Quantum Scalar Corrections to the Gravitational Potentials on de Sitter Background:

(How MMC scalars and gravitons affect gravity during inflation?)

Sohyun Park

arXiv: 1007.2662, 1101.5804, 1109.4187 SP and Woodard
arXiv: 1403.0896 Leonard, SP Prokopec and Woodard
arXiv: 1510.03352 SP, Prokopec and Woodard

Korea Astronomy and Space Science Institute (KASI)

CosPA 2015, Daejeon, Korea

Outline

- Motivation: Why we consider MMC (massless, minimally coupled) scalars and gravitons during inflation
- Message: How quantum fluctuations during inflation affect gravity:
  - dynamical gravitons and the gravitational force law
- A two-step procedure to study this effect:
  1. Compute and renormalize the graviton self-energy $-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ from a MMC scalar on de Sitter background;
  2. Use the graviton self-energy to quantum correct the linearized Einstein field equation and solve it.

$$\mathcal{D}^{\mu\nu\rho\sigma}\kappa h_{\rho\sigma}(x) - \int d^4x' \left[^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')\kappa h_{\rho\sigma}(x') = \mathcal{T}^{\mu\nu}_{\text{lin}}(x)$$

- Summary and Discussion

Why MMC scalars and gravitons during Inflation

- A generic prediction of inflation: particle production

  "The highly accelerated expansion causes quantum fluctuations to rip out of the vacuum and become real particles.

  These thought be the seeds of the large scale structure of the universe."

- Expansion of spacetime can lead to particle creation by delaying the annihilation of virtual pairs ripped out of the vacuum:

  Schrödinger, Physica 6 (1939) 899

- The effect is maximized if
  - the expansion is accelerated:
    (the cosmological patch of) de Sitter $a(t) = e^{Ht}$, $H =$ const. $\epsilon = 0$
    Quantum effects during inflation $\rightarrow$ Quantum field theory in de Sitter
  - the virtual particles are massless
    The persistent time gets longer if their mass is smaller, so massless particles live the longest
  - no conformal symmetry.
    The emergence rate of conformally invariant particles is suppressed as $1/a$.

  Parker 1968 - 1971

Why MMC scalars and gravitons during Inflation (continued)

- Only two types of particles with no mass & no conformal symmetry:
  - gravitons
  - massless, minimally coupled (MMC) scalar
- Occupation number of these two particles grows with time:

  $N(k,t) = \left(\frac{Ha(t)}{2k}\right)^2$ significant for infrared wave numbers $k \leq Ha$
- One consequence of this being infrared effect:

  Perturbative general relativity can be used reliably as a low-energy effective field theory, even though it is not renormalizable.

  "General Relativity as an effective field theory: The leading quantum corrections",

  John F. Donughue gr-qc/9405057

Overview: Effect of inflationary scalars on gravitons and force of gravity

- $\mathcal{L} = \frac{1}{16\pi G}\Big[R - 2\Lambda\Big]\sqrt{-g} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g}$
- Linearize: $g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa h_{\mu\nu}, \quad \overline{g}_{\mu\nu} = a^2\eta_{\mu\nu}, \kappa^2 = 16\pi G, \quad a = -\frac{1}{H\eta}$
  (the open conformal patch of de Sitter space)
- Linearized quantum effective field equations for gravitons:

$$\mathcal{D}^{\mu\nu\rho\sigma}\kappa h_{\rho\sigma}(x) - \int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')\kappa h_{\rho\sigma}(x') = \mathcal{T}^{\mu\nu}_{\rm lin}(x)$$

- Lichnerowicz operator: linearized Einstein tensor, $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}(R - 2\Lambda)$ acting on $h_{\mu\nu}$
  $\mathcal{D}^{\mu\nu\rho\sigma} =$
  $D^{(\rho}\overline{g}^{\sigma)(\mu}D^{\nu)} - \frac{1}{2}\Big[\overline{g}^{\rho\sigma}D^\mu D^\nu + \overline{g}^{\mu\nu}D^\rho D^\sigma\Big] + \frac{1}{2}\Big[\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\Big]D^2 + (D-1)\Big[\frac{1}{2}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\Big]H^2$
- $-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x')$: graviton self-energy = 1PI graviton 2-point function
  : quantum correction to the Lichnerowicz operator
  the self-energy of any particle: the quantum correction to that particle's kinetic operator
  e.g. For photons, the vacuum polarization: quantum correction to Maxwell's eqns

1. Calculate the graviton self-energy and renormalize so as to be integrable
2. Solve the quantum corrected field eqn for dynamical gravitons and the force of gravity

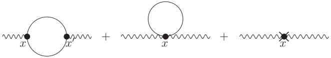
One Loop Scalar Contributions to Graviton Self-Energy

- Interaction between MMC scalars and gravitons:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} \\ &= -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi\overline{g}^{\mu\nu}\sqrt{-\overline{g}} - \frac{\kappa}{2}\partial_\mu\varphi\partial_\nu\varphi\Big(\frac{1}{2}h\overline{g}^{\mu\nu} - h^{\mu\nu}\Big)\sqrt{-\overline{g}} \\ &\quad -\frac{\kappa^2}{2}\partial_\mu\varphi\partial_\nu\varphi\bigg\{\Big[\frac{1}{8}h^2 - \frac{1}{4}h^{\rho\sigma}h_{\rho\sigma}\Big]\overline{g}^{\mu\nu} - \frac{1}{2}hh^{\mu\nu} + h^\mu_{\ \rho}h^{\rho\nu}\bigg\}\sqrt{-\overline{g}} + O(\kappa^3)\ .\end{aligned}$$

- One loop contribution to the graviton self-energy from MMC scalars on de Sitter background:

$$\begin{aligned}-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') &= \frac{1}{2}\sum_{I=1}^{2}T_I^{\mu\nu\alpha\beta}(x)\sum_{J=1}^{2}T_J^{\rho\sigma\gamma\delta}(x')\times\partial_\alpha\partial'_\gamma i\triangle(x;x')\times\partial_\beta\partial'_\delta i\triangle(x;x') \\ &\quad + \frac{1}{2}\sum_{I=1}^{4}F_I^{\mu\nu\rho\sigma\alpha\beta}(x)\times\partial_\alpha\partial'_\beta i\triangle(x;x')\times\delta^D(x-x') \\ &\quad + 2\sum_{I=1}^{2}C_I^{\mu\nu\rho\sigma}(x)\times\delta^D(x-x')\ .\end{aligned}$$



MMC scalar propagator

- The MMC scalar propagator obeys $\partial_\mu\Big[\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Big] i\triangle(x;x') = i\delta^D(x-x')$
- No de Sitter invariant solution for the propagator
- A solution preserving the homogeneity and isotropy:

$$\begin{aligned} i\triangle(x;x') &= A\Big(y(x;x')\Big) + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\ln(aa') \\ &= \text{de Sitter invariant function of } y + \text{de Sitter breaking term} \end{aligned}$$

$$\text{where}\quad A(y) \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-2} - \pi\cot\Big(\frac{\pi D}{2}\Big)\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$$
$$+\sum_{n=1}^{\infty}\Bigg[\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^{n} - \frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2}\Bigg]\Bigg\}.$$

- The de Sitter breaking term drops differentiated by $\partial_\alpha\partial'_\beta$
- $y(x;x') \equiv aa'H^2\Delta x^2$: the de Sitter invariant length function
  $Z = \cos(\mu) = 1 - \frac{y}{2}$: Fröb and Verdaguer

  μ: the geodesic distance

Primitive Diagrams

- Contribution from 4-point vertices

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{4\text{pt}}(x;x') \equiv \frac{1}{2}\sum_{I=1}^{4} F_I^{\mu\nu\rho\sigma\alpha\beta}(x) \times \partial_\alpha \partial'_\beta i\triangle(x;x') \times \delta^D(x-x')$$

$$= \Big(\frac{D-4}{4}\Big)\frac{i\kappa^2 H^D}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D)}{\Gamma(\frac{D}{2}+1)}\sqrt{-\overline{g}}\left\{\frac{1}{2}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\right\}\delta^D(x-x') = 0 \text{ for } D=4$$

- Contribution from 3-point vertices

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{3\text{pt}}(x;x') = \sqrt{-\overline{g}}\sqrt{-\overline{g}'}\left\{\frac{\partial^2 y}{\partial x_\mu \partial x'_{(\rho}}\frac{\partial^2 y}{\partial x'_{\sigma)}\partial x_\nu} \times \alpha(y) + \frac{\partial y}{\partial x_{(\mu}}\frac{\partial^2 y}{\partial x_{\nu)}\partial x'_{(\rho}}\frac{\partial y}{\partial x'_{\sigma)}} \times \beta(y)\right.$$

$$\left. + \frac{\partial y}{\partial x_\mu}\frac{\partial y}{\partial x_\nu}\frac{\partial y}{\partial x'_\rho}\frac{\partial y}{\partial x'_\sigma} \times \gamma(y) + \overline{g}^{\mu\nu}\overline{g}'^{\rho\sigma}H^4 \times \delta(y) + \left[\overline{g}^{\mu\nu}\frac{\partial y}{\partial x'_\rho}\frac{\partial y}{\partial x'_\sigma} + \frac{\partial y}{\partial x_\mu}\frac{\partial y}{\partial x_\nu}\overline{g}'^{\rho\sigma}\right]H^2 \times \epsilon(y)\right\}$$

$$\propto \frac{1}{y^4} \sim \frac{1}{\Delta x^8} \text{ in } D=4 \quad \longrightarrow \int d^4x' \frac{1}{\Delta x^8}$$ quartically divergent

Correspondence with flat space limit

- Flat space limit $H \to 0$:
  $\Delta x^0 \longrightarrow t - t'$, $\quad y(x;x') \longrightarrow H^2 \Delta x^2$
  $\frac{\partial y}{\partial x_\mu} \longrightarrow 2H^2 \Delta x^\mu$, $\frac{\partial y}{\partial x'_\nu} \longrightarrow -2H^2 \Delta x^\nu$, $\frac{\partial y^2}{\partial x_\mu \partial x'_\nu} \longrightarrow -2H^2 \eta^{\mu\nu}$ gives

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\rm flat}(x;x') = \frac{\kappa^2 \Gamma^2(\frac{D}{2})}{16\pi^D} \Bigg\{ \eta^{\mu(\rho}\eta^{\sigma)\nu} \times \Big[-\frac{2}{\Delta x^{2D}}\Big] + \Delta x^{(\mu}\eta^{\nu)(\rho}\Delta x^{\sigma)} \times \Big[\frac{4D}{\Delta x^{2D+2}}\Big]$$
$$+ \Delta x^\mu \Delta x^\nu \Delta x^\rho \Delta x^\sigma \times \Big[-\frac{2D^2}{\Delta x^{2D+4}}\Big] + \eta^{\mu\nu}\eta^{\rho\sigma} \times \Big[-\frac{1}{2}\frac{(D^2-D-4)}{\Delta x^{2D}}\Big]$$
$$+ \Big[\eta^{\mu\nu}\Delta x^\rho \Delta x^\sigma + \Delta x^\mu \Delta x^\nu \eta^{\rho\sigma}\Big] \times \Big[\frac{D(D-2)}{\Delta x^{2D+2}}\Big] \Bigg\}$$

- This agrees with G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré **XX** (1974) 69.

Correspondence with stress tensor correlators

- The graviton self-energy is related to the 2-point correlator of the stress tensor as

$$-i\Big[{}^{\mu\nu}\Delta^{\rho\sigma}\Big](x;x') = -\frac{1}{4}\kappa^2\sqrt{-\overline{g}(x)}\,\sqrt{-\overline{g}(x')}\Big\langle\Omega\Big|\delta\mathcal{T}^{\mu\nu}(x)\delta\mathcal{T}^{\rho\sigma}(x')\Big|\Omega\Big\rangle + O(\kappa^4)$$

- The stress tensor correlator obtained by Perez-Nadal, Roura and Verdaguer (JCAP **1005** (2010) 036, arXiv:0911.4870) agrees with out result.

$$\Big\langle\Omega\Big|\delta\mathcal{T}^{\mu\nu}(x)\delta\mathcal{T}^{\rho\sigma}(x')\Big|\Omega\Big\rangle = F_{\mu\nu\rho\sigma} = P(\mu)n_\mu n_\nu n_\rho n_\sigma + Q(\mu)(n_\mu n_\nu \overline{g}_{\rho\sigma} + n_\rho n_\sigma \overline{g}_{\mu\nu})$$
$$+R(\mu)(n_\mu n_\rho \overline{g}_{\nu\sigma} + n_\nu n_\sigma \overline{g}_{\mu\rho} + n_\mu n_\sigma \overline{g}_{\nu\rho} + n_\nu n_\rho \overline{g}_{\mu\sigma}) + S(\mu)(\overline{g}_{\mu\rho}\overline{g}_{\nu\sigma} + \overline{g}_{\nu\rho}\overline{g}_{\mu\sigma}) + T(\mu)\overline{g}_{\mu\nu}\overline{g}_{\rho\sigma}$$

- Note: their 5 basis tensors are converted into ours as

$$\begin{aligned}
n_a n_b n_{c'} n_{d'} &= \frac{1}{H^4(4y-y^2)^2}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}}\ , \\
n_a n_b \overline{g}_{c'd'} + n_{c'} n_{d'} \overline{g}_{ab} &= \frac{1}{H^2(4y-y^2)}\Big[\overline{g}_{ab}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}} + \frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\overline{g}_{c'd'}\Big]\ , \\
4n_{(a}\overline{g}_{b)(c'}n_{d')} &= -\frac{2}{H^4(4y-y^2)}\frac{\partial y}{\partial x^{(a}}\frac{\partial^2 y}{\partial x^{b)}\partial x'^{(c'}}\frac{\partial y}{\partial x'^{d')}} - \frac{2}{H^4(4y-y^2)(4-y)}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}}\ , \\
2\overline{g}_{a(c'}\overline{g}_{d')b} &= \frac{1}{2H^4}\frac{\partial^2 y}{\partial x^a\partial x'^{(c'}}\frac{\partial^2 y}{\partial x^{d')}\partial x'^b} + \frac{1}{H^4(4-y)}\frac{\partial y}{\partial x^{(a}}\frac{\partial^2 y}{\partial x^{b)}\partial x'^{(c'}}\frac{\partial y}{\partial x'^{d')}} \\
&\quad + \frac{1}{2H^4}\frac{1}{(4-y)^2}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}}\ , \\
\overline{g}_{ab}\overline{g}_{c'd'} &= \overline{g}_{ab}\overline{g}_{c'd'}
\end{aligned}$$

One Loop Counterterms

- For quantum gravity at one loop order the necessary counterterms are R^2 and C^2 first derived by t Hooft and Veltman, 1974
  Graviton 2-point function $\rightarrow$ 2 graviton fields

  S.D.D. $= 4 \rightarrow 4\partial$'s, with general coord. invariance 3 possibilities:

  $R^2, R^{\mu\nu}R_{\mu\nu}, R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$ with the Gauss-Bonet relation, only 2 are linearly indep.
- For calculational convenience, reorganize R^2 as
  $R^2 = \Big[R - D(D-1)H^2\Big]^2 + 2D(D-1)H^2R - D^2(D-1)^2H^4$
  So we employ four counterterms:

$$\Delta\mathcal{L}_1 \equiv c_1\Big[R - D(D-1)H^2\Big]^2\sqrt{-g}\;, \Delta\mathcal{L}_3 \equiv c_3H^2\Big[R - (D-1)(D-2)H^2\Big]\sqrt{-g}\;, \Delta\mathcal{L}_4 \equiv c_4H^4\sqrt{-g}\;.$$
$$\Delta\mathcal{L}_2 \equiv c_2C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}\sqrt{-g}\;,$$

  Note: the divergences can really be eliminated with just $\Delta\mathcal{L}_2$ and the particular linear combination of $\Delta\mathcal{L}_1$, $\Delta\mathcal{L}_3$ and $\Delta\mathcal{L}_4$ which is proportional to $R^2\sqrt{-g}$.
- We define two 2nd order differential operators by expanding the scalar and Weyl curvatures around de Sitter background

$$R - D(D-1)H^2 \equiv \mathcal{P}^{\mu\nu}\kappa h_{\mu\nu} + O(\kappa^2h^2)\;,$$
$$C_{\alpha\beta\gamma\delta} \equiv \mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\kappa h_{\mu\nu} + O(\kappa^2h^2)\;.$$

Counterterms in terms of two projection operators

- Spin zero projection operator:

$$\mathcal{P}^{\mu\nu} = D^{\mu} D^{\nu} - \overline{g}^{\mu\nu} \Big[D^2 + (D-1) H^2 \Big] \; ,$$

- Spin two projection operator

$$\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta} = \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} + \frac{1}{D-2} \Big[\overline{g}_{\alpha\delta} \mathcal{D}^{\mu\nu}_{\beta\gamma} - \overline{g}_{\beta\delta} \mathcal{D}^{\mu\nu}_{\alpha\gamma} - \overline{g}_{\alpha\gamma} \mathcal{D}^{\mu\nu}_{\beta\delta} + \overline{g}_{\beta\gamma} \mathcal{D}^{\mu\nu}_{\alpha\delta} \Big] + \frac{1}{(D-1)(D-2)} \Big[\overline{g}_{\alpha\gamma} \overline{g}_{\beta\delta} - \overline{g}_{\alpha\delta} \overline{g}_{\beta\gamma} \Big] \mathcal{D}^{\mu\nu} \; ,$$

where we define,

$$\mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} \equiv \frac{1}{2} \Big[\delta^{(\mu}_{\alpha} \delta^{\nu)}_{\delta} D_{\gamma} D_{\beta} - \delta^{(\mu}_{\beta} \delta^{\nu)}_{\delta} D_{\gamma} D_{\alpha} - \delta^{(\mu}_{\alpha} \delta^{\nu)}_{\gamma} D_{\delta} D_{\beta} + \delta^{(\mu}_{\beta} \delta^{\nu)}_{\gamma} D_{\delta} D_{\alpha} \Big] \; ,$$

$$\mathcal{D}^{\mu\nu}_{\beta\delta} \equiv \overline{g}^{\alpha\gamma} \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} = \frac{1}{2} \Big[\delta^{(\mu}_{\delta} D^{\nu)} D_{\beta} - \delta^{(\mu}_{\beta} \delta^{\nu)}_{\delta} D^2 - \overline{g}^{\mu\nu} D_{\delta} D_{\beta} + \delta^{(\mu}_{\beta} D_{\delta} D^{\nu)} \Big] \; ,$$

$$\mathcal{D}^{\mu\nu} \equiv \overline{g}^{\alpha\gamma} \overline{g}^{\beta\delta} \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} = D^{(\mu} D^{\nu)} - \overline{g}^{\mu\nu} D^2 \; .$$

Counterterms in terms of two projection operators

- The counterterms are expressed in terms of these two operators:

$$
\begin{aligned}
\frac{i\delta\Delta S_1}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} &= 2c_1\kappa^2\sqrt{-\overline{g}}\,\mathcal{P}^{\mu\nu}\mathcal{P}^{\rho\sigma}i\delta^D(x-x') \longrightarrow 2c_1\kappa^2\Pi^{\mu\nu}\Pi^{\rho\sigma}i\delta^D(x-x'),\\
\frac{i\delta\Delta S_2}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} &= 2c_2\kappa^2\sqrt{-\overline{g}}\,\overline{g}^{\alpha\kappa}\overline{g}^{\beta\lambda}\overline{g}^{\gamma\theta}\overline{g}^{\delta\phi}\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}i\delta^D(x-x')\\
&\longrightarrow 2c_2\kappa^2\Big(\frac{D-3}{D-2}\Big)\Big[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\Big]i\delta^D(x-x')\\
\frac{i\delta\Delta S_3}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} &= -c_3\kappa^2H^2\sqrt{-\overline{g}}\,\mathcal{D}^{\mu\nu\rho\sigma}i\delta^D(x-x') \longrightarrow 0\\
\frac{i\delta\Delta S_4}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} &= c_4\kappa^2H^4\sqrt{-\overline{g}}\Big[\frac{1}{4}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \frac{1}{2}\overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\Big]i\delta^D(x-x') \longrightarrow 0
\end{aligned}
$$

where we define $\Pi^{\mu\nu} \equiv \partial^\mu\partial^\nu - \eta^{\mu\nu}\partial^2$ in flat space limit.

Renormalizing the Flat Space Result: a guide for de Sitter

- Reorganize the primitive terms in the terms of two projection operators so as to be in the form of counterterms:

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\rm flat}(x;x') = \Pi^{\mu\nu}\Pi^{\rho\sigma}F_0(\Delta x^2) + \Big[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\Big]F_2(\Delta x^2)\ .$$

- Find the structure functions F_0 and F_2 comparing this with the previous primitive result:

$$\begin{aligned} F_0(\Delta x^2) &= \frac{\kappa^2\Gamma^2(\frac{D}{2})}{16\pi^D} \times -\frac{1}{8(D-1)^2}\Big(\frac{1}{\Delta x^2}\Big)^{D-2} \\ F_2(\Delta x^2) &= \frac{\kappa^2\Gamma^2(\frac{D}{2})}{16\pi^D} \times -\frac{1}{4(D-2)^2(D-1)(D+1)}\Big(\frac{1}{\Delta x^2}\Big)^{D-2} \end{aligned}$$

- Note: $\Pi^{\mu\nu}\Pi^{\rho\sigma} \sim \partial^4$ are w.r.t x Extract these outside the integral w.r.t x'. Now the factor of $1/\Delta x^{2D-4}$ is logarithmically divergent. Then extract one more d'Alembertian

$$\Big(\frac{1}{\Delta x^2}\Big)^{D-2} = \frac{\partial^2}{2(D-3)(D-4)}\Big(\frac{1}{\Delta x^2}\Big)^{D-3}\ .$$

- Now the integrand converges, however, we still cannot take the $D=4$ limit owing to the factor of $1/(D-4)$. The solution is to add zero in the form of the identity

$$\partial^2\Big(\frac{1}{\Delta x^2}\Big)^{\frac{D}{2}-1} - \frac{4\pi^{\frac{D}{2}}\, i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} = 0\ .$$

- Rewrite it by adding zero:

$$\left(\frac{1}{\Delta x^2}\right)^{D-2} = \frac{\partial^2}{2(D-3)(D-4)}\left\{\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}}\right\} + \frac{4\pi^{\frac{D}{2}}\mu^{D-4} i\delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)}$$

$$= -\frac{1}{4}\partial^2\left\{\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2} + O(D-4)\right\} + \frac{4\pi^{\frac{D}{2}}\mu^{D-4} i\delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)} .$$

: nonlocal finite term : local divergent term

- The divergence now segregated on the delta function: remove them with counterterms:

$$-i\left[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\right]_{\rm flat}(x;x') = \Pi^{\mu\nu}\Pi^{\rho\sigma}\left\{2c_1\kappa^2 i\delta^D(x-x')\right\} + \left[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\right]\left\{2\left(\frac{D-3}{D-2}\right)c_2\kappa^2 i\delta^D(x-x')\right\}$$

- by choosing the constants c_1 and c_2 as ,

$$c_1 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{\frac{D}{2}}}\frac{(D-2)}{(D-1)^2(D-3)(D-4)} , \quad c_2 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{\frac{D}{2}}}\frac{2}{(D+1)(D-1)(D-3)^2(D-4)} .$$

- The fully renormalized graviton self-energy for flat space background is,

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{\substack{\rm ren\\ \rm flat}} = \lim_{D\to 4}\left\{-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{\rm flat}(x;x') - i\left[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\right]_{\rm flat}(x;x')\right\} ,$$

$$= \Pi^{\mu\nu}\Pi^{\rho\sigma}\partial^2\left\{\frac{\kappa^2}{2^9 3^2\pi^4}\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right\} + \left[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{1}{3}\Pi^{\mu\nu}\Pi^{\rho\sigma}\right]\partial^2\left\{\frac{\kappa^2}{2^{10}3^1 5^1\pi^4}\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right\}$$

Again, this agrees with 't Hooft and Veltman

Renormalizing de Sitter result

- Reorganize the primitive result in terms of the projection operators as for flat space:

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x') = \sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big\{\mathcal{F}_0(y)\Big\}$$
$$+\sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big\{\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\Big(\frac{D-2}{D-3}\Big)\mathcal{F}_2(y)\Big\},$$

where the bitensor is $\mathcal{T}^{\alpha\kappa}(x;x') \equiv -\frac{1}{2H^2}\frac{\partial^2 y(x;x')}{\partial x_\alpha \partial x'_\kappa}$. Note: $\mathcal{T}^{\alpha\kappa}(x;x') \quad \leftarrow \quad \eta^{\alpha\kappa}$ in flat space

- Find the structure functions F_0 and F_2 comparing this with the previous primitive result:

$$\mathcal{F}_0(y) = \frac{\kappa^2 H^{2D-4}\Gamma^2(\frac{D}{2})}{(4\pi)^D}\Big\{\frac{-1}{8(D-1)^2}\Big(\frac{4}{y}\Big)^{D-2} + \dots\Big\}$$
$$\mathcal{F}_2(y) = \frac{\kappa^2 H^{2D-4}\Gamma^2(\frac{D}{2})}{(4\pi)^D}\Big\{\frac{-1}{4(D-3)(D-2)(D-1)(D+1)}\Big(\frac{4}{y}\Big)^{D-2} + \dots\Big\}$$

- Add zero in the form of the identity

$$\Big[\square - \frac{D}{2}\Big(\frac{D}{2}-1\Big)H^2\Big]\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} - \frac{(4\pi)^{\frac{D}{2}}\, i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)H^{D-2}\sqrt{-\overline{g}}} = 0\ .$$

- Then

$$\Big(\frac{4}{y}\Big)^{D-2} = -\Big[\frac{\square}{H^2}-2\Big]\Big\{\frac{4}{y}\ln\Big(\frac{y}{4}\Big)\Big\} - \frac{4}{y} + O(D-4) + \frac{2(4\pi)^{\frac{D}{2}}\, i\delta^D(x-x')/\sqrt{-\overline{g}}}{(D-4)(D-3)\Gamma(\frac{D}{2}-1)H^D}$$

nonlocal finite term — local divergent term

Renormalizing de Sitter result

- Add the counterterms to subtract the divergences off:

$$-i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big](x;x') = \sqrt{-\overline{g}}\Big[2c_1\kappa^2\mathcal{P}^{\mu\nu}\mathcal{P}^{\rho\sigma} + 2c_2\kappa^2\overline{g}^{\alpha\kappa}\overline{g}^{\beta\lambda}\overline{g}^{\gamma\theta}\overline{g}^{\delta\phi}\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}$$
$$-c_3\kappa^2H^2\mathcal{D}^{\mu\nu\rho\sigma} + c_4\kappa^2H^4\sqrt{-\overline{g}}\left[\frac{1}{4}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \frac{1}{2}\overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\right]\Big]i\delta^D(x-x')\ .$$

- The fully renormalized graviton self-energy for de Sitter is :

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\Big](x;x') = \lim_{D\to4}\Big\{-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x') - i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big](x;x')\Big\}\ ,$$
$$= \sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big[\mathcal{F}_{0R}(y)\Big]$$
$$+2\sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-\overline{g}(x')}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big[\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_{2R}(y)\Big]\ .$$

where $\mathcal{F}_{0R} = \frac{\kappa^2H^4}{(4\pi)^4}\Big\{\frac{\square}{H^2}\Big[\frac{1}{72}\times\frac{4}{y}\ln\Big(\frac{y}{4}\Big)\Big]+\cdots\Big\}$, $\mathcal{F}_{2R} = \frac{\kappa^2H^4}{(4\pi)^4}\Big\{\frac{\square}{H^2}\Big[\frac{1}{240}\times\frac{4}{y}\ln(\Big(\frac{y}{4}\Big)\Big]+\cdots\Big\}$

Note 1: The leading terms agree with the corresponding flat results.

Note 2: $\mathcal{F}_{0R}$ and $\mathcal{F}_{2R}$ are the first fully renormalized results for the graviton structure functions on de Sitter.

Spin zero structure function

$$\mathcal{F}_{0R} = \frac{\kappa^2 H^4}{(4\pi)^4}\Bigg\{\frac{\Box}{H^2}\Big[\frac{1}{72}\times\frac{4}{y}\ln\Big(\frac{y}{4}\Big)\Big] - \frac{1}{12}\times\frac{4}{y}\ln\Big(\frac{y}{4}\Big) + \frac{1}{72}\times\frac{4}{y} + \frac{1}{6}\ln^2\Big(\frac{y}{4}\Big)$$
$$+\frac{1}{45}\times\frac{4}{4-y}\ln(\frac{y}{4}) - \frac{1}{45}\ln(\frac{y}{4}) + \frac{43}{216}\times\frac{4}{4-y} - \frac{5}{6}\times\frac{y}{4}\ln(1-\frac{y}{4})$$
$$+\frac{7}{90}\times\frac{4}{y}\ln(1-\frac{y}{4}) - \frac{1}{20}\ln(1-\frac{y}{4}) - \frac{7(12\pi^2+265)}{540}\times\frac{y}{4}$$
$$+\frac{84\pi^2-131}{1080} - \frac{1}{3}\times\frac{y}{4}\ln^2\Big(\frac{y}{4}\Big) + \frac{4}{9}\times\frac{y}{4}\ln\Big(\frac{y}{4}\Big)$$
$$-\frac{1}{30}(2-y)\Big[7\mathrm{Li}_2(1-\frac{y}{4}) - 2\mathrm{Li}_2(\frac{y}{4}) + 5\ln(1-\frac{y}{4})\ln(\frac{y}{4})\Big]\Bigg\}\ .$$

Spin two structure function

$$
\begin{aligned}
\mathcal{F}_{2R} = \frac{\kappa^2 H^4}{(4\pi)^4} \Bigg\{ & \frac{\Box}{H^2}\left[\frac{1}{240}\times\frac{4}{y}\ln\left(\frac{y}{4}\right)\right] + \frac{3}{40}\times\frac{4}{y}\ln\left(\frac{y}{4}\right) - \frac{11}{48}\times\frac{4}{y} + \frac{1}{4}\ln^2\left(\frac{y}{4}\right) - \frac{119}{60}\ln\left(\frac{y}{4}\right) \\
& + \frac{4096}{(4y-y^2-8)^4}\Bigg[\Bigg[-\frac{47}{15}\left(\frac{y}{4}\right)^8 + \frac{141}{10}\left(\frac{y}{4}\right)^7 - \frac{2471}{90}\left(\frac{y}{4}\right)^6 + \frac{34523}{720}\left(\frac{y}{4}\right)^5 \\
& - \frac{132749}{1440}\left(\frac{y}{4}\right)^4 + \frac{38927}{320}\left(\frac{y}{4}\right)^3 - \frac{10607}{120}\left(\frac{y}{4}\right)^2 + \frac{22399}{720}\left(\frac{y}{4}\right) - \frac{3779}{960}\Bigg]\frac{4}{4-y} \\
& + \left[\frac{193}{30}\left(\frac{y}{4}\right)^4 - \frac{131}{10}\left(\frac{y}{4}\right)^3 + \frac{7}{20}\left(\frac{y}{4}\right)^2 + \frac{379}{60}\left(\frac{y}{4}\right) - \frac{193}{120}\right]\ln\left(2-\frac{y}{2}\right) \\
& + \left[-\frac{14}{15}\left(\frac{y}{4}\right)^5 - \frac{1}{5}\left(\frac{y}{4}\right)^4 + \frac{19}{2}\left(\frac{y}{4}\right)^3 - \frac{889}{60}\left(\frac{y}{4}\right)^2 + \frac{143}{20}\left(\frac{y}{4}\right) - \frac{13}{20} - \frac{7}{60}\left(\frac{4}{y}\right)\right]\ln\left(1-\frac{y}{4}\right) \\
& + \Bigg[-\frac{476}{15}\left(\frac{y}{4}\right)^9 + 160\left(\frac{y}{4}\right)^8 - \frac{5812}{15}\left(\frac{y}{4}\right)^7 + \frac{8794}{15}\left(\frac{y}{4}\right)^6 - \frac{18271}{30}\left(\frac{y}{4}\right)^5 + \frac{54499}{120}\left(\frac{y}{4}\right)^4 \\
& \quad - \frac{59219}{240}\left(\frac{y}{4}\right)^3 + \frac{1917}{20}\left(\frac{y}{4}\right)^2 - \frac{1951}{80}\left(\frac{y}{4}\right) + \frac{367}{120}\Bigg]\frac{4}{4-y}\ln\left(\frac{y}{4}\right) \\
& + \left[4\left(\frac{y}{4}\right)^7 - 12\left(\frac{y}{4}\right)^6 + 20\left(\frac{y}{4}\right)^5 - 20\left(\frac{y}{4}\right)^4 + 15\left(\frac{y}{4}\right)^3 - 7\left(\frac{y}{4}\right)^2 + \left(\frac{y}{4}\right)\right]\frac{4-y}{4}\ln^2\left(\frac{y}{4}\right) \\
& + \left[\frac{367}{30}\left(\frac{y}{4}\right)^4 - \frac{4121}{120}\left(\frac{y}{4}\right)^3 + \frac{237}{16}\left(\frac{y}{4}\right)^2 + \frac{1751}{240}\left(\frac{y}{4}\right) - \frac{367}{120}\right]\ln\left(\frac{y}{2}\right) \\
& + \frac{1}{64}(y^2-8)\left[4(2-y)-(4y-y^2)\right]\left[\frac{1}{5}\mathrm{Li}_2\left(1-\frac{y}{4}\right) + \frac{7}{10}\mathrm{Li}_2\left(\frac{y}{4}\right)\right]\Bigg]\Bigg\}\,.
\end{aligned}
$$

Solving the quantum-corrected linearized Einstein equation

- Use the renormalized self-energy for the quantum correction term:

$$\sqrt{-g}\,\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x)-\int d^4x'\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\right](x;x')h_{\rho\sigma}(x')=\frac{1}{2}\kappa\sqrt{-g}\,T^{\mu\nu}_{\rm lin}(x)\ ,$$

- Only know the self-energy at one loop order (at order $\kappa^2=16\pi G$), solve it perturbatively:

$$h_{\mu\nu}(x)=h^{(0)}_{\mu\nu}(x)+\kappa^2 h^{(1)}_{\mu\nu}(x)+O(\kappa^4)\ ,\quad \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\right](x;x')=\kappa^2\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x')+O(\kappa^4)\ .$$

- The corresponding one loop correction is

$$\int d^4x'\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x')h^{(0)}_{\rho\sigma}(x')=i\int d^4x'\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-g(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big\{\mathcal{F}_0\Big\}h^{(0)}_{\rho\sigma}(x')$$
$$+2i\int d^4x'\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-g(x')}\,\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big\{\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_2\Big\}h^{(0)}_{\rho\sigma}(x')\ .$$

- Simplification stratedy: partial integration

Step 1: pull the projectors $\mathcal{P}^{\mu\nu}(x)$ and $\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)$, which act on a function of x^μ outside the integration over x'^μ

Step 2: partially integrate the projectors $\mathcal{P}^{\rho\sigma}(x')$ and $\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')$ on $h^{(0)}_{\rho\sigma}(x')$.

$$\int d^4x'\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x')h^{(0)}_{\rho\sigma}(x')=i\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}(x)\int d^4x'\sqrt{-g(x')}\,\mathcal{F}_0\Big\{\mathcal{P}^{\rho\sigma}(x')h^{(0)}_{\rho\sigma}(x')\Big\}$$
$$+2i\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\int d^4x'\sqrt{-g(x')}\,\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_2\Big\{\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')h^{(0)}_{\rho\sigma}(x')\Big\}.$$

Solving the quantum-corrected linearized field eqn for dynamical gravitons

- For dynamical gravitons, that is for zero stress-energy $T^{\mu\nu}_{\text{lin}}(x) = 0$:

$$h^{(0)}_{\rho\sigma}(x) = \epsilon_{\rho\sigma}(\vec{k}) a^2 u(\eta, k) e^{i\vec{k}\cdot\vec{x}} \,, \quad u(\eta, k) = \frac{H}{\sqrt{2k^3}} \Big[1 - \frac{ik}{Ha}\Big] \exp\Big[\frac{ik}{Ha}\Big], \quad 0 = \epsilon_{0\mu} = k_i \epsilon_{ij} = \epsilon_{jj} \quad \text{and} \quad \epsilon_{ij}\epsilon^*_{ij} = 1$$

- The result is zero!

$$\int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_1\right](x; x') h^{(0)}_{\rho\sigma}(x') = 0$$

"Inflationary Scalars Don't Affect Gravitons at One Loop," SP and Woodard, arXiv: 1109.4187

- Gravitons interact with MMC scalar only through their kinetic energies which are redshifted. (Gravitons couple minimally only to differentiated scalars.)
- A little Doubt about this result: ignoring surface terms is really legitimate?
  - Ignored surface terms based on the assumption that either they fall off or can be absorbed into corrections of the initial state.
  - Found a counterexample to this assumption: A certain surface terms cannot be ignored.

  Leonard, Prokopec and Woodard, arXiv: 1210.6968

Noncovariant representation of the graviton self-energy

- A noncovariant representation of the conformally rescaled graviton field
  - Noncovariant Rep includes de Sitter breaking basis vectors in terms of $u(x;x') \equiv \ln(aa')$
  - Covariant Rep: $g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa\chi_{\mu\nu}$ vs Noncovariant Rep: $g_{\mu\nu} = a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$
  - Covariant Rep: 5 basis tensors - 3 relations = 2 structure ftns vs Noncovariant Rep: 14 - 10 = 4

$$
-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') = \mathcal{F}^{\mu\nu}(x) \times \mathcal{F}^{\rho\sigma}(x')\left[F_0(x;x')\right]
+\mathcal{G}^{\mu\nu}(x) \times \mathcal{G}^{\rho\sigma}(x')\left[G_0(x;x')\right] + \mathcal{F}^{\mu\nu\rho\sigma}\left[F_2(x;x')\right] + \mathcal{G}^{\mu\nu\rho\sigma}\left[G_2(x;x')\right]
$$

Leonard, SP, Prokopec and Woodard, arXiv: 1403.0896

- Checked that surface terms really fall off like powers of the scale factor
- Confirmed the previous result: no effect on dynamical gravitons from MMC scalars
- Much simpler than the de Sitter covariant representation, so can be easily employed to study the force of gravity

Structure functions in the noncovariant representation

$$F_{0R}(x;x') = \frac{\kappa^2(aa'H^2)^2}{2304\pi^4}\left\{\frac{\partial^2}{2(aa'H^2)^2}\left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right] - \frac{6}{y} + 6 + \left[-\frac{2}{y} + 6 - \frac{2}{4-y}\right]\ln\left(\frac{y}{4}\right) + \frac{3}{2}(2-y)\Psi(y)\right\}$$

$$F_{2R}(x;x') = \frac{\kappa^2(H^2aa')^2}{(4\pi)^4}\left\{\frac{\partial^2}{30(H^2aa')^2}\left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right] + \frac{2}{3}\left[\frac{1}{y} - \frac{1}{4-y}\right]\ln\left(\frac{y}{4}\right) - \frac{1}{3}\Psi(y)\right\}$$

$$G_0(x;x') = 0$$

$$G_2(x;x') = \frac{\kappa^2(H^2aa')^2}{(4\pi)^4}\left\{-2 + \frac{8}{3}\frac{\ln(\frac{y}{4})}{(4-y)} + \frac{2}{3}\Psi(y)\right\}$$

where

$$\Psi(y) \equiv \frac{1}{2}\ln^2\left(\frac{y}{4}\right) - \ln\left(1-\frac{y}{4}\right)\ln\left(\frac{y}{4}\right) - \mathrm{Li}_2\left(\frac{y}{4}\right)$$

One loop corrections from MMC scalar to the Newtonian potential

- For the linearized response to a stationary point mass M

$$h_{00}^{(0)}(x) = \frac{2GM}{a\|\vec{x}\|} = -2\Phi^{(0)} \; , \; h_{0i}^{(0)}(x) = 0 \; , \; h_{ij}^{(0)}(x) = \frac{2GM}{a\|\vec{x}\|}\delta_{ij} = -2\Psi^{(0)}\delta_{ij} \; , \quad T^{\mu\nu} = Ma\delta^3(\vec{x})\delta^0_\mu\delta^0_\nu$$

in flat space $h_{00}^{(0)}(x) = \frac{2GM}{\|\vec{x}\|}$, $h_{0i}^{(0)}(x) = 0$, $h_{ij}^{(0)}(x) = \frac{2GM}{\|\vec{x}\|}\delta_{ij}$, $T_{\mu\nu} = M\delta^3(\vec{x})\delta^0_\mu\delta^0_\nu$

- One loop corrections

$$h_{00}^{(1)}(x) \equiv f_1 \; , \; h_{0i}^{(1)}(x) = 0 \; , \; h_{ij}^{(1)}(x) \equiv f_3\delta_{ij}$$

The solutions are

$$f_1(x) = -\frac{\kappa^2 M}{2a^2}S_0^1(x) + \frac{\kappa^2 M}{a^2}\Big[-\frac{2}{3} + \nabla^{-2}(\partial_0^2 - aH\partial_0)\Big]S_2^1(x) = -2\Phi^{(1)}$$

$$f_3(x) = \frac{\kappa^2 M}{2a^2}S_0^1(x) + \frac{\kappa^2 M}{a^2}\Big[-\frac{1}{3} - \nabla^{-2}aH\partial_0\Big]S_2^1(x) = -2\Psi^{(1)}$$

where $\nabla^{-2}f(\eta,\vec{x}) = -[1/(4\pi)]\int d^3x' f(\eta,\vec{x}')/\|\vec{x}-\vec{x}'\|$

$$S_0^1(x) = \int\frac{d\eta'}{a(\eta')}[iF_0^1(x,x')]_{\vec{x}'=0} \; ,$$

$$S_2^1(x) = \int\frac{d\eta'}{a(\eta')}\Big[F_2^1(x;x') + \frac{1}{2}G_2^1(x;x')\Big]_{\vec{x}'=0}$$

One loop corrections from MMC scalar to the Newtonian potential

- In flat space

$$\Phi_{flat} = -\frac{GM}{r}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{r^2} + O\Big(G^2\Big)\right\}$$

$$\Psi_{flat} = -\frac{GM}{r}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{r^2} + O\Big(G^2\Big)\right\}$$

SP and Woodard, arXiv:1007.2662, Marunovic and Prokopec, arXiv: 1101.5059

Not the first for this result, but the first to solve the effective field eqns using the Schwinger-Keldysh or in-in formalism.

The previous calculations e.g. Radkowski, 1970, John F. Donoghue 1993, ... were done by the scattering amplitude technique.

- In de Sitter space

$$\Phi_{dS} = -\frac{GM}{ar}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\left[-\frac{1}{30}\ln(a) - \frac{3}{10}\ln\Big(\frac{Har}{c}\Big)\right] + O\Big(G^2 H^4\Big)\right\}$$

$$\Psi_{dS} = -\frac{GM}{ar}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\left[-\frac{1}{30}\ln(a) - \frac{3}{10}\ln\Big(\frac{Har}{c}\Big) + \frac{2}{3}\frac{Har}{c}\right] + O\Big(G^2 H^4\Big)\right\}$$

the de Sitterized version of the flat space correction + intrinsic de Sitter correction

SP, Prokopec and Woodard, arXiv:1510.03352

Summary and Discussion

- Derived one loop contributions to the graviton self-energy from MMC scalar on de Sitter background: covariant and noncovariant representations for the tensor structure.
- Used these representations to quantum correct the linearized Einstein field equations for dynamical gravitons and the force of gravity.
- The noncovariant representation is much easier to use in the effective field equations than the covariant one.
- Inflationary production of MMC scalars has no effect on dynamical gravitons.
- Inflationary production of MMC scalars gives quantum corrections to the force of gravity: a time dependent renormalization of the mass term,

$$M \rightarrow M\left[1 - \frac{\hbar}{c^5}\frac{GH^2}{30\pi}\ln(a)\right],$$

or equivalently a time dependent renormalization of the Newton's constant,

$$G \rightarrow G\left[1 - \frac{\hbar}{c^5}\frac{GH^2}{30\pi}\ln(a)\right].$$

THE END

- Thank you for your attention!