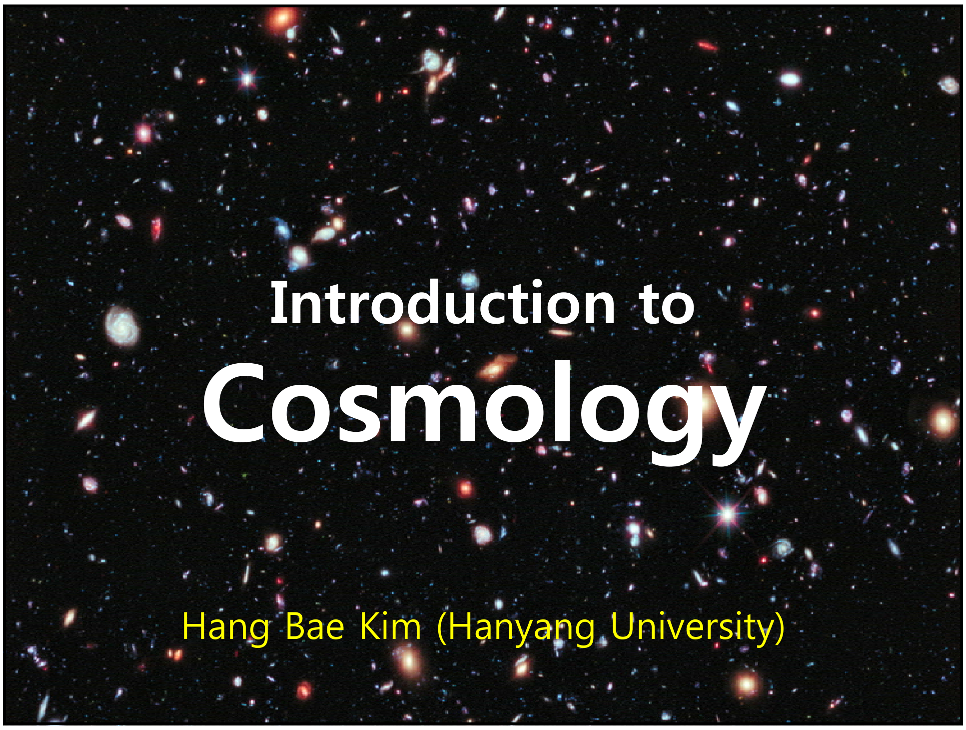
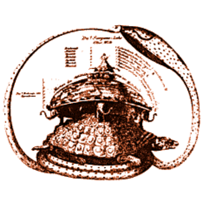


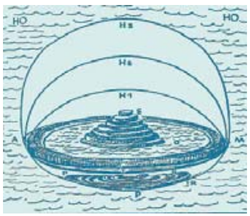
Introduction to Cosmology

Hang Bae Kim (Hanyang University)

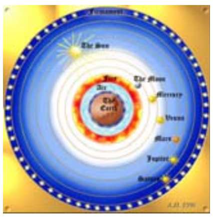
How Is the World Shaped?



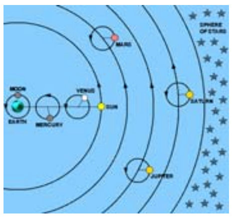
India



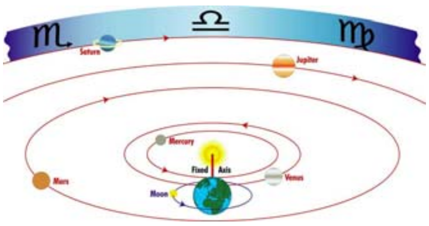
Babylon



Aristotle



Ptolemy



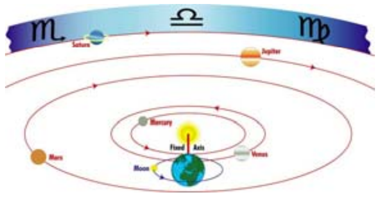
17thC, Western

Current

Understanding the Universe
A Success Story

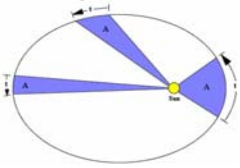
Tycho's observation

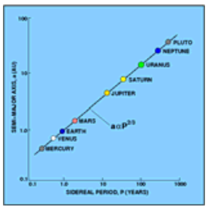
5 planets in the night sky



View of the solar system

Kepler's laws





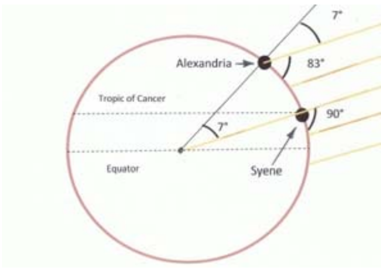
Newton's Law of Motion and Law of Gravitation

$$m\frac{d^2\mathbf{r}}{dt} = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$

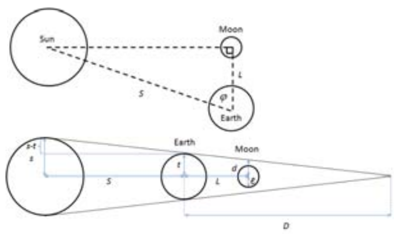
Compete understanding of planetary motion

Size of the World

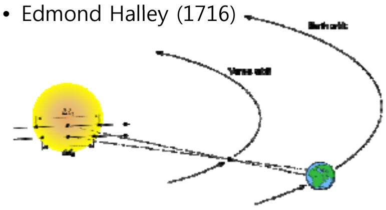
- Knowing the size or the distance is the first step to understand the world.
- Earth
  - Eratosthenes of Cyrene (276–195BC)



- Solar system
  - Aristarchus of Samos (310-230BC)

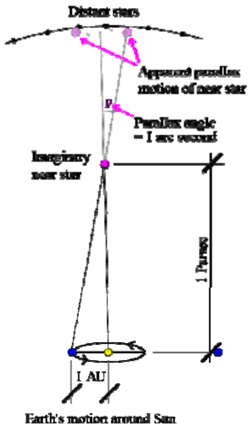


  - Edmond Halley (1716)



Distances to Stars

- **Annual Parallax**



1° = 60′ = 3600″ 1°

- Annual parallax was seriously searched as a crucial evidence for heliocentricism.
- F. Bessel, 1838 – measured successfully the annual parallax of Cygni 61 to be 0.314″. (The current value is 0.286″.)

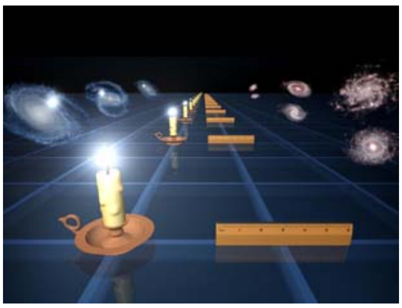
F. Bessel and his heliometer used to measure annual parallax

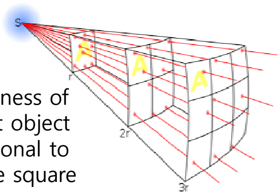
- Annual parallax of 1″ defines 1 parsec.

$$1\,\mathrm{pc} = 3.26\,\mathrm{ly} = 3.1 \times 10^{16}\,\mathrm{m}$$

- Angular diameter of the sun = 32′
- Annual parallax of the nearest star from the sun = Proxima Centauri, 0.769″

Standard Candles and Rulers





The brightness of the distant object is proportional to the inverse square of the distance

- How to measure the distance to farther objects?
- **Luminosity Distance**
  - Brightness of the astronomical objects

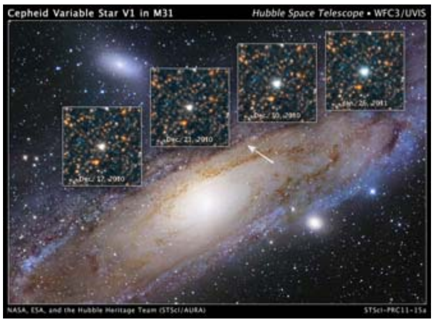
$$F = \frac{L}{4\pi r^2} \quad \Rightarrow \quad d_L^2 \equiv \frac{L}{4\pi F}$$

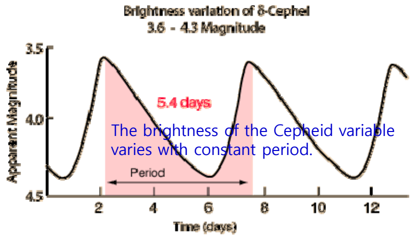
  - objects with known luminosity.
  - brighter objects for larger distance
- **Angular Diameter Distance**
  - Angular diameter of the object

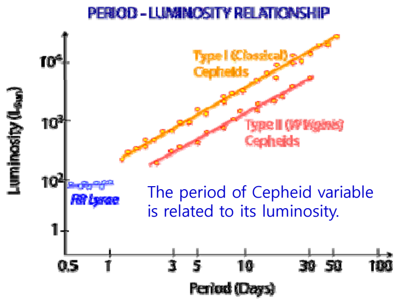
$$\theta = \frac{l}{r} \quad \Rightarrow \quad d_A = \frac{l}{\theta}$$

  - objects with known size.
  - larger objects for larger distance

Cepheid Variables







- John Goodricke, 1784
  - Discovery of δ-Cepheid variable
- **Henrietta Leavitt, 1908**
  - **Discovery of the period-luminosity relation**
- Harlow Shapley, 1915
  - Size and shape of our galaxy from Cepheids
- **Edwin Hubble, 1924**
  - Confirmed Andromeda galaxy is extra-galactic
- **Edwin Hubble & Milton Humason, 1929**
  - **Discovery of the expansion of the universe**

Henrietta Swan Leavitt

Pickering's Harem

In 1908-1912, **Henrietta Leavitt** studied the variables in Magellanic clouds and found that **Cepheid variables** can be **standard candles**.

1777 VARIABLES IN THE MAGELLANIC CLOUDS.

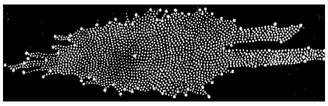
By Henrietta S. Leavitt.

In the spring of 1904, a comparison of two photographs of the Small Magellanic Cloud, taken with the 24-inch Bruce Telescope, led to the discovery of a number of faint variable stars. As the region appeared to be interesting, other plates were examined, and although the quality of most of these was below the usual high standard of excellence of the later plates, 57 new variables were found, and announced

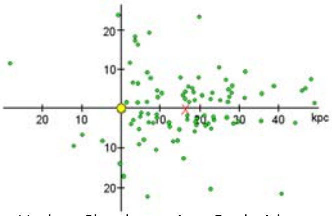
Large and Small Magellanic clouds

Shape of Our Galaxy

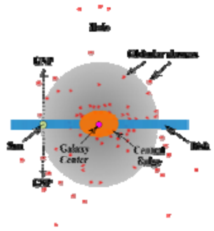
- Milky Way
  - Galileo – using telescope, confirmed that Milky Way is an aggregation of faint stars.
  - Kant – identified the disk shape of star distribution in Milky Way



William Herschel's drawing of star distribution (1785). He assumed that all stars have the same luminosity, so that their distance can be known from their brightness. The sun is located at the center and the star distribution has the disk shape.



Harlow Shapley, using Cepheid variables, obtained the distribution of globular clusters in our galaxy (1915). The yellow circle is sun's location and the X mark is the center of distribution.



Current view of our galaxy. Stars are mainly distributed in the disk and the sun is 8.5 kpc distant from the galactic center.

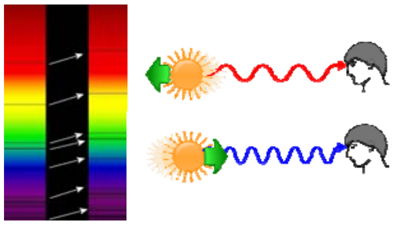
Discovery of Outer Galaxies

- Identity of spiral nebulae
  - Shapley-Curtis debate (1920s) – identity of spiral nebulae, aggregations of stars inside our galaxy versus another galaxies outside our galaxy
  - Edwin Hubble measured the distance to Andromeda nebula (1925), which is much larger than the size of our galaxy, proving the existence of outer galaxies.

Andromeda Galaxy (M31), a big spiral galaxy, nearest (70 Mpc) to our galaxy.

Discovery of Expansion

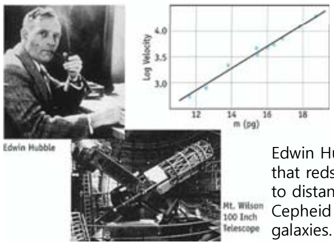
- **Redshift**
  - Vesto Slipher discovered the red shift of nebulae (1912) – Absorption spectra from distant galaxies are red shifted.
  - Interpretation – Distant galaxies are receding from us (Doppler shift).
- **Evidence for the expansion**
  - **Red shift proportional to Distance (Hubble's law, 1929)**



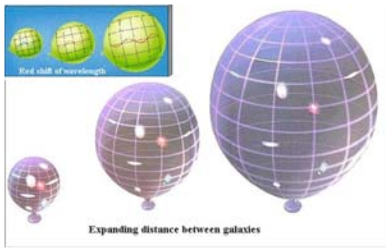
$\vec{v} = H_0 \vec{r}$

There is no center in the expansion.

Discovery of Expanding Universe



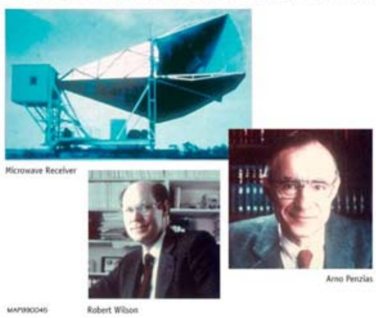
Edwin Hubble discovered that redshift is proportional to distance by observing Cepheid variables in distant galaxies.



Discovery of CMB

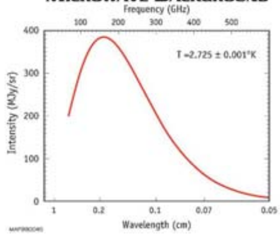
- Cosmic Microwave Background Radiation (CMB)
  - George Gamow and Ralph Alpher's prediction (1948)
  - Arno Penzias and Robert Wilson's discovery (1965)
  - **Very isotropic, perfect black body spectrum with T=2.73 K**
  - Eq. of state $p = \frac{1}{3}\rho$ (Ideal gas of photons)

Discovery of Cosmic Background

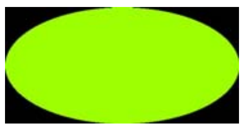


Penzias and Wilson, and the antenna they used in the discovery of CMB

Spectrum of the Cosmic Microwave Background

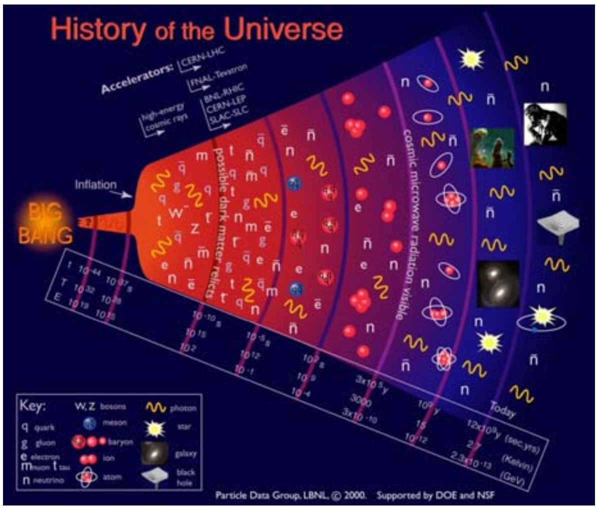


CMB has the blackbody spectrum.



CMB is very isotropic. ($\delta T/T \sim 10^{-5}$)

Hot Big Bang

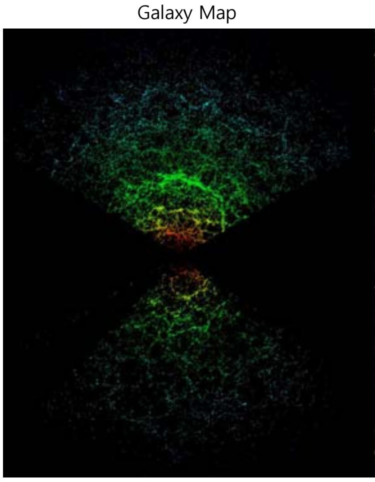


- CMB – Evidence for thermal equilibrium
- Expansion makes the universe cool down.
- Our universe was started in hot thermal state.

Galaxy Distribution

- Relatively Small Scales - Structures
- Very Large Scales - Homogeneous

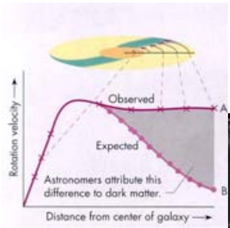
Galaxy Map



Hubble Deep Field

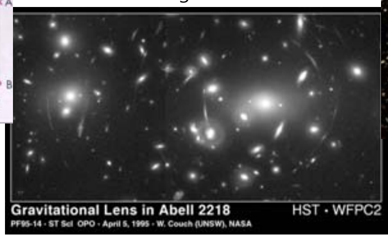
Discovery of Dark Matter

- To explain the formation of LSS, dark matter is a necessity.
  - The amount of dark matter in our universe : $\Omega_{\rm CDM} \sim 0.25$
- Evidences for dark matter
  - Motion of superclusters, Rotation curves of galaxies
  - Gravitational lensing, Mismatch in baryon and matter distribution



Rotation curve of galaxy shows the existence of dark matter outside the visible disk of galaxy.

Gravitational lensing effect reveals the existence of matter unseen between far galaxies and us.



Colliding clusters Baryon(red, X-ray) and dark matter (blue, gravitational lensing) reside separately. Galaxies follow the dark matter distribution.

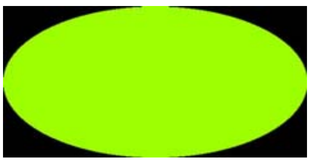
Discovery of CMB Anisotropies

- Is there any other evidence or probe for primordial density perturbation?
- CMB Anisotropies (CMBA)
  - CMB has **temperature fluctuations of order 10^{-5}**. $\delta T/T \sim 10^{-5}$
- Origin of CMBA – depending on scale
  - Gravitational potential due to **density perturbation of CDM**

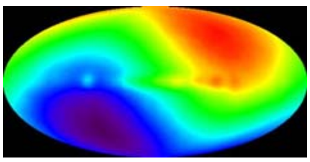
$$\delta\rho \longrightarrow \delta\Phi \longrightarrow \delta T$$

  - **Baryon Acoustic Oscillation**
    - Oscillation of strongly coupled baryon-photon plasma

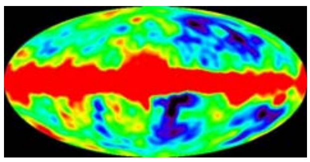
- Observations of CMBA



2.73 K



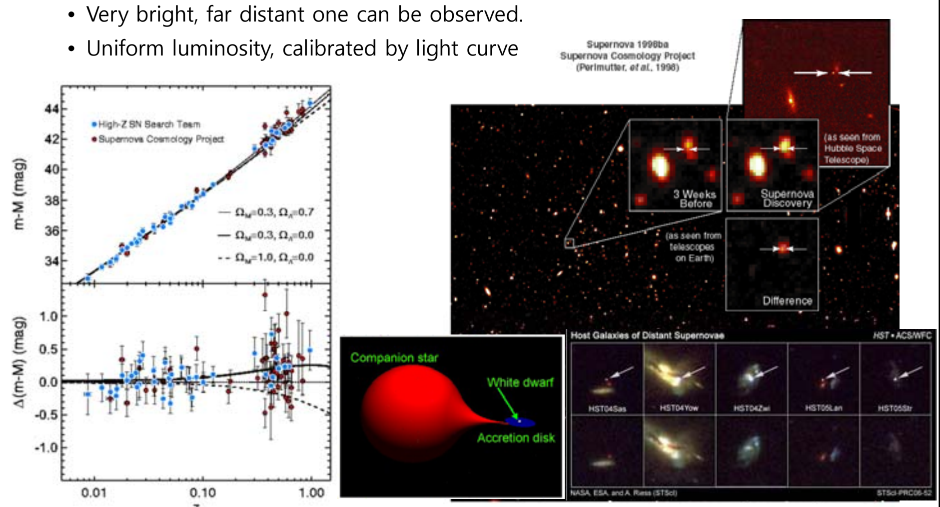
1/1,000 K



1/100,000 K (COBE)

Discovery of Accelerating Expansion

- SN Ia observations
  - Very bright, far distant one can be observed.
  - Uniform luminosity, calibrated by light curve



Ia형 초신성 관측

Units and Conversion Factors

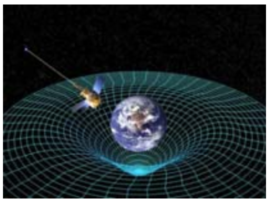
- Basic units
  - **Natural unit** $\hbar = c = k_B = 1$
  - Planck unit $\hbar = c = k_B = G = 1$
  - Reduced Planck mass $M_P = (8\pi G)^{-1/2} = 2.4 \times 10^{18}\,\mathrm{GeV}$
  - Solar mass $M_\odot = 2 \times 10^{30}\,\mathrm{kg}$
  - parsec $1\,\mathrm{pc} = 3.26\,\text{light-year} = 3.1 \times 10^{16}\,\mathrm{m}$
- Conversion factors in Natural unit
  - Energy-Mass $1\,\mathrm{eV}^{-1}/c^2 = 1.78 \times 10^{-36}\,\mathrm{kg}$
  - Energy-Time $1\,\mathrm{eV}^{-1}\hbar = 6.58 \times 10^{-16}\,\mathrm{s}$
  - Energy-Length $1\,\mathrm{eV}^{-1}\hbar c = 1.97 \times 10^{-7}\,\mathrm{m}$
  - Energy-Temperature $1\,\mathrm{eV}/k_B = 1.16 \times 10^{4}\,\mathrm{K}$

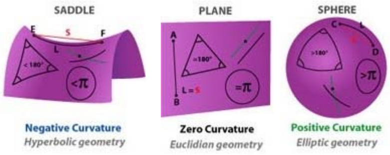
General Relativity

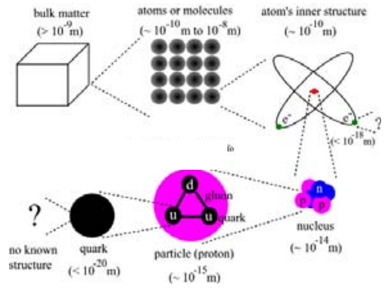
- Einstein Equation

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

SpaceTime **Einstein Tensor** **Stress-Energy Tensor** Matter







$$\Delta x\, \Delta p \geq \frac{1}{2}\hbar$$

- Most of cosmology can be learned with only a passing knowledge of general relativity : metric, geodesics, Einstein equation, …
  - Metric $g_{\mu\nu}$, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
  - Connection $\Gamma^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right)$
  - Curvature $R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}$
  - Ricci and Einstein tensor $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$, $R = R^\mu{}_\mu$, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
  - Einstein equation can be derived from the Einstein-Hilbert action.

$$S_{\rm EH} = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R + \mathcal{L}_{\rm M} \right) \quad \Rightarrow \quad G_{\mu\nu} = M_P^{-2}\, T_{\mu\nu}$$

  - Geodesic equation – path of a freely falling particle

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0$$

  - Isometry : Symmetry of manifold ⇔ Killing vector

    Maximally symmetric space

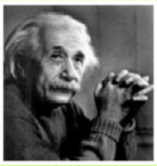
Exercise 1

Derive the Einstein equation from the Einstein-Hilbert action.

$$S_{\mathrm{EH}} = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \mathcal{L}_{\mathrm{M}}\right) \quad \Rightarrow \quad G_{\mu\nu} = M_P^{-2}\,T_{\mu\nu}$$

FLRW Universe

- **Two important observational facts about our universe**
  - The distribution of matter (galaxies) and radiation (CMB) in the observable universe is homogeneous and isotropic.
  - Distant galaxies are receding. - The universe is expanding.
- **Cosmological principle**
  - The universe is pretty much the same **everywhere**.



Belief of Einstein

- **Friedmann-Lemaitre-Robertson-Walker metric**

Our local Hubble volume during Hubble time

~ **spacetime with homogeneous and isotropic spatial sections**

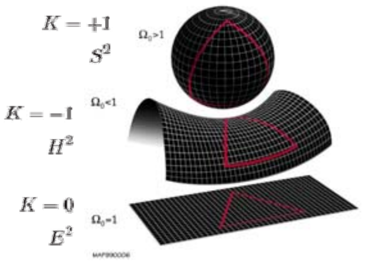
$$M = \underset{\text{Time}}{\underset{\uparrow}{\mathbf{R}}} \times \underset{\text{3D, maximally symmetric space}}{\underset{\uparrow}{\Sigma}}$$

$$ds^2 = -dt^2 + a(t)^2 \left[\overbrace{\frac{dr^2}{1 - Kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right)}^{\text{Maximally symmetric 3D space}} \right]$$

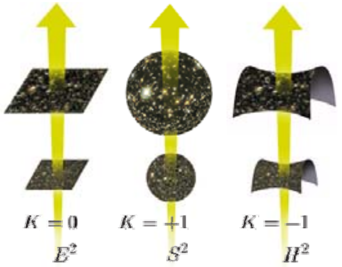
Cosmic Time

Scale Factor
the only dynamical variable of RW metric

Comoving Coordinates
fixed on the expanding space



Examples of maximally symmetric space in 2D



Change of Scale factor

Kinematics of FLRW Metric

- **Features of expanding space**
  - Momentum of a particle red shifts as the space expands.
  - Measuring distance in the expanding space is a little bit tricky.
    - Comoving distance – (fixed) coordinate distance
    - Physical distance – comoving distance x scale factor
    - Luminosity distance – measured by light intensity
    - Angular diameter distance – measured by angular size
  - Hubble's law – For small red shift, red shift is proportional to distance.
  - Growth of horizon (Visible universe) is different from the non-expanding universe.

- **Free (free-falling) particle - geodesic equation in RW metric**
  - Energy-momentum vector of a particle : $p^\mu = \frac{dx^\mu}{d\tau} = (E, \vec{p})$
  - 0-component of geodesic eq. $E\frac{dE}{dt} = -\Gamma^0_{ij}p^i p^j = -\delta_{ij} a\dot{a} p^i p^j$

    $\frac{1}{|\vec{p}|}\frac{d|\vec{p}|}{dt} + \frac{\dot{a}}{a} = 0 \quad \Rightarrow \quad |\vec{p}| \propto \frac{1}{a}$ Momentum red-shifts as the scale factor increases.
  - For light, $|\vec{p}| = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \frac{\lambda_0}{\lambda(t)} = \frac{a_0}{a(t)} \equiv 1 + z(t)$

    red shift parameter
  - Red shift in the light from far distant galaxies is actually not due to Doppler effect, but due to momentum red shift caused by the expansion of space.
  - Red shift in the light from near galaxies is a mixture of Doppler effect and momentum red shift, and we cannot distinguish between them.
  - Due to momentum red shift, the temperature ($\propto$ the average kinetic energy) of hot idea gas (consisting of free particles) cools down as the space expands.
  - Red shift parameter z can be used to parameterize the time, instead of the cosmic time t or the scale factor a(t).

- **Hubble's law**
  - Luminosity distance : $F = \frac{L}{4\pi d^2} \quad \Rightarrow \quad d_L^2 \equiv \frac{L}{4\pi F}$

    Energy conservation requires that the flux decreases by distance square.
  - Effect of expansion : $F = \frac{L}{4\pi(a_0 r(z))^2}\frac{1}{(1+z)^2}$
    - Red shift of light
    - Dilation of arrival time

    Comoving distance to the light source

    $\int_0^r \frac{dr'}{\sqrt{1-Kr'^2}} = \int_t^{t_0} \frac{dt'}{a(t')}$

    $r(z) \equiv f(z),\ \sin f(z), \sinh f(z)$ for $K = 0, +1, -1$, repectively, where

    $f(z) = \int_t^{t_0} \frac{dt'}{a(t')} = \int_0^z \frac{dz'}{a_0 H(z')}$
  - Luminosity distance – Red shift relation : $d_L = a_0 r(z)(1+z)$

    For small z, $H(z) = H_0 + H_0' z + \cdots$

    $$\underline{H_0 d_L = z} + \frac{1}{2}(1 - q_0)z^2 + \cdots$$

    Hubble's law

    Deceleration parameter

    $q_0 = -\frac{a_0 \ddot{a}_0}{\dot{a}_0^2}$

- **Comoving horizon**
  - Total comoving distance light ($ds^2 = 0$) have traveled since t=0

$$\eta(t) \equiv \int_0^{r_H} \frac{dr}{\sqrt{1-Kr^2}} = \int_0^t \frac{dt'}{a(t')}$$

  - No information could have propagated further than this.
    ⇒ The size of the universe we can see at present ⇒ **comoving horizon**
  - Physical distance to the horizon $d_H(t) = \int_0^{r_H} \sqrt{g_{rr}}\, dr = a(t)\eta$
  - Comparison to non-expanding universe : $\eta_{\rm NE}(t) = d_{\rm H,NE}(t) = t$
    For $a(t) \propto t^\alpha$ $(0 < \alpha < 1)$, comoving horizon grows slower and physical horizon grows faster.

$$\eta_{\rm E}(t) = \int_0^t \frac{dt'}{a(t')} = \frac{t^{1-\alpha}}{1-\alpha} \qquad d_{\rm H,E}(t) = a(t)\int_0^t \frac{dt'}{a(t')} = \frac{t}{1-\alpha}$$

Dynamics of the Universe

- **How is the evolution of the universe determined?**
- **Einstein equation for FLRW metric**

$$G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}$$

- SE tensor should have this form to be consistent with FLRW metric.

- Geometry of our universe

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega\right]$$

Scale Factor

- Matter distribution

$$T^\mu{}_\nu = \begin{pmatrix} -\rho(t) & 0 \\ 0 & p(t)\delta_{ij} \end{pmatrix}$$

Energy density **Pressure**

- **Friedmann equation**

$$\left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} = \frac{1}{3M_P^2}\sum_i \rho_i \qquad \frac{\ddot a}{a} = -\frac{1}{6M_P^2}\sum_i(\rho_i + 3p_i)$$

$H(t) \equiv \frac{\dot a}{a}$ **Hubble parameter** - expansion rate

- Positive energy density makes the universe expand or shrink.
- Pressure also gravitates.
- The combination ρ+3p makes the expansion decelerate or accelerate.

Exercise 2

Calculate the Einstein tensor of the FLRW metric and show that the Einstein equation leads to the Friedmann equations.

- **Evolution of the scale factor is determined by the matter content.**

Einstein Eq. : Spacetime Geometry ↔ Matter Distribution

Friedmann Eq. : scale factor change ↔ matter species and amount

- **Species** – **equation of state** (relation of energy density(ρ) and pressure(p)
  - Simple form of eq. of state : $p = p(\rho) = w\,\rho$ ⇨ $\rho = \rho_0\,(a/a_0)^{-3(1+w)}$

Name		Radiation	Matter	Vacuum E
Eq. of state	$p = w\rho$	$1/3$	0	-1
Energy density	$\rho \propto a^{-3(1+w)}$	a^{-4}	a^{-3}	constant
Scale Factor (K=0)	$a \propto t^{2/3(1+w)}$	$t^{1/2}$	$t^{2/3}$	e^{Ht}

- **Amount** – **Density parameter**, ratio to the critical density

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

- The critical density is determined by the Hubble constant.
- The present value is roughly - 6 protons per 1m^3

$$\rho_c = 3M_P^2 H_0^2 = 1.9h^2 \times 10^{-26}\,\mathrm{kg/m^3}$$

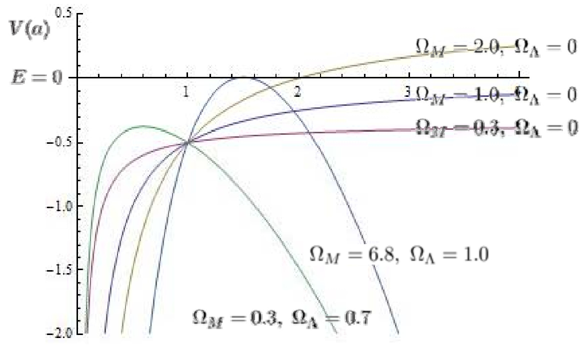
- **Solving Friedmann eq.**

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{1}{3M_P^2}\sum_i \rho_i = \frac{1}{3M_P^2}\sum_i \rho_{i0} a^{-3(1+w_i)} \qquad \Omega_0 \equiv \sum_i \Omega_i$$

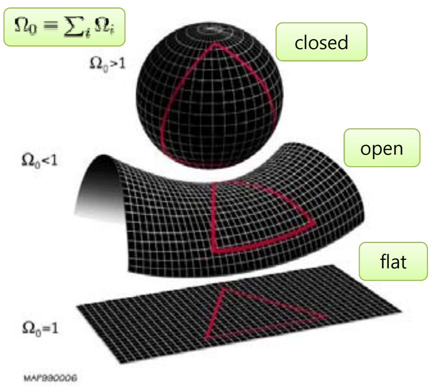
$$\Omega_K \equiv \Omega_0 - 1$$

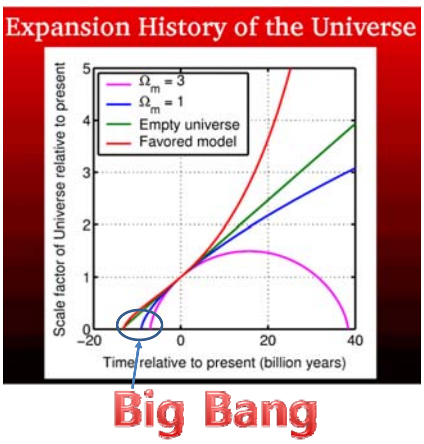
$$\frac{1}{2}\dot{a}^2 + V(a) = 0, \quad V(a) = \frac{1}{2}H_0^2\left[\Omega_K - \sum_i \Omega_i a^{-1-3w_i}\right]$$

⇒ the motion of a particle with E=0 under the potential V(a)



- **Expansion history of the universe depends on the species and amounts of matter in the universe.**



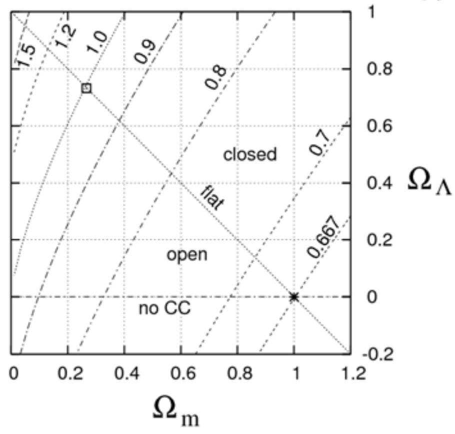


- **Our universe has the beginning.**
  If we trace back the expansion history, we meet a singular (infinite energy density) point of a=0 in a finite time.

- **Age of the universe**

$t_0 = H_0^{-1} f(\Omega_i)$ **Hubble time** $H_0^{-1} = \left(\frac{0.71}{h}\right) \times 13.8\,\mathrm{Gyr}$

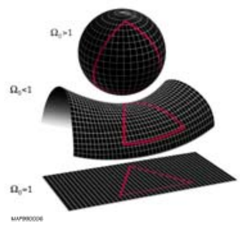
Age times current Hubble parameter ← $f(\Omega_i) = \int_0^1 \left[-\Omega_K + \sum_i \Omega_i x^{-1-3w_i} \right]^{-1/2} dx$



- **Red shift – Luminosity distance relation**
  - Distance depends on geometry.

$$r_1 = f_K(z) \equiv \begin{cases} \sin f(z), & \text{for } K = +1\ (\Omega > 1), \\ f(z), & \text{for } K = 0\ (\Omega = 1), \\ \sinh f(z), & \text{for } K = -1\ (\Omega < 1), \end{cases}$$

$$f(z) = \int_0^z \frac{dz'}{a_0 H(z')} = \frac{1}{a_0 H_0} \int_0^z \frac{dz'}{\left[\Omega_K (1+z)^2 + \sum_i \Omega_i (1+z)^{3(1+w_i)}\right]^{1/2}}$$



  - Luminosity distance – Red shift relation : $d_L = a_0 r(z)(1+z)$

$$H_0 d_L = z + \frac{1}{2}(1 - q_0) z^2 + \cdots$$

Distance modulus is frequently used instead of lum. distance.
$\mu_0 = m - M = 5 \log (d_L / 10\,\mathrm{pc})$

Hubble's law, Hubble Cons.

$$H_0 = \frac{\dot{a}_0}{a_0} = 73.8\,(\mathrm{km/s})/\mathrm{Mpc}$$

$$H_0^{-1} = 13.8\,\mathrm{Gy} = 4230\,\mathrm{Mpc}$$

Determine the age and the size of the universe

Deceleration parameter

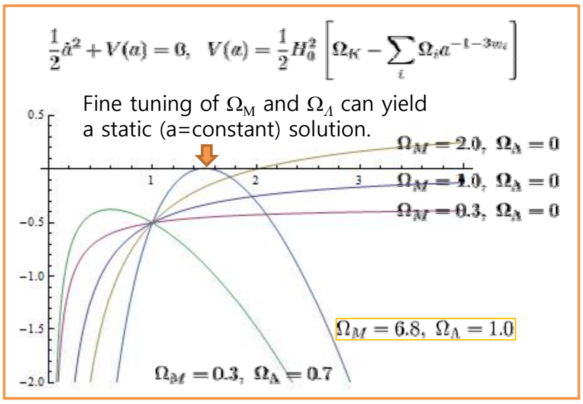
$$q_0 = -\frac{a_0 \ddot{a}_0}{\dot{a}_0^2} = \frac{1}{2} \sum_i \Omega_i (1 + 3w_i)$$

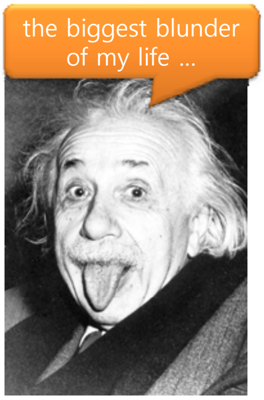
Determined by species and amounts of matter.

- For acceleration, matter with $w_i < -\frac{1}{3}$ must dominate.

Einstein's Biggest Blunder

- **Introduced the cosmological constant to obtain the static universe** (1917)





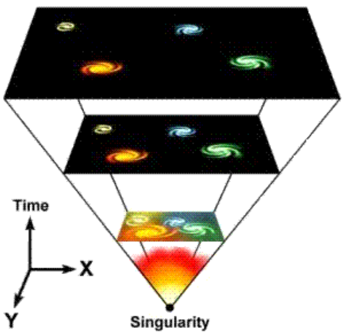
- Gave up the static universe after Hubble's discovery of expansion (1929)
- Resurrection of CC to explain the accelerating expansion (1998)

Consequences of Expansion

- Meaning of the existence of CMB
  - Black body spectrum of 2.73K
    ⇒ Our universe was in **thermal equilibrium** in the past.
- Relation between the scale factor and the temperature in thermal equilibrium

$$a(t)T(t) = \text{constant}.$$

- Temperature and expansion
  - small **a** in the past → high **T** in the past.
  - **Hot Big Bang : Our universe started in thermal equilibrium with high temperature.**



- High Temperature (T) ⇔ High Energy (E) ⇔ Short Distance (quantum principle)
  - **To understand the high temperature state of the early universe, we need the knowledge at short distance (high energy, that is particle physics).**

Particles in Equilibrium

- Direct evidence for thermal equilibrium in the early universe

  CMB : isotropic, accurate black body spectrum

  The early universe is filled with **hot ideal gases in thermal equilibrium**.

- Energy density and pressure of ideal gas at T

$$\rho_i(T) = g_i \int \frac{d^3\vec{p}}{(2\pi)^3} f_i(\vec{p}) E(\vec{p}) \qquad E = \sqrt{|\vec{p}|^2 + m^2}$$

$$p_i(T) = g_i \int \frac{d^3\vec{p}}{(2\pi)^3} f_i(\vec{p}) \frac{\vec{p}^2}{3E(\vec{p})}$$

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

\- boson;
+, [] fermion

Momentum distribution in thermal equilibrium

- Relativistic, non-degenerate : $T \gg m, \mu$

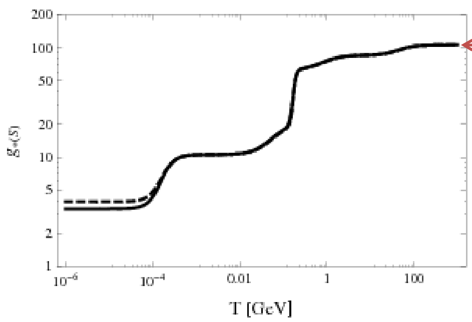
$$n = \left[\frac{3}{4}\right] \frac{\zeta(3)}{\pi^2} g T^3, \quad \rho = \left[\frac{7}{8}\right] \frac{\pi^2}{30} g T^4, \quad p = \frac{1}{3}\rho$$

- Non-relativistic : $T \ll m$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}, \quad \rho = mn + \frac{3}{2}p, \quad p = nT \ll \rho$$

- Total energy density and pressure
  - In equilibrium, the energy density of non-relativistic species is exponentially smaller than that of relativistic species.

$$\rho_R = \frac{\pi^2}{30} g_*(T)\, T^4, \quad p_R = \frac{1}{3}\rho_R, \quad g_*(T) = \sum_{\text{bosons}} g_b + \frac{7}{8} \sum_{\text{fermions}} g_f$$



- Standard Model : $\frac{427}{4} = 106.75$

  F: $\frac{7}{8}(4\times3\times6 + 4\times3 + 2\times3) = \frac{315}{8}$

  B: $2\times8 + 3\times3 + 2 + 1 = 28$

Exercise 3

A. Calculate the effective degrees of relativistic species (g_*) for the minimal supersymmetric standard model (MSSM).

B. Draw the graph of the effective degrees of relativistic species $g_*(T)$ as a function of the temperature T.

- Entropy
  - The entropy in a comoving volume is conserved in thermal equilibrium.
  - Entropy in the early universe is dominated by relativistic species

    Entropy density $s = \sum \frac{\rho_i + p_i}{T_i} = \frac{2\pi^2}{45} g_* T^3$

  - Evolution of the temperature T

$$T \propto g_*^{-1/3} a^{-1}$$

  - Since $n \propto a^{-3}$ and $s \propto a^{-3}$, $Y_i \equiv n_i / s$ is a convenient quantity for representing the abundance of decoupled species.

Exercise 4

By integrating the Friedmann equation, derive the time-temperature relation during the radiation dominated era.

$$\left(\frac{\dot a}{a}\right)^2 + \cancel{\frac{k}{a^2}} = \frac{1}{3M_P^2}\rho_R = \frac{1}{3M_P^2}\frac{\pi^2}{30}g_* T^4, \qquad aT \propto g_*^{-1/3}$$

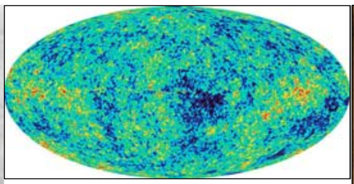
$$\Downarrow$$

$$\frac{\dot a}{a} = -\frac{\dot T}{T}$$

$$t = \left(\frac{90}{\pi^2 g_*}\right)^{1/2}\frac{M_P}{T^2} = 1.32\,\mathrm{s}\left(\frac{1\,\mathrm{MeV}}{T}\right)^2$$

Remnants of Expansion

- Thermal equilibrium and its break-down
  - To keep thermal equilibrium, the reaction rate must be larger than the expansion rate.
  - As temperature goes down, the reaction rate decreases faster than the expansion rate and thermal equilibrium is broken.
  - If thermal equilibrium is kept on, no remnant from the past can be found.
- **Out-of-equilibrium make the history of the universe.**
  - **Baryogenesis, Big Bang Nucleosynthesis**
  - **Decoupling of Dark Matter, Neutrinos, Photons**



Cosmology is similar to archeology in the sense that it deduces the past from the remnants.

The expansion of the universe makes the history.

Out-of-Equilibrium

- The universe has been very nearly in thermal equilibrium for most of its history.
- Departure from thermal equilibrium might make fossil record of the early universe.
- **Rule of thumb for thermal equilibrium**

$$\textbf{Interaction rate } \Gamma_{\rm int} > \textbf{Expansion rate } H$$

$$\Gamma_{\rm int}(T) = n(T)\langle\sigma|v|\rangle^T \qquad H(T) \approx \frac{T^2}{M_P}$$

- Rough understanding of decoupling of species
  - Interaction mediated by a massive gauge boson

$$\sigma \sim \frac{\alpha^2 s}{m_X^2} \quad \Rightarrow \quad \Gamma_{\rm int} \sim T^3 \cdot \frac{\alpha^2 T^2}{m_X^4} = \frac{\alpha^2 T^5}{m_X^4}$$

$$T \lesssim \left(\frac{m_X^4}{\alpha^2 M_P}\right)^{1/3} \sim \left(\frac{m_X}{100\,\mathrm{GeV}}\right)^{4/3} \mathrm{MeV} \quad \Rightarrow \quad \text{freeze out}$$

Boltzmann Eq. for Annihilation

- Boltzmann Eq. : The rate of abundance change
  = the rate of production – the rate of elimination

- Consider the particle 1 in a process 1 + 2 ↔ 3 + 4

$$\underbrace{\frac{1}{a^3}\frac{d(n_1 a^3)}{dt}}_{\text{Change in comoving volume}} = \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}$$

$$\times (2\pi)^4\delta^4(p_1+p_2-p_3-p_4)|\mathcal{M}(1+2\leftrightarrow 3+4)|^2$$

$$\times \{\underbrace{f_3 f_4(1\pm f_1)(1\pm f_2)}_{\substack{\text{Production of 1} \\ 3+4\to 1+2}} - \underbrace{f_1 f_2(1\pm f_3)(1\pm f_4)}_{\substack{\text{Elimination of 1} \\ 1+2\to 3+4}}\}$$

Distribution function and number density

$$n_1(t) = \int \frac{d^3\vec{p}_1}{(2\pi)^3} f_1(\vec{p}_1, t)$$

Particle physics enters here !
Scattering amplitude
CP(or T) symmetry assumed

- Simplifying assumptions
  - Kinetic equilibrium

    Rapid elastic scattering ➡ $f(\vec{p},t) = \dfrac{1}{e^{(E(\vec{p})-\mu(t))/T(t)} \pm 1}$
  - Annihilation in equilibrium : $\mu(t)$ → chemical potential

    $f(\vec{p},t)$: described by chemical potential (and temperature)
  - Low temperature approximation : $T \ll E - \mu$

    $f \approx e^{-(E-\mu(t))/T}, \quad 1 + f \approx 1$
  - Change of variables : chemical potential → number density

    $$\mu_i(t) \;\rightarrow\; n_i(t) = g_i e^{\mu_i(t)/T} \int \frac{d^3\vec{p}_i}{(2\pi)^3} e^{-E_i/T} = e^{\mu_i(t)/T} n_i^{(0)}$$

    ➡ Ordinary differential equation for $n_i(t)$

$$\begin{aligned} f_3 f_4 (1 \pm f_1)(1 \pm f_2) - f_1 f_2 (1 \pm f_3)(1 \pm f_4) & \\ \approx e^{-(E_1+E_2)/T} \left(e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T} \right) & \\ = e^{-(E_1+E_2)/T} \left\{ \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} \right\} & \end{aligned}$$

- Define the thermally averaged cross section

$$\langle \sigma v \rangle \equiv \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{p}_4}{(2\pi)^3 2E_4} e^{-(E_1+E_2)/T} \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \, |\mathcal{M}(1+2 \leftrightarrow 3+4)|^2$$

- Simplified Boltzmann equation

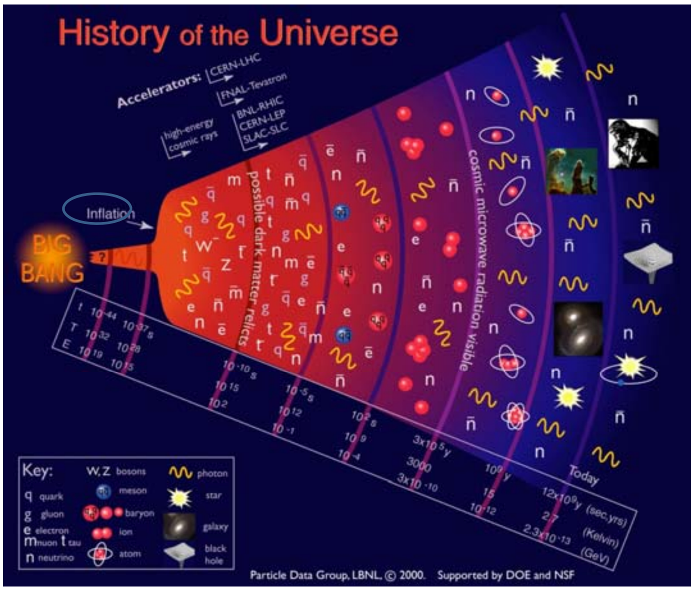
$$\Rightarrow \quad \frac{1}{a^3} \frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right)$$

$$\sim \frac{n_1}{t_H} \sim n_1 H \qquad n_1 n_2 \langle \sigma v \rangle \sim n_1 \Gamma_{\text{int}}$$

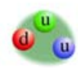
$$H \ll \Gamma_{\text{int}} \quad \Rightarrow \quad \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} = 0$$

chemical equilibrium

History of the Universe



Baryon Asymmetry



- Matter content of the universe – **Baryons**
  - Matter forming our body : Baryon(protons, neutron), lepton(electron)
  - Stars, planets, dust, gas, … (Most baryons are in intergalactic gases.)




- Baryon Asymmetry
  - SM of particle physics is very symmetric in baryon and anti-baryon.
  - But our universe is dominated by baryons, with little anti-baryon.

$$\frac{B-\bar{B}}{B+\bar{B}} \approx 1$$

- The amount of baryon in our universe
  - Good agreement in required amounts from BBN and CMBA

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 4 \times 10^{-9}$$ (4 baryons to 1 billion photons)

Exercise 5

Suppose that there were no baryon asymmetry so that the number density of baryons exactly equaled that of anti-baryons. Determine the final relic density of (baryons + anti-baryons). At what temperature is this asymptotic value reached? (from Exercise 12 of Ch.3, Dodelson)

$$\Gamma_{\rm int}(T) = n(T)\langle\sigma|v|\rangle^T \approx (m_N T)^{3/2} e^{-m_N/T} \cdot \frac{\alpha^2}{T^2}$$

$$H(T) \approx \frac{T^2}{M_P}$$

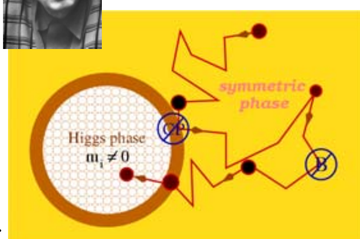
$$(m_N/T)^{5/2} e^{-m_N/T} \approx \frac{m_N}{\alpha^2 M_P} \quad \Rightarrow \quad \frac{m_N}{T} \approx 41.6$$

$$\Rightarrow \quad \frac{n_b}{n_\gamma} \approx 10^{-16}$$

Baryogenesis

- **Baryogenesis**
  - In the beginning, the universe was supposed to be baryon symmetric.
  - Baryon asymmetry was produced in the early universe.
  - Sakharov conditions for baryogenesis
    1. B violation
    2. C & CP violation
    3. Out-of-equilibrium
- **Standard Model**
  cannot make sufficient baryon asymmetry.
  → New theory is needed.
  - SM satisfies all three conditions.
  - But, CP violation is too small and out-of-equilibrium is not strong enough.

Andrei Sakharov



Neutrino Decoupling

- Equilibrium maintained by weak interaction is broken around T ~ 1 MeV.
- Species in equilibrium around T ~ 1 MeV :
  - baryon : proton, neutron (baryon asymmetry, non-relativistic)
  - lepton : electron, positron, 3 types of neutrinos
  - weak interaction $e^+ + e^- \leftrightarrow \nu_e + \bar{\nu}_e$

$$\Gamma(T) = n(T)\langle\sigma v\rangle_T \sim T^5/M_W^4$$

- Neutrino decoupling
  - decoupling temperature $T_{\rm dec} \sim 1\,{\rm MeV}$
  - relic abundance $Y_\nu = n_\nu/s = g_\nu/g_*(T_{\rm dec}) = 21/43$ $\quad \Omega_\nu h^2 = 1.68 \times 10^{-5}$ (massless)
    $\Omega_\nu h^2 = \sigma_{\nu_i} m_{\nu_i}/94\,{\rm eV}$ (massive)
  - temperature difference between photons and neutrinos
    Below T=m_e=0.52 MeV, e^+, e^- annihilate and dump energy only to photons, and thus photons cools slower than neutrinos.
    $T_\gamma/T_\nu = (4/11)^{1/3} = 0.71$

Neutron-Proton Ratio

- Baryon number density is fixed by baryon asymmetry.
- Neutron – Proton equilibrium is maintained by weak interaction.

$$n + e^+ \leftrightarrow p + \bar{\nu}_e\,, \quad n + \nu_e \leftrightarrow p + e^-\,, \quad n \leftrightarrow p + e^- + \bar{\nu}_e$$

- When temperature goes down below the neutron-proton mass difference, the neutron-proton equilibrium shift to proton.
  - neutron-proton mass difference $\Delta m = m_n - m_p = 1.3\,{\rm MeV}$
  - equilibrium neutron-proton ratio $n/p = e^{-\Delta m/T}$
- Below 1 MeV, neutron-proton conversion freeze out.
  - freeze-out temperature $T_f \approx 0.75\,{\rm MeV}$
  - neutron-proton ratio frozen $(n/p)_f = e^{-\Delta m/T_f} \approx 0.18$
  - neutron decay slowly reduces the neutron-proton ratio, reaching 0.13 at the beginning of big bang nucleosynthesis (t=200 s, T=0.07 MeV).

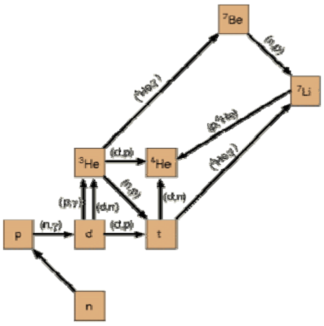
Big Bang Nucleosynthesis

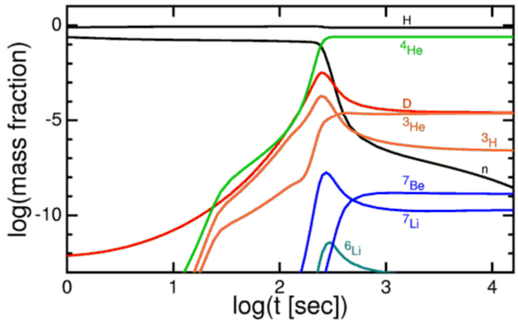
- As the universe cools down, light nuclei are synthesized from protons and neutrons. (Heavy nuclei are produced in the process of star evolution.)
- BBN is one of supporting evidences of Big Bang, by explaining very well the ratios of light nuclei in our universe.
- The ratios depends on the amount of baryon and the expansion rate at the time of BBN. BBN is a good probe of baryon amount.

- Universe at T ~ 1 MeV
  - Species in equilibrium : $\text{(photons)}\ \gamma,\ \text{(leptons)}\ e^+,\ e^-,\ \text{(baryons)}\ p,\ n$
  - Species decoupled : $\text{(neutrinos)}\ \nu_e,\ \nu_\mu,\ \nu_\tau$
  - Initial baryon asymmetry : $\eta_b \equiv \dfrac{n_b}{n_\gamma} = 5.5 \times 10^{-10} \left(\dfrac{\Omega_b h^2}{0.020} \right)$

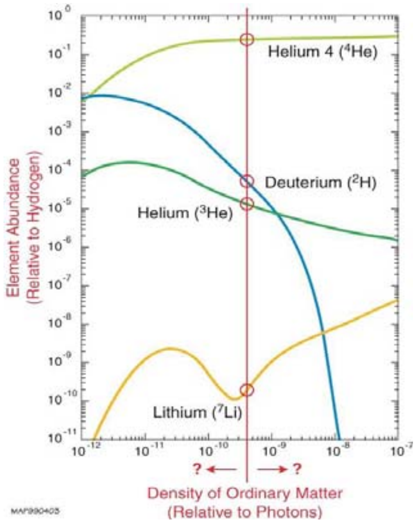
- How the baryons end up?
  - Nuclear binding energies are of order MeV, but the nucleosyntheis is delayed until T ~ 0.1 MeV by the effect of small η_b.
  - If thermal equilibrium is kept through out, the nuclear state with the lowest energy per baryon (iron nucleus) will dominates.
  - BBN produced no elements heavier than beryllium due to a bottleneck: the absence of a stable nucleus with 8 or 5 nucleons.

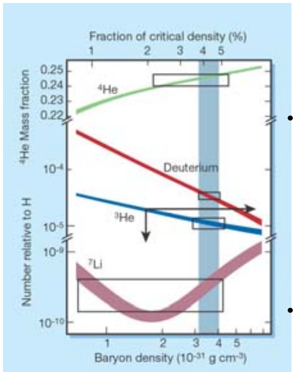
- Numerical solution of Boltzmann equations (BBN code)

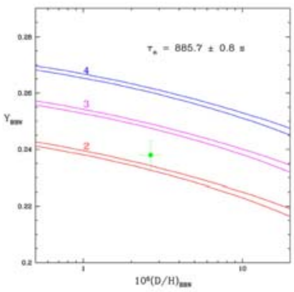




- Comparison with observations







- Observed abundances of light nuclei constrain the baryon density to be

$$\eta_b \approx 4 \times 10^{-9}$$
$$\Omega_b \approx 0.05$$

- BBN and CMBA give consistent results on the baryon density.
- The number of neutrino species affects the energy density at BBN, which can change the neutron-proton ratio, and thereby the helium fraction.

$$N_\nu \sim 3$$

Exercise 6

1. Explain why the neutron fraction rises when the number of neutrino-like species increases in big bang nucleosynthesis.
2. What would the present universe look like, if big bang nucleosynthesis proceed to turn all nucleons into iron nuclei which are most stable?

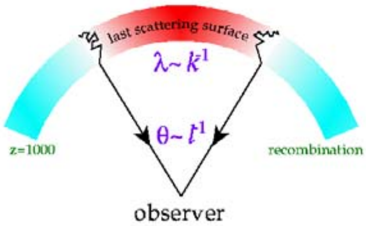
Decoupling of CMB

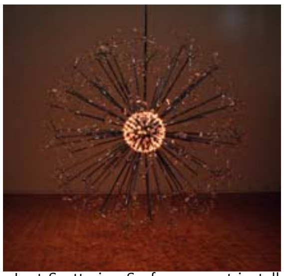
- Decoupling of cosmic microwave background radiation
  - Equilibrium between protons, electron, hydrogen atoms, and photons (about 300,000 years after big bang)

$$p + e^- \leftrightarrow H + \gamma \qquad (E_{\text{bind}} = 13.6\ \text{eV})$$
$$e^- + \gamma \leftrightarrow e^- + \gamma \qquad (\text{Thomson scattering})$$

Below $T \sim E_{\text{bind}}$,
H is preferred $\rightarrow$ Reduction of e^- $\rightarrow$ Decoupling of γ

  - CMB we see today comes from the last scattering surface.





The Last Scattering Surface, an art installation at the Henry Art Gallery on the University of Washington campus in Seattle

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- Species at temperature T ~ eV : $\gamma,\ e^-,\ p$ Tightly coupled $\qquad \nu_e,\ \nu_\mu,\ \nu_\tau$ decoupled

Compton scattering	$e^- + \gamma \leftrightarrow e^- + \gamma$
Coulomb scattering	$e^- + p \leftrightarrow H + \gamma$

- Evolution of the free electron fraction

$$X_e \equiv \frac{n_e}{n_e + n_H} = \frac{n_p}{n_p + n_H}$$

  - If $e^- + p \leftrightarrow H + \gamma$ remains in equilibrium

$$\frac{n_e n_p}{n_H} = \frac{n_e^{(0)} n_p^{(0)}}{n_H^{(0)}} \Rightarrow \frac{X_e^2}{1 - X_e} = \frac{1}{n_e + n_H}\left[\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-[m_e + m_p - m_H]/T}\right]$$

$$\approx n_b = \eta_b n_\gamma \sim 10^{-9} T^3$$

$$\approx 10^9 \left(\frac{m_e}{2\pi T}\right)^{3/2} e^{-\epsilon_0/T}$$

$X_e \approx 1$ at $T \sim \epsilon_0$ $\qquad$ As $X_e \rightarrow 0$, out of equilibrium.

- Boltzmann equation

$$\frac{dX_e}{dt} = \left[(1 - X_e)\beta - X_e^2 n_b \alpha^{(2)}\right]$$

$$\beta = \langle \sigma v \rangle \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\epsilon_0/T}$$

ionization rate

$$\alpha^{(2)} \equiv \langle \sigma v \rangle_{n=2}$$

recombination rate (to n=2 state)

- Decoupling of photon occurs
  when Compton scattering rate ~ Expansion rate

$$n_e \sigma_T = X_e n_b \sigma_T = 7.5 \times 10^{-30}\ \mathrm{cm}^{-1}\ X_e \Omega_b h^2 a^{-3}$$

$$\frac{n_e \sigma_T}{H} = 113 X_e \left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{0.15}{\Omega_m h^2}\right)^{1/2}\left(\frac{1+z}{1000}\right)^{3/2}\left[1 + \frac{1+z}{3600}\frac{0.15}{\Omega_m h^2}\right]^{-1/2}$$

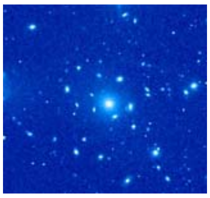
Decoupling of photons occurs during recombination ($X_e \lesssim 10^{-2}$)

- Reionization

Inhomogeneity

- On very large scale, our universe is very homogeneous.
- On smaller scales, we see inhomogeneities, such as stars, galaxies, clusters.
- **Without inhomogeneity, we cannot explain our existence itself.**

Galaxy: Andromeda



Cluster: Coma

Supercluster: Perseus

Measure of inhomogeneity – **density contrast**

depending on scale

$$\frac{\delta\rho}{\rho} \sim 10^{-5} - 10^{5}$$

- Analogy with Earth
  - On large scale, Earth is a nearly perfect sphere.
  - On small scales, we see surface fluctuations, such as mountains, valleys, trenches.
  - Measure of surface fluctuations : $\frac{\delta R}{R} \sim 10^{-3}$
  - What makes Earth a sphere?
    What creates surface fluctuations?

- **Understanding the formation of large scale structures**
  - Basic Ideas : Small initial density perturbations grow to form structures.

Action of gravity

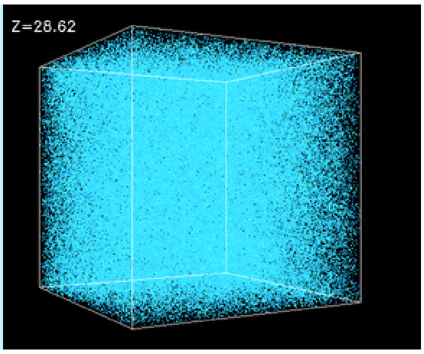
Initial density perturbation → Evolution → Large scale structures

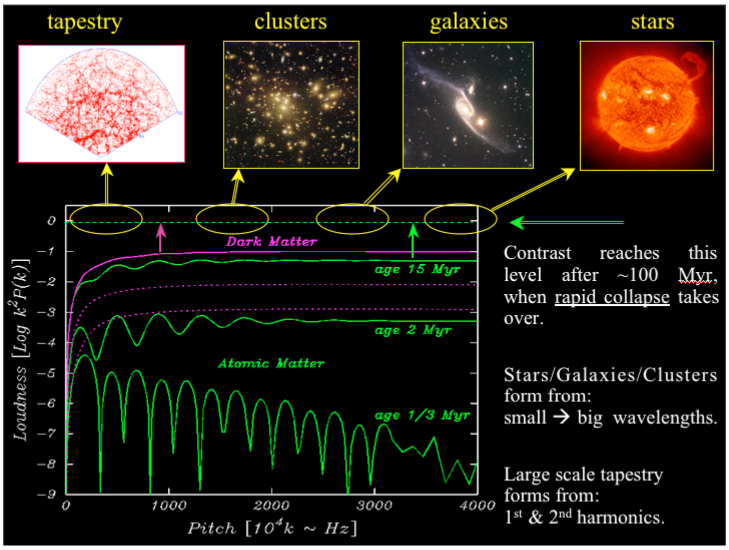
Origin of initial density perturbation?
- Inflation
- Topological defects

Comparison with observation
- Galaxy distribution
- CMB anisotropies

- **Observed facts**
  - The size of initial density perturbation is about 10^{-5}.
  - Cold Dark Matter must be there.

Structure can grows after matter domination. If matter consists solely of baryons, structures can grows only after photon decoupling and there is not enough time for structures to grow.



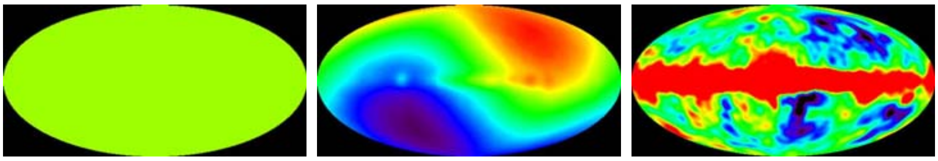


CMB Anisotropies

- Is there any other evidence or probe for primordial density perturbation?
- CMB Anisotropies (CMBA)
  - CMB has **temperature fluctuations of order 10^{-5}**. $\delta T/T \sim 10^{-5}$
- Origin of CMBA – depending on scale
  - Gravitational potential due to **density perturbation of CDM**

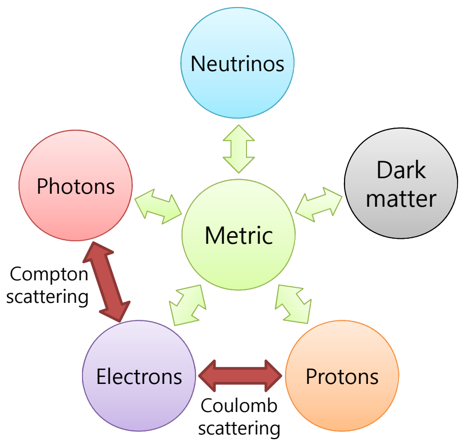
$$\delta\rho \longrightarrow \delta\Phi \longrightarrow \delta T$$

  - **Baryon Acoustic Oscillation**
    - Oscillation of strongly coupled baryon-photon plasma
- Observations of CMBA



2.73 K   1/1,000 K   1/100,000 K (COBE)

Evolution of Inhomogeneities



- Linear and Nonlinear regime
  - Linear regime – linear perturbation eqs.
  - Nonlinear regime – Numerical simulations
- Linear regime – Perturbations
  - Scalar perturbations - 9 perturbation variables
  - Tensor perturbations
- Initial conditions
  - At early time, all modes are super-horizon and all variables depend on the gravitational potential Φ.
  - Types of perturbation

$$\delta = -\frac{3}{2}\Phi + C$$

C=0 : **Adiabatic**

C ≠0 : **Isocurvature**

  - What produce Φ initially?

- Scalar perturbations – Coupled first order differential equations for 9 scalar perturbation variables $\Theta, \Theta_p,\ N,\ \delta, v,\ \delta_b, v_b,\ \Phi, \Psi$
  - Photons Θ, Θ_P
  - Neutrinos N
  - Cold Dark Matter δ, v
  - Baryon δ_b, v_b
  - Metric Φ, Ψ

$$n_{\rm dm}(\vec{x},t) = n_{\rm dm}^{(0)}\left[1+\delta(\vec{x},t)\right] = \int \frac{d^3\vec{p}}{(2\pi)^3} f_{\rm dm}(\vec{x},\vec{p},t)$$

$$\vec{v}(\vec{x},t) = \frac{1}{n_{\rm dm}} \int \frac{d^3\vec{p}}{(2\pi)^3} f_{\rm dm}(\vec{x},\vec{p},t)\frac{\vec{p}}{E}$$

$$f_\gamma(\vec{x},\vec{p},t) = \left[e^{p/T(t)[1+\Theta(\vec{x},\vec{p},t)]} - 1\right]^{-1}$$

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 + \Theta + \mu v_b - \tfrac{1}{2}P_2(\mu)\Pi\right]$$

$$\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}\left[-\Theta_P + \tfrac{1}{2}\left(1 - P_2(\mu)\right)\Pi\right]$$

$$\dot{\delta} + ikv = -3\dot{\Phi}, \quad \dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$$

$$\dot{\delta}_b + ikv_b = -3\dot{\Phi}, \quad \dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}\left[v_b + 3i\Theta\right]$$

$$\dot{N} + ik\mu N = -\dot{\Phi} - ik\mu\Psi$$

$$k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G a^2\left[\rho_{\rm dm}\delta + \rho_b\delta_b + 4\rho_\gamma\Theta_0 + 4\rho_\nu N_0\right]$$

$$k^2(\Phi + \Psi) = -32\pi G a^2\left[\rho_\gamma\Theta_2 + \rho_\nu N_2\right]$$

$\mu \equiv \hat{k}\cdot\hat{p}_\gamma$

$\dot{\tau} \equiv -n_e\,\sigma_T\,a$ optical depth

$\Pi \equiv \Theta_2 - \Theta_{P2} + \Theta_{P0}$

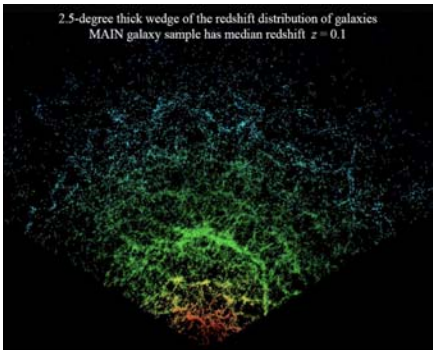
$R \equiv \dfrac{3\rho_b^{(0)}}{4\rho_\gamma^{(0)}}$

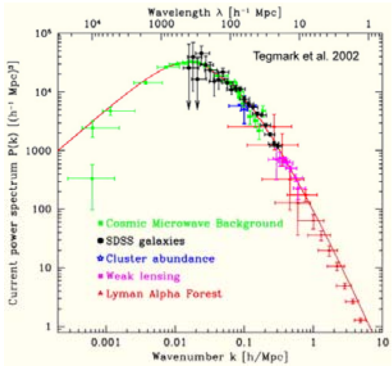
- Gravitational instability – Matter accumulates in initially overdense region.
- Equation governing overdensities in simplified form

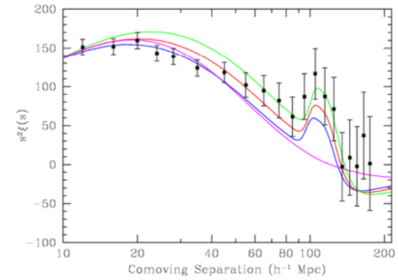
$$\ddot{\delta} + [\ \underbrace{\text{Pressure}}_{\substack{\text{random thermal motion}\\ \text{causing loss, oscillation}}} - \underbrace{\text{Gravity}}_{\substack{\text{increase overdensity}\\ \text{causing exponential growth}}}\ ]\,\delta = 0$$

- 3 stages of evolution
  - Early on – all modes are outside the horizon
  - Intermediate times – the mode fall within the horizon and the universe evolves from RD to MD.
  - Late times – all the modes evolve identically.
- Comparison with observations
  - Galaxy distribution – Matter power spectrum
  - CMB anisotropies – CMB power spectrum

- **Matter power spectrum**

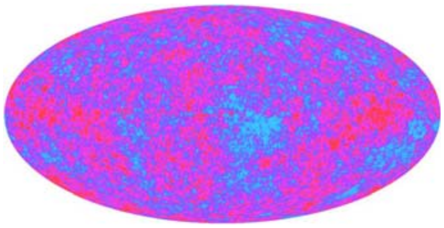


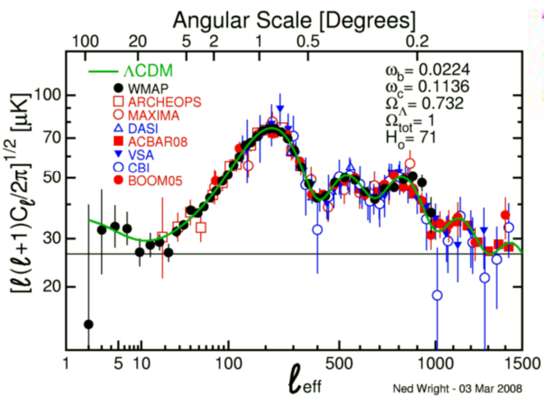


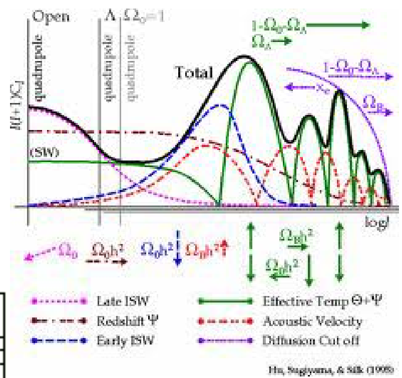


- Dependence on Ω_{DM} (beginning of MD era)
- Effect of BAO

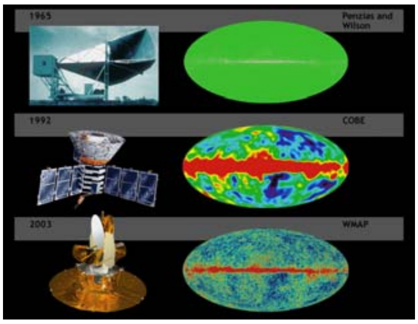
- **CMB Anisotropies**

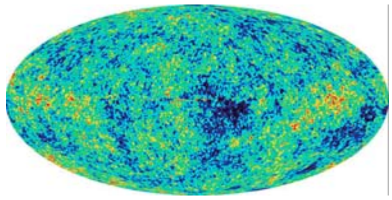


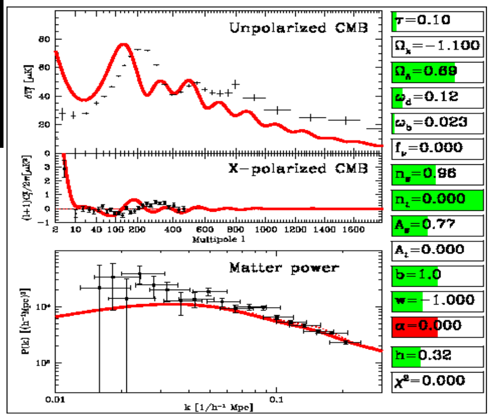




- Cosmic parameters



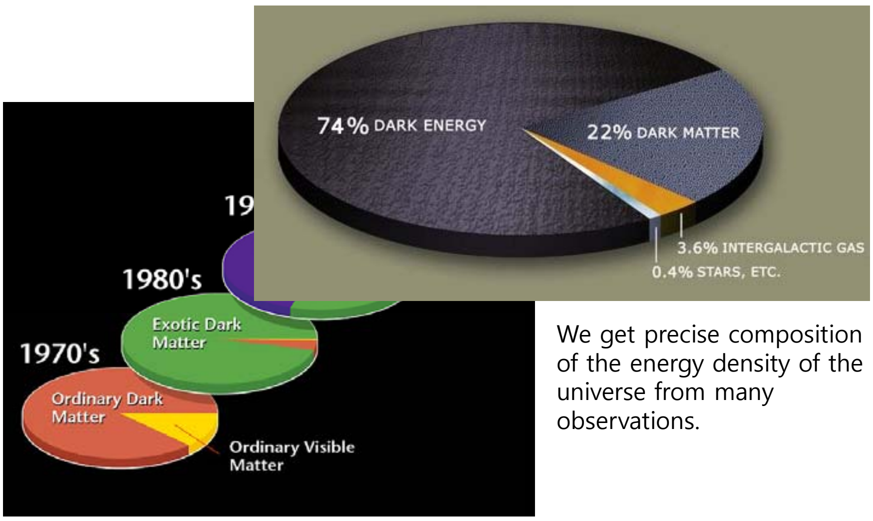




- Discovery of the origin of structures in the universe
- Cosmology became precise science.

CMBA contain much information about our universe.

Cosmic Inventory



We get precise composition of the energy density of the universe from many observations.

How Did Big Bang Begin?

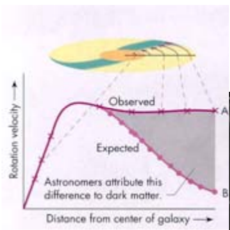
- When and how was **baryon asymmetry** made?
- What is **dark matter** and How was it created?
- What is **dark energy**?
- Why is our universe so flat and so homogenous?
  - The **flatness problem**
  - the **horizon problem**
  - Scale Factor versus Horizon size The current Hubble volume was not causally connected in the past.
- How was the **initial density perturbations** created?
  - Density perturbations at large scales which were not causally connected in the past cannot be created.

All problems are related to the initial state of the big bang universe.

What gives the solution, cosmology or particle physics?

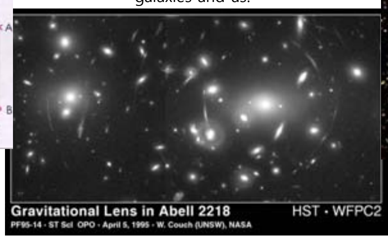
Dark Matter

- To explain the formation of LSS, dark matter is a necessity.
  - The amount of dark matter in our universe : $\Omega_{\mathrm{CDM}} \sim 0.25$
- Evidences of dark matter at various scales
  - Motion of galaxy clusters
  - Rotation curves of galaxies
  - Gravitational lensing, mismatch in baryon and matter distribution



Rotation curve of galaxy shows the existence of dark matter outside the visible disk of galaxy.

Gravitational lensing effect reveals the existence of matter unseen between far galaxies and us.



Colliding clusters : Baryon(red, X-ray) and dark matter (blue, gravitational lensing) reside separately. Galaxies follow the dark matter distribution.

- Required properties of dark matter
  - Darkness – Shedding no light, weak interaction with ordinary particles.
  - Matter – Pressureless at the time of structure formation (MD era)
  - Stability – It must survive until now.
  - Right amount

- Dark matter candidates
  - LSP – Neutralino
  - Axion
  - Gravitino, Axino, LKK, …

- Dark matter search
  - Direct search : DM – Nucleon scattering
  - Indirect search : Annihilation or Decay product

Dark Matter - WIMP

- **Generic WIMP scenario**

heavy particle $X + \bar{X} \longleftrightarrow \ell + \bar{\ell}$ light particle, tightly coupled to cosmic plasma (in equilibrium)

weak interaction

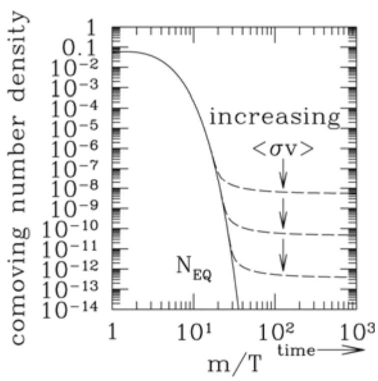
- Boltzmann eq. for X

$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = \langle \sigma v \rangle \left(n_X^{(0)} - n_X^2 \right)$$

$$\Omega_X = \left(\frac{4\pi^3 G g_*(m)}{45} \right)^{1/2} \frac{x_f T_0^3}{30 \langle \sigma v \rangle \rho_c}$$

$$= 0.3 h^{-2} \left(\frac{x_f}{10} \right) \left(\frac{g_*(m)}{100} \right)^{1/2} \frac{10^{-39}\ \mathrm{cm}^2}{\langle \sigma v \rangle}$$

✓ No explicit mass dependence.
✓ Relic abundance is mainly determined by cross section.



Dark Matter - Axion

- **Axion**
  - Strong CP problem – Non-trivial vacuum structure of QCD makes $\theta_s F\tilde{F}$ observable, which breaks CP symmetry. Neutron EDM constrain $\bar{\theta} = \theta_s + \arg\det M_q < 10^{-10}$
  - Spontaneously broken PQ symmetry dynamically relaxes θ to zero.
  - QCD instantons break PQ symmetry and gives the axion (Goldstone boson) a small mass $m_a = 6\,\mu\mathrm{eV}\,(10^{12}\,\mathrm{GeV}/f_a)$.
- **Coherent oscillation of scalar field**
  - massive scalar field in expanding universe : $\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$
  - friction-dominated ($H \gg m$) : $\phi \approx \phi_0$ (constant)
  - mass-dominated ($H \ll m$) : oscillation about the minimum, matter-like (condensate)
- **Relic density of axion**
  - Setting of initial misalignment
  - Relic density

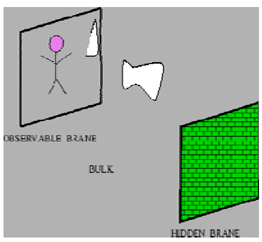
$$\Omega_a h^2 = 0.7\left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{7/6}\left(\frac{\bar{\theta}_i}{\pi}\right)^2$$

Why is the Expansion Accelerating?

- **Option 1 – The energy density is dominated by Dark Energy.**
  - **What is dark energy?**
    - Negative pressure ($w < -\frac{1}{3}$) accelerates the expansion.
    - No interaction with ordinary matter (other than gravity)
  - Candidates for dark energy
    - **Vacuum energy (Cosmological constant)**
    - Slow-rolling scalar field (quintessence)
- **Option 2 – Gravity deviates from GR at scales larger than galaxies.**
  - Ex) DGP model (5D brane-world model)

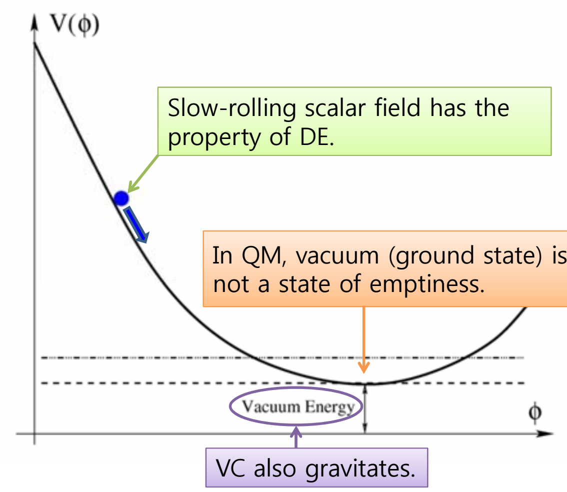
$$S_{\rm DGP} = -\underbrace{\frac{M_5^2}{2}\int d^5x\sqrt{-g}\,R_5}_{\text{5D gravity}} - \underbrace{\frac{M_P^2}{2}\int d^4x\sqrt{-h}\,R_4}_{\text{Magic piece of DGP}} + \underbrace{\int d^4x\sqrt{-h}\,\mathcal{L}_M}_{\text{Matter living on 4D brane}} + S_{\rm GH}$$

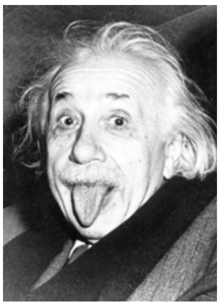
Gives acceleration, but with many other troubles …



What is Dark Energy?

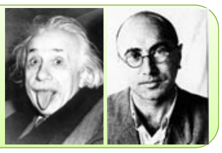
- **Vacuum energy density - Cosmological constant**
  - Vacuum energy density also gravitates.
- **Dynamical model**
  - Assume that V.E is set to 0 by some reason.
  - Slow-rolling scalar field has the property of DE.





Cosmological Constant

- Einstein, 1917 – Introduced the cosmological constant to get a static universe from GR
- Zeldovich, 1968 – Identified the cosmological constant as the vacuum energy density and raised the cosmological constant problem



- **Can we calculate the vacuum energy density?**
  - QFT : VED is the sum of zero-point energy and subject to renormalization.

$$\rho_{vac} = \sum_{\text{all fields}} (-1)^F g_i \underbrace{\int \frac{d^3\vec{k}}{(2\pi)^3}}_{\text{sum over all modes}} \underbrace{\frac{1}{2}\sqrt{k^2+m^2}}_{\text{zero-point energy of each mode}} = \sum_{\text{all fields}} (-1)^F g_i \frac{k_{max}^4}{16\pi^2}$$

Fermions contribute negative.

∞, diverge.

Energy cutoff
The highest energy at which QFT holds

- **Cosmological constant problem**

$\rho_{observed} \sim \rho_{crit} \Rightarrow k_{max} = 0.01\,\mathrm{eV}$

QFT prediction $k_{max} \sim M_P \approx 10^{19}\,\mathrm{GeV}$ ⇒ $\dfrac{\rho_{expected}}{\rho_{observed}} \approx 10^{120}$! most serious naturalness problem

- **Need for Quantum Gravity !**

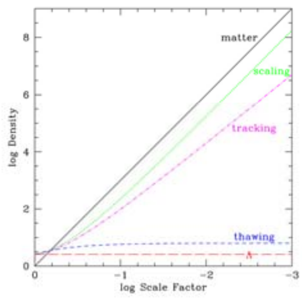
Slow-Rolling Scalar

- Dynamical model for dark energy
- Dynamics of homogeneous scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \Rightarrow \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\frac{\dot{\phi}^2}{V} \ll 1 \Rightarrow w \approx -1 \Leftarrow w(\phi) = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$



- Merits of slow-rolling scalar field
  - Eq. of state that varies in time
  - Possibility to explain the present ratio of D.E. and D.M. densities.

- Troubles of slow-rolling scalar field
  - The question why V.E.D is zero is still remained.
  - The mass of scalar is extremely small. $m_\phi = V''(\phi)^{1/2} \sim 3H_0 \approx 10^{-33}\,\mathrm{eV}$

Flatness Problem

- Physical radius of curvature $R_{\text{cur}} = \dfrac{H^{-1}}{|\Omega - 1|^{1/2}}$
- Ω as a function of a

$$\Omega - 1 = \frac{k}{H^2 a^2} \propto \frac{1}{\rho a^2} \propto \begin{cases} a, & \text{MD} \\ a^2, & \text{RD} \end{cases}$$

$$t = t_{\text{BBN}} \sim 1\,\text{s} \;:\; |\Omega - 1| \lesssim 10^{-16},\; R_{\text{cur}} \gtrsim 10^{8} H^{-1}$$

$$t = t_{\text{P}} \sim 10^{-43}\,\text{s} \;:\; |\Omega - 1| \lesssim 10^{-60},\; R_{\text{cur}} \gtrsim 10^{30} H^{-1}$$

- Big bang universe requires **a very special initial condition**.
- If $\Omega \sim 1$ and $R_{\text{cur}} \sim H^{-1}$ at Planck time,
  $k > 0$: the universe re-collapse within few $\times 10^{-43}\,\text{s}$
  $k < 0$: temperature 3K reached at $t \approx 10^{-11}\,\text{s}$
  The natural time scale for cosmology ~ $10^{-43}\,\text{s}$
  The age of our universe ~ $10^{60} \times 10^{-43}\,\text{s}$

Horizon Problem

- Comoving horizon grows during RD and MD eras.
  This means that the particle horizon grows faster than the scale factor.

$$\eta = \int_0^t \frac{dt'}{a(t')} \propto \begin{cases} a^{1/2}, & \text{MD} \\ a, & \text{RD} \end{cases} \qquad a \propto \begin{cases} t^{2/3}, & \text{MD} \\ t^{1/2}, & \text{RD} \end{cases} \qquad d_H \propto t$$

- **Horizon problem**
  - **Large-Scale Smoothness Problem**
    CMB we see today is very close to isotropy ($\delta T/T \approx 10^{-5}$).
    How can this be? – The largest scales observed today have entered the horizon just recently, long after decoupling. – Microscopic causal physics cannot make it !
  - **Small-Scale Inhomogeneity Problem**
    Where does the density perturbation ($\delta\rho/\rho \approx \delta T/T \approx 10^{-5}$) come from?
    E.g. $(\delta\rho/\rho)_{\lambda_{\text{galaxy}}}$ - This scale was outside the horizon in the past.

- The entropy within a horizon volume

$$S_{\rm H} = s \cdot \frac{4\pi}{3} d_{\rm H}^3 \approx \begin{cases} 0.05 g_*^{-1/2} (M_P/T)^3, & \text{RD} \\ 3 \times 10^{87} (\Omega_0 h^2)^{-3/2} (1+z)^{-3/2}, & \text{MD} \end{cases}$$

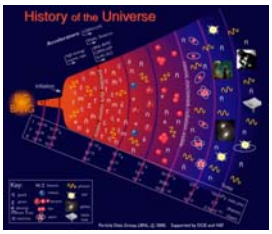
$$S_{\rm H}(t = t_0) = 10^{88} \ \leftarrow \ 10^5 \text{ Hubble volumes at recombination}$$

$$S_{\rm H}(t = t_{\rm rec}) = 10^{83}$$

- Monopole problem
  - Phase transition in the early universe can leave topological defects.
  - Among topological defects, string is not harmful, but domain walls and monopoles can over-close the universe.
  - Many GUTs predict the existence of magnetic monopoles, which must be avoided in cosmology.

Inflation

- What is inflation?
  - Period during which the scale factor grows rapidly (faster than the horizon)
  - Inflation can make the universe flat and homogeneous.
  - If the scale factor grows by more than e^{60}, the universe we see today were causally connected.
- How does inflation occur?
  - Big vacuum energy
  - Slow rolling scalar field along flat potential
- Inflation generates initial density perturbation. (Inflation can explains our existence.)
  - **Quatum fluctuations $\delta\phi$**
    → density perturbations $\delta\rho$
- Transition from inflation to Hot Big Bang
  - (re)heating process
- How did inflation begin?
  - Endless questions again.... ?



▪ **Basic ideas**
- An early epoch of rapid expansion solves the horizon problem, etc.
- For this, negative pressure is required.
- Negative pressure can be realized in a scalar field theory.

▪ **How to solve the horizon problem**
- Comoving horizon

$$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{a'} \frac{1}{a'H(a')}$$

Comoving Hubble radius H^{-1}/a
= the distance over which particles can travel in one expansion time

$r > \eta$ never have communicated

$r > H^{-1}/a$ cannot communicate now

- It is possible to have $\eta \gg H^{-1}/a|_{t_0}$: $H^{-1}/a|_{\text{early}} \gg H^{-1}/a|_{\text{now}}$

That is, η get contribution mostly from early epoch.
In RD or MD, 1/aH increase with time, so the latter epoch contributions dominate.

- In the early epoch, the comoving Hubble radius decreased.
1/aH must decrease ⇒ aH must increase.

$$\frac{d}{dt}(aH) = \frac{d^2}{dt^2}a > 0$$ accelerating expansion, **inflation**

- Quantitative understanding
Suppose the energy scale of inflation ~ 10^{15} GeV.

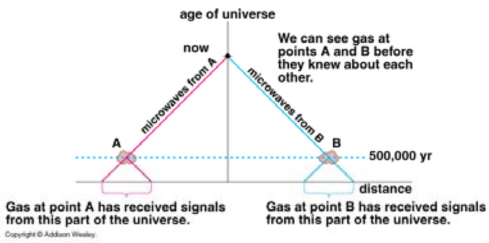
$$(aH)^{-1}|_{T\approx 10^{15}\,\text{GeV}} = 10^{-28}\,(aH)^{-1}|_{T=T_0}$$

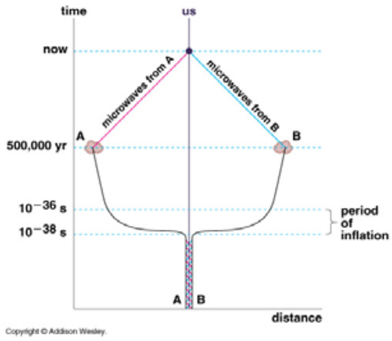
During inflation, the comoving Hubble radius had to decrease by, at least, 28 orders of magnitude.

Most common way to construct a model – H ~ constant

$$H = \frac{\dot{a}}{a} = \text{const.} \;\Rightarrow\; a(t) = a_e e^{H(t-t_e)}$$

$$(aH)^{-1} \propto e^{-Ht}, \quad 10^{28} \approx e^{64}$$ More than 60 e-folds are needed.





- **Negative pressure** is required for acceleration.

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}(\rho + 3p) > 0 \quad \Rightarrow \quad p < -\frac{1}{3}\rho < 0$$

- **Implementation** using a scalar field $\phi(x^\mu)$

Lagrangian $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

Energy-momentum tensor

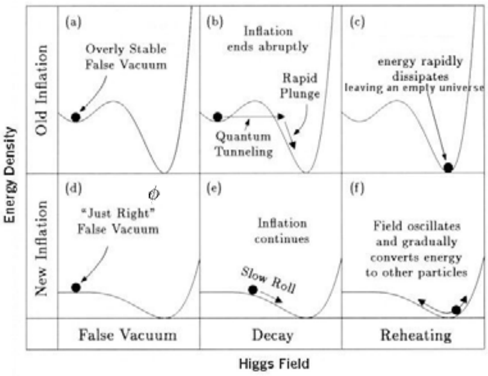
$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L} \quad \Rightarrow \quad \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Negative pressure : potential energy dominates over kinetic energy.

$$V(\phi) > \frac{1}{2}\dot{\phi}^2$$

- Scalar field trapped in a false vacuum
- Scalar field slow-rolling toward its true vacuum

- **False vacuum models**
  - On-set : Thermal phase transition
  - Exit : Decay to true vacuum via bubble nucleation
  - Graceful exit problem



- **Slow-roll models**
  - Evolution of the universe dominated by a homogeneous scalar field

$$H^2 = \frac{1}{3M_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

  - Slow-rolling : dynamics dominated by the friction, $\ddot{\phi} \ll 3H\dot{\phi}$, $H \approx \text{constant (slow-varying)}$
  - Consistency requires two **slow-roll parameters** are small.

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = M_P^2\left(\frac{V''}{V}\right) \qquad\qquad \epsilon \equiv \frac{d}{dt}H^{-1}, \quad \eta = \frac{\ddot{\phi}}{H\dot{\phi}}$$

  - On-set : ??
  - Exit : Break-down of slow-roll condition

- **Gravitational wave (tensor perturbation) production**
  - Tensor perturbations (h_+, $h_\times$) satisfy the linear equation $\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} + k^2 h = 0$
  - Quantization of tensor perturbations
    Introduce the field having mass dimension $\tilde{h} = \frac{M_P}{\sqrt{2}}a h$, $\ddot{\tilde{h}} + \left(k^2 - \frac{\ddot{a}}{a}\right)\tilde{h} = 0$

$$\hat{\tilde{h}}(\vec{k},\eta) = v(k,\eta)\,\hat{a}_{\vec{k}} + v^*(k,\eta)\,\hat{a}^\dagger_{\vec{k}}, \quad \ddot{v} + \left(k^2 - \frac{\ddot{a}}{a}\right)v = 0, \quad [\hat{a}_{\vec{k}}, \hat{a}^\dagger_{\vec{k}'}] = \delta(\vec{k} - \vec{k}')$$

$$\langle \hat{\tilde{h}}^\dagger(\vec{k},\eta)\hat{\tilde{h}}(\vec{k}',\eta)\rangle = |v(k,\eta)|^2\,(2\pi)^3\delta^3(\vec{k} - \vec{k}')$$

vacuum fluctuation $\langle \hat{h}^\dagger(\vec{k},\eta)\hat{h}(\vec{k}',\eta)\rangle = \frac{2}{M_P^2 a^2}\,|v(k,\eta)|^2\,(2\pi)^3\delta^3(\vec{k} - \vec{k}')$

power spectrum $P_h(k) = \frac{2}{M_P^2 a^2}\,|v(k,\eta)|^2$

  - During inflation : $\frac{\ddot{a}}{a} \approx \frac{2}{\eta^2}$ $\qquad \ddot{v} + \left(k^2 - \frac{2}{\eta^2}\right)v = 0$ $\qquad v = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)$
  - After horizon crossing ($k\eta \sim 1$, or $aH \sim k$)
    Power spectrum approaches to constant

$$P_h(k) = \left.\frac{H^2}{M_P^2 k^3}\right|_{aH\sim k}$$

For $k\eta \gg 1$ (sub-horizon), $h \propto \frac{v}{a} \propto \frac{1}{a}$
For $k\eta \ll 1$ (super-horizon), $h \propto \frac{v}{a} \propto H$

  - Inflation produces gravitons (gravitational waves).

- **Generation of density perturbations**
  - Quantum fluctuation of the inflaton $\phi(\vec{x},t) = \phi^{(0)}(t) + \delta\phi(\vec{x},t)$
    satisfy the linear equation
    - same as gravitational wave $\delta\ddot{\phi} + 2aH\delta\dot{\phi} + (k^2 + a^2V'')\delta\phi = 0$

    negligible during slow-roll inflation
  - Power spectrum

$$P_{\delta\phi}(k) = \frac{H^2}{2k^3}\bigg|_{aH\sim k} \qquad \text{cf.} \quad P_h(k) = \frac{H^2}{M_P^2 k^3}\bigg|_{aH\sim k}$$

  - Perturbation spectrum of Ψ
    - Why are we justified in neglecting Ψ until horizon crossing?
    - How do the perturbations get transferred from $\delta\phi$ to Ψ ?

  - Curvature perturbation – conserved for super-horizon mode $\zeta \equiv -\frac{ik_i \delta T^0{}_i H}{k^2(\rho+p)} - \Psi$
    - For sub-horizon and just-left-horizon modes, Ψ is negligible.
    - Post-inflation, perturbation shared between $\delta T^0{}_i$ and Ψ .

$$\zeta|_{\text{horizon crossing}} = -\frac{aH\delta\phi}{\dot{\phi}^{(0)}}, \quad \zeta|_{\text{post inflation}} = -\frac{3}{2}\Psi \quad \Rightarrow \quad \Psi|_{\text{post inflation}} = \frac{2}{3}aH\frac{\delta\phi}{\dot{\phi}^{(0)}}\bigg|_{\text{horizon crossing}}$$

$$P_\Psi = \frac{4}{9}\left(\frac{aH}{\dot{\phi}^{(0)}}\right)^2 P_{\delta\phi}\bigg|_{aH=k} = \frac{2}{9k^3}\left(\frac{aH^2}{\dot{\phi}^{(0)}}\right)^2\bigg|_{aH=k} = \frac{H^2}{9\epsilon M_P^2 k^3}\bigg|_{aH=k} = \frac{16\pi}{9M_P^2 k^3}\left(\frac{H^2V^2}{V'^2}\right)\bigg|_{aH=k}$$

- **Density perturbation in slow-roll inflation**
  - Slow-roll parameters : $\epsilon = \frac{M_P^2 V'^2}{2V^2}, \quad \eta = \frac{M_P^2 V''}{V}, \quad \xi = \frac{M_P^4 V' V'''}{V^2}$
  - Spectral index : $n_s - 1 = 2\eta - 6\epsilon$
  - Running of spectral index : $n_s' \equiv dn_s/d\ln k = 16\epsilon\eta - 24\epsilon^2 - 2\xi$
  - Tensor to scalar ratio : $r = 16\epsilon$
  - Number of e-folds : $N(\phi) = \int_t^{t_{\text{end}}} H\,dt = \frac{1}{M_P^2}\int_{\phi_{\text{end}}}^{\phi} \frac{V(\phi)}{V'(\phi)}d\phi$
- **Chaotic inflation**
  - Single scalar field with a potential $V(\phi) = \Lambda^4\left(\frac{\phi}{M_P}\right)^p$
  - Slow-roll (condition) is achieved for $\phi \gtrsim \phi_{\text{end}} \sim M_P$ $\quad \epsilon = \frac{p}{4N_*}, \ \eta = \frac{p-1}{2N_*}$
    (trans-Planckian field value)
  - Primordial density perturbation requires $\frac{\Lambda}{M_P} \approx 10^{-2}\epsilon^{1/4}$
    (fine tuning of parameter)
  - Spectral index and its running : $n_s - 1 = -\frac{2+p}{2N_*} \qquad n_s' = -\frac{2+p}{2N_*^2}$
  - Tensor to scalar ratio : $r = \frac{4p}{N_*}$
  - Setting of initial condition for inflation – thermal fluctuation?

Exercise 7

Verify the following results of chaotic inflation.

- **Chaotic inflation**
  - Single scalar field with a potential $V(\phi) = \Lambda^4 \left(\frac{\phi}{M_P}\right)^p$
  - Slow-roll (condition) is achieved for $\phi \gtrsim \phi_{\rm end} \sim M_P$ (trans-Planckian field value) $\quad \epsilon = \frac{p}{4N_*},\ \eta = \frac{p-1}{2N_*}$
  - Primordial density perturbation requires $\frac{\Lambda}{M_P} \approx 10^{-2}\epsilon^{1/4}$ (fine tuning of parameter)
  - Spectral index and its running : $n_s - 1 = -\frac{2+p}{2N_*} \qquad n_s' = -\frac{2+p}{2N_*^2}$
  - Tensor to scalar ratio : $r = \frac{4p}{N_*}$

- **Higgs inflation**
  - Higgs field **non-minimally coupled to gravity**

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R - \xi H^\dagger H R + \mathcal{L}_{\rm SM}\right) \quad \Rightarrow \quad \int d^4x\sqrt{-g}\left(-\frac{M_P^2+\xi\phi^2}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda}{4}\phi^4\right)$$

  - Jordan frame ⇒ Einstein frame by conformal transformation $\quad g_{\mu\nu} = \Omega^{-2}\tilde{g}_{\mu\nu},\quad \Omega = 1 + \xi\phi^2/M_P^2$

$$S = \int d^4x\sqrt{-\tilde{g}}\left(-\frac{M_P^2}{2}\tilde{R} + \frac{1+(6\xi+2)\xi\phi^2/M_P^2}{2(1+\xi\phi^2/M_P^2)^2}(\partial_\mu\phi)^2 - \frac{\lambda\phi^4}{4(1+\xi\phi^2/M_P^2)^2}\right)$$

  - Canonical kinetic term by field redefinition $\quad \frac{d\chi}{d\phi} = \frac{\sqrt{1+(6\xi+1)\xi\phi^2/M_P^2}}{1+\xi\phi^2/M_P^2}$

$$S = \int d^4x\sqrt{-\tilde{g}}\left(-\frac{M_P^2}{2}\tilde{R} + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{\lambda\phi(\chi)^4}{4(1+\xi\phi(\chi)^2/M_P^2)^2}\right)$$

  - Slow-roll condition is satisfied for $\phi > \phi_e \approx M_P/\sqrt{\xi}$
  - Primordial density perturbation requires $\xi \approx 47{,}000\sqrt{\lambda}$

- R^2 **(curvature-squared) inflation – Starobinsky model**
  - Einstein gravity + **R^2 term**

$$S = \int d^4x \sqrt{-g} \left(-\frac{M_P^2}{2} R + \alpha R^2 \right)$$

  - Does quantum gravity induce more derivatives?
  - R^2 term introduces an additional degree of freedom, because it contains more derivatives
  - Conformal transformation : $\tilde{g}_{\mu\nu} = [1 + 2\alpha R] g_{\mu\nu}$
  - Field redefinition : $\psi = \sqrt{3/2} \ln(1 + 2\alpha R)$

$$S = \int d^4x \sqrt{-\tilde{g}} \left(-\frac{M_P^2}{2} \tilde{R} + (\tilde{\partial}\psi)^2 - \frac{1}{4\alpha}(1 - e^{\sqrt{2/3}\psi})^2 \right)$$

  - Slow-roll condition is satisfied for …
  - Primordial density perturbation requires …

- **(Re)Heating, or Thermalization**
  - Inflation cools down the universe, almost to temperature zero.
  - After the end of inflation, hot thermal radiations needs to be produced, starting the hot big bang universe.
  - Because the nature of the inflation is not known, this process is still poorly understood.
  - Energy source of – Large potential energy of the inflaton field
  - **Inflaton decay** Inflaton decay into relativisitic (standard model) particles during it oscillates around the potential minimum.

$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} + m^2\phi = 0$ During oscillation, $\langle \dot{\phi}^2 \rangle = \langle m\phi^2 \rangle$, pressureless matter

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi \quad \dot{\rho}_R + 4H\rho_R = \Gamma_\phi \rho_\phi \quad H^2 = \frac{1}{3M_P^2}(\rho_\phi + \rho_R) \quad T_R \approx 0.2\sqrt{M_P \Gamma_\phi}$$

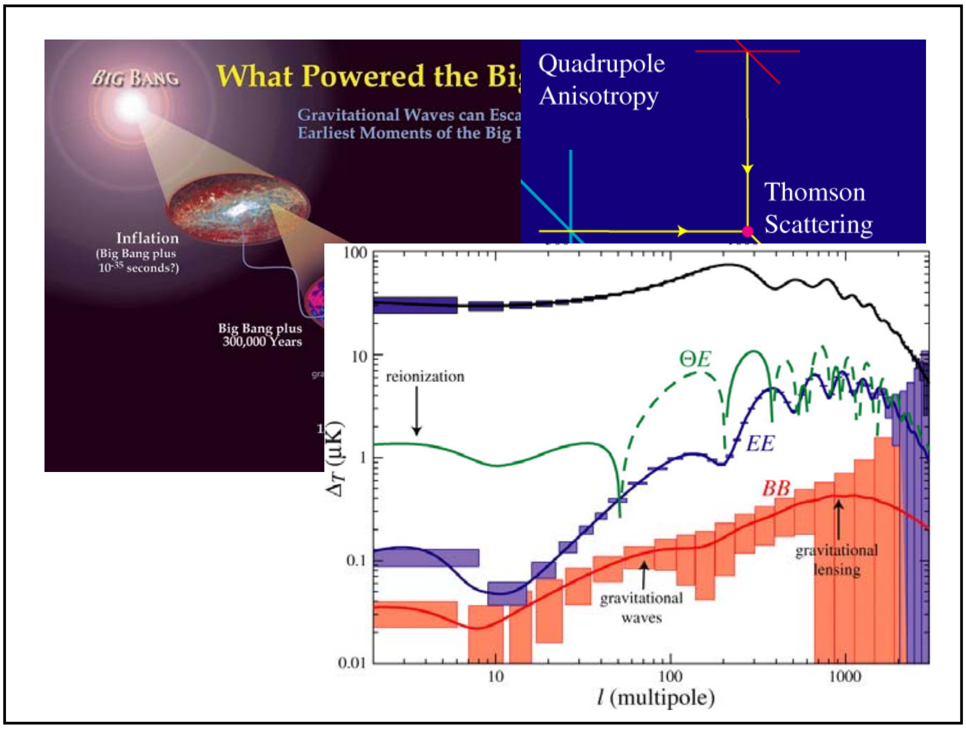
  - **Parametric resonance** Particles can be produced more efficiently through parametric resonance.

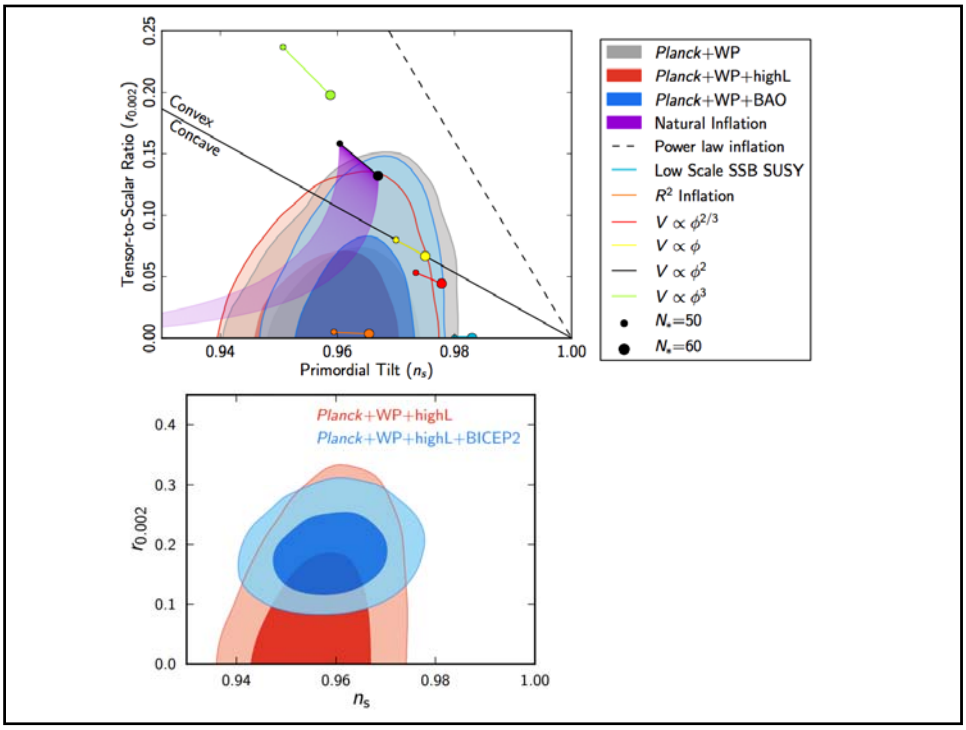
$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + g^2\phi^2 \right) \chi_k = 0$$

Oscillation of the inflaton field
may cause parametric resonance.

Non-Gaussian Perturbation

CMB Polarization

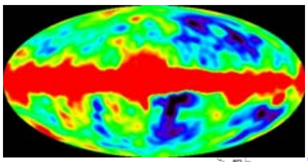

BIG BANG
What Powered the Bi
Gravitational Waves can Esca
Earliest Moments of the Big
Inflation
(Big Bang plus
10^{-35} seconds?)
Big Bang plus
300,000 Years
Quadrupole
Anisotropy
Thomson
Scattering
ΘE
EE
BB
reionization
gravitational
waves
gravitational
lensing
Δ_T (μK)
l (multipole)

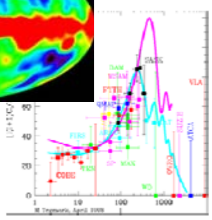

Tensor-to-Scalar Ratio ($r_{0.002}$)
Primordial Tilt (n_s)
Convex
Concave
Planck+WP
Planck+WP+highL
Planck+WP+BAO
Natural Inflation
Power law inflation
Low Scale SSB SUSY
R^2 Inflation
$V \propto \phi^{2/3}$
$V \propto \phi$
$V \propto \phi^2$
$V \propto \phi^3$
N_*=50
N_*=60
Planck+WP+highL
Planck+WP+highL+BICEP2
$r_{0.002}$
n_s

Multiverse

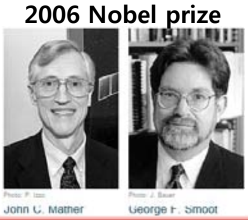
Great Discoveries in 1990s

CMB Anisotropies

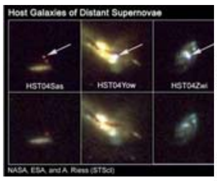


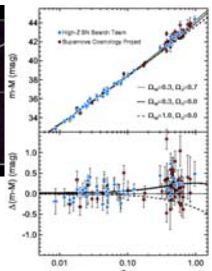


2006 Nobel prize

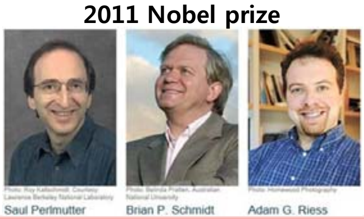


Accelerating Universe

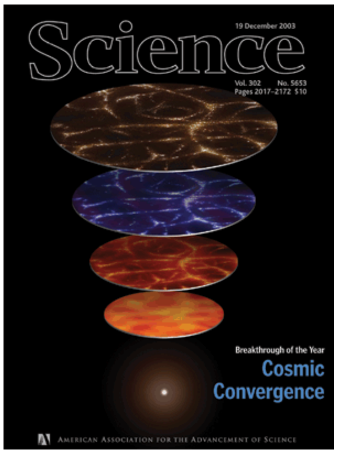




2011 Nobel prize

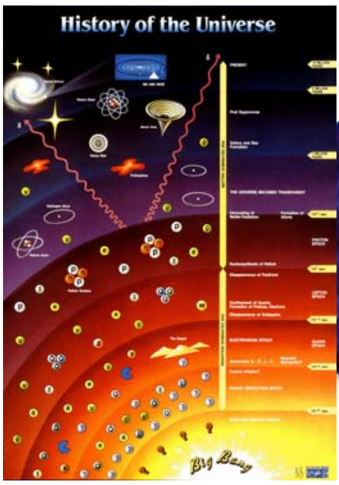


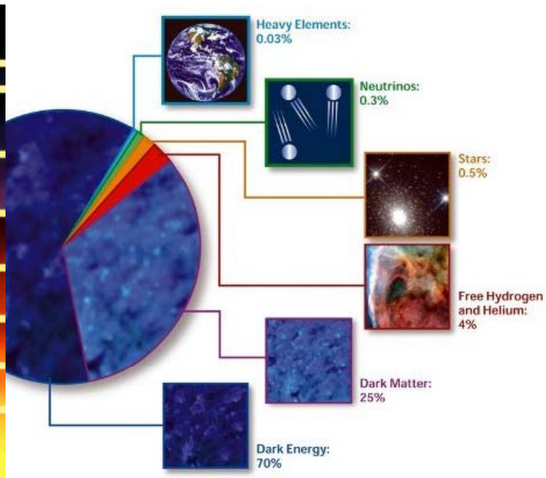
- SCIENCE Breakthrough of the Year 1998 – The Accelerating Universe
- SCIENCE Breakthrough of the Year 2003 – Cosmic Convergence



Opened a new horizon in understanding our universe.

Understanding the Universe

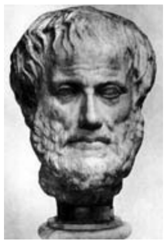




It seems that we understand our universe rather well …

But, we don't know what are really 95% of the matter filling our universe

Epilogue




The eternal universe
I have experienced eternity
I am an insignificant part of it all.
But each part flares, dies and lives again
Adding its small part to the whole.
There is beauty and complexity,
Pulsing eternity.
We are all a part,
We 'are' God.
But it's so hard to let go,
To believe it,
It's so wonderful,
Inexplicable.
The doors to heaven have been opened.
I 'am'.
Alan Potter 2010
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