Will quantum cosmology resurrect chaotic inflation model?

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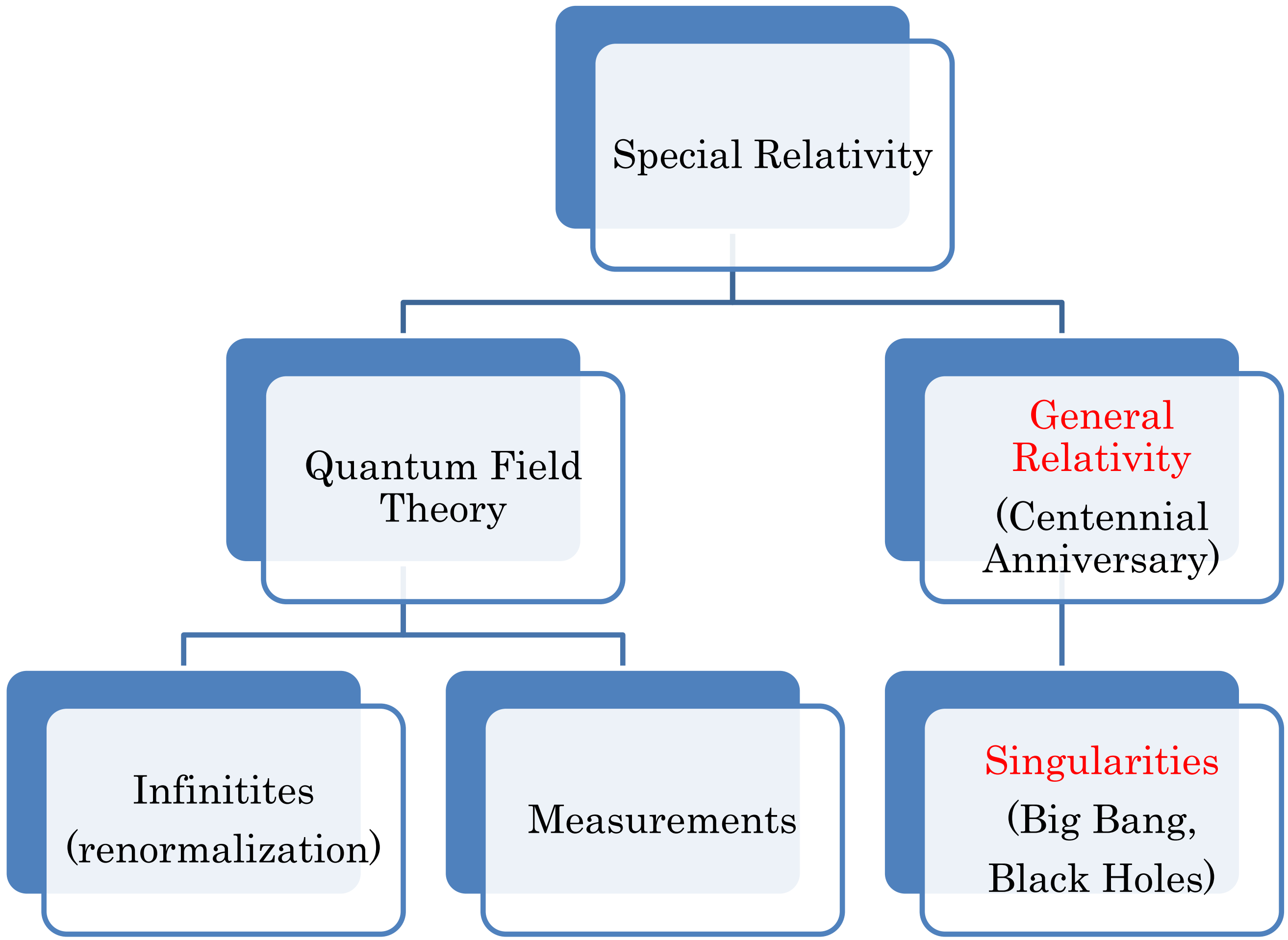
Outline

- Motivation
  - Why quantum cosmology?
  - Why quantum cosmology now?
  - Why massive scalar quantum cosmology?
- Second quantized universes
- Semiclassical quantum cosmology
- Gauge invariant quantum cosmology
- Conclusion

Why Quantum Cosmology?

Pathologies of Modern Physics



Big Bang, an Ingredient of Cosmology

- The singularity theorem implies the Big Bang (BB) [Hawking, Penrose, Proc. R. Soc. Lond. A 314 ('70)].
- Inflationary spacetimes have the singularity [Borde, Guth, Vilenkin, PRL 90 ('03)].
- What is the spacetime geometry including the BB?
- How to quantize the spacetime as well as matter fields, that is, what is quantum gravity and quantum cosmology?
- How do a classical universe and the unitary quantum field theory emerge from quantum cosmology?

What does HH Wavefunction Predict?

[D. N. Page, hep-th/0610121]

- Lorentzian spacetime can emerge in a WKB limit of an analytical continuation of the Hartle-Hawking (HH) wave function [Hartle, Hawking, PRD 28 ('83); Hawking, NPB 239 ('84)].
- The universe can inflate to large size [Hawking, NPB 239 ('84)].
- Models can predict the near-critical density [Hawking, NPB 239 ('84); Hawking, Page, NPB 264 ('86)].
- Models can predict low anisotropies [Hawking, Luttrell, PLB 143 ('84)].
- Inhomogeneities start in ground states and so can fit CMB data [Halliwell, Hawking, PRD 31 ('85)].
- Entropy starts low and grows with time (arrow of time) [Hawking, PRD 32 ('85); Page, PRD 32 ('85); Hawking, Laflamme, Lyons, PRD 47 ('93); SPK, S. W. Kim, PRD 49 ('94); SPK, S. W. Kim, PRD 51 ('95); …]
- Decoherence and classicality of the Universe [Kiefer, CQG 4 ('87); SPK, PRD 46 ('92)]

Why Quantum Cosmology Now?

Starobinsky View of Inflation Scenario

- Talks at ICGC (Beijing, ‘15), ICGAC12 (Moscow, ‘15), HTGRG2 (Quy Nhon, ‘15), LeCosPA2 (Taipei, Dec.14-18, ‘15)
- Aesthetic elegance.
- Predictability, proof and/or falsification.
- Naturalness of the hypothesis.
- Relate quantum gravity and quantum cosmology to astronomical observations.
- Produces (non-universal) arrow of time for our universe.

Emergence of CG from QG & Quantum Effects

Quantum Gravity/Cosmology	
$\hat{G}_{\mu\nu} = 8\pi G \hat{T}_{\mu\nu}$	WDW, HH wave function, tunneling wave function

$$G = 1/m_P^2 << 1$$

Semiclassical Quantum Gravity/Cosmology	
$G_{\mu\nu}^{C} + G_{\mu\nu}^{Q}[G] = 8\pi G \left\langle \hat{T}_{\mu\nu} \right\rangle$	QFT in curved spacetime, Hawking radiation, pair production

$$\hbar << 1$$

Classical Gravity/Cosmology*	
$G_{\mu\nu}^{C} + G_{\mu\nu}^{Q}[G] = 8\pi G \left(T_{\mu\nu}^{C} + T_{\mu\nu}^{Q}[\hbar]\right)$	Inflationary models

Effect of Quantum Gravity in Cosmological Observations?

- Wheeler-DeWitt equation for FRW universe with scale factor $a = e^{\alpha}$, inflaton ϕ and Fourier-modes f_k of ϕ-fluctuations has the wavefunction $\Psi(\alpha, \phi, f_k)$; Assume the ϕ-derivatives to be much smaller than the α-derivatives (a slow-roll approximation); Born-Oppenheimer interpretation
- Quantum cosmology corrected power spectrum [Kiefer, Kramer, PRL 108 ('12); 1st prize in 2012 essay competition of Gravity Research Foundation]

$$\Delta^2_{(1)}(k) = \Delta^2_{(0)}(k)\left(1 - \frac{43.56}{k^3}\frac{H^2}{m_P^2}\right)^{-3/2}\left(1 - \frac{189.18}{k^3}\frac{H^2}{m_P^2}\right)$$

- Suppression of power spectrum at large scales and weaker upper bound on H than tensor-to-scalar ratio.

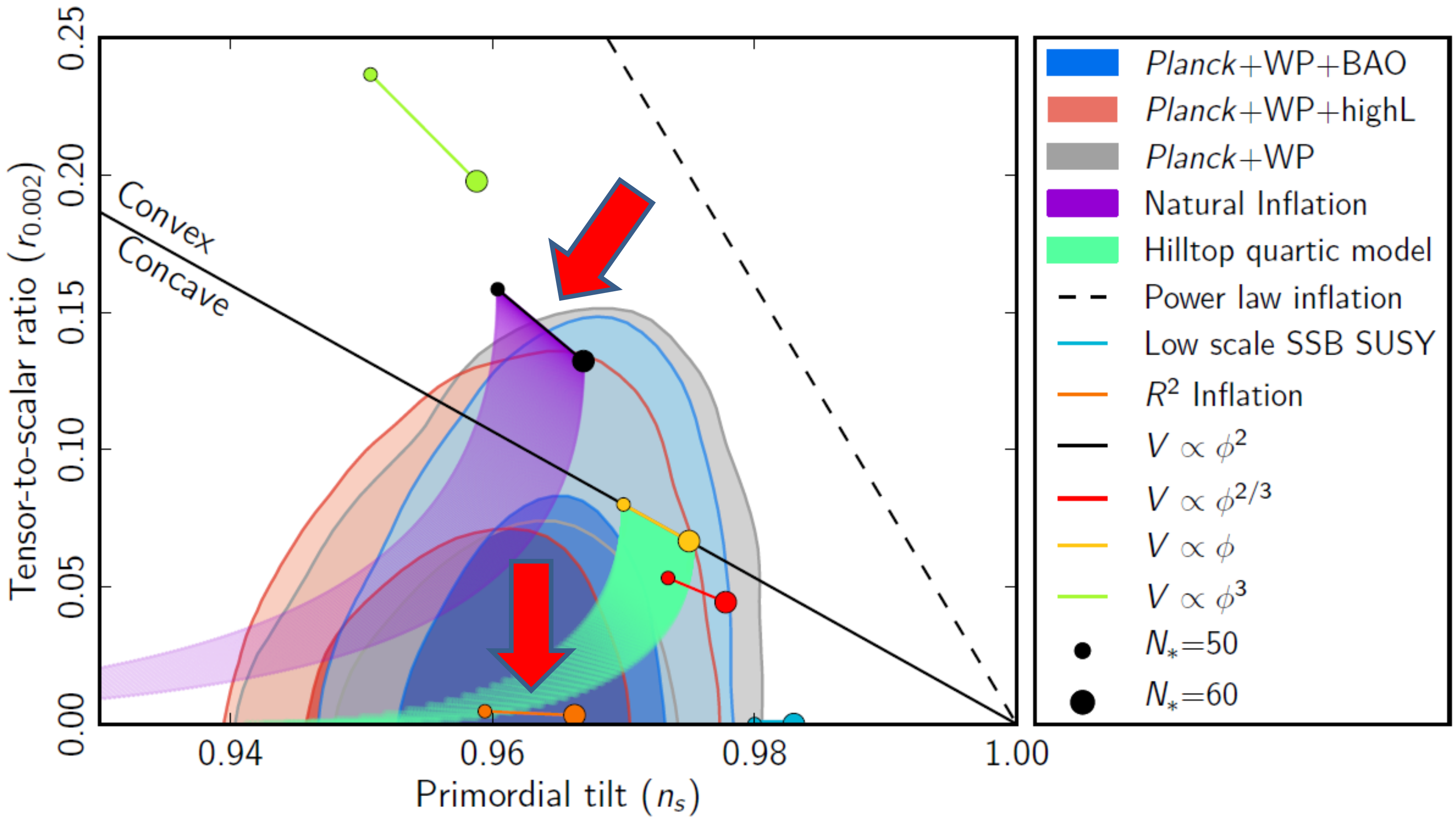
Gauge Invariant Perturbations in QC

- Earlier quantum cosmological models are minisuperspace ones, for instance, $a = e^{\alpha}$and ϕ only.
- Inhomogeneity model by Halliwell and Hawking assumes perturbations (scalar, vector & tensor) around a and ϕ.
- Gauge invariant super-Hamiltonian & super-momenta constraints in terms of Mukhanov-Sasaki variables [Pinto-Nieto et al, PRD 76 ('07); 79 ('09); Mena Marugan et al, PRD 90 ('14); Pinto-Neto, ICGA12 ('15); Mena Marugan, HTGRG2 ('15)].
- Then, classical cosmology from quantum cosmology may give a complete description of density perturbations with quantum effects included for CMB data.

Why Massive Scalar Quantum Cosmology?

Planck 2013 Results

[arXiv:1303.5082v2]



R^2 Inflation Model

- Starobinsky inflation model $(R + \alpha R^2)$ [PLB 91 ('80)]: a de Sitter-type acceleration .
- Mukhanov and Chibisov [JETP Lett 33 ('81)]:
  - approximately scale-invariant quantum fluctuations

$$n_s - 1 = \left.\frac{d \ln \Delta_R^2(k)}{d \ln k}\right|_{\mathrm{WMAP}} = -\frac{2}{N}$$

  - N = number of e-folds of expansion between the end of inflation and the epoch at which a fluctuation with k_{WMAP} left the horizon.
- Whitt [PLB 145 ('84)] and Maeda [PRD 37 ('87)]: equivalent to a scalar field under a conformal transformation

$$\tilde{g}_{\mu\nu} = (1 + 2\alpha R) g_{\mu\nu}, \qquad \Psi = \sqrt{3/2} \ln(1 + 2\alpha R)$$

Second Quantized Universes

Relativistic Wave in Superspace

[SPK, thesis ('91); SPK, Page, PRD 45 ('92); SPK, PRD 46 ('92)]

- Supermetric for FRW geometry and a minimal scalar

$$ds^2 = -da^2 + a^2 d\phi^2$$

- Hamiltonian constraint and Wheeler-DeWitt equation

$$H(a,\phi) = -\underbrace{\left(\pi_a^2 + V_G(a)\right)}_{\text{gravity part}} + \underbrace{\frac{1}{a^2}\left(\pi_\phi^2 + 2a^6 V(\phi)\right)}_{\text{matter part}} = 0$$

$$\left[-\nabla^2 - V_G(a) + 2a^4 V(\phi)\right]\Psi(a,\phi) = 0$$

$$\nabla^2 = -a^{-1}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) + \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2}\ , \quad V_G(a) = ka^2 - 2\Lambda a^4$$

- The universe scatters from an initial surface to a final one in superspace of 3-geometries. A prescription of the boundary condition?

Quantum Universes in the Superspace

- Single-field inflation model

$$V(\phi) = \lambda_{2p}\phi^{2p}/(2p)$$

- Eigenfunctions and Symanzik scaling-law

$$H_M(\phi, a)\Phi_n(\phi, a) = E_n(a)\Phi_n(\phi, a)$$

$$E_n(a) = \left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\varepsilon_n$$

$$\Phi_n(\phi, a) = \left(\lambda_{2p}a^6/p\right)^{1/4(p+1)}F_n\left(\left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\phi\right)$$

- Coupling matrix among energy eigenfunctions

$$\frac{\partial}{\partial a}\vec{\Phi}(\phi, a) = \Omega(a)\vec{\Phi}(\phi, a)$$

$$\Omega_{mn}(a) = \left(3/4(p+1)a\right)\left(\varepsilon_m - \varepsilon_n\right)\int d\varsigma F_m(\varsigma)F_n(\varsigma)\varsigma^2$$

Quantum Universes in the Superspace

- Gravitational part of WDW equation

$$\Psi(a,\phi) = \vec{\Phi}^T(\phi,a)\cdot\vec{\psi}(a)$$

$$\left[\frac{d^2}{da^2} - V_G(a) + \frac{E(a)}{a^2} - \left(2\Omega(a)\frac{d}{da} - \Omega^2(a) - \frac{1}{a}\Omega(a)\right)\right]\vec{\psi}(a) = 0$$

- Transition matrix and Cauchy problem

$$\Psi(a,\phi) = \vec{\Phi}^T(\phi,a)T(a)\vec{\psi}(a);\; T(a) = \exp\left[\int^a da'\Omega(a')\right]$$

$$\frac{d}{da}\begin{pmatrix} \vec{\psi}(a) \\ d\vec{\psi}(a)/da \end{pmatrix} = \begin{pmatrix} 0 & I \\ T^{-1}\left(V_G - \frac{E}{a^2}\right)T & 0 \end{pmatrix}\begin{pmatrix} \vec{\psi}(a) \\ d\vec{\psi}(a)/da \end{pmatrix}$$

Quantum Universes in the Superspace

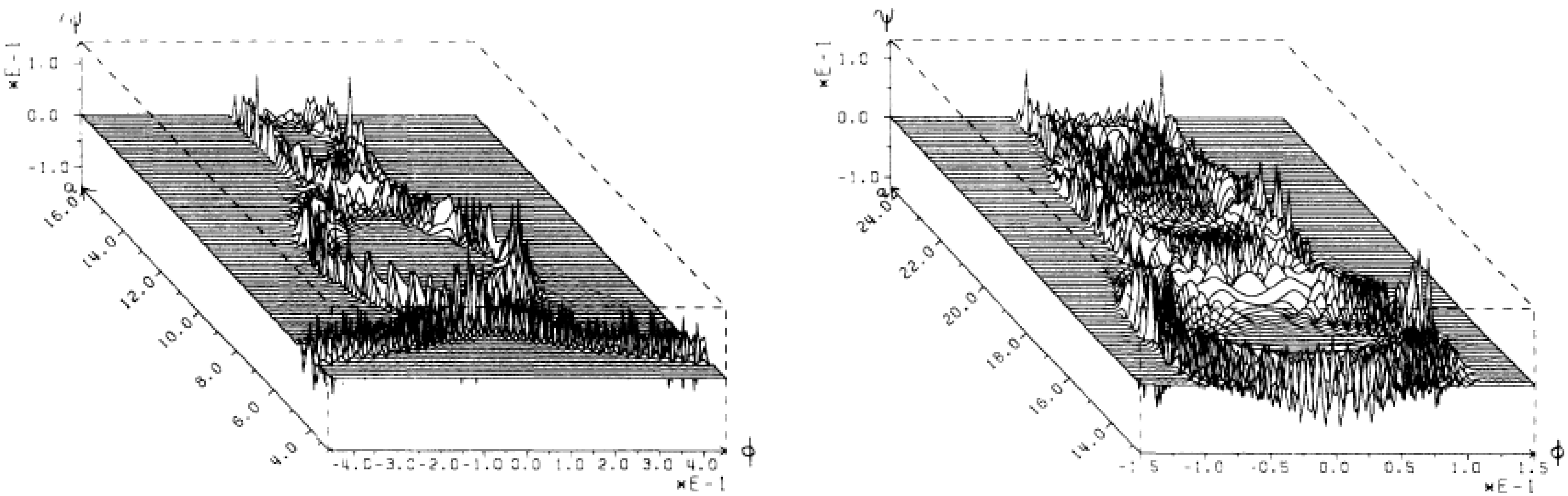
- The two-component wave function

$$\begin{pmatrix} \Psi(a,\phi) \\ \partial\Psi(a,\phi)/\partial a \end{pmatrix} = \begin{pmatrix} \vec{\Phi}^T(\phi,a) & 0 \\ 0 & \vec{\Phi}^T(\phi,a) \end{pmatrix}$$

$$\times T\exp\left[\int \begin{pmatrix} \Omega(a') & I \\ V_G(a') - E/a'^2 & \Omega(a') \end{pmatrix} da'\right] \begin{pmatrix} \vec{\psi}(a_0) \\ d\vec{\psi}(a_0)/da_0 \end{pmatrix}$$

- Off-diagonal components are gravitational part equation only with $V_G(a) - E/a^2$.
- Continuous transitions among energy eigenfunctions.
- Construct the present Cauchy data based on observations and evolve back to the early universe to understand the origin of the universe.

Wave Packet for FRW with a Minimal Scalar



A closed universe (k=1), m = 6, and n =120 (harmonic quantum number)
[Fig. from Kiefer, PRD 38 ('88)]

Semiclassical Quantum Cosmology

de Broglie-Bohm Pilot-Wave Theory & Born-Oppenheimer Idea

- Wave functions are peaked around wave packets and allow de Broglie-Bohm pilot-wave theory

$$\left[-\frac{\hbar^2}{2M}\nabla^2 - MV_{\mathrm{G}}(h_a) + \hat{H}(\phi, -i\frac{\delta}{\delta\phi}, h_a)\right]\Psi(h_a,\phi) = 0\ , \quad (h_a = h_{ij})$$

- Apply Born-Oppenheimer idea that separates a slow moving massive particle (M: Planck mass squared) from a fast moving light particle (matter field, perturbations) and then expand quantum state for fast moving variable by a certain basis to be determined

$$|\Psi(h_a,\phi)\rangle = \psi(h_a)|\Phi(\phi,h_a)\rangle$$

$$|\Phi(\phi,h_a)\rangle = \sum_k c_k(h_a)|\Phi_k(\phi,h_a)\rangle$$

Semiclassical Quantum Gravity

[SPK, PRD 52 ('95); CQG 13 ('96); PRD 55 ('97)]

- Apply de Broglie-Bohm pilot-wave theory to gravity part only

$$\psi(h_a) = F(h_a) e^{iS(h_a)/\hbar}$$

- Then, in a semiclassical regime, WDW equation is equivalent to

$$\frac{1}{2M}(\nabla S)^2 - MV_G(h_a) + H_{nn} - \frac{\hbar^2}{2M}\frac{\nabla^2 F}{F} - \frac{\hbar^2}{M}\mathrm{Re}(Q_{nn}) = 0$$

$$\frac{1}{2}\nabla^2 S + \frac{\nabla F}{F}\cdot\nabla S + \mathrm{Im}(Q_{nn}) = 0$$

$$H_{nk}(h_a) := \langle \Phi_n(\phi, h_a) | \hat{H} | \Phi_k(\phi, h_a) \rangle \; ; \; \vec{A}_{nk}(h_a) := i \langle \Phi_n(\phi, h_a) | \nabla | \Phi_k(\phi, h_a) \rangle$$

$$Q_{nn}(h_a) := \frac{\nabla F}{F}\cdot\left(\frac{\nabla c_n}{c_n} - i\sum_k \vec{A}_{nk}\frac{c_k}{c_n}\right)$$

Semiclassical FRW Universe

- Extended superspace for a FRW with a minimal scalar and the cosmological time:

$$ds^2 = -ada^2 + a^3 d\phi^2$$

$$\frac{\partial}{\partial \tau} = -\frac{1}{Ma}\frac{\partial S(a)}{\partial a}\frac{\partial}{\partial a}, \left(\frac{\partial a(\tau)}{\partial \tau} = -\frac{1}{Ma}\frac{\partial S(a)}{\partial a} \right)$$

- Heisenberg (matrix) equation for scalar field

$$i\hbar \frac{\partial c_n}{\partial \tau} = \sum_{k \neq n} H_{nk} c_k - \hbar \sum_k B_{nk} c_k - \frac{\hbar^2}{2Ma} \sum_{k \neq n} \Omega_{nk} c_k$$

$$H_{nk}(a(\tau)) := \langle \Phi_n | \hat{H} | \Phi_k \rangle \; ; \; B_{nk}(a(\tau)) := i \langle \Phi_n | \frac{\partial}{\partial \tau} | \Phi_k \rangle$$

$$\Omega_{nk}(a(\tau)) := -\frac{1}{\dot{a}^2} \left[\left(\frac{\partial^2}{\partial \tau^2} - \frac{\ddot{a}}{\dot{a}} \frac{\partial}{\partial \tau} \right) \delta_{nk} - 2iB_{nk} \frac{\partial}{\partial \tau} + \langle \Phi_n | \frac{\partial^2}{\partial \tau^2} - \frac{\ddot{a}}{\dot{a}} \frac{\partial}{\partial \tau} | \Phi_k \rangle \right]$$

Semiclassical Chaotic Models

- Semiclassical chaotic models necessarily contain (higher) curvature terms

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \Lambda = \frac{8\pi}{3m_P^2 a^3}\left[H_{nn} - \frac{4\pi\hbar^2}{3m_P^2 a\dot{a}} U_{nn} \operatorname{Re}(R_{nn}) + \frac{2\pi\hbar^2}{3m_P^2 a}\left(U_{nn}^2 + \frac{1}{\dot{a}}\dot{U}_{nn}\right)\right]$$

$$R_{nn} = \frac{\dot{c}_n}{c_n} - i\sum_k B_{nk}\frac{c_k}{c_n}$$

$$U_{nn} := \frac{\partial F / \partial a}{F} = -\frac{1}{2}\frac{(a\dot{a})^{\cdot}}{a\dot{a}^2 + (4\pi\hbar / 3m_P^2)\operatorname{Im}(R_{nn})}$$

- Effective energy density contains curvature terms

$$\rho_{nn} = H_{nn} - \frac{4\pi\hbar^2}{3m_P^2 a\dot{a}} U_{nn} \operatorname{Re}(R_{nn}) + \frac{2\pi\hbar^2}{3m_P^2 a}\left(U_{nn}^2 + \frac{1}{\dot{a}}\dot{U}_{nn}\right)$$

Semiclassical Massive Model

- Semiclassical chaotic model with a massive scalar at order of $\hbar/M$

$$\hat{H} = -\frac{\hbar^2}{2a^3}\frac{\partial^2}{\partial\phi^2} + \frac{m^2 a^3}{2}\phi^2$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \Lambda = \frac{8\pi}{3m_P^2 a^3}\left[H_{nn} + \frac{2\pi\hbar^2}{3m_P^2 a}\left(U_{nn}^{(0)2} + \frac{1}{\dot{a}}\dot{U}_{nn}^{(0)}\right)\right]$$

$$R_{nn}^{(0)} = 0 \;\; ; \;\; U_{nn}^{(0)} = -\frac{1}{2}\frac{(a\dot{a})^{\bullet}}{a\dot{a}^2}$$

$$H_{nn} = \hbar a^3\left(n + \frac{1}{2}\right)\left[\dot{\varphi}^*\dot{\varphi} + m^2\varphi^*\varphi\right] \;;\;\; \ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + m^2\varphi = 0$$

Gauge Invariant Quantum Cosmology

Mukhanov-Sasaki Hamiltonian

- Action for scalar perturbations of metric and field & Muhkanov-Sasaki variable up to quadratic order [Mena Marugan et al, JCAP ('15)] has the total Hamiltonian

$$H = \overline{N}_0 \left[H_0 + \sum_{\vec{n},\pm} \breve{H}_2^{\vec{n},\pm} \right] + \sum_{\vec{n},\pm} \breve{G}_{\vec{n},\pm} \breve{H}_1^{\vec{n},\pm} + \sum_{\vec{n},\pm} \breve{K}_{\vec{n},\pm} \tilde{H}_1^{\vec{n},\pm}$$

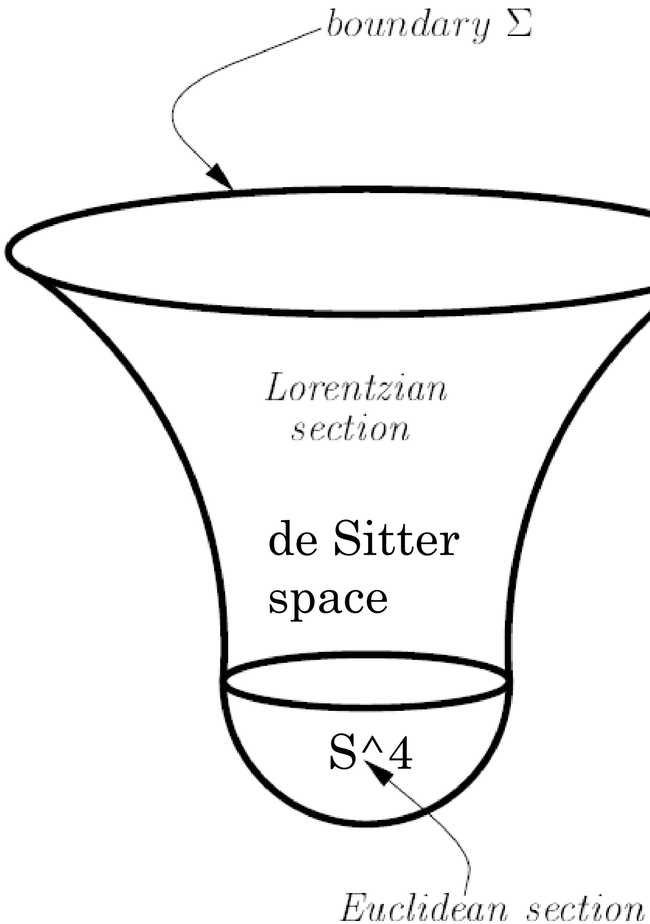
- Semiclassical cosmology from WDW equation provides the master equation for power spectrum of primordial scalar perturbations (vector and tensor perturbations)
- Semiclassical cosmology with (higher) curvatures via de Broglie-Bohm pilot theory and Born-Oppenheimer idea [SPK, Mena Maruga, work in progress].

Conclusion

- In classical theory chaotic inflation models with convex power-law seem to be excluded by CMB data.
- Starobinsky model with higher curvature term such as R^2 or $f(R)$ seems to be a viable model. However, such terms originate from quantum fluctuations.
- Quantum cosmology is a natural framework to probe both quantum nature of spacetime and inflaton and their fluctuations and predicts power spectrum with quantum effects.
- Gauge invariant quantum cosmology using Mukhanov-Sasaki Hamiltonian with a massive scalar field may yield semiclassical chaotic inflation model with (higher) curvature terms and may resurrect the chaotic scenario.

Hartle-Hawking No-Boundary Wavefunction

Hartle-Hawking No-Boundary Wave Function of the Universe



[Fig. Bousso, Hawking, gr-qc/9608009]

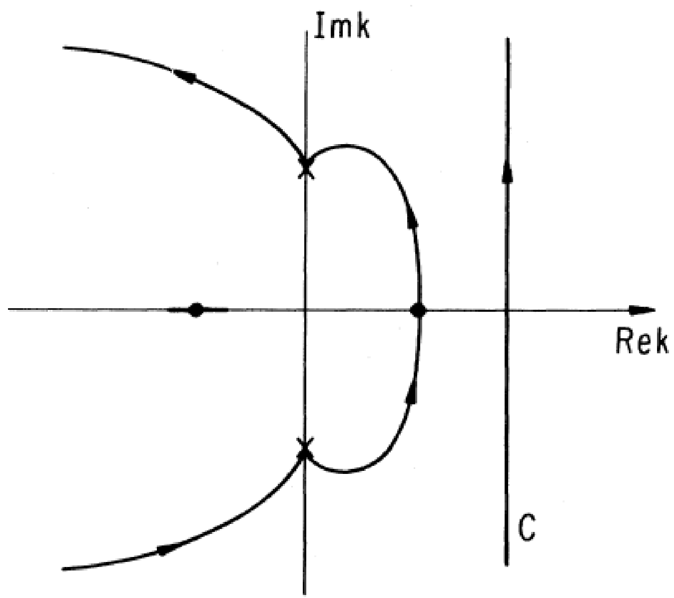
- The sum over all 4-compact Euclidean manifolds with the 3-surface boundary

$$\Psi\left[h_{ij},\phi\right]= N\int D[g]D[\phi]\exp\left(-I_E[g,\phi]\right)$$

- The HH wave function prescribes the boundary condition, i.e. "no-boundary" but one-boundary at the 3-surface.

Semiclassical Ground-State Gravitational Wave Function

$$\psi_0(a_0) = -\frac{N}{2\pi i}\int_C dk \exp\left[k a_0^3 + \frac{1}{3H^2}\left(1 - \frac{3k}{\sqrt{9k^2 + H^2}}\right)\right]$$

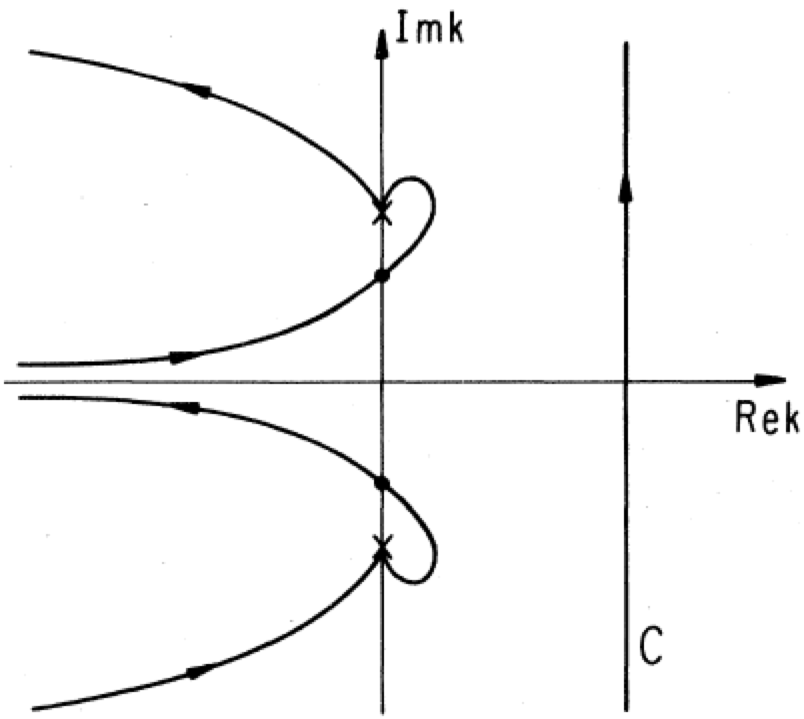


- The wave function for FRW geometries with $k = (H \cot\theta)/3$
- For $\Lambda > 0$, $Ha_0 < 1$ the contour integral leads to

$$\psi_0(a_0) = \exp\left(\frac{1}{2}a_0^2 - \frac{1}{3H^2}\right)$$

[Fig. Hartle, Hawking, PRD 28 (1983)]

Semiclassical Ground-State Gravitational Wave Function



- For $\Lambda > 0$, $Ha_0 > 1$ the contour integral leads to

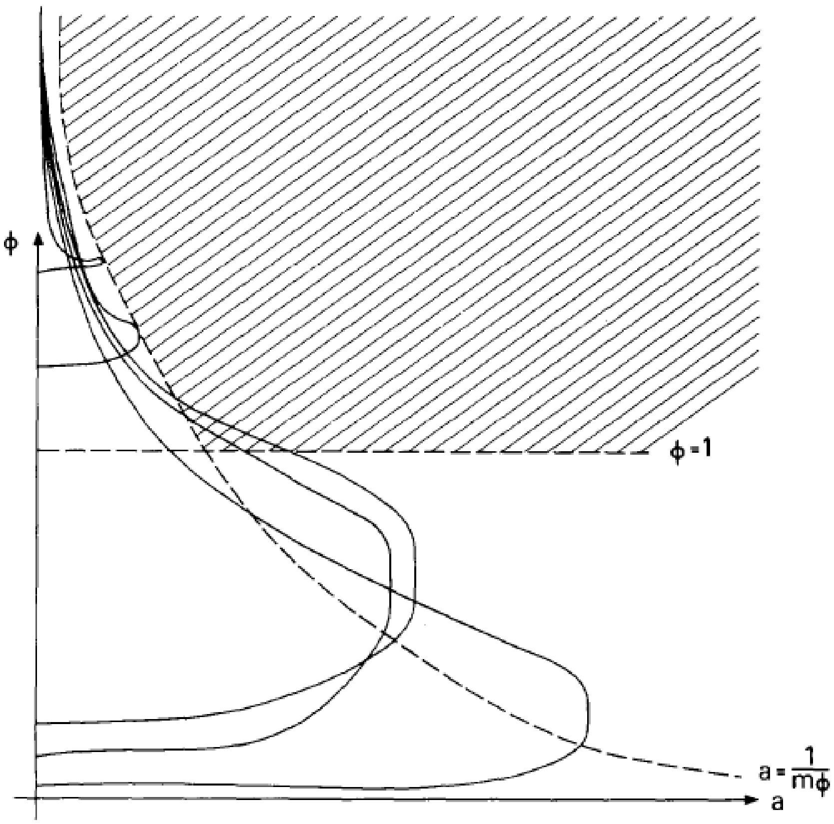
$$\psi_0(a_0) = 2\cos\left[\frac{\left(H^2 a_0^2 - 1\right)^{3/2}}{3H^2} - \frac{\pi}{4}\right]$$

- For $\Lambda > 0$, $Ha_0 >> 1$

$$\psi_0(a_0) \approx e^{iHa_0^3/3} + e^{-iHa_0^3/3}$$

[Fig. Hartle, Hawking, PRD 28 (1983)]

Hartle-Hawking No-Boundary Wave Function



Euclidean solutions for a FRW coupled to a massive field scalar [Hawking, NPB 239 ('84)]