Evolution of plasma wave In Laser Plasma Acceleration

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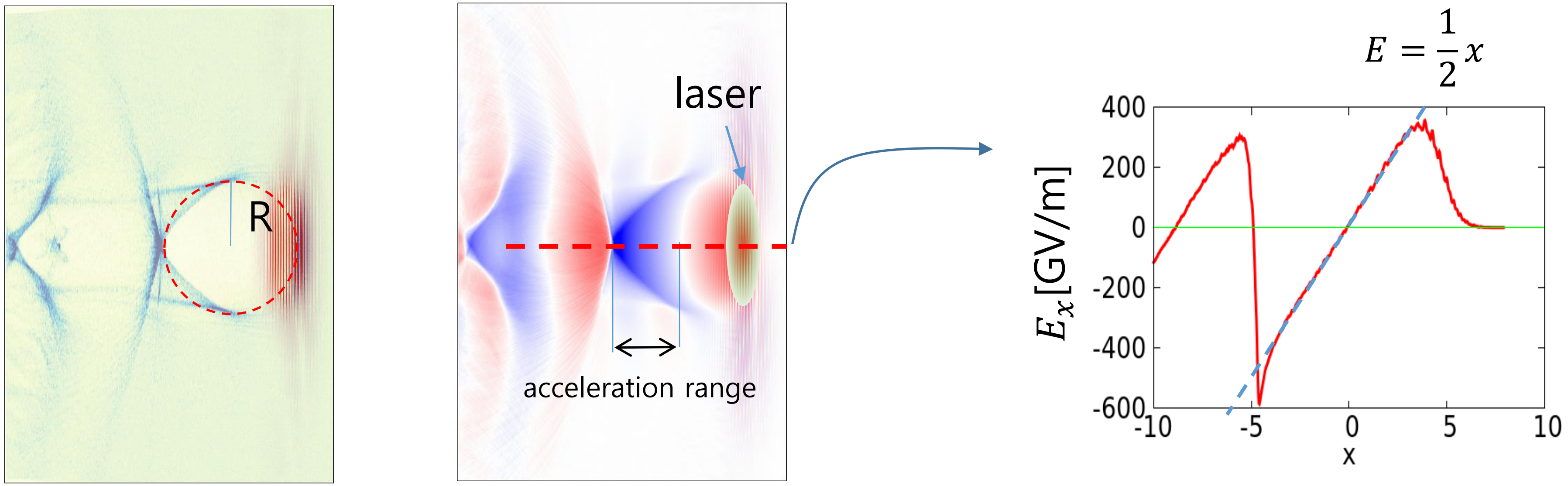
Laser wake field acceleration

'Bubble' has strong electric field in the order of GV/cm.
LWFA can accelerate electrons in high acceleration gradient.



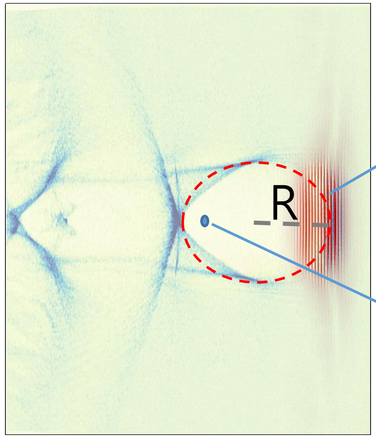
Linear bubble speed is approximated as $\gamma_0 = \omega_0/\omega_p$.
For example, assuming $0.8\mu m$ laser wavelength

$n_0 = 1 \times 10^{19} cc \rightarrow \gamma_0 = 13$

$n_0 = 1 \times 10^{18} cc \rightarrow \gamma_0 = 42$

Electron energy scale

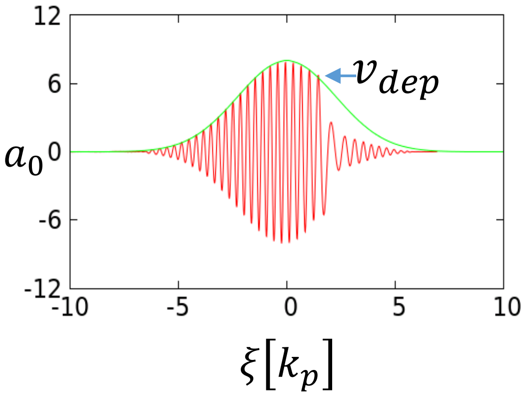
Applying the speed of laser local, electron energy is scaled as $\Delta\mathcal{E}/mc^2 = 2a_0\gamma_0^2/3$.

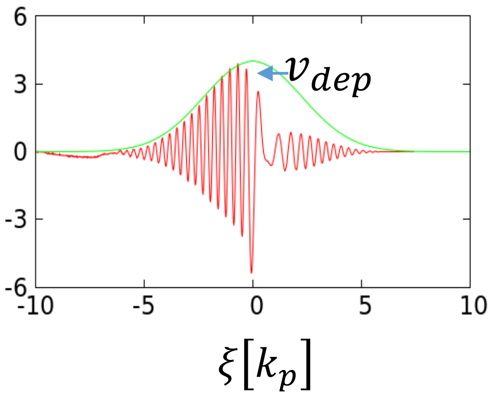


$v_{head} \approx 1 - \frac{3}{2}\frac{\omega_p^2}{\omega_0^2}$

*Decker, POP **3** 2047 (1996)*

$\gamma_e > \gamma_0$





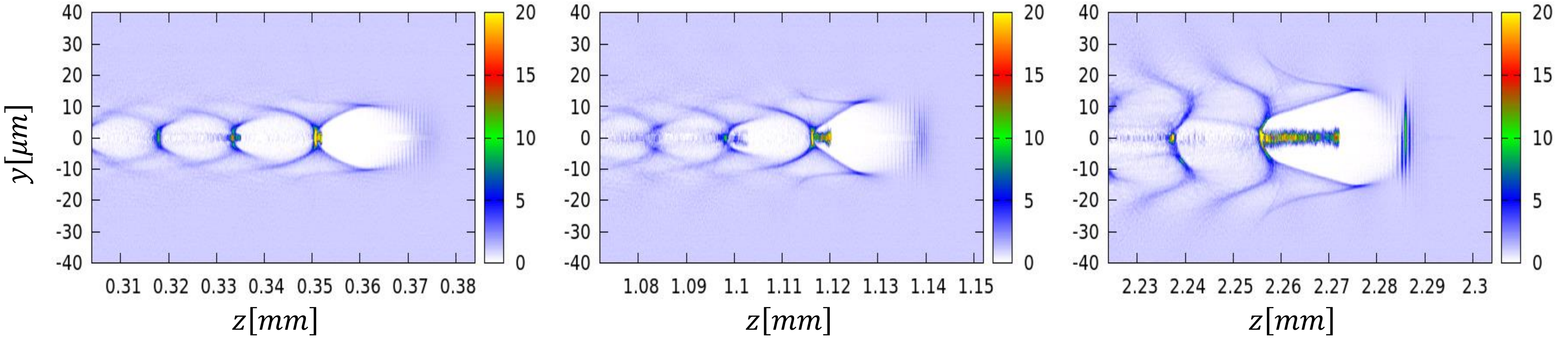
In the assumption of non-evolving bubble, considering pulse local depletion
The electron energy is

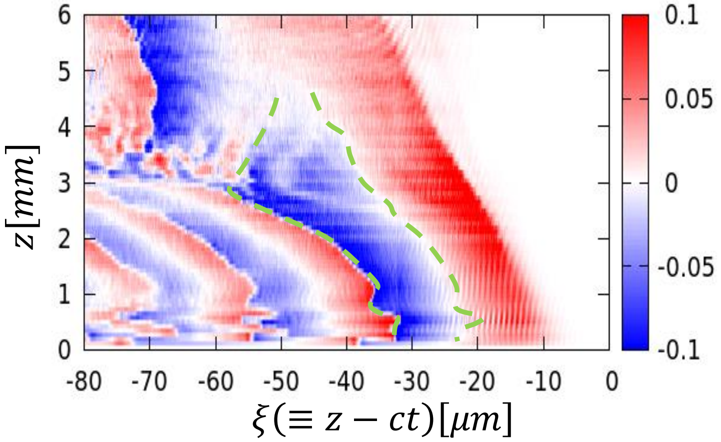
$$\gamma(t) = -\frac{3t^2}{8\gamma_0^2} + \frac{Rt}{2} + \gamma_0, \qquad R = 2\sqrt{a_0}$$

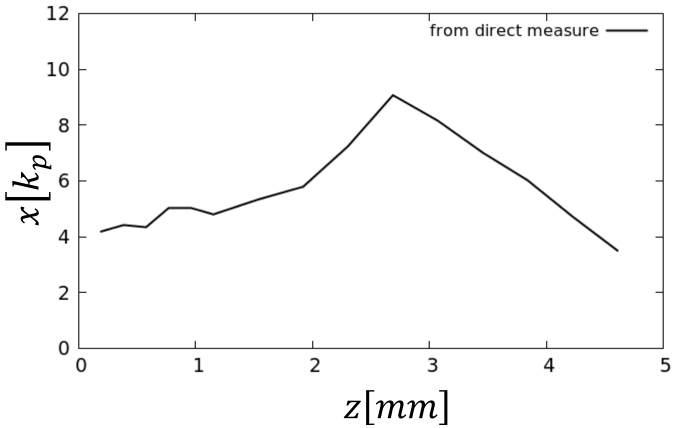
, which comes to the maximum energy in form of $\Delta\mathcal{E}/mc^2 = 2a_0\gamma_0^2/3$. *Lu, Phys. Rev. STAB **10** 061301 (2007)*

Evolving bubble

During laser propagation in plasma, the bubble evolves by expanding its size.





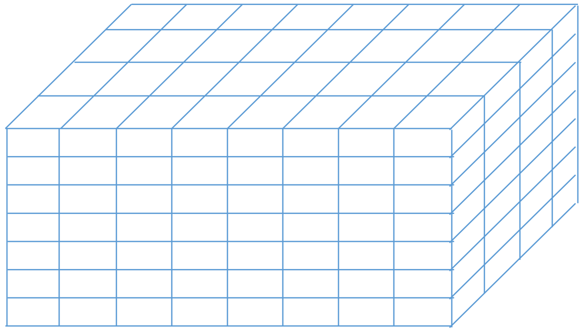


Can we predict the bubble evolution?
Can we use this bubble evolution for enhancing electron energy?

JoPIC-CYL

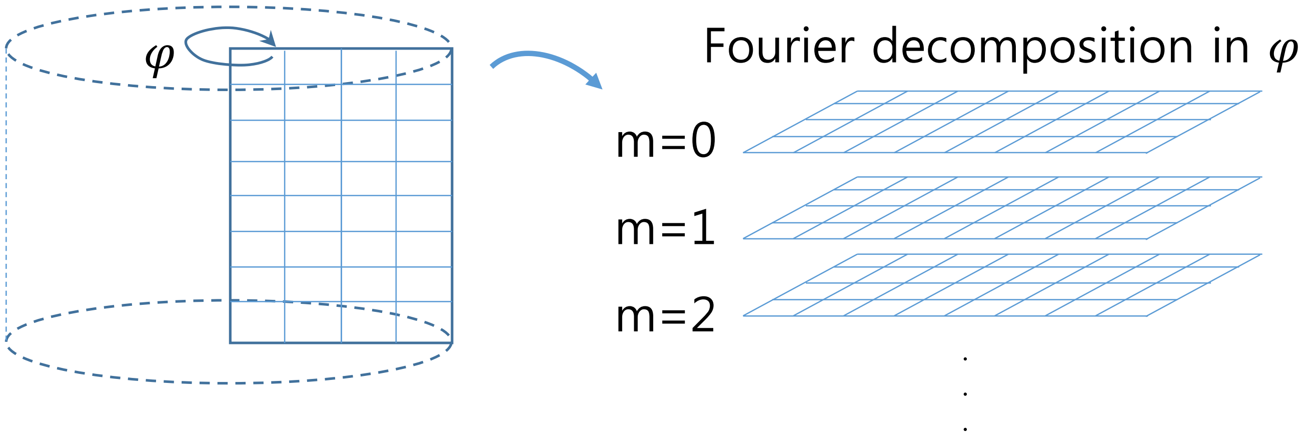
PIC simulations are done using in-house cylindrical PIC code (JoPIC-CYL).
The code is developed in similar way of CALDER-CIR.

3D Cartesian

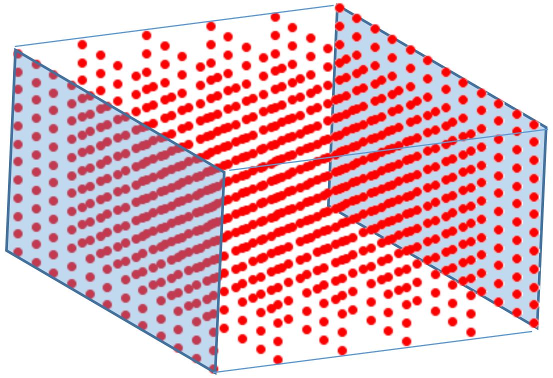


3D cylindrical

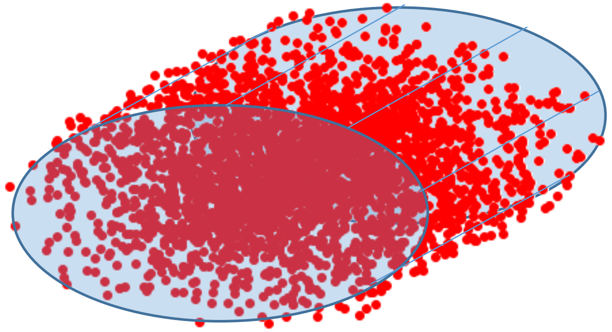
Fourier decomposition in φ



Particle distribution



Particle distribution

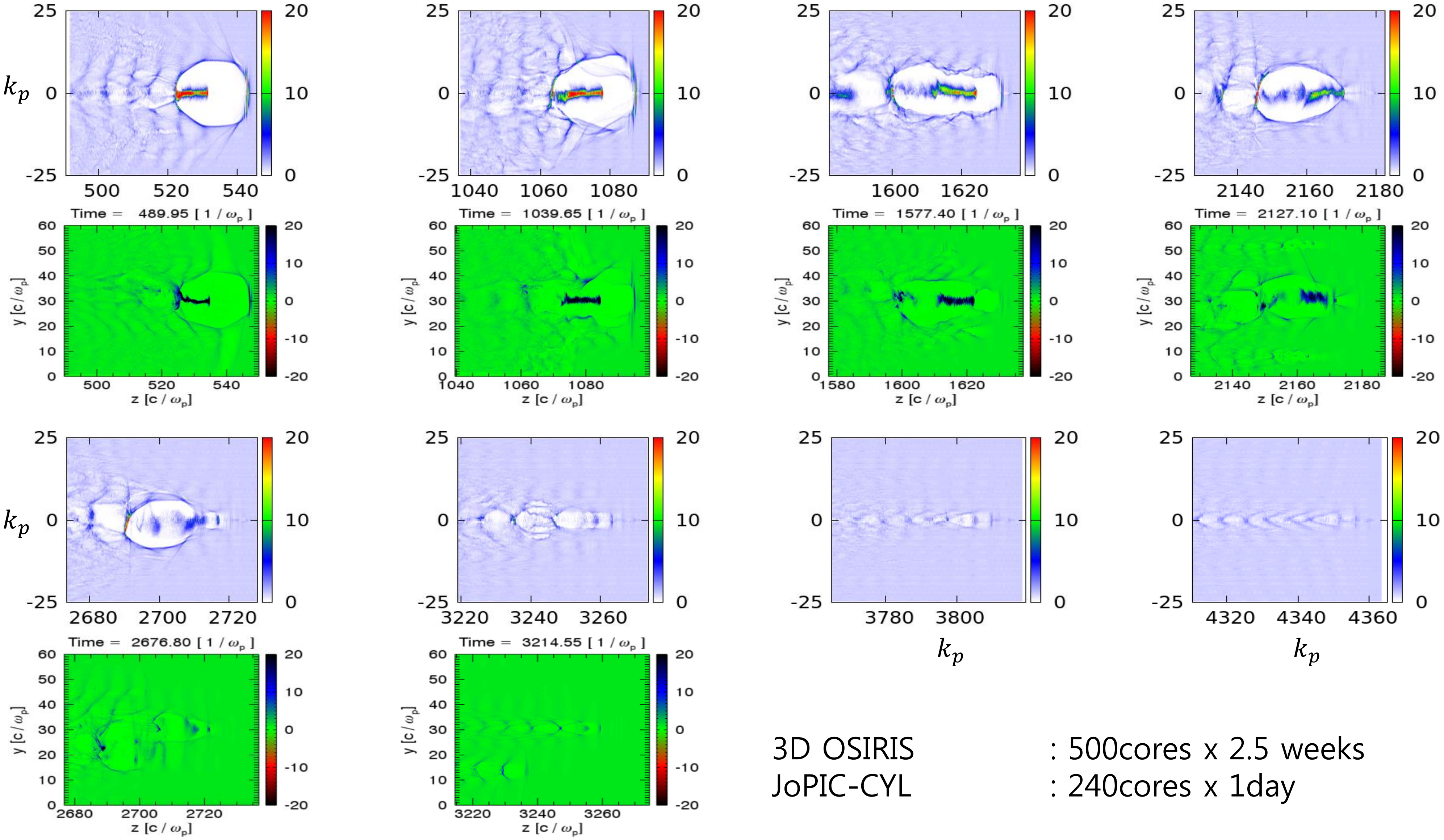


A.F. Lifschitz, Journal of Comp. Phys. ***228*** *1803 (2009)*

JoPIC-CYL

JoPIC-CYL

OSIRIS



3D OSIRIS : 500cores x 2.5 weeks
JoPIC-CYL : 240cores x 1day

Bubble field

When a laser pulse evacuates electrons in plasma (creating bubble), an ideal bubble field (fully evacuating and high wakefield phase velocity close to the light velocity) has form of $E_{x,nor} = \xi/2$.

From Maxwell equations,

$$\Delta\Phi = \frac{3}{2}(1-n) + n\frac{p_x}{\gamma} - \frac{1}{2}\frac{\partial}{\partial\xi}(\nabla_\perp \cdot A_\perp)$$

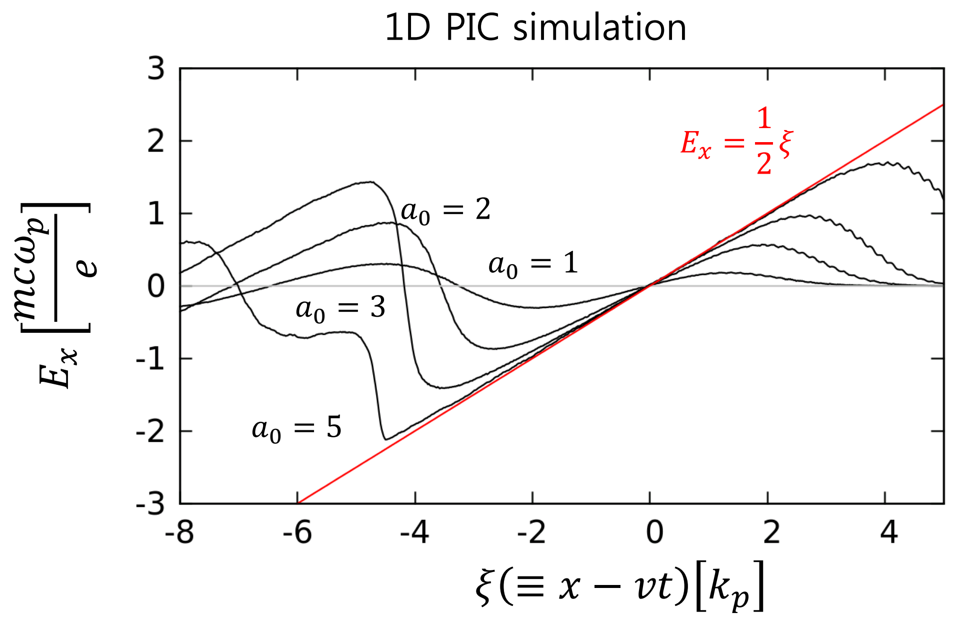
$$\nabla_\perp A_\perp - \nabla_\perp(\nabla_\perp \cdot A_\perp) = n\frac{p_\perp}{\gamma} + \frac{1}{2}\nabla_\perp\frac{\partial\Phi}{\partial\xi}$$

when a residual density inside of bubble is zero (n=0) and $A_\perp = 0$,

$$\Delta\Phi = \frac{3}{2}$$

the solution is $\quad \Phi = 1 - \frac{R^2}{4} + \frac{r^2}{4}, \qquad A_x = \Phi$

*Kostyukov, POP **11** 5256 (2004)*



The bubble field slope does not exceed 0.5, even though the laser intensity is stronger.

$$E_{nor,max} = \frac{k_p R}{2} \rightarrow E_{x,max} = \frac{1}{2}\frac{m_e c}{e}\omega_p^2 R \propto n_0$$

Bubble field generation

Laser pulse energy is transformed to wakefields by consuming laser energy to push electrons.

The laser wave equation is

$$\frac{\partial}{\partial t}\int\left|\left(1+i\frac{k_p}{k_0}\frac{\partial}{\partial\xi}\right)\hat{a}\right|^2 d\xi=-\frac{k_p^2}{k_0^2}{E_{x,max}}^2$$

*Shadwick, POP **16** 056704 (2009)*

laser pulse energy the maximum bubble field

$$-\frac{\partial\mathcal{E}}{\partial t}=\frac{E_{x,max}^2}{\gamma_0^2}\approx\frac{R^2}{4\gamma_0^2} \longleftarrow E_{x,max}=\frac{R}{2}$$

where, $\gamma_0=k_0/k_p$ and R is the bubble radius.

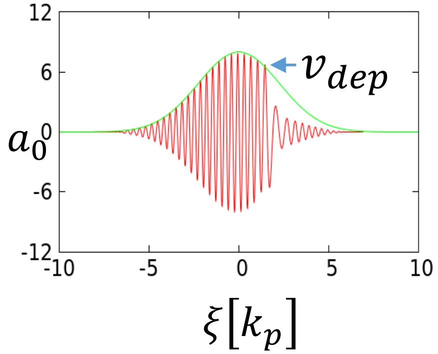
Generally speaking, changes in energy loss rate of laser pulse evolves a bubble.

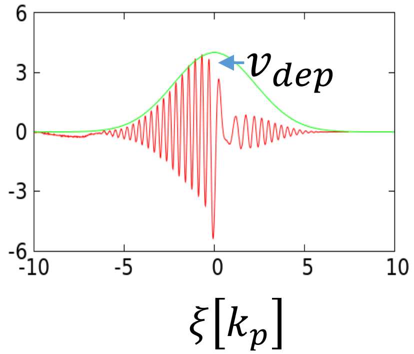
$$R(t)\approx 2\gamma_0\sqrt{\frac{\partial\mathcal{E}}{\partial t}}$$

Laser local depletion

We set a simple model to predict laser loss rate using a constant local depletion speed of laser pulse.

Once local depletion starts, its etching velocity is just dependent on the plasma density.



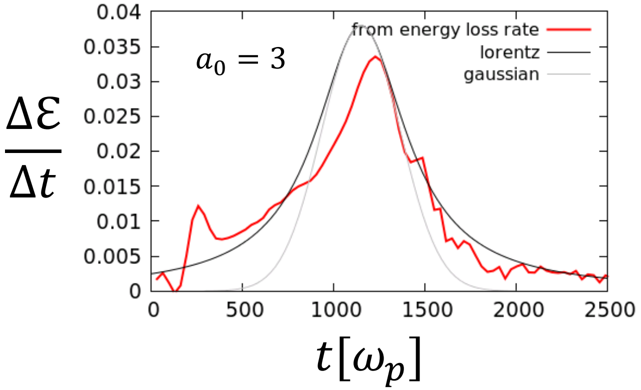


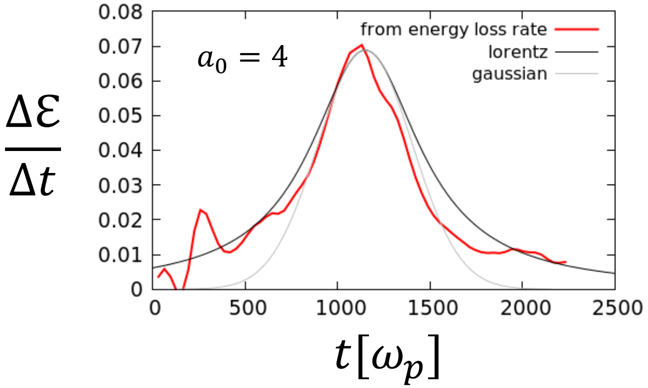
*Decker, POP **3** 2047 (1996)*

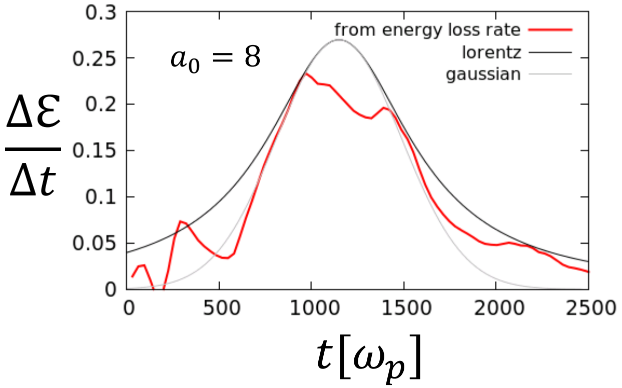
To describe the energy loss rate, we suggest a Lorentz distribution function

$$\frac{\partial \mathcal{E}}{\partial t} = v_{dep} a_0^2 \frac{1}{(t-t_0)^2/\alpha^2 + 1}$$

empirically $v_{dep} = \frac{3}{2}\frac{1}{\gamma_0^2}$ $\qquad \alpha = \frac{x_0}{2 v_{dep}}$







Laser deformation criteria

Deformation range in laser pulse is $2\sigma\sqrt{ln a_0 - \left(\frac{ln a_0}{4\sigma^2}\right)^{1/5}}$ which determines the depletion criteria.

Laser energy equation

$$\frac{\partial}{\partial\tau}\int\left|\left(1+i\frac{k_p}{k_0}\frac{\partial}{\partial\xi}\right)\hat{a}\right|^2 d\xi = -\frac{k_p^2}{2k_0^2}\int\rho\frac{\partial|\hat{a}|^2}{\partial\xi}d\xi \Rightarrow \frac{\partial\mathcal{E}}{\partial\tau} = \frac{1}{2}\int|\hat{a}|^2\frac{\partial\rho}{\partial\xi}d\xi$$

Wake potential equation

$$\frac{\partial^2\phi}{\partial\xi^2} = \frac{1}{2}\left[\frac{1+\hat{a}^2}{(1+\phi)^2} - 1\right]$$

$$\frac{\partial}{\partial\xi}\left(\frac{\partial\phi}{\partial\xi}\right)^2 = 2\frac{\partial\phi}{\partial\xi}\frac{\partial^2\phi}{\partial\xi^2} = \frac{\partial\phi}{\partial\xi}\frac{1+\hat{a}^2}{(1+\phi)^2} - \frac{\partial\phi}{\partial\xi} = -(1+\hat{a}^2)\frac{\partial}{\partial\xi}\frac{1}{(1+\phi)} - \frac{\partial\phi}{\partial\xi}$$

$$\Rightarrow \hat{a}^2\frac{\partial}{\partial\xi}\frac{1}{(1+\phi)} = \hat{a}^2\frac{\partial\rho}{\partial\xi} = -\frac{\partial}{\partial\xi}\left[\left(\frac{\partial\phi}{\partial\xi}\right)^2 + (\phi+1) + \frac{1}{(1+\phi)}\right]$$

Combine two equations

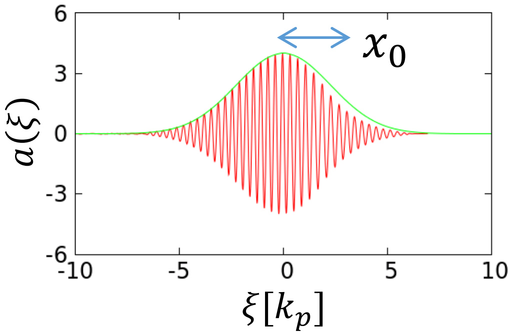
$$\frac{\partial}{\partial\tau}\left|\left(1+i\frac{k_p}{k_0}\frac{\partial}{\partial\xi}\right)\hat{a}\right|^2 = -\frac{1}{2}\frac{{\omega_p}^2}{{\omega_0}^2}\frac{\partial}{\partial\xi}\left[\left(\frac{\partial\phi}{\partial\xi}\right)^2 + \phi + \frac{1}{(1+\phi)} - 1\right] \xrightarrow{1 \gg i\frac{k_p}{k_0}\frac{\partial}{\partial\xi}} \frac{\partial ln(|\hat{a}|^2)}{\partial\tau} + \frac{\partial}{\partial\xi}\frac{-\frac{1}{2\gamma_0^2}}{(1+\phi)} = 0$$

transferring velocity

$$\Rightarrow v(\xi) = -\frac{1}{2\gamma_0^2}\frac{1}{(1+\phi)ln(|\hat{a}|^2)}$$

Well-known fact is v of the leading edge of laser pulse is linear group velocity ~ $-1/2\gamma_0^2$

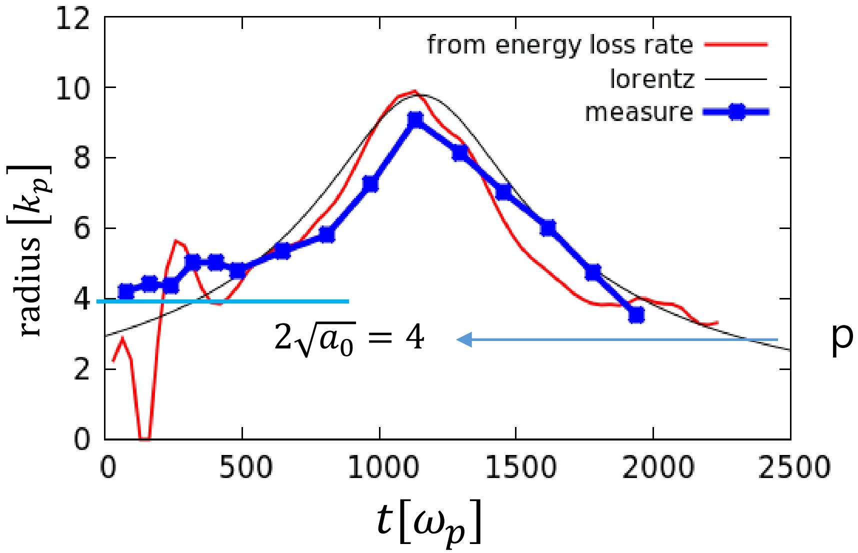
$$(1+\phi)ln(|\hat{a}|^2) = 1 \xrightarrow{a(\xi)>1} \boldsymbol{x_0} = \sigma\sqrt{ln a_0 - \left(\frac{ln a_0}{4\sigma^2}\right)^{1/5}}$$



Prediction on bubble evolution

Bubble radius evolves from $2\sqrt{a_0}$ to $\sqrt{6}a_0$.

$$R(t) = 2\gamma_0\sqrt{\frac{\partial \mathcal{E}}{\partial t}} = 2\gamma_0\sqrt{f(t)} \approx \sqrt{6}a_0\sqrt{\frac{1}{(t-t_0)^2/\alpha^2+1}} \quad \rightarrow \quad R_{max} = \sqrt{6}a_0$$

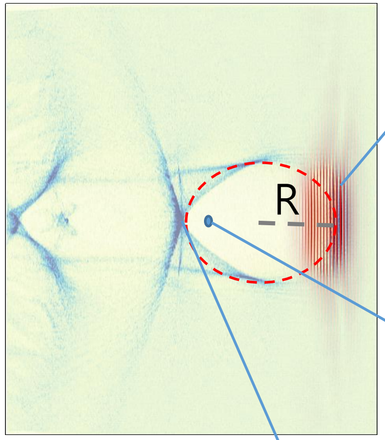


prediction of initial bubble size

Lu, Phys. Rev. STAB ***10*** *061301 (2007)*

Electron Injection

Using the injection condition of electron,
we can expect the injected beam size resulted by continuous injections.



$$v_{head} \approx 1 - \frac{3}{2}\frac{\omega_p^2}{\omega_0^2}$$

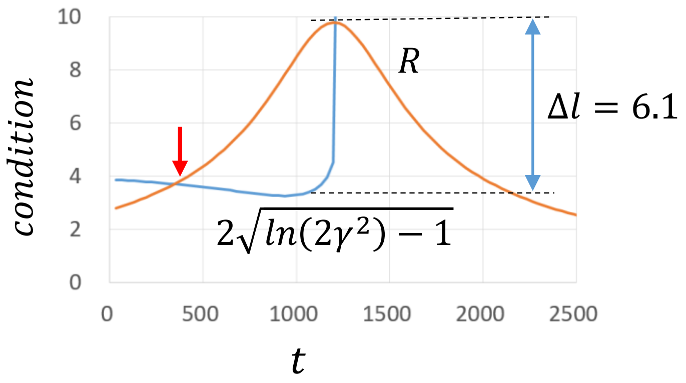
$$\gamma_e > \gamma_{back}$$

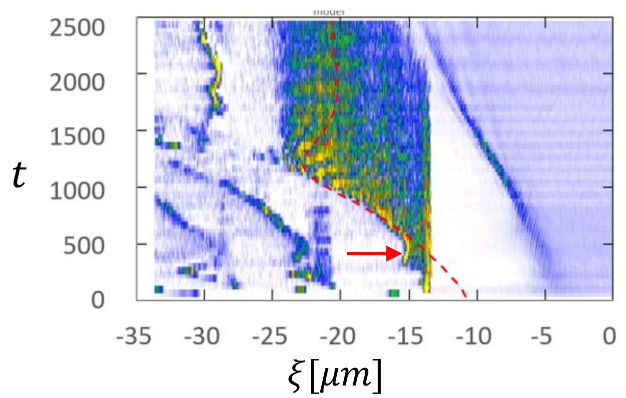
$$v_{back} \approx 1 - \frac{3}{2}\frac{\omega_p^2}{\omega_0^2} - 2\frac{dR}{d\tau}$$

$$\gamma_{back} \approx \frac{1}{\sqrt{3\frac{\omega_p^2}{\omega_0^2} + 4\frac{dR}{d\tau}}}$$

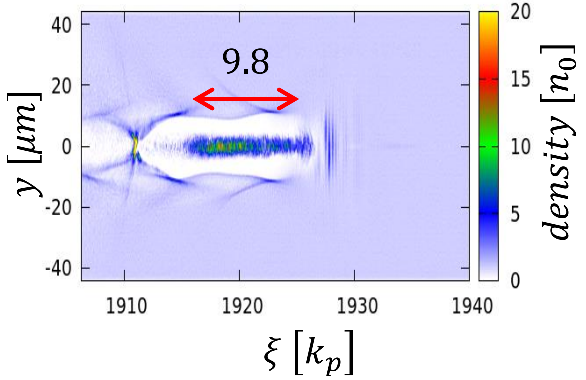
$$2\sqrt{ln(2\gamma^2) - 1} < R$$

Thomas, POP 17 056708 (2010)





Beam length in bubble is stretched during injection period.
A mount of stretch is $shift = v_{dep}\Delta t$.



$$beam\ length = \Delta l + v_{dep}(t_0 - t_1)$$

Electron Acceleration

Modified electron momentum indicates a possible phase locking.

Equation of motion :

$$\frac{\partial p_x}{\partial t} = -(1+v_0)\frac{\xi}{4}$$
$$\frac{\partial \xi}{\partial t} = \frac{p_x}{\gamma} - v_0$$
$$v_0 = 1 - \frac{3}{2}\frac{1}{\gamma_0^2} - A\frac{dR}{dt}$$

$\Rightarrow$

$$\xi = \frac{3}{2}\frac{1}{\gamma_0^2}(t-t_1) + AR(t) - (A+1)R(t_1)$$
$$p_x(t) \cong -\frac{3}{8}\frac{1}{\gamma_0^2}(t-t_1)^2 - \frac{A}{2}\int_{t_1}^{t} R(t)dt + \frac{A+1}{2}R(t_1)(t-t_1)$$

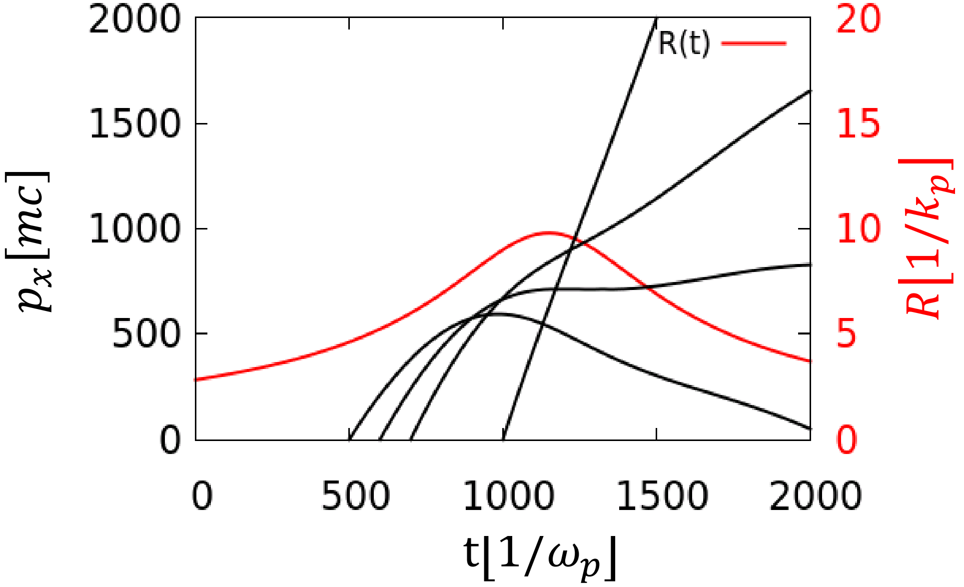
Ref. stationary model

$$p_x(t) \cong -\frac{3}{8}\frac{1}{\gamma_0^2}t^2 + \frac{1}{2}Rt$$

Using $R(t) = R_{max}[(t-t_0)^2/\alpha^2+1]^{-1/2}$

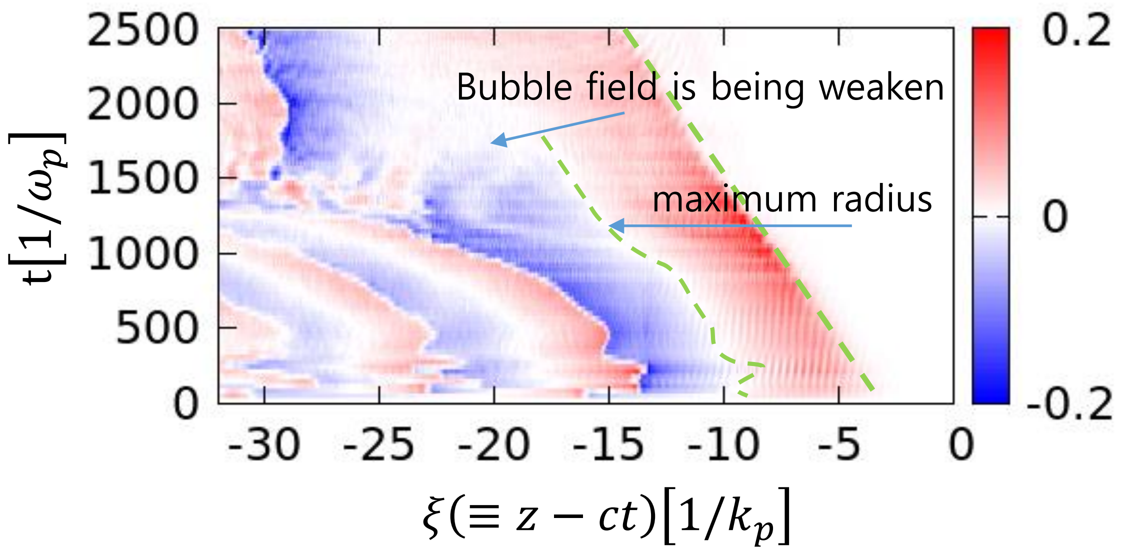
$$p_x(t) \cong -\frac{3}{8}\frac{1}{\gamma_0^2}(t-t_1)^2 + \frac{A+1}{2}R(t_1)(t-t_1) - \frac{A}{2}\alpha R_{max}\left[arsinh\left(\frac{t-t_0}{\alpha}\right) - arsinh\left(\frac{t_1-t_0}{\alpha}\right)\right]$$

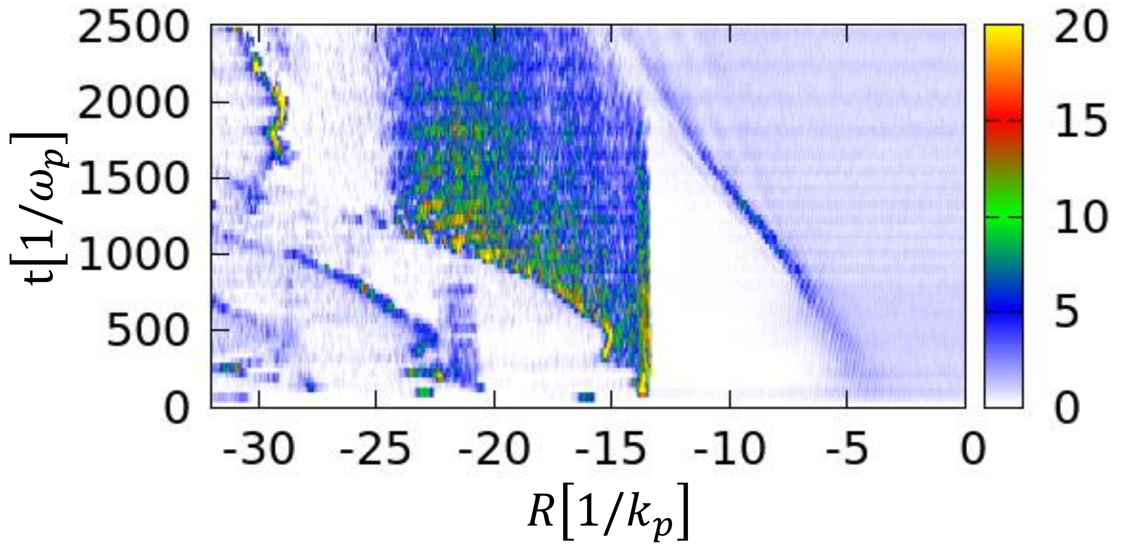
when $A = 0.6$



Electron Acceleration

In normal case, it does not show phase locking phenomenon.





1) The velocity of bubble center did not go down after full expansion.
2) The bubble field starts to decrease after full expansion.

➔ Because of injected electron beam in the bubble.

➔ Injected beam plays a residual electron density in a bubble that makes weaken bubble field

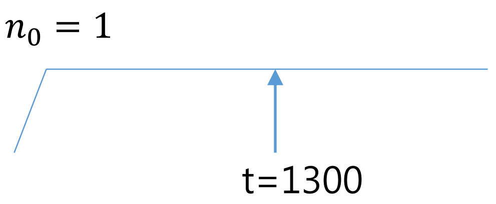
$$\Delta\Phi \sim \frac{1}{2}\left(1-\frac{n}{2}\right) \Rightarrow E_x \sim \frac{1}{2}\left(1-\frac{n}{2}\right)x$$

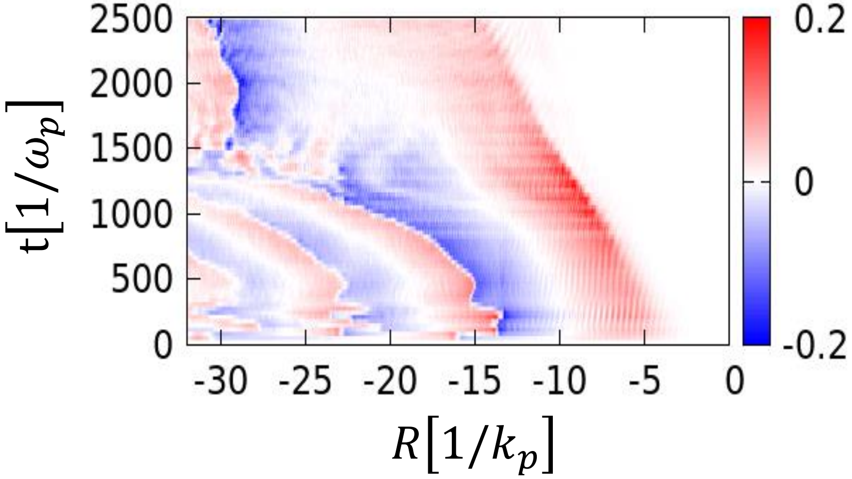
bubble field slope

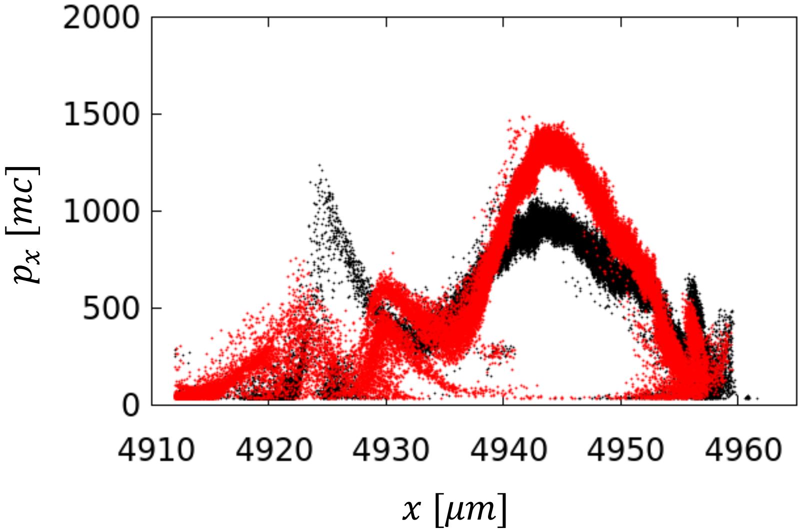
Enforcement of phase locking

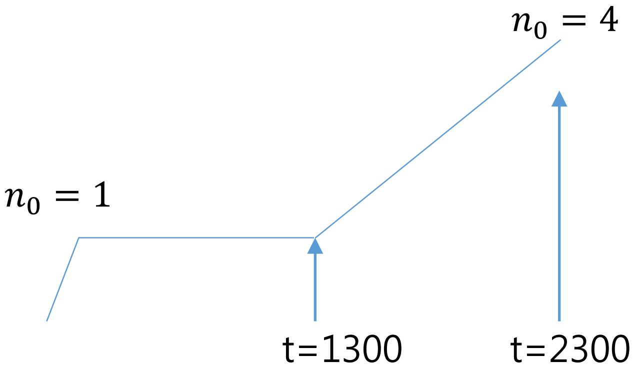
In normal case, it does not show phase locking phenomenon.

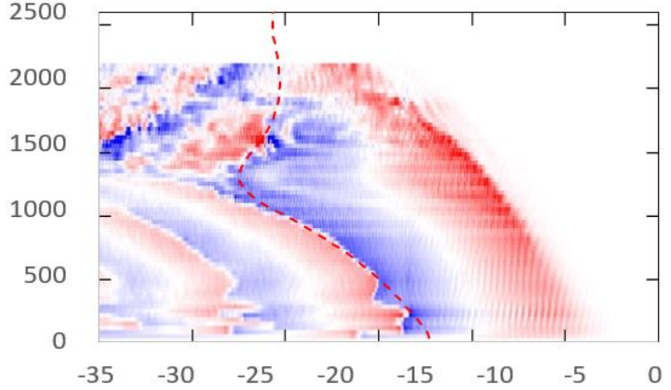
$a_0 = 4, n_0 = 5 \times 10^{18} cc$
$P = 80TW, \mathcal{E} = 10J$

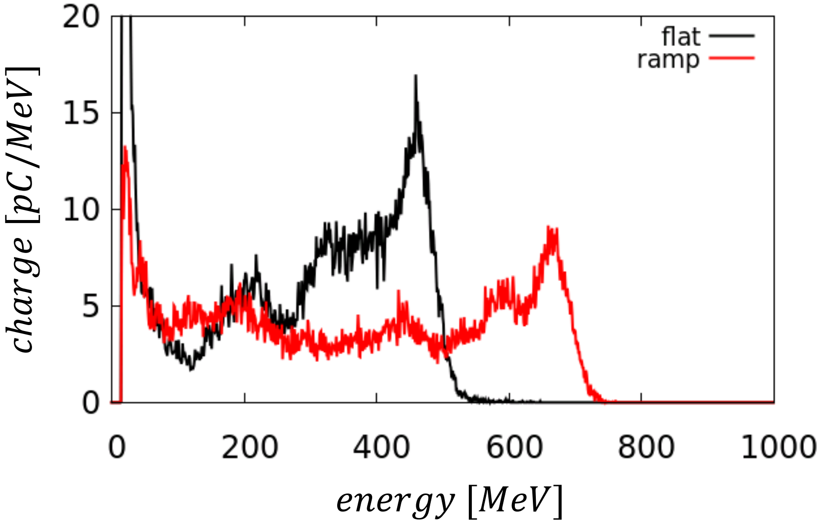












Conclusion

1. Bubble size is related the energy loss rate of laser pulse.
2. Considering laser local depletion, we suggested the function of bubble radius, which agrees with PIC simulation.
3. Predictions on bubble size, electron injection and longitudinal beam size are compared with PIC simulation.
4. Modified electron acceleration potentially has phase locking to enhance electron energy.
5. We couldn’t find clues of phase locking in a normal condition, however, the phase locking effect is possible adding plasma ramp where the maximum bubble radius was reached.