Quantum Primordial Standard Clocks

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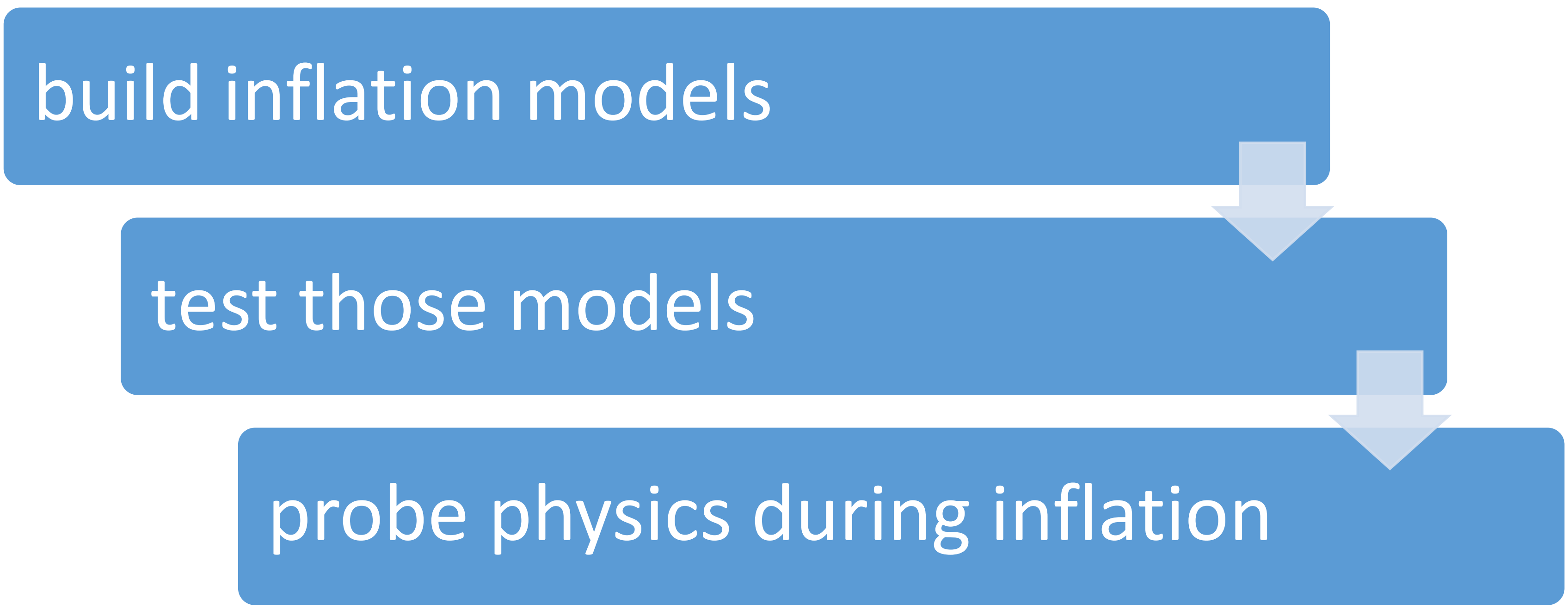
With Xingang Chen and Mohammad Hossein Namjoo, 1509.03930

Inflation: $a \propto e^{Ht}$

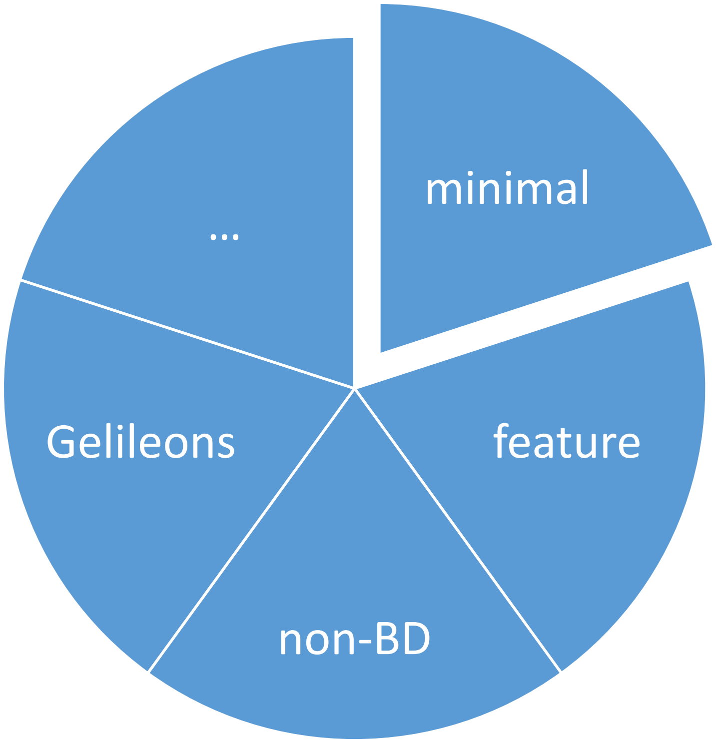

build inflation models
test those models
probe physics during inflation

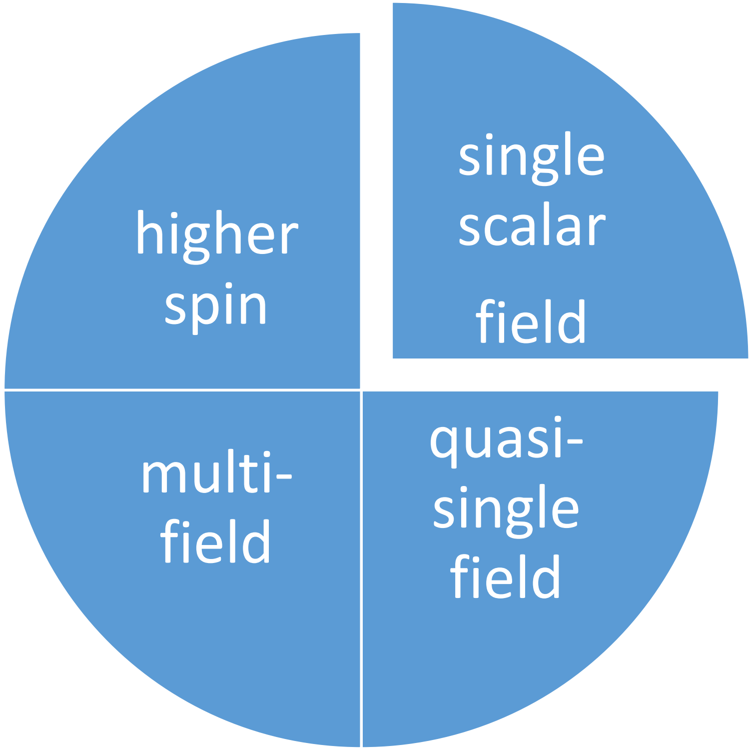
Let’s see how bad it works …

Incomplete list of single field inflation from Encyclopedia Inflationaris

Martin, Ringeval, Vennin, 2013

Name	Parameters	Sub-models	$V(\phi)$
HI	0	1	$M^4\left(1-e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)$
RCHI	1	1	$M^4\left(1-2e^{-\sqrt{2/3}\phi/M_{\rm Pl}}+\frac{A_{\rm I}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right)$
LFI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p$
MLFI	1	1	$M^4\frac{\phi^2}{M_{\rm Pl}^2}\left[1+\alpha\frac{\phi^2}{M_{\rm Pl}^2}\right]$
RCMI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-2\alpha\frac{\phi^2}{M_{\rm Pl}^2}\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RCQI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^4\left[1-\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
NI	1	1	$M^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$
ESI	1	1	$M^4\left(1-e^{-q\phi/M_{\rm Pl}}\right)$
PLI	1	1	$M^4e^{-\alpha\phi/M_{\rm Pl}}$
KMII	1	2	$M^4\left(1-\alpha\frac{\phi}{M_{\rm Pl}}e^{-\phi/M_{\rm Pl}}\right)$
HF1I	1	1	$M^4\left(1+A_1\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-\frac{2}{3}\left(\frac{A_1}{1+A_1\phi/M_{\rm Pl}}\right)^2\right]$
CWI	1	1	$M^4\left[1+\alpha\left(\frac{\phi}{Q}\right)^4\ln\left(\frac{\phi}{Q}\right)\right]$
LI	1	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RpI	1	3	$M^4e^{-2\sqrt{2/3}\phi/M_{\rm Pl}}\left\lvert e^{\sqrt{2/3}\phi/M_{\rm Pl}}-1\right\rvert^{2p/(2p-1)}$
DWI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-1\right]^2$
MHI	1	1	$M^4\left[1-\mathrm{sech}\left(\frac{\phi}{\mu}\right)\right]$
RGI	1	1	$M^4\frac{(\phi/M_{\rm Pl})^2}{\alpha+(\phi/M_{\rm Pl})^2}$
MSSMI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\left(\frac{\phi}{\phi_0}\right)^6+\frac{1}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
RIPI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\left(\frac{\phi}{\phi_0}\right)^3+\frac{1}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
RMI	3	4	$M^4\left[1-\frac{c}{2}\left(-\frac{1}{2}+\ln\frac{\phi}{\phi_0}\right)\frac{\phi^2}{M_{\rm Pl}^2}\right]$
VHI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^p\right]$
DSI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^{-p}\right]$
GMLFI	3	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^q\right]$
LPI	3	3	$M^4\left(\frac{\phi}{\phi_0}\right)^p\left(\ln\frac{\phi}{\phi_0}\right)^q$
CNDI	3	3	$\frac{M^4}{\left\{1+\beta\cos\left[\alpha\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)\right]\right\}^2}$
AI	1	1	$M^4\left[1-\frac{2}{\pi}\arctan\left(\frac{\phi}{\mu}\right)\right]$
CNAI	1	1	$M^4\left[3-(3+\alpha^2)\tanh^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)\right]$
CNBI	1	1	$M^4\left[(3-\alpha^2)\tan^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
OSTI	1	1	$-M^4\left(\frac{\phi}{\phi_0}\right)^2\ln\left[\left(\frac{\phi}{\phi_0}\right)^2\right]$
WRI	1	1	$M^4\ln\left(\frac{\phi}{\phi_0}\right)^2$
SFI	2	1	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^p\right]$
II	2	1	$M^4\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta}-M^4\frac{\beta^2}{6}\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta-2}$
KMIII	2	1	$M^4\left[1-\alpha\frac{\phi}{M_{\rm Pl}}\exp\left(-\beta\frac{\phi}{M_{\rm Pl}}\right)\right]$
LMI	2	2	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^\alpha\exp\left[-\beta(\phi/M_{\rm Pl})^\gamma\right]$
TWI	2	1	$M^4\left[1-A\left(\frac{\phi}{\phi_0}\right)^2e^{-\phi/\phi_0}\right]$
GMSSMI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^6+\frac{\alpha}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
GRIPI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^3+\frac{\alpha}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
BSUSYBI	2	1	$M^4\left(e^{\sqrt{6}\frac{\phi}{M_{\rm Pl}}}+e^{\sqrt{6}\gamma\frac{\phi}{M_{\rm Pl}}}\right)$
TI	2	3	$M^4\left(1+\cos\frac{\phi}{\mu}+\alpha\sin^2\frac{\phi}{\mu}\right)$
BEI	2	1	$M^4\exp_{1-\beta}\left(-\lambda\frac{\phi}{M_{\rm Pl}}\right)$
PSNI	2	1	$M^4\left[1+\alpha\ln\left(\cos\frac{\phi}{f}\right)\right]$
NCKI	2	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^2\right]$
CSI	2	1	$\frac{M^4}{\left(1-\alpha\frac{\phi}{M_{\rm Pl}}\right)^2}$
OI	2	1	$M^4\left(\frac{\phi}{\phi_0}\right)^4\left[\left(\ln\frac{\phi}{\phi_0}\right)^2-\alpha\right]$
CNCI	2	1	$M^4\left[(3+\alpha^2)\coth^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
SBI	2	2	$M^4\left\{1+\left[-\alpha+\beta\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right\}$
SSBI	2	6	$M^4\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^2+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right]$
IMI	2	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^{-p}$
BI	2	2	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^{-p}\right]$


minimal
...
Gelileons
feature
non-BD

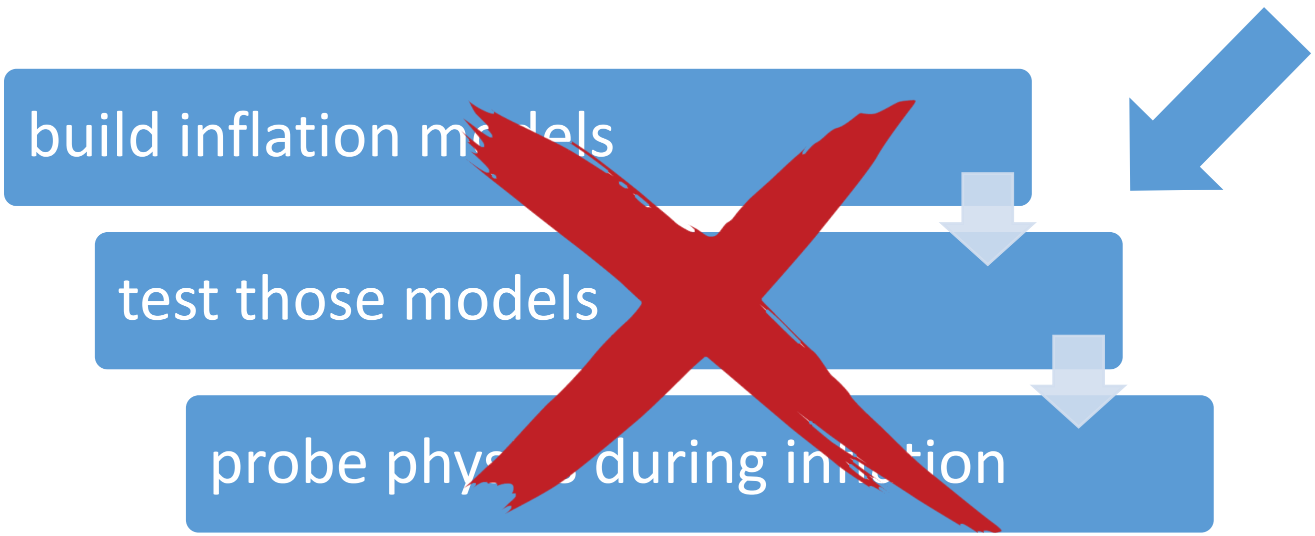

single
scalar
field
higher
spin
multi-
field
quasi-
single
field

Lots of data coming to test inflation

(polarization, LSS, 21cm, …)

But data may never be enough to distinguish those models

We would be stuck here forever!



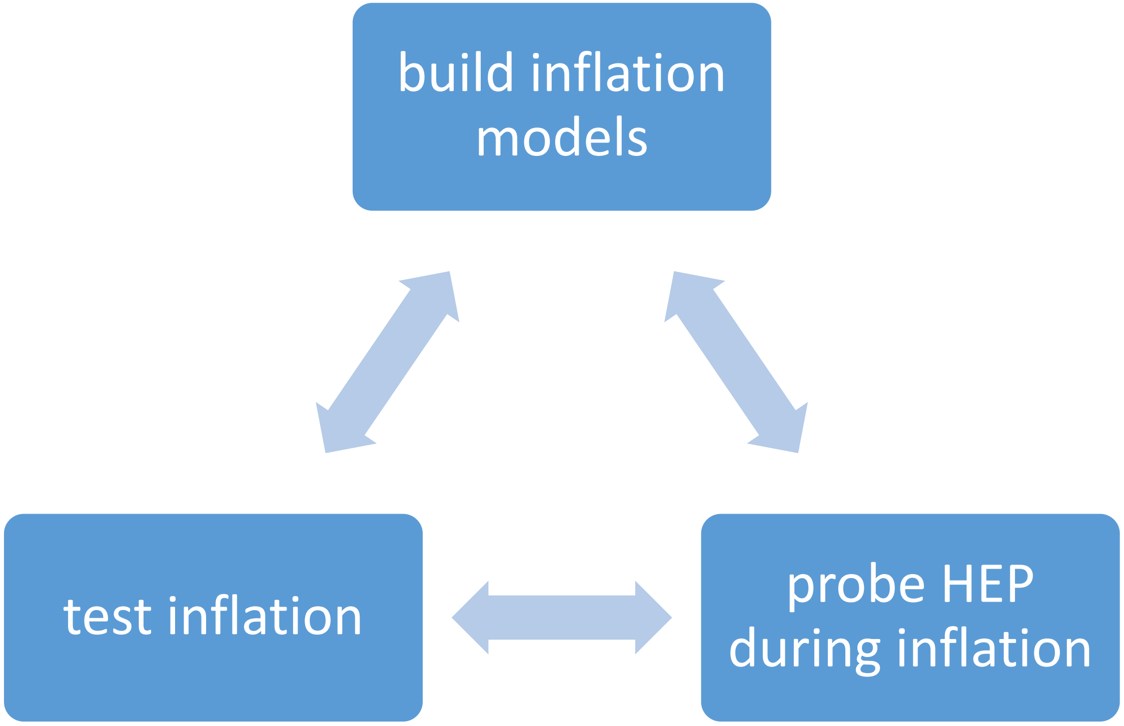
But things are not so unlucky – Remember $a \propto e^{Ht}$

But things are not so unlucky – Remember $a \propto e^{Ht}$

The ugly: Why so many models

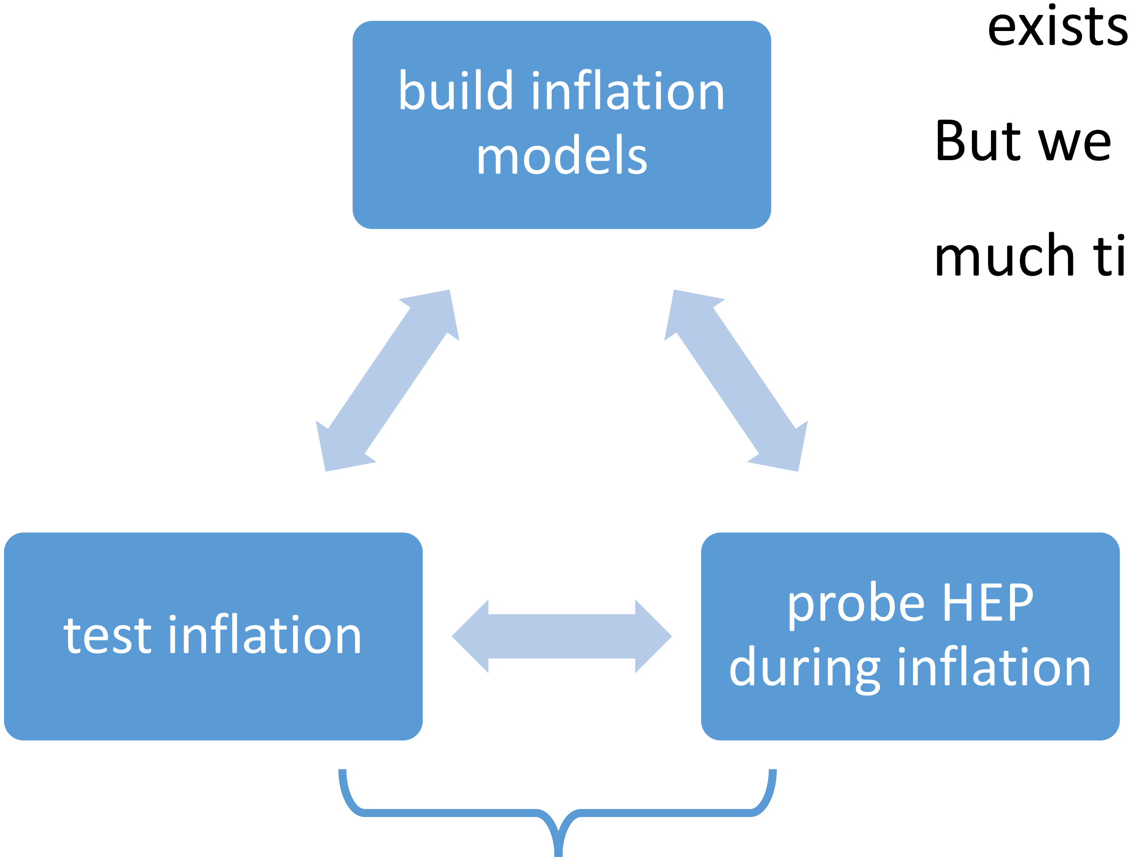
The bad: Why hard to distinguish

The good: Universality allows us to do things in parallel


build inflation models
test inflation
probe HEP during inflation

Definitely all those researches exists in the literature.

But we probably spent too much time at the first step.



Both to be done in a model-independent way

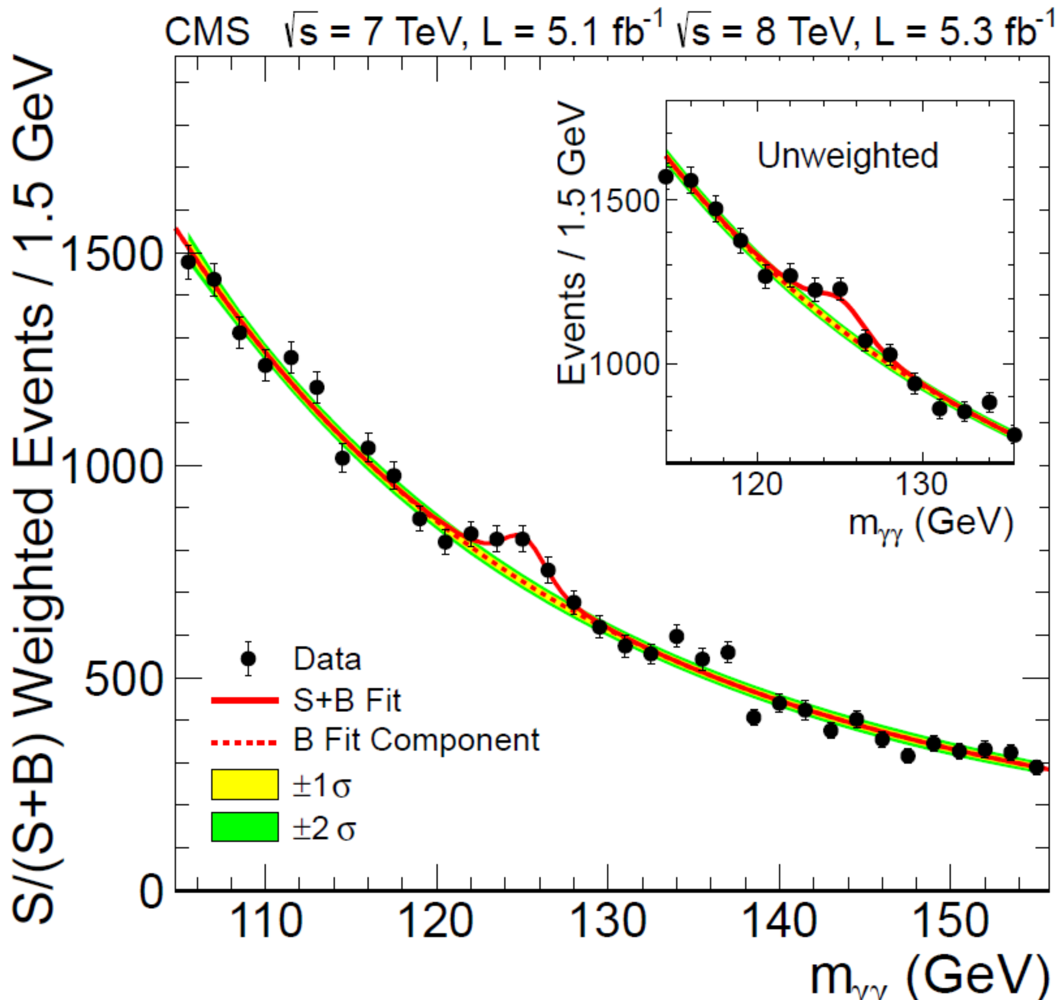
How to probe HEP during inflation?

How to probe HEP during inflation?

Similarly, one may ask: How to probe HEP on colliders?

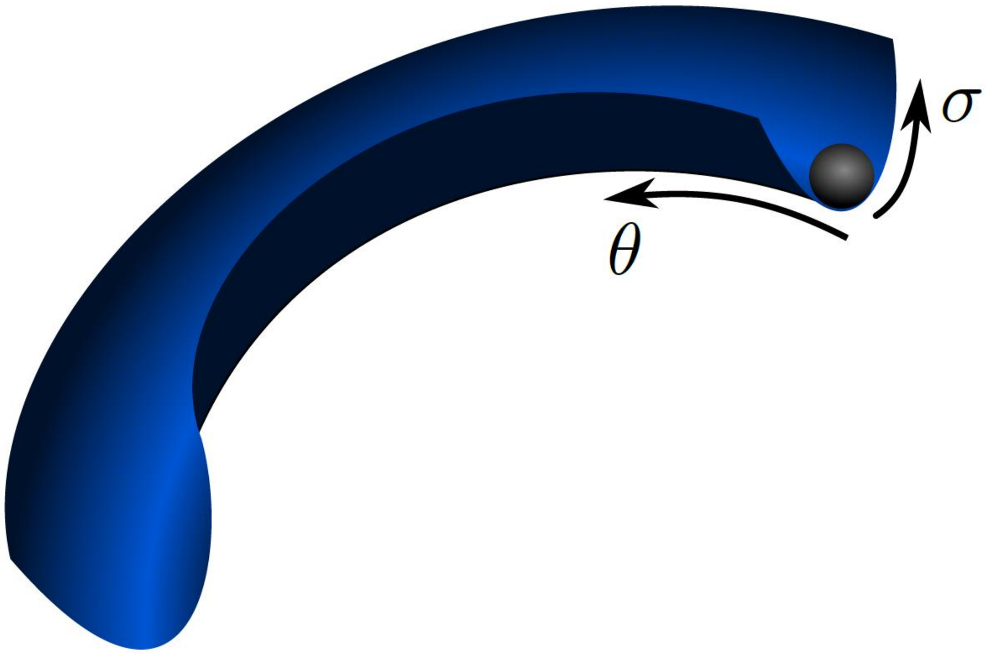
Search for massive

new particles



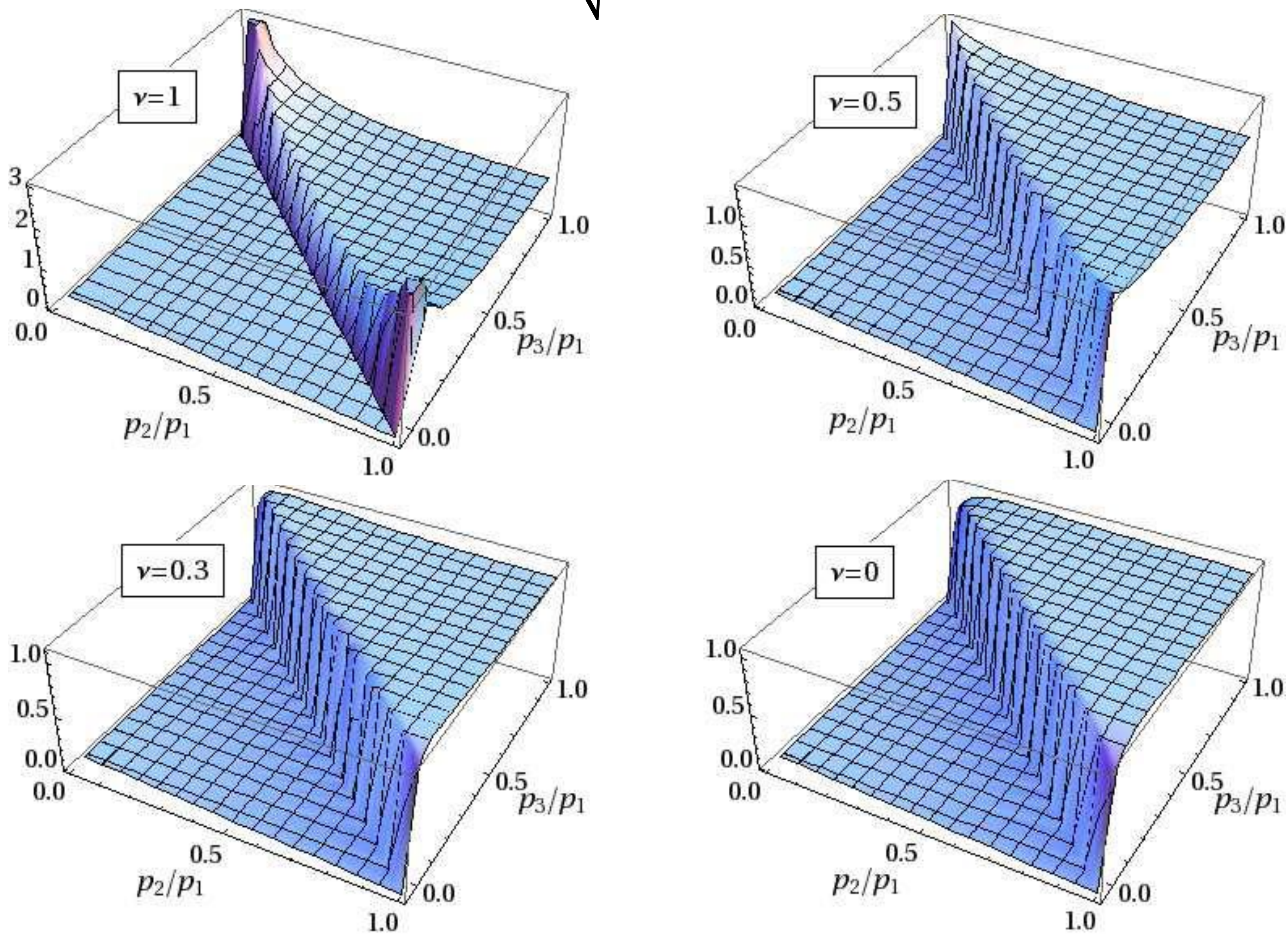
CMS, 2012

Quasi-single field inflation

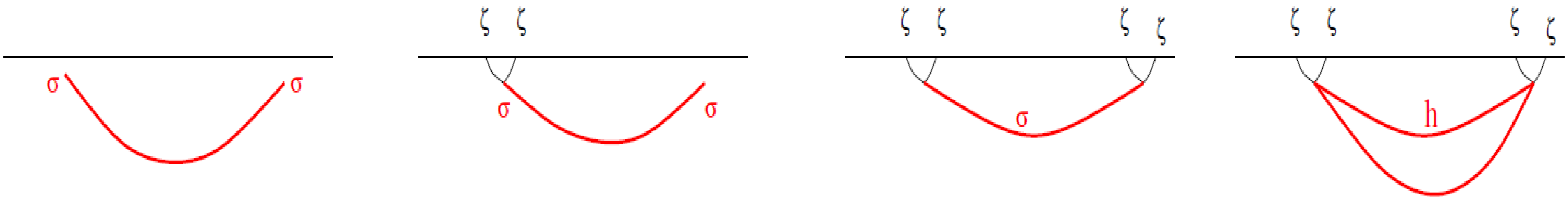


X. Chen, YW, 2009

$$\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$



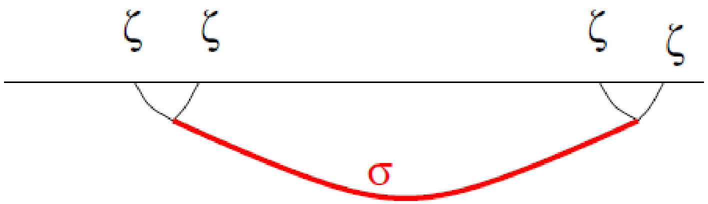
Cosmological double slit experiment



Arkani-Hamed, Maldacena 2015, Maldacena 2015

X. Chen, YW 09, 12, Pi, Sasaki 12, Gong, Pi, Sasaki 13

Arkani-Hamed, Maldacena 15



Contributions to correlation functions:

type	meaning	depencence on $\theta(\tau-\tau')$	analytic in k?	integrate out?	suppression at large μ	suppression at large x
“local”	vacuum correlation	Yes	Yes	Yes	$1/\mu^2$	vanish outside lightcone
“non-local”	thermal particle production	No	No	No	$e^{-\pi\mu}$	non-vanishing

$$\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

$$\frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle_{\rm short}\langle\zeta\zeta\rangle_{\rm long}} \sim \epsilon e^{-\pi\mu}|c(\mu)| \left[e^{i\delta(\mu)} \left(\frac{k_{\rm long}}{k_{\rm short}}\right)^{\frac{3}{2}+i\mu} + e^{-i\delta(\mu)} \left(\frac{k_{\rm long}}{k_{\rm short}}\right)^{\frac{3}{2}-i\mu} \right] P_s(\cos\theta)$$

Arkani-Hamed, Maldacena 15

How to test inflation?

How to test inflation?

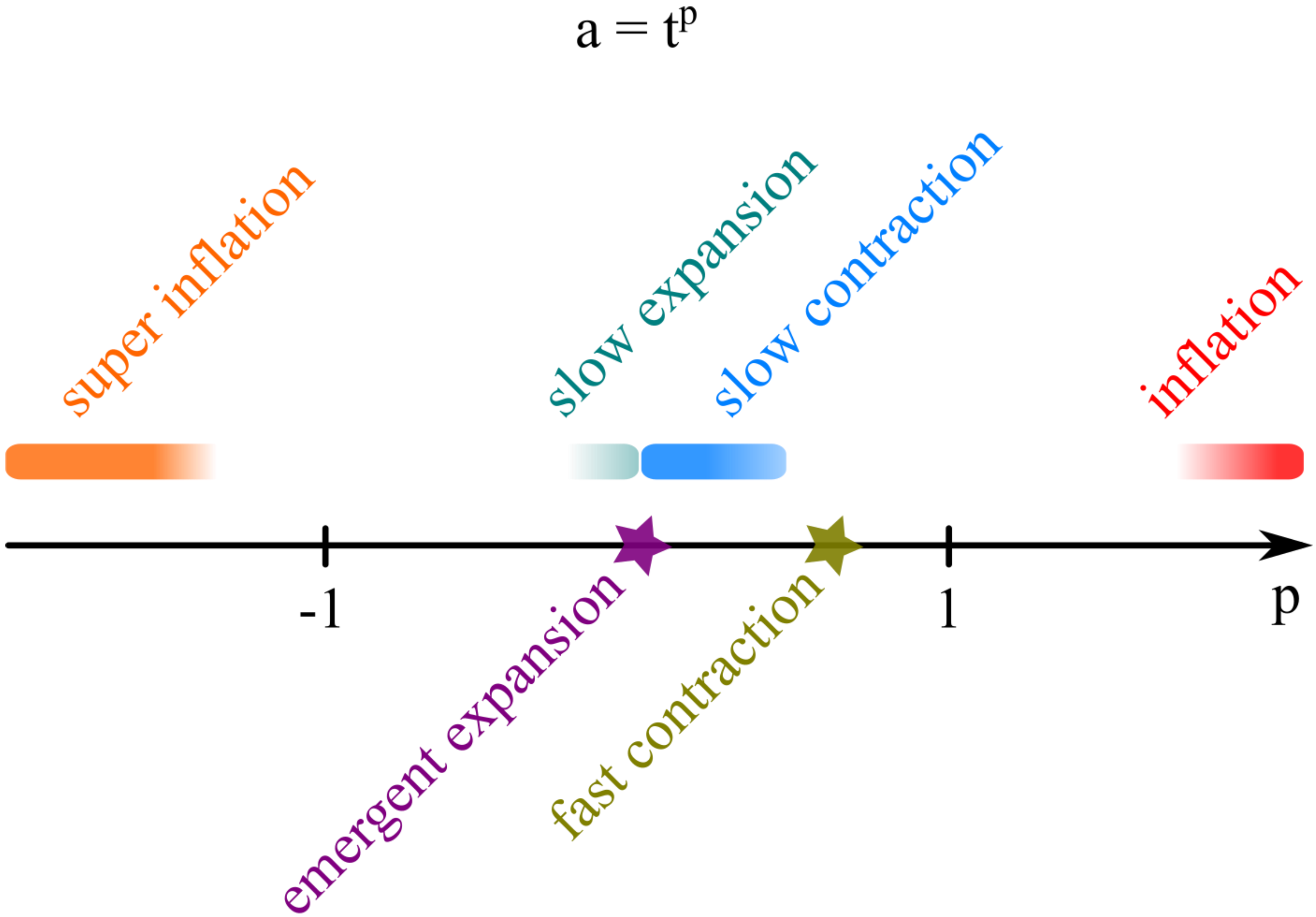
We don’t have to test model by model.

Better to just test $a \propto e^{Ht}$

How to?

Instead of testing inflation against itself,

we should test against alternatives.



a = t^p
super inflation
slow expansion
slow contraction
inflation
-1
1
p
emergent expansion
fast contraction

How to test inflation?

We don’t have to test model by model.

Better to just test $a \propto e^{Ht}$

How to?

Conventional argument: Search for gravitational waves (GW).

Because GW “directly” probes the scale factor.

Definitely important direction.

However, there are important assumptions.

Assumption 1: Observe constant mode

Exception: Matter bounce

Wands 1999, Finelli and Brandenberger 2002

Assumption 2: Vacuum fluctuations

Exception: String gas cosmology

Brandenberger and Vafa 1989

Brandenberger, Nayeri, Patil and Vafa, 2007

Towards a model independent direct “proof”

Search for physical “clock” signals during inflation.

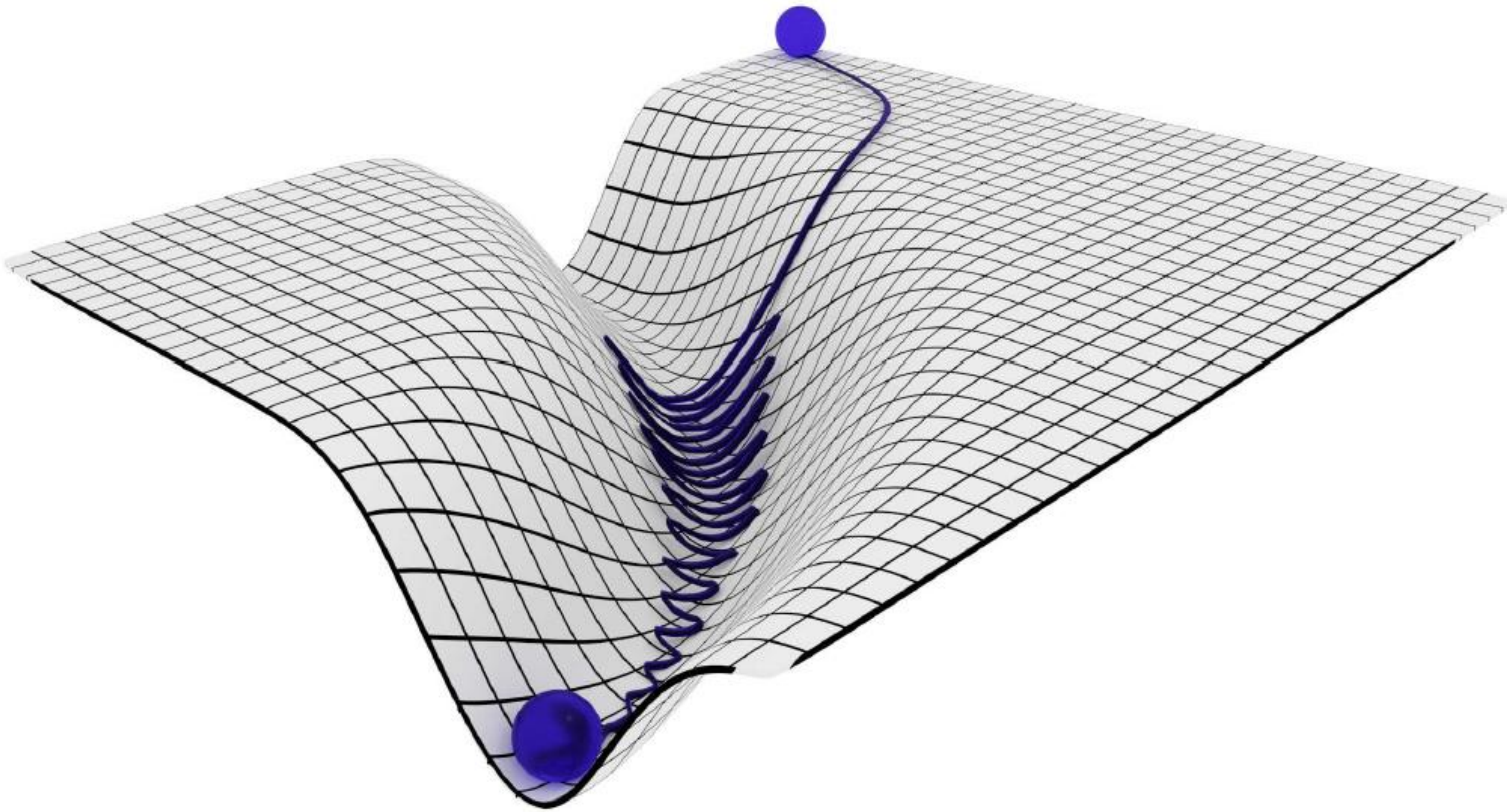
CMB fluctuations are generated at horizon crossing $k \sim aH$

Thus probing different cosmological scales $k \leftrightarrow$ probing aH.

Once there is a physical clock to probe $t \leftrightarrow a(t)$ is known.

Chen 11; Chen, Namjoo, YW 15

Massive fields are clocks of physical time



Chen, Namjoo, YW 15

But to realize the model, luck is needed,

e.g. massive field fall into its minimal potential,

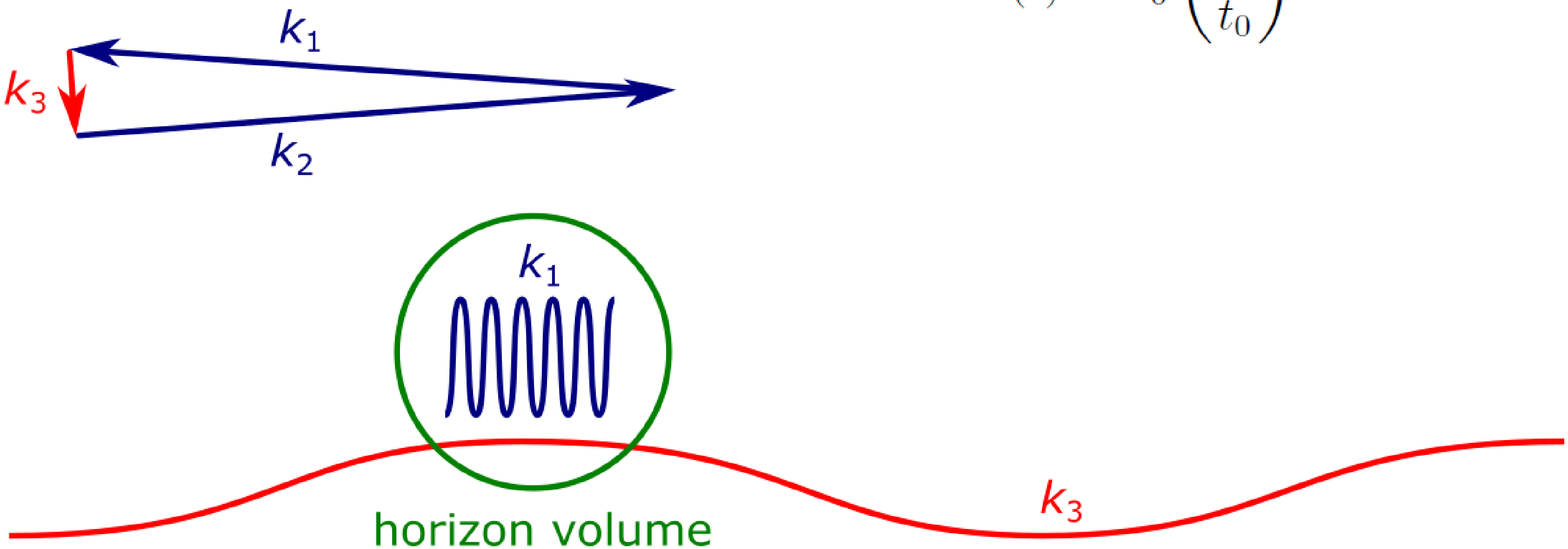
during the observable stage of inflation.

More universal test of inflation?

Massive fields always have quantum zero point fluctuations.

$\langle \zeta^3 \rangle$

$$a(t) = a_0 \left(\frac{t}{t_0} \right)^p$$



To see the clock signal (massive quantum oscillation), we need

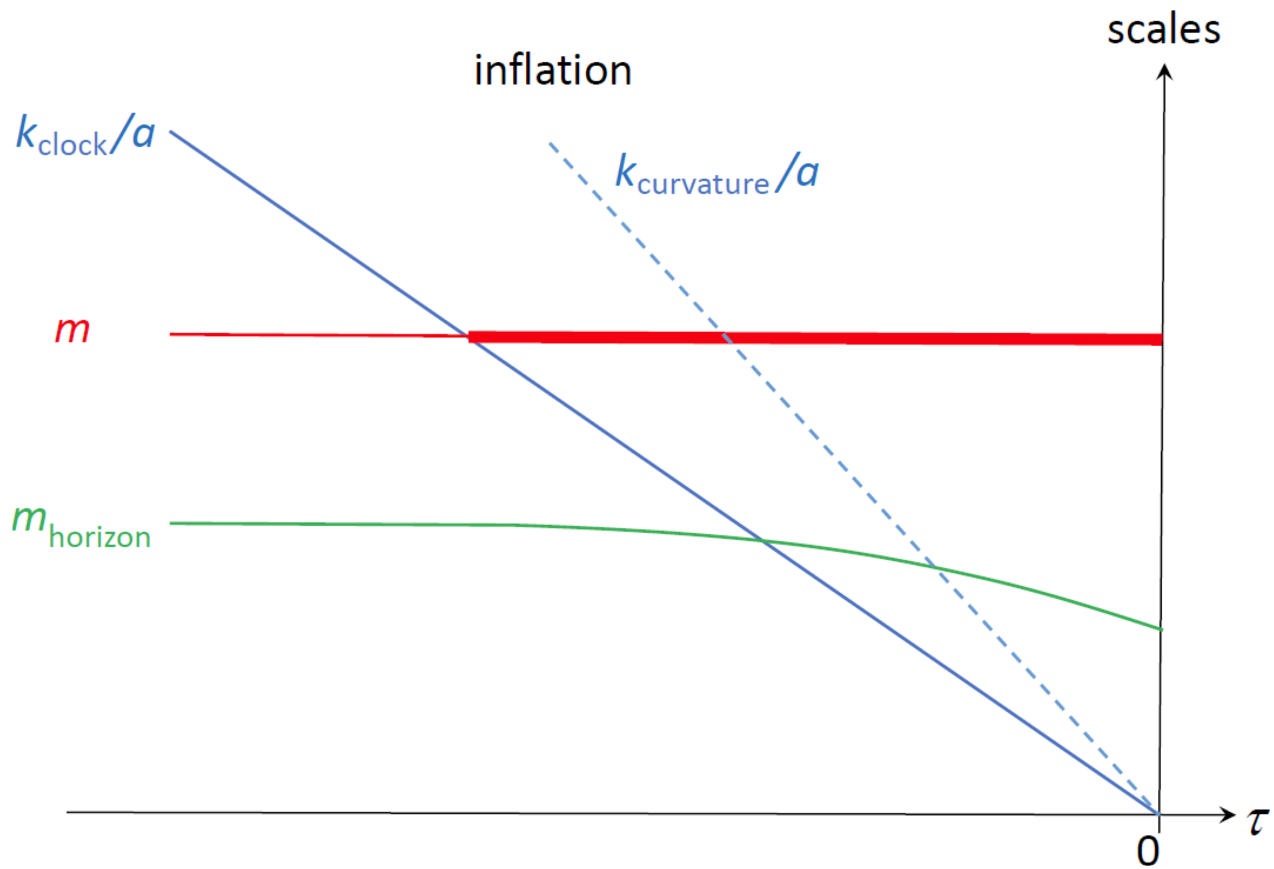
1. $m \gg k/a$ (exist homogeneous oscillation)
2. $m \gg m_h$, where m_h is the horizon mass scale

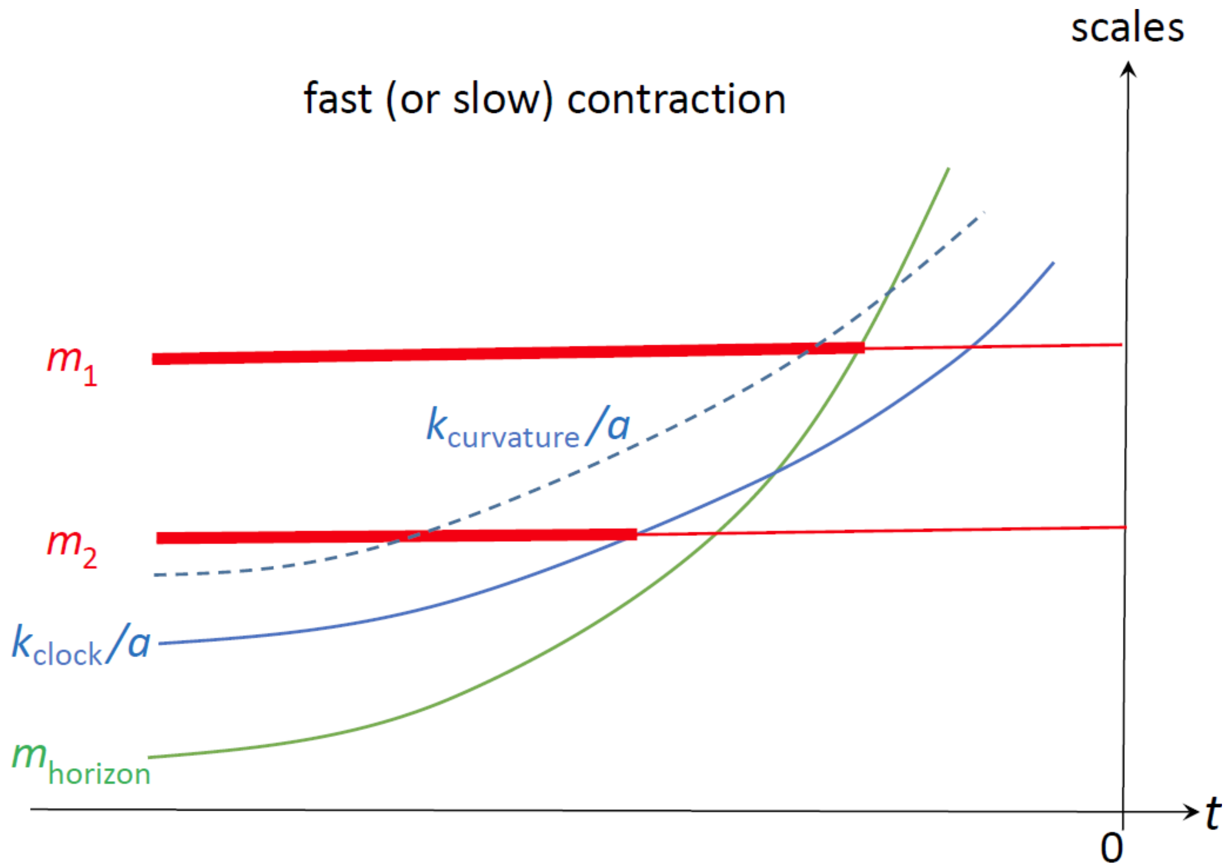
$$\frac{1}{m_h} \equiv a(t) \int_t^{t_{\mathrm{end}}} \frac{dt}{a(t)} = |a\tau| = \left|\frac{t}{1-p}\right|$$

in order to imprint oscillations on inflaton's horizon crossing

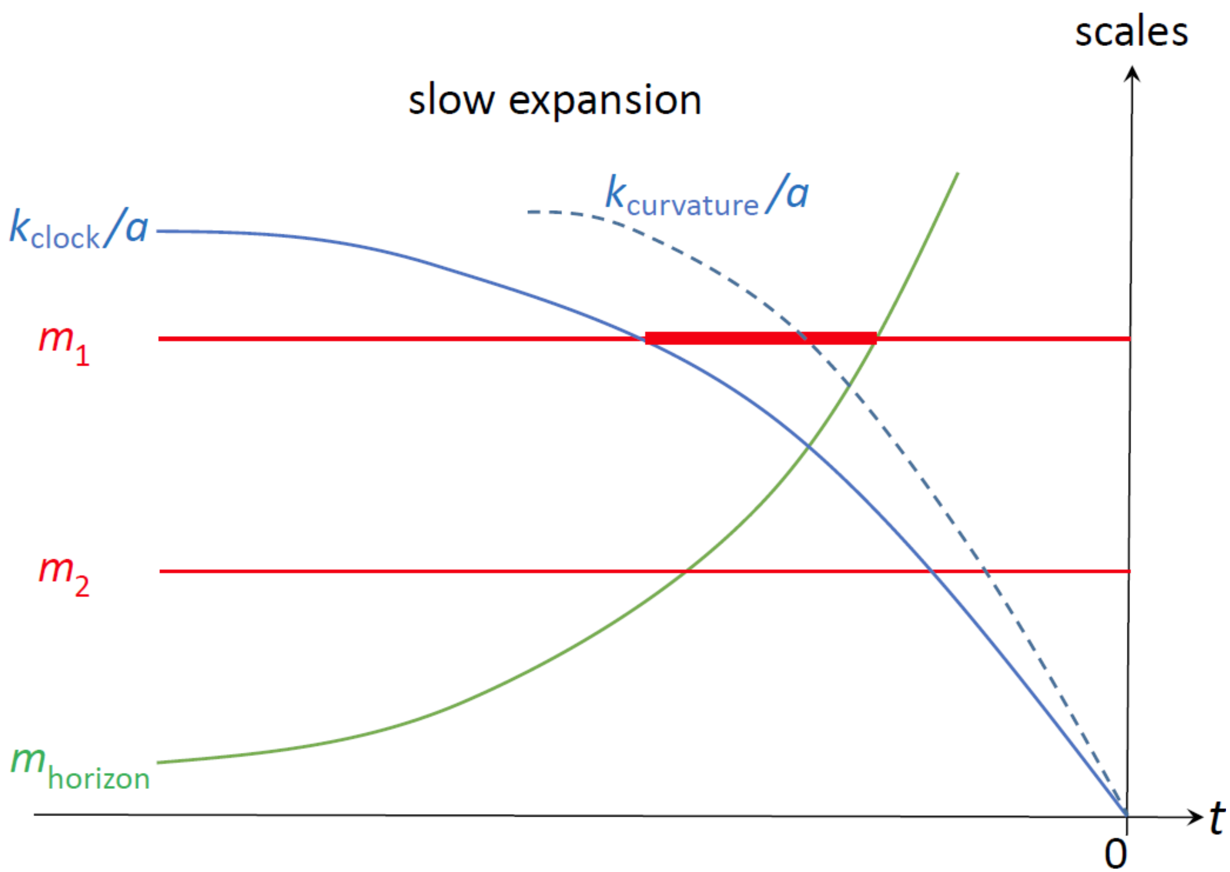
(also, too small mass → over-damp oscillator)

We call such regime the "classical regime"


scales
inflation
k_{clock}/a
$k_{curvature}/a$
m
$m_{horizon}$
0
τ



scales
fast (or slow) contraction
m_1
$k_{\rm curvature}/a$
m_2
$k_{\rm clock}/a$
$m_{\rm horizon}$
t
0


scales
slow expansion
$k_{curvature}/a$
k_{clock}/a
m_1
m_2
$m_{horizon}$
t
0

Calculation:

(1) Resonant approximation

(2) Explicit calculation in an explicit model

The resonance of an integral

Chen, Easther, Lim, 2008

Flauger & Pajer, 2010

$$\int_{\tau_{\rm begin}}^{\tau_{\rm end}} d\tau g(t) e^{imt} e^{-iK\tau} \rightarrow g(t_*) \int_{\tau_{\rm begin}}^{\tau_{\rm end}} d\tau e^{imt-iK\tau}$$

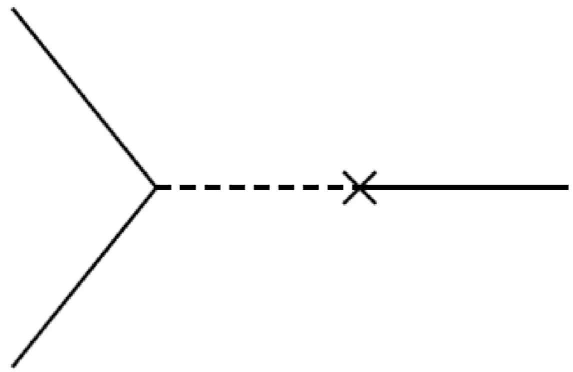
$$\frac{d}{d\tau}\left(mt - K\tau\right)\Big|_{t=t_*} = 0$$

$$a(t_*) = K/m$$

$$\int_{\tau_{\rm begin}}^{\tau_{\rm end}} d\tau g(t) e^{imt} e^{-iK\tau}$$

$$\rightarrow \sqrt{2\pi} g(t_*) \left(\frac{m}{|H_{k_0}|}\right)^{1/2} K^{-1} \left(\frac{K}{k_0}\right)^{1/2p} \exp\left[-i\frac{p^2}{1-p}\frac{m}{H_{k_0}}\left(\frac{K}{k_0}\right)^{1/p} \mp i\frac{\pi}{4}\right]$$

A model to illustrate the Primordial Quantum Standard Clock



$$\mathcal{L}_2 \sim c_2 a^3 \dot{\zeta} \delta\sigma$$

$$\mathcal{L}_3 \sim c_3 a^3 \dot{\zeta}^2 \delta\sigma$$

$$\langle\zeta^3\rangle' \supset \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{t} dt_1 \langle 0|H_I(\tilde{t}_1)\zeta_I^3 H_I(t_1)|0\rangle' - 2\mathrm{Re}\left[\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \langle 0|\zeta_I^3 H_I(t_1) H_I(t_2)|0\rangle'\right]$$

$$= 2u_{k_3}^* u_{k_1} u_{k_2}|_{\tau=0} \left(\int_{-\infty}^{0} d\tilde{\tau} c_2 a^3 v_{k_3} u'_{k_3}\right)\left(\int_{-\infty}^{0} d\tau c_3 a^2 v_{k_3}^* u_{k_1}'^* u_{k_2}'^*\right) \qquad (3.8)$$

$$- 2u_{k_3} u_{k_1} u_{k_2}|_{\tau=0}$$

$$\times \left[\int_{-\infty}^{0} d\tau_1 c_2 a^3 v_{k_3} u_{k_3}'^* \int_{-\infty}^{\tau_1} d\tau_2 c_3 a^2 v_{k_3}^* u_{k_1}'^* u_{k_2}'^* + \int_{-\infty}^{0} d\tau_1 c_3 a^2 v_{k_3} u_{k_1}'^* u_{k_2}'^* \int_{-\infty}^{\tau_1} d\tau_2 c_2 a^3 v_{k_3}^* u_{k_3}'^*\right] \qquad (3.9)$$

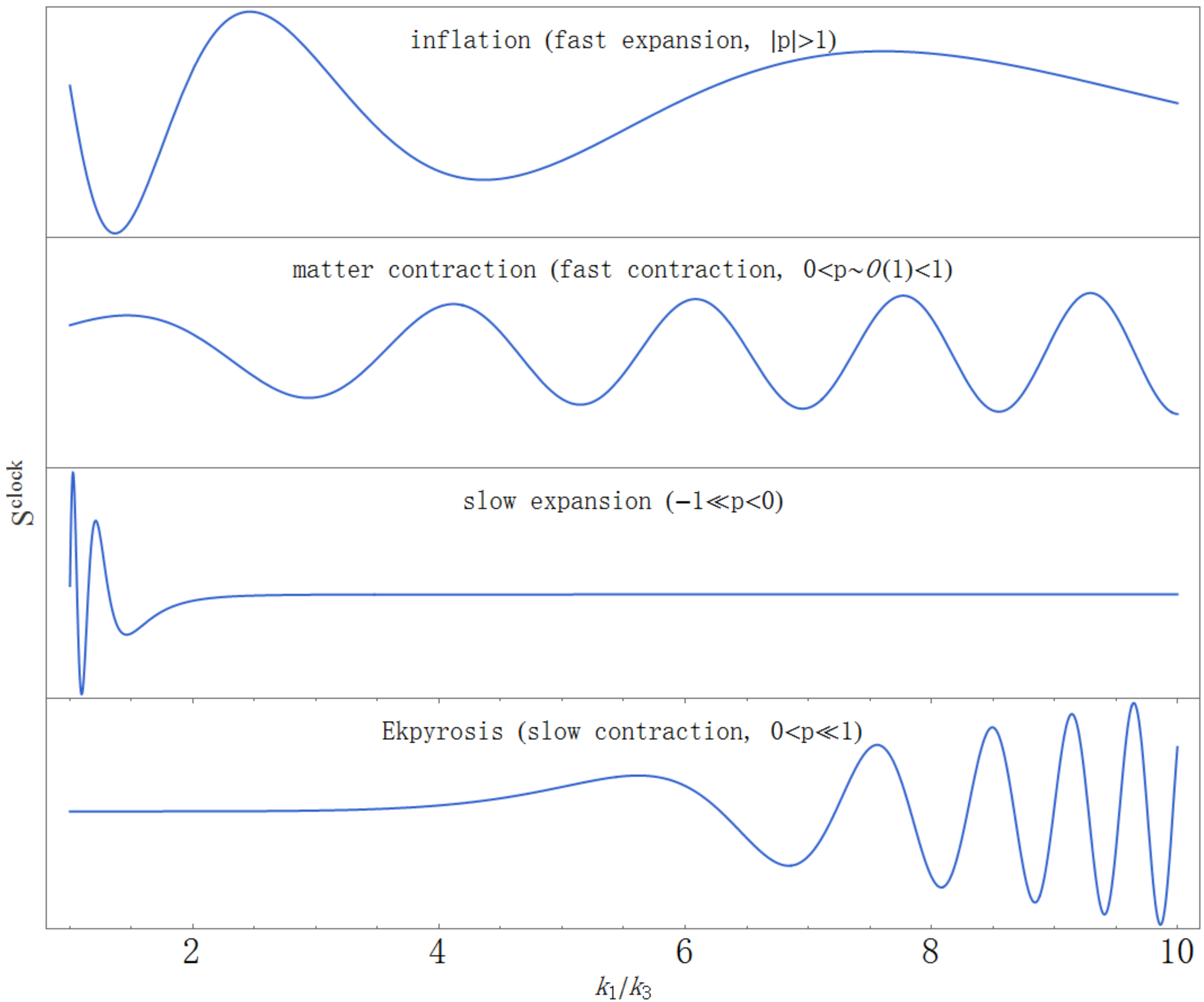
$$+ \text{c.c.} + 2 \text{ perm.}$$

Resonance approximation: $a(t) = a_0 \left(\frac{t}{t_0} \right)^p$

$$\langle \zeta^3 \rangle \equiv S(k_1, k_2, k_3) \frac{1}{(k_1 k_2 k_3)^2} \tilde{P}_\zeta^2 (2\pi)^7 \delta^3 (\sum_{i=1}^{3} \mathbf{k}_k)$$

$$S^{\text{clock}} \propto \left(\frac{2k_1}{k_3} \right)^{-\frac{1}{2}+\frac{1}{2p}} \sin \left[\frac{p^2}{1-p} \frac{m}{H_{k_3}} \left(\frac{2k_1}{k_3} \right)^{1/p} + \varphi(k_3) \right]$$

$$\propto \left(\frac{2k_1}{k_3} \right)^{-\frac{1}{2}+\frac{1}{2p}} \sin \left[p \frac{m}{m_{\text{h},k_3}} \left(\frac{2k_1}{k_3} \right)^{1/p} + \varphi(k_3) \right]$$


inflation (fast expansion, |p|>1)
matter contraction (fast contraction, 0<p~O(1)<1)
slow expansion (−1≪p<0)
Ekpyrosis (slow contraction, 0<p≪1)
S^clock
2
4
6
8
10
k1/k3

Explicit calculation for inflation:

Non-time ordered integral:

Has clock signal. Can be calculated precisely.

Time ordered integral:

We integrate once, and decompose the second integral into two parts

Integrand has hypergeometric functions

No clock signal. We can calculate at large μ.

Integrand has no hypergeometric functions

Has clock signal. Can be calculated precisely.

Explicit calculation for inflation:

$$\langle\zeta^3\rangle' \supset \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{t} dt_1 \langle 0|H_I(\tilde{t}_1)\zeta_I^3 H_I(t_1)|0\rangle' - 2\mathrm{Re}\left[\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \langle 0|\zeta_I^3 H_I(t_1) H_I(t_2)|0\rangle'\right]$$

$$= 2u_{k_3}^* u_{k_1} u_{k_2}|_{\tau=0} \left(\int_{-\infty}^{0} d\tilde{\tau} c_2 a^3 v_{k_3} u'_{k_3}\right)\left(\int_{-\infty}^{0} d\tau c_3 a^2 v_{k_3}^* u_{k_1}'^* u_{k_2}'^*\right) \tag{3.8}$$

$$- 2u_{k_3} u_{k_1} u_{k_2}|_{\tau=0}$$

$$\times \left[\int_{-\infty}^{0} d\tau_1 c_2 a^3 v_{k_3} u_{k_3}'^* \int_{-\infty}^{\tau_1} d\tau_2 c_3 a^2 v_{k_3}^* u_{k_1}'^* u_{k_2}'^* + \int_{-\infty}^{0} d\tau_1 c_3 a^2 v_{k_3} u_{k_1}'^* u_{k_2}'^* \int_{-\infty}^{\tau_1} d\tau_2 c_2 a^3 v_{k_3}^* u_{k_3}'^*\right] \tag{3.9}$$

$$+ \text{c.c.} + 2 \text{ perm.} \ ,$$

Non-time ordered integral

$$S_{(3.8)} = \frac{c_2 c_3}{\epsilon H M_{\rm P}^2}(k_1 k_2 k_3)^{1/2}\mathcal{I}_1 + \text{c.c.} + 2 \text{ perm.}$$

$$\mathcal{I}_1 \equiv -\frac{\sqrt{\pi}}{16\sqrt{2}}\frac{\text{sech}(\pi\mu)}{\Gamma(1-i\mu)}\frac{(k_1 k_2)^{1/2}}{(k_1+k_2)^{5/2}}$$
$$\times\left\{-\left[\frac{2(k_1+k_2)}{k_3}\right]^{-i\mu}\Gamma(\frac{5}{2}+i\mu)\Gamma(1-i\mu)\Gamma(-i\mu)\ {}_2F_1\left(\frac{5}{4}+\frac{i\mu}{2},\frac{7}{4}+\frac{i\mu}{2},1+i\mu,\left(\frac{k_1+k_2}{k_3}\right)^{-2}\right)\right.$$
$$\left.+i\pi\left[\frac{2(k_1+k_2)}{k_3}\right]^{i\mu}\text{csch}(\pi\mu)\Gamma(\frac{5}{2}-i\mu)\ {}_2F_1\left(\frac{5}{4}-\frac{i\mu}{2},\frac{7}{4}-\frac{i\mu}{2},1-i\mu,\left(\frac{k_1+k_2}{k_3}\right)^{-2}\right)\right\}\ .$$

Non-time ordered integral: very squeezed limit

$$S^{\text{clock}}_{(3.8)} \xrightarrow[\text{large mass}]{\text{very squeezed}} e^{i\frac{\pi}{4}} \frac{\pi^{3/2}}{32} \frac{c_2 c_3}{\epsilon H M_{\text{P}}^2} \mu^{3/2} e^{-2\pi\mu} \left(\frac{k_1}{k_3}\right)^{-1/2} \left[-\left(\frac{4k_1}{k_3}\right)^{-i\mu} + i \left(\frac{4k_1}{k_3}\right)^{i\mu} \right] + \text{c.c.}$$

$$= -\frac{\pi^{3/2}}{8} \frac{c_2 c_3}{\epsilon H M_{\text{P}}^2} \mu^{3/2} e^{-2\pi\mu} \left(\frac{k_1}{k_3}\right)^{-1/2} \sin\left[\mu \ln \frac{4k_1}{k_3} + \frac{\pi}{4}\right] .$$

Time ordered integral without hypergeometric functions

$$S_{(3.9)} = \frac{c_2 c_3}{\epsilon H M_{\rm P}^2} (k_1 k_2 k_3)^{1/2} \mathcal{I}_2 + \text{c.c.} + 2 \text{ perm.}$$

$$\begin{aligned}\mathcal{I}_{\rm Hankel} = &- \frac{\sqrt{\pi}\ \text{sech}(\pi\mu)}{8\sqrt{2}\ \Gamma(i\mu+1)} \frac{(k_1 k_2)^{1/2}}{(k_1+k_2)^{5/2}} \\ &\times \left\{ \left[\frac{2(k_1+k_2)}{k_3}\right]^{-i\mu} \Gamma\left(\frac{5}{2}+i\mu\right) \left[\pi - i\cosh(\pi\mu)\Gamma(i\mu+1)\Gamma(-i\mu)\right] {}_2\hat{F}_1 \right. \\ &\left. - i \left[\frac{2(k_1+k_2)}{k_3}\right]^{i\mu} \Gamma\left(\frac{5}{2}-i\mu\right) \Gamma(i\mu+1)\Gamma(i\mu) e^{-\pi\mu}\, {}_2\tilde{F}_1 \right\} ,\end{aligned}$$

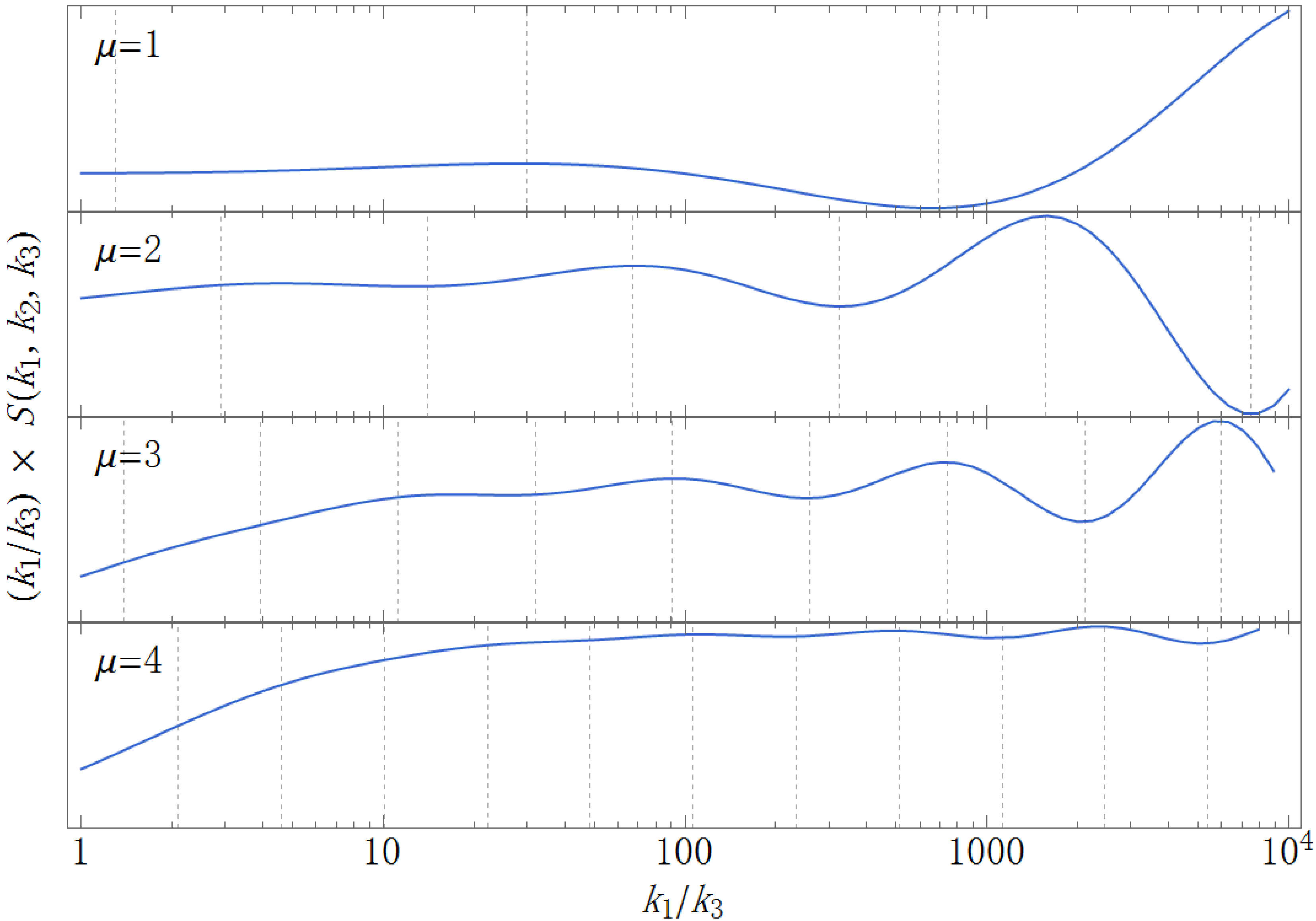
$${}_2\hat{F}_1 \equiv {}_2F_1\left(\frac{5}{4}+\frac{i\mu}{2}, \frac{7}{4}+\frac{i\mu}{2}; 1+i\mu; \left(\frac{k_1+k_2}{k_3}\right)^{-2}\right)$$

$${}_2\tilde{F}_1 \equiv {}_2F_1\left(\frac{5}{4}-\frac{i\mu}{2}, \frac{7}{4}-\frac{i\mu}{2}; 1-i\mu; \left(\frac{k_1+k_2}{k_3}\right)^{-2}\right)$$

Time ordered integral: very squeezed limit

$$S^{\rm clock}_{(3.9)} \xrightarrow[\rm large\ \ mass]{\rm very\ \ squeezed} -\frac{\pi^{3/2}}{32} e^{i\frac{3\pi}{4}} \frac{c_2 c_3}{\epsilon H M_{\rm P}^2} \mu^{3/2} e^{-\pi\mu} \left(\frac{k_1}{k_3}\right)^{-1/2} \left(\frac{4k_1}{k_3}\right)^{-i\mu} + {\rm c.c.}$$

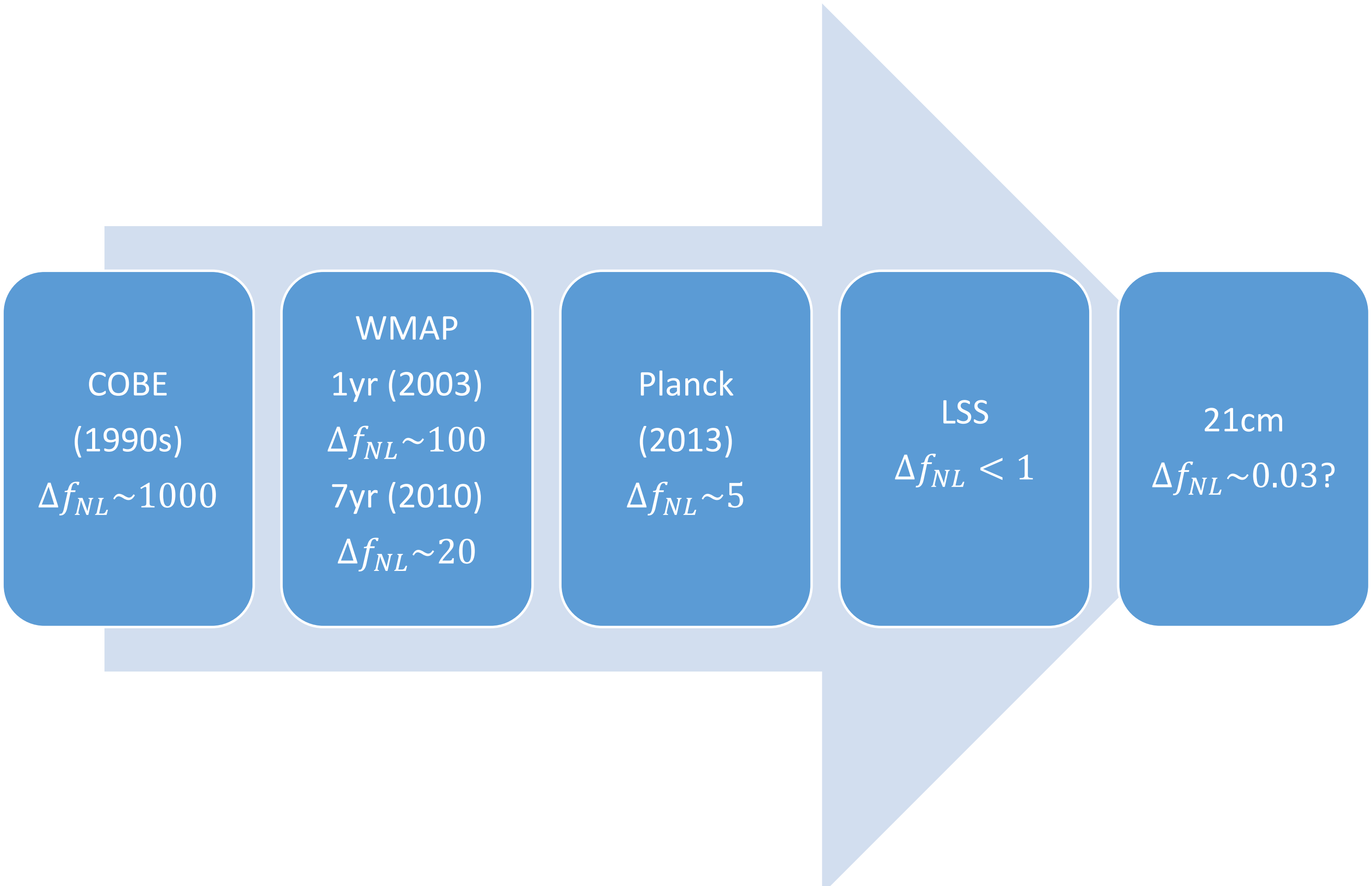
$$= \frac{\pi^{3/2}}{16} \frac{c_2 c_3}{\epsilon H M_{\rm P}^2} \mu^{3/2} e^{-\pi\mu} \left(\frac{k_1}{k_3}\right)^{-1/2} \sin\left[\mu \ln\frac{4k_1}{k_3} + \frac{3\pi}{4}\right] ,$$



μ=1
μ=2
μ=3
μ=4
$(k_1/k_3) \times S(k_1, k_2, k_3)$
1
10
100
1000
10^4
k_1/k_3

Back to non-technical features

The signal is interesting, unique, but challenging to detect


COBE
(1990s)
$\Delta f_{NL} \sim 1000$
WMAP
1yr (2003)
$\Delta f_{NL} \sim 100$
7yr (2010)
$\Delta f_{NL} \sim 20$
Planck
(2013)
$\Delta f_{NL} \sim 5$
LSS
$\Delta f_{NL} < 1$
21cm
$\Delta f_{NL} \sim 0.03?$

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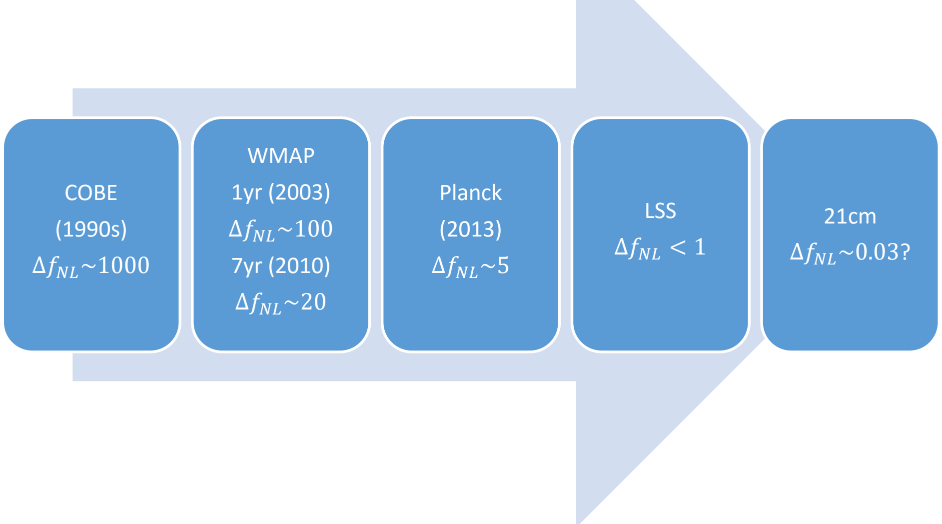
$\Delta f_{NL} \sim 0.03?$

See, e.g. Cooray's talk

Munoz, Ali-Haimoud,
Kamionkowski 2015



Different shape

Different challenge

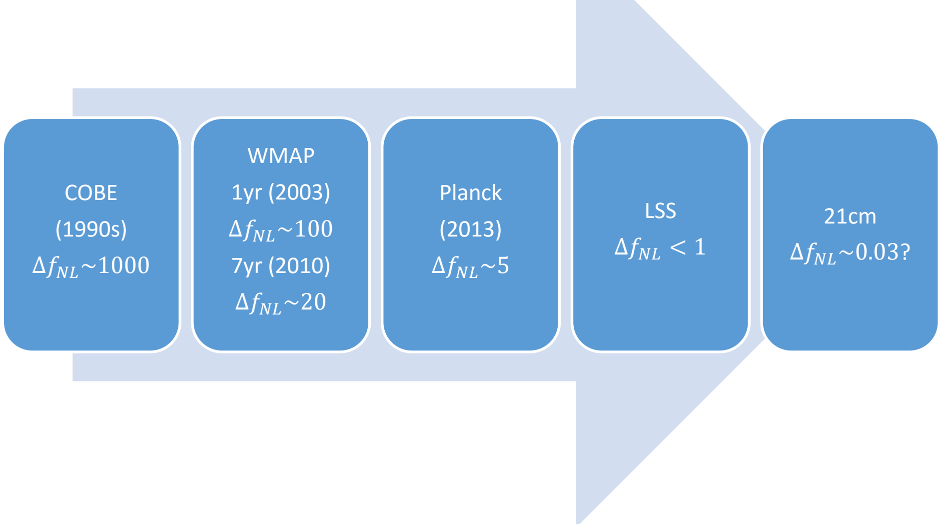
For inflation

$$\langle \zeta^3 \rangle \propto \frac{1}{k_1^{9/2} k_3^{3/2}} \left(\frac{k_1}{k_3} \right)^{\pm i\mu}$$

Greater k

Smaller k

Oscillation factor



Different shape

Different challenge

$$\langle \zeta^3 \rangle \propto \frac{1}{k_1^{9/2} k_3^{3/2}} \left(\frac{k_1}{k_3} \right)^{\pm i\mu}$$

Drops faster than local
in the squeezed limit
(but more slowly than the EFT part)

Oscillation may be
cleaner to be separated
from noise

Further comparison with gravitational waves:

Pros:

- Both real existing effects
- Both as proof* of inflation
- Both characteristic observational features
- Both go together with significant new physics

Cons:

- Both hard to detect

Detectability: Depends on luck

Direct coupling is better than gravitational coupling

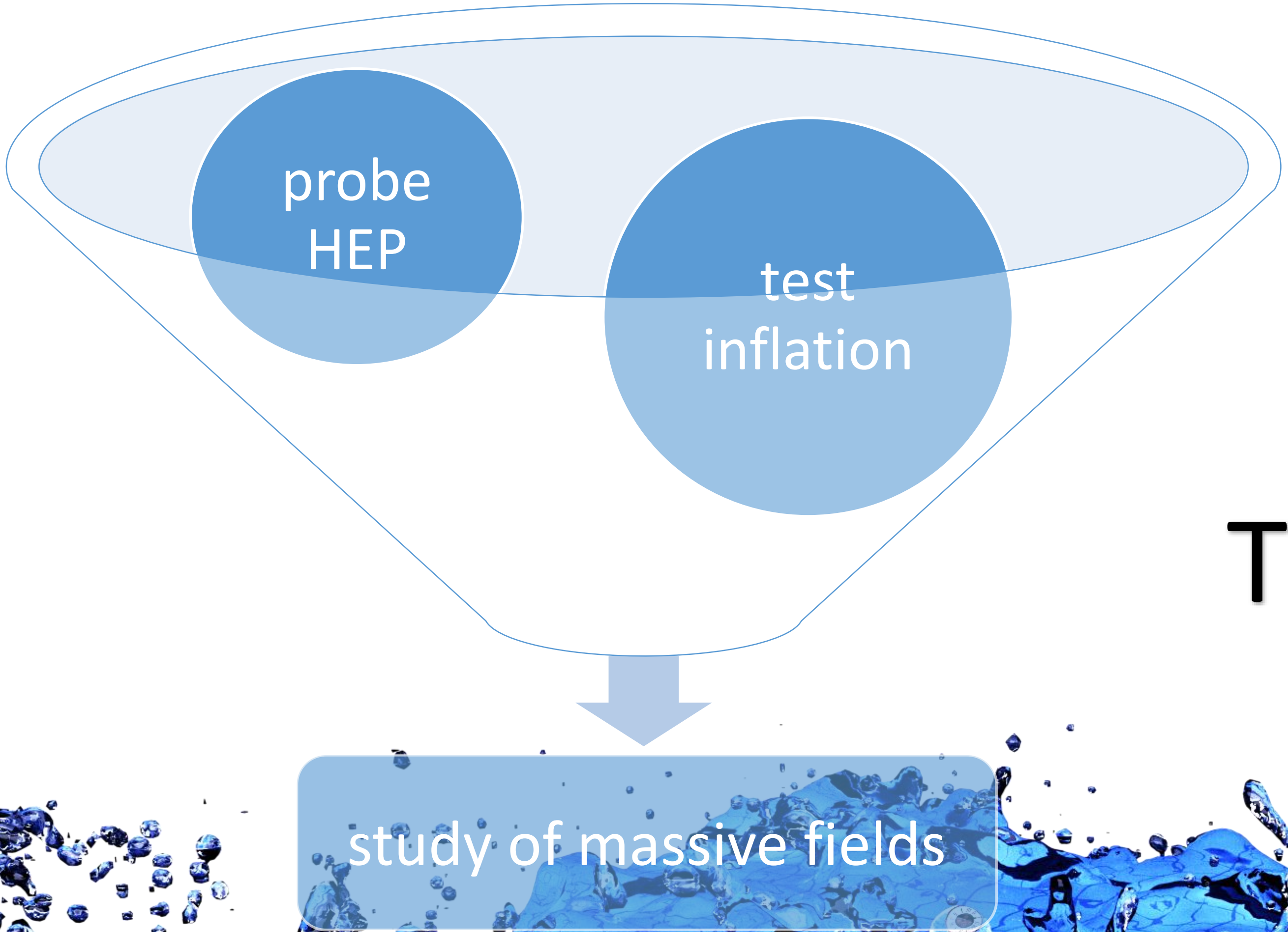
(but gravitational coupling is not hopeless)

Exponentially hard if $m \gg H$

c.f. primordial gravitational waves,

r may be exponentially small

Gravitational waves decay when return to horizon



Thank you!