Small scale isocurvature perturbation of WIMP

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Contents

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   - chemical/kinetic decoupling

2. Generation of isocurvature perturbation
   - Early matter domination

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Dark Matter

No motivation or need in particle physics

It is necessary to explain the inconsistencies between visible matter and gravitational matter

Astrophysics and Cosmology

Existence of some EM/color neutral matters
Evidences for dark matter

In 1933, F. Zwicky first discovered Dark Matter in the velocity dispersion of galaxies in the COMA cluster.

Now there are many evidences for the existence of dark matter:

- **Galactic scales**: rotation curves of galaxies

- **Galaxy cluster scales**: distribution of velocities, weak gravitational lensing, profile of X-ray emission

- **Cosmological scales**: acoustic peaks of CMB, large scale structure formation

**Only Gravitational!**

Single component of dark matter can explain all of them!
1. exist in the early Universe and also now around galaxies, clusters
   stable or lifetime longer than the age of universe
2. neutral: NO electromagnetic interaction
   Only upper bounds on the self interaction
   \[ \sigma / m \lesssim 10^{-24} \text{ cm}^2 / \text{GeV} \]
   from bullet cluster
   No lower bound down to gravity!
   In fact all the evidences of DM are gravitational.
3. 25\% of the present energy density of the universe
4. cold (or warm): non-relativistic to seed the structure formation
What is the production mechanism of dark matter? You can think about how about the relic density is the most non-trivial problem in dark matter. The coldness of dark matter was produced in the early Universe within the expanding system. However, in the larger scales, such as galaxy, clusters of galaxies or in their behaviors, the properties of the materials or the ultimate ingredients of their behaviors, the properties of the materials or the ultimate ingredients of dark matter have remained. Maybe that is the same for dark matter, and even for dark matter.

Several well-motivated candidates of DM are shown in the log–log plane of DM relic mass and number density and the average energy in the phase distribution. Holes in the allowed parameter space are produced by upper limits from weak interactions and lower limits from direct detection.

**DM candidates**

- **Interact** 10^40

**Mass** 10^{30}
WIMP Dark Matter
**WIMP**: Weakly Interacting Massive Particle

[B. W. Lee and S. Weinberg, PRL 1977]

Freeze-out temperature < Mass

\[ Y(x) = \frac{n}{s} \]

\[ n = \frac{\zeta(3)}{\pi^2} g T^3 \]

\[ n = g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T} \]

\[ \dot{n}_X + 3Hn_X = -\langle \sigma v \rangle \left( n_X^2 - (n_X^{eq})^2 \right) \]

\[ Y \approx \frac{H}{s\langle \sigma v \rangle} \propto \frac{x_f}{\sqrt{g_*} \langle \sigma v \rangle m} \]

**annihilation cross section**

**thermal equilibrium**
**WIMP**: Weakly Interacting Massive Particle  
[B. W. Lee and S. Weinberg, PRL 1977]  

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\[ Y \simeq \frac{H}{s \langle \sigma v \rangle} \propto \frac{x_f}{\sqrt{g_*} \langle \sigma v \rangle m} \]

\[ x_f \sim 20 - 25 \]
Weakly Interacting Massive Particles

\[ \Omega h^2 \simeq \frac{2.5 \times 10^{-10} \text{ GeV}}{\langle \sigma_{\text{ann}} v \rangle} \simeq \frac{3 \times 10^{-27} \text{ cm}^3 \text{ sec}^{-1}}{\langle \sigma_{\text{ann}} v \rangle} \]

New physics at weak scale!

Massive LH neutrino: ruled-out by Direct Detection.

RH sneutrino, many WIMP models in SUSY

Kaluza-Klein DM, minimal DM

Neutralino: Lightest SUSY Particle in the extension of SM.

......
CMSSM and direct DM searches

\[ \mu > 0 \]

1405.4289 (update of 1302.5956)

CMSSM, \( \mu > 0 \)
Posterior pdf
Log Priors

~1 TeV higgsino DM
A-funnel
Stau coan’n

Higgs boson inspired

~1 TeV higgsino DM: exciting prospects for LUX, X100 & 1t detectors

Focus point region ruled out by LUX
(already tension with X100)

L. Roszkowski, Nordita, 3 June '15
• The scattering cross section and chemical/kinetic decoupling

Inelastic scatterings: Number changing interactions

\[ T_{fr} \approx \frac{m}{20} \]

- Chemical equilibrium
- Relic density

Elastic scatterings: change momentum (number conserved)

- Kinetic equilibrium
- Structure formation

- The density perturbation can grow after kinetic decoupling
- This or free-streaming scale determines the minimum scale for the structure formation
- Smaller scales are damped during kinetic decoupling
Kinetic decoupling temperature of neutralinos

Kinetic decoupling takes place much later than chemical decoupling by a factor of $10 - 1000$. 

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**Figure 1**

*Z_{g}* : gaugino fraction

- Higgsino ($Z_g < 0.05$)
- Mixed ($0.05 \leq Z_g \leq 0.95$)
- Gaugino ($Z_g > 0.95$)

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**Figure 2**

$x_{cd} = m_{\chi}/T_{cd}$

- Higgsino ($Z_g < 0.05$)
- Mixed ($0.05 \leq Z_g \leq 0.95$)
- Gaugino ($Z_g > 0.95$)
Typical size of the smallest proto-halos: $10^{-11} M_\odot$ to a few times $10^{-4} M_\odot$, 

Earth mass ~

![Graph showing typical size of the smallest proto-halos](graph.png)

Legend:
- Higgsino ($Z_g < 0.05$)
- Mixed (0.05 $\leq Z_g \leq 0.95$)
- Gaugino ($Z_g > 0.95$)

[Bringmann, 2009]
Isocurvature perturbation

\[ S_i \equiv \frac{\delta \rho_i}{\rho_i} - \frac{3 \delta \rho_\gamma}{4 \rho_\gamma}. \]

After BBN, there are 4 different species:

- photon, neutrino (w/ velocity), dark matter, baryon

Isocurvature perturbations between these components. We are focused on the DM isocurvature perturbation and consider the neutrinos and baryons are produced from the photons in the standard scenario in radiation dominated era.

\[ S \equiv 3H \left( \frac{\delta \rho_m}{\dot{\rho}_m} - \frac{\delta \rho_r}{\dot{\rho}_r} \right) = \delta_m - \frac{3}{4} \delta_r, \]
Constraints on the correlated matter isocurvature mode $\alpha$ and the amplitude parameter $S_m$, $n_s$, $\alpha = -0.0025^{+0.0035}_{-0.0047}$ (95%, Planck TT+lowP), $S_m = \text{sgn}(\alpha) \sqrt{|\alpha|/1-|\alpha|} \xi$. 

Planck Collaboration XXII

Fig. 24.
• The primordial power spectrum of the curvature perturbation is adiabatic, Gaussian, and almost scale invariant.

• In Fourier space, with power law spectrum,

\[
P_R(k) = A_s \left( \frac{k}{k_0} \right)^{n_s-1+(1/2)(d n_s/d \ln k) \ln(k/k_0)},
\]

\[
P_\zeta = (2.198 \pm 0.056) \times 10^{-9}
\]

\[
n_\zeta = 0.959 \pm 0.007
\]

• The above measurement is only for the range of the scale of

\[
10^{-3} \lesssim k/\text{Mpc}^{-1} \lesssim 10^{-1}
\]

• No measurement on the smaller scales.
Isocurvature perturbation of WIMP

WIMP dark matter is adiabatic on both large and small scales
- since it is produced from the relativistic thermal plasma
  when it was in the thermal equilibrium in standard Universe.
- It is consistent with observation, adiabatic power spectrum

Adiabatic perturbation is damped during the kinetic decoupling,
for the smaller scales than the kinetic decoupling. [Peebles 1987]

However the isocurvature perturbation is not damped.

\[ \dot{S} = \theta_r - \theta_m, \]

 divergence of velocity

- Is it possible to have a large isocurvature perturbation at small scales
  with adiabatic at large scales?
- No in the standard WIMP.
- However there is a one case, where the generation is possible.
COSMIC BACKGROUND TEMPERATURE ANISOTROPY IN A MINIMAL ISOCURVATURE MODEL FOR GALAXY FORMATION

P. J. E. Peebles
Joseph Henry Laboratories, Princeton University
Received 1987 January 2; accepted 1987 January 26

ABSTRACT

If the dominant components of the universe were radiation and baryons, and the primeval baryon distribution had a roughly flat spectrum normalized to galaxy clustering on scales $\sim 20$ Mpc, and young stars were able to keep the bulk of the matter ionized at redshifts $z \gtrsim 20$, then several encouraging results would follow. The first generation that starts to form when Compton drag becomes unimportant would have masses and radii comparable to galaxies. Mass fluctuations on scales $\sim 200$ Mpc could be relatively large and so perhaps favorable for development of large-scale structure. And the residual fluctuations in the background temperature would have coherence length $\sim 3^\circ - 5^\circ$ and standard deviation $\delta T/T \approx 10^{-5}$, close to but below the observational bounds.
**WIMP isocurvature perturbation and small scale structure**

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The adiabatic perturbation of dark matter is damped during the kinetic decoupling due to the collision with relativistic component on sub-horizon scales. However the isocurvature part is free from damping and could be large enough to make a substantial contribution to the formation of small scale structure. We explicitly study the weakly interacting massive particles as dark matter with an early matter dominated period before radiation domination and show that the isocurvature perturbation is generated during the phase transition and leaves imprint in the observable signatures for small scale structure.

[1507.03871]

The generation of the isocurvature perturbation in the subhorizon scales can happen when [KY Choi, Gong, Shin, 1507.03871]

1. During the matter domination, WIMP is chemically decoupled but still kinetically coupled to radiation,

2. After chemical decoupling, the phase transition happens from the matter-dom. to the radiation-dom.
Early matter dominated epoch with low-reheating temperature

Non-standard cosmology with early matter-dominated era and all radiation and DM are generated from the scalar again:

- Inflaton oscillation
- Thermal inflation
- Curvaton domination

- During this period, the perturbation grows linearly inside horizon.
- The signatures of the large density perturbation on small scales
- It is not suppressed by the kinetic equilibrium
- It can be erased by free streaming
Non-thermal Universe dominated by a non-relativistic field (scalar domination)

\[ T_{fr} > T_{reh} > T_{kd} \]

10 GeV 100 MeV 1 MeV

Background evolution

Scalar-dominated

Radiation

DM

Rad.-dominated

\[ \rho/\rho_i \]

scale factor \((a/a_i)\)

\[ 10^{-20} \quad 10^{-16} \quad 10^{-12} \quad 10^{-8} \quad 10^{-4} \quad 1 \]

\[ 1 \quad 10 \quad 100 \quad 1000 \quad 10^4 \quad 10^5 \quad 10^6 \]

decay \(a/a_i \approx 300\)

thermal freeze-out \(a/a_i \approx 20\)
Background evolution

\[ \dot{\rho}_\phi + 3H \rho_\phi = -\Gamma_\phi \rho_\phi , \]

\[ \dot{\rho}_r + 4H \rho_r = (1 - f_m) \Gamma_\phi \rho_\phi + \frac{\langle \sigma_a v \rangle}{M} \left[ \rho_m^2 - (\rho_m^{eq})^2 \right] \]

\[ \dot{\rho}_m + 3H \rho_m = f_m \Gamma_\phi \rho_\phi - \frac{\langle \sigma_a v \rangle}{M} \left[ \rho_m^2 - (\rho_m^{eq})^2 \right] , \]

We consider that the radiation is generated by the decay of the scalar and quickly thermalized. The DMs are produced from the annihilation of radiations like WIMP.

\[ f_m = 0 . \]

\[ \langle \sigma_a v \rangle \quad \text{Thermal averaged annihilation cross section of DM} \]
\textbf{Perturbation equations}

\[ ds^2 = -(1 + 2\Phi) dt^2 + a^2 (1 - 2\Psi) \delta_{ij} dx^i dx^j \]

\[ \dot{\delta}_\alpha + (1 + w_\alpha) \frac{\theta_\alpha}{a} - 3(1 + w_\alpha) \dot{\Psi} = \frac{1}{\rho_\alpha} (\delta Q_\alpha - Q_\alpha \delta_\alpha + Q_\alpha \Phi) , \]

\[ \dot{\theta}_\alpha + (1 - 3w_\alpha) H \theta_\alpha + \frac{\Delta \Phi}{a} + \frac{w_\alpha}{1 + w_\alpha} \frac{\Delta \delta_\alpha}{a} = \frac{1}{\rho_\alpha} \left[ \partial_i Q_{(\alpha)}^i - Q_\alpha \theta_\alpha \right] , \]

with

\[ Q_\phi = -\Gamma_\phi \rho_\phi , \]

\[ Q_r = \Gamma_\phi \rho_\phi + \frac{\langle \sigma v \rangle}{M} \left[ \rho_m^2 - (\rho_m^{\text{eq}})^2 \right] , \]

\[ Q_m = -\frac{\langle \sigma v \rangle}{M} \left[ \rho_m^2 - (\rho_m^{\text{eq}})^2 \right] , \]

\[ \delta Q_\phi = -\Gamma_\phi \rho_\phi \delta_\phi , \]

\[ \delta Q_r = \Gamma_\phi \rho_\phi \delta_\phi + \frac{2\langle \sigma v \rangle}{M} \left[ \rho_m^2 \delta_m - (\rho_m^{\text{eq}})^2 \frac{M}{T} \delta_r \right] , \]

\[ \delta Q_m = -\frac{2\langle \sigma v \rangle}{M} \left[ \rho_m^2 \delta_m - (\rho_m^{\text{eq}})^2 \frac{M}{T} \delta_r \right] , \]

\[ \partial_i Q_{(\phi)}^i = -\Gamma_\phi \rho_\phi \theta_\phi \]

\[ \partial_i Q_{(r)}^i = \Gamma_\phi \rho_\phi \theta_\phi + \frac{\langle \sigma v \rangle}{M} \left[ \rho_m^2 \theta_m - (\rho_m^{\text{eq}})^2 \left( \frac{M}{2\pi T} \right)^{1/2} \theta_r \right] - \frac{4 \sigma_e}{3 M} \rho_m \rho_r (\theta_r - \theta_m) , \]

\[ \partial_i Q_{(m)}^i = -\frac{\langle \sigma v \rangle}{M} \left[ \rho_m^2 \theta_m - (\rho_m^{\text{eq}})^2 \left( \frac{M}{2\pi T} \right)^{1/2} \theta_r \right] + \frac{4 \sigma_e}{3 M} \rho_m \rho_r (\theta_r - \theta_m) , \]
The modes that enter the horizon during SD

\( k^{-1} < k_{\text{reh}}^{-1} \)

|\[ \frac{\delta}{\Phi_0} \]| vs. scale factor (a)

- **Scalar**
- **Radiation**
- **WIMP**

- **No suppression**
- **do not oscillate**

- **horizon reentry**
- **reheating**
- **kinetic decoupling**

**Questions:**
1. All the perturbation below \( k \) is suppressed?
2. Why not the case for \( T_{\text{reh}} < T < k_{\text{fr}} \)?
The growth and suppression of the density perturbation of the dark matter. 

The figure shows the evolution of the density perturbation $|\delta_{dm}(a=10^7)|$ as a function of $k/k_{RH}$. The blue line represents WIMP DM, while the orange line represents E-WIMP DM. The graph highlights different epochs and their effects on the density perturbation:

1. **Super horizon**: The density perturbation begins to grow at this stage.
2. **Kinetic decoupling**: The perturbation experiences oscillations and damping due to kinetic decoupling.
3. **Enhancement during SD**: The density perturbation is enhanced during the standard decoupling phase.
4. **Suppression by kinetic eq.**: The density perturbation is suppressed by the kinetic equations during the standard decoupling phase.

The figure also indicates the importance of the dark matter relic density in the matter dominant epoch (MD) and the subsequent behavior. The growth and suppression of the density perturbation are critical in understanding the evolution of the universe in different epochs.
Small scale suppression of power spectrum

- Free streaming

The free streaming (collision-less damping, Landau damping) of dark matter from dense regions to under-dense regions smoothes out inhomogeneities for the smaller than the free streaming length scale.

\[ \lambda_f = \int_{t_i}^{t} \frac{v(t')}{R(t')} dt'. \]

For WIMPs,

\[ k_{fs} \sim \left( \frac{0.01}{c_s(T_{kd})} \right) \left( \frac{14}{1 + 0.07 \ln\left( \frac{T_{kd}}{\text{MeV}} \right)} \right) \left( \frac{T_{kd}}{\text{MeV}} \right) \left( \frac{1}{12 \text{ pc}} \right) \]

with effective sound speed squared of DM \( c_s^2(T_{kd}) \sim \frac{T_{kd}}{m} \).
Density perturbation later times

$|\delta_{dm}/\Phi_0|$ at $a=10a_{eq}$

$|\delta_{dm}/\Phi_0|$ at $a=100a_{kd}$
Discussion

1. Isocurvature perturbation of DM is not suppressed. and can be probed in the small scale dark matter clumps.

2. The isocurvature perturbation of WIMP can be generated during the early matter domination.

3. The WIMP DM inside the small scale objects can be observed in the gamma-ray, cosmic rays or neutrinos and in the direct detection rate.

3. The power spectrum of WIMP shows non-trivial shape of structure formation at small scales. It implies one way to see the early Universe before BBN.