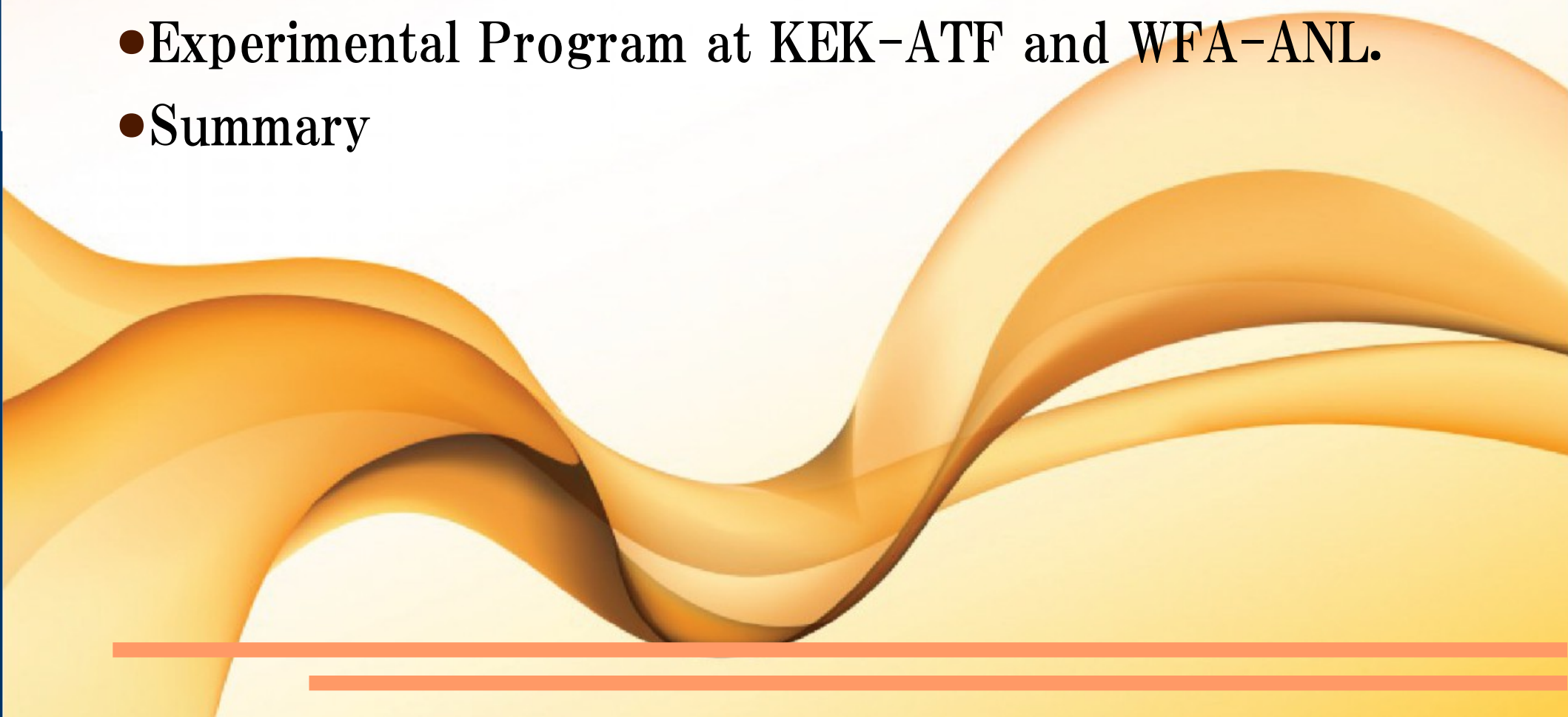


Emittance Exchange Program at KEK STF

Masao KURIKI
Hiroshima University

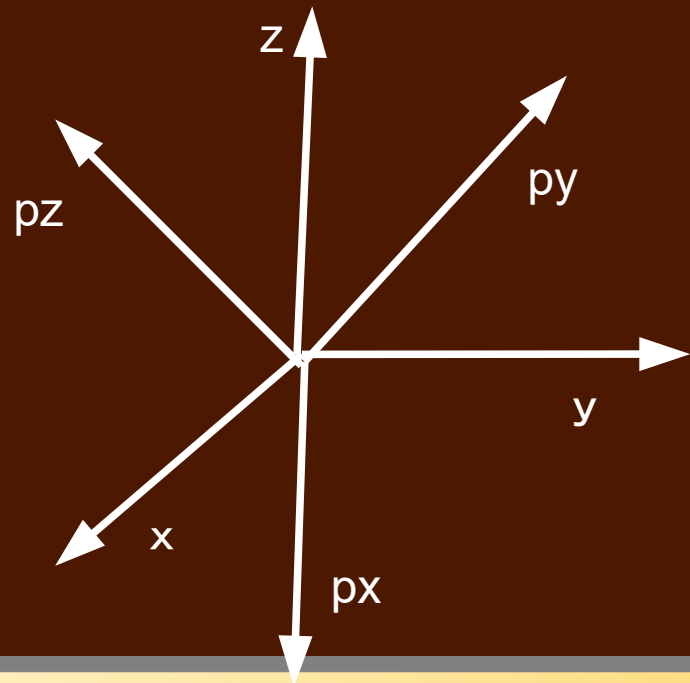
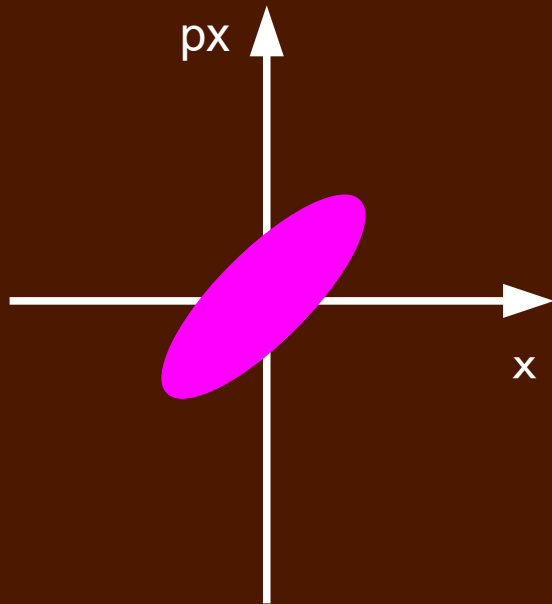
AFAD 2018, 29-31 Jan, at Daejeon Convention Center, Daejeon, Korea

Content

- Introduction
 - Emittance Exchange
 - Experimental Program at KEK-ATF and WFA-ANL.
 - Summary
- 

Introduction

- Particles occupy the area of the phase-space: emittance.
- Emittance is invariant.
- In principle, the emittance is the volume in 6D phase-space.
- The 6D emittance is invariant (Liouville's theorem).



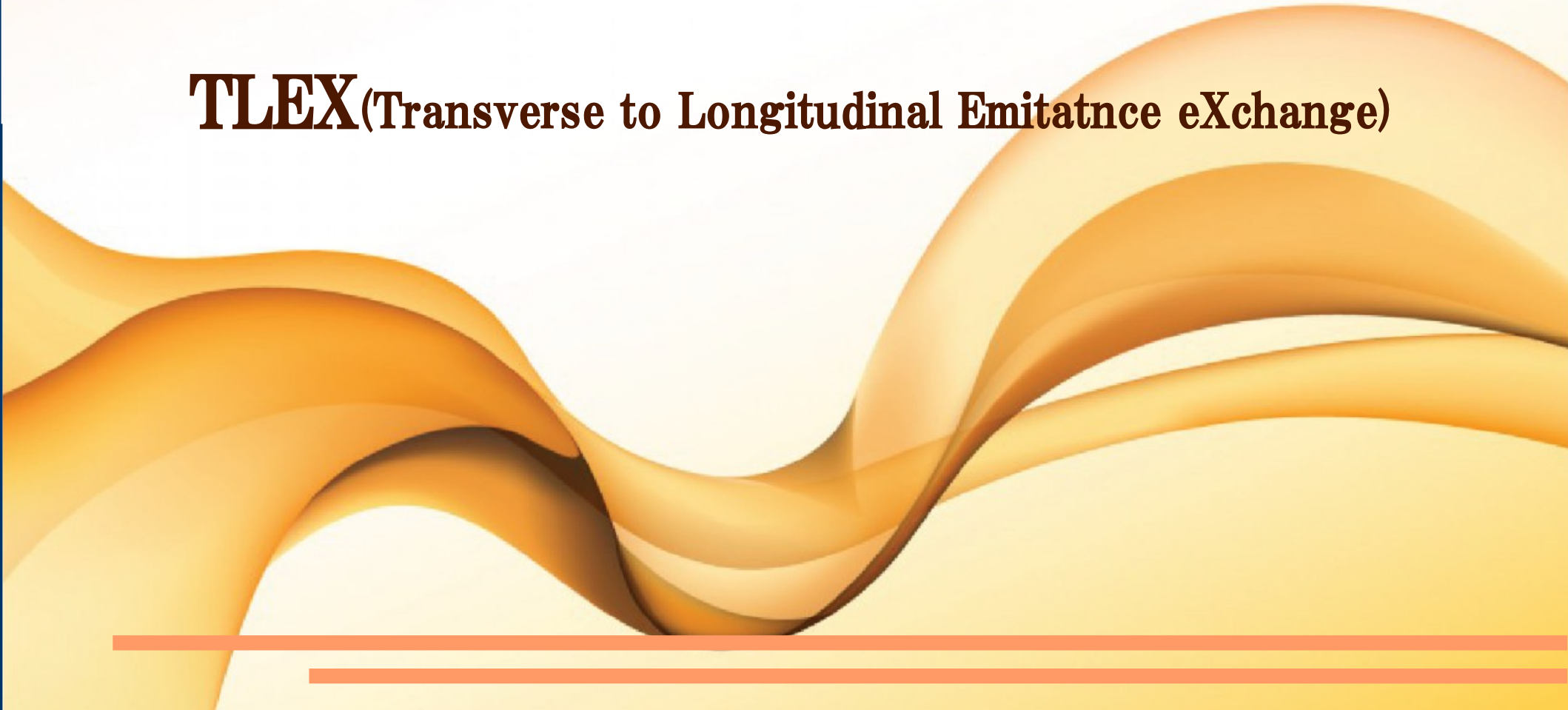
Louisville's theorem

- **6D emittance is invariant.**
- **2D emittance (e_x , e_y , e_z) are not invariant.**
- **Can the partitioning of x , y , and z be modified as we want? (Emittance Exchange).**

Emittance	e_x	e_y	e_z	$e_x * e_y * e_z$
No coupling	conserved	conserved	conserved	Invariant
With coupling	varied	varied	varied	Invariant

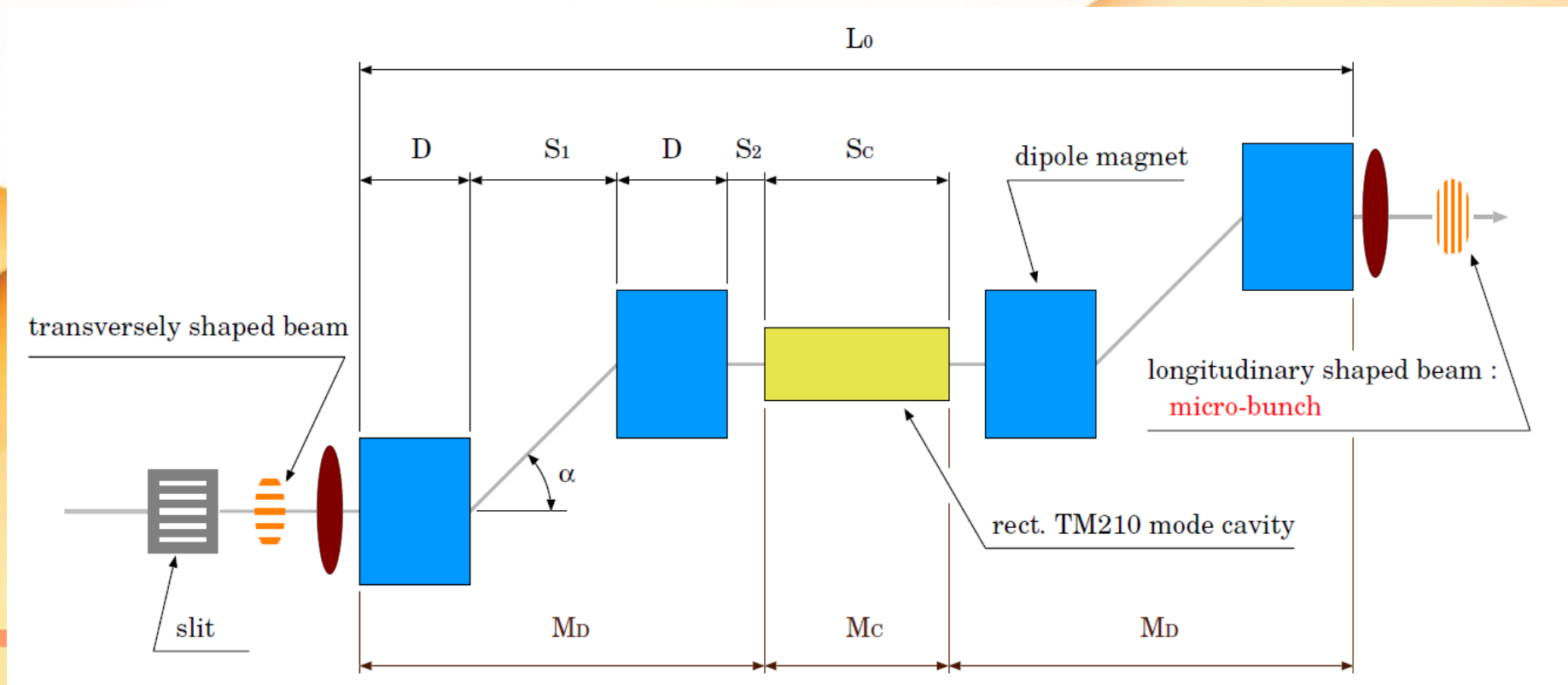
XY-Z Emittance Exchange

TLEX(Transverse to Longitudinal Emittance eXchange)



XY-Z Emittance Exchange

- TLEX can be made with chicane or two doglegs and a dipole mode RF cavity.
- The phase spaces of X(transverse direction in deflecting plane) and Z (longitudinal direction) are swapped.



Transfer Matrix of a Dogleg

One dog-leg section

$$M_D(\eta, \xi, L) = \begin{bmatrix} 1 & L & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Dispersion

$$\eta = S_1 \frac{\sin \alpha}{\cos^2 \alpha} + \frac{2D}{\sin \alpha} \left(\frac{1}{\cos \alpha} - 1 \right)$$

Momentum compaction

$$\xi = S_2 \frac{\sin^2 \alpha}{\cos^3 \alpha} + \frac{2D}{\sin \alpha - \alpha}$$

Effective length

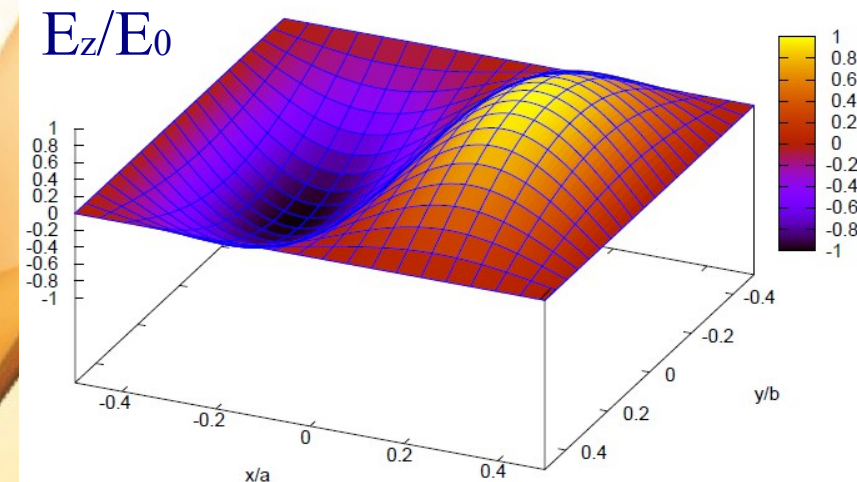
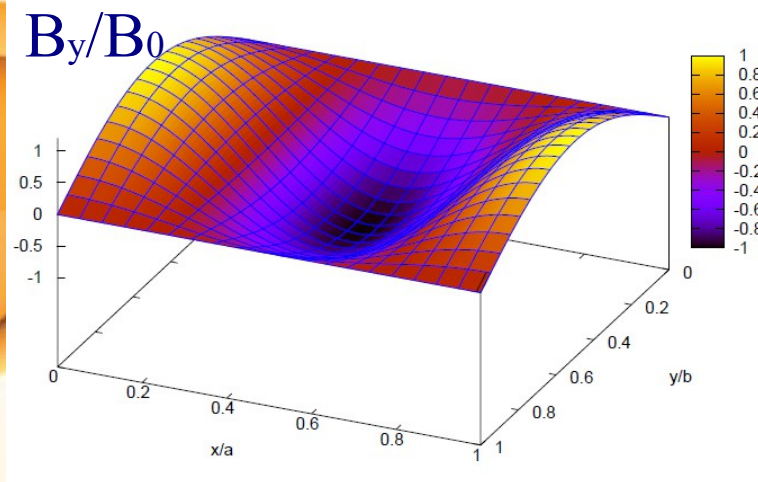
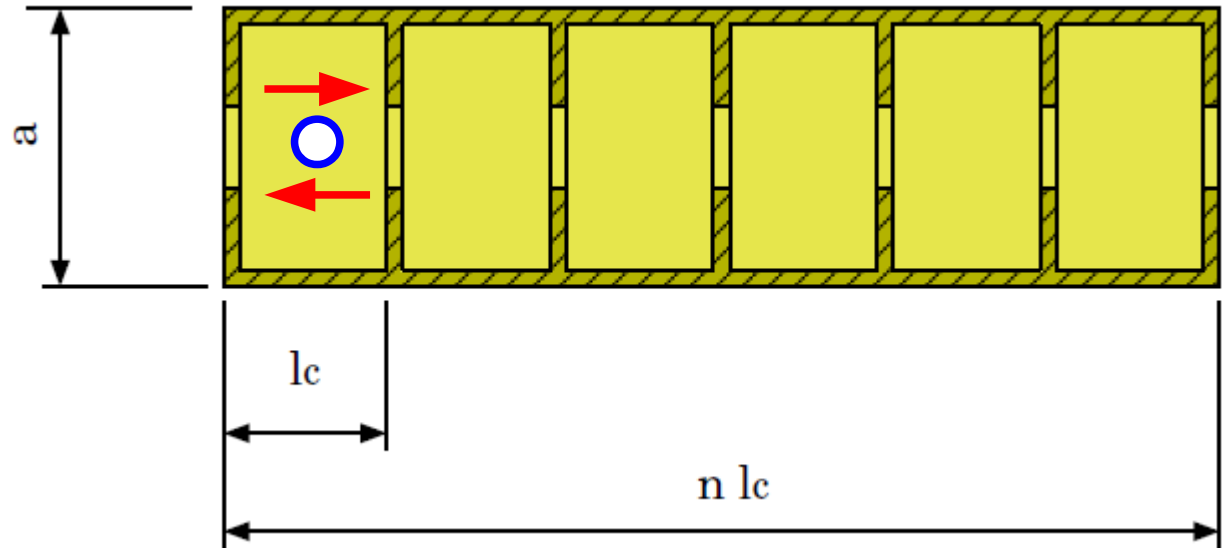
$$L = S_1 \frac{1}{\cos^3 \alpha} + \frac{2D}{\cos \alpha} + S_2$$

Transfere Matrix of Diple RF

TM011cavity

$$M_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{bmatrix}$$

$$k = \frac{V_0}{aE_0}$$



Matrices -EEX section-

$$M_{EEX} = M_D(\eta, \xi, L) M_C M_D(\eta, \xi, L)$$

$$= \begin{bmatrix} 0 & 0 & -L/\eta & \eta - L\xi/\eta \\ 0 & 0 & -1/\eta & -\xi/\eta \\ -\xi/\eta & \eta - L\xi/\eta & 0 & 0 \\ -1/\eta & -\frac{L}{\eta} & 0 & 0 \end{bmatrix}$$

matching condition

$$1 + \eta k = 0$$

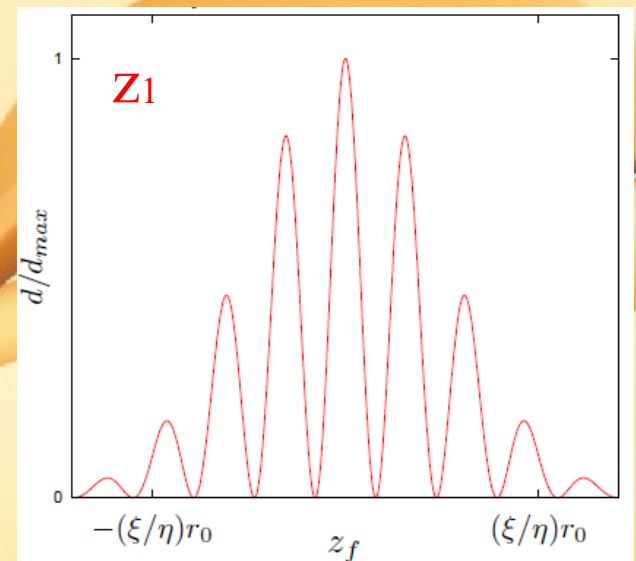
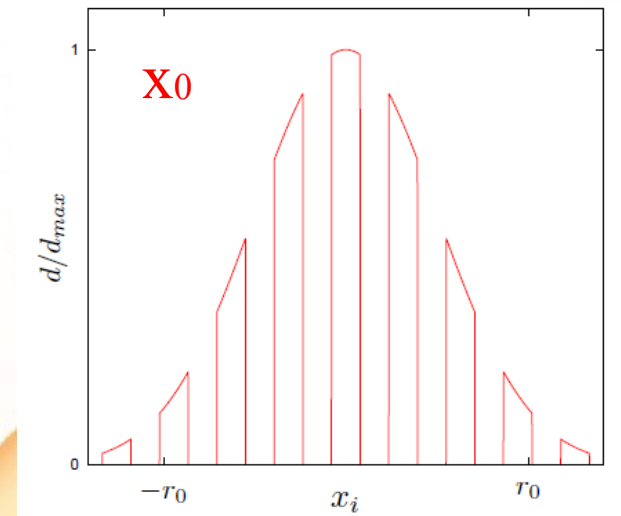
compression
factor

dilution
factor

$$z_1 = -\frac{\xi}{\eta} x_0 + \left(\eta - L \frac{\xi}{\eta} \right) x'$$

$$\delta_1 = -\frac{1}{\eta} x_0 - \frac{L}{\eta} x'$$

x-z space exchange
by the 4D space rotation.



XY Emittance Exchange

RFTB(Round to Flat Beam Transformation)

Magnetized Beam

- Beam Generation in solenoid field (B_z)
- Canonical momentum by Vector potential has the angular momentum.
- By the fringe field, the canonical momentum becomes the kinetic momentum .
- The x-py and y-px correlated beam is obtained.
- The ex and ey are quite bad and the beam is useless in a usual sense.

Vector Potential

$$A_x = -\frac{B_0}{2} y \quad A_y = \frac{B_0}{2} x$$

Magnetized Beam

$$\vec{P}_c = \vec{P} - e \vec{A} \quad \vec{P}_c \sim \frac{eB_0}{2} \begin{pmatrix} y \\ -x \end{pmatrix}$$

In sigma matrix

$$\Sigma_0 = \sigma^2 \begin{bmatrix} 1 & 0 & 0 & -\kappa \\ 0 & \kappa^2 & \kappa & 0 \\ 0 & \kappa & 1 & 0 \\ -\kappa & 0 & 0 & \kappa^2 \end{bmatrix} \quad \kappa \equiv \frac{eB_0}{2mc}$$

Restoration Matrix

- By ignoring the intrinsic emittance, the beam at the exit of the solenoid field is

$$\vec{Y}_0 = \vec{S} \vec{X}_0 \quad \vec{S} = \begin{pmatrix} 0 & \frac{2}{eB_0} \\ \frac{-eB_0}{2} & 0 \end{pmatrix}$$

- Beam transfer with a matrix M is

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix} = M \begin{pmatrix} 1 \\ S \end{pmatrix} X_0$$

- If the condition is satisfied, Y_1 is independent from X_0 .

$$M_{21} + M_{22}S = 0$$

Restoration Matrix

- The solution is the skew Q channel composed from three quadrupoles.

$$M = R^{-1} Q(q_3) O(D) Q(q_2) O(D) Q(q_1) R$$

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} I & I \\ -I & I \end{pmatrix} \quad Q(q) = \sqrt{2} \begin{pmatrix} Q(q) & 0 \\ 0 & Q(-q) \end{pmatrix} X_0$$

- The skew quadrupole strengths are given by the initial sigma matrix.

$$q_1 = \pm \sqrt{\frac{-Ds_{11} + s_{12} + 2D^2 s_{21} + 2Ds_{22}}{2D^2 s_{12}}}$$

$$q_2 = -\frac{s_{12} + 2Ds_{22}}{D^2(1 + q_1 s_{12})}$$

$$q_3 = \frac{-q_1 - q_2 - Dq_1 q_2 s_{11} - s_{21}}{1 + (2Dq_1 + Dq_2)s_{11} + D^2 q_2(q_1 + s_{21})}$$

Emittance Ratio

- The emittance after the skew Q-channel is

$$\epsilon_n^{\pm} \sqrt{(\epsilon_n^u)^2 + (\gamma \beta L)^2} \pm (\gamma \beta L)$$
$$L = \frac{e B_0 \sigma_c^2}{p_z}$$

- If $\epsilon_n^u \leq \gamma \beta L$ the emittance and the ratio are

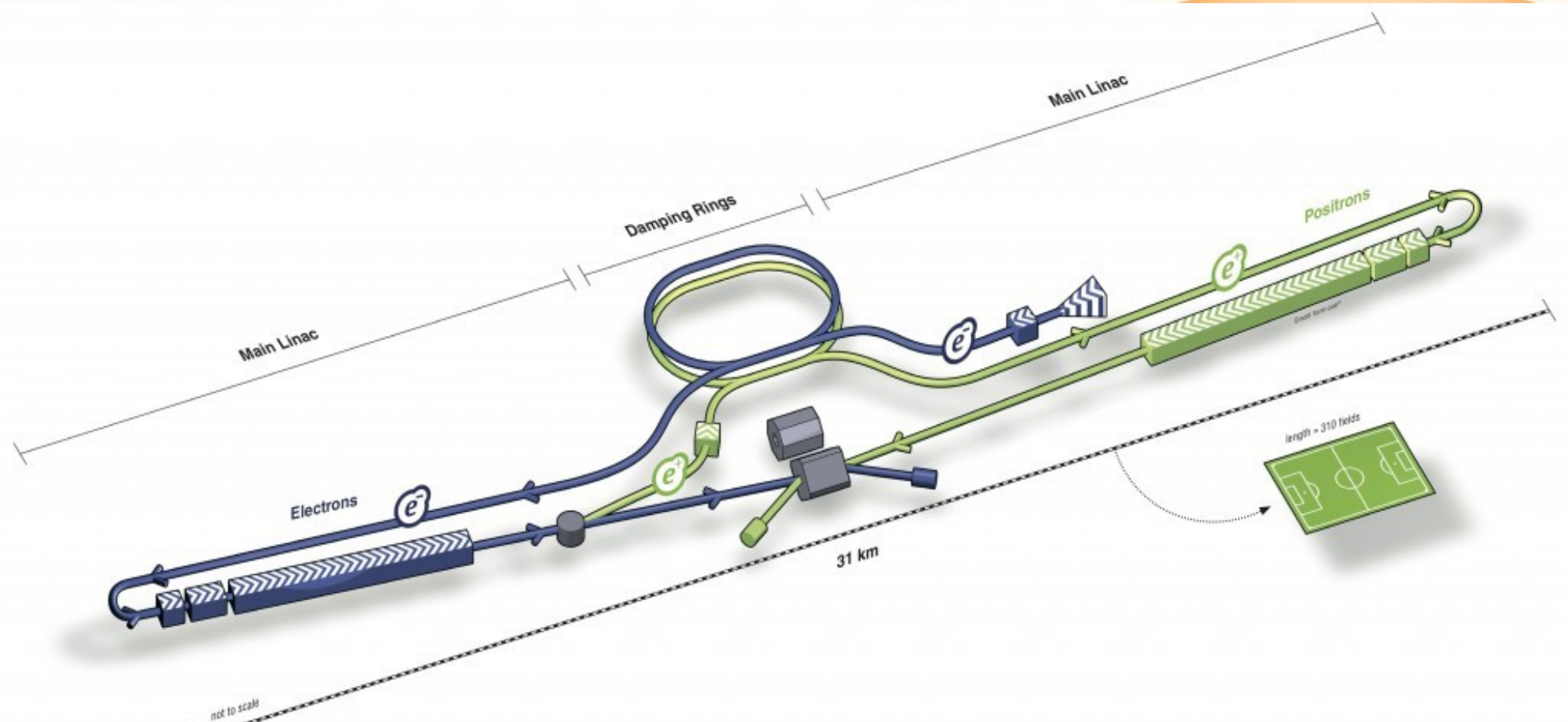
$$\epsilon_n^+ \sim 2 \gamma \beta L$$

$$\epsilon_n^- \sim \frac{(\epsilon_n^u)^2}{(2 \gamma \beta L)}$$

$$\frac{\epsilon_n^+}{\epsilon_n^-} \sim \frac{(\epsilon_n^u)^2}{(2 \gamma \beta L)^2}$$

Asymmetric Beam for Linear Collider

- a) Linear Collider is an only solution for e^+e^- collision beyond the limit of ring colliders by the huge energy loss by synchrotron radiation.
- b) The beam is “one pass” and the beam current is very limited. In order of 10mA for linear colliders, and in order of A for a modern ring collider .
- c) To obtain an enough luminosity, the beam is focused down to nm.



High Aspect Ratio Beam in LC

Event Rate

$$N = \sigma \times L$$

$\sigma_y \ll \sigma_x$ (Asymmetric Beam)

Luminosity

$$L = \frac{f_{rep} n_b N^2}{4\pi \sigma_x \sigma_y}$$

Beamstrahlung

$$\frac{\Delta E}{E} \propto \frac{N^2 E}{(\sigma_x^2 + \sigma_y^2) \sigma_z}$$

Disruption

$$D_{x,y} = \frac{2Nr_e}{\gamma} \frac{\sigma_z}{\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Parameter

Value

Horizontal size

640 nm

Vertical size

5.7 nm

Bunch length

300 μm

Vertical Disruption

19.4

RMS energy by BS

2.4%

Horizontal emi.

10 mm.mrad

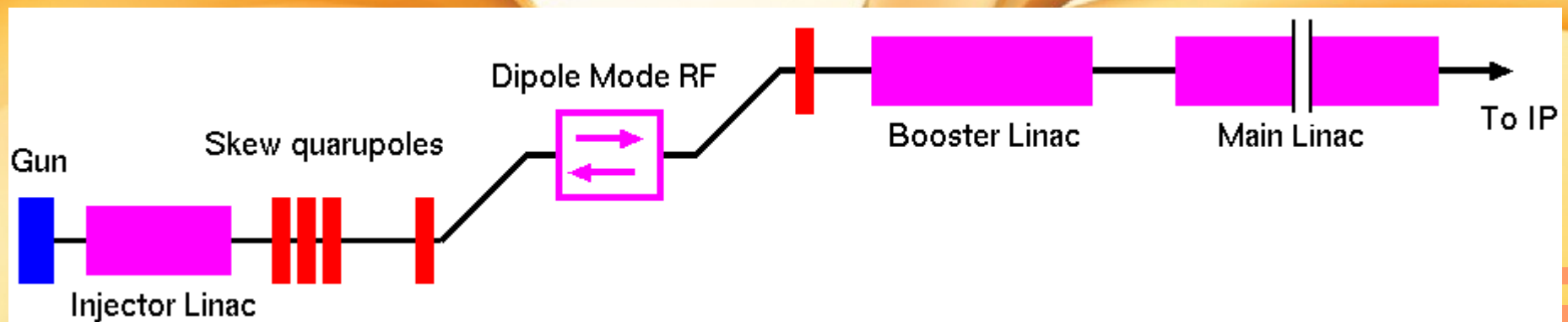
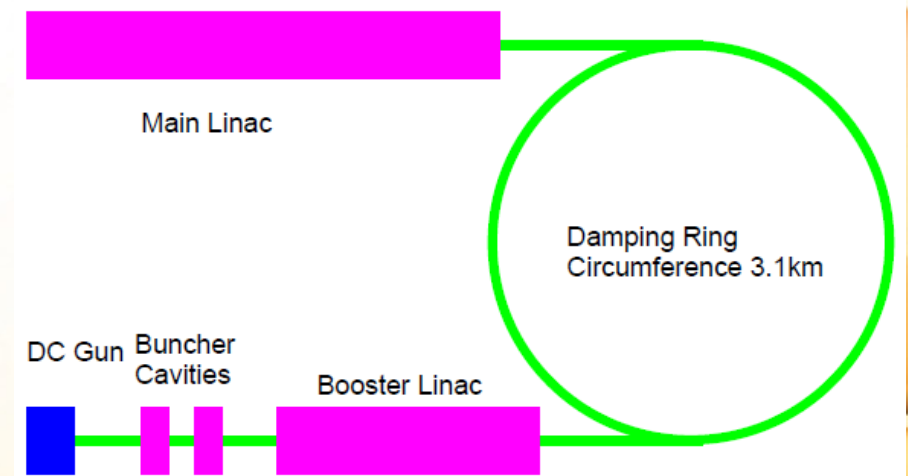
Vertical emi.

0.04 mm.mrad

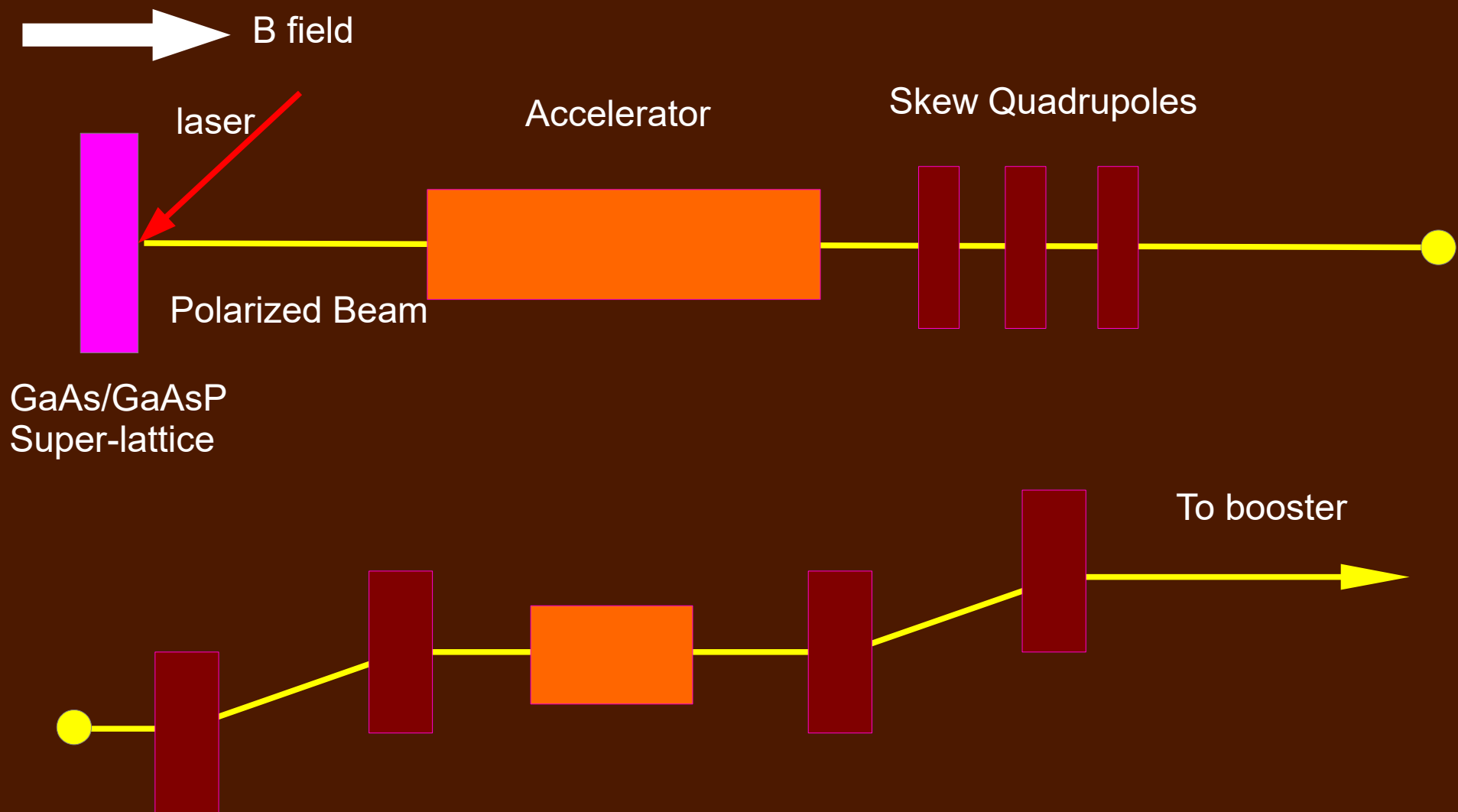
aspect ratio 250
is made up with 3km DR

High Aspect Ratio Beam by EMIX

- The high aspect ratio beam is generated by a 3km DR (Damping Ring) in the current design.
- If the beam can be generated directly from the injector with EmiX technique, the design can be much simpler.

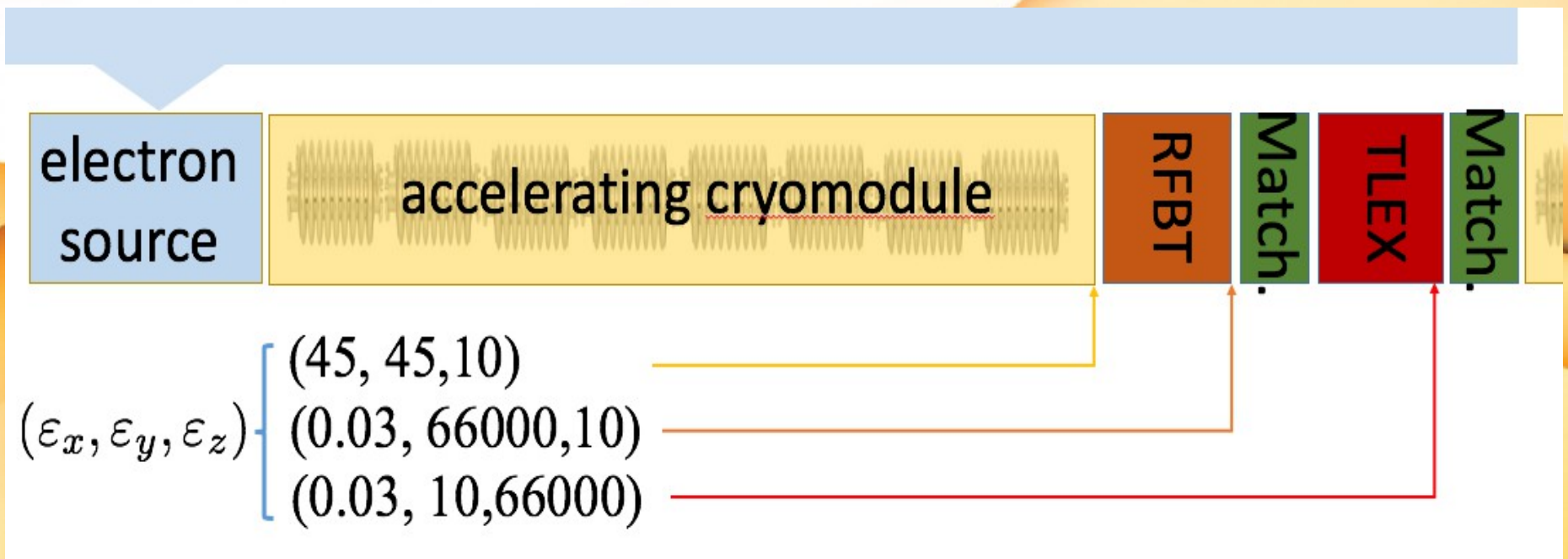


High Aspect Ratio Beam Generation with EmiX



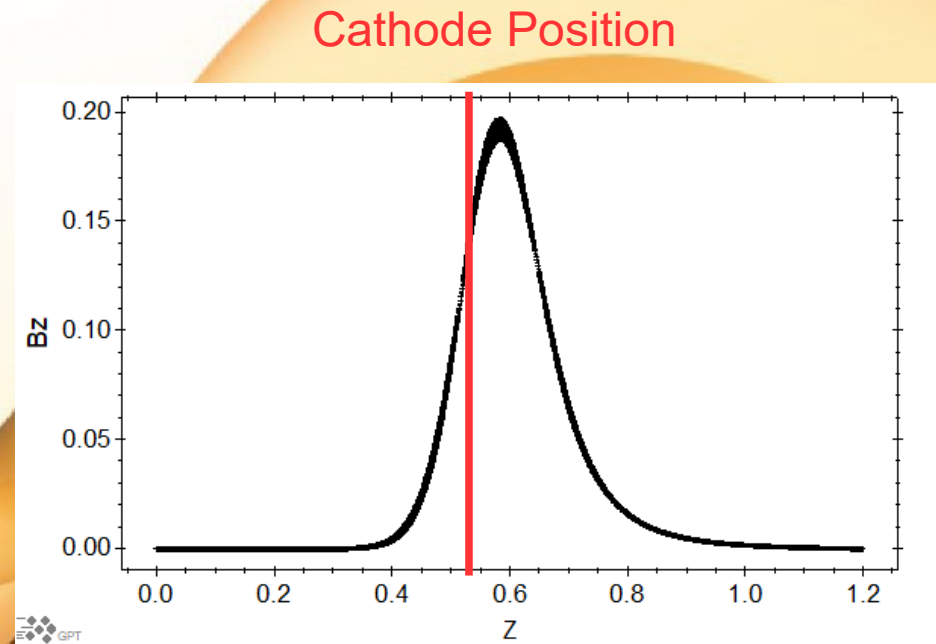
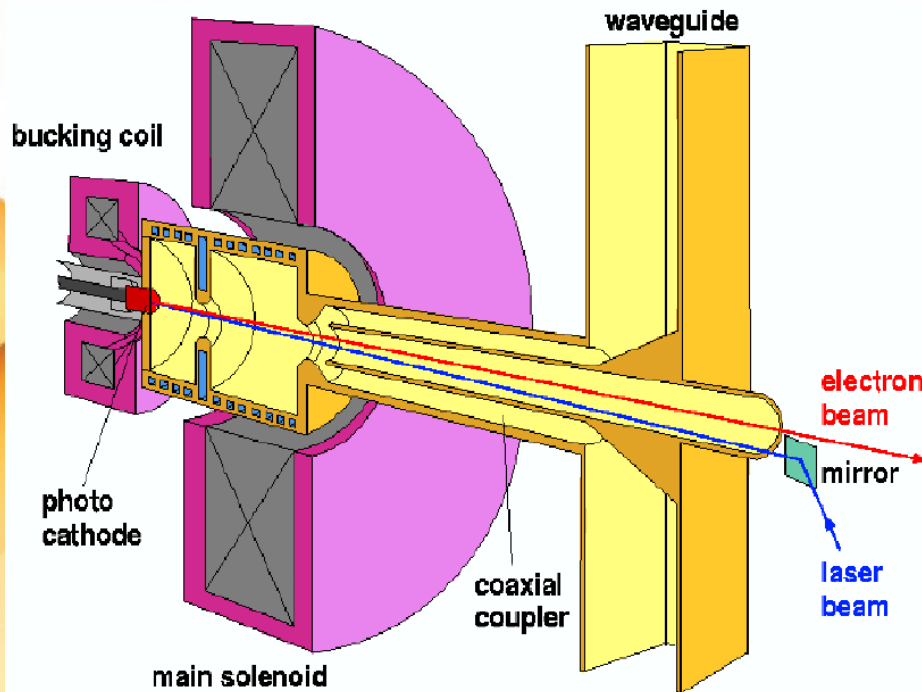
Flat Beam Generation for LC

- The flat beam ($\epsilon_x=10\mu\text{m}$, $\epsilon_y=0.04\mu\text{m}$) can be generated with RFTB and TLEX.
- The beam is generated in a large size to compensate the space charge non-linearity. $\epsilon_x=\epsilon_y=45\mu\text{m}$.
- By RFTB, it can be $\epsilon_x=66000\mu\text{m}$ and $\epsilon_y=0.03$.
- The ϵ_x is too large for LC. It should be exchanged with ϵ_z by TLEX.



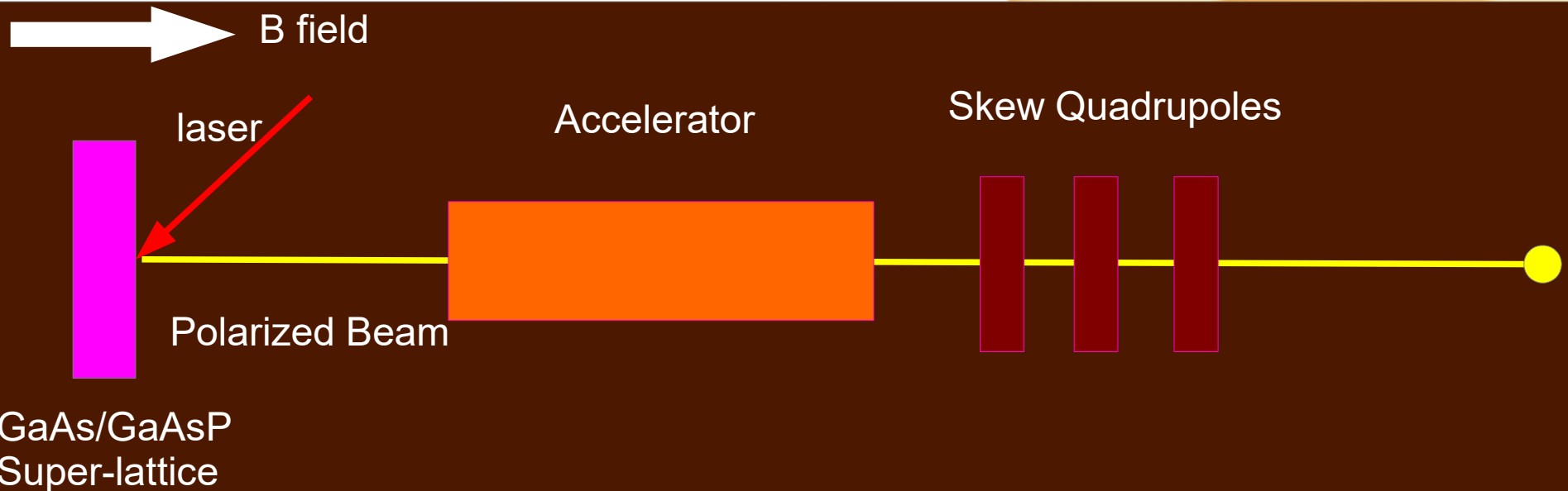
STF RF Gun

- L-band (1.3 GHz) normal conducting RF gun developed for FLASH/XFEL at DESY.
- By switching current of the bucking coil, solenoid field can be made on the cathode surface.
- In the simulation, that was 0.1 Tesla.



RFTB Section

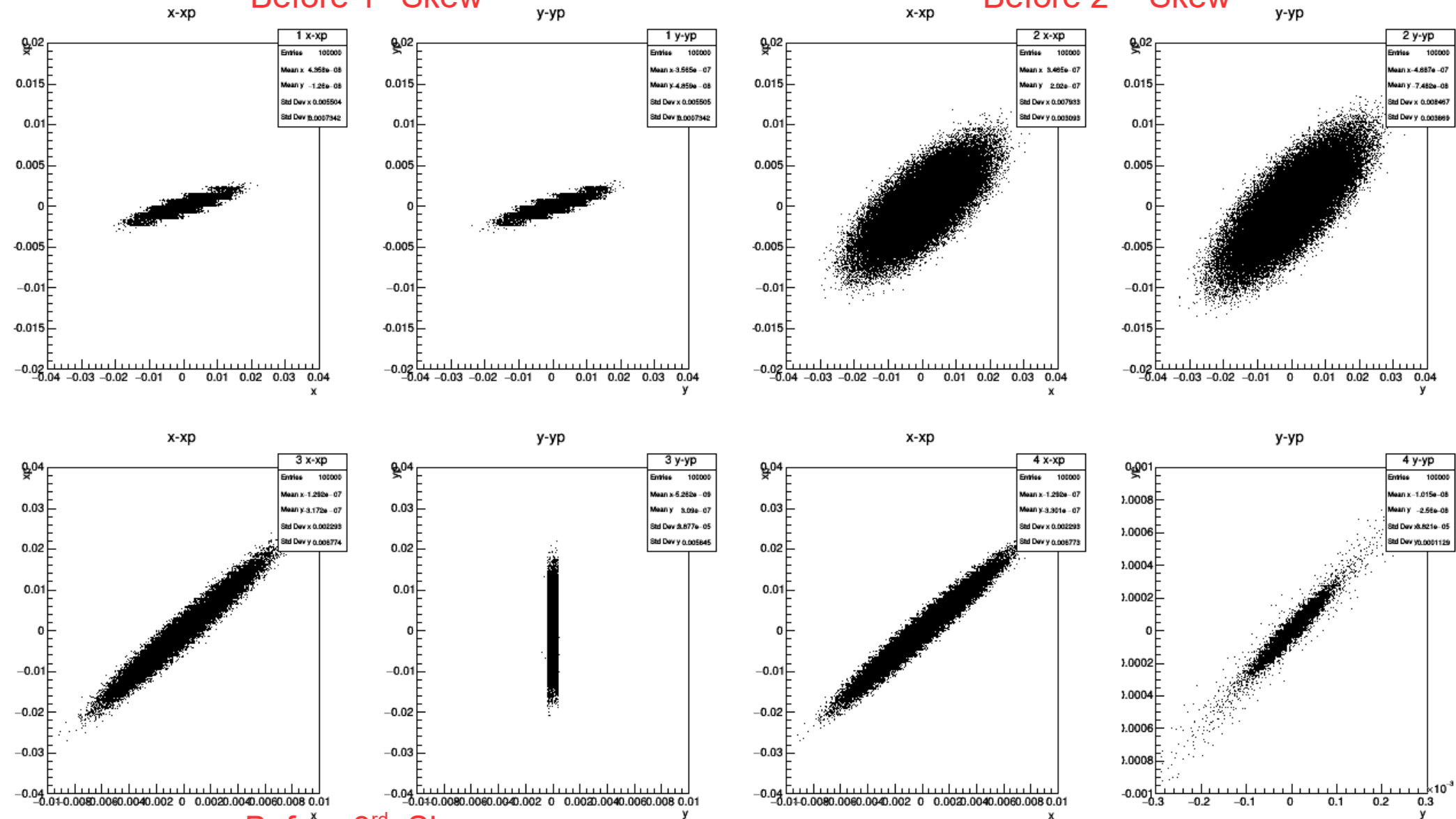
- The beam is generated in 0.1 Tesla selenoid field.
- The canonical angular momentum is converted in the free-space.
- The beam is accelerated and the correlation is restored by skew-Q channel.



Phase-space in RFTB

Before 1st Skew

Before 2nd Skew

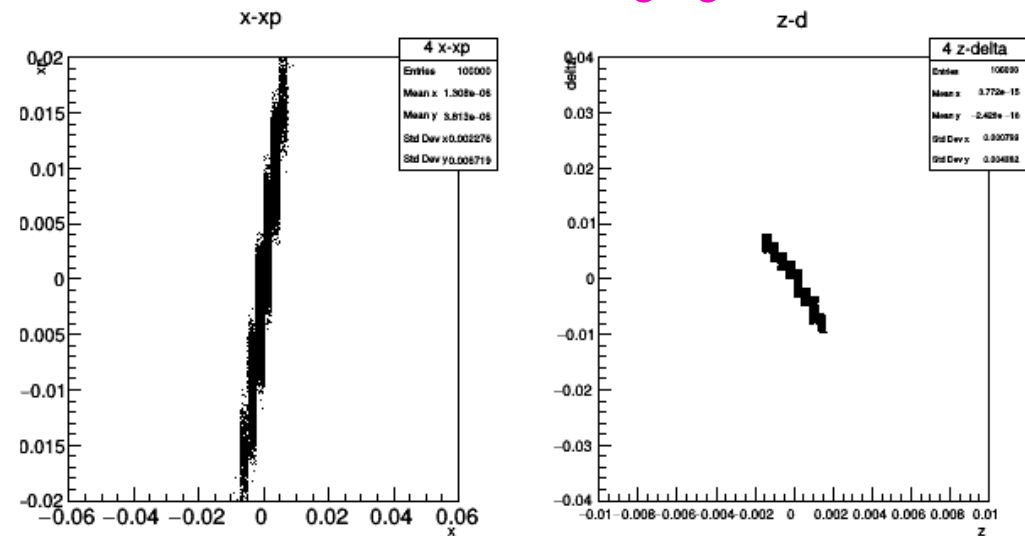


Before 3rd Skew

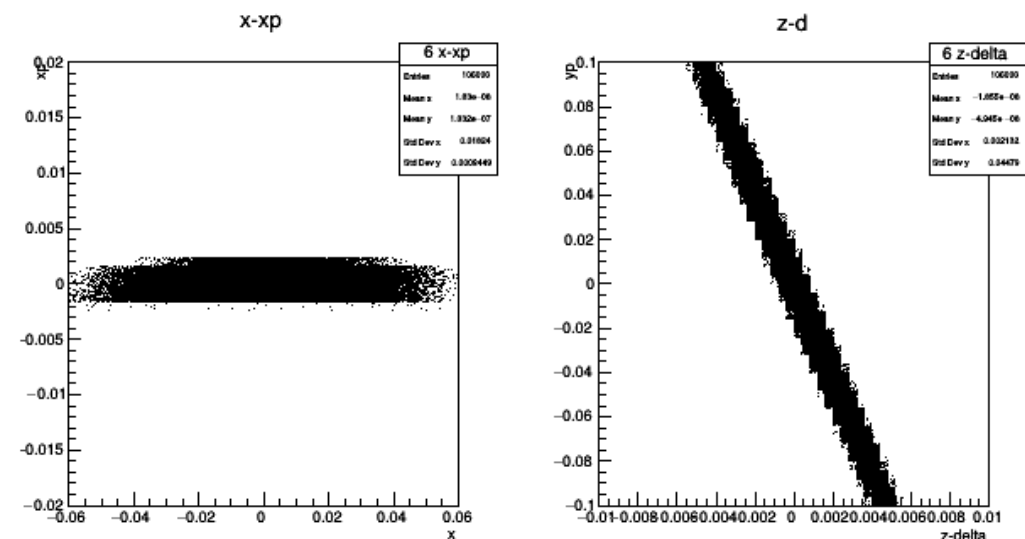
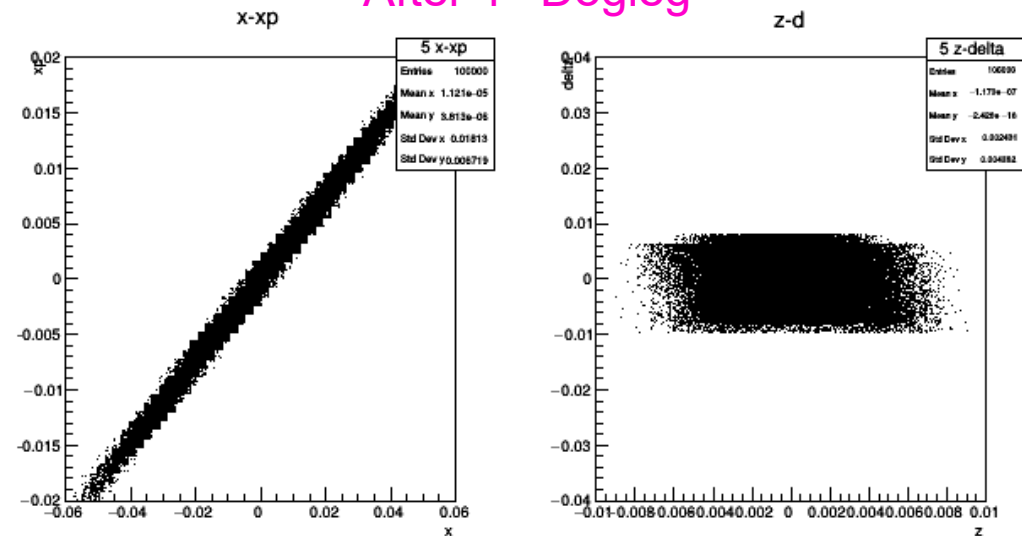
After 3rd Skew

Phase-space in TLEX

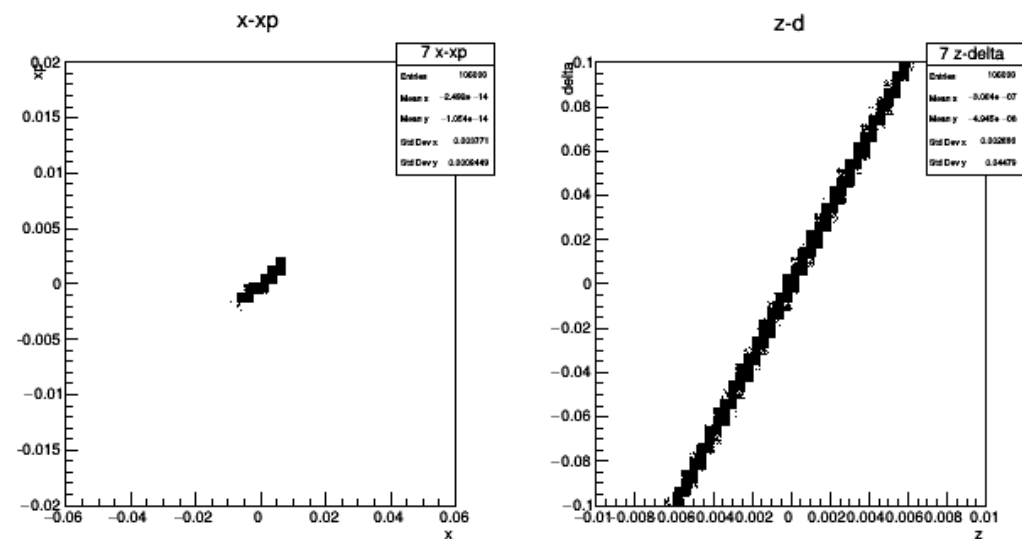
Before 1st Dogleg



After 1st Dogleg

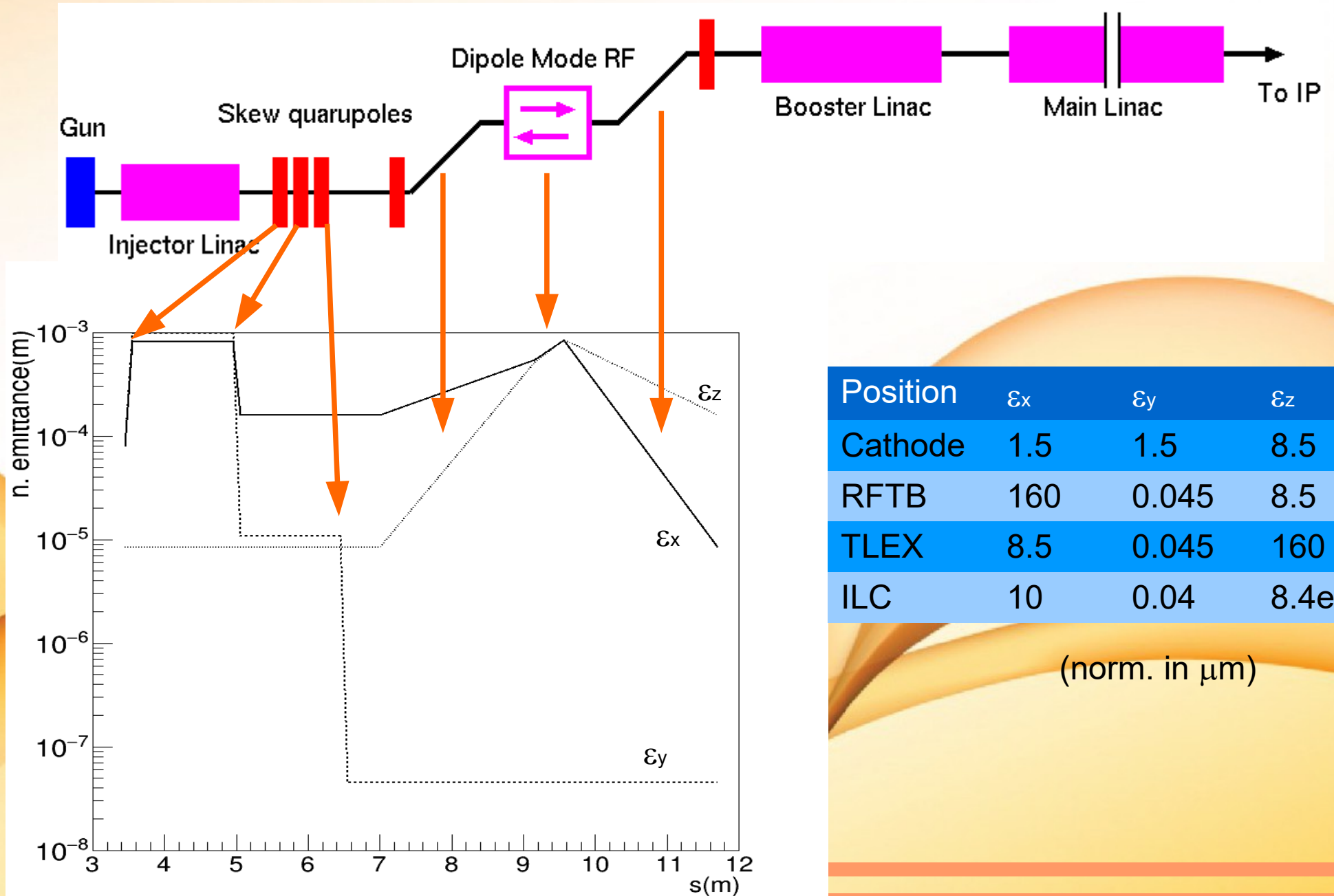


After dipole RF



After 2nd Dogleg

Emittance Evolution



Parameters

Parameter	Value	Unit
Bsol	0.1	T
Initial emittance	1.5	mm.mrad
Bunch length	12	ps (full width)
Beam size	1.6	mm (rms)
Gamma after acc.	49	
g1	-0.503	T/m
g2	0.926	T/m
g3	2.058	T/m
alpha	0.3	rad
η (dispersion)	0.355	
ξ (mom. compaction)	0.106	
RF voltage	1.29	MV (at λ)

Flat Beam Generation at STF

- RFBT experiment will be carried at KEK-STF.
- TLEX experiment will be carried out at ANL WFA.
- The flat beam compatible to LC will be demonstrated in STF by introducing the TLEX beam line as a future proposal.



Summary

- Emittance exchange gives a freedom to optimize the emittance partitioning among degree of freedom.
- ILC compatible beam can be made with these techniques.
- A tiny ε_y can be made by RFTB with a mm size beam without strong space charge effect.
- ε_x after RFTB is too large. TLEX fix the problem by swap it to the small ε_z .
- Simulation with space charge, non-linearity, thick lens effects... are issues.
- Pilot experiments will be carried out at KEK-STF and ANL-WFA.
- It can be a proposal for a future plan of STF.

Acknowledgement

P. PIOT (NORTHRN ILLINOIS UNIVERSITY)

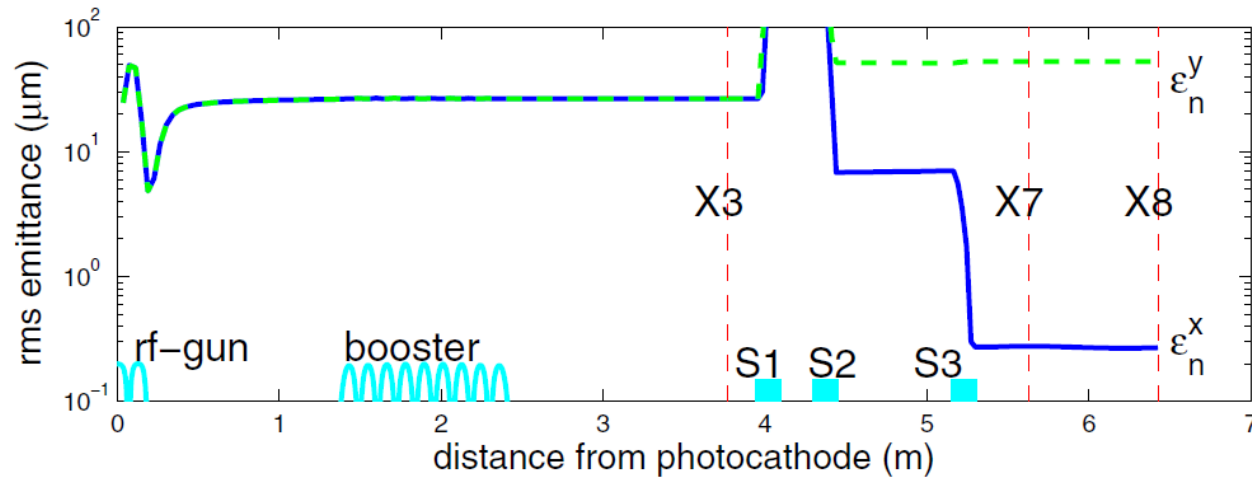
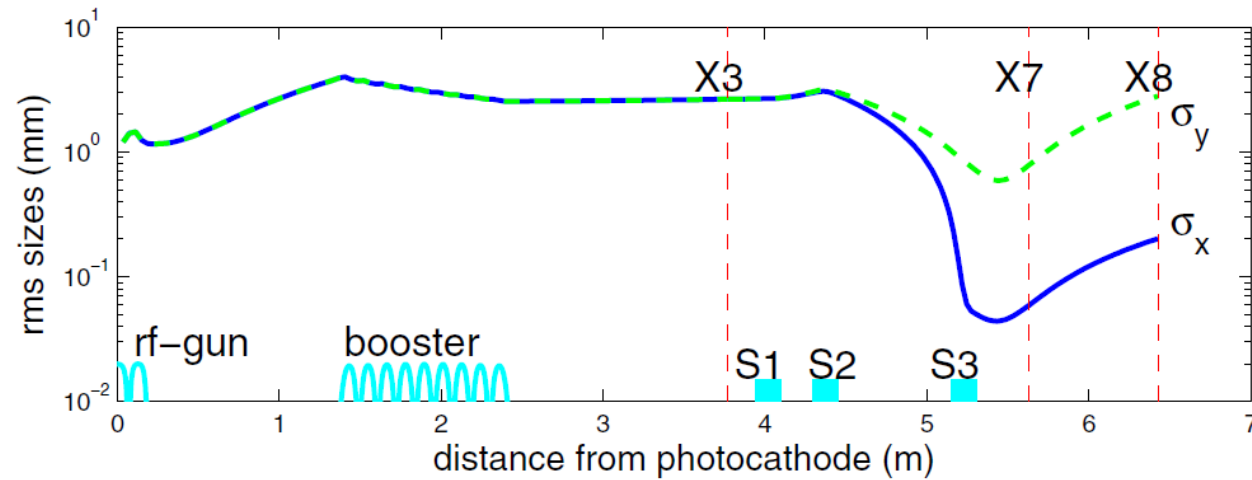
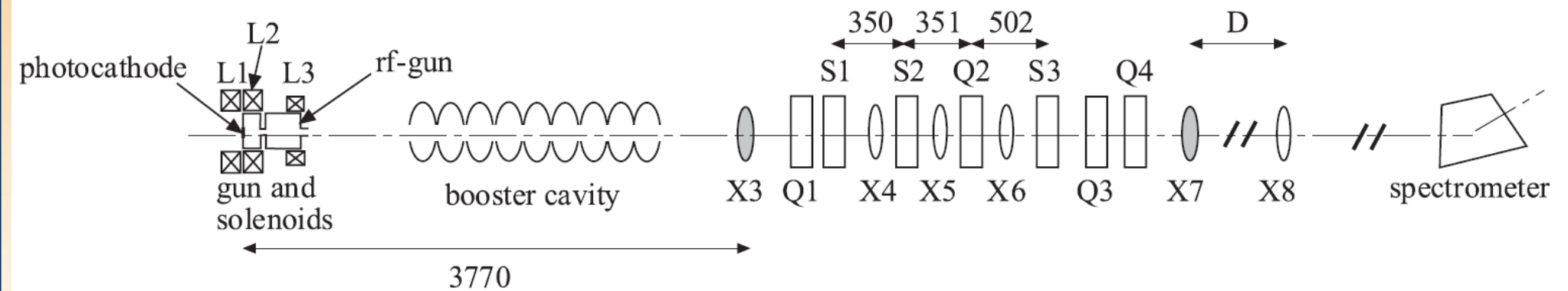
J. POWER (ANL)

H. HAYANO, N. YAMAMOTO, Y. SEIMIYA (KEK)

S. KASHIWAGI (TOHOKU UNIVERSITY)

K. SAKAUE, M. WAHIO (WASEDA UNIVERSITY)

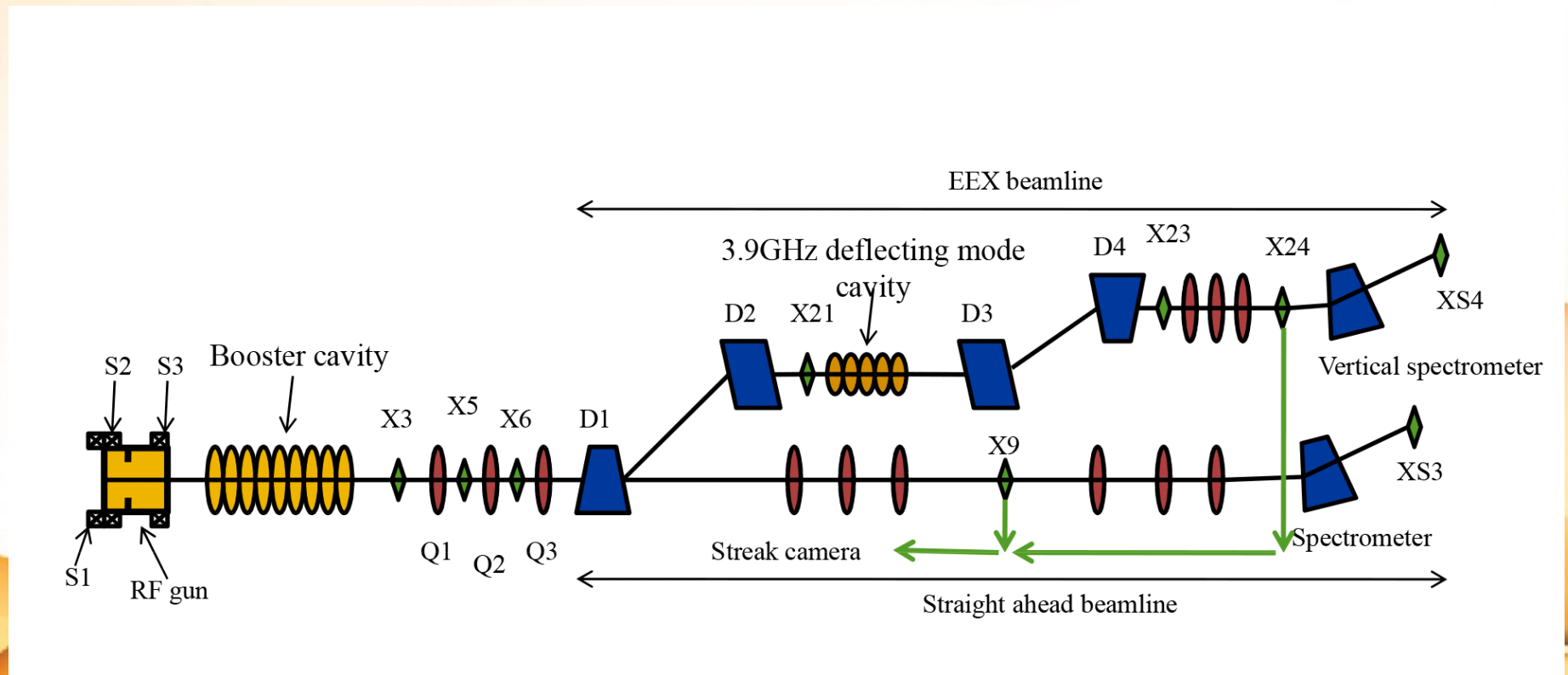




P.Piot, Y.-E Sun, K.-J. Kim
PRSTAB(9)031001(2006)

Emittance Ratio 100
was achieved.

1st observation of TLEX



J. Ruan, *et al.* "First observation of the exchange of transverse and longitudinal emittances." FERMILAB-PUB-10-468-AD (2011).

