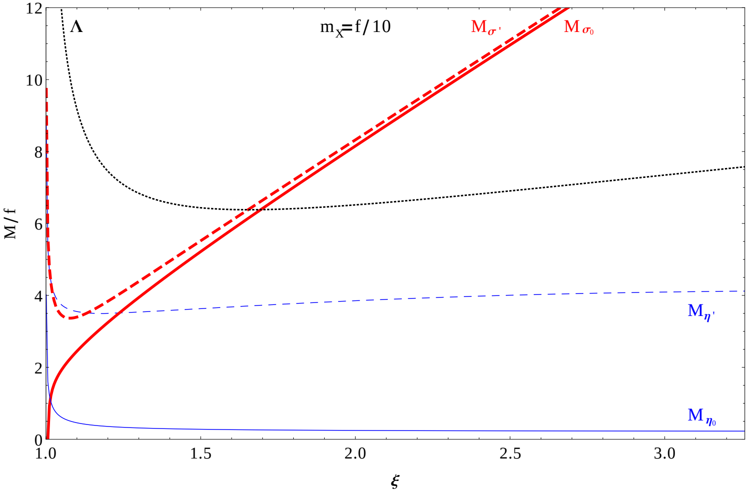
⇒ One linear combination of eta', η'_c is massless for $m_X = 0$: $U(1)$ Goldstone

$$\mathbf{K}_{\eta_\psi \eta_X} = \begin{pmatrix} K_{\eta_\psi} & -K_{\psi X} & 0 & 0 \\ -K_{\psi X} & K_{\eta_X} & 0 & 0 \\ 0 & 0 & K_a & 0 \\ 0 & 0 & 0 & K_{a_c} \end{pmatrix}, \qquad \mathbf{\Pi}_{\eta_\psi \eta_X} = \begin{pmatrix} \tilde{\Pi}^\psi_P & 0 & \sqrt{p^2}\tilde{\Pi}^\psi_{AP} & 0 \\ 0 & \tilde{\Pi}^X_P & 0 & \sqrt{p^2}\tilde{\Pi}^X_{AP} \\ \sqrt{p^2}\tilde{\Pi}^\psi_{AP} & 0 & \tilde{\Pi}^{L\psi}_A & 0 \\ 0 & \sqrt{p^2}\tilde{\Pi}^X_{AP} & 0 & \tilde{\Pi}^{LX}_A \end{pmatrix}$$

Mixed states may couple both to EW gauge boson ($\phi \to \gamma\gamma$) and gluons ($gg \to \phi$)

⇒ Potential discovery channel

Singlet masses



- η_0 is a pNGB associated to spontaneously broken $U(1)$ symmetry
  $\Rightarrow M^2_{\eta_0} \sim m_X$: could be very light
- η' obtains a mass through the anomaly
  $\Rightarrow M^2_{\eta_0} \sim A_{\psi X}$: No way to estimate $A_{\psi X}$, η' could also be very light

- Fermionic UV completions of CHMs require:

pNGB Higgs + top partners

$\Rightarrow$ In general two species of fundamental fermions should be present
(EW fermions ψ and coloured fermions X)

- Additional PNGBs have in general anomalous couplings to SM gauge bosons and couplings to tops (thanks to PC)

Singlets η, η_0 and η' as well as the octet $\mathcal{O}_c$ are present in all UV completions
$\Rightarrow$ Interesting to focus on them
[Cacciapaglia, Flacke et al, 1507.02283, 1610.06591]

- η and η' produced by gluon fusion through the anomaly and decay to dibosons ($gg, WW, ZZ, Z\gamma, \gamma\gamma$) and di-top
- η decays into diboson ($ZZ, \gamma Z, WW$) and produced by EW interactions as no anomalous coupling to gg
$\Rightarrow$ More difficult to produce it
- $\mathcal{O}_c$ mainly pair produced by QCD interactions and decays through the anomalous couplings gg, γg and Zg or deacys into $t\bar{t}$

General idea of a composite Nambu-Goldstone Higgs particle provides a very attractive framework for the EWSB
$\Rightarrow$ Gauge theory confining at the multi-TeV scale has the potential to provide a UV-complete framework to study composite Higgs phenomenology

$\Rightarrow$ Minimal model features 4 flavours $[SU(4)/Sp(4)]$ which condense as the hypercolour interaction becomes strong

- $SU(4)$ flavour symmetry unavoidably broken spontaneously to Sp(4) as required for NGB Higgs ('t Hooft anomaly matching)

- NJL well describes SSB: non-perturbative computation of $M_{\psi,\chi}$ and f
$\Rightarrow$ f can be as small as $\Lambda/10$ $\rightarrow$ large hierarchy could explain that no new states have been observed so far at LHC

- Computation of the composite masses (consistent with lattice results)
$\Rightarrow$ spectrum belong to multi-TeV range but few states can be relatively light (e.g. EW and coloured pNGBs including η_0, η' for small κ_B/κ_A, vectors for intermediate ξ, σ for small ξ)

- Only few parameters (ξ, κ_{A6}/κ_A, κ_B/κ_A, N, m_χ) if current-current hypothesis is assumed $\Rightarrow$ Phenomenologically simple

First thorough analysis of the spectrum of meson resonances in confining gauge theory with fermions in two different representations of the gauge group
$\Rightarrow$ Main limitation: absence of interactions with SM fermion fields
(to generate Yukawa couplings between the composite Higgs and the SM fermions and induce radiatively Higgs potential that realizes EWSB)

- Calculation of top partners masses within NJL framework [work in progress]
$\Rightarrow$ Relevant for LHC studies

- Generate Higgs potential by realizing partial compositeness

- Consider other UV completions (other cosets and/or hyperfermions)
$\Rightarrow$ Completions with $f \sim N^2$ imply lighter composite resonances in EW sector

- Explore minimal fundamental partial compositeness paradigm
[Sanino, Strumia, Tesi, '16]
$\Rightarrow$ baryons made of 1 scalar and 1 fermion ie $B = (S\psi)$: easy to compute top partners masses

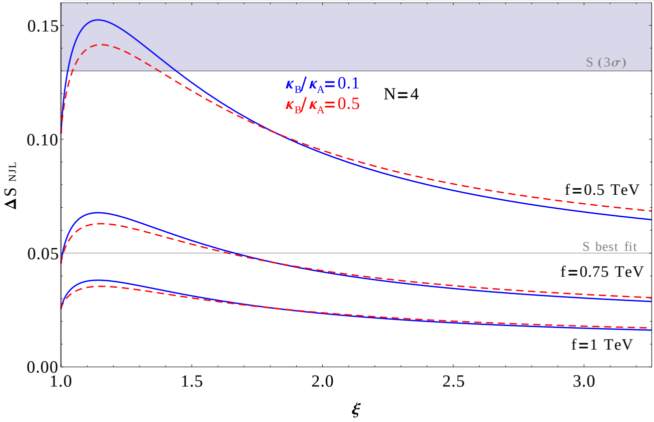
Thanks for your attention!

S parameter

Need only to assume vev for the Higgs (No need to explicitly consider details of breaking terms)

$$\Delta S = 16\pi \left.\frac{d\Pi^{(v)}_{3Y}(q^2)}{dq^2}\right|_{q^2=0} = 8\pi \frac{v^2}{f^2} \frac{d}{dq^2}\left(q^2\Pi_{V-A}(q^2)\right)\Big|_{q^2=0},\ \frac{v}{f} = \sin\left(\frac{\langle h\rangle}{f}\right)$$

Correlator $\Pi_{V-A}(q^2)$ can be estimated in the NJL approximation



3σ limit assumes $\Delta T = 0$

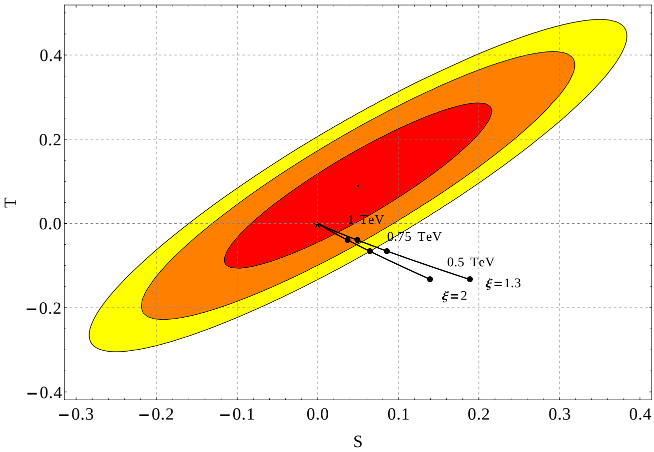
ΔS decreases when strong sector decouples (ie increase of f)

No corresponding shift in T parameter due to custodial symmetry

IR contributions

Composite sector also modifies Higgs couplings to EW gauge bosons by factor $\sqrt{1 - v^2/f^2}$

$$\Delta S_{\mathrm{IR}} = \frac{1}{6\pi}\frac{v^2}{f^2}\ln\left(\frac{\mu}{M_h}\right), \qquad \Delta T_{\mathrm{IR}} = -\frac{3}{8\pi}\frac{1}{\cos^2\theta_W}\frac{v^2}{f^2}\ln\left(\frac{\mu}{M_h}\right) = -\frac{9}{4}\frac{\Delta S_{\mathrm{IR}}}{\cos^2\theta_W}$$



One expects additional contributions from partial compositeness

⇒ Not complete prediction, only shown is specific contributions

Vector-like gauge theories

Asymptotically free and confining gauge theory with a set of N_f Dirac fermions (even number of Weyl) that can all be made massive in a gauge invariant way

$\Rightarrow$ Exact results concerning non-perturbative dynamical aspects exist

Vafa-Witten theorem

In any vector-like gauge theory with massless fermions and vanishing vacuum angles, the subgroup H_m of the flavour group G that corresponds to the remaining global symmetry when all fermion flavours are given identical gauge invariant masses, cannot be spontaneously broken

$\Rightarrow$ If H_m corresponds to a maximal subgroup of G: either G is not spontaneously broken at all or G is spontaneously broken towards H_m

Three cases in vector-like theories: [Peskin, '80]

- $G = SU(N_f)_L \times SU(N_f)_R$ and $H_m = SU(N_f)_V$ (complex rep. of $\mathcal{G}$)
- $G = SU(2N_f)$ and $H_m = SO(2N_f)$ (real rep.)
  $H_m = Sp(2N_f)$ (pseudo-real rep.)