Can Higgs be the inflaton?

Jinsu Kim

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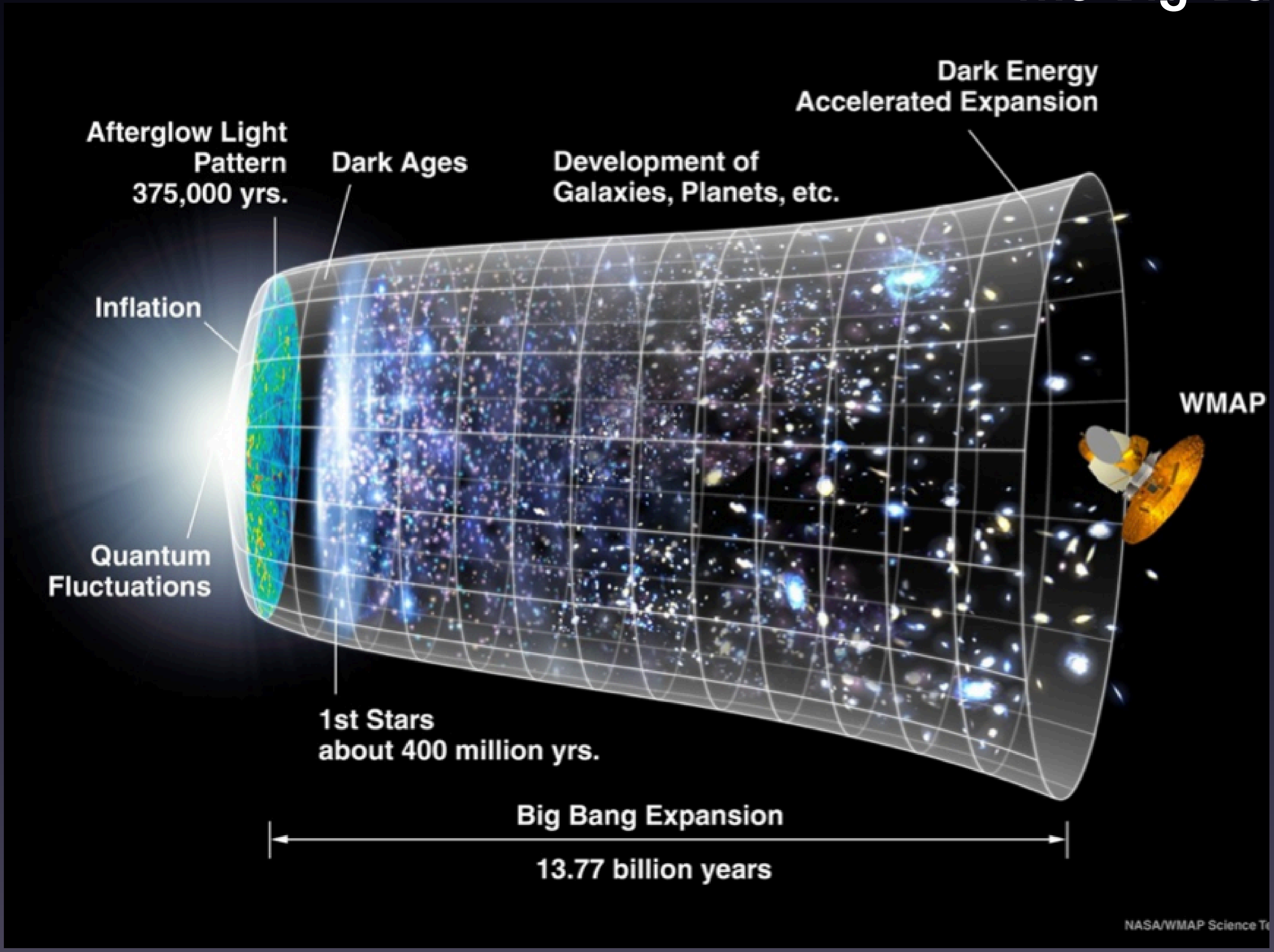
Apr. 17, 2015 @ Wonju, Korea

Our whole universe was in a hot and dense state, then nearly 14 billion years ago **expansion** started, … …, That all stated with the Big Bang!

- The Big Bang Theory



Can Higgs be the inflaton?

[Bezrukov Shaposhnikov 2008]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}(M_P^2 + \xi\phi^2)R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

Inflaton as a known particle
Small tensor-to-scalar ratio
[favoured by Planck+BKP (??)]

$$V(\phi) = \frac{\lambda}{4}\phi^4$$

$$\lambda \simeq 0.13$$

Potential unstable against radiative corrections

Non-minimal coupling ~ 10000

Small tensor-to-scalar ratio
[disfavoured by BICEP2 (??)]

The Standard Model Higgs is at EW scale.
For inflationary physics, quantum correction (i.e., RGE) is needed.

Can Higgs be the inflaton?

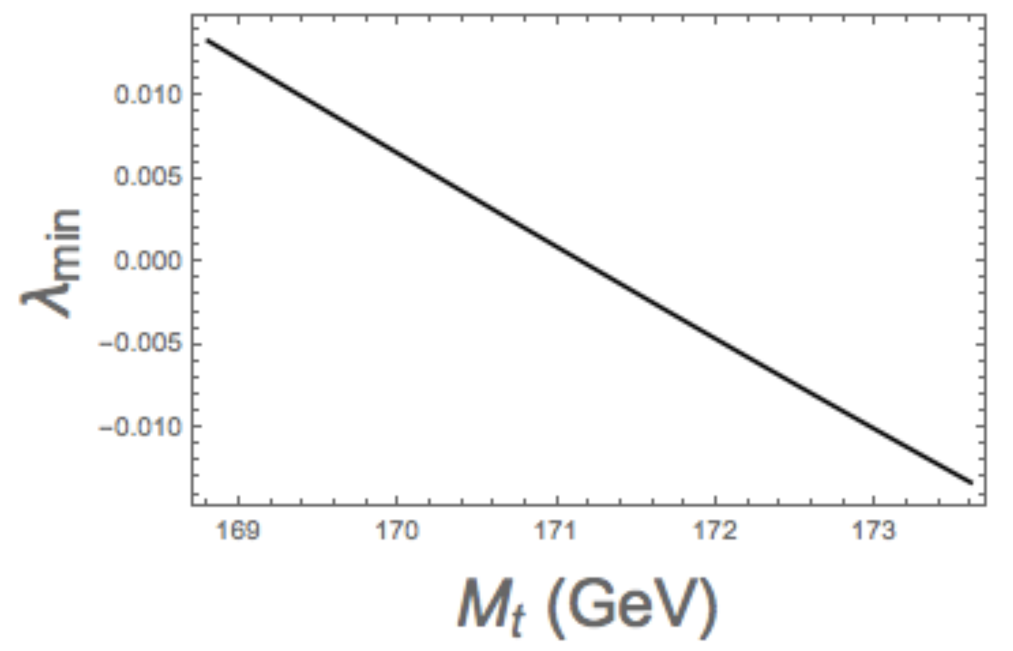
[Cook et al. 2014]
[Hamada et al. 2014]
[Allison 2014]
[Simone et al. 2009]

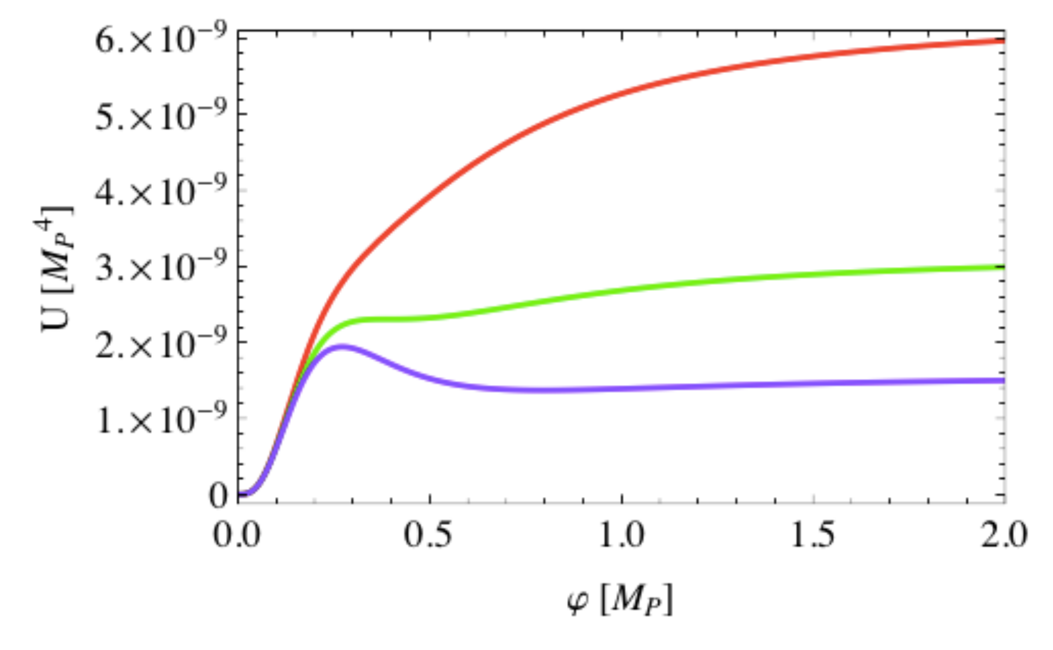
$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}(M_P^2 + \xi\phi^2)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

In the Einstein frame,

$$U = \frac{\lambda(\mu)}{4}\frac{\phi^4}{(1+\xi\phi^2/M_{\rm P}^2)^2}$$

$$\lambda = \lambda_{\rm min} + \frac{\beta_2}{(4\pi)^4}\left[\ln\left(\frac{1}{c}\sqrt{\frac{\xi\phi^2/M_{\rm P}^2}{1+\xi\phi^2/M_{\rm P}^2}}\right)\right]^2 + \cdots$$

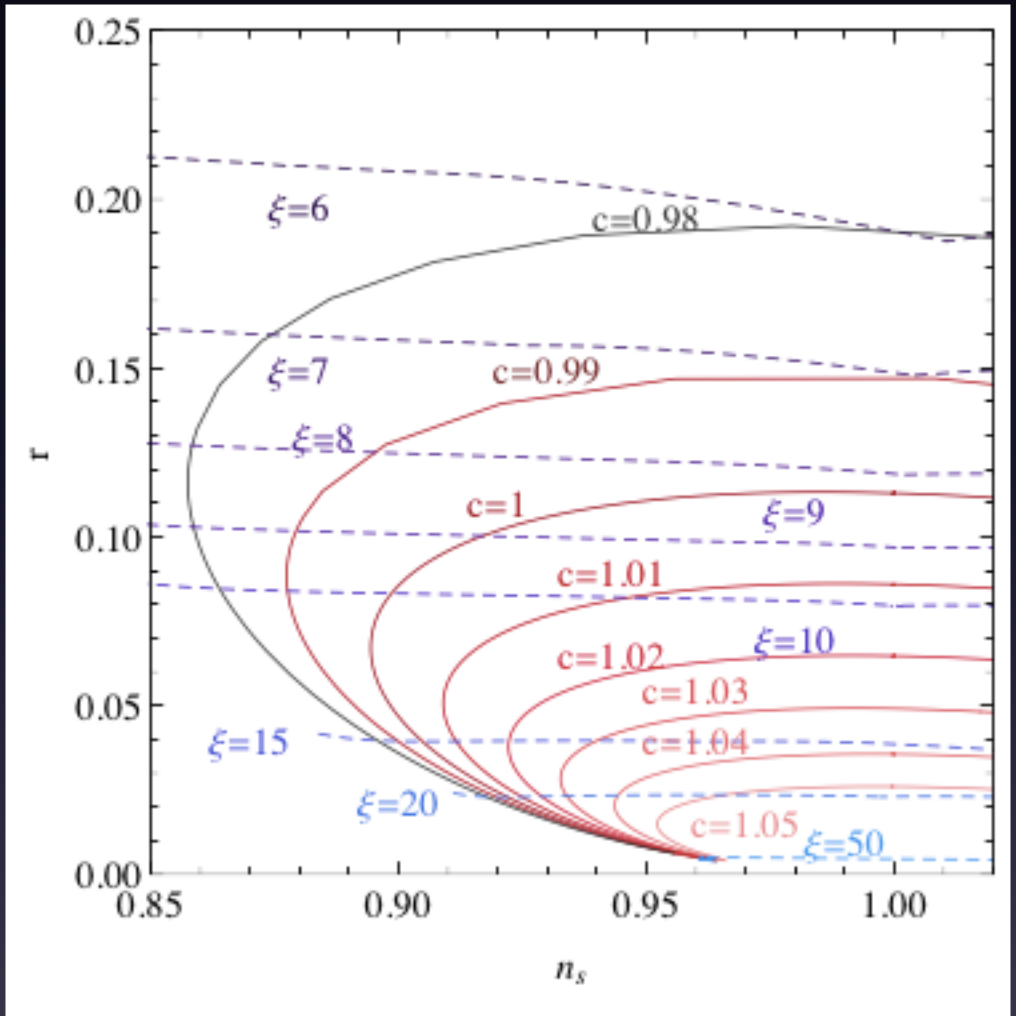




Assumption:
SM is valid up to the ~Planck scale
(i.e., no new physics)

Can Higgs be the inflaton?

[Hamada et al. 2014]

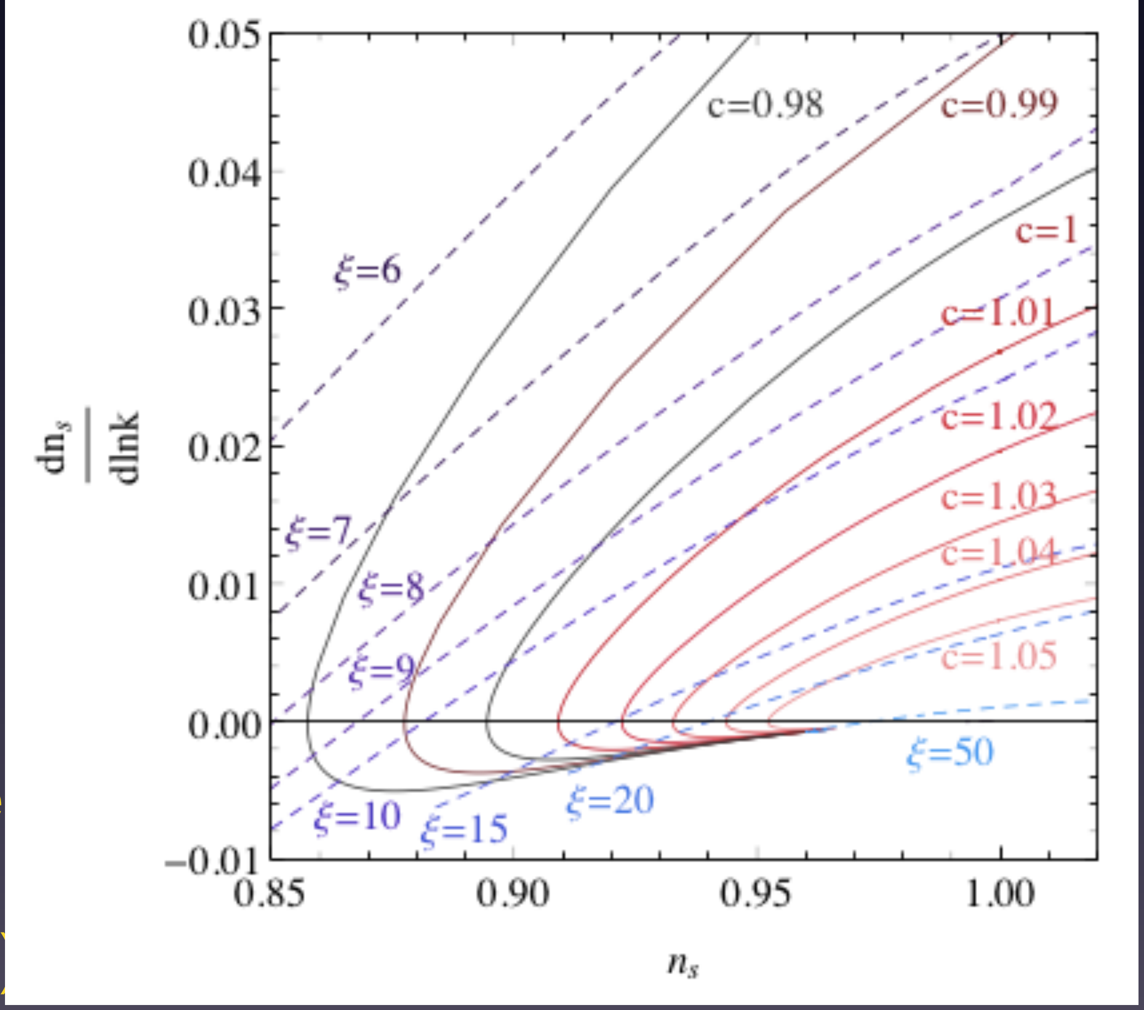


[from Hamada et al. 2014]

Large tensor-to-scalar ratio is possible.

Small non-minimal coupling is possible.

But........



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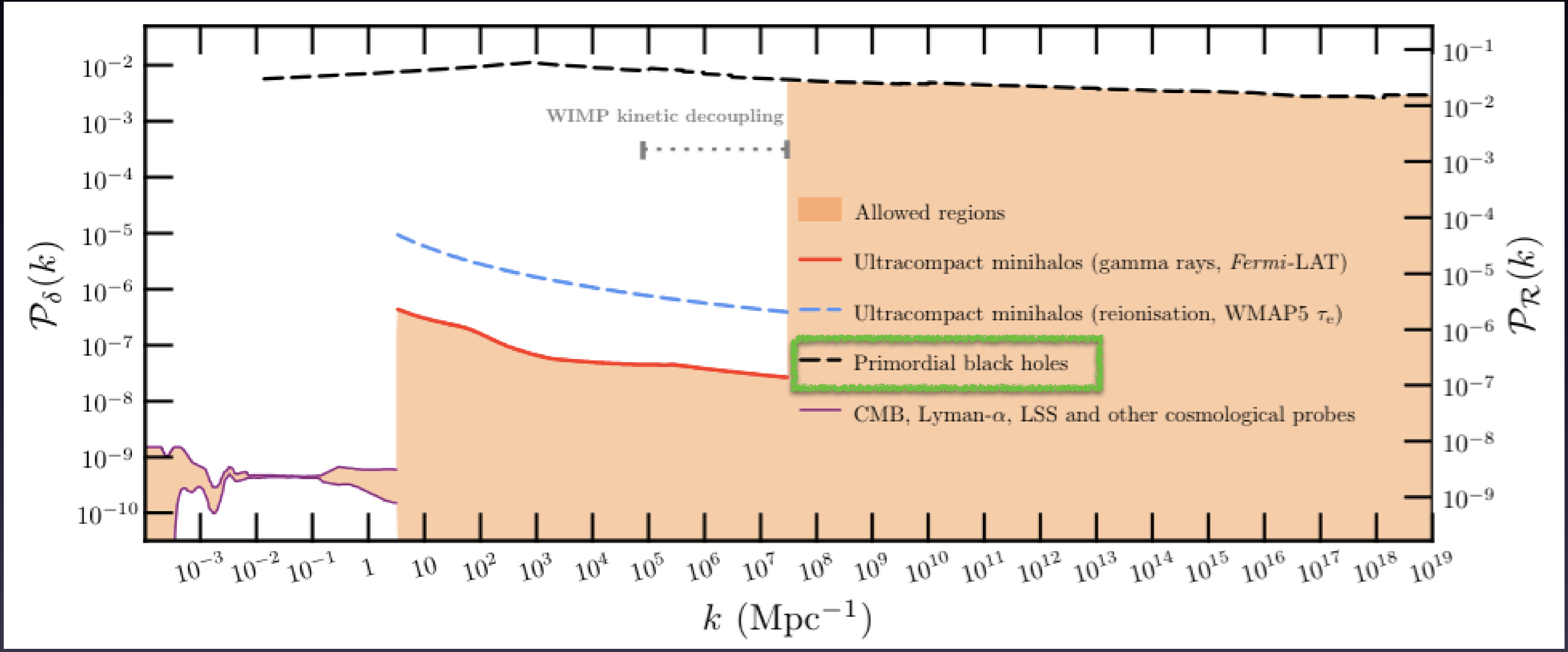
Primordial Power Spectrum

Power Spectrum: Constraint

[Bringmann et al. 2013]



Numerical Analysis: Method

$M_{\mathrm{P}} = 1$

1. Background

$$0 = \ddot{\chi} + 3H\dot{\chi} + \frac{dU}{d\chi}$$

$$3H^2 = \frac{1}{2}\dot{\chi}^2 + U$$

$$\dot{H} = -\frac{1}{2}\dot{\chi}^2$$

convenient to use :

$$dN = H\,dt$$

$$\phi \text{ instead of } \chi$$

$$0 = \frac{d^2\phi}{dN^2} + \left(3 + \frac{1}{H}\frac{dH}{dN} + \frac{1}{d\chi/d\phi}\frac{d^2\chi}{d\phi^2}\frac{d\phi}{dN}\right)\frac{d\phi}{dN} + \frac{1}{H^2}\frac{1}{(d\chi/d\phi)^2}\frac{dU}{d\phi}$$

$$H^2 = \frac{U}{3 - \frac{1}{2}\left(\frac{d\chi}{d\phi}\right)^2\left(\frac{d\phi}{dN}\right)^2}$$

$$\frac{dH}{dN} = -\frac{1}{2}H\left(\frac{d\chi}{d\phi}\right)^2\left(\frac{d\phi}{dN}\right)^2$$

$$\frac{d\chi}{d\phi} = \frac{1}{\Omega^2}\left[\Omega^2 + 6\xi^2\left(\frac{\phi}{M_{\mathrm{P}}}\right)^2\right]^{1/2}$$

Numerical Analysis: **Method**

2. Perturbation

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0$$: Mukhanov-Sasaki equation

$$z = \frac{a\dot{\chi}}{H} = a\frac{d\chi}{dN}$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2$$

3. Initial condition

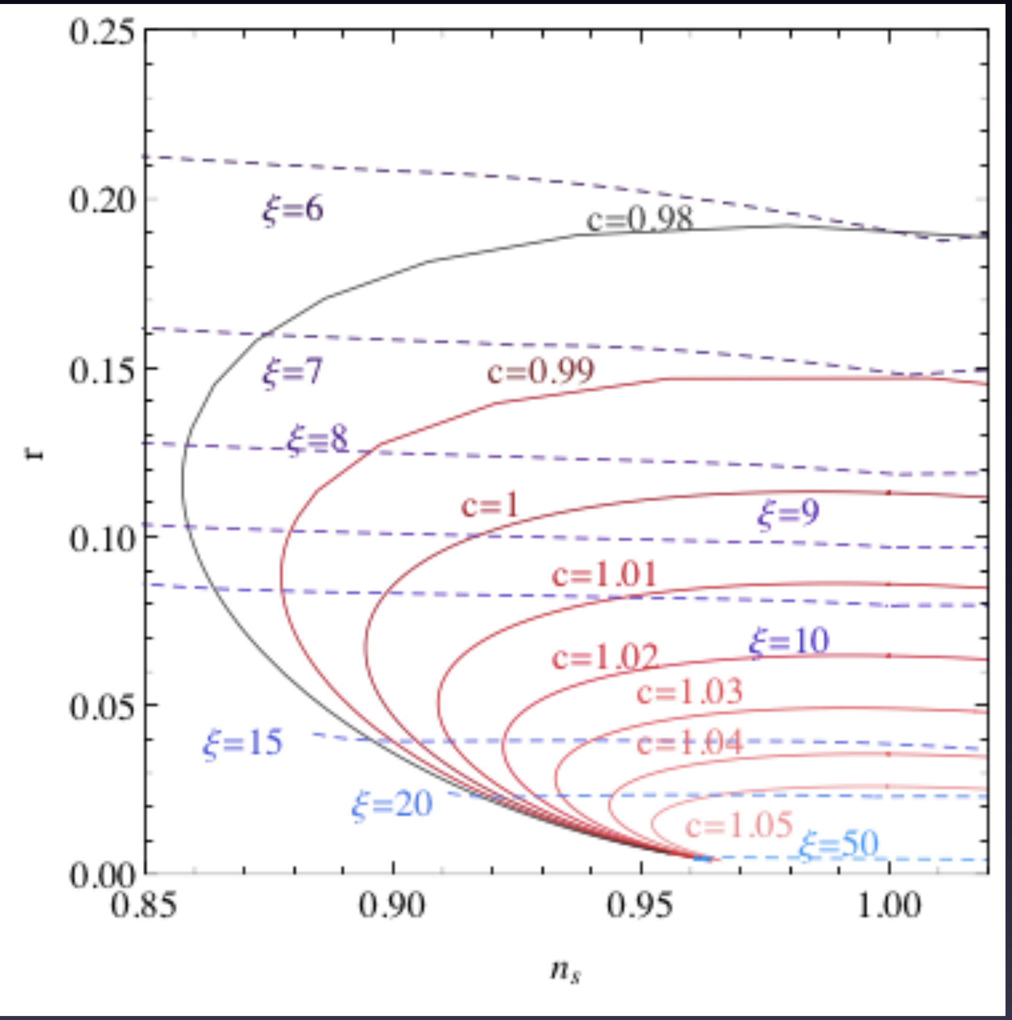
Bunch-Davies vacuum

$$u_k \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$$ well inside the horizon

[Special thanks to Dr. Hazra, Dr. Choi and Dr. Kawai]

Numerical Analysis: Results

1. Benchmark points



$$\lambda_{\rm min} = 1.5185 \times 10^{-6} \quad r = 0.1126$$
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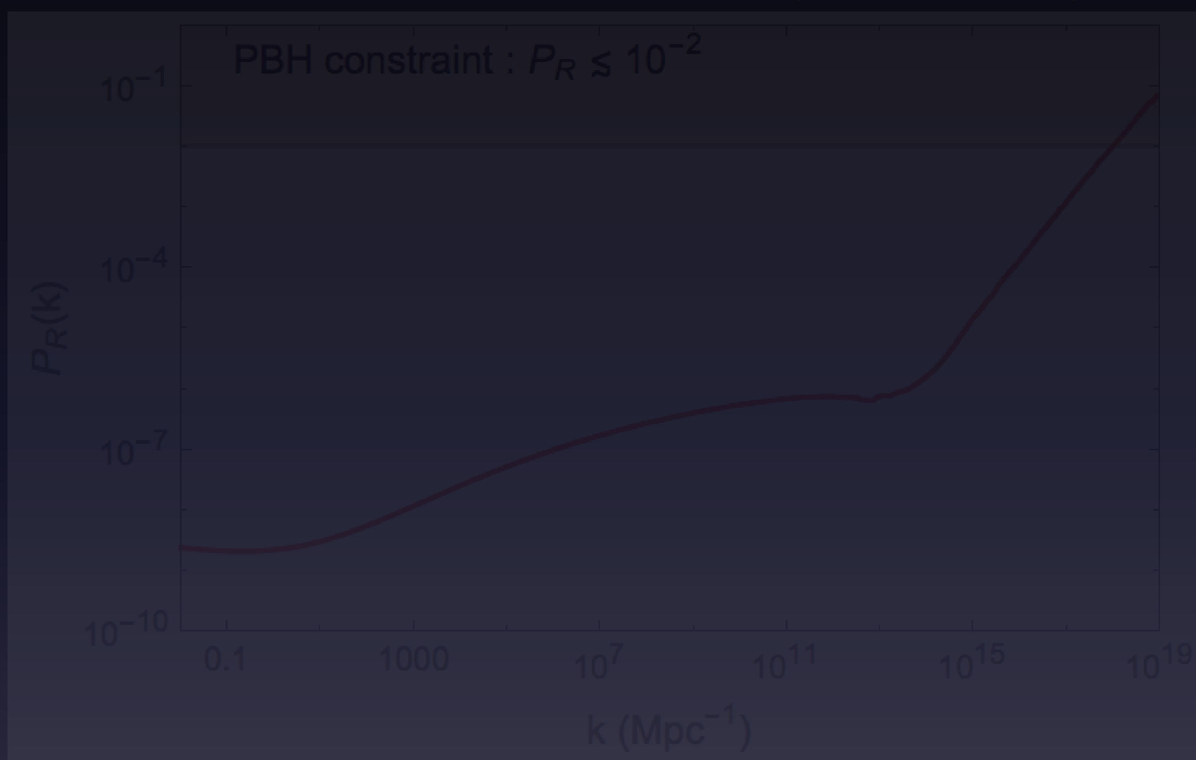
$$\lambda_{\rm min} = 1.57 \times 10^{-6} \quad r = 0.01$$

Numerical Analysis: Results

2. Power spectrum

$\lambda_{\min} = 1.5185 \times 10^{-6} \qquad (r = 0.1126)$



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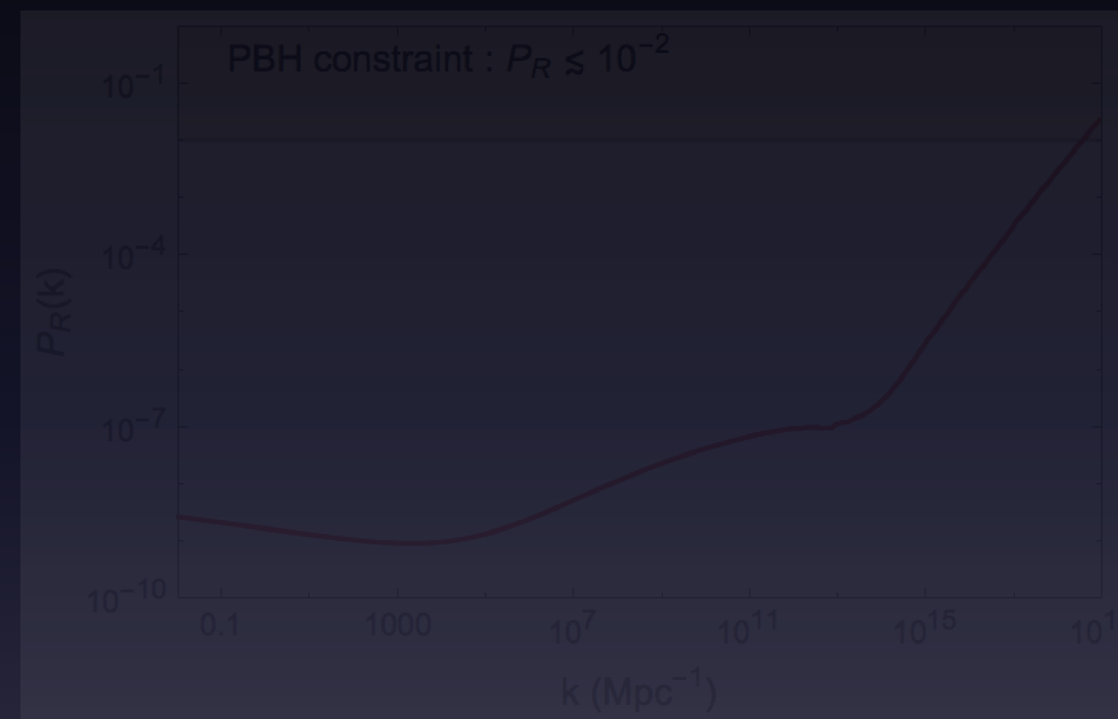
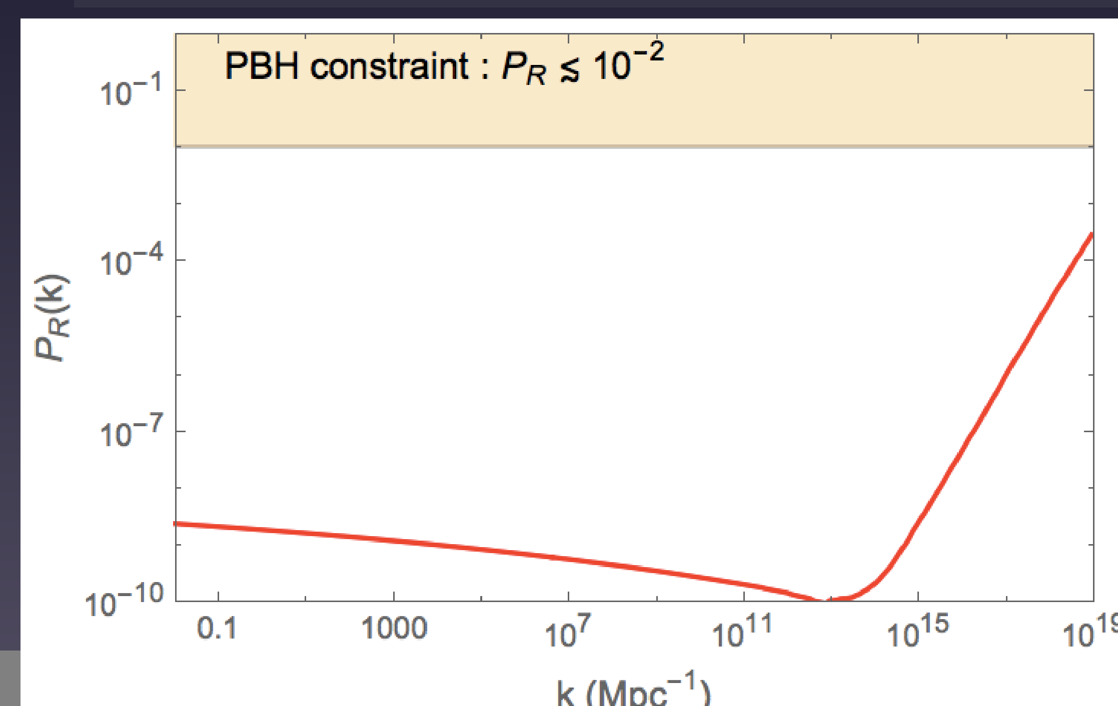


$$\lambda_{\min} = 1.57 \times 10^{-6} \qquad (r = 0.01)$$



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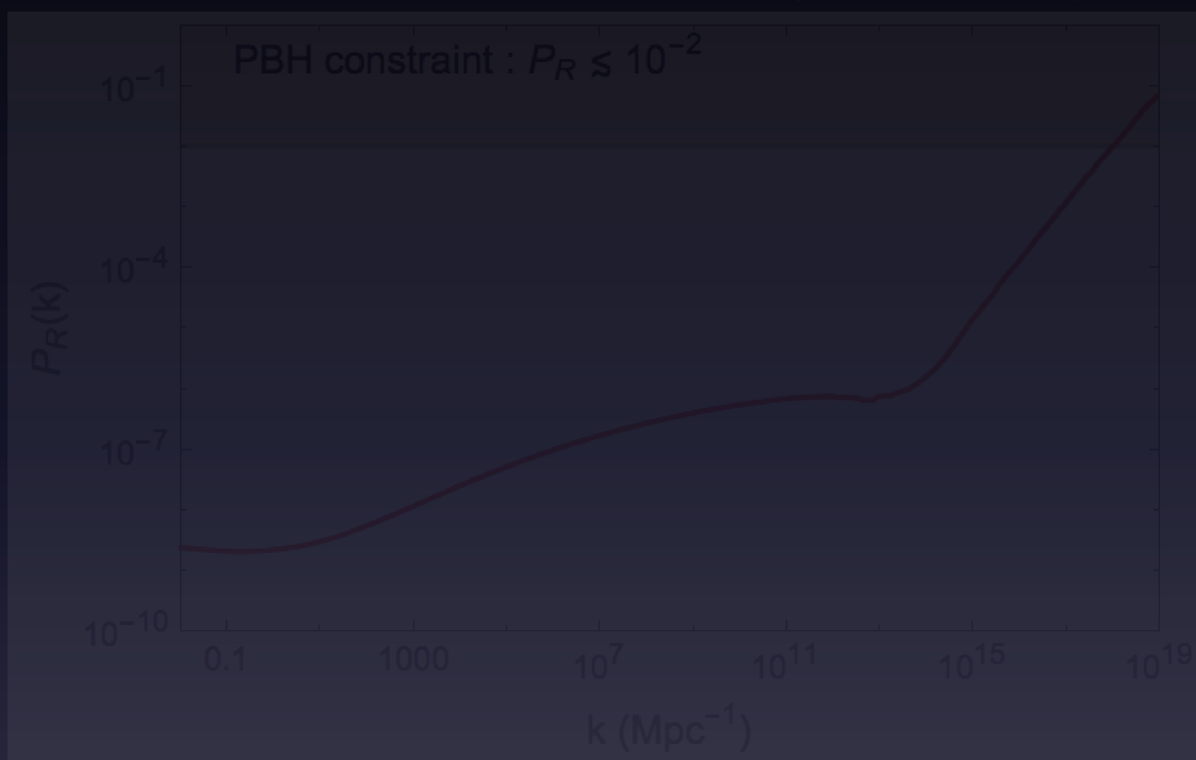

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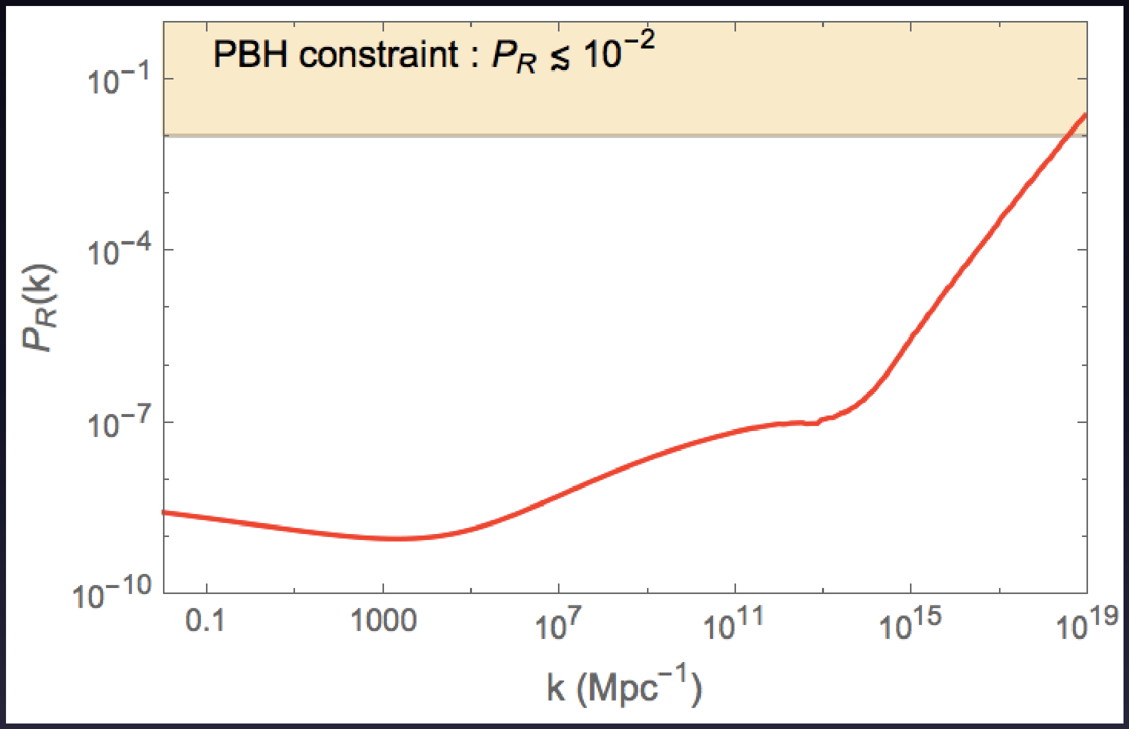

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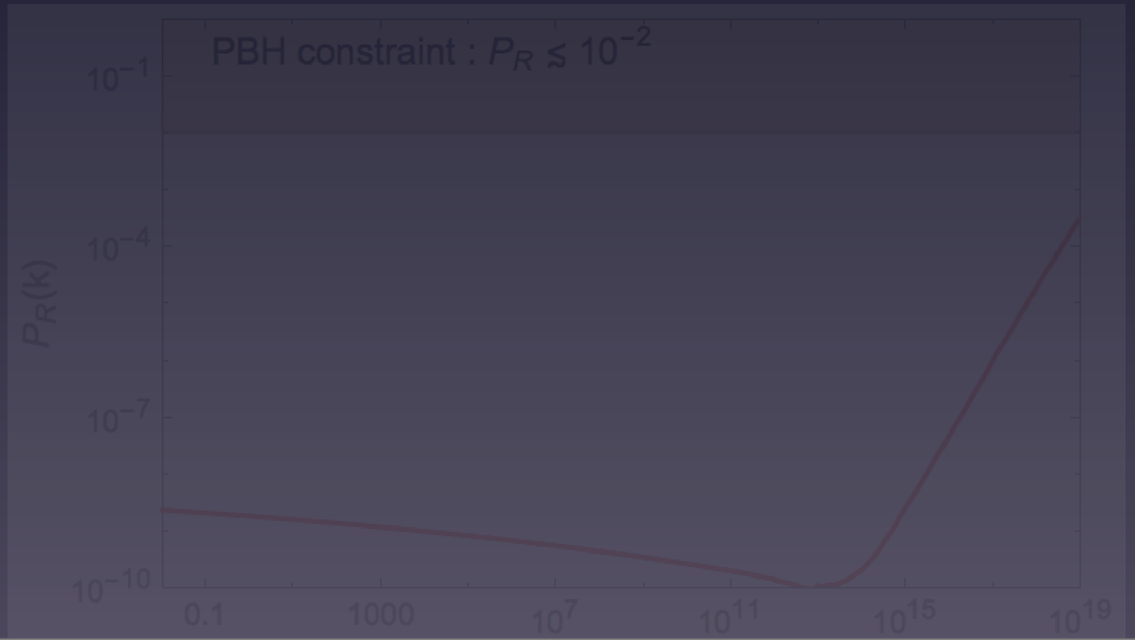

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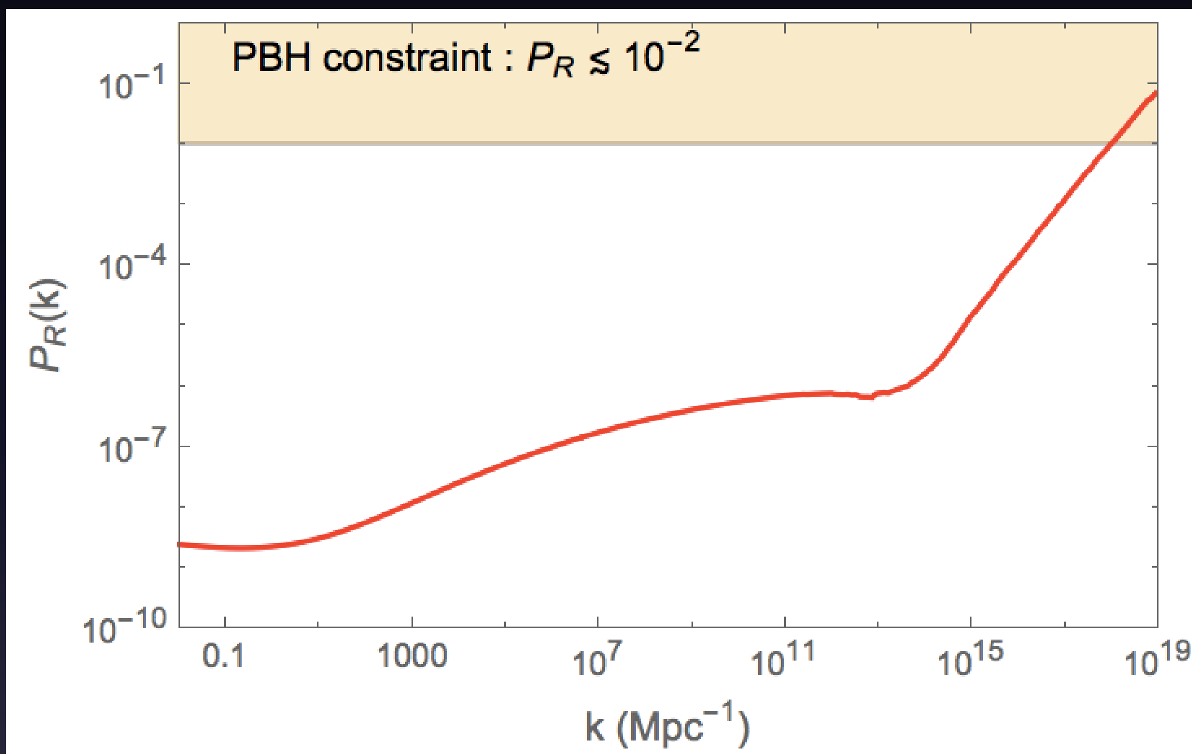

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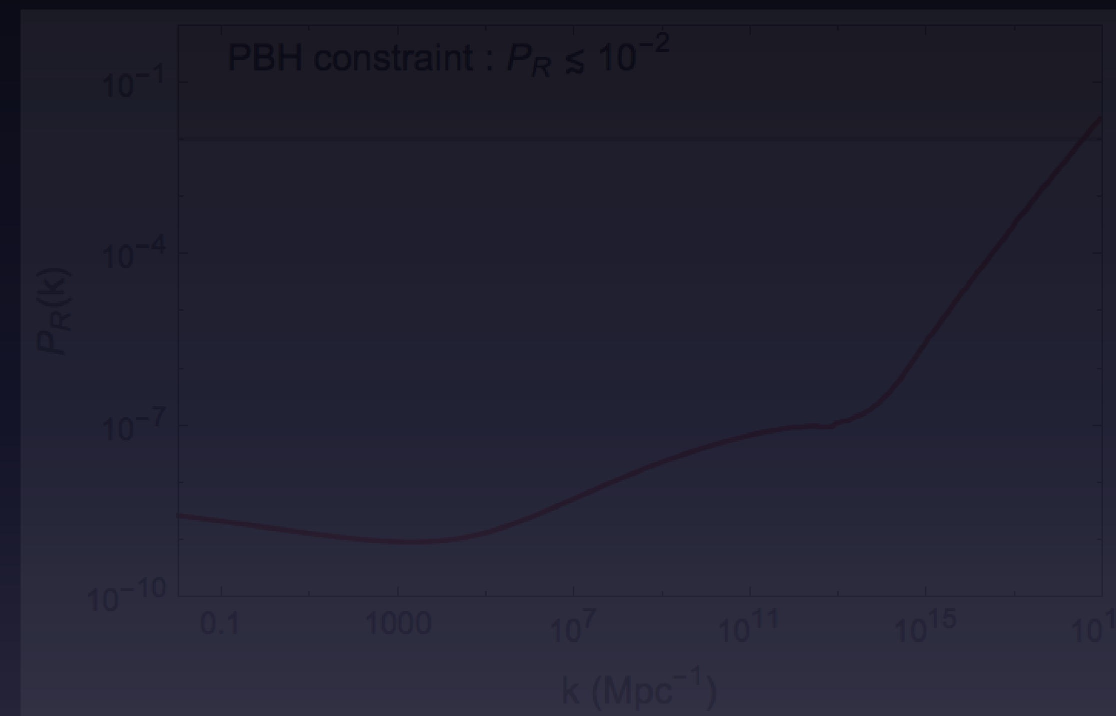

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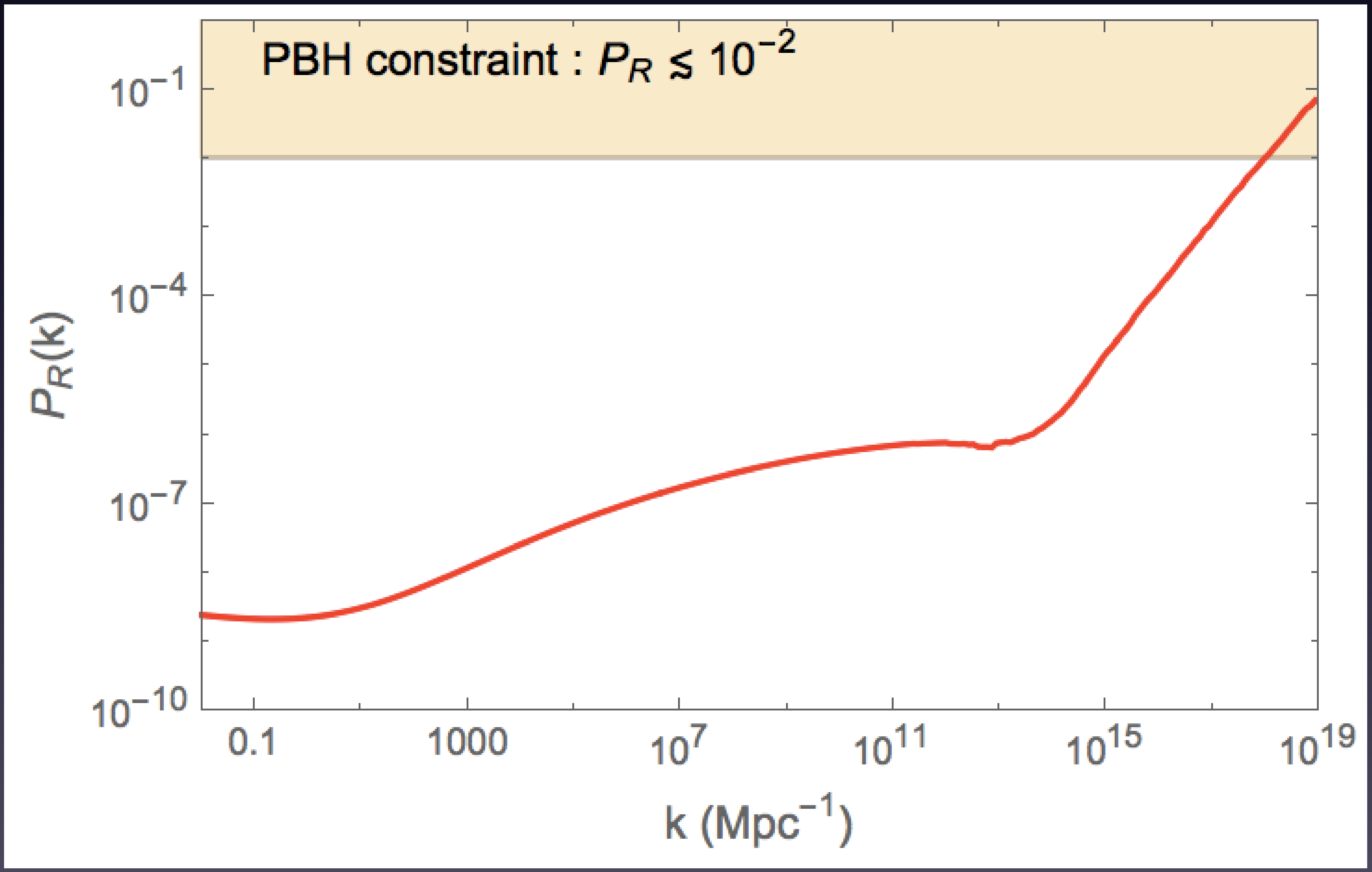
[WARNING!! Need to check the code]

Numerical Analysis: **Results**

stringent bound on tensor-to-scalar ratio?



Supersymmetric Extension

Can Higgs be the inflaton?

[Bezrukov Shaposhnikov 2008]

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Inflaton as a known particle
Small tensor-to-scalar ratio
[favoured by Planck+BKP (??)]

Potential unstable against radiative corrections

Non-minimal coupling ~ 10000

Small tensor-to-scalar ratio
[disfavoured by BICEP2 (??)]

The Standard Model is not the end of the story.
We need to account for

Baryogenesis and Neutrino oscillations

→ SUSY-seesaw

SUSY-seesaw : MSSM+N_R

[Arai Kawai Okada 2011, 2012]

Superpotential

$$W = W_{MSSM} + \frac{1}{2} M_R N_R^c N_R^c + y_D N_R^c L H_u$$

D-flat direction

$$L = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi \\ 0 \end{pmatrix} \qquad H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$$

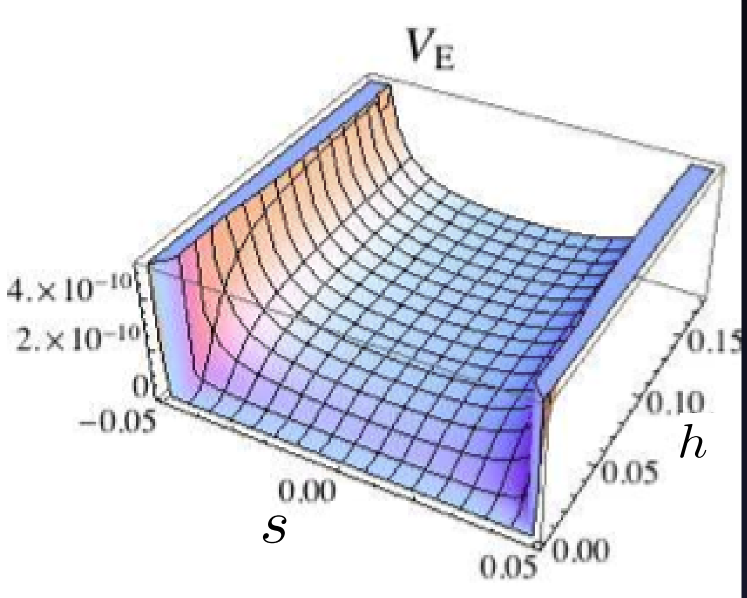
Kahler potential

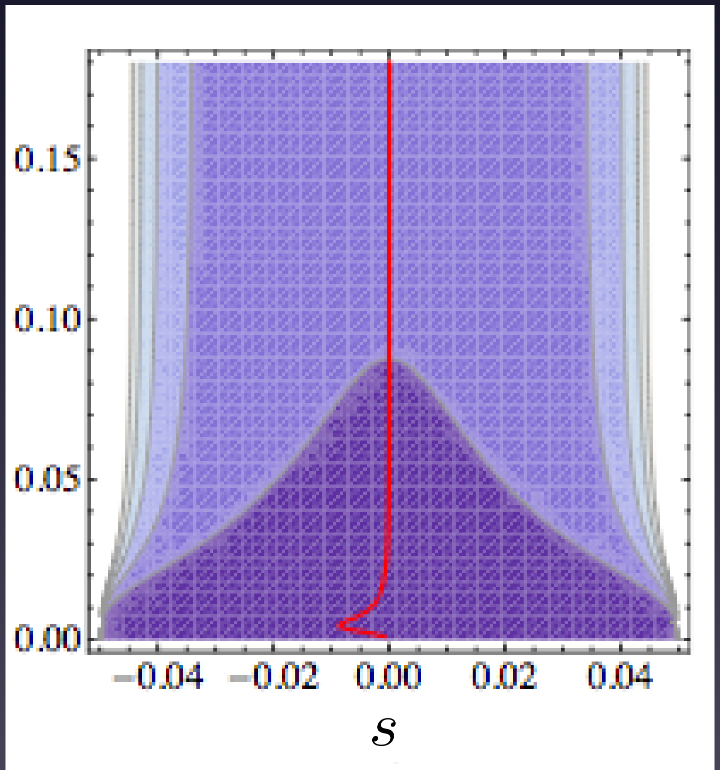
$$\Phi = 1 - \frac{1}{3}\left(|N_R^c|^2 + |\varphi|^2\right) + \frac{1}{4}\gamma\left(\varphi^2 + c.c.\right) + \frac{1}{3}\zeta |N_R^c|^4$$

$$K = -3\Phi$$

Seesaw relation

$$m_\nu = \frac{y_D^2 \langle H_u \rangle^2}{M_R} \qquad y_D = \left(\frac{M_R}{6.14 \times 10^{14}\,\mathrm{GeV}}\right)^{1/2}$$



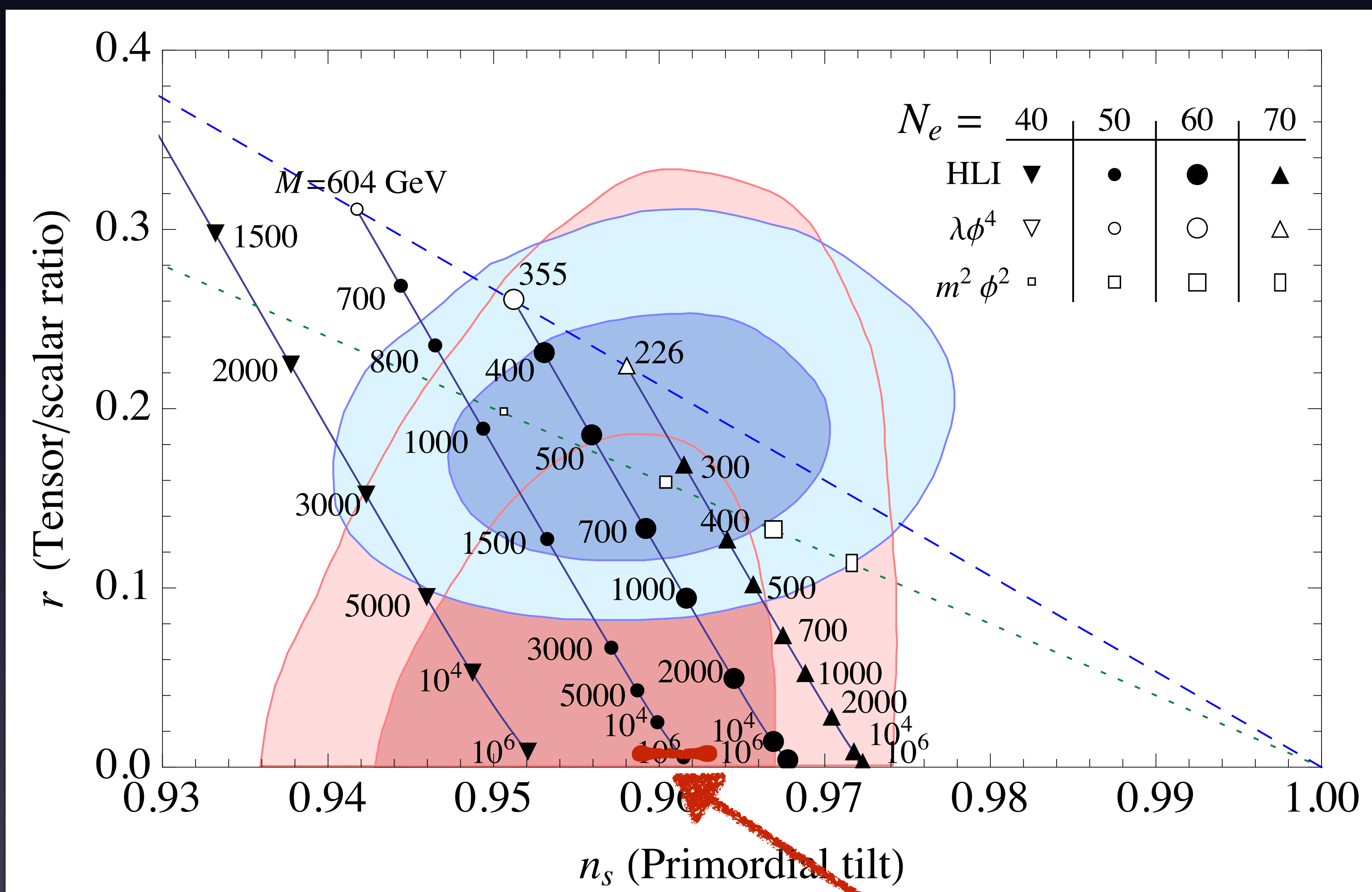


$\varphi = h/\sqrt{2}$

$N_R^c = s/\sqrt{2}$

Higgs-lepton inflation in SUSY-seesaw

[Arai Kawai Okada 2011, 2012]

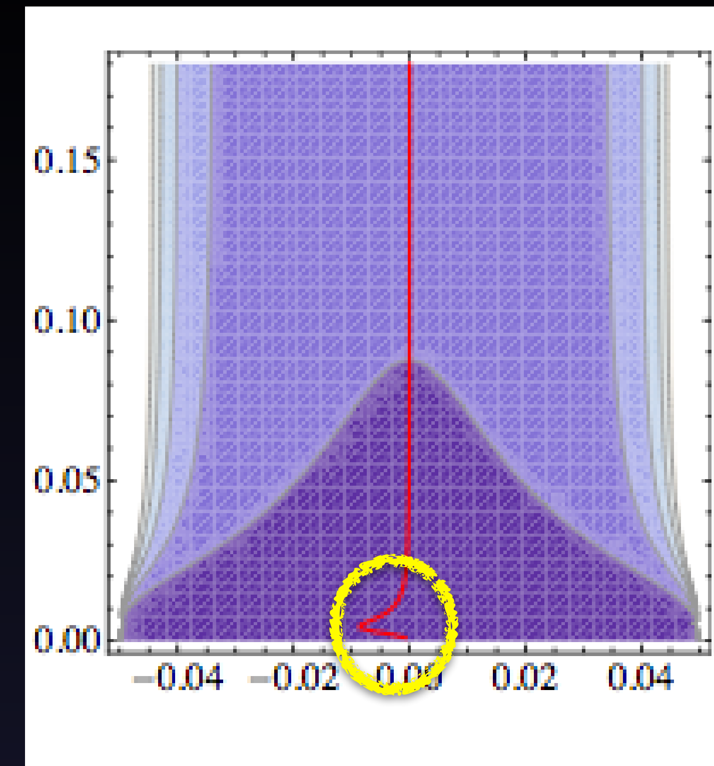


Higgs inflation

M_R (GeV)	ξ
10^{13}	2566
10^{11}	257
10^{9}	25.6
10^{6}	0.730
10^{5}	0.184
10^{4}	0.0303
5000	0.0152
2000	4.97×10^{-3}
1000	1.33×10^{-3}
644	0

Higgs-lepton *inflation*

- Based on a well-motivated particle theory MSSM
- Radiative corrections under control SUSY
- Small non-zero neutrino masses Seesaw
- Falsifiability and predictability Planck/BICEP/LHC



Multi-field effect?

[JK & S. Kawai, 2015]

to investigate how the non-Gaussianity restricts noncanonical terms of the Kahler potential of the underlying supergravity theory.

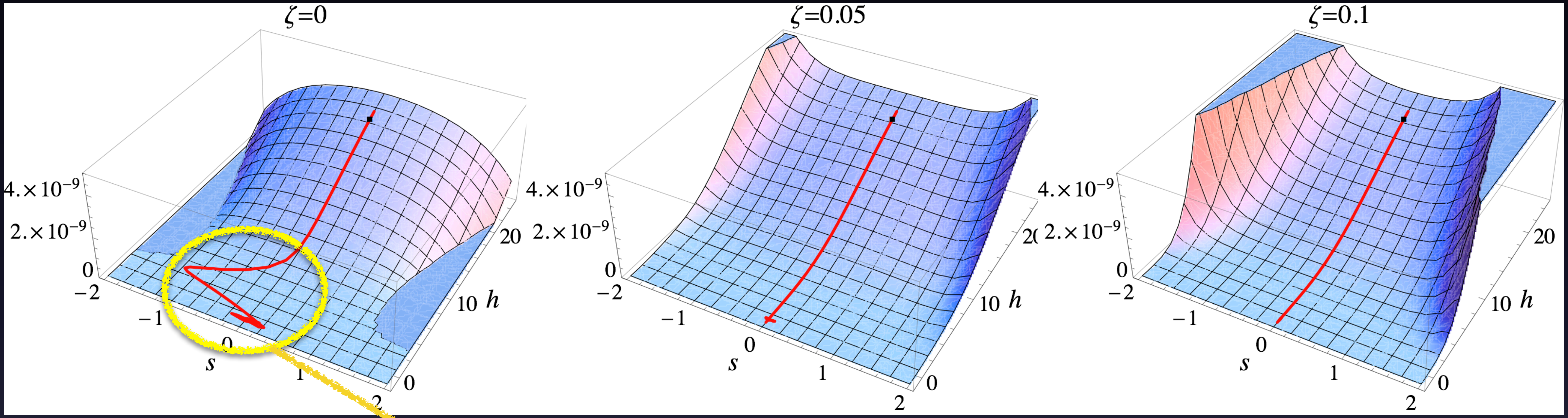
→ ζ - effect

$$\Phi = 1 - \frac{1}{3}\left(|N_R^c|^2 + |\varphi|^2\right) + \frac{1}{4}\gamma\left(\varphi^2 + c.c.\right) + \frac{1}{3}\zeta|N_R^c|^4$$

$$K = -3\Phi$$

Higgs-lepton *inflation*

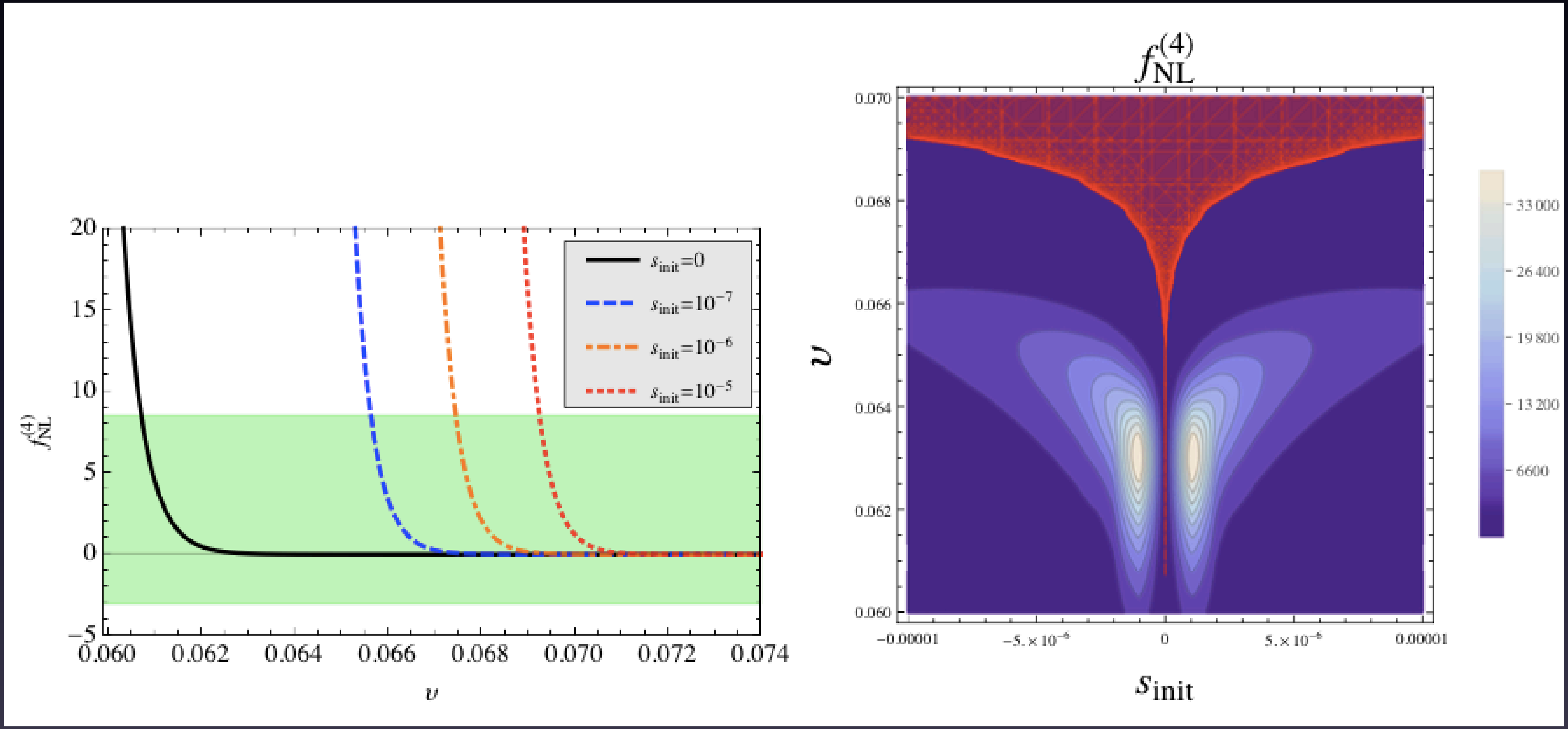
$$M_R = 1\,\mathrm{TeV}$$



Multi-field inflation !

Backward formalism !

Results (1 TeV case)
Nonlinearity parameter



- effect of the change of the transfer function $\Delta T_{\zeta S} \sim N_{ab}$
- most stringent bound $v \gtrsim 0.06925$

Higer-dimensional Extension

Higher-order dim'l operator?

f(R) gravity?

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm P}^2}{2} \left(R + \frac{\alpha}{M_{\rm P}^2} R^2 \right) + \frac{1}{2}\xi h^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} h^4 \right]$$

Why just R^2 ?

$$S = \frac{M_{\rm P}^2}{2} \int d^4x \sqrt{-g} \left(R + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right)$$

In 4-dim. FRW universe,

$$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = 0$$

Higher-order dim'l operator?

A $f(R)$ gravity is equivalent to scalar-tensor theories by introducing a new scalar. Let ϕ be the new scalar:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} F(\phi, h) R - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - V(\phi, h) \right] ,$$

where

$$F(\phi, h) \equiv 1 + \zeta \left(\frac{\phi}{M_p} \right) + \xi \left(\frac{h}{M_p} \right)^2 ,$$

$$V(\phi, h) \equiv \frac{\lambda}{4} h^4 + \frac{\zeta^2 M_p^4}{8\alpha} \left(\frac{\phi}{M_p} \right)^2 .$$

Thus, this action can be interpreted as a two-field inflationary model with non-minimal coupling terms.

Higher-order dim'l operator?

One can go to the Einstein frame via Weyl transformation,

$$g_{\mu\nu} \to g^E_{\mu\nu} = \Omega^2(x) g_{\mu\nu} \, .$$

The resultant Einstein action is given by

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_p^2}{2} \tilde{R} - \frac{1}{2} \mathcal{G}_{IJ} \tilde{g}^{\mu\nu} \partial_\mu \varphi^I \partial_\nu \varphi^J - U \right] ,$$

where

$$\varphi^1 \equiv \phi \, , \quad \varphi^2 \equiv h \, , \quad \mathcal{G}_{IJ} = \begin{pmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} \\ \mathcal{G}_{21} & \mathcal{G}_{22} \end{pmatrix} ,$$

$$\mathcal{G}_{11} \equiv \frac{3\zeta^2}{2F^2} \, , \quad \mathcal{G}_{22} \equiv \frac{1}{F} \left(1 + \frac{6\xi^2}{F} \left(\frac{h}{M_p} \right)^2 \right) , \quad \mathcal{G}_{12} = \mathcal{G}_{21} \equiv \frac{3\xi\zeta}{F^2} \left(\frac{h}{M_p} \right) ,$$

$$U(\phi, h) = \frac{\frac{\lambda}{4} h^4 + \frac{\zeta^2 M_p^4}{8\alpha} \left(\frac{\phi}{M_p} \right)^2}{\left[1 + \zeta \left(\frac{\phi}{M_p} \right) + \xi \left(\frac{h}{M_p} \right)^2 \right]^2} \, .$$

?

Summary

Summary

Can Higgs be the inflaton?

A bit of Advertisement !

Developed numerical codes:

Single-field inflation $S = \int d^4x \sqrt{-g}\left[\frac{1+K(\phi)}{2}R - \frac{1}{2}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$

Fortran / Mathematica

scale dependency of power spectrum

Multi-field inflation 1 $S = \int d^4x \sqrt{-g}\left[\frac{1+K(\phi^I)}{2}R - \frac{1}{2}h_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$

Mathematica

non-Gaussianity (delta N formalism)

Multi-field inflation 2 $S = \int d^4x \sqrt{-g}\left[\frac{1+K(\phi^I)}{2}R - \frac{1}{2}h_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$

Mathematica

isocurvature fraction (covariant formalism)

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Mathematica

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Multi-field inflation 2

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Mathematica

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Thank you

Jinsu Kim

Sungkyunkwan University

@ 2015 Wonju Spring School

Back-up slides

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[Bezrukov Shaposhnikov 2008]

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$$\Omega^2 = 1 + \frac{\xi\phi^2}{M_{\mathrm{P}}^2}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\mathrm{P}}^2}{2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi) \right].$$

$$\frac{d\chi}{d\phi} = \frac{1}{\Omega^2}\left[\Omega^2 + 6\xi^2\left(\frac{\phi}{M_{\mathrm{P}}}\right)^2\right]^{1/2}$$

$U(\chi) = \frac{V}{\Omega^4}$: Einstein frame potential

[Sasaki Stewart 1996]

[Tanaka Stewart 1996]

[Yokoyama Suyama Tanaka 2007, 2008]

Backward formalism

In the Einstein frame, the action takes :

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}G_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V\right]$$

spacetime metric field-space metric

There are many formalisms which enable us to study multi-field inflation.

A particularly convenient description of curvature perturbation is due to the delta N formalism.

=> a powerful tool to compute cosmological observables for the super-horizon evolution of the nonlinear curvature perturbation.

Backward formalism

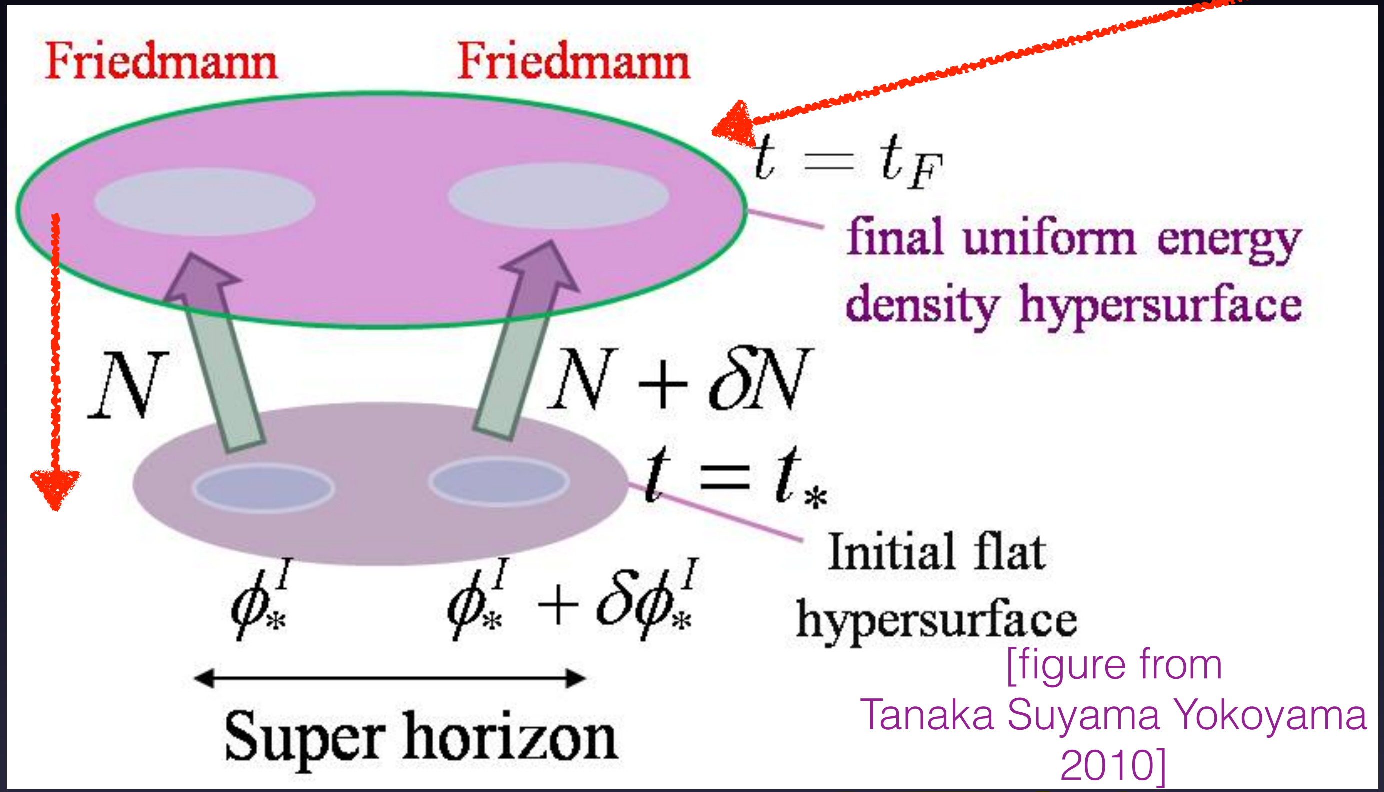
[Sasaki Stewart 1996]

[Tanaka Stewart 1996]

[Yokoyama Suyama Tanaka 2007, 2008]

Delta N formalism:

$$H(\varphi^a(N_F + \delta N(N_F))) = H(\overset{(0)}{\varphi^a}(N_F))$$



[figure from Tanaka Suyama Yokoyama 2010]

$$N^F_a \qquad \& \qquad N^F_{ab}$$

$$\frac{D}{dN} N_a(N) = -N_b(N) P^b_a(N)$$

$$f^{(4)}_{\rm NL} = \frac{5}{6} \frac{A^{ac}_* A^{bd}_* N^*_c N^*_d N^*_{ab}}{(A^{ab}_* N^*_a N^*_b)^2}$$

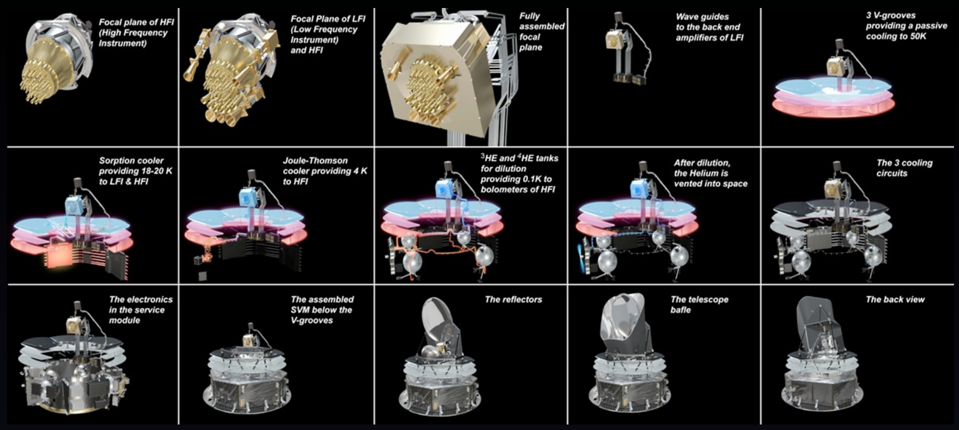
(local-type) non-linearity

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \rangle \equiv \delta(\mathbf{k}_1 + \mathbf{k}_2) P_{\mathcal{R}}(k_1)$$

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle \equiv \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_{\mathcal{R}}(k_1, k_2, k_3)$$

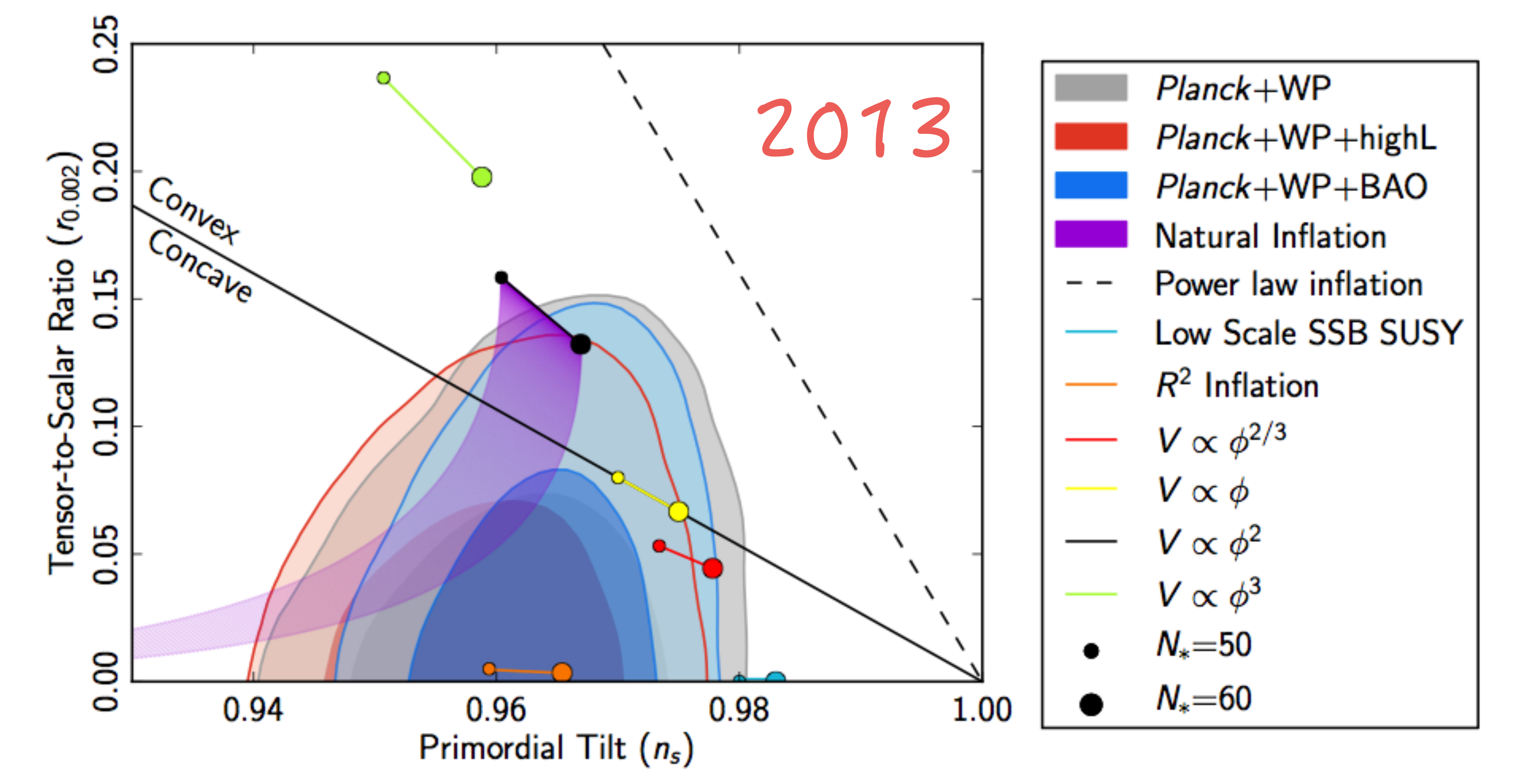
$$B_{\mathcal{R}}(k_1, k_2, k_3) = \frac{6}{5} \frac{f^{(4)}_{\rm NL}}{(2\pi)^{3/2}} \left[P_{\mathcal{R}}(k_1) P_{\mathcal{R}}(k_2) + P_{\mathcal{R}}(k_2) P_{\mathcal{R}}(k_3) + P_{\mathcal{R}}(k_3) P_{\mathcal{R}}(k_1) \right]$$

Results (1 TeV case)

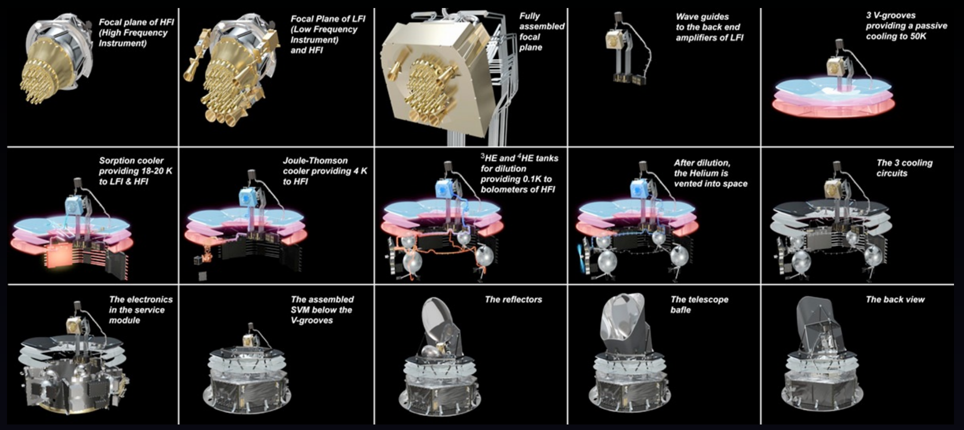


Experimental windows: [Planck Collaboration]

$$
\begin{aligned}
n_s &= 0.9603 \pm 0.0073 \quad (68\%\ \text{C.L.}), \\
r &< 0.12 \quad (95\%\ \text{C.L.}), \\
f_{\rm NL}^{\rm local} &= 2.7 \pm 5.8 \quad (68\%\ \text{C.L.}).
\end{aligned}
$$



Results (1 TeV case)



Experimental windows: [Planck Collaboration]

$$n_s = 0.9603 \pm 0.0073 \quad (68\%\ \text{C.L.}),$$
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$$f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8 \quad (68\%\ \text{C.L.}).$$