Primordial gravitational waves from the space-condensate inflationary model

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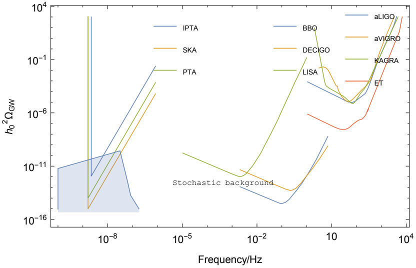
- Observational detection of primordial GWs would not only verify the success of inflation, but would also open a new window to physics of the early universe.
- There are several observational constraints on the energy spectrum of a stochastic GWs background at different frequency ranges. For example:
  - Ground-based interferometric detectors such as aLIGO , aVIGRO and KAGRA.
  - Space-based interferometeric detectors such as eLISA/NGO, BBO or DECIGO.
  - Pulsar timing experiments such as PTA, EPTA or SKA.
  - CMB anisotropies in temperature and polarizations, $f < 10^{-15}$.



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- by assuming the abrupt phase transition between two consecutive regimes, we calculate the energy spectrum of the primordial GWs in the full frequency range.
- constrain the model parameter by the several observational upper bound and CMB angular power spectrum for BB-mode.

Review: Inflationary prediction

The inflation predicts almost gaussian and nearly scale-invariant primordial power spectrum for the scalar perturbation,

$$\mathcal{P}_S(k) = \mathcal{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S - 1 + \frac{\alpha_S}{2}\ln(k/k_*)}, \tag{1}$$

as well as the existence of the primordial GWs. The primordial tensor power spectrum can always be described as

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_*)\left(\frac{k}{k_*}\right)^{n_T + \frac{\alpha_T}{2}\ln(k/k_*)}. \tag{2}$$

In the context of single-field slow-roll inflation models, the observable parameters are obtained in terms of slow-roll parameters

$$n_S - 1 \simeq 2\eta_V - 6\epsilon_V, \quad n_T \simeq -\tfrac{r}{8}, \quad r \simeq -\tfrac{8}{3}(n_S - 1) + \tfrac{16}{3}\eta_V, \tag{3}$$

$$\alpha_S \simeq \tfrac{r}{8}\left[(n_S - 1) + \tfrac{r}{8}\right], \quad \alpha_T \simeq \tfrac{r}{8}\left[(n_S - 1) + \tfrac{r}{8}\right] \tag{4}$$

where $r \equiv \mathcal{P}_T(k_*)/\mathcal{P}_S(k_*)$ is the tensor-to-scalar ratio at the certain pivot scale k_* and slow-roll parameters

$$\epsilon_V \simeq \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2, \quad \eta_V \simeq \frac{V_{\phi\phi}}{V}M_p^2. \tag{5}$$

Review: Primordial gravitational waves

In the flat FRW background, the GWs are described by the tensor perturbation, h_{ij}, of metric defined as

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right] \tag{6}$$

where h_{ij} is symmetric, and satisfies $\delta^{ij}h_{ij} = 0 \,\&\, \partial_i h^{ij} = 0$; and $dt \equiv a(\tau)d\tau$.
For $|h_{ij}| \ll 1$, the linearized Einstein equation

$$\partial_\mu(\sqrt{-g}\partial^\mu h_{ij}) = 16\pi G a^2(\tau)\Pi_{ij} \tag{7}$$

where Π_{ij} is the anisotropic stress tensor, satisfies $\Pi_{ii} = 0$ & $\partial_i \Pi_{ij} = 0$; which includes the contribution of large-scale magnetic field, free-streaming relativistic particle and so on.
In the Fourier space, we expand

$$h_{ij}(\tau, \mathbf{x}) = \sqrt{8\pi G}\sum_\lambda \int \frac{d\,\mathbf{k}}{(2\pi)^{3/2}}\epsilon_{ij}^{(\lambda)}(\mathbf{k})h_{\mathbf{k}}^\lambda e^{i\mathbf{kx}} \tag{8}$$

$$\Pi_{ij}(\tau, \mathbf{x}) = \sqrt{8\pi G}\sum_\lambda \int \frac{d\,\mathbf{k}}{(2\pi)^{3/2}}\epsilon_{ij}^{(\lambda)}(\mathbf{k})\Pi_{\mathbf{k}}^\lambda e^{i\mathbf{kx}} \tag{9}$$

where $\lambda = +/\times$ is the each polarization states of the GWs, and $\epsilon_{ij}^{(\lambda)}$ is the polarization tensor and satisfies $\epsilon_{ij}^{(\lambda)}\epsilon^{ij(\lambda')} = 2\delta^{\lambda\lambda'}$.

The linearized evolution equation of GWs can be written in the Fourier space as

$$h''_{\lambda,k} + 2\mathcal{H}h'_{\lambda,k} + k^2 h_{\lambda,k} = 16\pi G a^2(\tau)\Pi_{\lambda,k} \tag{10}$$

where $\mathcal{H} = a'/a$ and $' \equiv d/d\tau$ and we denote previous $h_{\mathbf{k}}^{(\lambda)}(\tau)$ and $\Pi_{\mathbf{k}}^{(\lambda)}(\tau)$ as $h_{\lambda,k}(\tau)$ and $\Pi_{\lambda,k}(\tau)$, respectively; and $k = |\mathbf{k}|$ is the wavenumber of the GWs.
The strength of primordial GWs is characterized by their energy spectrum

$$\Omega_{GW}(k) = \frac{1}{\rho_{\rm crit}}\frac{d\rho_{GW}}{d\ln k} \tag{11}$$

where $\rho_{\rm crit} = 3H_0^2/8\pi G$ is the critical density and H_0 is the present Hubble constant. The energy density of a GWs background, ρ_{GW}, is defined as

$$\rho_{GW} = \frac{M_p^2}{32\pi}\int k^2 P_T(k) d\ln k. \tag{12}$$

where $P_T(k)$ is the power spectrum of primordial GWs observed today and is defined as

$$P_T(k) \equiv \frac{32k^3}{\pi M_p^2}\sum_{\lambda}\langle h^{\dagger}_{\lambda,\,k} h_{\lambda,\,k}\rangle. \tag{13}$$

By using Eq. (12) and Eq. (13), we obtain

$$\Omega_{GW}(k) = \frac{k^2}{12H_0^2} P_T(k), \tag{14}$$

The tensor power spectrum observed today can relate to the that of inflationary one by the transfer transfer function $\mathcal{T}(k)$ as follows

$$P_T = \mathcal{T}^2(k)\mathcal{P}_T(k) \tag{15}$$

By using this relation, we can reexpress Eq. (14) in terms of the inflationary tensor power spectrum

$$h_0^2\Omega_{GW}(k) = \frac{h_0^2 k^2}{12H_0^2}\mathcal{T}^2(k)\mathcal{P}_T(k). \tag{16}$$

where $\mathcal{T}(k)$ reflects the damping effect of the GWs when evolving in the expansion universe.

There are several damping effects that we may have take an account.

- the damping effect of the cosmic expansion
- the damping effect of the free-streaming relativistic particles
- the damping effect of the successive changes in the relativistic d.o.f during RD epoch

In this work, for simplicity, we consider the cosmic expansion as the only damping effect since

- it is the most important one among the others
- it approximately shows the evolution of primordial GWs in the expanding universe.

Thus, the evolution of primordial GWs can be described by following equation

$$h''_{\lambda,k} + 2\mathcal{H}h'_{\lambda,k} + k^2 h_{\lambda,k} = 0 \tag{17}$$

The mode solutions to this equation have qualitative behavior in two regimes:

- far outside the horizon ($k \ll aH$) where the amplitude of $h_{\lambda,k}$ keeps constant;
- far inside the horizon ($k \gg aH$) where they damp as $h_{\lambda,k} \sim 1/a$.

Transfer function for the modes which are well inside the horizon ($k \gg 10^{-18}$Hz) has been calculated by integrating Eq. (17) numerically from $\tau = 0$ to τ_0; a good fit to the transfer function is

$$\mathcal{T}(k) = \frac{3}{k^2\tau_0^2}\frac{\Omega_m}{\Omega_\Lambda}\sqrt{1 + \frac{4}{3}\left(\frac{k}{k_{eq}}\right) + \frac{5}{2}\left(\frac{k}{k_{eq}}\right)^2} \quad (18)$$

where $k_{eq} = 0.073\Omega_m h^2\text{Mpc}^{-1}$ is the wavenumber corresponding to a Hubble radius at the time that matter and radiation have equal energy density. The factor Ω_m/Ω_Λ is the effect of the accelerated expansion of the universe.

- For modes that re-entered the horizon during matter dominated era where $k \ll k_{eq}$, Eq. (18) evolves as $\mathcal{T}(k) \sim k^{-2}$,
- For modes that re-entered the horizon during radiation dominated era where $k \gg k_{eq}$, it evolves as $\mathcal{T}(k) \sim k^{-1}$

By combining Eq. (16) with Eq. (18), we obtain the gravitational waves energy spectrum

$$h_0^2\Omega_{GW}(k) \simeq \frac{3h_0^2}{4H_0^2\tau_0^4}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2\frac{1}{k^2}\left[1 + \frac{4}{3}\left(\frac{k}{k_{eq}}\right) + \frac{5}{2}\left(\frac{k}{k_{eq}}\right)^2\right]\mathcal{P}_T(k). \quad (19)$$

As a result, we obtain

$$h_0^2\Omega_{GW} \simeq \frac{3h_0^2}{4H_0^2\tau_0^4}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2\frac{1}{k^2}\left[1 + \frac{4}{3}\left(\frac{k}{k_{eq}}\right) + \frac{5}{2}\left(\frac{k}{k_{eq}}\right)^2\right]\mathcal{P}_S(k_*)r\left(\frac{k}{k_*}\right)^{n_T + \frac{\alpha_T}{2}\ln(k/k_*)} \quad (20)$$

We start with the following action motivated in the nonlinear sigma model

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\tilde{h}_{mn}(\sigma)\partial_\mu\sigma^m\partial_\nu\sigma^n - V(\sigma)\right] \tag{21}$$

where $\tilde{h}_{mn}$ can be considered as an internal metric of a four-dimensional Riemannnian manifold.
To construct an inflationary model, we choose following internal metric and potential

$$\tilde{h}_{ab} = f(\phi)\delta_{ab}, \;\; \tilde{h}_{a4} = 0, \;\; \tilde{h}_{44} = 1, \;\; V(\sigma) = V(\phi) \tag{22}$$

where $\phi \equiv \sigma^4$ and $f(\phi)$ is an arbitrary positive function of ϕ and we fix it to $f(\phi) = 1$ through out this work for simplicity. Then the resulting action,

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\delta_{ab}\partial_\mu\sigma^a\partial_\nu\sigma^b - V(\phi)\right] \tag{23}$$

can be considered as the single scalar field model interacting with a triad of scalar fields σ^a.

We read the energy-momentum tensor and EoM of the scalar field ϕ and σ^a as

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi + \delta_{ab}\partial_\mu\sigma^a\partial_\nu\sigma^b - \frac{1}{2}g_{\mu\nu}\left[g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \delta_{ab}g^{\alpha\beta}\partial_\alpha\sigma^a\partial_\beta\sigma^b + 2V(\phi)\right] \quad (24)$$

$$\partial_\mu\partial^\mu\phi - V_\phi = 0, \quad \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\sigma^a) = 0 \quad (25)$$

The background EoM for $g_{\mu\nu}$ and ϕ with following FRW metric and an ansatz for the scalar field σ^a,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \quad \text{and} \quad \sigma^a = \xi x^a, \quad (26)$$

are given by

$$H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V + \frac{3\xi^2}{2a^2}\right), \quad \dot{H} = -\frac{1}{2M_p^2}\left(\dot{\phi}^2 + \frac{\xi^2}{a^2}\right) \quad (27)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0. \quad (28)$$

Background EoM and slow-roll EoM:

In the context of the slow-roll inflation with potential satisfying the slow-roll approximations,

$$\dot{\phi}^2/2 \ll V, \quad \ddot{\phi} \ll 3H\dot{\phi} \tag{29}$$

one gets the background EoM approximately,

$$H^2 \simeq \frac{V}{3M_p^2} + \frac{\xi^2}{2a^2M_p^2}, \quad 3H\dot{\phi} + V_\phi \simeq 0. \tag{30}$$

The feature of the model is that there exists a phase transition, during inflation, of the background evolution and it is connected to the slow-roll phase of the single field inflation model at the late time.

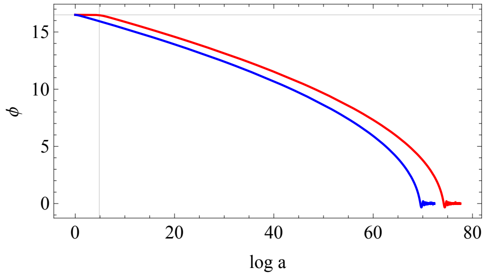

Figure 1: Potential $V = m^2\phi^2/2$. We chose: $\phi_0 = 16.5M_p$, $\dot{\phi}_0 = 0$, $a_0 = 1$, $m = 5.85 \times 10^{-6}M_p$ and $\xi = 10^{-2}$.

Transition takes a place at different time depending on the values of ξ. However, following condition from the slow-roll EoM above must be hold

$$aH \sim \frac{\xi}{M_p} \sim k_{\min}. \tag{31}$$

Observable parameters:

The way to reflect these slow-roll approximations is to introduce the slow-roll parameters,

$$\epsilon_V \simeq \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2\left(1-\frac{3\xi^2 M_p^2}{a^2 V}\right), \quad \eta_V \simeq \frac{V_{\phi\phi}}{V}M_p^2\left(1-\frac{3\xi^2 M_p^2}{2a^2 V}\right) \tag{32}$$

The corresponding observable parameters during inflation are given by

$$\mathcal{P}_S(k) \simeq \frac{H^2}{\pi\epsilon_V M_p^2}\left(1+(6C-2)\epsilon_V - 2C\eta_V - \frac{\xi^2}{\epsilon_V M_p^2 k^2}\right) \tag{33}$$

$$\mathcal{P}_T(k) \simeq \frac{16H^2}{\pi M_p^2}\left(1+(2C-2)\epsilon - \frac{37\xi^2}{12M_p^2 k^2}\right) \tag{34}$$

$$n_S - 1 \simeq 2\eta_V - 6\epsilon_V + \frac{2\xi^2}{\epsilon_V k^2 M_p^2} \tag{35}$$

$$n_T \simeq -2\epsilon_V + \frac{31\xi^2}{6k^2 M_p^2} \tag{36}$$

$$r \simeq 16\epsilon_V + \frac{16\xi^2}{k^2 M_p^2} \tag{37}$$

$$\alpha_S = -\frac{4\xi^2}{\epsilon_V k^2 M_p^2}, \quad \alpha_T = -\frac{31\xi^2}{3k^2 M_p^2} \tag{38}$$

(39)

At the end of the inflation, our model asymptotically approaches to the standard single field inflationary model with the small contribution from the ξ dependent term.
By using the observable parameters shown earlier, we obtain the energy spectrum of primordial GWs in terms of physical frequency observed today

$$h_0^2\Omega_{GW} = \frac{3h_0^2}{16\pi^2 H_0^2 \tau_0^4}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2 \frac{1}{f^2}\left[1+\frac{4}{3}\left(\frac{f}{f_{eq}}\right)+\frac{5}{2}\left(\frac{f}{f_{eq}}\right)^2\right] \times \mathcal{P}_S(k_*) r \left(\frac{f}{f_*}\right)^{-\frac{r}{8}+\frac{43\xi^2}{24\pi^2 f^2}-\frac{31\xi^2}{24\pi^2 f^2}\ln(f/f_*)} \quad (40)$$

where we use $M_p^2 = 1$, $f_{eq} \equiv k_{eq}/2\pi$ and $f_* \equiv k_*/2\pi$.
We consider the standard model of cosmology and cosmological parameters adopted in this work are listed as follows:

- $\Omega_m = 0.315$, $\Omega_\Lambda = 0.685$, $a(\tau_0) = 1$, $\tau_0 = 1.41 \times 10^4$Mpc, $k_{eq} = 0.073\Omega_m h_0^2\,\mathrm{Mpc}^{-1}$, $H_0 = 100h_0\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, reduced Hubble parameter $h_0 \simeq 0.67$, $\mathcal{P}_S(k_*) \simeq 2.19 \times 10^{-9}$ at pivot scale $k_* = 0.002\mathrm{Mpc}^{-1}$, so that $f_* = 3.092 \times 10^{-18}$Hz.
  The relation between wavenumber k and physical frequency, $f = k/2\pi$, is used as well.

$$h_0^2\Omega_{GW} = \frac{3h_0^2}{16\pi^2 H_0^2 \tau_0^4}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2 \frac{1}{f^2}\left[1 + \frac{4}{3}\left(\frac{f}{f_{eq}}\right) + \frac{5}{2}\left(\frac{f}{f_{eq}}\right)^2\right]$$
$$\times \mathcal{P}_S(k_*) r \left(\frac{f}{f_*}\right)^{-\frac{r}{8} + \frac{43\xi^2}{24\pi^2 f^2} - \frac{31\xi^2}{24\pi^2 f^2}\ln(f/f_*)} \quad (41)$$

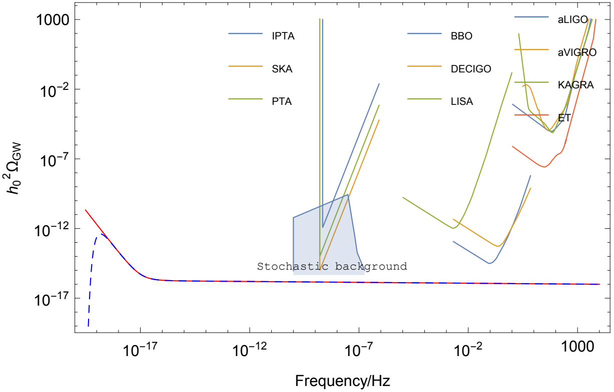

Figure 2: The dashed line corresponds to $\xi = 10^{-19}$ while the solid line is for $\xi = 0$ where $r = 10^{-1}$.

Numerical results:

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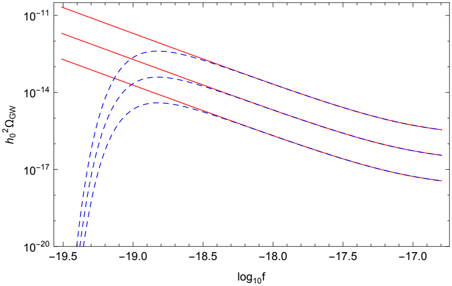
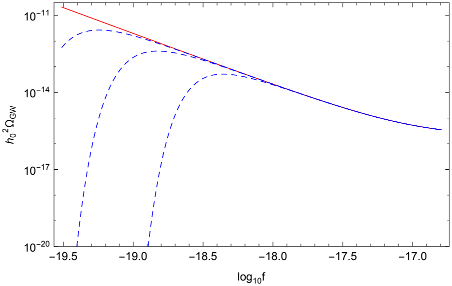


(a) $r = 10^{-1}$, 10^{-2}, and 10^{-3}; for $\xi = 10^{-19}$



(b) $\xi = 3.1 \times 10^{-20}$, 10^{-19}, and 4×10^{-19}; for $r = 10^{-1}$.

Figure 3: The low frequency range of Figure 2. The solid line for all profiles indicates $\xi = 0$.

$$n_S - 1 \simeq 2\eta_V - 6\epsilon_V + \frac{2\xi^2}{\epsilon_V k^2 M_p^2}, \quad n_T \simeq -2\epsilon_V + \frac{31\xi^2}{6k^2 M_p^2}, \quad r \simeq 16\epsilon_V + \frac{16\xi^2}{k^2 M_p^2}, \tag{43}$$

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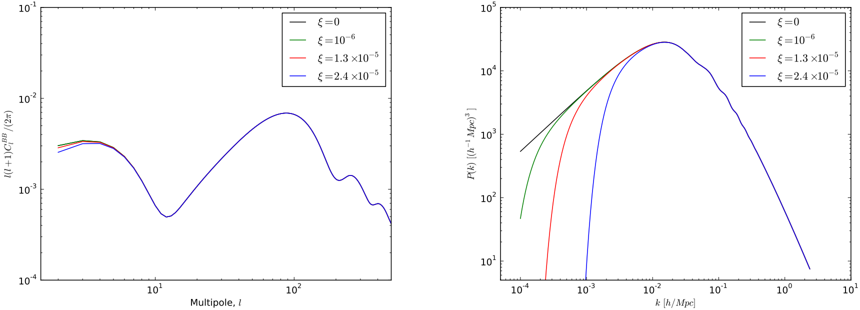

(a) Preliminary (b) Preliminary

Figure 4: We use the power law form of the both scalar and tensor power spectrum.

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- to consider other damping effects in other to study about the detection possibility of the primordial GWs.
- to consider $f(\phi)$ as some function of ϕ rather than fixing to 1.

Thank you for your attention!