Theory for Neutrino Mixing

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SJTU/NTU/NCTS

The Nobel Prize in Physics 2015

Ill. N. Elmehed. © Nobel Media AB 2015.

Takaaki Kajita

Prize share: 1/2

Ill. N. Elmehed. © Nobel Media AB 2015.

Arthur B. McDonald

Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

β decay

December 4, 1930

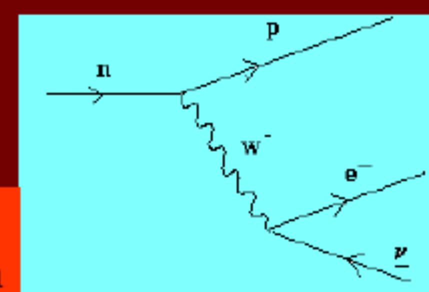
Dear radioactive ladies and gentlemen,

...I have hit upon a 'desperate remedy' to save...the law of conservation of energy. Namely the possibility that there exists in the nuclei electrically neutral particles, that I call neutrons...I agree that my remedy could seem incredible...but only the one who dare can win...

Unfortunately I cannot appear in person, since I am indispensable at a ball here in Zurich.

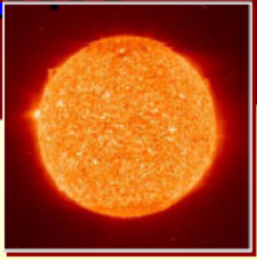
Your humble servant
W. Pauli

Note: this was before the discovery of the real neutron



Where do Neutrinos come from?

✓ Nuclear Reactors (power stations, ships)

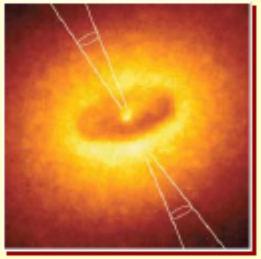


Sun ✓

✓ Particle Accelerator

Supernovae (star collapse)
SN 1987A ✓

✓ Earth's Atmosphere (Cosmic Rays)



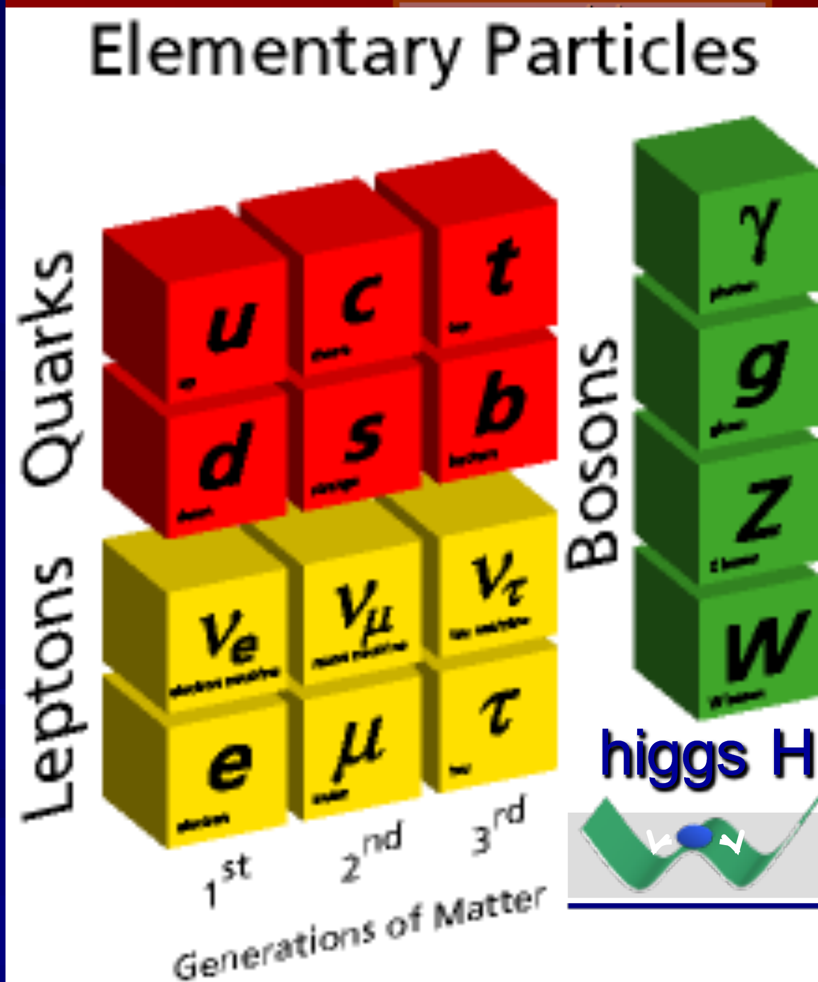
Astrophysical Accelerators
Soon ?

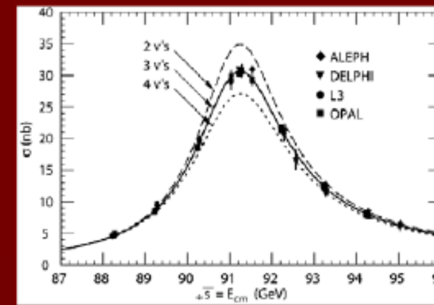
✓ Earth's Crust (Natural Radioactivity)

Big Bang (here 330 ν/cm^3)
Indirect Evidence

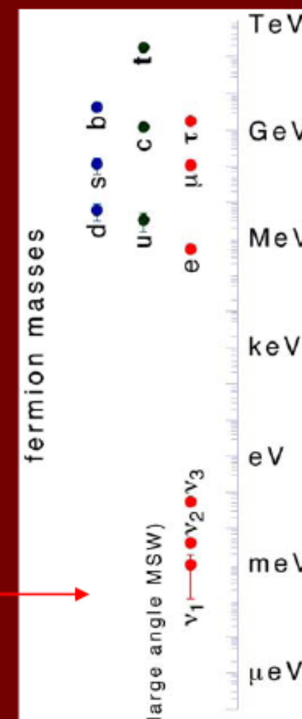
Three types of active light neutrinos

Nature's building blocks





Three flavors or generations, and no more, and we do not know why.



Some mass, but curiously little.

The Nobel Prize in Physics 2015

Ill. N. Elmehed. © Nobel Media AB 2015.

Takaaki Kajita

Prize share: 1/2

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Mixing and non-zero neutrino mass

$$\begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

At t=0, neutrino $\nu_1(0)$ produced, at t, the state becomes

$$|\nu_1(0) >= \quad \cos\theta e^{-i(Et-p_1L)}|\nu_{m1}> \; -\sin\theta e^{-i(Et-p_2L)}|\nu_{m2}>.$$

Using $t \approx L$ and $E - P_i = E - \sqrt{E^2 - m_i^2} \approx m_1^2/2E$,
the probability amplitude of find $\nu_2(0)$ at time t is given by

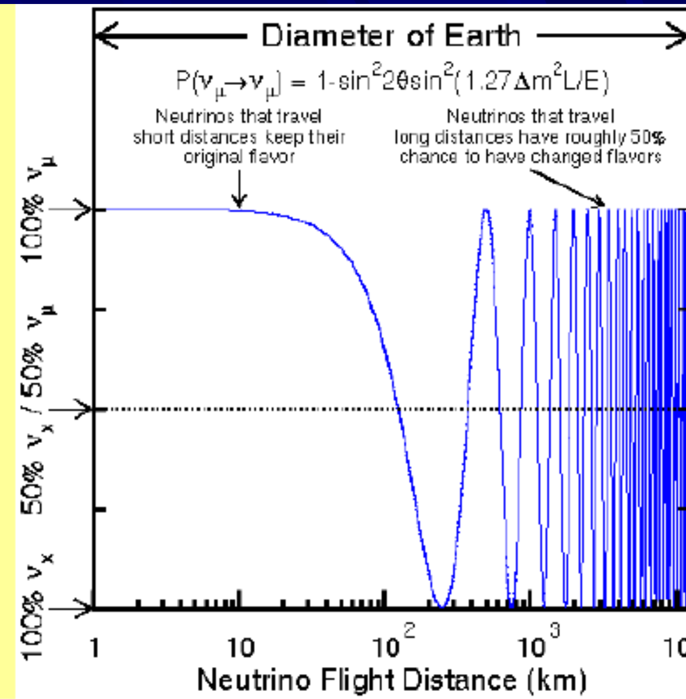
$$< \nu_2(0)|\nu_1(t) >= \cos\theta\sin\theta(e^{-im_1^2L/2E} - e^{-im_2^2L/2E})$$

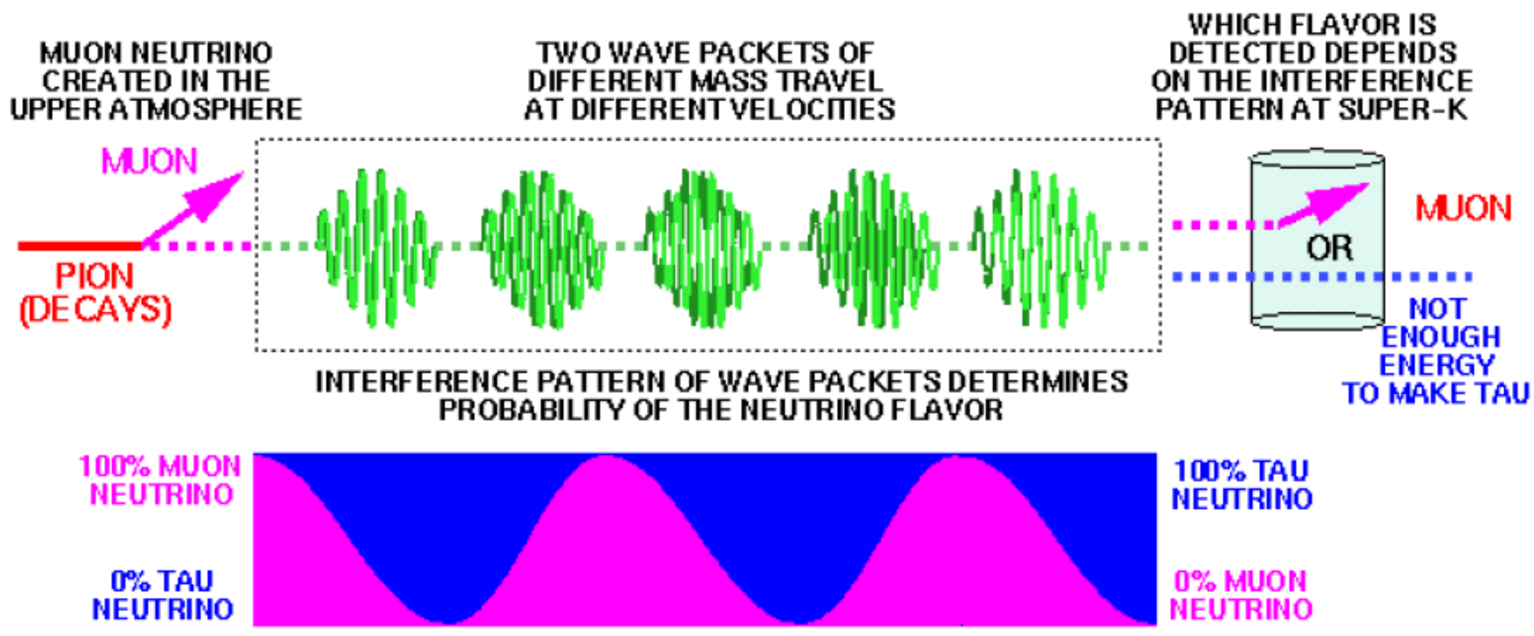
which leads to the probability of finding $\nu_2(0)$ is

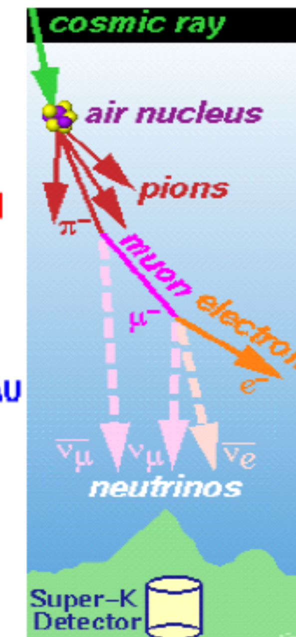
$$P(\nu_1 \to \nu_2) = |<\nu_1(0)|\nu_2(t)>|^2 = \sin^2(2\theta)\sin^2(\Delta m_{21}^2 L/4E),$$
$$\Delta m_{21}^2 = m_2^2 - m_1^2.$$

$$P(\nu_e \to \nu_\mu) \cong \sin^2(2\theta)\sin^2[\frac{\Delta m^2}{4}\frac{L}{E}]$$

$$m_1 \neq m_2$$







- Active neutrinos have small mass, they mix with each other
- There may be additional light sterile neutrinos
- Electrically neutral, has the possibility of being its own anti-particle, Majorana particle. Dirac or Majorana particle?

Theoretical Models for Neutrinos

In the minimal SM: Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$G(8,1)\ (0), W(1,3)\ (0), B\ (1,1)(0)\ ,$$
$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} (3,2)(1/6)\ ,\ \ U_R\ (3,1)(2/3)\ ,\ \ D_R\ (3,1)(-1/3)\ ,$$
$$L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} (1,2)(-1/2)\ ,\ \ E_R\ (1,1)(-1)\ ,$$
$$H = \begin{pmatrix} h^+ \\ (v+h^0)/\sqrt{2} \end{pmatrix} (1,2,1/2)\ ,\ \ \text{v - vev of Higgs}\ .$$

Quark and charged lepton masses are from the following Yukawa coupolings

$$\bar{Q}_L \tilde{H} U_R\ ,\ \ \bar{Q}_L H D_R\ ,\ \ \bar{L}_L H E_R\ .$$

Nothing to pair up with $L_L(\nu_L)$. In minimal SM, neutrinos are massless!

Extensions needed: Give neutrino masses and small ones!

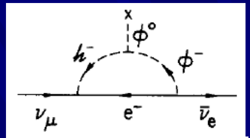
To have Dirac mass, need to introduce right handed neutrinos ν_R: (1,1)(0)

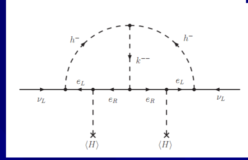
Dirac neutrino mass term

$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C\ , \to\ \ -\bar{\nu}_L m_\nu \nu_R\ , \to m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$

$m_{\nu_e} < 0.3$ eV, $\to$ $Y_{\nu_e}/Y_e < 10^{-5}$, very much fine tuned!

Some theoretical models for neutrino masses





Loop generated neutrino masses:

The Zee Model(1980); Zee-Babu Model (zee 1980; Babu, 1988)

Other loop models: Babu&He; E. Ma; Mohapatra et al; Geng et al

Seesaw Models:

$$M_\nu = \begin{pmatrix} 0 & Y_\nu v/\sqrt{2} \\ Y_\nu^T v/\sqrt{2} & M_R \end{pmatrix}$$

Type I Introduce singlet neutrinos

(Minkowski (1977); Gell-Mann, Ramond, and Slansky (1979); Yanagida (1979); Glashow (1980); Mohapatra and Senjanovic(1980))

$$L = \bar{\nu}_L (Y_\nu v/\sqrt{2}) \nu_R + \bar{\nu}_R^c M_R \nu_R / 2$$

Type II: Introduce triplet Higgs representation $\Delta : (1,3,1)$, (W. Konetschny, and W. Kummer, 1977; L.-F. Li and T.-P. Cheng, 1980; Gelmini and Roncadelli, 1981)

Type-III: Introduce triplet lepton representations Σ: (1,3,0))
(Foot, Lew, He and Joshi, 1989).

Neutrino Mixing

B. Pontecorvo (1957). "Mesonium and anti-mesonium". *Zh. Eksp. Teor. Fiz.* 33: 549–551.
B. Pontecorvo (1967). "Neutrino Experiments and the Problem of Conservation of Leptonic Charge". *Zh. Eksp. Teor. Fiz.* 53: 1717

Z. Maki, M. Nakagawa, and S. Sakata (1962). "Remarks on the Unified Model of Elementary Particles". *Progress of Theoretical Physics* 28 (5): 870.

S. Sakata Z. Maki M. Nakagawa

Three generation mixing in quarks and leptons

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix V_{CKM},

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix U_{PMNS}

$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\mathrm{CKM}} D_L W_\mu^+ - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\mathrm{PMNS}} N_L W_\mu^- + H.C.\ ,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\mathrm{CKM}}$ or U_{PMNS} is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

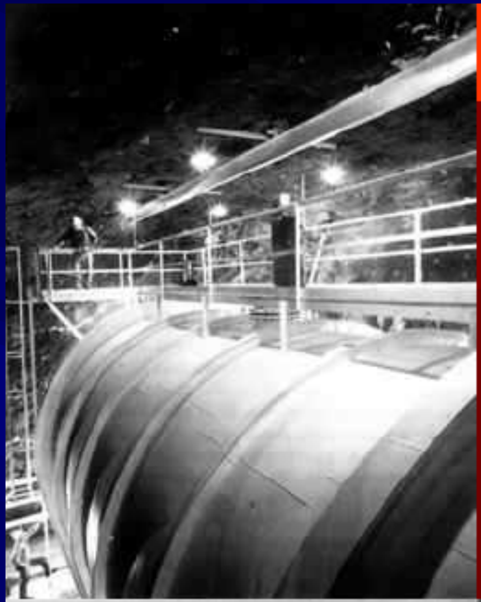
Where are we now? PDG2014

Table 14.7: The best-fit values and 3σ allowed ranges of the 3-neutrino oscillation parameters, derived from a global fit of the current neutrino oscillation data (from [174]) . The values (values in brackets) correspond to $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$). The definition of Δm^2 used is: $\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus, $\Delta m^2 = \Delta m_{31}^2 - \Delta m_{21}^2/2 > 0$, if $m_1 < m_2 < m_3$, and $\Delta m^2 = \Delta m_{32}^2 + \Delta m_{21}^2/2 < 0$ for $m_3 < m_1 < m_2$.

Parameter	best-fit ($\pm 1\sigma$)	3σ
Δm_{21}^2 $[10^{-5}$ eV $^2]$	$7.54^{+0.26}_{-0.22}$	$6.99 - 8.18$
$\lvert\Delta m^2\rvert$ $[10^{-3}$ eV $^2]$	2.43 ± 0.06 (2.38 ± 0.06)	$2.23 - 2.61$ $(2.19 - 2.56)$
$\sin^2\theta_{12}$	0.308 ± 0.017	$0.259 - 0.359$
$\sin^2\theta_{23}$, $\Delta m^2 > 0$	$0.437^{+0.033}_{-0.023}$	$0.374 - 0.628$
$\sin^2\theta_{23}$, $\Delta m^2 < 0$	$0.455^{+0.039}_{-0.031}$,	$0.380 - 0.641$
$\sin^2\theta_{13}$, $\Delta m^2 > 0$	$0.0234^{+0.0020}_{-0.0019}$	$0.0176 - 0.0295$
$\sin^2\theta_{13}$, $\Delta m^2 < 0$	$0.0240^{+0.0019}_{-0.0022}$	$0.0178 - 0.0298$
δ/π (2σ range quoted)	$1.39^{+0.38}_{-0.27}$ $(1.31^{+0.29}_{-0.33})$	$(0.00 - 0.16) \oplus (0.86 - 2.00)$
		$((0.00 - 0.02) \oplus (0.70 - 2.00))$

Where have we come from and where will we go?

Experimental discovery of nuetrino mixing



Homestake Gold Mine

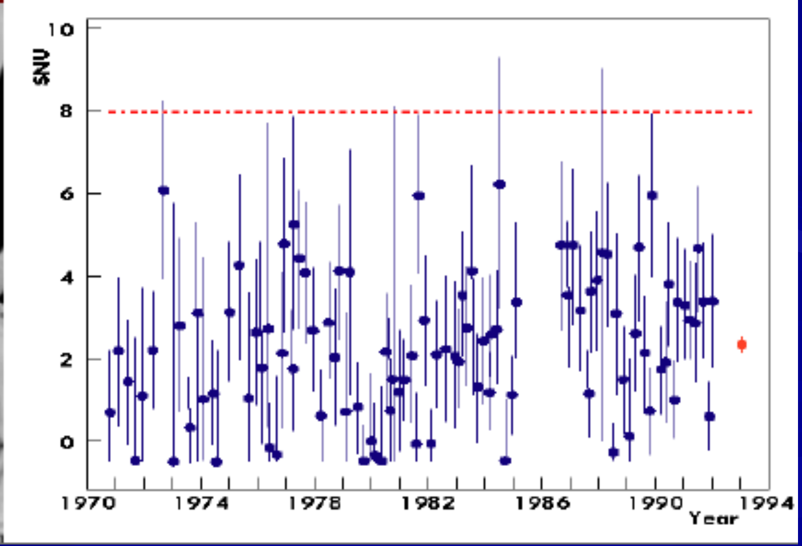
100,000 gallons of cleaning fluid C_2Cl_4

Expected 1.5 interactions per day
Measured 0.5 interactions per day

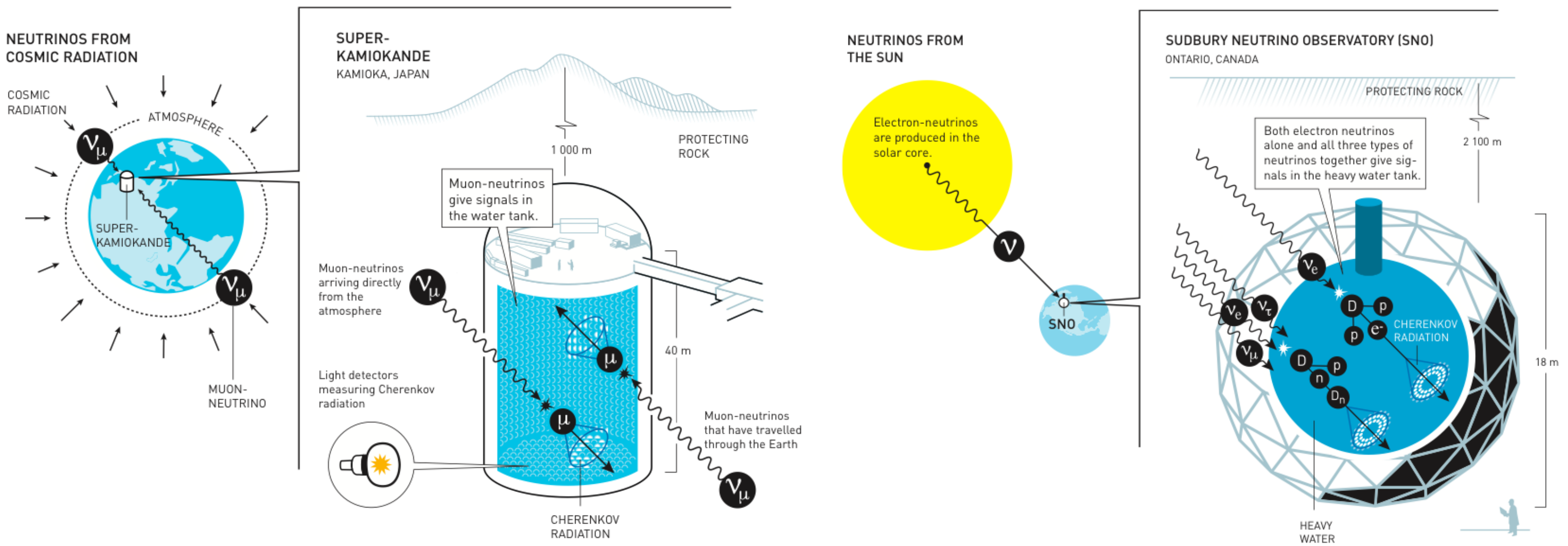
1968!

Sensitive to ^8B solar neutrinos only

$\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$



Super-K and SNO



Evidence for Oscillation of Atmospheric Neutrinos

Y. Fukuda *et al.* (Super-Kamiokande Collaboration)
Phys. Rev. Lett. **81**, 1562 – Published 24 August 1998

Measurement of the Rate of $\nu_e + d \rightarrow p + p + e^-$ Interactions Produced by ^8B Solar Neutrinos at the Sudbury Neutrino Observatory

Q. R. Ahmad *et al.* (SNO Collaboration)
Phys. Rev. Lett. **87**, 071301 – Published 25 July 2001

Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory

Q. R. Ahmad *et al.* (SNO Collaboration)
Phys. Rev. Lett. **89**, 011301 – Published 13 June 2002

Abundant data show that neutrinos have non-zero masses and mix with each other.

Solar neutrino oscillation: Homestake, Sage+Gallex/GNO, Super-K, SNO,Borexino …

Atmospherical neutrino oscillation: Super-Kamokande, …

Accelerator neutrino source: K2K, Minos , Nova …

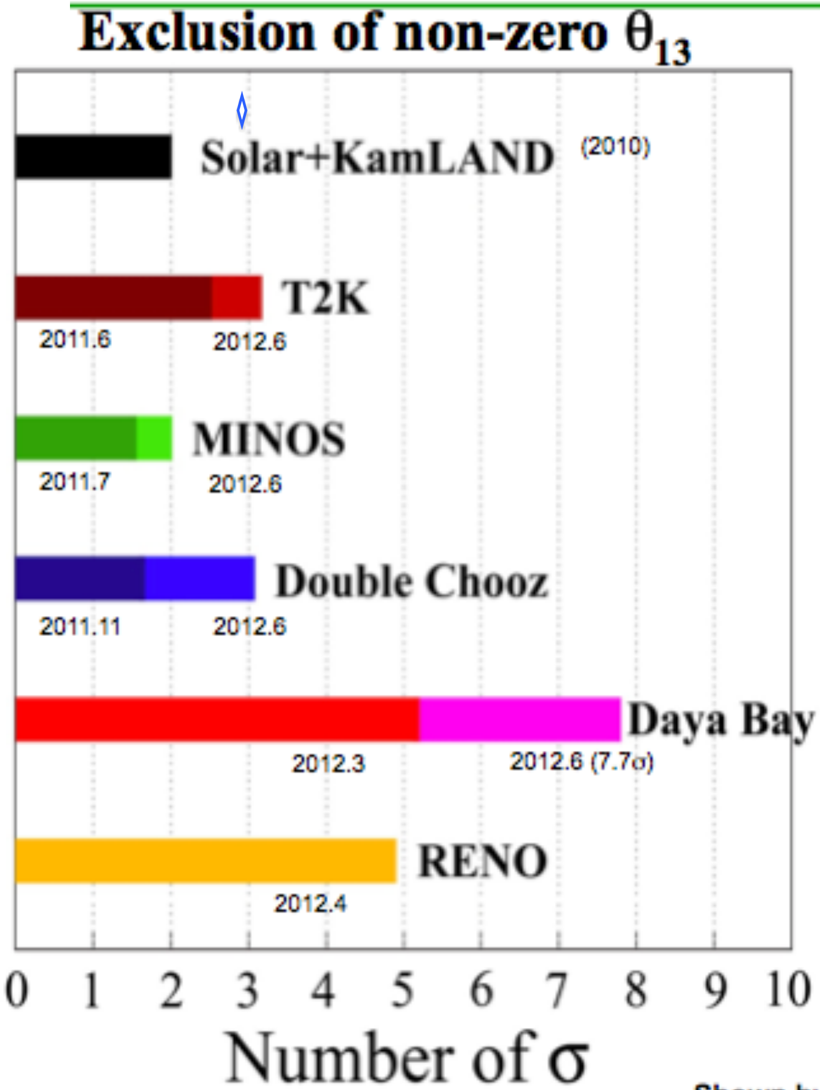
Reactor neutrino source: Kamland, T2K, Chooz, Daya-Bay, Reno…

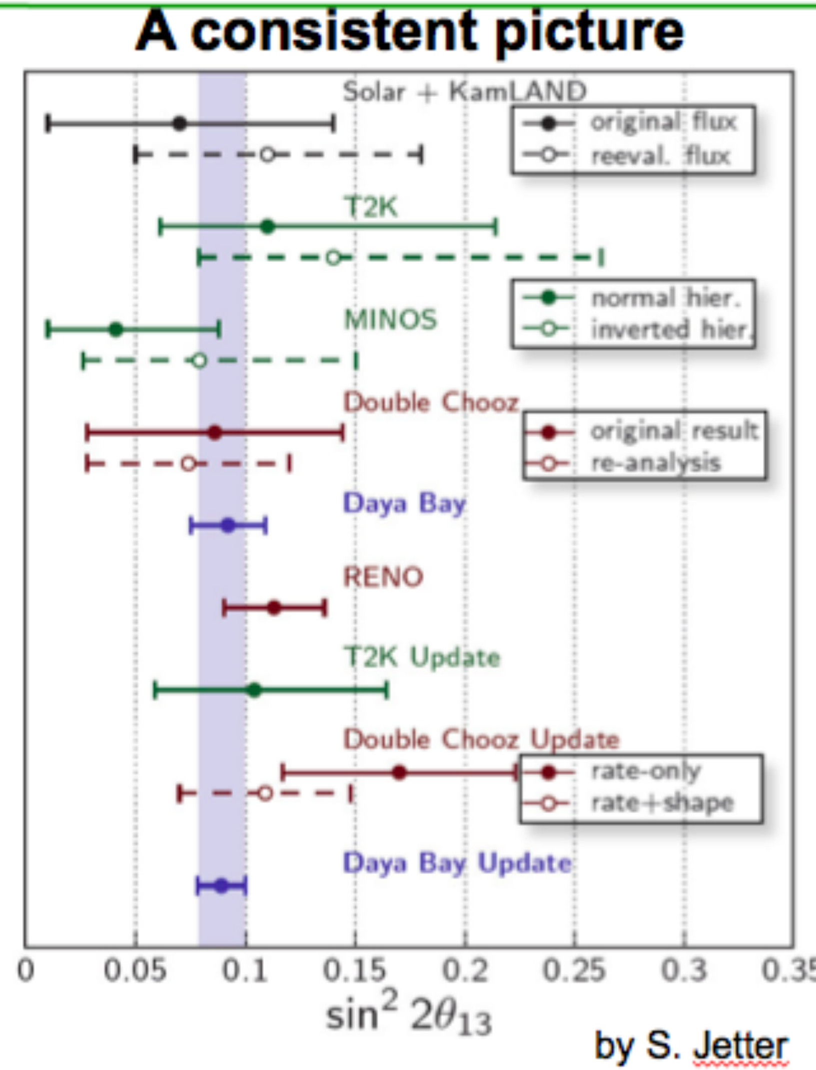
have observed neutrino oscillation phenomenon.

LSND and Miniboon…?

θ_{13} has been measured

Global Picture





Shown by Jun Cao @ ICHEP2012

Neutrino oscillations

Status

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric | reactor | solar

Remaining unknowns in the 3-flavor picture

Masses

$$m_1,\ m_2,\ m_3 \leftrightarrow \Delta m^2_{12}, |\Delta m^2_{23}|, sign(\Delta m^2_{23}), m_i$$

✔ ✔ ? ?

Angles

(plus Majorana phases)

$$\theta_{12},\ \theta_{23},\ \theta_{13},\ \delta$$

Known to good precisions

?

Neutrino mixing in 2012 after θ_{13} measurement

NuFIT 1.0 (2012)

	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	0.30 ± 0.013	$0.27 \to 0.34$	0.31 ± 0.013	$0.27 \to 0.35$
$\theta_{12}/^\circ$	33.3 ± 0.8	$31 \to 36$	33.9 ± 0.8	$31 \to 36$
$\sin^2\theta_{23}$	$0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$	$0.34 \to 0.67$	$0.41^{+0.030}_{-0.029} \oplus 0.60^{+0.020}_{-0.026}$	$0.34 \to 0.67$
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$	$36 \to 55$	$40.1^{+2.1}_{-1.7} \oplus 50.7^{+1.1}_{-1.5}$	$36 \to 55$
$\sin^2\theta_{13}$	0.023 ± 0.0023	$0.016 \to 0.030$	0.025 ± 0.0023	$0.018 \to 0.033$
$\theta_{13}/^\circ$	$8.6^{+0.44}_{-0.46}$	$7.2 \to 9.5$	$9.2^{+0.42}_{-0.45}$	$7.7 \to 10.$
$\delta_{CP}/^\circ$	300^{+66}_{-138}	$0 \to 360$	298^{+59}_{-145}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	7.50 ± 0.185	$7.00 \to 8.09$	$7.50^{+0.205}_{-0.160}$	$7.04 \to 8.12$
$\frac{\Delta m^2_{31}}{10^{-3}\ \mathrm{eV}^2}$ (N)	$2.47^{+0.069}_{-0.067}$	$2.27 \to 2.69$	$2.49^{+0.055}_{-0.051}$	$2.29 \to 2.71$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ (I)	$-2.43^{+0.042}_{-0.065}$	$-2.65 \to -2.24$	$-2.47^{+0.073}_{-0.064}$	$-2.68 \to -2.25$

Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}\ \mathrm{eV}^2$ (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}\ \mathrm{eV}^2$ (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}\ \mathrm{eV}^2$ (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 $\oplus$ 5.43 – 6.41	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

Fogli et al, arXiv:1205.3204

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34 - 2.69$ $-(2.25 - 2.59)$	$2.26 - 2.77$ $-(2.15 - 2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$(0.83^{+0.54}_{-0.64})\,\pi$ 0.07π [a]	$0 - 2\pi$	$0 - 2\pi$

Forero et al, arXiv:1205.4018

Not much can be said about CP violation and mass hierarchy.

Next ?

Mixing pattern in quark sector

$$V_{CKM} \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix} .$$

$$\theta_{12}^{Q} = 13.021^\circ \pm 0.039^\circ, \quad \theta_{23}^{Q} = 2.350^\circ \pm 0.052^\circ,$$

$$\theta_{13}^{Q} = 0.199^\circ \pm 0.008^\circ. \quad \delta^{Q} = 68.9^\circ$$

Theory for neutrino mixing

Early days, expecting neutrino mixing might be following a similar pattern as quarks, mixing angles are small.

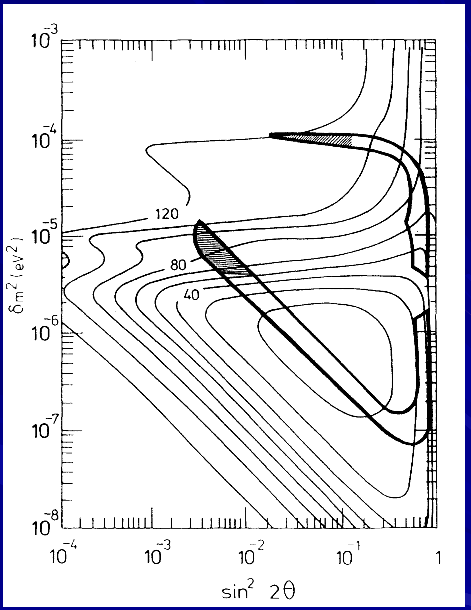
For example, 1992 people are

trying to produce mixing on the right

Simultaneous solutions to the solar and atmospheric neutrino problems via Fritzsch-type lepton mass matrices

A. J. Davies* and Xiao-Gang He†

(Davies and He, PRD46, 3208)



But both solar and atmospheric show large mixing angles after 1998!

Theory before and after Daya-Bay/Reno results

Before: popular mixing -The Tribimaximal Mixing

Harrison, Perkins, Scott (2002) , Z-Z. Xing (2002), He& Zee (2003)

The mixing pattern is consistent, within 2σ, with the tri-bimaximal mixing

$$V_{tri-bi} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

A4 a promising model (Ma&Ranjasekara, 2001) and realizations (Altarelli&Feruglio 2005, Babu&He 2005). Later many realizations: S4, D3, S3,D4, D7,A5,T',S4, Δ(27, 96), $PSL_2(7)$ … discrete groups Altarelli&Feruglio for review. (H. Lam; Mohapatra et al), T. Mahanthanpa&M-C. Chen; Frampton&Kephart; Y-L Wu, ….

After: Need to have a nonzero θ_{13}

Modification to tri-bimaximal mixing pattern need to be made. (Keum&He&Volkas; He&Zee, 2006).

In fact, more generically, A_4 symmetry leads to

$$V = \begin{pmatrix} \frac{2c}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \end{pmatrix} .$$

Tri-bimximal at higher scales and generate none zero θ_{13} at low energies?

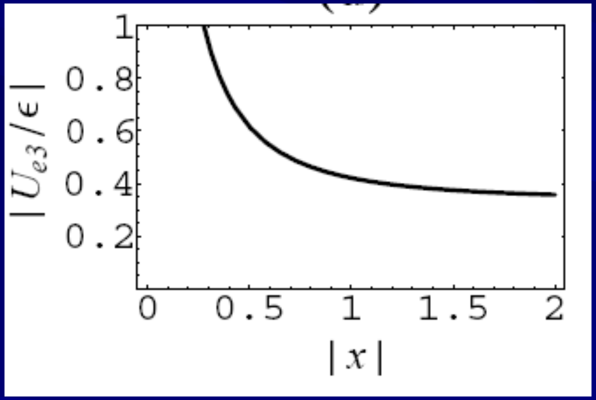
Baub and He, arxiv:0507217(hep-ph): A susy A4 model

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \, .$$

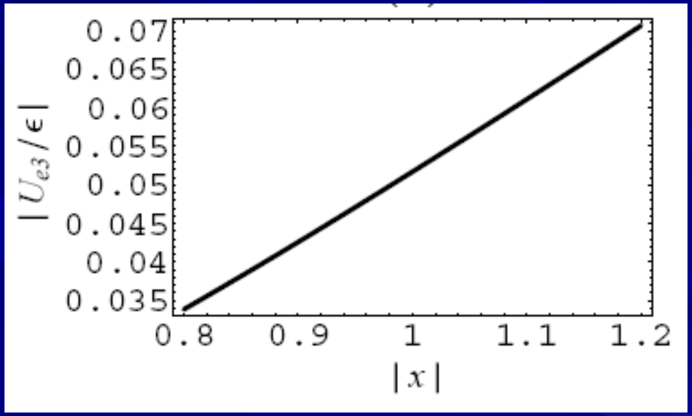
one–loop RGE $\frac{dM^e_\nu}{d\ln t} = \frac{1}{32\pi^2}[M^e_\nu Y^\dagger_e Y_e + (Y^\dagger_e Y_e)^T M^e_\nu] + \ldots$

leading to the entries $M_{13,23}(1-\epsilon)$ and $M_{33}(1-2\epsilon)$

$\epsilon \simeq Y^2_\tau \ln(M_{\rm GUT}/M_{EW})/32\pi^2$.



Inverted hierarchy



Normal hierarchy

Susy model, Y_t ~O(1), U_{e3} for inverted hierarchy, can be as large a 0.1, with RG effects!

Many other matrix form come back

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

An interesting coincidence:

$\theta_{13} = \theta_C/\sqrt{2}$

$V_{PMNS} = UV_{KM}; V_{Tr}V_{KM}$

B-Q. Ma et al: Ramond et al.; King et al.…

Bi-Maximal Mixing

(Barger, Pakvasa, Weiler, Whisnant)

The CP violating phase gradually becomes the attention of theoretical studies.

Group theoretical studies of neutrino mass matrix, correlations between mixing angle and CP violation.

Ishimori&Kobayashi, Henandez & Smirnov, G-J Ding&King and etc..

SO(10)Grand Unification

SO(10) Yukawa couplings: $16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$

Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\,\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

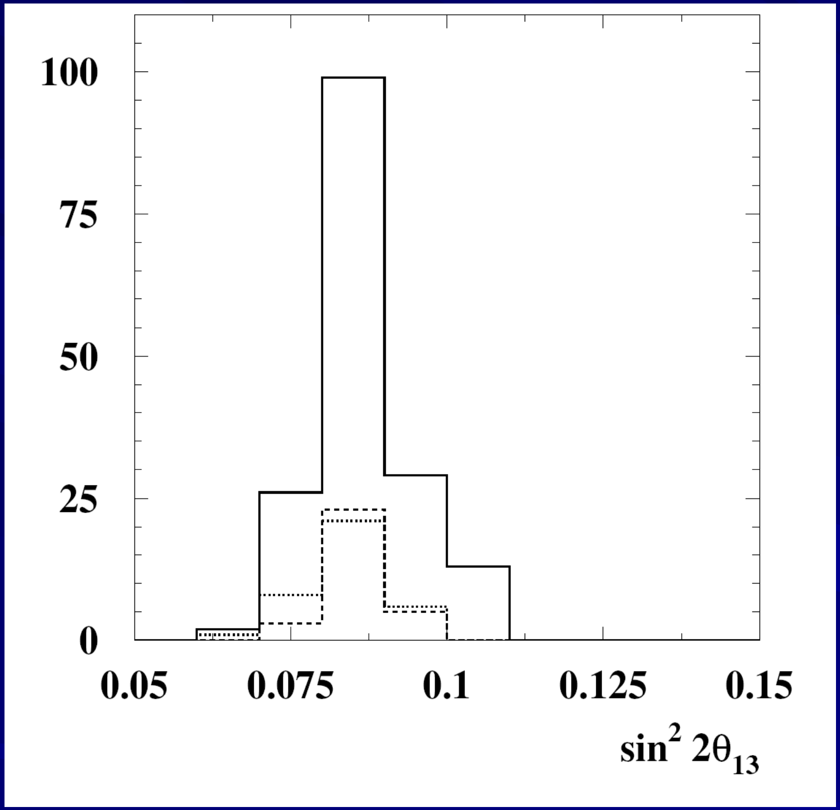
$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

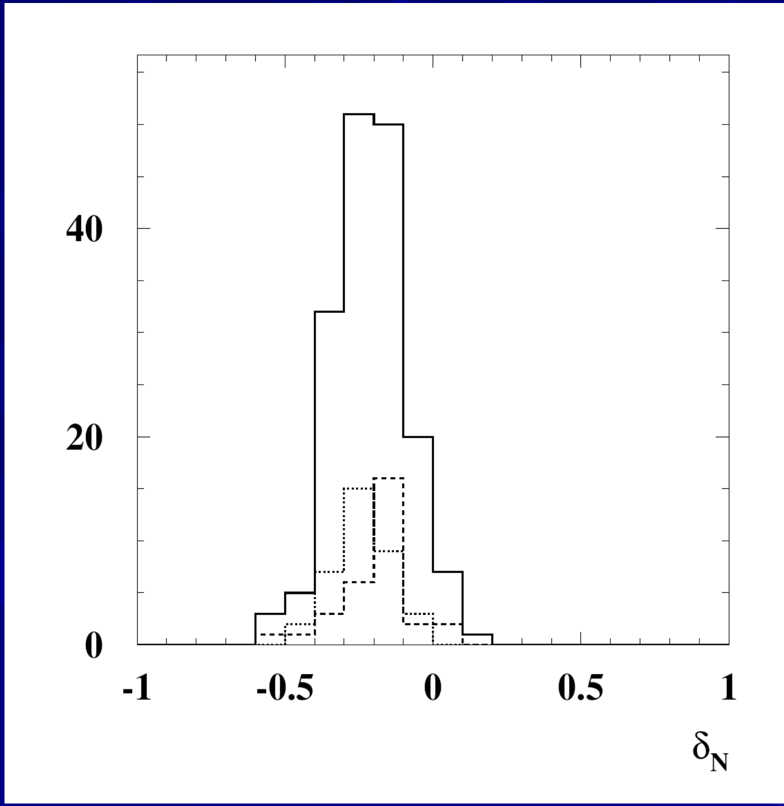
Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004)
Aulakh et al (2004)

Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006)
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek
Jushipura, Patel (2011)

θ_{13} in Minimal SO(10)

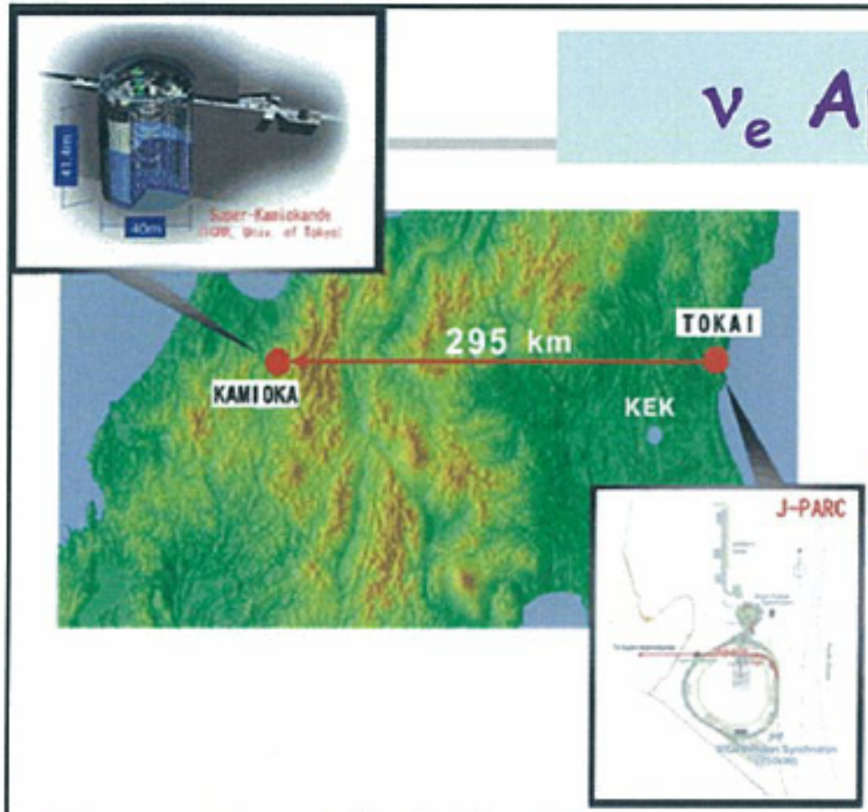




$\sin^2 2\theta_{13}$ and CP violating phase δ_N

K.S. Babu and C. Macesanu (2005)

ν_e Appearance



T2K- From Tokai To Kamioka

Mass hierarchy (+/-)

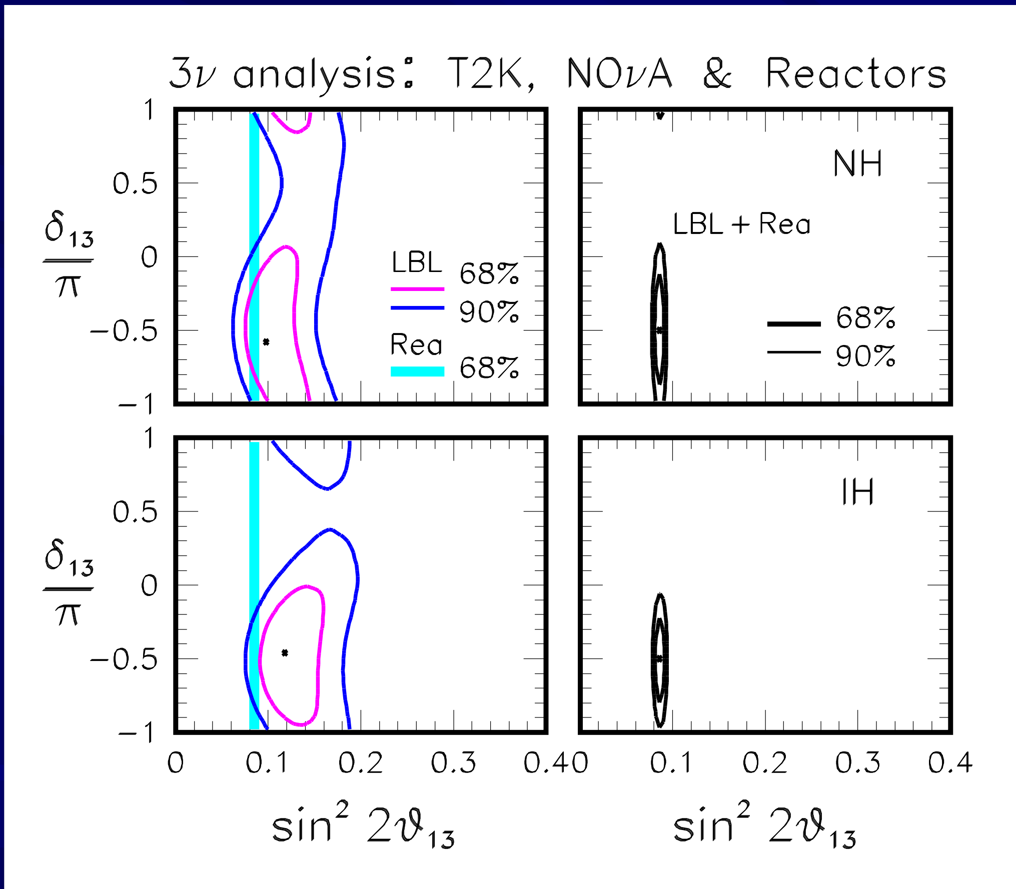
CP violation

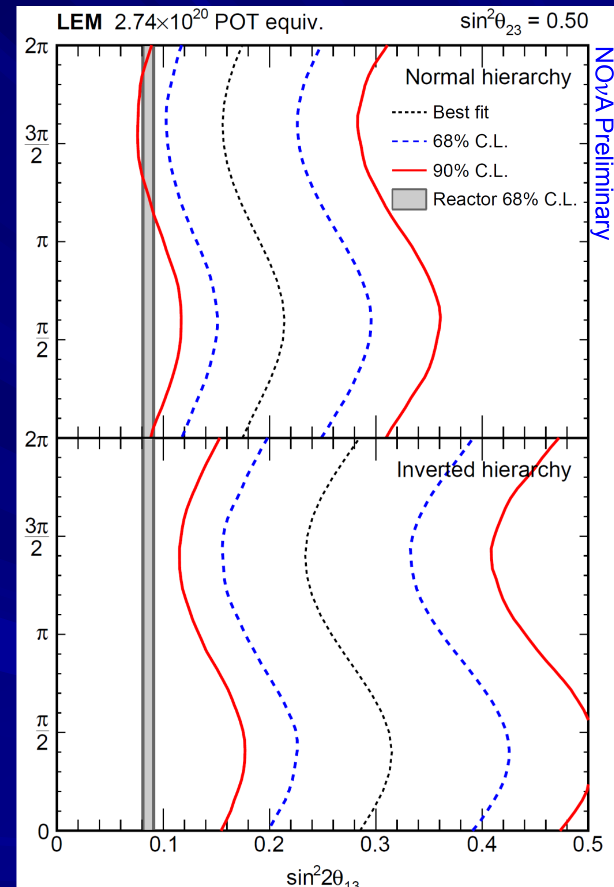
$$
\begin{aligned}
P(\nu_\mu \to \nu_e) = {} & 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31} \\
& + 8c_{13}^2 s_{13} s_{23} c_{23} s_{12} c_{12} \sin \Delta_{31} [\cos \Delta_{32} \cos \delta - \sin \Delta_{32} \sin \delta] \sin \Delta_{21} \\
& - 8c_{13}^2 s_{13}^2 s_{23}^2 s_{12}^2 \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \\
& + 4c_{13}^2 s_{12}^2 [c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2 c_{12} c_{23} s_{12} s_{23} s_{13} \cos \delta] \sin^2 \Delta_{21} \\
& - 8c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2s_{13}^2) \frac{aL}{4E_\nu} \sin \Delta_{31} \left[\cos \Delta_{32} - \frac{\sin \Delta_{31}}{\Delta_{31}} \right] .
\end{aligned}
$$

matter

Possible way of measuring mass hierarchy and CP phase

T2K, Nova new data fitting





arXiv:1509.03148
A. Palazzo

Fermilab seminar, August 6 2015
R. Patterson

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. See also Fig. 3 for a graphical representation of the results. We remind that Δm^2 is defined herein as $m_3^2-(m_1^2+m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH. The CP violating phase is taken in the (cyclic) interval $\delta/\pi \in [0, 2]$. The overall χ^2 difference between IH and NH is insignificant ($\Delta\chi^2_{\mathrm{I-N}} = -0.3$).

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.37 – 2.49	2.30 – 2.55	2.23 – 2.61
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.38	2.32 – 2.44	2.25 – 2.50	2.19 – 2.56
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15 – 2.54	1.95 – 2.74	1.76 – 2.95
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18 – 2.59	1.98 – 2.79	1.78 – 2.98
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14 – 4.70	3.93 – 5.52	3.74 – 6.26
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24 – 5.94	4.00 – 6.20	3.80 – 6.41
δ/π (NH)	1.39	1.12 – 1.77	0.00 – 0.16 $\oplus$ 0.86 – 2.00	—
δ/π (IH)	1.31	0.98 – 1.60	0.00 – 0.02 $\oplus$ 0.70 – 2.00	—

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	Normal Ordering ($\Delta\chi^2 = 0.97$)		Inverted Ordering (best fit)		Any Ordering
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.270 \to 0.344$
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$31.29 \to 35.91$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \to 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \to 0.644$	$0.385 \to 0.644$
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	$38.2 \to 53.3$	$49.5^{+1.5}_{-2.2}$	$38.6 \to 53.3$	$38.3 \to 53.3$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0186 \to 0.0250$	$0.0219^{+0.0011}_{-0.0010}$	$0.0188 \to 0.0251$	$0.0188 \to 0.0251$
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	$7.85 \to 9.10$	$8.51^{+0.20}_{-0.21}$	$7.87 \to 9.11$	$7.87 \to 9.11$
$\delta_{\mathrm{CP}}/^\circ$	306^{+39}_{-70}	$0 \to 360$	254^{+63}_{-62}	$0 \to 360$	$0 \to 360$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.02 \to 8.09$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$+2.317 \to +2.607$	$-2.449^{+0.048}_{-0.047}$	$-2.590 \to -2.307$	$\begin{bmatrix}+2.325 \to +2.599\\ -2.590 \to -2.307\end{bmatrix}$

Table 1. Three-flavor oscillation parameters from our fit to global data after the NOW 2014 conference. The results are presented for the "Free Fluxes + RSBL" in which reactor fluxes have been left free in the fit and short baseline reactor data (RSBL) with $L \lesssim 100$ m are included. The numbers in the 1st (2nd) column are obtained assuming NO (IO), *i.e.*, relative to the respective local minimum, whereas in the 3rd column we minimize also with respect to the ordering. Note that $\Delta m^2_{3\ell} \equiv \Delta m^2_{31} > 0$ for NO and $\Delta m^2_{3\ell} \equiv \Delta m^2_{32} < 0$ for IO.

parameter	best fit $\pm$ 1σ	2σ range	3σ range
Δm^2_{21} [10^{-5}eV^2]	$7.60^{+0.19}_{-0.18}$	7.26–7.99	7.11–8.18
$\lvert\Delta m^2_{31}\rvert$ [10^{-3}eV^2] (NH)	$2.48^{+0.05}_{-0.07}$	2.35–2.59	2.30–2.65
$\lvert\Delta m^2_{31}\rvert$ [10^{-3}eV^2] (IH)	$2.38^{+0.05}_{-0.06}$	2.26–2.48	2.20–2.54
$\sin^2\theta_{12}/10^{-1}$	3.23$\pm$0.16	2.92–3.57	2.78–3.75
$\theta_{12}/^\circ$	34.6$\pm$1.0	32.7–36.7	31.8–37.8
$\sin^2\theta_{23}/10^{-1}$ (NH)	$5.67^{+0.32\ a}_{-1.28}$	4.13–6.23	3.92 – 6.43
$\theta_{23}/^\circ$	$48.9^{+1.9}_{-7.4}$	40.0–52.1	38.8–53.3
$\sin^2\theta_{23}/10^{-1}$ (IH)	$5.73^{+0.25}_{-0.43}$	4.32–6.21	4.03–6.40
$\theta_{23}/^\circ$	$49.2^{+1.5}_{-2.5}$	41.1–52.0	39.4–53.1
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34$\pm$0.20	1.95–2.74	1.77–2.94
$\theta_{13}/^\circ$	8.8$\pm$0.4	8.0–9.5	7.7–9.9
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40$\pm$0.19	2.02–2.78	1.83–2.97
$\theta_{13}/^\circ$	8.9$\pm$0.4	8.2–9.6	7.8–9.9
δ/π (NH)	$1.34^{+0.64}_{-0.38}$	0.0–2.0	0.0–2.0
$\delta/^\circ$	241^{+115}_{-68}	0–360	0–360
δ/π (IH)	$1.48^{+0.34}_{-0.32}$	0.0–0.14 & 0.81-2.0	0.0–2.0
$\delta/^\circ$	266^{+61}_{-58}	0–25 & 146–360	0–360

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Phys.Rev. D90 (2014) 9, 093006

Model Building with $\theta_{23}=\pi/4$ and $\delta_{CP} = 3\pi/2(-\pi/2)$

Structure of the mass matrix (charged lepton is already diagonal)

Assuming neutrinos are Majorana particles,

$$L = -\frac{1}{2}\bar{\nu}_L m_\nu \nu_L^C$$
$$m_\nu = V_{PMNS}\hat{m}_\nu V_{PMNS}^T \ ,$$

$\hat{m}_\nu = diag(m_1, m_2, m_3)$ with $m_i = |m_i| exp(i\alpha_i)$.

With $\delta = -\pi/2$ and $\theta_{23} = \pi/4$,
m_ν has the following form

$$m_\nu = \begin{pmatrix} a & c+i\beta & -(c-i\beta) \\ c+i\beta & d+i\gamma & \tilde{b} \\ -(c-i\beta) & \tilde{b} & d-i\gamma \end{pmatrix} \ ,$$

$$a = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 - m_3 s_{13}^2 \ ,$$
$$\tilde{b} = -\frac{1}{2}\left(m_1(s_{12}^2 + c_{12}^2 s_{13}^2) + m_2(c_{12}^2 + s_{12}^2 s_{13}^2) - m_3 c_{13}^2\right) \ ,$$
$$c = -\frac{1}{\sqrt{2}}(m_1 - m_2) s_{12} c_{12} c_{13} \ ,$$
$$d = \frac{1}{2}\left(m_1(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2(c_{12}^2 - s_{12}^2 s_{13}^2) + m_3 c_{13}^2\right) \ ,$$
$$\beta = \frac{1}{\sqrt{2}} s_{13} c_{13}\left(m_1 c_{12}^2 + m_2 s_{12}^2 + m_3\right) \ ,$$
$$\gamma = -(m_1 - m_2) s_{12} c_{12} s_{13} \ .$$

Note that in the most general case, because non-zero Majorana phases, the parameters a, $\tilde{b}$, c, d, β and γ are all complex.

Equavilent forms

$$m_\nu = \begin{pmatrix} e^{ip_1} & 0 & 0 \\ 0 & e^{ip_2} & 0 \\ 0 & 0 & e^{ip_3} \end{pmatrix} \begin{pmatrix} a & c+i\beta & -(c-i\beta) \\ c+i\beta & d+i\gamma & \tilde{b} \\ -(c-i\beta) & \tilde{b} & d-i\gamma \end{pmatrix} \begin{pmatrix} e^{ip_1} & 0 & 0 \\ 0 & e^{ip_2} & 0 \\ 0 & 0 & e^{ip_3} \end{pmatrix},$$

where the phases p_i are arbitrary.

For $p_1 = p_2 = 0$ and $p_3 = \pi$,

$$m_\nu = \begin{pmatrix} a & c+i\beta & (c-i\beta) \\ c+i\beta & d+i\gamma & b \\ (c-i\beta) & b & d-i\gamma \end{pmatrix},$$

eq A

where $b = -\tilde{b}$.

If neutrinos do not have non-trivial Majorana phases, α, β, γ are real

The mass matrix can be written asand

$$m_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}.$$

eq B

Mass matrix in forms eq A and eq B

always predict $\theta_{23} = \pi/4$ and $\delta_{CP} = 3\pi/2$?

The answer is no!

For both is change $\delta_{CP} = \pi/2$, the mass matrix has the same form.

If α, β, γ are complex, θ_{23} and δ_{CP} may not be $\pi/4$ and $\pm\pi/2$.

This also points out a method to modify the solutions of $\theta_{23} = \pi/4$ and $\delta_{CP} = 3\pi/2$

Need to be careful!

How to obtain m_ν discussed before?

μ-τ exchange + charge conjugation symmetry

(Generalized CP symmetry: Grimus&Lavoura (2004), Gui-Jun Ding et al., Xing …)

$\nu_e \to \nu_e^C,\ \nu_\mu \to \nu_\tau^c,\ \nu_\tau \to \nu_\mu^C$

Under this transformation

require invarance under the above transformatin

$$m_\nu = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \to \quad m_\nu^\dagger = \begin{pmatrix} A_{11} & A_{13} & A_{12} \\ A_{13} & A_{33} & A_{23} \\ A_{12} & A_{23} & A_{22} \end{pmatrix} = \begin{pmatrix} A_{11}^* & A_{12}^* & A_{13}^* \\ A_{12}^* & A_{22}^* & A_{23}^* \\ A_{13}^* & A_{23}^* & A_{33}^* \end{pmatrix}$$

The above implies

$$A_{11} = A_{11}^* \quad A_{23} = A_{23}^* \qquad m_\nu = \begin{pmatrix} A_{11} & A_{12} & A_{12}^* \\ A_{12} & A_{33}^* & A_{23} \\ A_{12}^* & A_{23} & A_{33} \end{pmatrix}$$

$$A_{22} = A_{33}^* \quad A_{12} = A_{13}^*$$

The above is the same as eq B.

What about charged leptons since (ν_L, e_L) are in one doublet, need to have all considered together! Not a trivial task!

Realization with A_4 symmetry

Some general properties

(X-G He, arXiv:1504.01560 X-G He and G-N Li, arXiv:1505.01932

E Ma, arXiv:1504.02086

Assuming that the charged lepton mass matrix M_l

is diagonalized from left by U_l ,

$$M_l = U_l \hat{m}_l U_r \ , \quad U_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \ ,$$

where $\omega = exp(i2\pi/3)$ and $\omega^2 = exp(i4\pi/3)$.

A_4 models usually have the above characteristic U_i.

U_r is a unitary matrix, but does not play a role in determining V_{PMNS}.

If neutrinos are Majorana particles, the most general mass matrix is

$$M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix} \ ,$$

In the basis where charged lepton is digonalized,
the neutrino mass matrix is of the form given by $U_l^\dagger M_\nu U_l^*$ with

$$A_{11} = \frac{1}{3}(w_1 + w_2 + w_3 + 2(x + y + z)) ,$$

$$A_{22} = \frac{1}{3}(w_1 + \omega w_2 + \omega^2 w_3 + 2(\omega^2 x + \omega y + z)) ,$$

$$A_{33} = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 + 2(\omega x + \omega^2 y + z)) ,$$

$$A_{12} = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 - \omega x - \omega^2 y - z) ,$$

$$A_{13} = \frac{1}{3}(w_1 + \omega w_2 + \omega^2 w_3 - \omega^2 x - \omega y - z) ,$$

$$A_{23} = \frac{1}{3}(w_1 + w_2 + w_3 - (x + y + z)) .$$

If the parameter set $P = (w_1, w_2, w_3, x, y, z)$ is real, then

$$A_{11} = A_{11}^* \quad A_{23} = A_{23}^*$$

$$A_{22} = A_{33}^* \quad A_{12} = A_{13}^*$$

The mass matrix is of the form given by eq B, $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$

How to achieve in A_4 models?

A_4 group is defined as the set of all twelve even permutations of four object.
It has three one-dimensional representations 1, $1'$ and $1''$
and one three-dimensional irreduciblerepresentation 3.
Multiplication rules

$3 \times 3 = 3_s + 3_a + 1 + 1' + 1''$

$1 \times 1_i = 1_i$, $1' \times 1' = 1''$, $1'' \times 1'' = 1'$, $1' \times 1'' = 1$

$$
\begin{aligned}
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_s} &= (x_2y_3 + x_3y_2\,,\; x_3y_1 + x_1y_3\,,\; x_1y_2 + x_2y_1) \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_a} &= (x_2y_3 - x_3y_2\,,\; x_3y_1 - x_1y_3\,,\; x_1y_2 - x_2y_1) \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}} &= x_1y_1 + x_2y_2 + x_3y_3, \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}'} &= x_1y_1 + \omega\, x_2y_2 + \omega^2\, x_3y_3, \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}''} &= x_1y_1 + \omega^2\, x_2y_2 + \omega\, x_3y_3,
\end{aligned}
$$

A model with Type II seesaw

Particle contents and their transformation properties under standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge and A_4 family symmetry properties

$$
\begin{aligned}
&l_L : (1,2,-1)(3)\ , \quad l_R : (1,1,-2)(1+1''+1')\ ,\\
&\phi : (1,2,-1)(1)\ , \quad \Phi : (1,2,-1)(3)\ ,\\
&\Delta^{0,\prime,\prime\prime} : (1,3,-2)(1+1'+1'')\ , \quad \chi : (1,3,-2)(3)\ .
\end{aligned}
$$

The Lagrangian responsible for the lepton mass matrix is

$$
\begin{aligned}
L = &\, y_e \bar{l}_L \tilde{\Phi} l_R^1 + y_\mu \bar{l}_L \tilde{\Phi} l_R^2 + y_\tau \bar{l}_L \tilde{\Phi} l_R^3 \\
&+ Y_\nu^0 \bar{l}_L \Delta^0 l_L^c + Y_\nu^{\prime} \bar{l}_L \Delta^{\prime} l_L^c + Y_\nu^{\prime\prime} \bar{l}_L \Delta^{\prime\prime} l_L^c + y_\nu \bar{l}_L \chi l_L^c + H.C.
\end{aligned}
$$

If the structure of the vacuum expectation value (vev) is of the form

$< \Phi_{1,2,3} >= v^{\Phi}_{1,2,3} = v^{\Phi}$, $< \chi_i >= v^{\chi}_i$, $< \phi >= v_\phi$, and $< \Delta^{0,',''} >= v^{0,',''}_{\Delta}$,

$$M_l = U_l \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \ , \quad M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix} \ ,$$

In general $w_i, \ x, \ y, \ z$ are complex!

where $m_{e,\mu,\tau} = \sqrt{3} y_{e,\mu,\tau} v^{\Phi}$ and

$$\begin{aligned} w_1 &= Y^0_\nu v^0_\Delta + Y^{'}_\nu v^{'}_\Delta + Y^{''}_\nu v^{''}_\Delta \ , \\ w_2 &= Y^0_\nu v^0_\Delta + \omega^2 Y^{'}_\nu v^{'}_\Delta + \omega Y^{''}_\nu v^{''}_\Delta \ , \\ w_3 &= Y^0_\nu v^0_\Delta + \omega Y^{'}_\nu v^{'}_\Delta + \omega^2 Y^{''}_\nu v^{''}_\Delta \ , \\ x &= y_\nu v^{\chi}_3 \ , \quad y = y_\nu v^{\chi}_2 \ , \quad z = y_\nu v^{\chi}_1 \ . \end{aligned}$$

In general $w_i, \ x, \ y, \ z$ are complex!

Making parameters in the set P real

To obtain real w_i, x, y, and z,
one needs to make the Yukawa couplings and scalar vevs to be real
one can require the following generalized CP symmetry under

$$(l_{e,L}\ ,\ \ l_{\mu,L}\ ,\ l_{\tau,L}) \to (l_{e,L}^{CP}\ ,\ \ l_{\tau,L}^{CP}\ ,\ \ l_{\mu,L}^{CP})\ ,\quad \Phi = (\Phi_1, \Phi_2, \Phi_3) \to (\Phi_1^\dagger, \Phi_3^\dagger, \Phi_2^\dagger)\ ,$$
$$(\Delta^0\ ,\ \Delta^{'}\ ,\ \ \Delta^{''}) \to (\Delta^{0\dagger}\ ,\ \ \Delta^{'\dagger}\ ,\ \ \Delta^{''\dagger})\ ,\quad (\chi_1\ ,\ \chi_2\ ,\ \ \chi_3) \to (\chi_1^\dagger\ ,\ \ \chi_3^\dagger\ ,\ \ \chi_2^\dagger)\ ,$$

and all other fields transform the same as those under the usual CP symmetry.
The above transformation properties will transform relevant terms
into their complex conjugate ones.
Requiring the Lagragian to be invariant under the above transformation
dictates the Yukawa couplings to be real.
The same requirement will dictates the scalar potential
to forbid spontaneous CP violation and vevs to be real.

One, however, notices that the parameters $w_{2,3}$ are in general complex even if the Yukawa couplings and the vevs of the scalar fields are made real because the appearance of ω^i.
To make them real, it is therefore required that

$$Im(\omega^2 Y_\nu^{'} v_\Delta^{'} + \omega Y_\nu^{''} v_\Delta^{''}) = Im(\omega Y_\nu^{'} v_\Delta^{'} + \omega^2 Y_\nu^{''} v_\Delta^{''}) = 0 \ .$$

The above can be achieved by
the absent of the scalar fields $\Delta^{','' }$ or $Y_\nu^{'} v_{\Delta'} = Y_\nu^{''} v_{\Delta''}$.

An example with Z_2 residual group unbroken

If vev of χ_2 component of χ is non-zero,

but the vevs of $\chi_{1,3}$ are zero, the vev structure breaks A_4 down to a Z_2.

The charge lepton and neutrino mass matrice are given by

$$M_l = U_l \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} , \qquad M_\nu = \begin{pmatrix} w_1 & 0 & y \\ 0 & w_2 & 0 \\ y & 0 & w_3 \end{pmatrix} .$$

The parameters in the set w_i, and y are in general complex.

Diagonalizing the mass matrices, we have

$$V_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} c + se^{i\rho} & 1 & ce^{i\rho} - s \\ c + \omega se^{i\rho} & \omega^2 & \omega ce^{i\rho} - s \\ c + \omega^2 se^{i\rho} & \omega & \omega^2 ce^{i\rho} - s \end{pmatrix} ,$$

$\tan\rho = Im(yw_1^* + y^*w_3)/Re(yw_1^* + y^*w_3)$,
$s = \sin\theta$ and $c = \cos\theta$,

$$\tan 2\theta = \frac{2|yw_1^* + w_3 y^*|}{|w_1|^2 - |w_3|^2} .$$

Majorana phases α_i of m_i

$$\alpha_{1,3} = Arg(w_i(1 \pm \cos 2\theta) + w_2 e^{-i2\rho}(1 \mp \cos 2\theta) \pm 2\sin 2\theta y e^{-i\rho} , \quad \alpha_2 = Arg(w_2)$$

Translate into standard parameterization

$$s_{12} = \frac{1}{\sqrt{2}(1+cs\cos\rho)^{1/2}} \;, \; s_{23} = \frac{(1+cs\cos\rho+\sqrt{3}cs\sin\rho)^{1/2}}{\sqrt{2}(1+cs\cos\rho)^{\frac{1}{2}}} \;,$$

$$s_{13} = \frac{(1-2cs\cos\rho)^{1/2}}{\sqrt{3}} \;.$$

and

$$\sin\delta = (1+\frac{4c^2s^2\sin^2\rho}{(c^2-s^2)^2})^{-1/2}(1-\frac{3c^2s^2\sin^2\rho}{(1+cs\cos\rho)^2})^{-1/2} \times \begin{cases} -1\;, & \text{if } \; c^2 > s^2\;, \\ +1\;, & \text{if } \; s^2 > c^2\;. \end{cases}$$

It is clearly that if $\sin\rho$ is not zero,

$|\delta|$ and θ_{23} deviate from $\pi/2$ and $\pi/4$, respectively.

In the limit ρ goes to zero, that real parameter set P

$\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$.

Predictions: $|V_{i2}| = 1/\sqrt{3}$

$J = \mathrm{Im}(V_{11}V^*_{12}V^*_{21}V_{22}) = (s^2 - c^2)/6\sqrt{3}$

independent of ρ

Comparison with data

	δ/π	s^2_{12}	$s^2_{13}/10^{-2}$	s^2_{23}
NH	$1.41^{+0.55}_{-0.44}$	0.323 ± 0.016	2.26 ± 0.12	$0.567^{+0.032}_{-0.124}$
2σ region	$0.0 \sim 2.0$	$0.292 \sim 0.357$	$2.02 \sim 2.50$	$0.414 \sim 0.623$
IH	1.48 ± 0.31	0.323 ± 0.016	2.29 ± 0.12	$0.573^{+0.025}_{-0.039}$
2σ region	$0.00 \sim 0.09 \& 0.86 \sim 2.0$	$0.292 \sim 0.357$	$2.05 \sim 2.52$	$0.435 \sim 0.621$

mass-squared differences are given by

	$\Delta m^2_{21}[10^{-5}]eV^2$	$\lvert\Delta m^2_{31}\rvert[10^{-3}eV^2]$
NH	$7.60^{+0.19}_{-0.18}$	$2.48^{+0.05}_{-0.07}$
2σ region	$7.26 \sim 7.99$	$2.35 \sim 2.59$
IH	$7.60^{+0.19}_{-0.18}$	$2.38^{+0.05}_{-0.06}$
2σ region	$7.26 \sim 7.99$	$2.26 \sim 2.48$

Forero et al, arXiv:1205.4018

The case with real w_i and y

The predictions for δ and θ_{23} are $\pm\pi/2$ and $\theta_{23} = \pi/4$,

Additional information for fixing the sign of δ_{CP}.

Since δ should be close to $-\pi/2$, should take $c^2 > s^2$.

$s_{13} = (1 - 2cs)^{1/2}/\sqrt{3}$ is not predicted.

Fix $cs = 0.497 \pm 0.018$ to predict $s_{12}^2 = 0.334 \pm 0.004$ for both NH and IH cases.

Note that $V_{e2}^2 = (s_{12}c_{13})^2 = 1/3$.

The s_{13} and $|V_{e2}|$ agree with data within 1σ.

But s_{23} outside 1σ, can be consistent with data at 2 σ level.

The case with complex w_i and y

If the parameters in the set P are complex,
therefore a new phase ρ appears in the model.
ρ can be used to improve agreement of the model with data.
In both NH and IH cases,
fixing cs and $\cos\rho$ to be 0.468 and 0.992, respectively.
s_{23} and δ are determined to:
0.534 and 1.426π, respectively.
These values are in agreement with data at 1σ level.

Interesting model building guidline

- predict $\theta_{23} = \pi/4$ and $\delta_{CP} = 3\pi/2$
- A non-zero θ_{13}

At least first order approximation.

Apply μ-τ exchange with CP symmetry on neutrino mass matrix can achieve this, but including charged leptons, the task is non-trivial.

A_4 family symmetry model provide an example to fully realize such mixing pattern. This class of A_4 models also provide direction for modifying the pattern, with complex Yukawa coefficients.

An A_4 model with Z_2 residual symmetry has been constructed. This al model also predicts

$|V_{i2}| = 1/\sqrt{3}$, $J = \mathrm{Im}(V_{11}V^*_{12}V^*_{21}V_{22}) = (s^2-c^2)/6\sqrt{3}$ independent of ρ.

Really has something to do with Nature?

New experimental data will provide more clue about what the mixing pattern is and how theoretical model should be constructed.

A lot more for theoretical neutrino models

Neutrino mass hierarchy, Dirac or Majorana

Sterile neutrinos LSND&MiniBoon, seem still alive

Lepton number violating FCNC connection

Cosmological and Astrophysical connection

Dark matter connection,

Leptogenesis connection.

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