Diluting the inflationary axion fluctuation by a stronger QCD in the early Universe

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Outline

• Axion Dark Matter vs. High Scale Inflation
• Stronger QCD phase in the early Universe
• An example of model
• Summary
Axion Dark Matter

- PQ breaking after inflation

☑ Axionic strings and domain walls are generated.
☑ Axion DM is mainly given by annihilation of them.

\[ \frac{\Omega_a}{\Omega_{DM}} \simeq (5 \pm 2) \times \left( \frac{f_a(t_0)}{10^{11}\text{GeV}} \right)^{1.19} \]

Hiramatsu, Kawasaki, Saikawa, Sekiguchi (2012)

But, it requires

\[ N_{DW} = \left| \sum_i 2q_{\psi_i} \text{Tr}(T_a^2(\psi_i)) \right| = 1 \]

(Axion Domain Wall problem)
Axion Dark Matter

- PQ breaking \textit{before/during} inflation

\begin{itemize}
  \item No domain wall problem
  \item Axion DM is given by coherent oscillation of the axion field.
\end{itemize}

\[
\frac{\Omega_a}{\Omega_{DM}} \approx 1.7 \theta^2_{\text{mis}} \left( \frac{f_a(t_0)}{10^{12} \text{ GeV}} \right)^{1.19}
\]

But, the axion field gets quantum fluctuation during inflation.

\[
\delta \theta = \frac{H(t_I)}{2\pi f_a(t_I)}
\]
Axion isocurvature perturbation

- The primordial axion quantum fluctuations turn into axion DM density perturbation after QCD phase transition.

\[
\frac{\Omega_a}{\Omega_{DM}} \approx 1.7 (\theta_0^2 + \delta \theta^2) \left( \frac{f_a(t_0)}{10^{12} \text{GeV}} \right)^{1.19}
\]

- This must generate the “isocurvature mode” of CMB perturbation.

\[
\left( \frac{\delta T}{T} \right)_{\text{iso}} \approx \frac{4}{5} \left( \frac{\Omega_a}{\Omega_{DM}} \right) \frac{\delta \theta}{\theta_{\text{mis}}} < 3.8 \times 10^{-6}
\]

Planck Collaboration (2014)
Tension with High scale inflation

• If axion is a major component of DM in the Universe, the axion field fluctuations must be very small.

\[
\left( \frac{\delta T}{T} \right)_{\text{iso}} \simeq \frac{4}{5} \left( \frac{\Omega_a}{\Omega_{DM}} \right) \frac{\delta \theta}{\theta_{\text{mis}}} < 3.8 \times 10^{-6}
\]

\[
\frac{\delta \theta}{\theta_{\text{mis}}} \lesssim 10^{-5}
\]

\[
\Omega_a \simeq \Omega_{DM}
\]

• This means that the primordial inflationary Hubble scale should be so suppressed compared to the axion scale at the inflationary epoch.

\[
\delta \theta = \frac{H(t_I)}{2\pi f_a(t_I)}
\]

\[
H(t_I) \lesssim 10^{-5} \theta_{\text{mis}} f_a(t_I)
\]
Upper bound on the inflation scale for $\Omega_a = \Omega_{DM}$ in the conventional scenario.
Suppressing the axion field fluctuation: A stronger QCD in the early Universe

- If the QCD confinement scale $\Lambda'_{\text{QCD}}$ in the early Universe was high enough compared to $\Lambda_{\text{QCD}}$ of the present Universe, there can be an epoch in which

$$m_a(t) \sim \frac{\Lambda'_{\text{QCD}}^2}{f_a(t)} > H(t)$$

- During the epoch, the axion field undergoes *damped* coherent oscillation toward the minimum of its potential.

$$\ddot{a} + 3H(t)\dot{a} + m_a^2(t)a = 0 \quad \Rightarrow \quad |a(t)| \sim \left(a_0 + \delta a_0\right) \left(\frac{R(t_i)}{R(t)}\right)^{3/2}$$

- If this period is long enough, then the axion field fluctuation $\delta a$ can be significantly reduced, though the ratio $\delta a/<a> = \delta \theta/\theta_{\text{mis}}$ is still *unchanged*.

- But *if the axion potential minimum of the present Universe is displaced from the stronger QCD phase’s one*, $\theta_{\text{mis}}$ becomes $O(1)$ while $\delta \theta$ remains reduced.

$\Rightarrow \delta \theta/\theta_{\text{mis}}$ suppressed!
Schematic picture

After primordial inflation

Late time QCD phase transition

$\delta \theta$ not so small

$\theta_{\text{mis}} \sim O(1) \Rightarrow \delta \theta / \theta_{\text{mis}}$ suppressed!

$\delta \theta$ reduced!

Stronger QCD phase

After the stronger QCD phase ends
Stronger QCD by large Higgs VEV

- A temporal stronger QCD phase in the early Universe can arise if the Higgs field has temporarily large vacuum expectation value so that quarks become heavy during the time.
- Heavy quarks make the QCD coupling run faster, and so QCD confinement scale becomes large.

\[ \Lambda_{QCD}' \approx 23 \text{ TeV} \left( \frac{v'}{10^{12} \text{ GeV}} \right)^{6/11} \times \left( \frac{m_{\tilde{g}}}{3 \text{ TeV}} \right)^{2/11} \left( \frac{\tan \beta}{10} \right)^{3/11} \]
Large Higgs VEV along flat directions

- Temporarily large Higgs VEV in the early Universe is plausibly realized in supersymmetric theories because of the existence of Higgs flat directions.

\[ V(\phi) = \begin{cases} m_\phi^2 |\phi|^2 & \text{for } 4\phi^4/M_{Pl} + \text{h.c.} \\ + 2|\mu|^2 |\phi|^2 & \text{for } (B\mu\phi^2 + \text{h.c.}) \end{cases} \]

\[ \phi^2 \equiv H_u H_d \]

D-flat direction

- We assume that \( m_\phi^2 < 0 \) and \( \mu \)-term is dynamically generated at the time of so-called “\( \mu \)-transition”.

- Before \( \mu \)-transition, \( \mu = B\mu = 0 \)

In the above setup, it is important to note that the coupling constants of Planck-suppressed terms for sim-
Large Higgs VEV along flat directions

\[ V(\phi) = m_\phi^2 |\phi|^2 - \left( A_\phi \frac{\phi^4}{M_{Pl}} + \text{h.c.} \right) + \frac{|\phi|^6}{M_{Pl}^2} + 2|\mu|^2 |\phi|^2 - (B \mu \phi^2 + \text{h.c.}) \]

\[ \phi^2 \equiv H_u H_d \]

- **After μ-transition**, non-zero \( \mu \) and \( B\mu \) are generated so that \( m_\phi^2 + 2|\mu|^2 > 2B\mu \).
- **Arg(B\mu)** changes the position of the axion potential minimum of the present Universe compared to the stronger QCD phase (\( \rightarrow \theta_{\text{mis}} \sim \mathcal{O}(1) \)).

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An example of model

\[ W = (\text{MSSM Yukawa terms}) + \{\lambda Y \Phi \Phi^c\} \text{ thermal mass } (|\lambda|^2 T^2/8) \text{ to } Y \]

\[ W = \left( \kappa_1 \frac{X^2}{M_{Pl}} + \kappa_2 \frac{X Y^3}{M_{Pl}} + \kappa_3 \frac{(H_u H_d) (L H_u)}{M_{Pl}} \right) \]

\[ \Rightarrow \mu \quad \text{Kim, Nilles (1984)} \]

\[ V_1 = \left( m_X^2 |X|^2 + \frac{1}{8} |\lambda|^2 T^2 \right) |Y|^2 + \left( \frac{\kappa_2 A_2}{M_{Pl}} X Y^3 + \text{h.c.} \right) \]

\[ + \frac{|\kappa_2|}{M_{Pl}^2} (|Y|^6 + 9 |X|^2 |Y|^4) \]

- \( \frac{1}{8} |\lambda|^2 T^2 > |m_Y^2| \) \( (T > T_c \sim m_Y) \rightarrow \langle X \rangle = \langle Y \rangle = 0 \rightarrow \mu = 0 \)

- \( \frac{1}{8} |\lambda|^2 T^2 < |m_Y^2| \) \( (T < T_c) \rightarrow \langle X \rangle \sim \langle Y \rangle \sim \sqrt{m_{\text{SUSY}} M_{Pl}} \rightarrow \mu \sim m_{\text{SUSY}} \)
\[ W = (\text{MSSM Yukawa terms}) + \lambda Y \Phi \Phi^c \]
\[ + \kappa_1 \frac{X^2}{M_{Pl}} H_u H_d + \kappa_2 \frac{X Y^3}{M_{Pl}} + \kappa_3 \frac{(H_u H_d)(LH_u)}{M_{Pl}} \]

- Higgs flat direction (F & D flat)

\[ H_d^T = (\phi_d, 0), \quad L^T = (\phi_\ell, 0), \quad H_u^T = (0, \sqrt{|\phi_d|^2 + |\phi_\ell|^2}), \]

\[ V_2 = (m_{\phi_d}^2 + 2|\mu|^2)|\phi_d|^2 + (m_{\phi_\ell}^2 + |\mu|^2)|\phi_\ell|^2 < 0 \]
\[ + \left( B \mu \phi_d \sqrt{\sum_i |\phi_i|^2} + \frac{\kappa_3 A_3 \phi_d \phi_\ell}{M_{Pl}} \left( \sum_i |\phi_i|^2 \right) + \text{h.c.} \right) \]
\[ + \frac{\kappa_3^2}{M_{Pl}^2} \left( \sum_i |\phi_i|^2 \right) \left( |\phi_d|^4 + 4|\phi_d \phi_\ell|^2 + |\phi_\ell|^4 \right) < 0 \]

Before \( \mu \)-transition (\( T > T_c \)), \( \mu = B \mu = 0 \), \( \phi_d \sim \phi_\ell \sim \sqrt{m_{\text{SUSY}}M_{Pl}} \) stronger QCD

After \( \mu \)-transition (\( T < T_c \)), \( \mu \sim \sqrt{B \mu} \sim m_{\text{SUSY}} \), \( \phi_d \sim m_Z \), \( \phi_\ell = 0 \)

ordinary QCD, present electroweak vacuum
Total damping over the stronger QCD phase

- Axion gets a large enough mass during $T_c < T < \Lambda'_{\text{QCD}}$ so that $m_a > H$.
- Then its amplitude and fluctuation get suppressed during its coherent oscillation.

\[
\left( \frac{R(t_i)}{R(t_i + \Delta t)} \right)^{3/2} \approx \left( \frac{T_c}{\Lambda'_{\text{QCD}}} \right)^{3/2} \approx \mathcal{O}(10^{-2} - 10^{-3})
\]
Upper bound on the inflation scale for $\Omega_a = \Omega_{DM}$

- Solid: $m_{\text{SUSY}}(t_0) = 1 \text{ TeV}$
- Dashed: $m_{\text{SUSY}}(t_0) = 10 \text{ TeV}$
- $T_c = 1 \text{ TeV}$
- $M_{\text{gluino}} = 3 \text{ TeV}$, $\tan \beta = 10$

Equation:

- Stronger QCD before the $\mu$-transition
- $f_a(t) \sim \sqrt{m_{\text{SUSY}}(t)} M_{\text{Pl}}$
- $f_a(t_I) = f_a(t_0)$

Graph:

- Log$_{10}[H(t_I)/\text{GeV}]$ vs. Log$_{10}[f_a(t_0)/\text{GeV}]$
Summary

- If one allows $N_{DW} > 1$ and assumes that axion explains the dominant part of DM in the Universe, high scale inflation ($> 10^{10}$ GeV) conflicts with the axion isocurvature constraint in the conventional scenario.

- If there exists a stronger QCD phase in the early Universe by temporarily large Higgs VEV, axion undergoes damped coherent oscillation while reducing its fluctuation.

- Together with the reduced fluctuation, the displacement of the axion potential minimum after $\mu$-transition suppresses the axion isocurvature perturbation.

- This allows that even the current Planck upper bound value ($10^{14}$ GeV) on the inflationary Hubble scale is compatible with the axion DM.