A Particle Probing Thermodynamics in Rotating AdS Black Hole

Based on arXiv:1509.06691 "Cosmic Censorship of Rotating Anti-de Sitter Black Hole with a Probe"

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Motivation

- We test the validity of cosmic censorship in a 4-dimensional rotating AdS black hole through a particle absorption.
- The particle conserved quantities are redefined to satisfy the 1st and 2nd law of thermodynamics of the black hole.
- We can show the validity of cosmic censorship in the extremal black hole.

4-dimensional rotating AdS black hole

- The metric:

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(a\,dt - \frac{r^2+a^2}{\Xi}d\phi\right)^2 ,$$

$$\rho^2 = r^2 + a^2\cos^2\theta\,,\ \Delta_r = (r^2+a^2)(1+\frac{r^2}{\ell^2}) - 2Mr\,,\ \Delta_\theta = 1 - \frac{a^2}{\ell^2}\cos^2\theta\,,\ \Xi = 1 - \frac{a^2}{\ell^2}\,.$$

- AdS radius, mass parameter, spin parameter.
- Horizon area and Bekenstein-Hawking entropy:

$$\mathcal{A}_h = \frac{4\pi\left(r_h^2+a^2\right)}{\Xi}\,, \quad S_{BH} = \frac{1}{4}\mathcal{A}_h = \frac{\pi\left(r_h^2+a^2\right)}{\Xi}\,,$$

The 1st law of thermodynamics in AdS BH

- The rotating velocity at the horizon: $\Omega_h = \frac{a\Xi}{r_h^2 + a^2}.$
- But, $dM_B \neq T_H dS_{BH} + \Omega_h dJ_B$
- The rotating velocity at $r \to \infty$: $\Omega_\infty = -\frac{a}{\ell^2}.$
- Redefine the rotating velocity at the horizon: $\Omega = \Omega_h - \Omega_\infty = \frac{a\left(1 + \frac{r_h^2}{\ell^2}\right)}{r_h^2 + a^2}$

  S. W. Hawking, C. J. Hunter, M. Taylor, Phys. Rev. D 59, 064005 (1999)
- Under this redefinition, the 1st law of thermodynamics:

$$dM_B = T_H dS_{BH} + \Omega dJ_B$$

- The black hole mass and angular momentum: $M_B = \frac{M}{\Xi^2}, \quad J_B = \frac{aM}{\Xi^2},$

  M. M. Caldarelli, G. Cognola, D. Klemm, Class. Quant. Grav. 17, 399 (2000)
  G. W. Gibbons, M. J. Perry, C. N. Pope, Class. Quant. Grav. 22, 1503 (2005)

The particle equations of motions

- The black hole properties can be changed by a particle quantities.
- Using Hamilton-Jacobi method:

$$\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu \quad S = \frac{1}{2} m^2 \lambda - Et + L\phi + S_r(r) + S_\theta(\theta)$$

- Equations of motions with a separate variable:

$$p^r = \dot{r} = \frac{\Delta_r}{\rho^2}\sqrt{R(r)}, \quad p^\theta = \dot{\theta} = \frac{\Delta_\theta}{\rho^2}\sqrt{\Theta(\theta)}.$$

$$R(r) = \frac{\mathcal{K}}{\Delta_r} + \frac{1}{\Delta_r^2}\left(a\Xi L - \left(r^2 + a^2\right) E\right)^2 - \frac{m^2 r^2}{\Delta_r},$$

$$\Theta(\theta) = -\frac{\mathcal{K}}{\Delta_\theta} - \frac{1}{\Delta_\theta^2}\left(\Xi L \csc\theta - aE\sin\theta\right)^2 - \frac{m^2 a^2 \cos^2\theta}{\Delta_\theta}.$$

The particle absorption

- The particle energy is obtained for given location and momenta:

$$\alpha E^2 + 2\beta E + \gamma = 0\,,$$

$$\alpha = \frac{(r^2+a^2)^2}{\Delta_r} - \frac{a^2 \sin^2\theta}{\Delta_\theta}\,, \quad \beta = -\frac{(r^2+a^2)(aL\Xi)}{\Delta_r} + \frac{aL\Xi}{\Delta_\theta}\,, \quad \gamma = -\frac{(p^r)^2\rho^4 - a^2L^2\Xi^2}{\Delta_r} - \frac{\left(p^\theta\right)^2\rho^4 + L^2\Xi^2\csc^2\theta}{\Delta_\theta} - m^2\rho^2\,.$$

- Solution at the horizon:

$$E_h = \frac{a\Xi}{r_h^2+a^2}L + \frac{\rho_h^2}{r_h^2+a^2}|p^r|\,.$$

- However, this equation is not consistent with thermodynamic laws.
- This can be resolved by energy redefinition.
- The particle energy moving φ-plane at the boundary: $E_\infty = -\frac{a}{\ell^2}L < 0$
- To avoid negative energy:

$$E = E_h - E_\infty = \frac{a\left(1+\frac{r_h^2}{\ell^2}\right)}{r_h^2+a^2}L + \frac{\rho_h^2}{r_h^2+a^2}|p^r|\,.$$

- The black hole infinitesimally changes by the particle:

$$E = \delta M_B \,, \quad L = \delta J_B \,,$$

- Under the changes, the black hole entropy always increases.

$$\delta S_{BH} = \frac{4\pi\rho_h^2}{\dot{D}} |p^r| \geq 0$$

- This satisfies the 2nd law of thermodynamics.
- Using this relation, the redefined particle energy becomes:

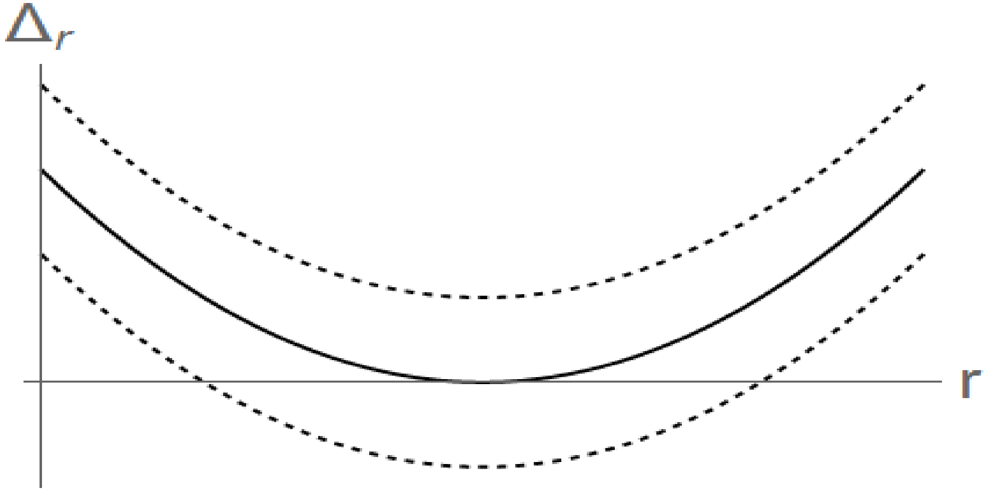
$$\delta M_B = \Omega \, \delta J_B + T_H \, \delta S_{BH}$$

- This is the 1st law of thermodynamics.
- The particle is now consistent with the laws of thermodynamics.

Testing the validity of cosmic censorship

- We will investigate whether the extremal black hole can be overspun through particle absorption.
- If the horizon disappears, the black hole becomes a naked singularity, and cosmic censorship is invalid.
- The black hole horizon only depends on the function Δ_r.
- The extremal black hole is changed as

$$(M, J) \to (M + \delta M, J + \delta J)$$



- The location of minimum point: $\delta r_e = -\frac{\dot{D}_M P_J + \dot{D}_J}{\ddot{D}} L - \frac{P_P}{\ddot{D}} |p^r| \,,$

$$\ddot{D} = \frac{\partial \dot{D}}{\partial r_h} = 2\left(1+\frac{r_h^2}{\ell^2}\right) + \frac{8r_h^2}{\ell^2} + \frac{2\left(r_h^2+a^2\right)}{\ell^2}\,,$$
$$\dot{D}_M = \frac{\partial \dot{D}}{\partial M_B} = -\frac{8a^2\Xi}{\ell^2} - 2\Xi^2 - \frac{4a^2 r_h \Xi^2}{M\ell^2}\,,$$
$$\dot{D}_J = \frac{\partial \dot{D}}{\partial J_B} = \frac{8a\Xi}{\ell^2} + \frac{4a r_h \Xi^2}{M\ell^2}\,.$$

- The function Δ_r: $\Delta_r(r_h + \delta r_e) = D_M P_P \, |p^r| \,, \leq 0$

$$D_M = \frac{\partial \Delta_h}{\partial M_B} = -\frac{8a^2 r_h \Xi}{\ell^2} - 2r_h\Xi^2 - \frac{2a^2\left(1+\frac{r_h^2}{\ell^2}\right)\Xi^2}{M}\,,$$
$$D_J = \frac{\partial \Delta_h}{\partial J_B} = -\frac{8a r_h \Xi}{\ell^2} + \frac{2a\left(1+\frac{r_h^2}{\ell^2}\right)\Xi^2}{M}\,.$$

- The black hole mass increases more than the angular momentum of the black hole, and the black hole becomes non-extremal one.
- Thus, cosmic censorship is valid under the particle absorption.

Summary

- We construct particle energy equation to satisfy the 1st and 2nd laws of thermodynamics using energy regularization.
- In the particle absorption, the extremal black hole mass increases more than the angular momentum of the black hole.
- It becomes non-extremal one, and the horizon still exists.
- Therefore, cosmic censorship is valid.

THANK YOU！