

# Aligned natural inflation: Oscillations in primordial power spectrum

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16 Oct 2015 @ CosPA 2015

Suppose we achieve ***super-Planckian axion scale***  
through ***alignment mechanism***

$$V(\phi) = \Lambda_1^4 \left[ 1 - \cos \frac{\phi}{f_{\text{eff}}} \right] + \Lambda_2^4 \left[ 1 - \cos \left( \frac{\phi}{f} + \delta \right) \right]$$

$$\Lambda_2 \ll \Lambda_1$$
$$f \ll f_{\text{eff}}$$

What would be the ***observational consequences*** of this model?

# Natural Inflation

$$V(\phi) = \Lambda^4 \left[ 1 - \cos \frac{\phi}{f} \right]$$

***The amplitude*** can be determined by observed temperature fluctuation

***The axion scale*** is required to be ***super-Planckian***

# Axion scale

Especially in string theory,

$$f \simeq \frac{g^2}{8\pi^2} M_P \sim 10^{16} \text{GeV}$$

Axion scale is normally **sub-Planckian** in weak coupling limit

Choi, Kim '85

Banks et. al '03

Svrcek, Witten '06

# Proposals

Let's introduce ***extra axions!***

Axion alignment

**Kim, Nilles, Peloso '04**

N-flation

**Dimopolous '05**

Other variations

**Choi, HK, Yun '14**

**Higaki, Takahashi '14**

**Tye, Wong '14**

**Ben-Dayan, Pedro, Westphal '14**

**Harigaya, Ibe '14**

**... ..**

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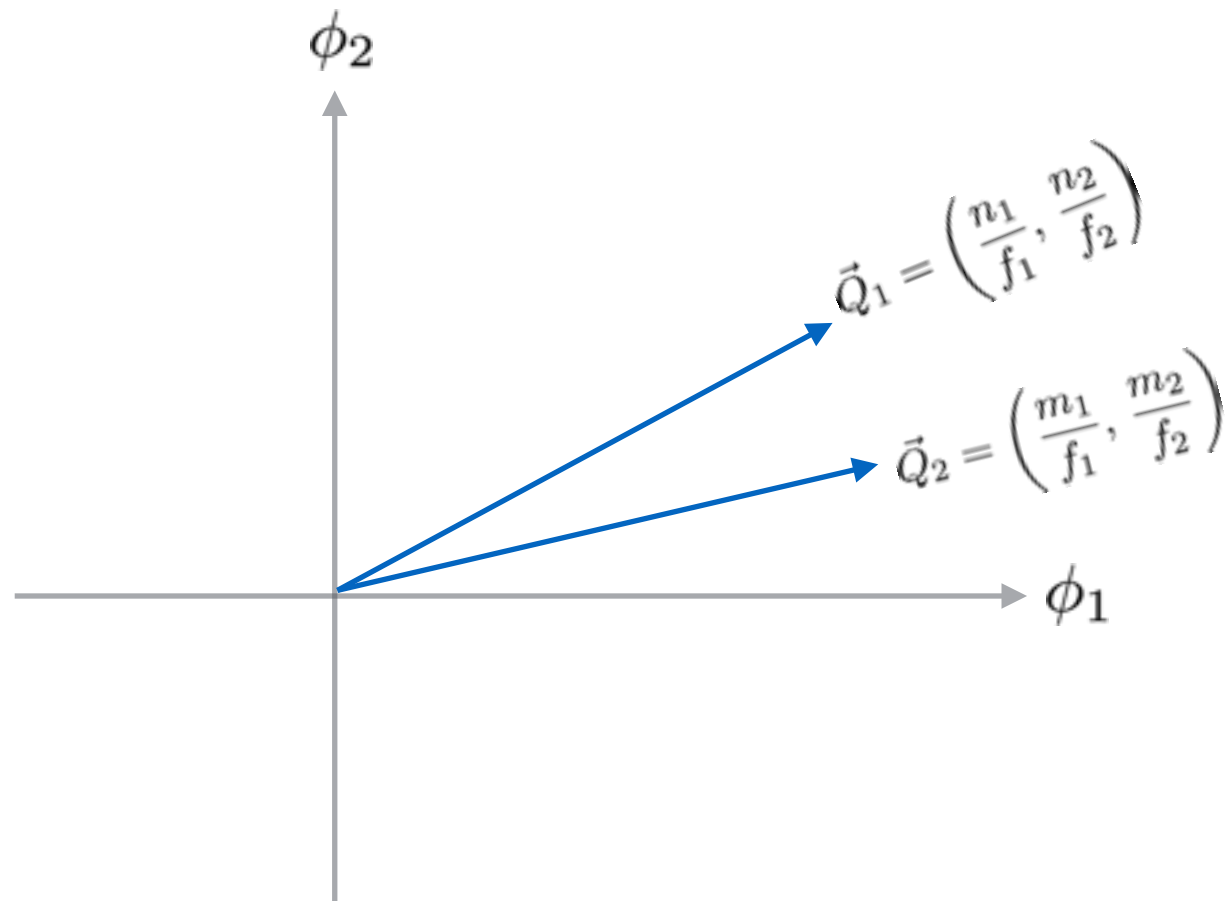
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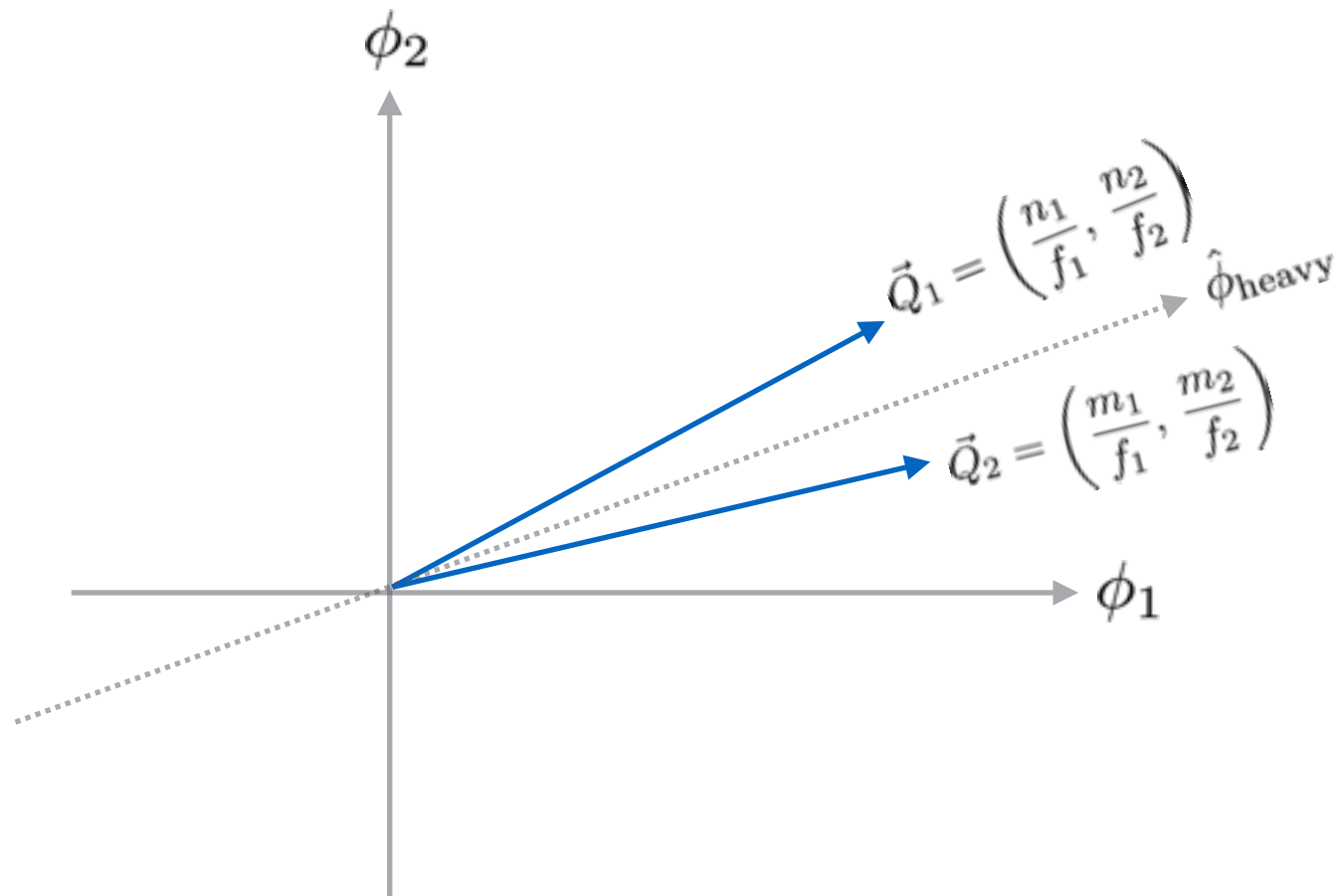
# Axion alignment

$$V = \Lambda^4 e^{-S_1} \left[ 1 - \cos \left( \frac{n_1}{f_1} \phi_1 + \frac{n_2}{f_2} \phi_2 \right) \right] + \Lambda^4 e^{-S_2} \left[ 1 - \cos \left( \frac{m_1}{f_1} \phi_1 + \frac{m_2}{f_2} \phi_2 \right) \right]$$



# Axion alignment

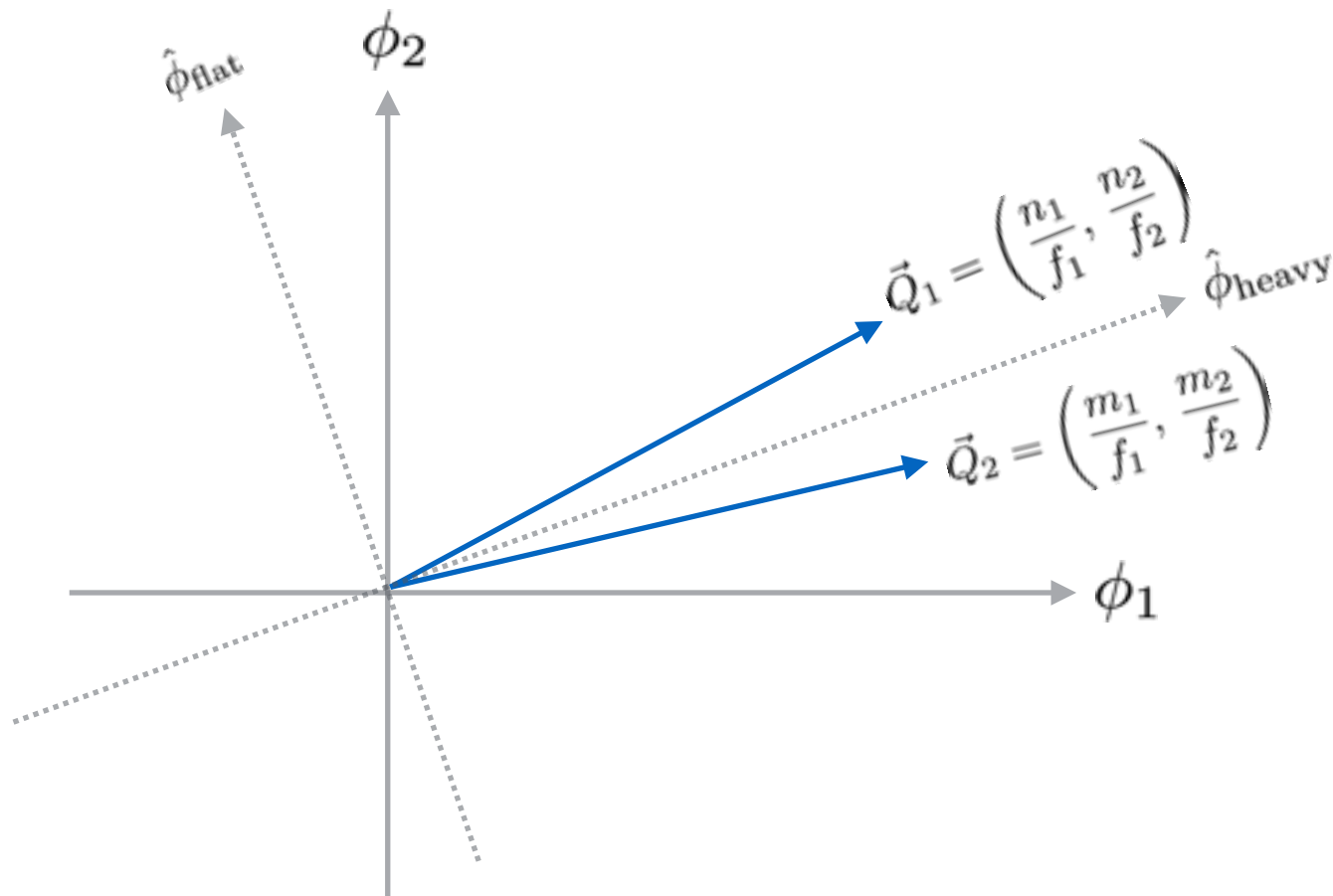
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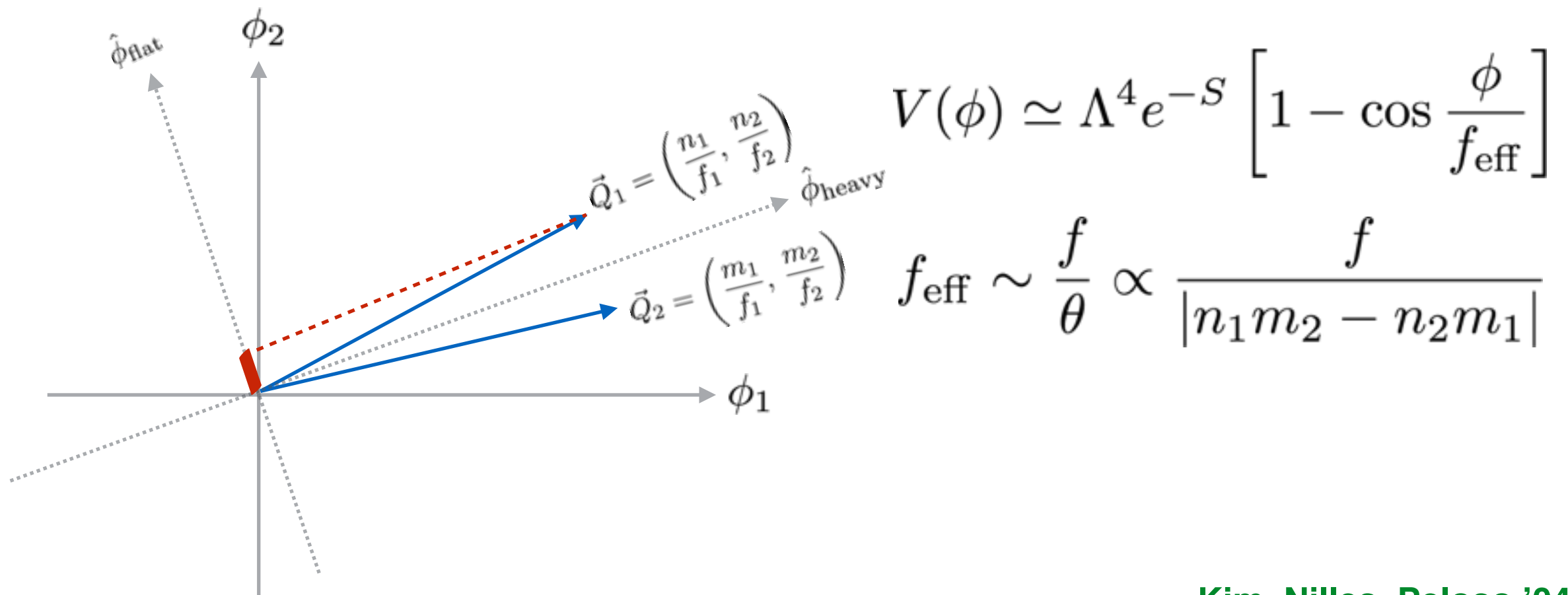
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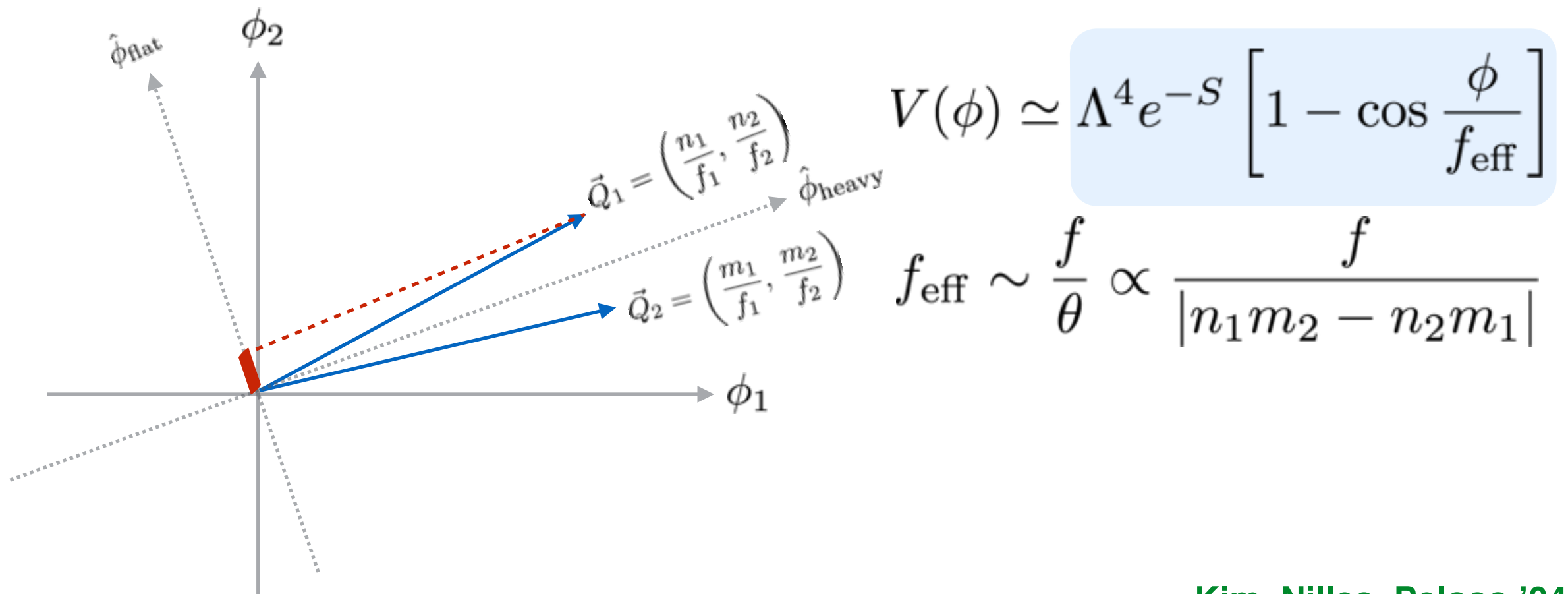
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# Weak gravity conjecture on axions

Consider a theory of ***U(1) gauge boson + gravity***

$$q \geq \frac{m}{M_P}$$

Consider a theory of ***axion + gravity***

$$\frac{1}{f} \geq \frac{S_{\text{inst}}}{M_P} \quad \Leftrightarrow \quad f \leq \frac{M_P}{S_{\text{inst}}}$$

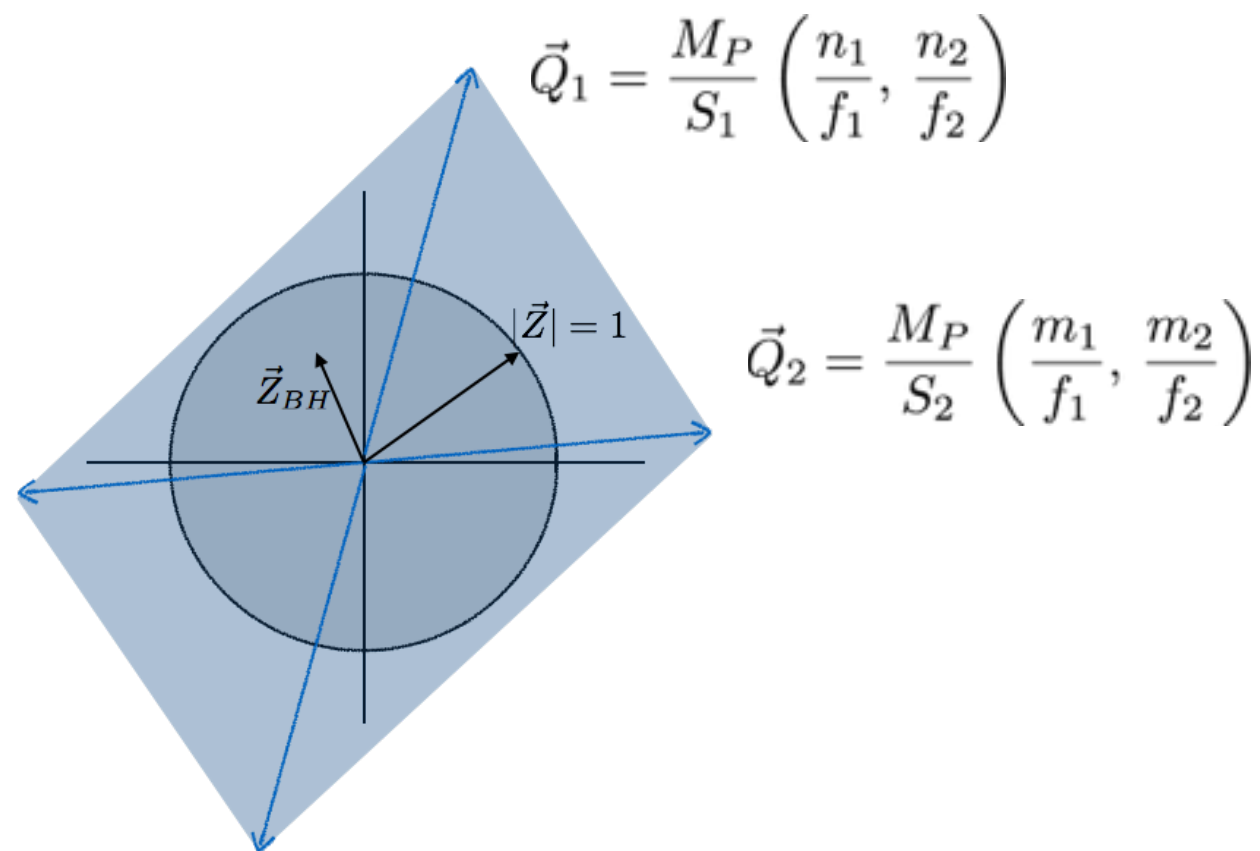
***axion scale*** is ***bounded*** from above!

# Weak gravity conjecture on axions

How about ***N axions?***

***Convex Hull condition*** Cheung et. al. '14

$$V = \Lambda^4 e^{-S_1} \left[ 1 - \cos \left( \frac{n_1}{f_1} \phi_1 + \frac{n_2}{f_2} \phi_2 \right) \right] + \Lambda^4 e^{-S_2} \left[ 1 - \cos \left( \frac{m_1}{f_1} \phi_1 + \frac{m_2}{f_2} \phi_2 \right) \right]$$



Brown et. al. '15

$$f_{\text{eff}} < M_P$$

Rudelius '15

Montero et. al. '15

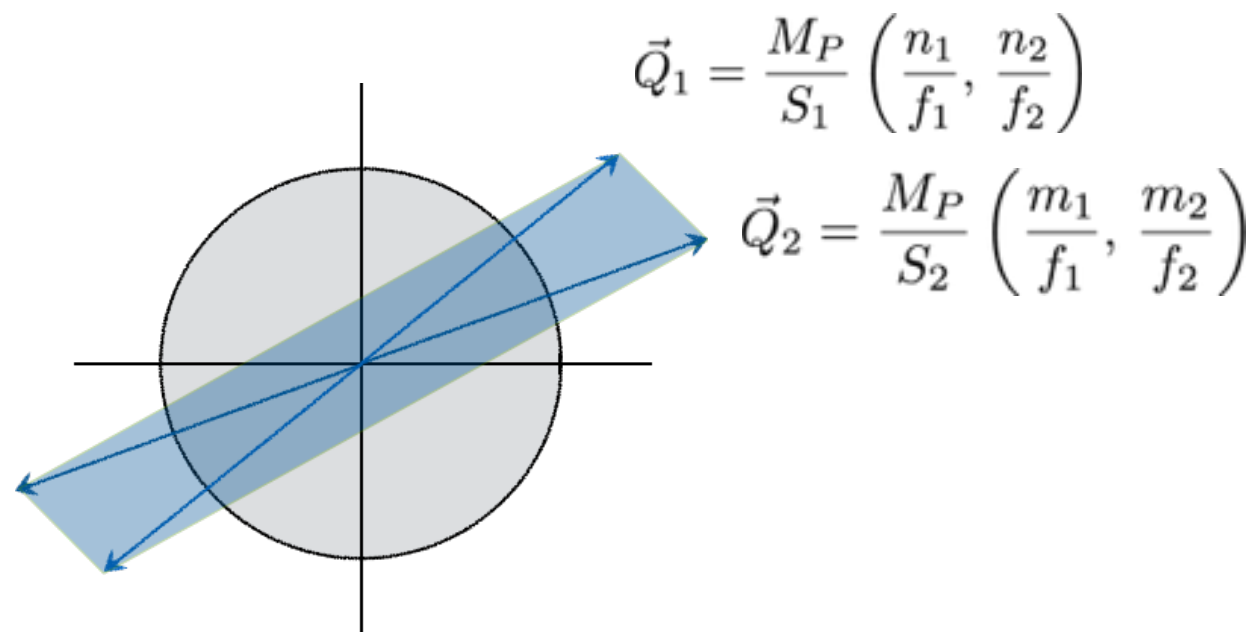
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Brown et. al. '15

***Alignment case***

$$f_{\text{eff}} < M_P$$

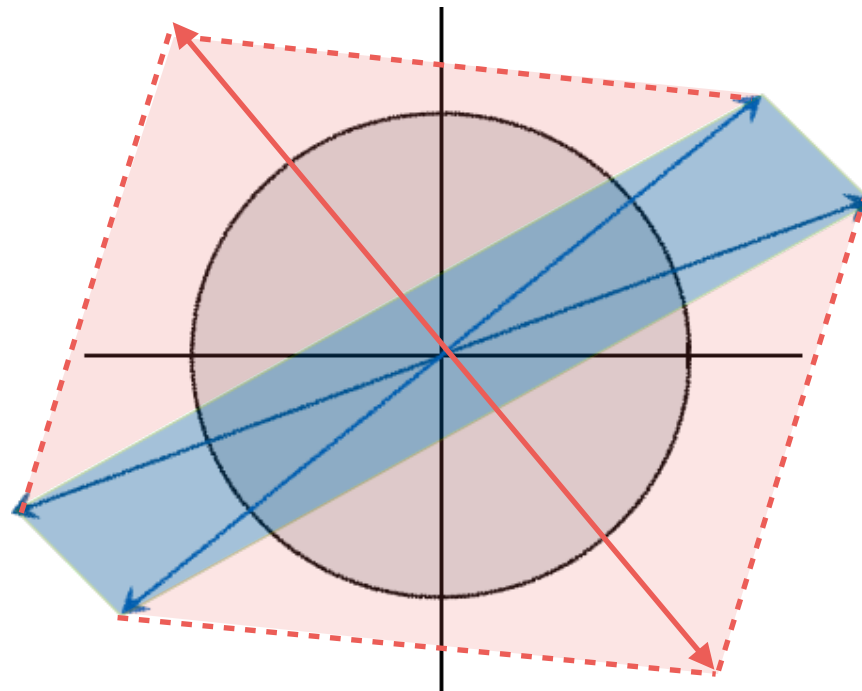
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...

# Weak gravity conjecture on axions

One way of circumventing this is to ***add extra instanton!***



$$f \ll f_{\text{eff}}$$

$$S \ll S_{\text{large}}$$

$$V(\phi) = \Lambda^4 e^{-S} \left[ 1 - \cos \frac{\phi}{f_{\text{eff}}} \right] + \Lambda^4 e^{-S_{\text{large}}} \left[ 1 - \cos \left( \frac{\phi}{f} + \delta \right) \right]$$

# Back to the original question

If the modulation is natural consequence  
of alignment mechanism,

$$V(\phi) = \Lambda_1^4 \left[ 1 - \cos \frac{\phi}{f_{\text{eff}}} \right] + \Lambda_2^4 \left[ 1 - \cos \left( \frac{\phi}{f} + \delta \right) \right]$$

$f \ll f_{\text{eff}}$

***How do they change the shape of primordial power spectrum?***

***How could they be constrained*** by the recent observational data?



# Former study: Axion monodromy

Axion monodromy potential with cosine modulation

$$V(\phi) = \mu^3 \phi + \Lambda^4 \cos \left( \frac{\phi}{f} + \delta \right)$$

***induces oscillations*** in primordial power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}*} \left( \frac{k}{k_*} \right)^{n_s-1} \left[ 1 + \delta n_s \cos \left( \frac{\phi_k}{f} + \alpha \right) \right]$$

# Natural inflation with modulations

Natural inflation with cosine modulation

$$V(\phi) = \Lambda_1^4 \left[ 1 - \cos \frac{\phi}{f_{\text{eff}}} \right] + \Lambda_2^4 \left[ 1 - \cos \left( \frac{\phi}{f} + \delta \right) \right]$$

***also induces oscillations*** in primordial power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}*} \left( \frac{k}{k_*} \right)^{n_s - 1} \left[ 1 + \delta n_s \cos \left( \frac{\phi_k}{f} + \alpha \right) \right]$$

# Natural inflation with modulations

Natural inflation with cosine modulation

$$\delta n_s = \left( \frac{\Lambda_2^4 f_{\text{eff}}}{\Lambda_1^4 f} \right) \frac{3 \sqrt{2\pi\gamma \coth \frac{\pi}{2\gamma}}}{\sin(\phi_*/f_{\text{eff}}) \sqrt{(1 + \frac{3}{2} \gamma^2 / f_{\text{eff}}^2)^2 + (3\gamma)^2}} \quad \gamma = f_{\text{eff}} f \tan \frac{\phi_*}{2f_{\text{eff}}}$$

*also induces oscillations*

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}*} \left( \frac{k}{k_*} \right)^{n_s - 1} \left[ 1 + \delta n_s \cos \left( \frac{\phi_k}{f} + \alpha \right) \right]$$

# Oscillations in power spectrum

For ordinary slow-roll inflation, power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = A_* \left( \frac{k}{k_*} \right)^{(n_s - 1) + \frac{1}{2} \frac{dn_s}{d \ln k} \ln(k/k_*) + \frac{1}{6} \frac{d^2 n_s}{d \ln k^2} \ln^2(k/k_*) + \dots}$$

is described by a set of parameters

$$A_* \quad n_s \quad \frac{dn_s}{d \ln k} \quad \frac{d^2 n_s}{d \ln k^2} \quad \dots$$

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# Oscillations in power spectrum

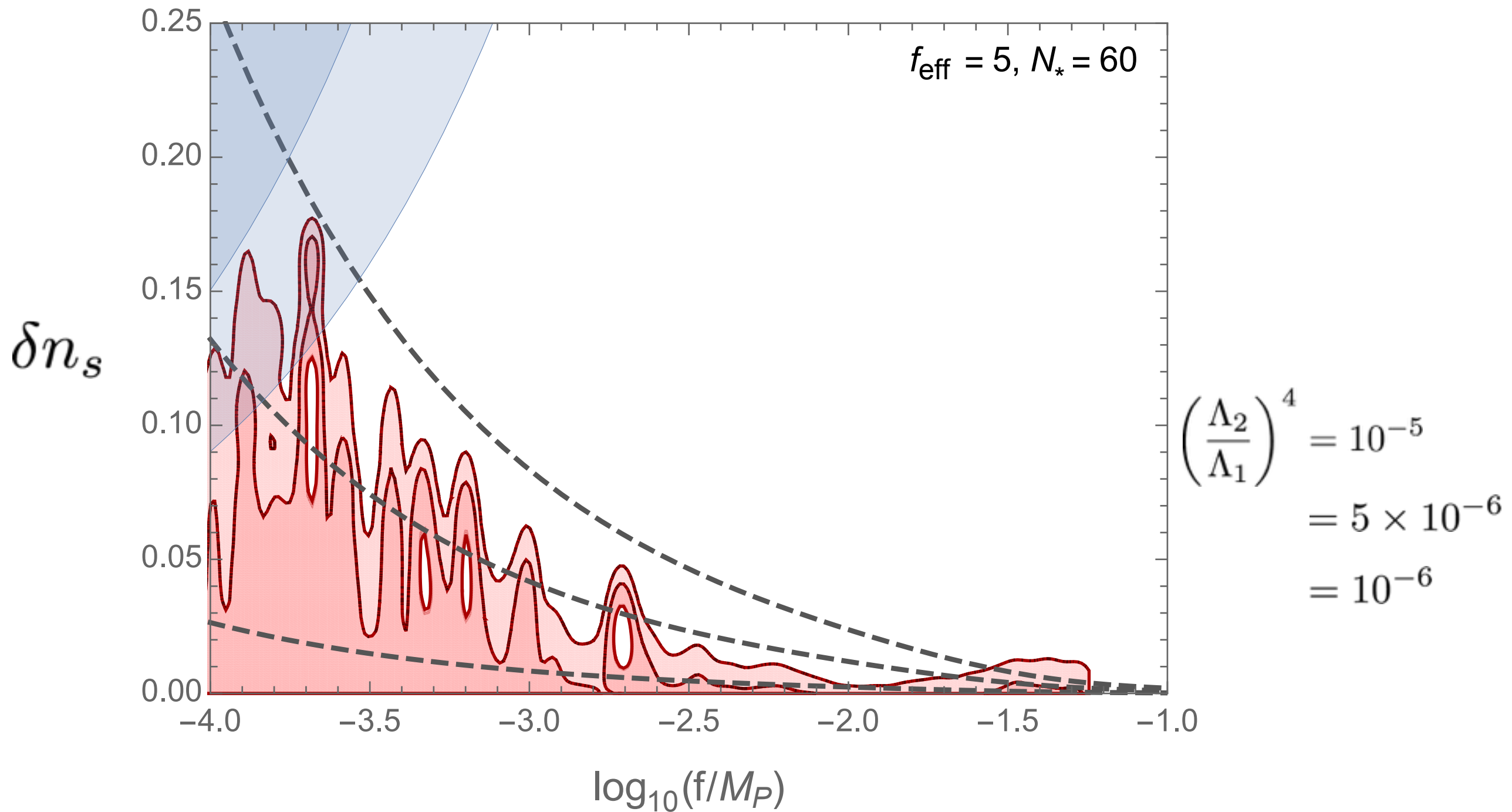
In this scenario, power spectrum looks like

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}*} \left( \frac{k}{k_*} \right)^{n_s-1} \left[ 1 + \delta n_s \cos \left( \frac{\phi_k}{f} + \alpha \right) \right]$$

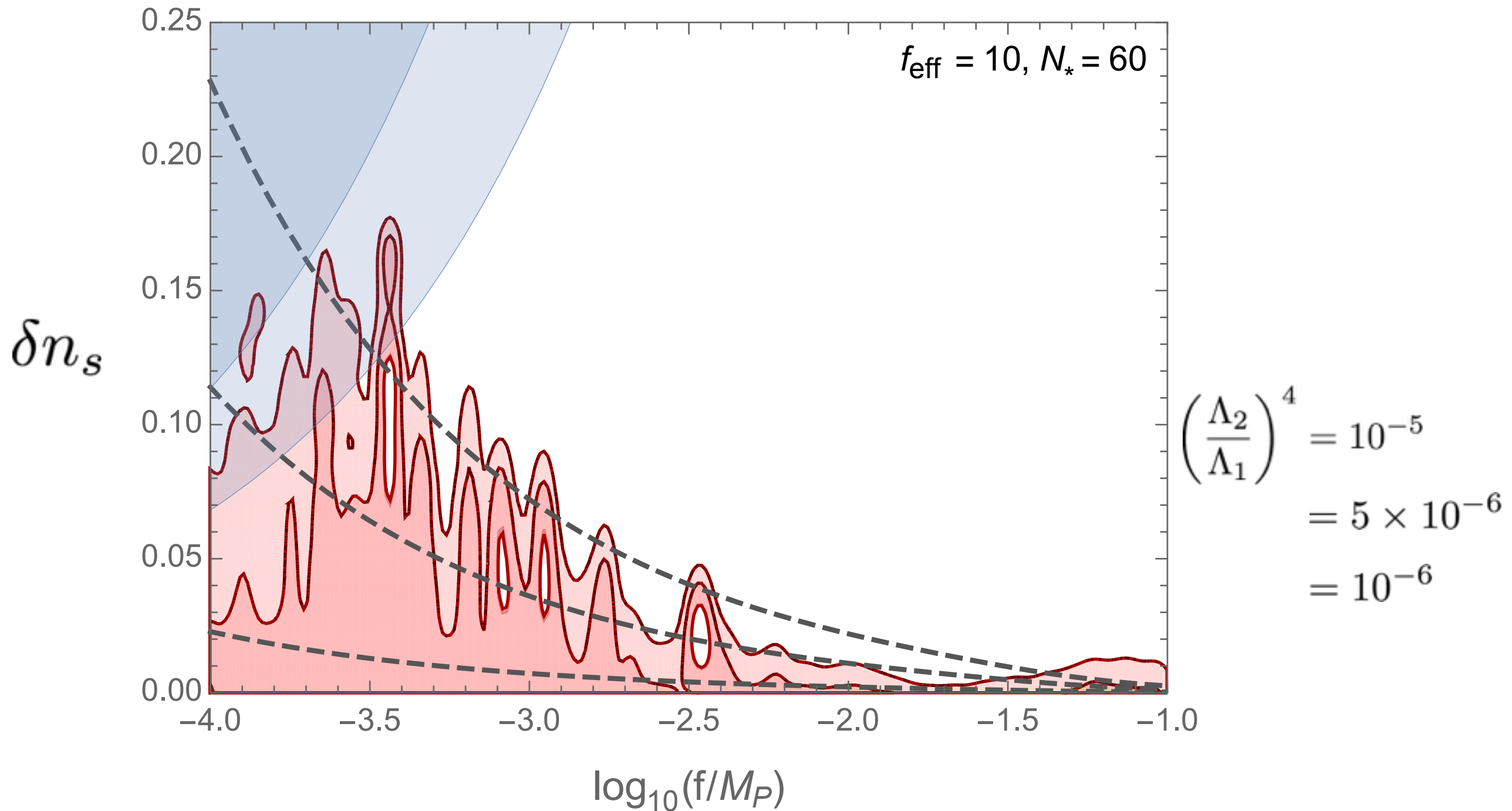
the standard template  $(A_{\mathcal{R}*}, n_s)$

***does not capture*** characteristic features of this scenario.

# Constraints on oscillations

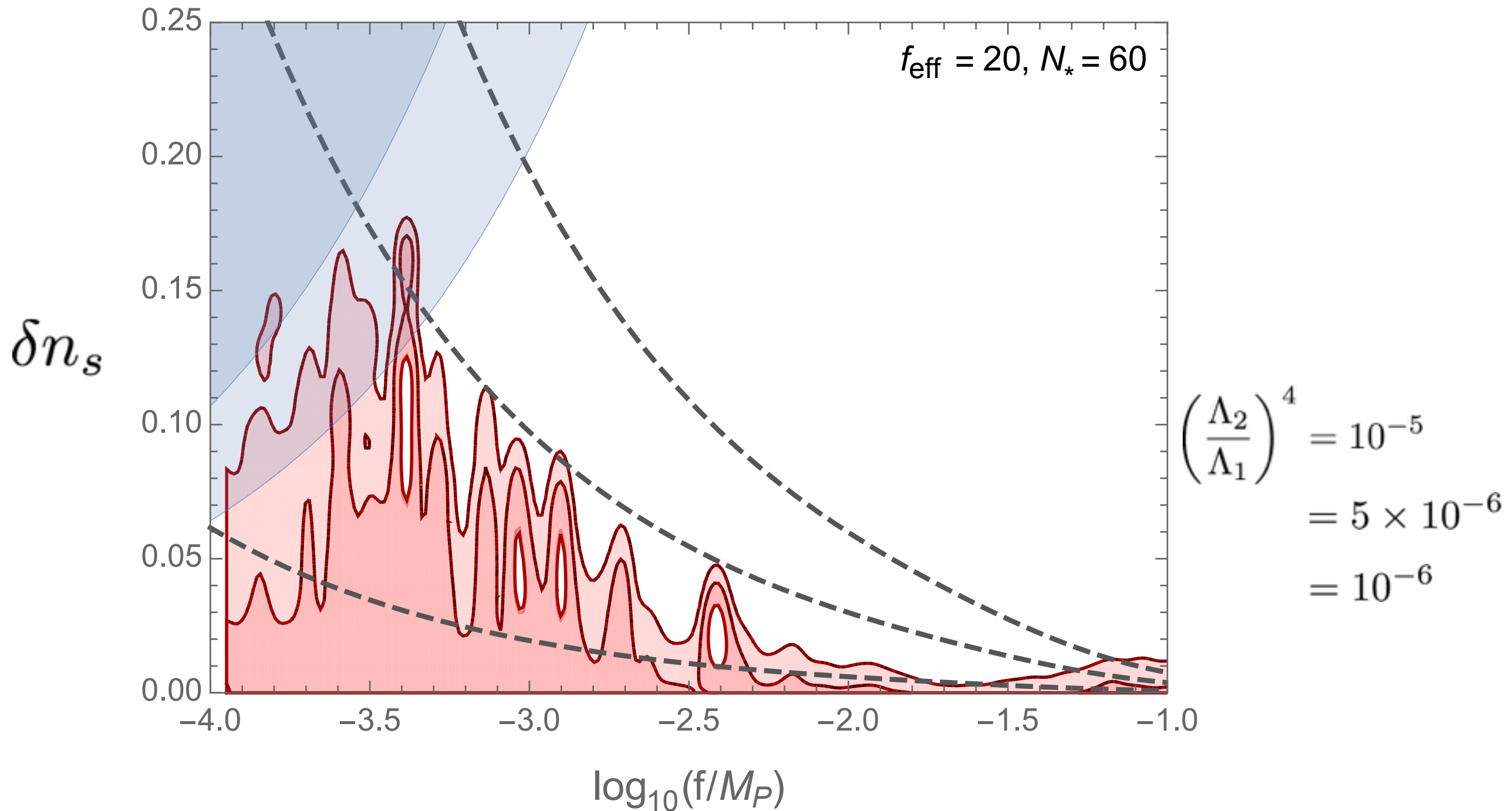


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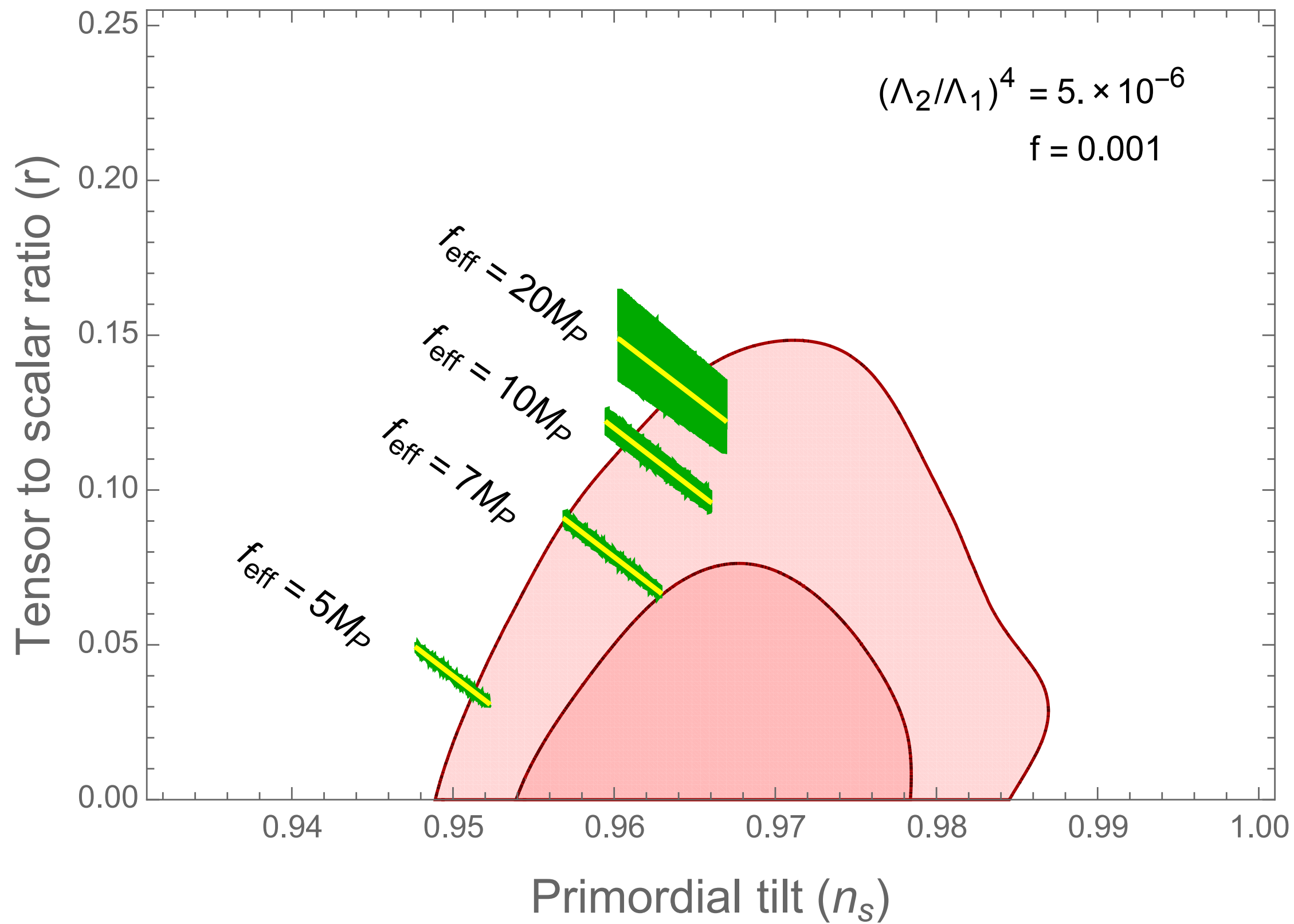


# tensor-to-scalar ratio?

tensor-to-scalar ratio can change, but only by

$$r = \frac{\mathcal{P}_t}{\mathcal{P}_{\mathcal{R}}} = \frac{r_0}{1 + \delta n_s \cos(\phi_k / f)}$$

as the amplitude of oscillation is constrained by observation,  
only **~10%** change is possible



# Summary

$$V(\phi) = \Lambda_1^4 \left[ 1 - \cos \frac{\phi}{f_{\text{eff}}} \right] + \Lambda_2^4 \left[ 1 - \cos \left( \frac{\phi}{f} + \delta \right) \right]$$

***Observational signature?***

***Oscillations!***

***Any constraints?***

***Amplitude of oscillation leads to  $(\Lambda_2/\Lambda_1)^4 \lesssim 5 \times 10^{-6}$***

***tensor-to-scalar ratio can change by  $\sim 10\%$***