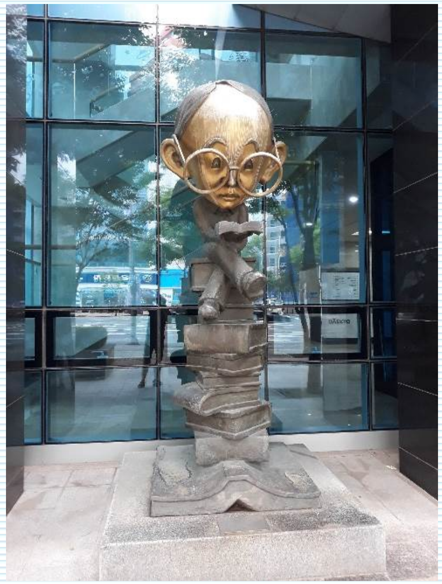
*6th Symposium on Neutrinos and Dark Matter (NDM 2018)*
*IBS HQ, Daejoen, Korea: June 29 – July 4, 2018*

# Neutrino masses, DBD NMEs and quenching of $g_A$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# OUTLINE

- ***Introduction***
  ***ν-oscillations and ν-masses***
- ***The 0νββ-decay scenarios due neutrinos exchange***
  ***(simpliest, sterile ν, LR-symmetric model, interpolating formula)***
- ***DBD NMEs – Current status***
  ***(deformed QRPA versus ISM, role of SU(4) symmetry … )***
- ***Is there a proportionality between 0νββ- and 2νββ-decay NMEs?***
- ***New modes of the double-beta decay with emission of a single electron from an atom***
- ***Conclusion***

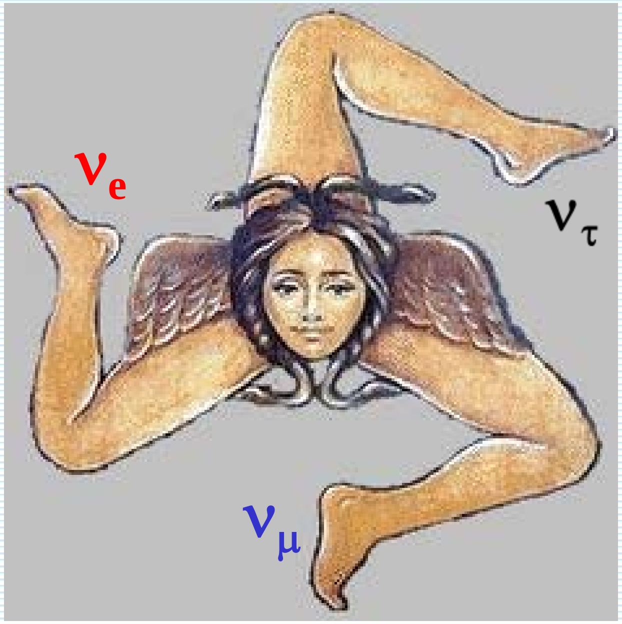
*Acknowledgements:* **A. Faesler** **(Tuebingen), P. Vogel (Caltech), S. Kovalenko (Valparaiso U.), M. Krivoruchenko (ITEP Moscow), D. Štefánik, R. Dvornický (Comenius U.), A. Babič, A. Smetana, J. Terasaki (IEAP CTU Prague), …**

After 59 years we know

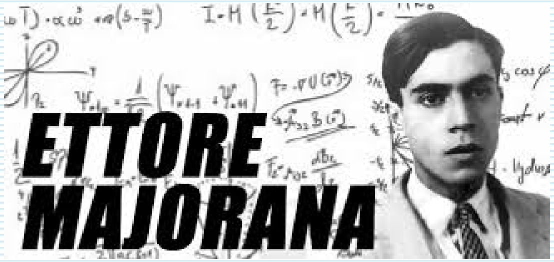
# Fundamental ν properties

No answer yet

- 3 families of light (V-A) neutrinos:
  $\nu_e, \nu_\mu, \nu_\tau$
- ν are massive:
  we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

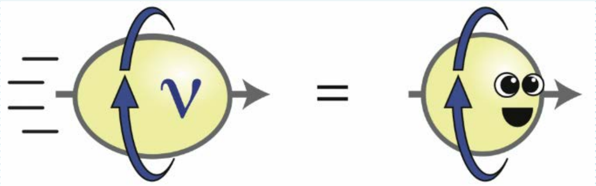



- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?



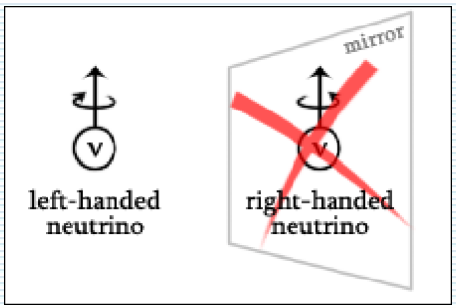

Currently main issue

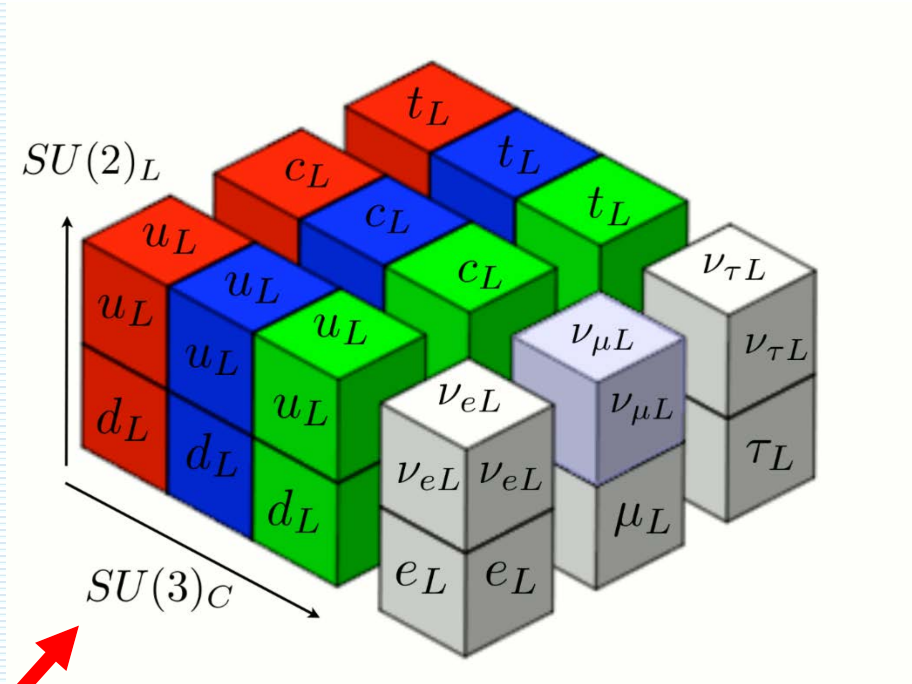
Nature, Mass hierarchy, CP-properties, sterile ν




The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

# Beyond the Standard model physics
## *(EFT scenario)*




$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \boxed{\frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)}$$

# Minimal SM + EFT

S.M. Bilenky,
Phys.Part.Nucl.Lett. 12 (2015) 453-461

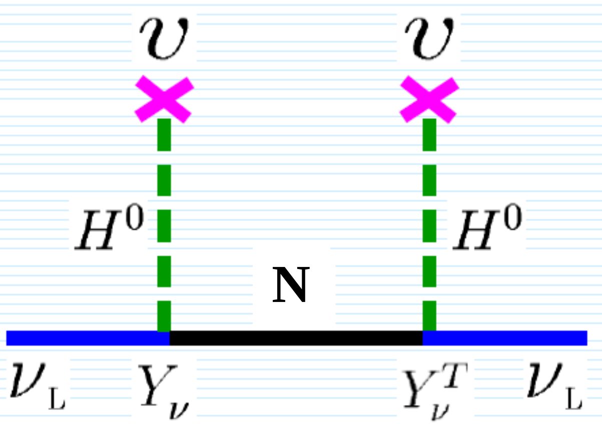
The **absence of the right-handed neutrino fields** in the Standard Model is the simplest, most economical possibility. In such a scenario **Majorana mass term** is the only possibility for neutrinos to be massive and mixed. This mass term is generated by the **lepton number violating Weinberg effective Lagrangian.**

$$\mathcal{L}_5^{eff} = -\frac{1}{\Lambda}\sum_{l_1 l_2}\left(\overline{\Psi}_{l_1 L}^{lep}\tilde{\Phi}\right)\acute{Y}_{l_1 l_2}\left(\tilde{\Phi}^T(\Psi_{l_2 L}^{lep})^c\right)$$

$$m_i = \frac{v}{\Lambda}\,(y_i v), \quad i = 1, 2, 3$$

$\Lambda \geq 10^{15}$ GeV

Heavy Majorana leptons $N_i$ ($N_i = N^c_i$)
singlet of $SU(2)_L x U(1)_Y$ group
Yukawa lepton number violating int.



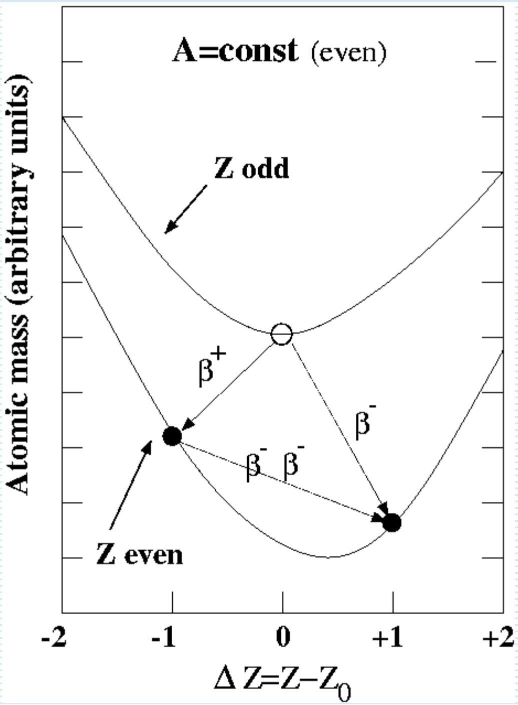

The three Majorana neutrino masses are suppressed by the ratio of the electroweak scale and a scale of a lepton-number violating physics.

The discovery of the ββ-decay and absence of transitions of flavor neutrinos into sterile states would be evidence in favor of this minimal scenario.

# I. *The simplest 0νββ-decay scenario*
## *(SM + EFT scenario)*

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \; g_A^4 \; \left|M^{0\nu}_{\nu}\right|^2 \; G^{0\nu}$$

**(A,Z) → (A,Z+2) + e⁻ + e⁻**

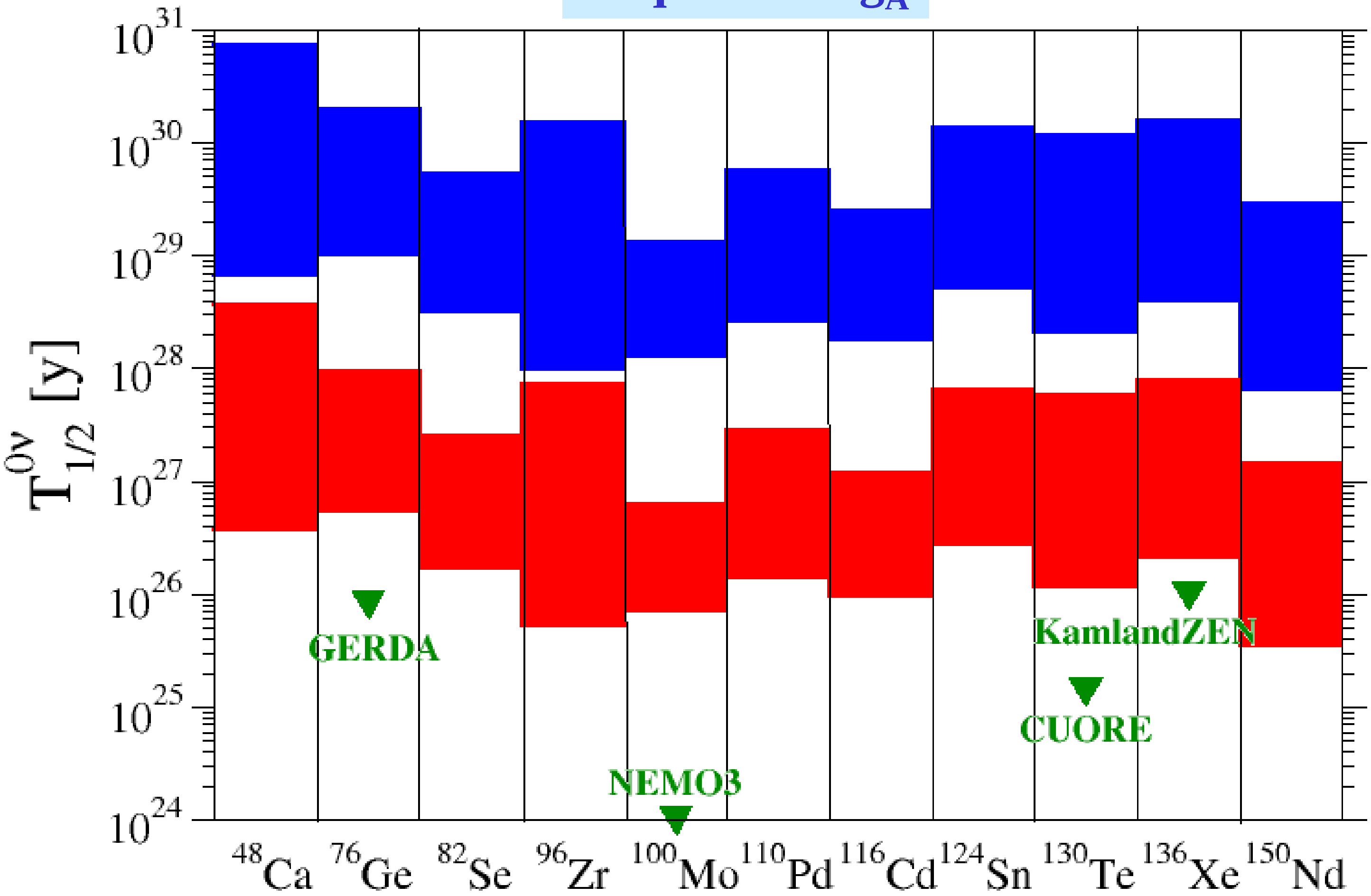



| transition | $G^{01}(E_0,Z)$ $\times 10^{14} y$ | $Q_{\beta\beta}$ [MeV] | Abund. (%) | $\lvert M^{0\nu}\rvert^2$ |
|---|---|---|---|---|
| $^{150}Nd \rightarrow {}^{150}Sm$ | 26.9 | 3.667 | 6 | ? |
| $^{48}Ca \rightarrow {}^{48}Ti$ | 8.04 | 4.271 | 0.2 | ? |
| $^{96}Zr \rightarrow {}^{96}Mo$ | 7.37 | 3.350 | 3 | ? |
| $^{116}Cd \rightarrow {}^{116}Sn$ | 6.24 | 2.802 | 7 | ? |
| $^{136}Xe \rightarrow {}^{136}Ba$ | 5.92 | 2.479 | 9 | ? |
| $^{100}Mo \rightarrow {}^{100}Ru$ | 5.74 | 3.034 | 10 | ? |
| $^{130}Te \rightarrow {}^{130}Xe$ | 5.55 | 2.533 | 34 | ? |
| $^{82}Se \rightarrow {}^{82}Kr$ | 3.53 | 2.995 | 9 | ? |
| $^{76}Ge \rightarrow {}^{76}Se$ | 0.79 | 2.040 | 8 | ? |

***The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory***

# $0\nu\beta\beta$ –half lives for NH and IH with included undertainties in NMEe

unquenched $g_A$




NH

IH

**NH:** $m_1 \ll m_2 \ll m_3$ $m_3 \simeq \sqrt{\Delta m^2}$

$m_1 \ll \sqrt{\delta m^2}.$ $m_2 \simeq \sqrt{\delta m^2}$

$1.4\ \text{meV} \le m_{\beta\beta} \le 3.6\ \text{meV}$

**IH:** $m_3 \ll m_1 < m_2$ $m_1 \simeq m_2 \simeq \sqrt{\Delta m^2}$

$m_3 \ll \sqrt{\Delta m^2}$

**Lightest ν-mass equal to zero**

$20\ \text{meV} \le m_{\beta\beta} \le 49\ \text{meV}$

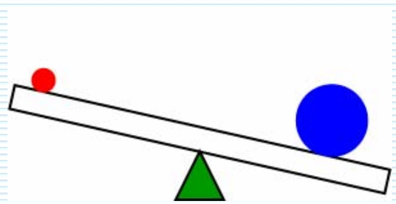
## II. *The sterile $\nu$ mechanism of the $0\nu\beta\beta$-decay*
## *(D-M mass term, V-A SM int.)*
## *Interpolating formula*

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha}\,\nu_\alpha$$

**Mixing of active-sterile neutrinos**

**Dirac-Majorana mass term**

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$



**Light $\nu$ mass $\approx$ $(m_D/m_{LNV})\, m_D$**
**Heavy $\nu$ mass $\approx m_{LNV}$**

**small $\nu$ masees due to see-saw mechanism**

# Possible lepton number violating scale - $m_{LNV}$

**Neutrinos masses may offer a great opportunity to jump beyond the EW framework via see-saw ...**

$10^{19}$ GeV — **Planck** — **How heavy are the heavy Majorana neutrinos?**

$10^{16}$ GeV — **GUT** — **to unify strong, weak & electromagnetic forces?**

**Conventional (Type-one) Seesaw Picture: close to the GUT scale**

$10^{3}$ GeV — **TeV** — **TeV Seesaw Idea: driven by testability at LHC**

**to solve the unnatural gauge hierarchy problem?**

GeV/5 — **Fermi**

$10^{-6}$ GeV — **kev** — **Hot dark mattter**

# Left-handed neutrinos: Majorana neutrino mass eigenstate N with arbitrary mass $m_N$

**Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]**

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_A^4 \left| \sum_N \left( U^2_{eN} m_N \right) m_p \, M'^{0\nu}(m_N, g_A^{\rm eff}) \right|^2$$

## General case

$$M'^{0\nu}(m_N, g_A^{\rm eff}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x\, d^3y\, d^3p$$

$$\times e^{ip\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+ | J^{\mu\dagger}(\mathbf{x}) | n \rangle \langle n | J^\dagger_\mu(\mathbf{y}) | 0_I^+ \rangle}{\sqrt{p^2 + m_N{}^2}\left(\sqrt{p^2 + m_N{}^2} + E_n - \frac{E_I - E_F}{2}\right)}$$

$$M'^{0\nu}(m_N \to 0, g_A^{\rm eff}) = \frac{1}{m_p m_e} M'^{0\nu}_\nu(g_A^{\rm eff})$$

$$M'^{0\nu}(m_N \to \infty, g_A^{\rm eff}) = \frac{1}{m_N^2} M'^{0\nu}_N(g_A^{\rm eff})$$

## Particular cases

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_A^4 \times$$

$$\times \begin{cases} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M'^{0\nu}_\nu(g_A^{\rm eff}) \right|^2 & \text{for } m_N \ll p_F \\ \left| \langle \frac{1}{m_N} \rangle m_p \right|^2 \left| M'^{0\nu}_N(g_A^{\rm eff}) \right|^2 & \text{for } m_N \gg p_F \end{cases}$$

$$\langle m_\nu \rangle = \sum_N U^2_{eN} m_N$$

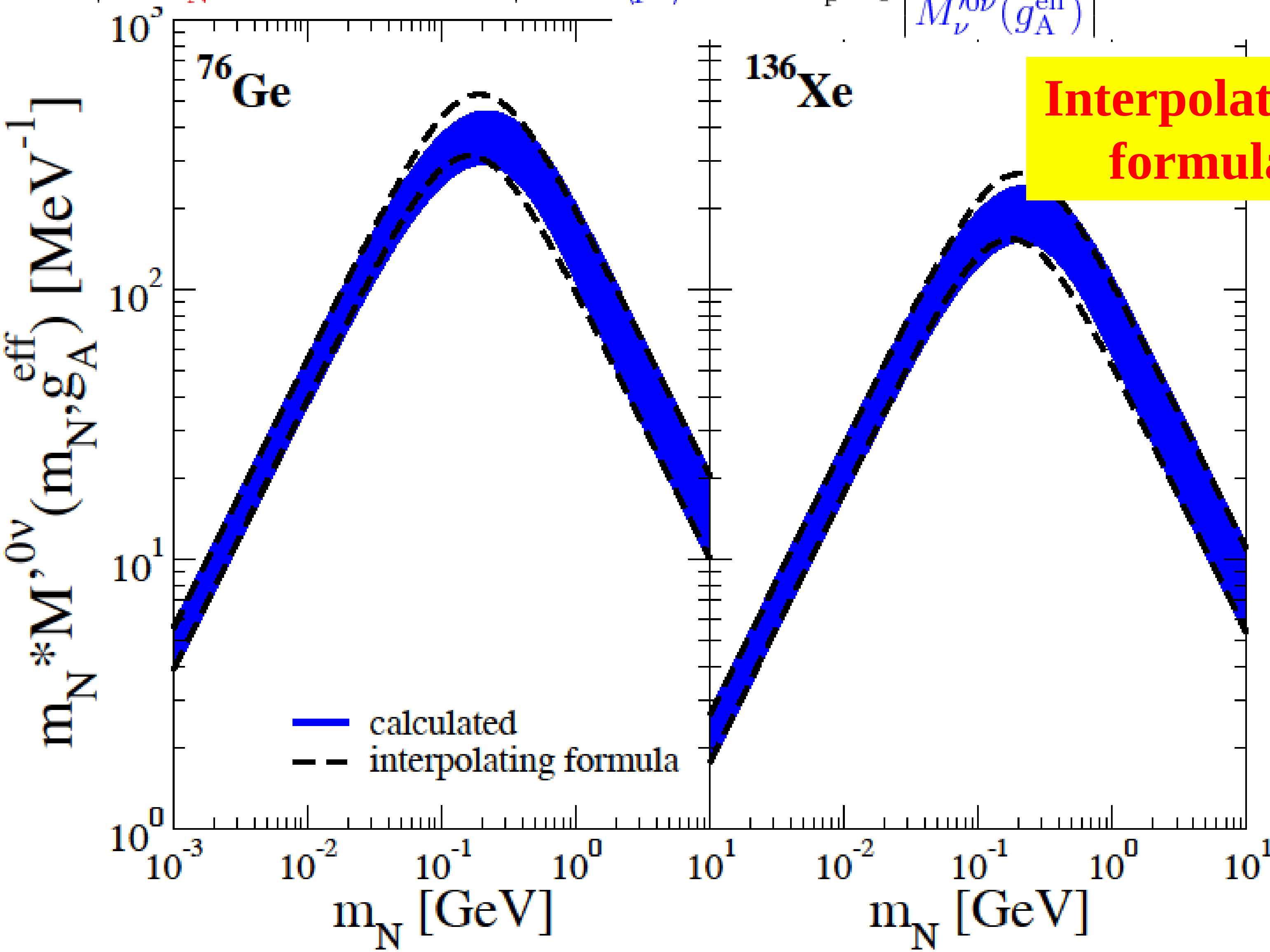
$$\left\langle \frac{1}{m_N} \right\rangle = \sum_N \frac{U^2_{eN}}{m_N}$$

$$[T^{0\nu}_{1/2}]^{-1} = \mathcal{A} \cdot \left| m_{\rm p} \sum_{\rm N} U^2_{e\rm N} \frac{m_{\rm N}}{\langle p^2 \rangle + m_{\rm N}^2} \right|^2 ,$$

$$\mathcal{A} = G^{0\nu} g_{\rm A}^4 \left| M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff}) \right|^2 ,$$

$$\langle p^2 \rangle = m_{\rm p} m_{\rm e} \left| \frac{M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff})}{M'^{0\nu}_{\nu}(g_{\rm A}^{\rm eff})} \right| \approx 200 \; MeV$$

**Interpolating formula**

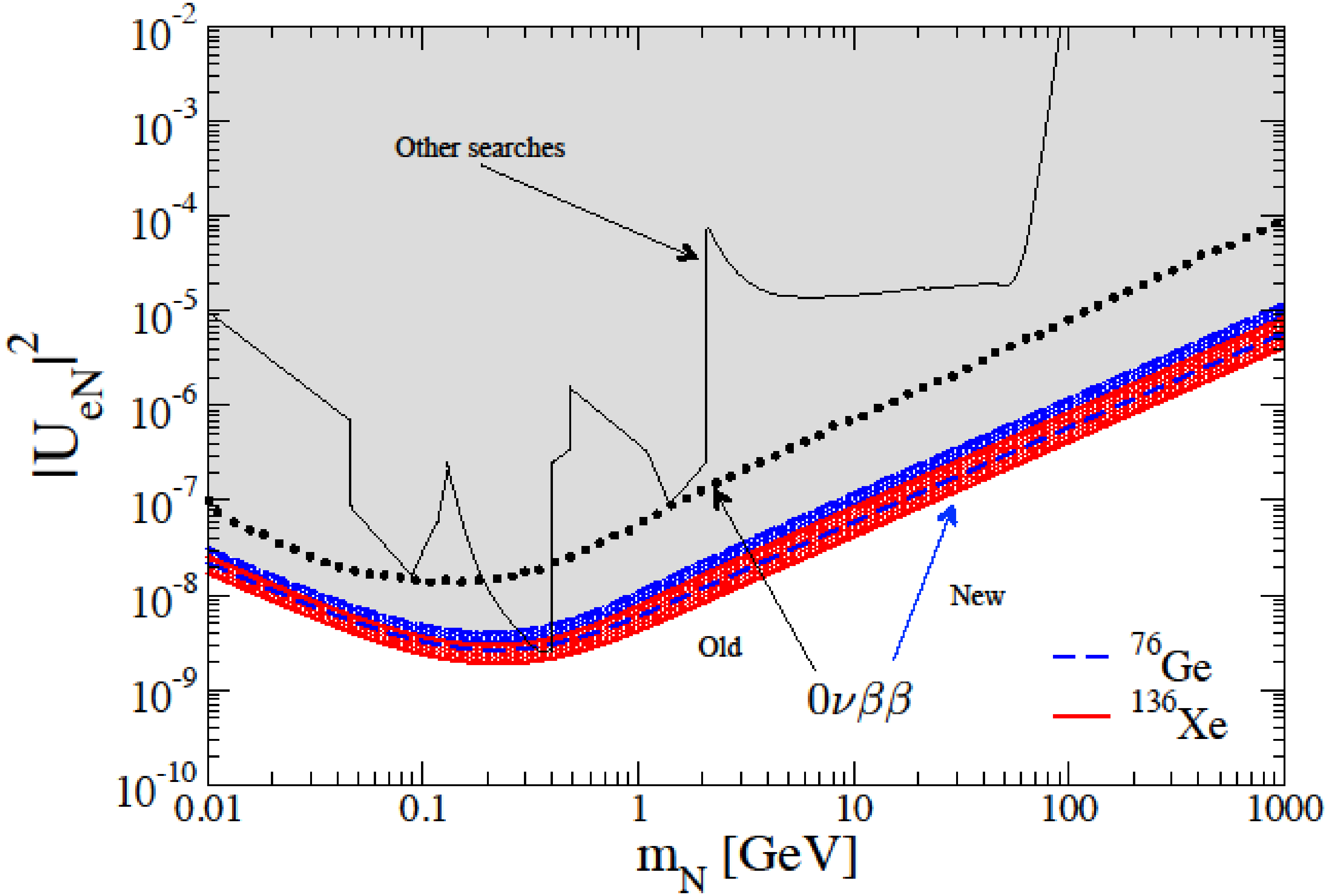





**Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]**

**Exclusion plot in $|U_{eN}|^2 - m_N$ plane**

$T^{0\nu}_{1/2}(^{76}Ge) \geq 3.0\ 10^{25}$ yr
$T^{0\nu}_{1/2}(^{136}Xe) \geq 3.4\ 10^{25}$ yr




**Improvements: i) QRPA (constrained Hamiltonian by 2νββ half-life, self-consistent treatment of src, restoration of isospin symmetry …), ii) More stringent limits on the 0νββ half-life**

# *III. The 0νββ-decay within L-R symmetric theories*
## *(interpolating formula)*
### *(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)*

**A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., arXiv:1804.04218[hep-ph], accepted in PRD**

$$[T^{0\nu}_{1/2}]^{-1} = \eta^2_{\nu N}\ C_{\nu N}$$

$$C_{\nu N} = g^4_{\mathrm{A}}\ \left| M'^{0\nu}_{\nu} \right|^2\ G^{0\nu}$$

**Mixing of light and heavy neutrinos**

$$\nu_{eL} = \sum_{j=1}^{3} \left( U_{ej}\nu_{jL} + S_{ej}(N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left( T^*_{ej}(\nu_{jL})^C + V^*_{ej}N_{jR} \right)$$

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

**Effective LNV parameter within LRS model**

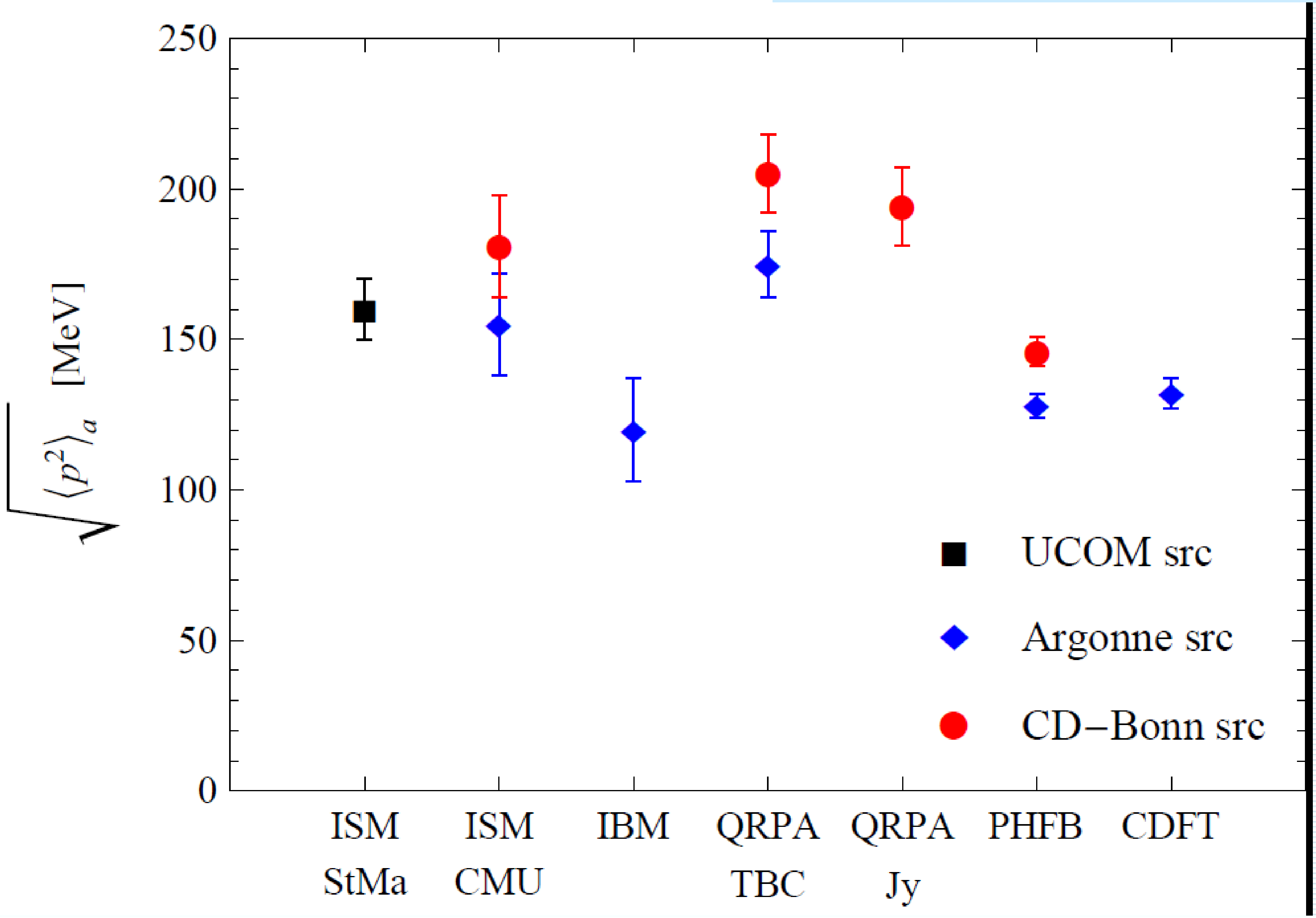
$$\eta^2_{\nu N} = \left| \sum_{j=1}^{3} \left( U^2_{ej}\ \frac{m_j}{m_e}\ +\ S^2_{ej}\ \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j}\ \frac{M_j}{m_e} \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^{3} \left( T^2_{ej}\ \frac{m_j}{m_e}\ +\ V^2_{ej}\ \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j}\ \frac{M_j}{m_e} \right) \right|^2$$

$$\langle p^2 \rangle = m_{\mathrm{p}} m_{\mathrm{e}} \frac{M'^{0\nu}_{\mathrm{N}}}{M'^{0\nu}_{\nu}}$$

**Effective neutrino momentum in the mass mechanism of the 0νββ-decay**

$$\langle p^2 \rangle = m_{\rm p} m_{\rm e} \frac{M'^{0\nu}_{\rm N}}{M'^{0\nu}_{\nu}}$$

**Practically no dependence on A**



# 6x6 PMNS see-saw ν-mixing matrix
**(the most economical one)**

**6x6 neutrino mass matrix**

**Basis**

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix} \qquad (\nu_L, (N_R)^c)^T \qquad \mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}$$

**6x6 matrix: 15 angles, 10+5 CP phases**
**3x3 matrix: 3 angles, 1+2 CP phases**

**3x3 block matrices U, S, T, V are generalization of PMNS matrix**

**The see-saw structure is assumed and mixing between different generations is neglected**

$$\mathcal{U}_{\mathrm{PMNS}} = \begin{pmatrix} U_{\mathrm{PMNS}} & \zeta\,\mathbf{1} \\ -\zeta\,\mathbf{1} & U^\dagger_{\mathrm{PMNS}} \end{pmatrix} \qquad \mathcal{U}_{\mathrm{PMNS}}\,\mathcal{U}^\dagger_{\mathrm{PMNS}} = \mathcal{U}^\dagger_{\mathrm{PMNS}}\,\mathcal{U}_{\mathrm{PMNS}} = 1$$

**see-saw parameter**

$$\zeta = \frac{m_{\mathrm{D}}}{m_{\mathrm{LNV}}}$$

**6x6 matrix: 3 angles, 1+2 CP phases, 1 see-saw par.**

**A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., arXiv:1804.04218[hep-ph], accepted in PRD**

# 6x6 PMNS see-saw ν-mixing matrix
**(the most economical one)**

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\,1 \\ -\zeta\,1 & V_0 \end{pmatrix}$$

$$U_0 = U_{\rm PMNS}$$

**Lost of meaning "Dirac" and "Majorana" CP violating phases**

$$V_0 = U^\dagger_{\rm PMNS} =$$

$$\begin{pmatrix} c_{12}\,c_{13}\,e^{-i\alpha_1} & \left(-s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{-i\delta}\right)e^{-i\alpha_1} & \left(s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{-i\delta}\right)e^{-i\alpha_1} \\ s_{12}\,c_{13}\,e^{-i\alpha_2} & \left(c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{-i\delta}\right)e^{-i\alpha_2} & \left(-c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{-i\delta}\right)e^{-i\alpha_2} \\ s_{13}\,e^{i\delta} & c_{13}\,s_{23} & c_{13}\,c_{23} \end{pmatrix}$$

**Assumption about heavy neutrino masses $M_i$ (by assuming see-saw)**

**Inverse proportional**

$$m_i M_i \simeq m_D^2$$

$$M^{\rm R}_{\beta\beta} = \lambda\,\frac{\langle p^2\rangle_a}{m_D^2}\,\left|\sum_{j=1}^{3}(U_0^\dagger)^2_{ej}\;m_j\right|$$

**Proportional**

$$m_i \simeq \zeta^2 M_i$$

$$M^{\rm R}_{\beta\beta} = \lambda\,\zeta^2\,\left|\sum_{j=1}^{3}(U_0^\dagger)^2_{ej}\;\frac{\langle p^2\rangle_a}{m_j}\right|$$

**Heavy Majorana mass $\mathbf{M}^{\mathbf{R}}{}_{\beta\beta}$ depends on the "Dirac" CP violating phase δ**

# Contribution from exchange of heavy neutrino to 0νββ-decay rate might be large

## Inverse proportional

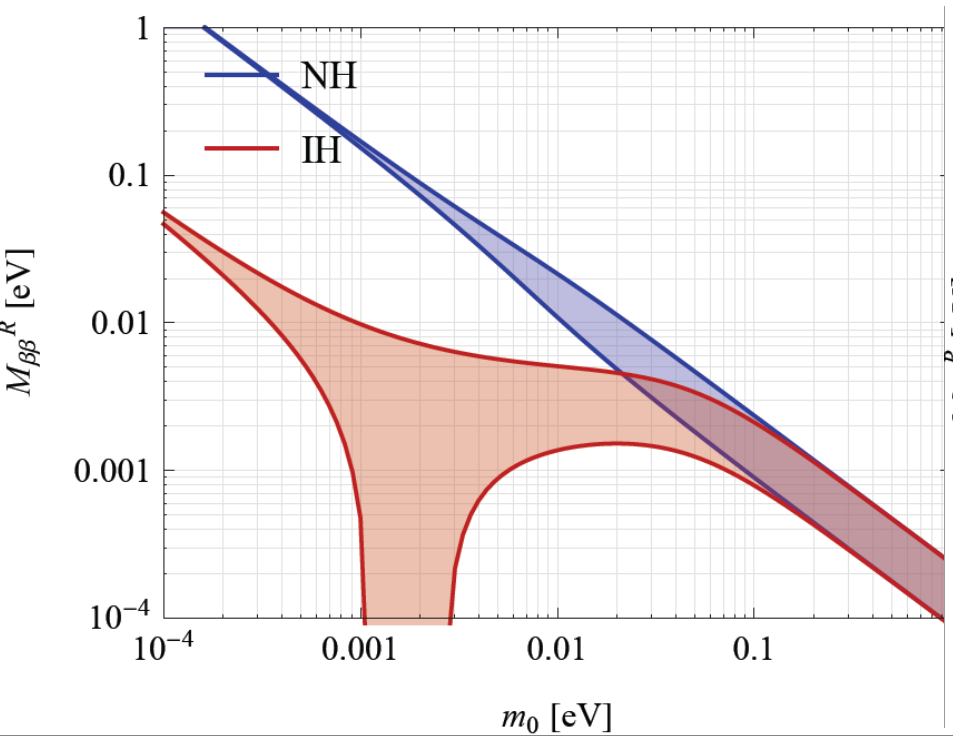
$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^{\rm R} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \, m_j \right|$$

$$V_0 = U_{PMNS}^\dagger$$

$$M_i = m_D^2/m_i \quad m_D \simeq 5 \text{ MeV}$$

$$\lambda = 7.7 \times 10^{-4}$$




## Proportional

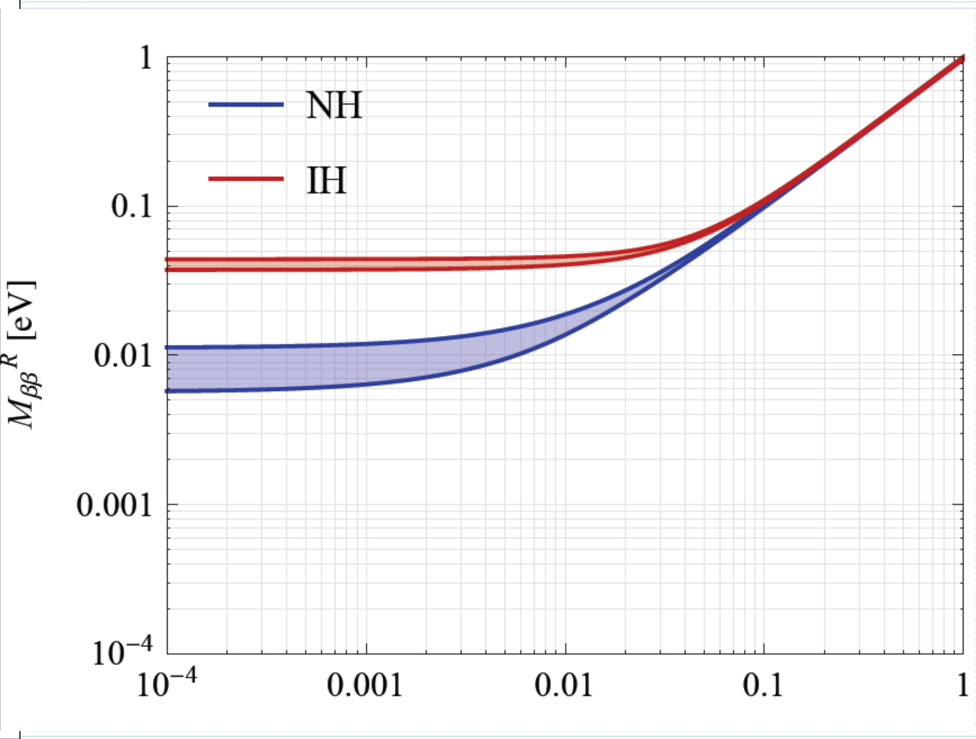
$$m_i \simeq \zeta^2 M_i$$

$$M_{\beta\beta}^{\rm R} = \lambda \, \zeta^2 \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \, \frac{\langle p^2 \rangle_a}{m_j} \right|$$

$$V_0 = U_{PMNS}^\dagger$$

$$\zeta = m_i/M_i \quad \zeta^2 \simeq 5 \times 10^{-17}$$

$$\lambda = 7.7 \times 10^{-4}$$



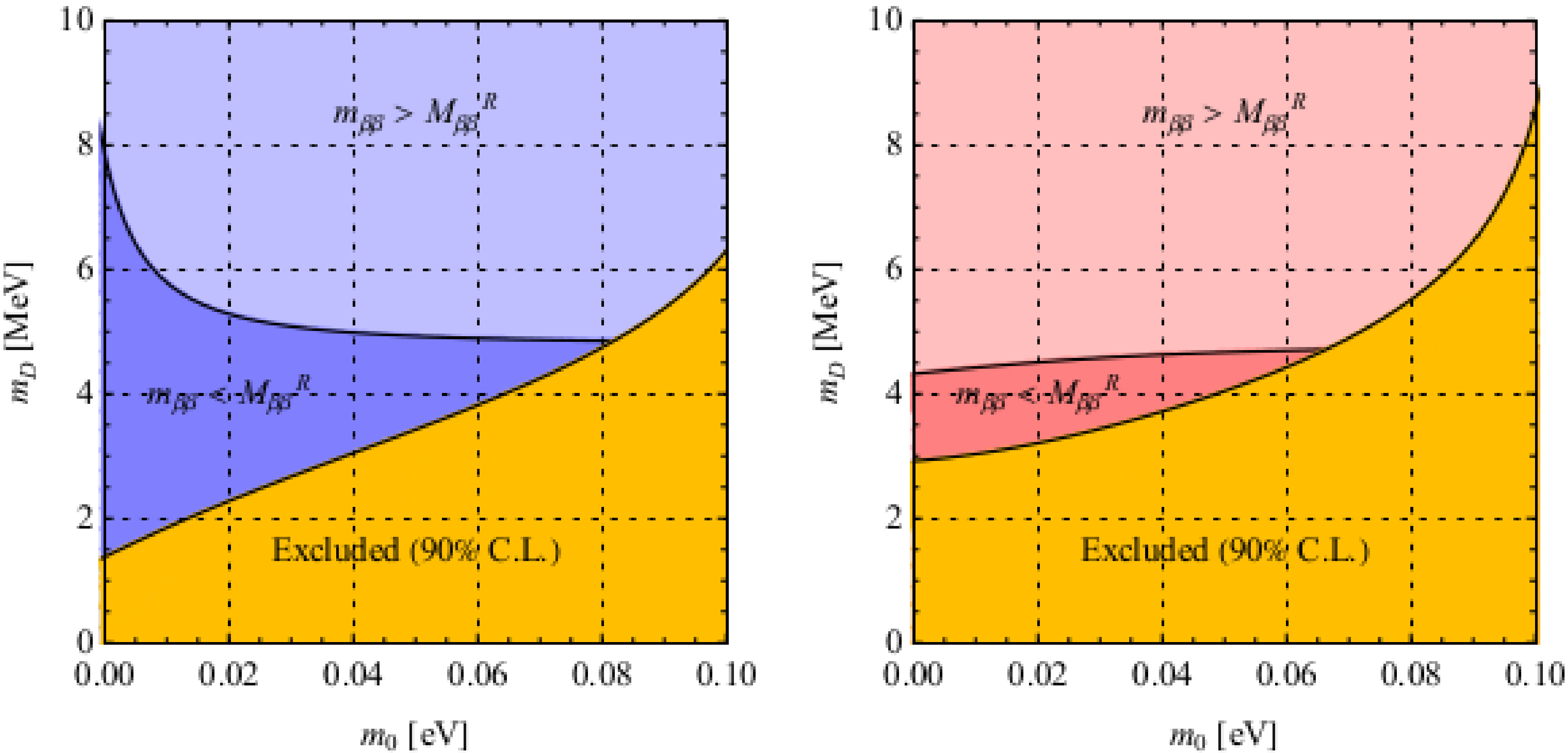
$$\eta^2_{\nu N} = \frac{1}{m_e^2}\left(m^2_{\beta\beta} + (M^R_{\beta\beta})^2\right)$$

**See-saw scenario**

$$m_i M_i \simeq m_D^2$$

$$M^{\mathrm{R}}_{\beta\beta} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\, m_j \right|$$

**Normal spectrum** **Inverted spectrum**




**A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., arXiv:1804.04218[hep-ph], accepted in PRD**

# IV. The $0\nu\beta\beta$-decay within L-R symmetric theories

*(D-M mass term, see-saw, V-A and V+A int., exchange of light neutrinos)*

## Effective β-decay Hamiltonian

$$H^\beta = \frac{G_\beta}{\sqrt{2}} \left[ j_L{}^\rho J_{L\rho} + \chi\, j_L{}^\rho J_{R\rho} + \eta\, j_R{}^\rho J_{L\rho} + \lambda\, j_R{}^\rho J_{R\rho} + h.c. \right].$$

## left- and right-handed lept. currents

$$j_L{}^\rho = \bar{e}\gamma^\rho (1-\gamma_5)\nu_{eL}$$
$$j_R{}^\rho = \bar{e}\gamma^\rho (1+\gamma_5)\nu_{eR}$$

## Mixing of vector bosons $W_L$ and $W_R$

$$\begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W_1^- \\ W_2^- \end{pmatrix}$$

$$\eta = -\tan\zeta, \quad \chi = \eta,$$
$$\lambda = (M_{W_1}/M_{W_2})^2$$

## The 0νββ-decay half-life

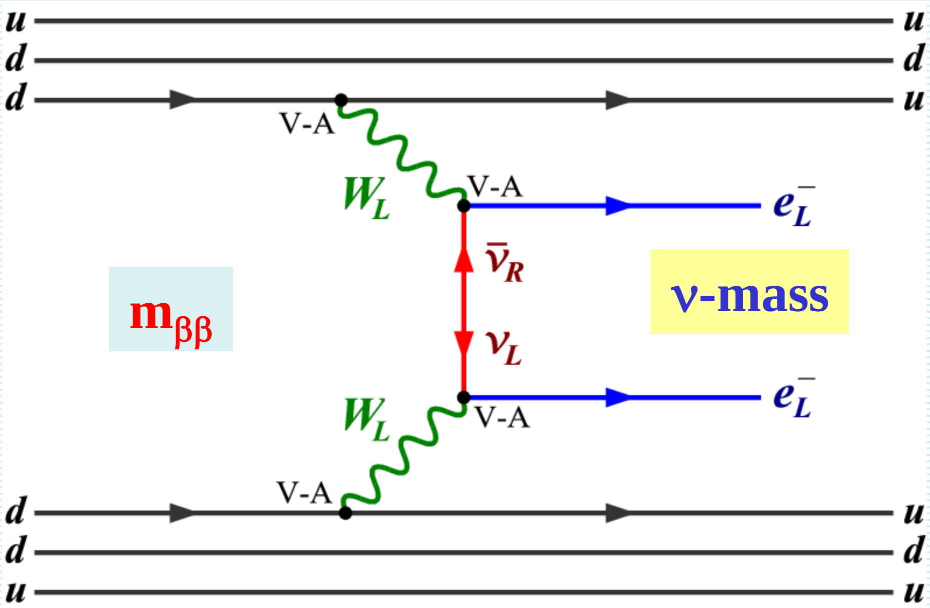
$$\left[T_{1/2}^{0\nu}\right]^{-1} = \frac{\Gamma^{0\nu}}{\ln 2} = g_A^4\, |M_{GT}|^2 \left\{ C_{mm} \frac{|m_{\beta\beta}|}{m_e}^2 + C_{m\lambda}\frac{|m_{\beta\beta}|}{m_e}\langle\lambda\rangle \cos\psi_1 + C_{m\eta}\frac{|m_{\beta\beta}|}{m_e}\langle\eta\rangle\cos\psi_2 + C_{\lambda\lambda}\langle\lambda\rangle^2 + C_{\eta\eta}\langle\eta\rangle^2 + C_{\lambda\eta}\langle\lambda\rangle\langle\eta\rangle \cos(\psi_1-\psi_2) \right\}$$

**$\langle\lambda\rangle$ - $W_L$-$W_R$ exch.**

**$\langle\eta\rangle$ - $W_L$-$W_R$ mixing**



**D. Štefánik, R. Dvornický, F.Š., P. Vogel, PRC 92, 055502 (2015)**

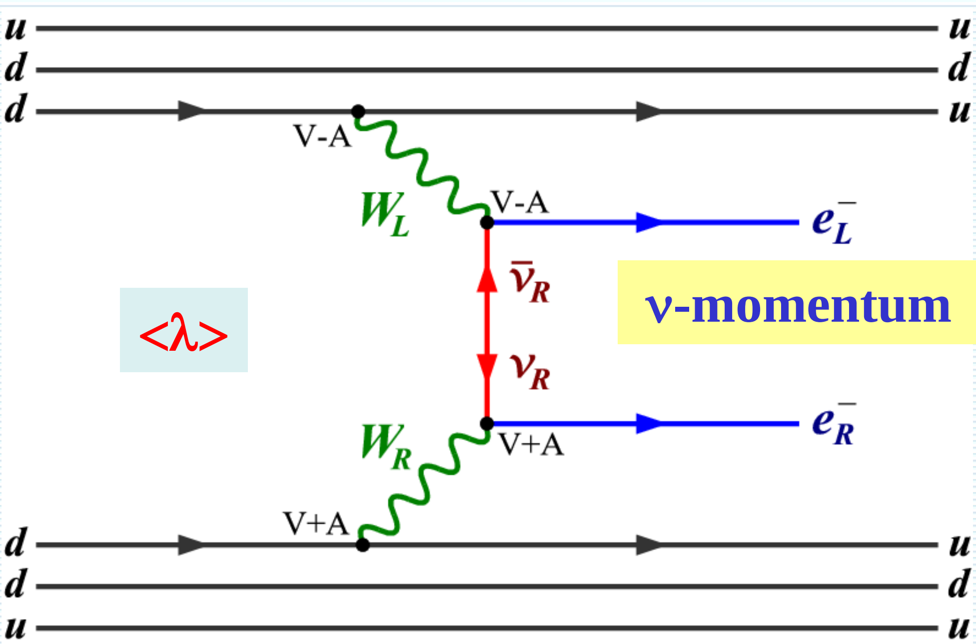



# *Left-right symmetric models SO(10)*

**Mixing of ligt and heavy neutrinos**

$$\nu_{eL} = \sum_{j=1} \left( U_{ej}\nu_{jL} + S_{ej}(N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left( T^*_{ej}(\nu_{jL})^C + V^*_{ej}N_{jR} \right)$$




**Effective LNV parameters due to RHC**

$$\langle\lambda\rangle = \lambda \left|\sum_{j=1}^{3} U_{ej}T^*_{ej}\right|$$

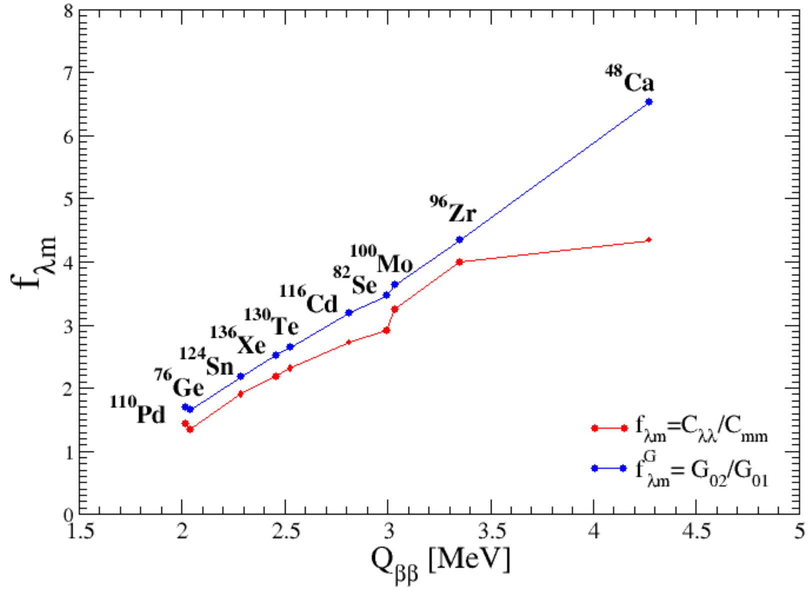
$$\langle\eta\rangle = \eta \left|\sum_{j=1}^{3} U_{ej}T^*_{ej}\right|$$

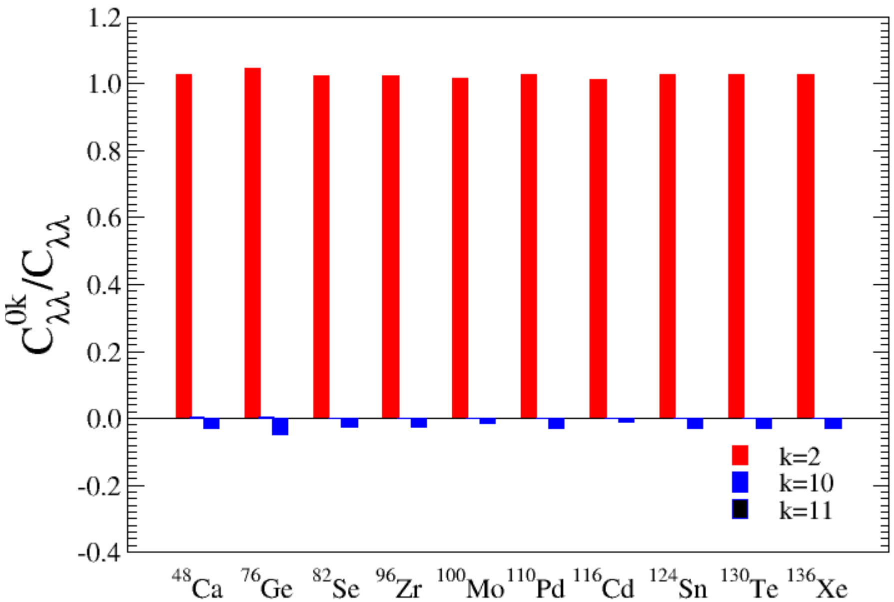
**Mixing and masses of vector bosons**

$$\eta = -\tan\zeta, \quad \chi = \eta,$$

$$\lambda = (M_{W_1}/M_{W_2})^2$$

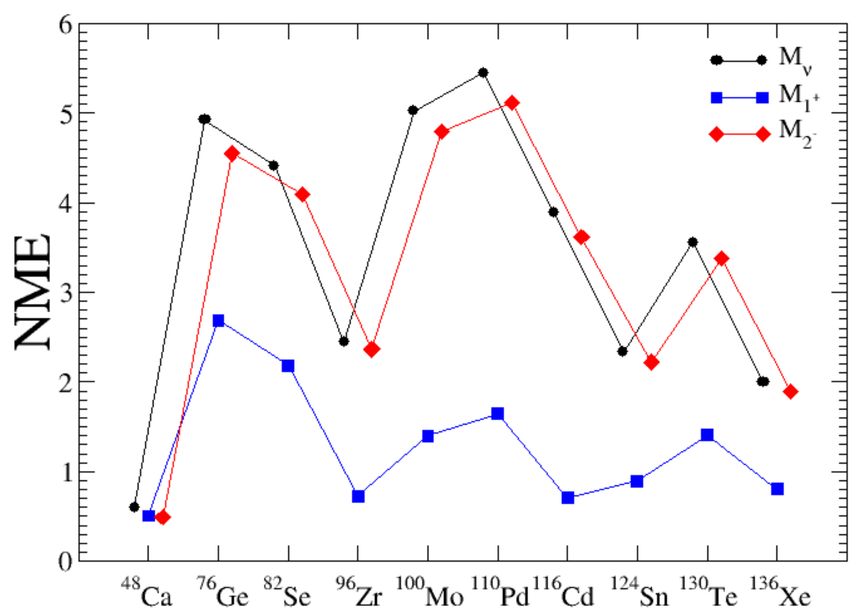
# $m_{\beta\beta}$ and $\lambda$ mechanisms




$$
\begin{aligned}
\left[T^{0\nu}_{1/2}\right]^{-1} &= \left(\eta_\nu^2 + \eta_\lambda^2 f_{\lambda m}\right) C_{mm} \\
&\simeq \left(\eta_\nu^2 + \eta_\lambda^2 f^G_{\lambda m}\right) g_A^4 M_\nu^2 G_{01}
\end{aligned}
$$

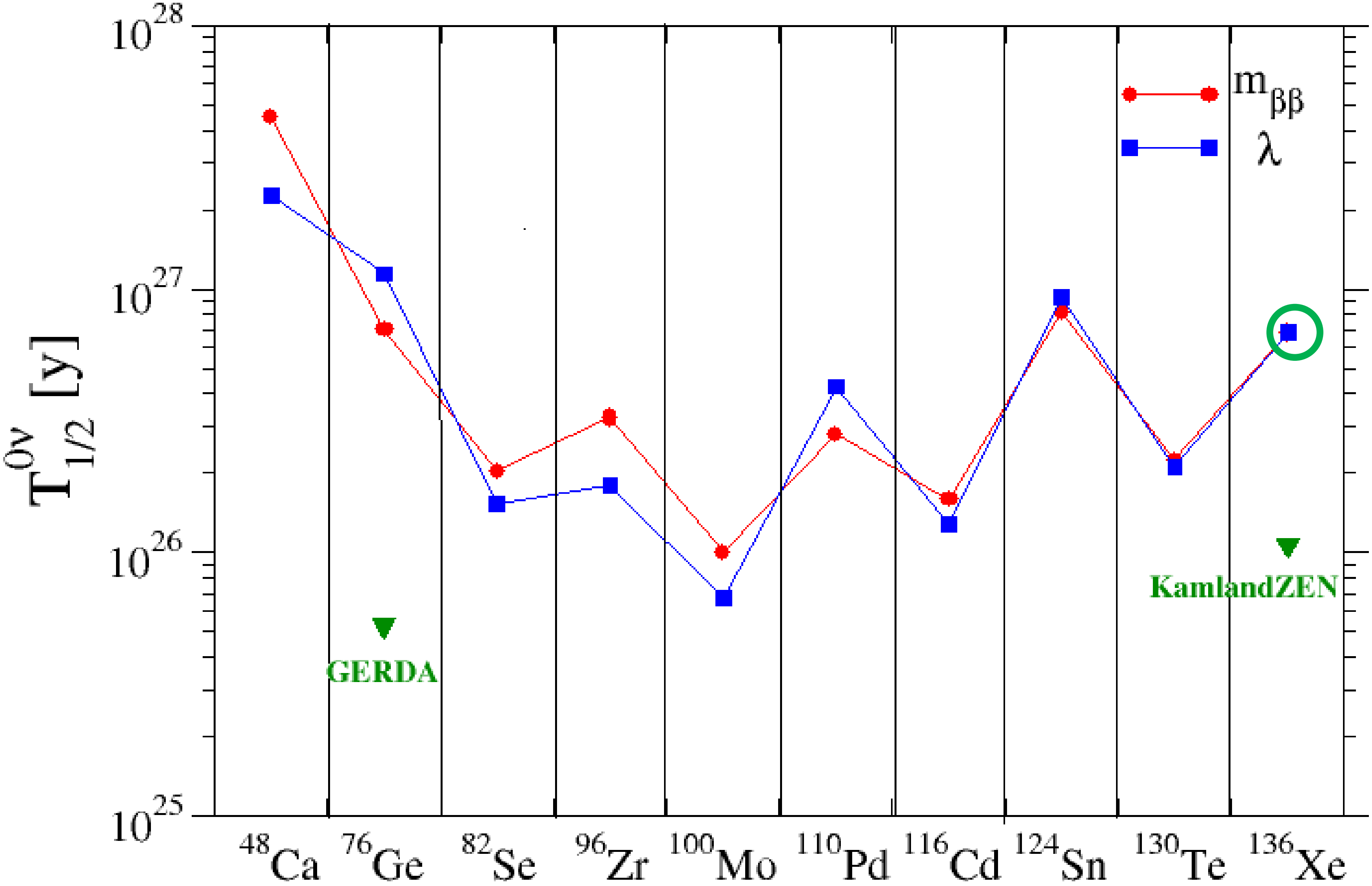
$$
\begin{aligned}
f_{\lambda m} &= \frac{C_{\lambda\lambda}}{C_{mm}} \\
&\simeq f^G_{\lambda m} = \frac{G_{02}}{G_{01}}
\end{aligned}
$$






**F.Š., R. Dvornický, R. Štefánik, Found. Phys. 5, 57 (2017)**

$m_{\beta\beta}$= 50 meV ( $^{136}$Xe), $g_A$=1.269, QRPA NMEs



# $0\nu\beta\beta$ *decay NMEs*

| Method | $g_A$ | src | $M_\nu^{0\nu}$ | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $^{48}$Ca | $^{76}$Ge | $^{82}$Se | $^{96}$Zr | $^{100}$Mo | $^{110}$Pd |
| ISM-StMa | 1.25 | UCOM | 0.85 | 2.81 | 2.64 | | | |
| ISM-CMU | 1.27 | Argonne | 0.80 | 3.37 | 3.19 | | | |
| | | CD-Bonn | 0.88 | 3.57 | 3.39 | | | |
| IBM | 1.27 | Argonne | 1.75 | 4.68 | 3.73 | 2.83 | 4.22 | 4.05 |
| QRPA-TBC | 1.27 | Argonne | 0.54 | 5.16 | 4.64 | 2.72 | 5.40 | 5.76 |
| | | CD-Bonn | 0.59 | 5.57 | 5.02 | 2.96 | 5.85 | 6.26 |
| QRPA-Jy | 1.26 | CD-Bonn | | 5.26 | 3.73 | 3.14 | 3.90 | 6.52 |
| dQRPA-NC | 1.25 | without | | 5.09 | | | | |
| PHFB | 1.25 | Argonne | | | | 2.84 | 5.82 | 7.12 |
| | | CD-Bonn | | | | 2.98 | 6.07 | 7.42 |
| NREDF | 1.25 | UCOM | 2.37 | 4.60 | 4.22 | 5.65 | 5.08 | |
| REDF | 1.25 | without | 2.94 | 6.13 | 5.40 | 6.47 | 6.58 | |
| Mean value | | | 1.34 | 4.55 | 4.02 | 3.78 | 5.57 | 6.12 |
| variance | | | 0.81 | 1.20 | 0.91 | 2.49 | 0.58 | 1.78 |

| Method | $g_A$ | src | $M_\nu^{0\nu}$ | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $^{116}$Cd | $^{124}$Sn | $^{128}$Te | $^{130}$Te | $^{136}$Xe | $^{150}$Nd |
| ISM-StMa | 1.25 | UCOM | | 2.62 | | 2.65 | 2.19 | |
| ISM-CMU | 1.27 | Argonne | | 2.00 | | 1.79 | 1.63 | |
| | | CD-Bonn | | 2.15 | | 1.93 | 1.76 | |
| IBM | 1.27 | Argonne | 3.10 | 3.19 | 4.10 | 3.70 | 3.05 | 2.67 |
| QRPA-TBC | 1.27 | Argonne | 4.04 | 2.56 | 4.56 | 3.89 | 2.18 | |
| | | CD-Bonn | 4.34 | 2.91 | 5.08 | 4.37 | 2.46 | 3.37 |
| QRPA-Jy | 1.26 | CD-Bonn | 4.26 | 5.30 | 4.92 | 4.00 | 2.91 | |
| dQRPA-NC | 1.25 | without | | | | 1.37 | 1.55 | 2.71 |
| PHFB | 1.27 | Argonne | | | 3.90 | 3.81 | | 2.58 |
| | | CD-Bonn | | | 4.08 | 3.98 | | 2.68 |
| NREDF | 1.25 | UCOM | 4.72 | 4.81 | 4.11 | 5.13 | 4.20 | 1.71 |
| REDF | 1.25 | without | 5.52 | 4.33 | | 4.98 | 4.32 | 5.60 |
| Mean value | | | 4.34 | 3.07 | 4.34 | 3.42 | 2.59 | 3.01 |
| variance | | | 0.79 | 1.01 | 0.23 | 1.67 | 1.10 | 1.34 |

# NMEs for unquenched value of $g_A$

J.D.Vergados, H. Ejiri, , F.Š.,
Int. J. Mod. Phys. E25, 1630007(2016)

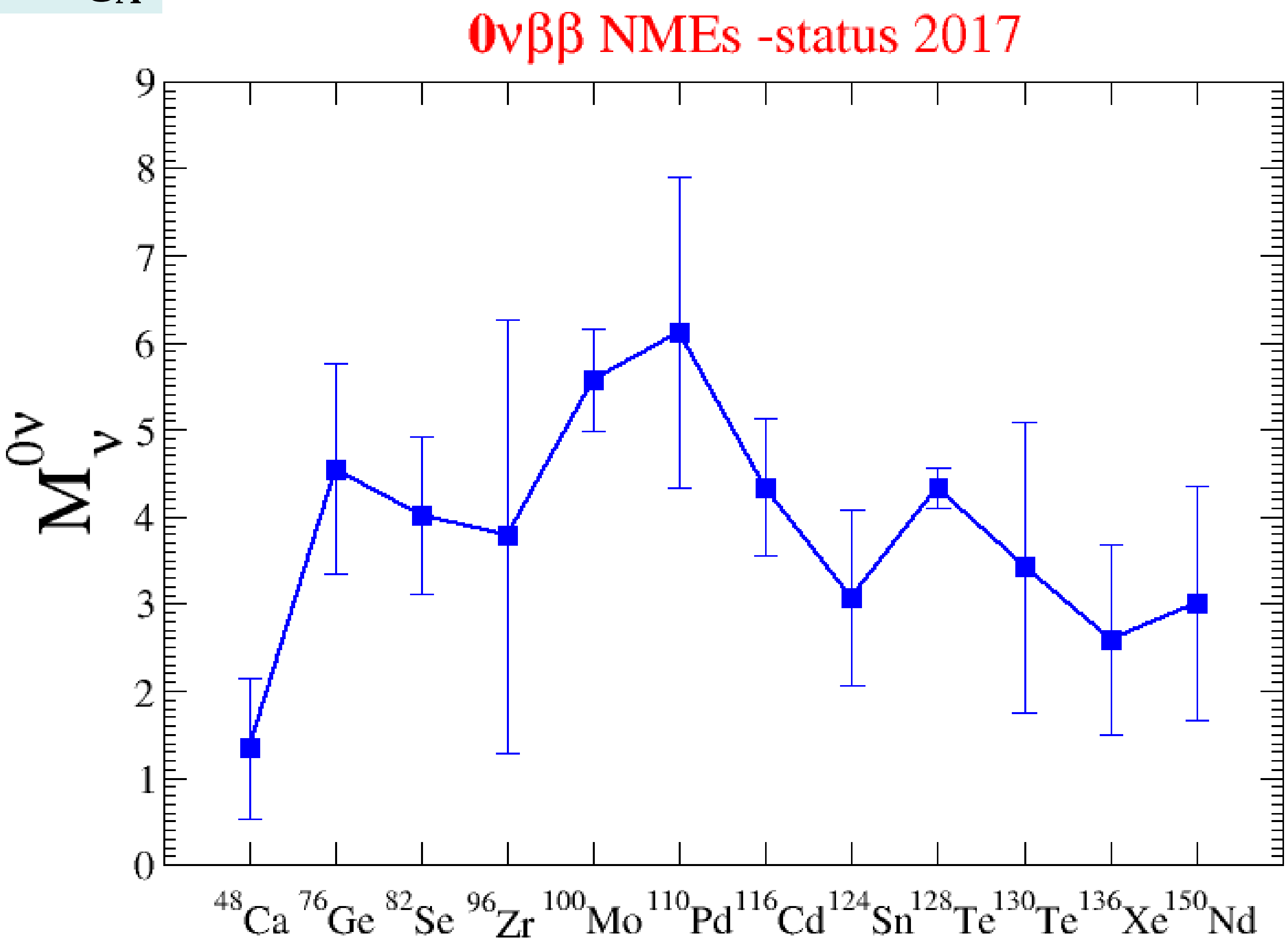
**Mean field approaches**
**(PHFB, NREDF, REDF)**
**⇒ Large NMEs**

**Interacting Shell Model**
**(ISM-StMa, ISM-CMU)**
**⇒ small NMEs**

**Quasiparticle Random Phase Approximation**
**(QRPA-TBC, QRPA-Jy, dQRPQ-NC)**
**⇒ Intermediate NMEs**

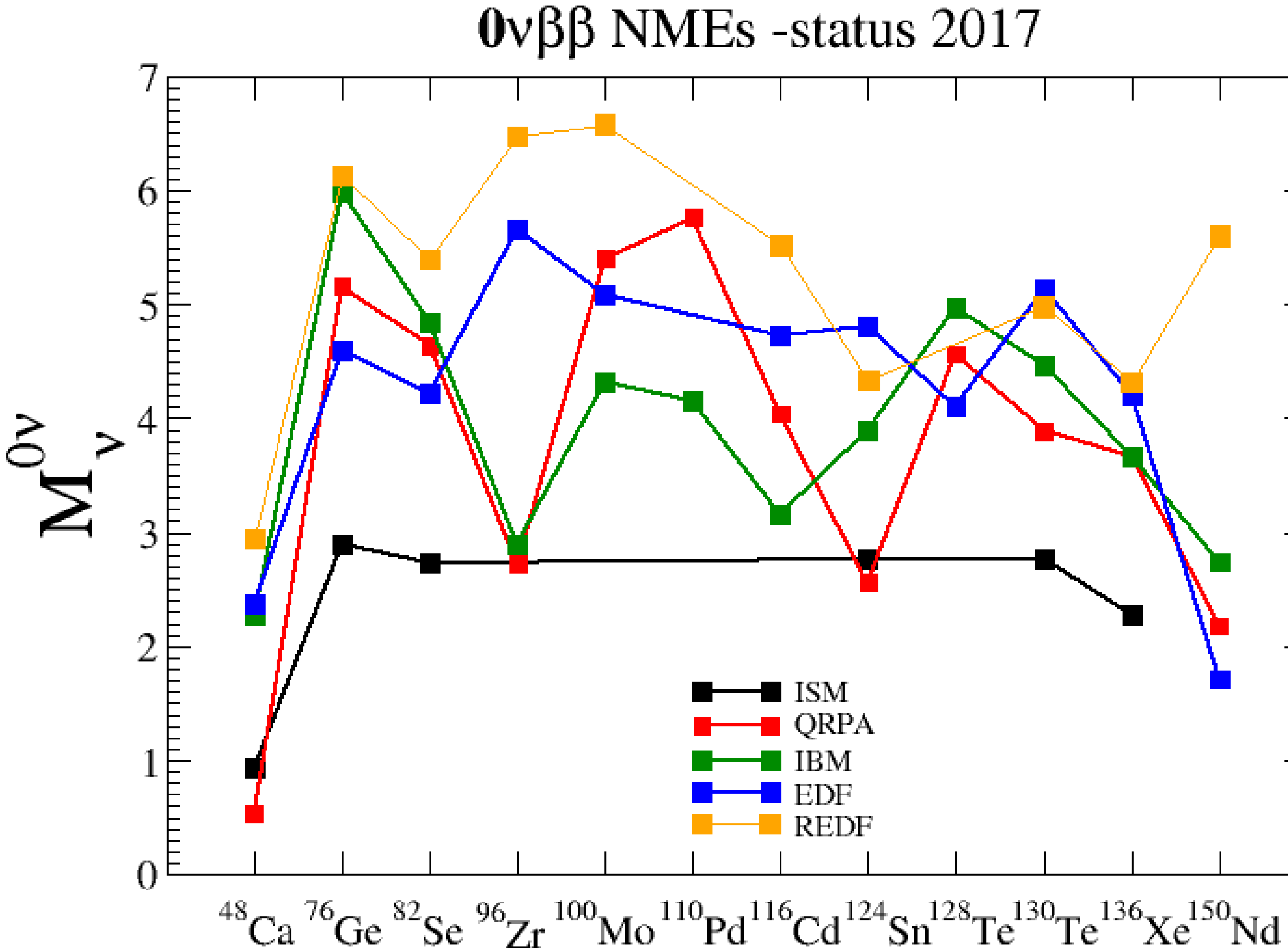
**Interacting Boson Model**
**(IBM)**
**⇒ Close to QRPA results**

**unquenched $g_A$**




J.D.Vergados, H. Ejiri, , F.Š., Int. J. Mod. Phys. E25, 1630007(2016)

| | mean field meth. | ISM | IBM | QRPA |
|---|---|---|---|---|
| Large model space | yes | no | yes | yes |
| Constr. Interm. States | no | yes | no | yes |
| Nucl. Correlations | limited | all | restricted | restricted |



0νββ NMEs -status 2017
$M_\nu^{0\nu}$
0
1
2
3
4
5
6
7
ISM
QRPA
IBM
EDF
REDF
$^{48}$Ca $^{76}$Ge $^{82}$Se $^{96}$Zr $^{100}$Mo $^{110}$Pd $^{116}$Cd $^{124}$Sn $^{128}$Te $^{130}$Te $^{136}$Xe $^{150}$Nd

# Nuclear deformation

$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$

**Exp. I (nuclear reorientation method)**
**Exp.II (based on measured E2 trans.)**
**Theor. I (Rel. mean field theory)**
**Theor. II (Microsc.-Macrosc. Model of Moeller and Nix)**

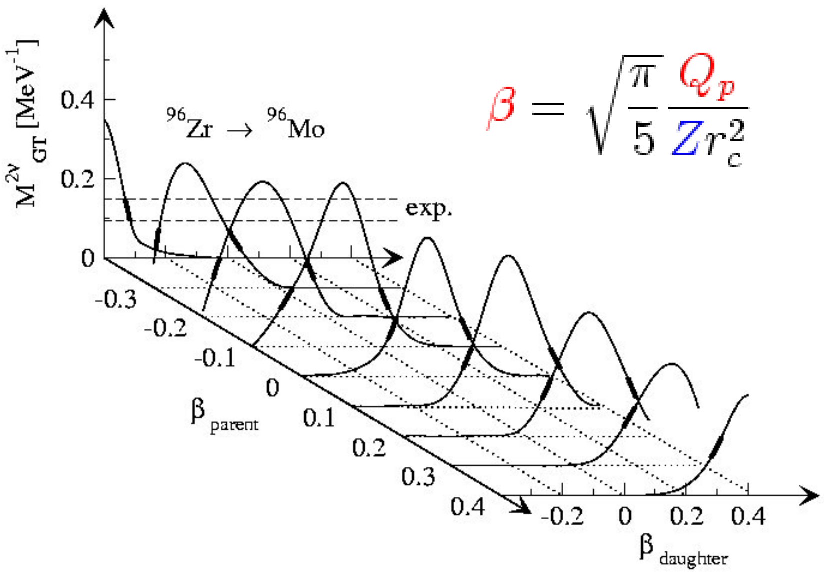
**Till now, in the QRPA-like calculations of the 0νββ-decay NME spherical symetry was assumed**

**The effect of deformation on NME has to be considered**

| Nucl. | Exp. I | Exp. II | Theor. I | Theor. II |
|---|---|---|---|---|
| $^{48}$Ca | 0.00 | 0.101 | 0.00 | 0.00 |
| $^{48}$Ti | +0.17 | 0.269 | -0.01 | 0.00 |
| $^{76}$Ge | +0.09 | 0.26 | 0.16 | 0.14 |
| $^{76}$Se | +0.16 | 0.31 | -0.24 | -0.24 |
| $^{82}$Se | +0.10 | 0.19 | 0.13 | 0.15 |
| $^{82}$Kr | | 0.20 | 0.12 | 0.07 |
| $^{96}$Zr | | 0.081 | 0.22 | 0.22 |
| $^{96}$Mo | +0.07 | 0.17 | 0.17 | 0.08 |
| $^{100}$Mo | +0.14 | 0.23 | 0.25 | 0.24 |
| $^{100}$Ru | +0.14 | 0.22 | 0.19 | 0.16 |
| $^{116}$Cd | +0.11 | 0.19 | -0.26 | -0.24 |
| $^{116}$Sn | +0.04 | 0.11 | 0.00 | 0.00 |
| $^{128}$Te | +0.01 | 0.14 | -0.00 | 0.00 |
| $^{128}$Xe | | 0.18 | 0.16 | 0.14 |
| $^{130}$Te | +0.03 | 0.12 | 0.03 | 0.00 |
| $^{130}$Xe | | 0.17 | 0.13 | -0.11 |
| $^{136}$Xe | | 0.09 | 0.00 | 0.00 |
| $^{136}$Ba | | 0.12 | 0.00 | 0.00 |
| $^{150}$Nd | +0.37 | 0.28 | 0.22 | 0.24 |
| $^{150}$Sm | +0.23 | 0.19 | 0.18 | 0.21 |

# Suppression of the 0νββ-decay NMEs due to different deformation of initial and final nuclei

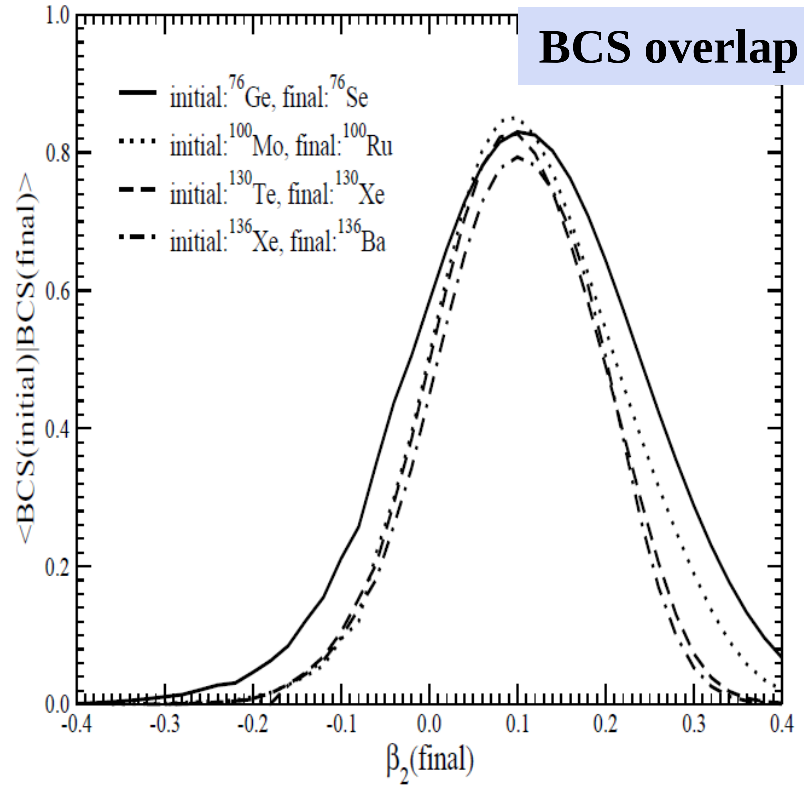
$$\beta = \sqrt{\frac{\pi}{5}}\frac{Q_p}{Z r_c^2}$$




**The suppression of the NME depends on the relative deformation of initial and final nuclei**

**F.Š., Pacearescu, Faessler, NPA 733 (2004) 321**

**BCS overlap**

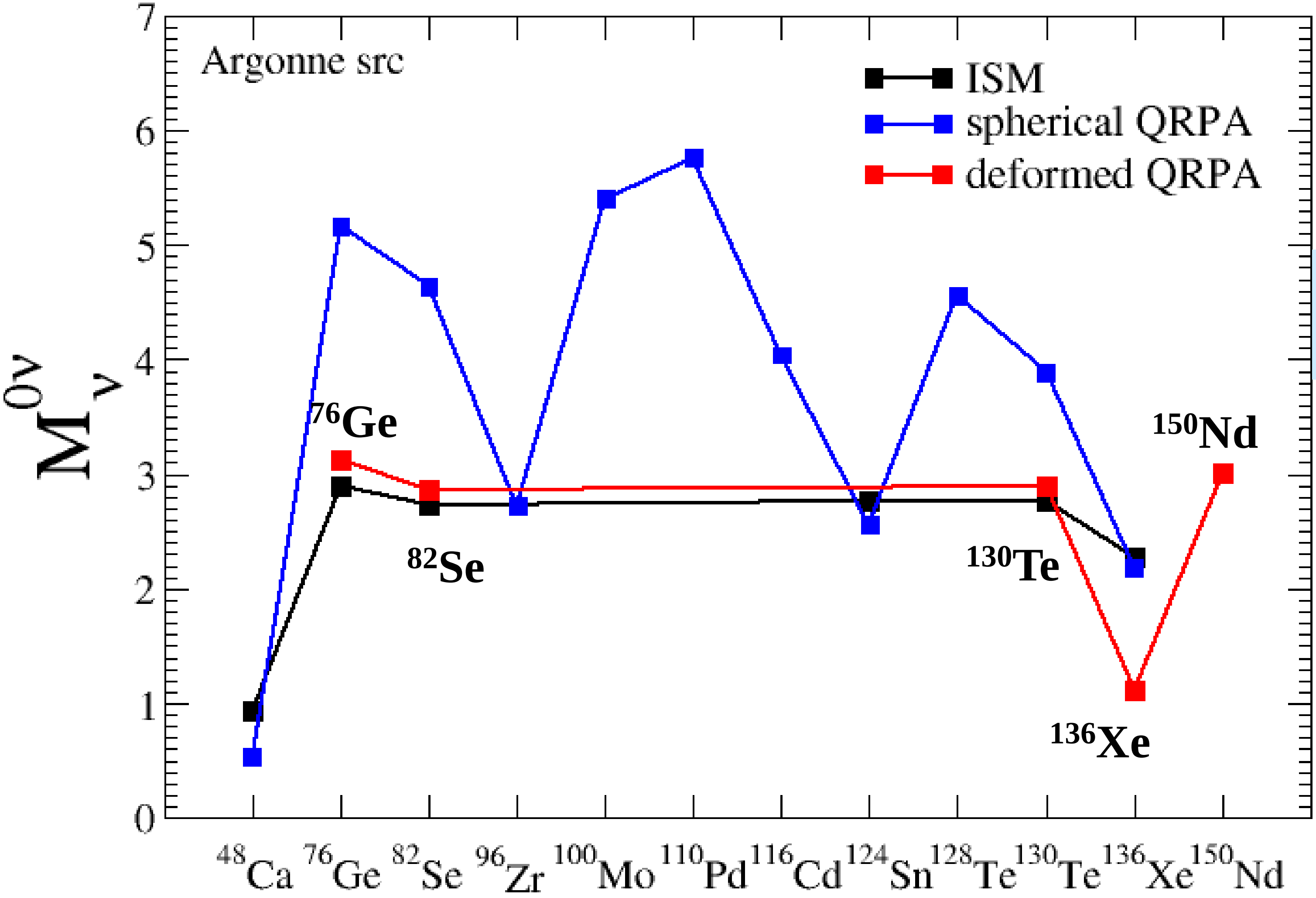



**Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA**

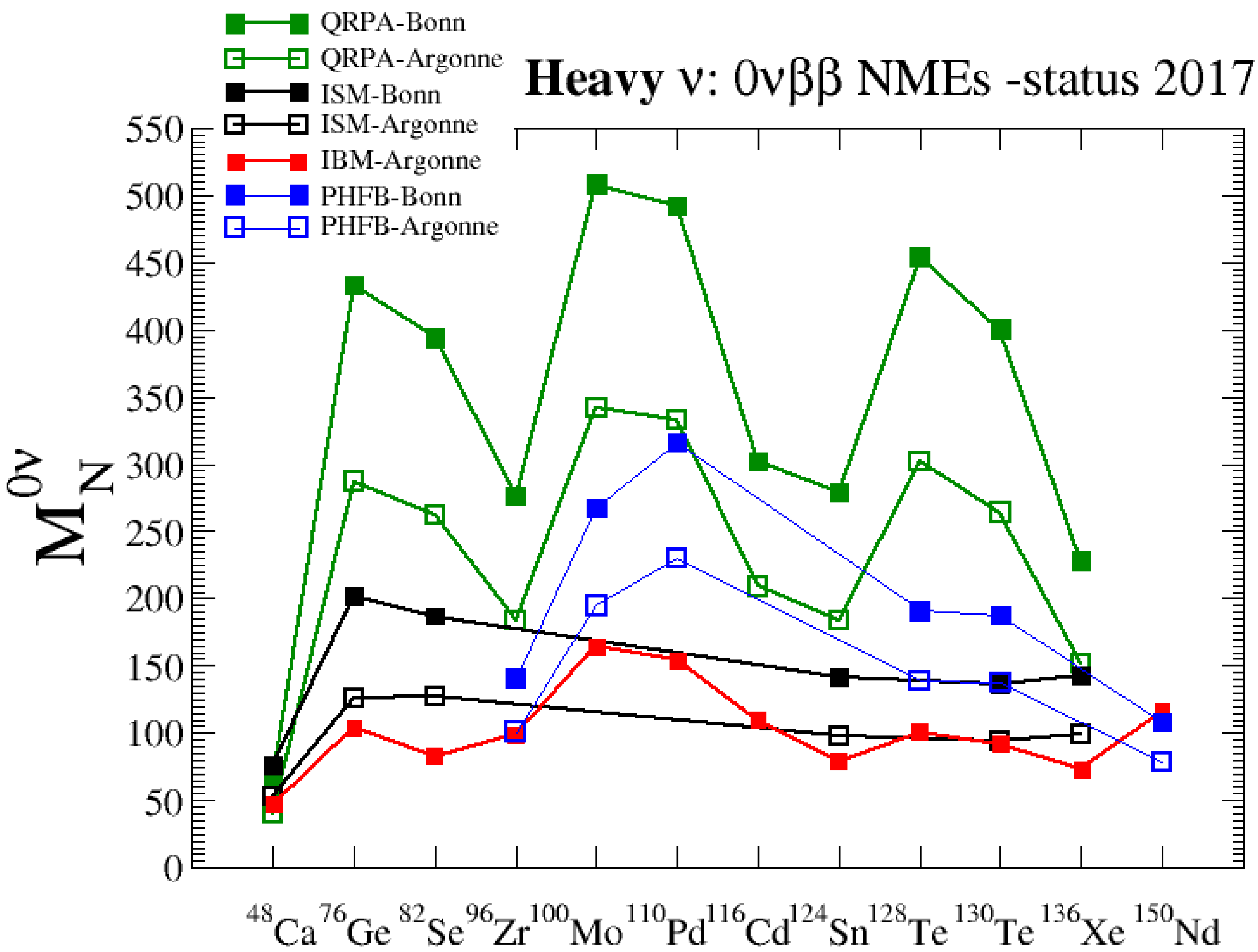
**Alvarez, Sarriguren, Moya, Pacearescu, Faessler, F.Š., Phys. Rev. C 70 (2004) 321**

# 0νββ-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (light neutrino exchange)

D. Fang, A. Faessler, F.Š., PRC 97, 045503 (2018)



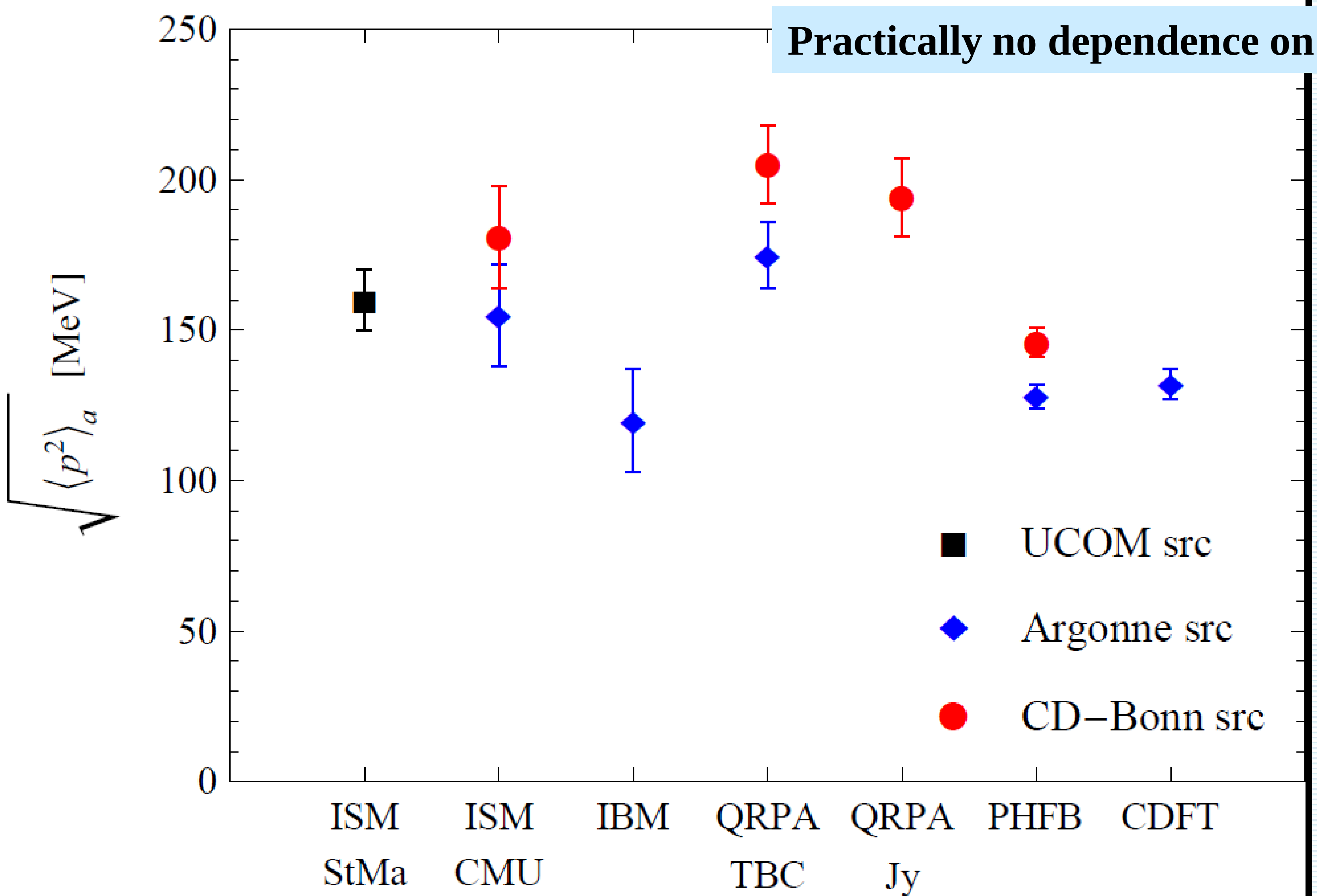

Agreement by a chance?


Heavy ν: 0νββ NMEs -status 2017
QRPA-Bonn
QRPA-Argonne
ISM-Bonn
ISM-Argonne
IBM-Argonne
PHFB-Bonn
PHFB-Argonne
$M^{0\nu}_{N}$
0
50
100
150
200
250
300
350
400
450
500
550
$^{48}$Ca $^{76}$Ge $^{82}$Se $^{96}$Zr $^{100}$Mo $^{110}$Pd $^{116}$Cd $^{124}$Sn $^{128}$Te $^{130}$Te $^{136}$Xe $^{150}$Nd

**Effective neutrino momentum**
**in the mass mechanism of the 0νββ-decay**

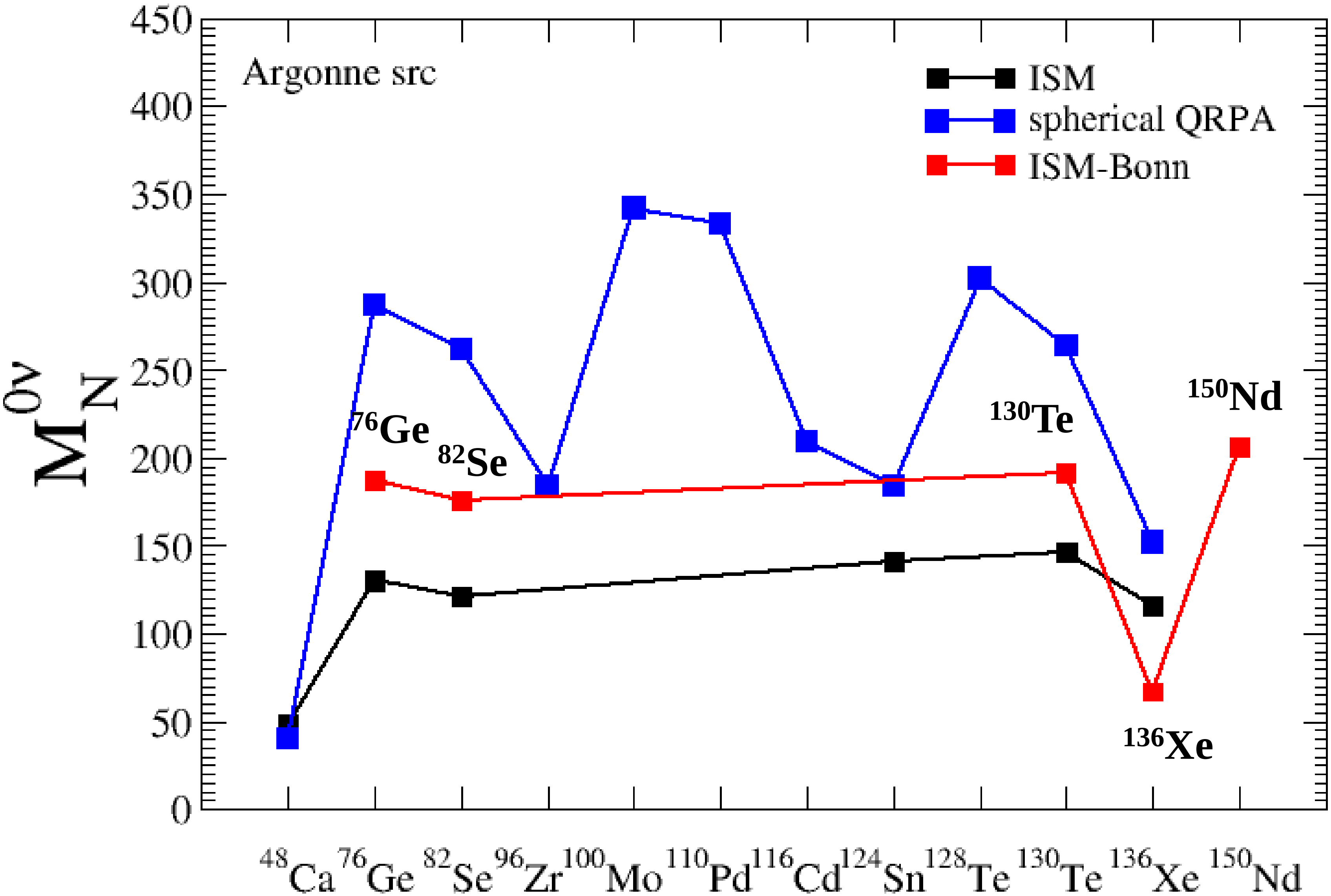
$$\langle p^2\rangle = m_\mathrm{p} m_\mathrm{e} \frac{M_\mathrm{N}^{\prime 0\nu}}{M_\nu^{\prime 0\nu}}$$

**Practically no dependence on A**




**A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., arXiv:1804.04218[hep-ph], accepted in PRD**

# 0νββ-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (heavy neutrino exchange)

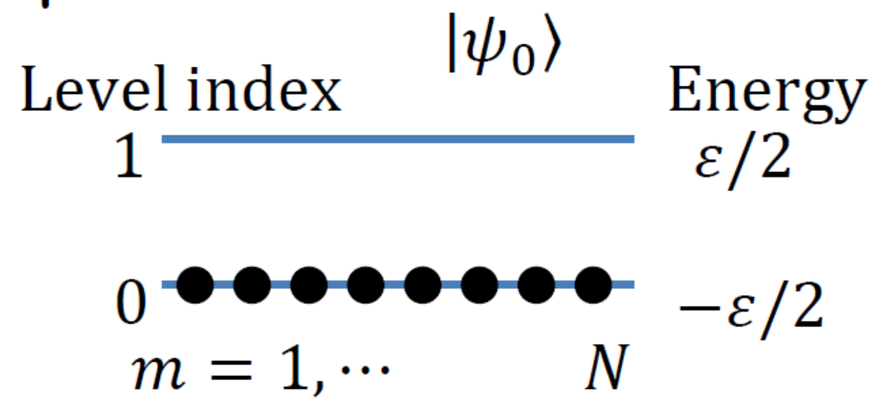


# Reproduction of exact solutions of Lipkin model by nonlinear higher random-phase approximation

J. Terasaki, A. Smetana, F. Š., M.I. Krivoruchenko, Int. J. Mod. Phys. E 26, 1750062 (2017)

Useful for test of theory often used.
H.J. Lipkin et al., N.P. **62**, 188 (1965)

**Lipkin model**




**Hamiltonian**

$$H = \varepsilon J_z + \frac{V}{2}\left(J_+^2 + J_-^2\right)$$

**Algebra**

$$[J_z, J_+] = J_+$$
$$[J_z, J_-] = -J_-$$
$$[J_+, J_-] = 2J_z$$

**The nonlinear phonon operator**

$$Q_k^{o\dagger} = \sum_{l=1}^{n}\left(X_{2l-1}^k \mathcal{J}_+^{2l-1} + Y_{2l-1}^k \mathcal{J}_-^{2l-1}\right),$$
(odd-order subspace)

$$Q_k^{e\dagger} = c_k + \sum_{l=1}^{n}\left(X_{2l}^k \mathcal{J}_+^{2l} + Y_{2l}^k \mathcal{J}_-^{2l}\right),$$
(even-order subspace)

**RPA ground state**

$$Q_k|\Psi_0\rangle = 0$$

**Eigen states, wave functions, total energies, excitation energies and phonon-creation operators obtained for N=2 by the nonlinear higher RPA.**

| Eigenstate | Wave function | Total energy |
|---|---|---|
| Ground | $\|\Psi_0\rangle = \frac{V}{\sqrt{2E^o_{10}(E^o_{10}-\varepsilon)}}\left(1 - \frac{E^o_{10}-\varepsilon}{2V}J^2_+\right)\|\psi_0\rangle$ | $-E^o_{10}$ |
| Odd-order excited | $Q^{o\dagger}_1\|\Psi_0\rangle = \frac{1}{\sqrt{2}}J_+\|\psi_0\rangle$ | 0 |
| Even-order excited | $Q^{e\dagger}_1\|\Psi_0\rangle = \frac{V}{\sqrt{2E^o_{10}(E^o_{10}+\varepsilon)}}\left(1 + \frac{E^o_{10}+\varepsilon}{2V}J^2_+\right)\|\psi_0\rangle$ | $E^o_{10}$ |

| Eigenstate | Excitation energy | Phonon-creation operator |
|---|---|---|
| Ground | 0 | |
| Odd-order excited | $E^o_{10} = \sqrt{\varepsilon^2 + V^2}$ | $Q^{o\dagger}_1 = \frac{\sqrt{E^o_{10}}}{2\varepsilon}\left(\frac{V}{\|V\|}\sqrt{E^o_{10}+\varepsilon}J_+ + \sqrt{E^o_{10}-\varepsilon}J_-\right)$ |
| Even-order excited | $E^e_{10} = 2E^o_{10}$ | $Q^{e\dagger}_1 = \frac{V}{\|V\|}\left(\frac{V}{2\varepsilon} + \frac{E^o_{10}+\varepsilon}{4\varepsilon}J^2_+ + \frac{E^o_{10}-\varepsilon}{4\varepsilon}J^2_-\right)$ |

J. Terasaki, A. Smetana, F. Š., M.I. Krivoruchenko, arXiv:1701.08368 [nucl-th]

***Is there a proportionality between 0νββ- and 2νββ-decay NMEs?***

## *Understanding of the 2νββ-decay NMEs is of crucial importance for correct evaluation of the 0νββ-decay NMEs*

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

***Both 2νββ and 0νββ operators connect the same states. Both change two neutrons into two protons.***

***Explaining 2νββ-decay is necessary but not sufficient***

**There is no reliable calculation of the *2νββ-decay NMEs***

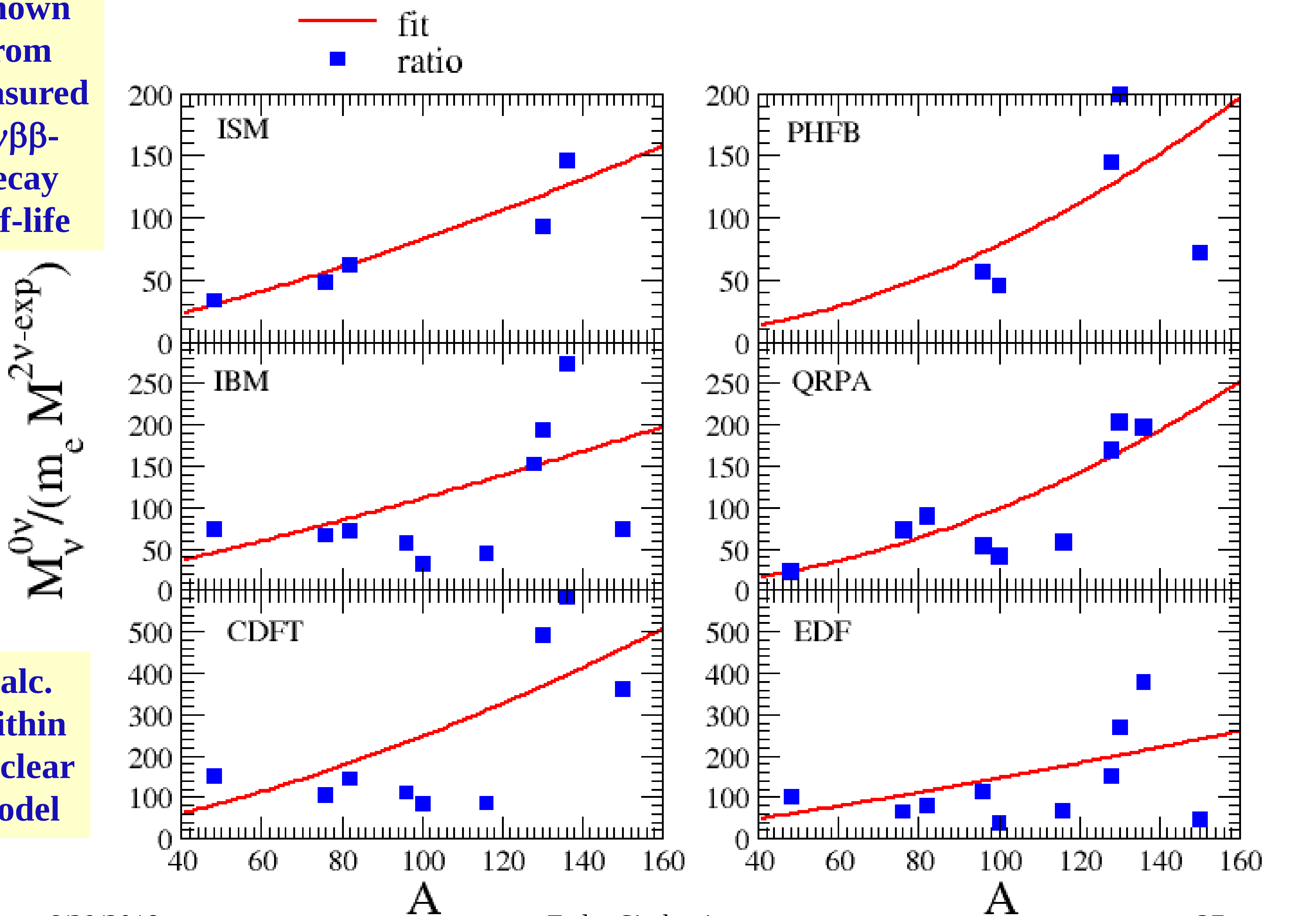
**Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.) ISM (quenching, truncation of model space, spin-orbit partners)**

**Calculation via closure NME: IBM, PHFB**

**No calculation: EDF**

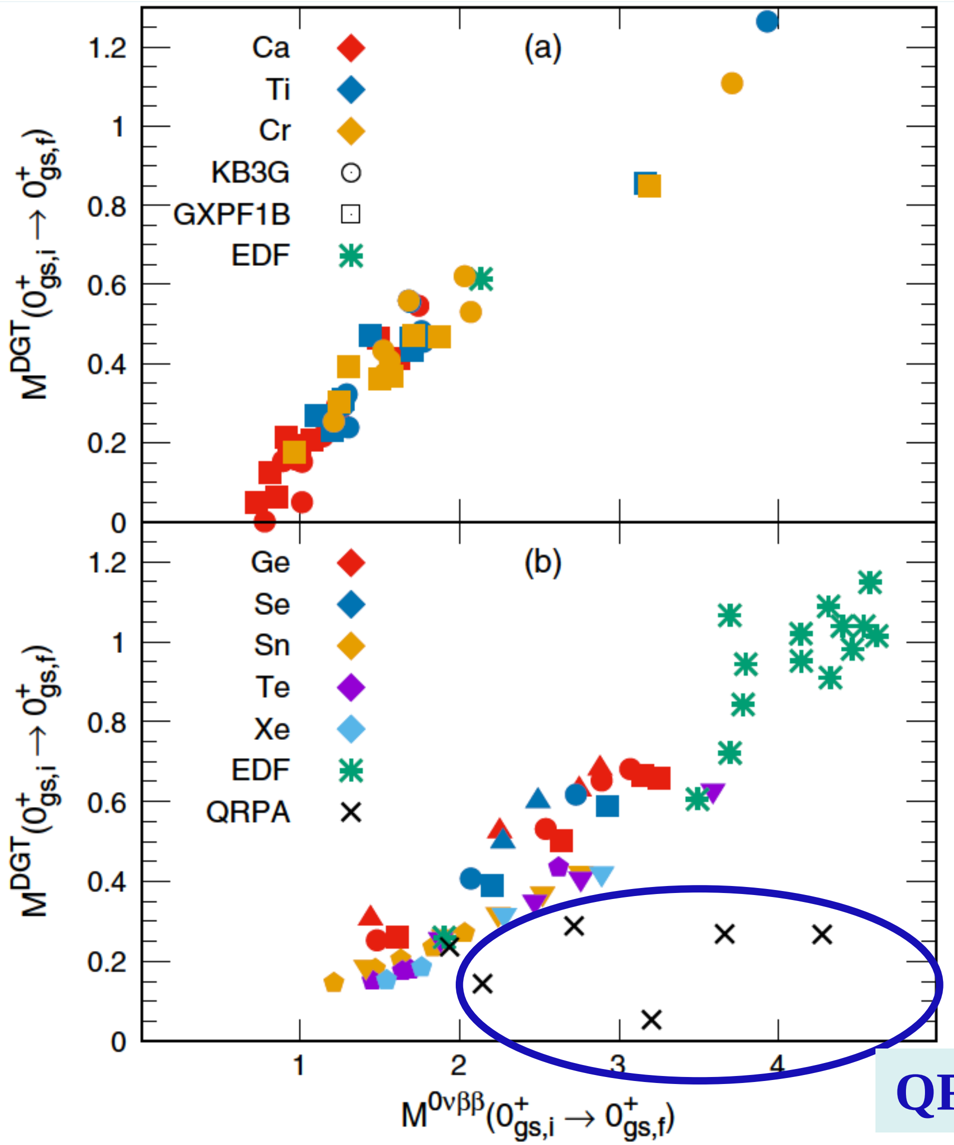
# Is there a proportionality between 0νββ- and 2νββ-decay NMEs?

Known krom measured 2νββ-decay half-life




Calc. within nuclear model

## $M^{0\nu} \propto M^{2\nu}_{GT-cl}$ : ISM, EDF




QRPA?

$M^{DGT}=M^{2\nu}_{GT}$

| | SSD | ChER |
|---|---|---|
| $^{48}Ca$ | | 0.22 |
| $^{76}Ge$ | | 0.52 |
| $^{96}Zr$ | | 0.22 |
| $^{100}Mo$ | 0.35 | |
| $^{116}Cd$ | 0.35 | 0.30 |
| $^{128}Te$ | 0.41 | |

EDF: $0.6 \to 1.2$
ISM: $0.1 \to 0.7$
IBM: $1.6 \to 4.4$
QRPA: $|0.1| \to |0.7|$

**IBM:** J. Barea, J. Kotila, F. Iachello, PRC 91, 034304 (2015)

**QRPA:** F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

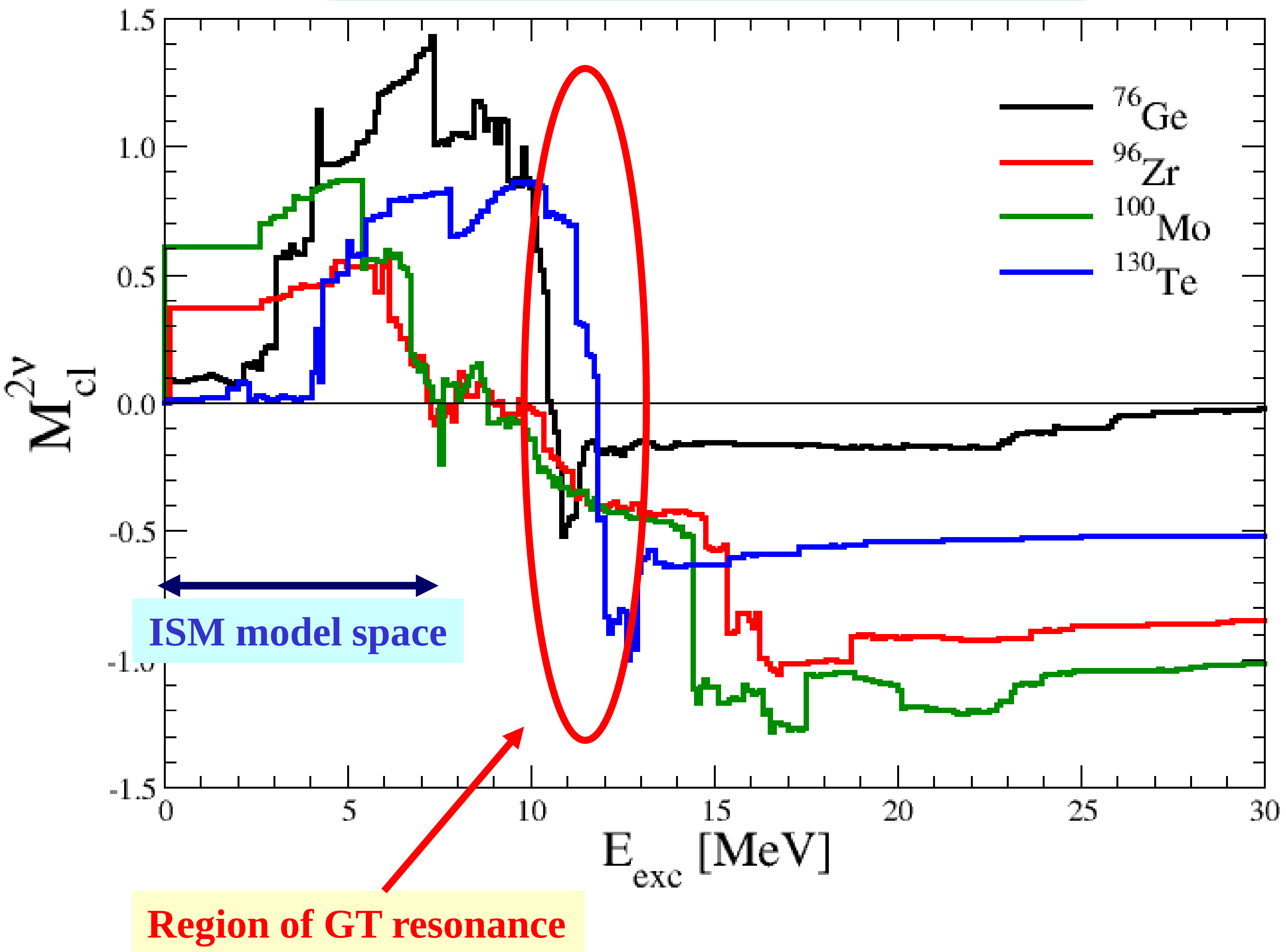
$M^{DGT}$ – only $1^+$
$M^{0\nu}$ - contribution from many $J^\pi$ (!)

**ISM:** N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

Fedor Simkovic

# QRPA: There is no proportionality between 0νββ-decay and 2νββ-decay NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)



## *Going to relative coordinates:*

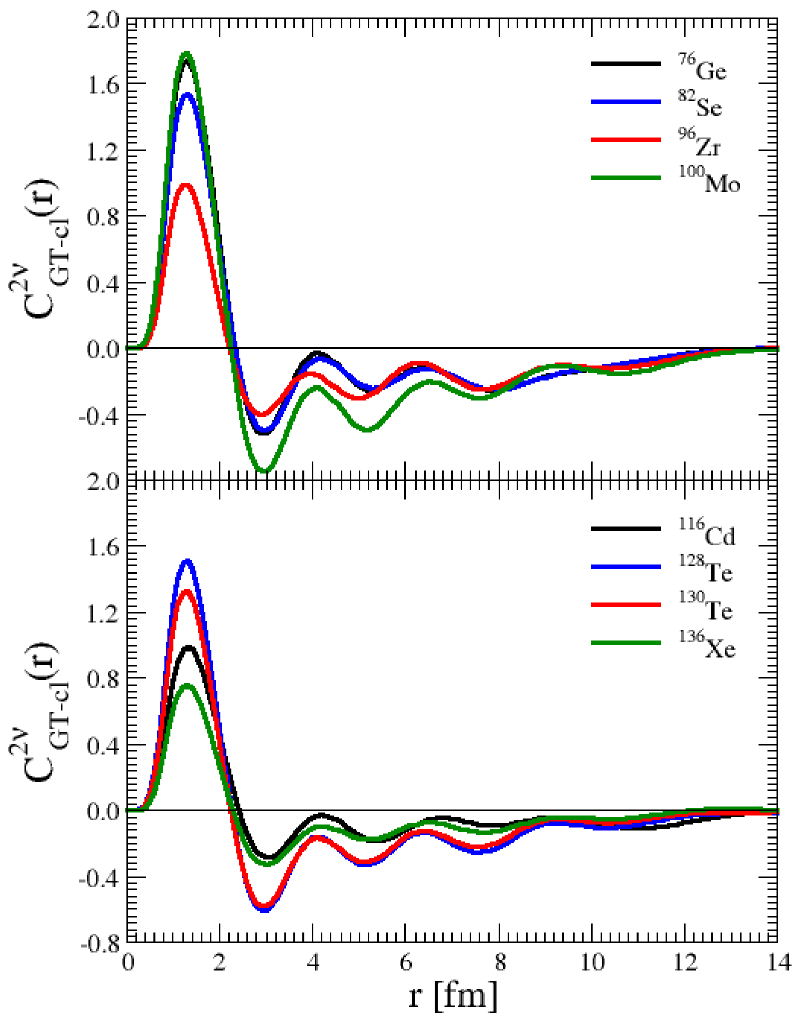
# *A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs*

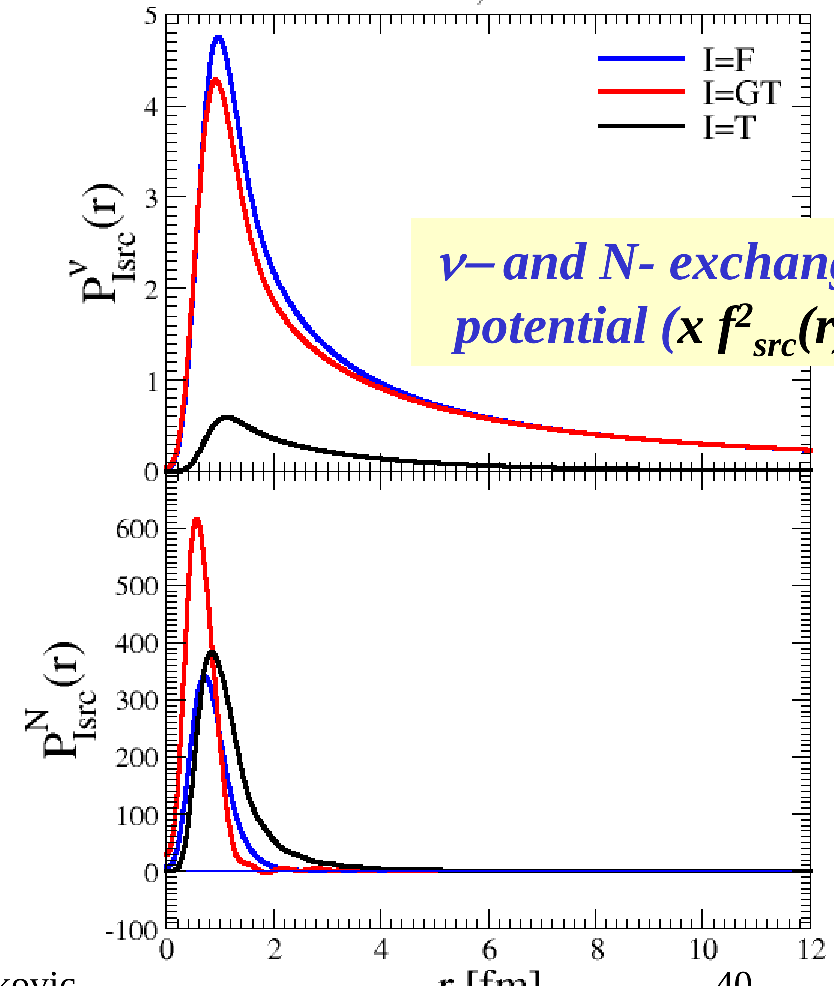
**F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)**

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r) dr$$

$$M^{0\nu}_{\nu,N-I} = \int_0^\infty P^{\nu,N}_{I-src}(r)\; C^{2\nu}_{I-cl}(r)\; dr = \int_0^\infty f^2_{src}(r)\; P^{\nu,N}_{I}(r)\; C^{2\nu}_{I-cl}(r)\; dr$$

$$I = F,\; GT \text{ and } T$$




*$\nu$– and N- exchange potential (x $f^2_{src}(r)$)*

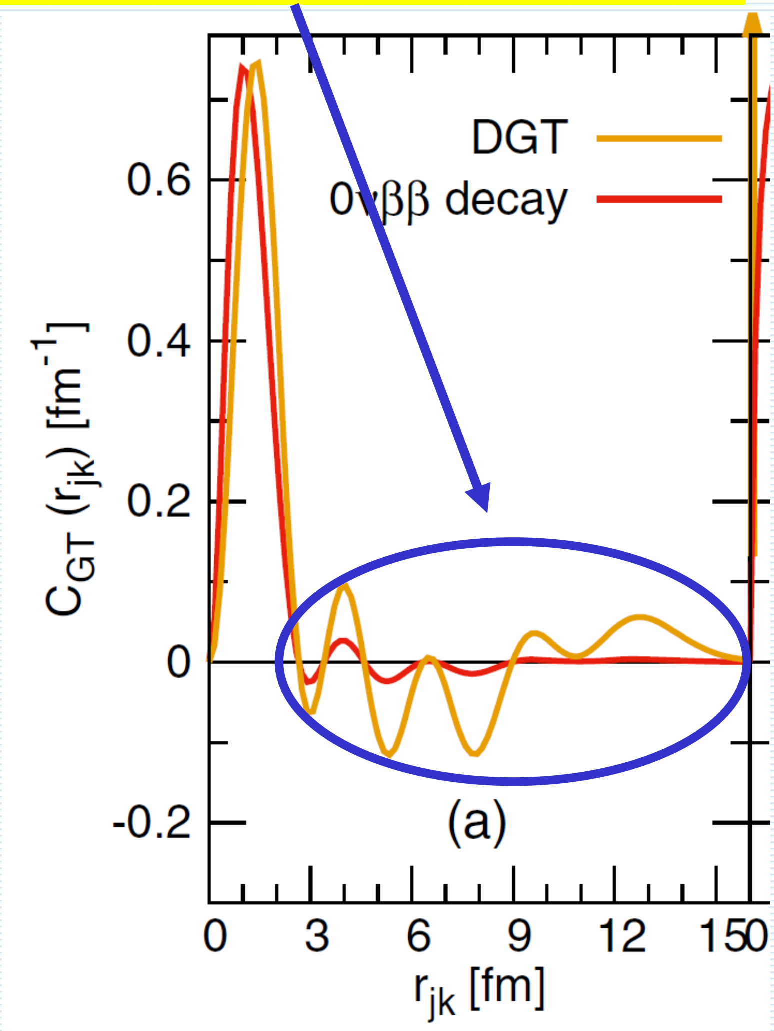
***r**- relative distance of two decaying nucleons*

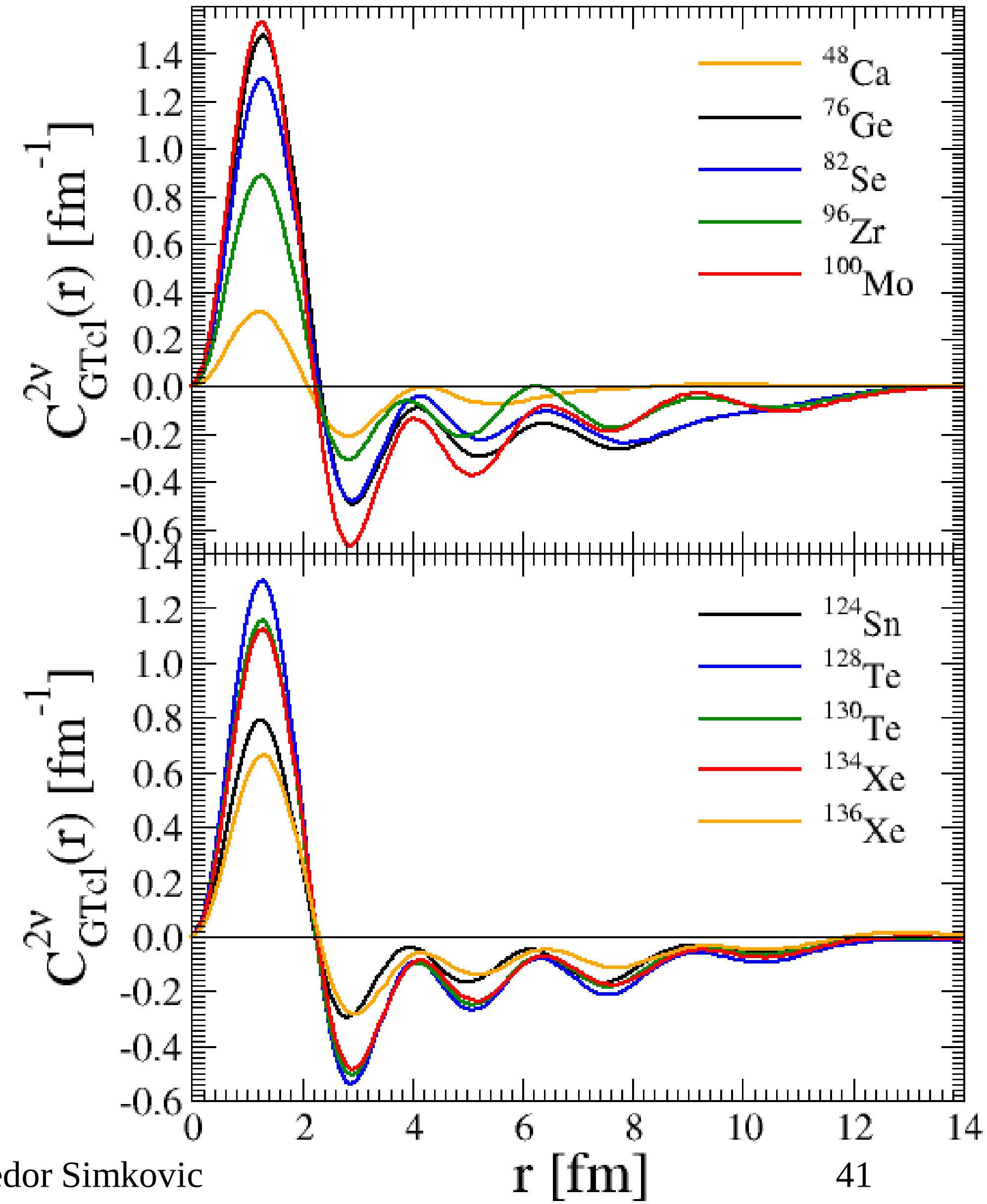
*Neutrino potential prefers short distances*

**QRPA: Bump ≈ - Tail => $M^{2\nu}_{cl} \approx 0$**

**Close to restoration of the SU(4) symmetry of residual Hamiltonian**

**ISM: Tail ≈ 0 (?!) => $M^{2\nu}_{cl}$ » 0**




N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

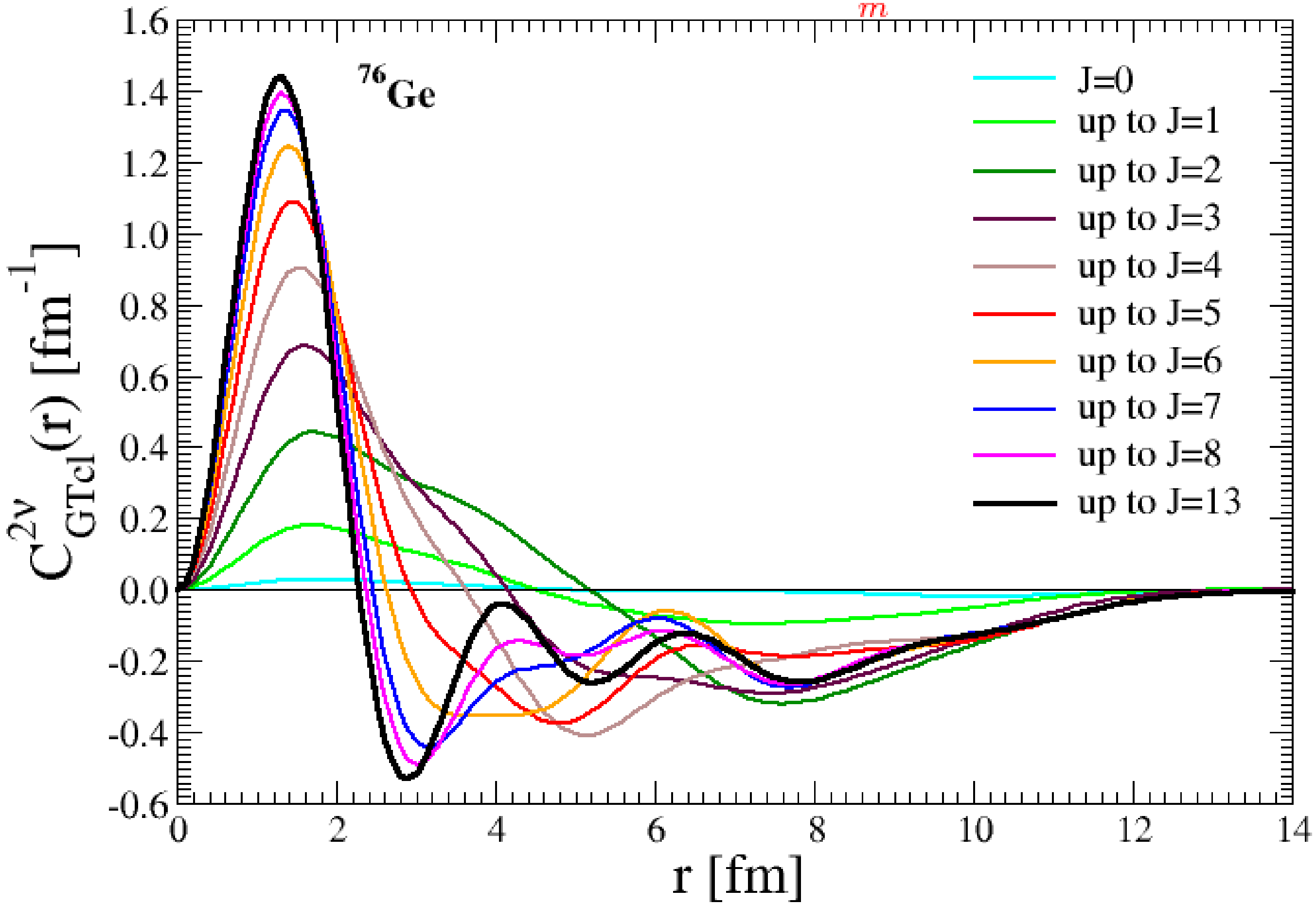
# Closure $2\nu\beta\beta$ GT NME

*The only non-zero contribution from $J^{\pi}=1^{+}$*

$$M^{2\nu}_{GT-cl} = \sum_{J^{\pi}} \int_0^{\infty} C^{2\nu}_{GT-J^{\pi}}(r)dr$$

$$M^{2\nu}_{GT-cl} = \sum_{J^{\pi},m} < 0_f^+|\tau^+\vec{\sigma}|J^{\pi},m > \cdot < J^{\pi},m|\tau^+\vec{\sigma}|0_i^+ >$$

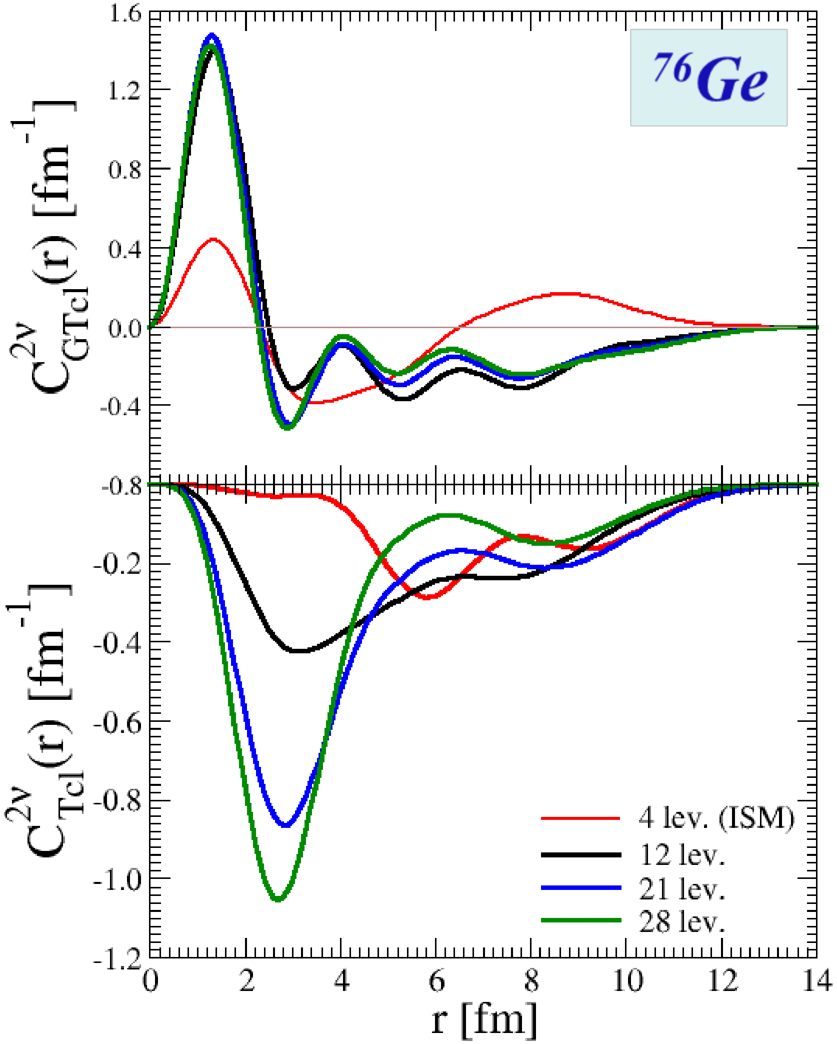
$$= \sum_{m} < 0_f^+|\tau^+\vec{\sigma}|1^+,m > \cdot < 1^+,m|\tau^+\vec{\sigma}|0_i^+ >$$

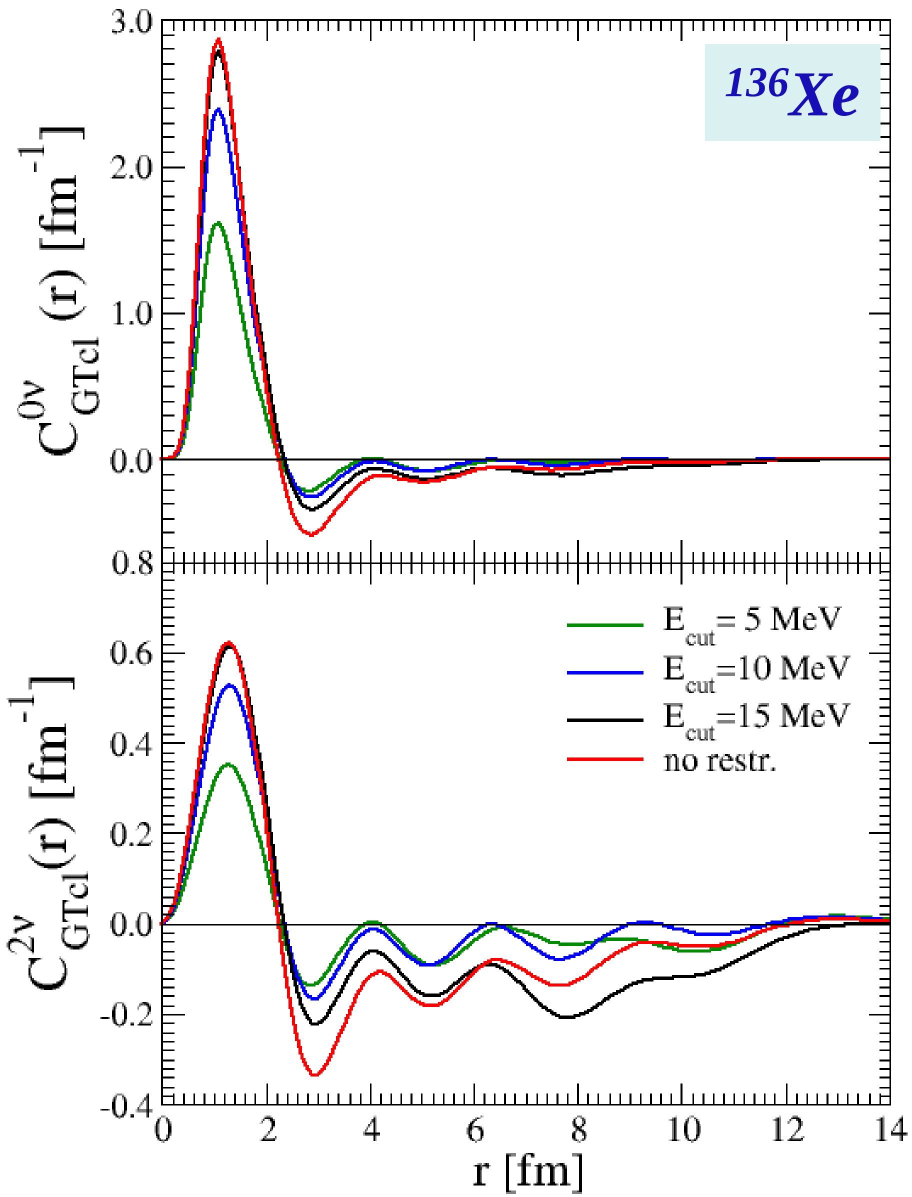



**Many multipole contributions not included within the ISM due to truncation of the model space**

## Contribution from the GT region might be important

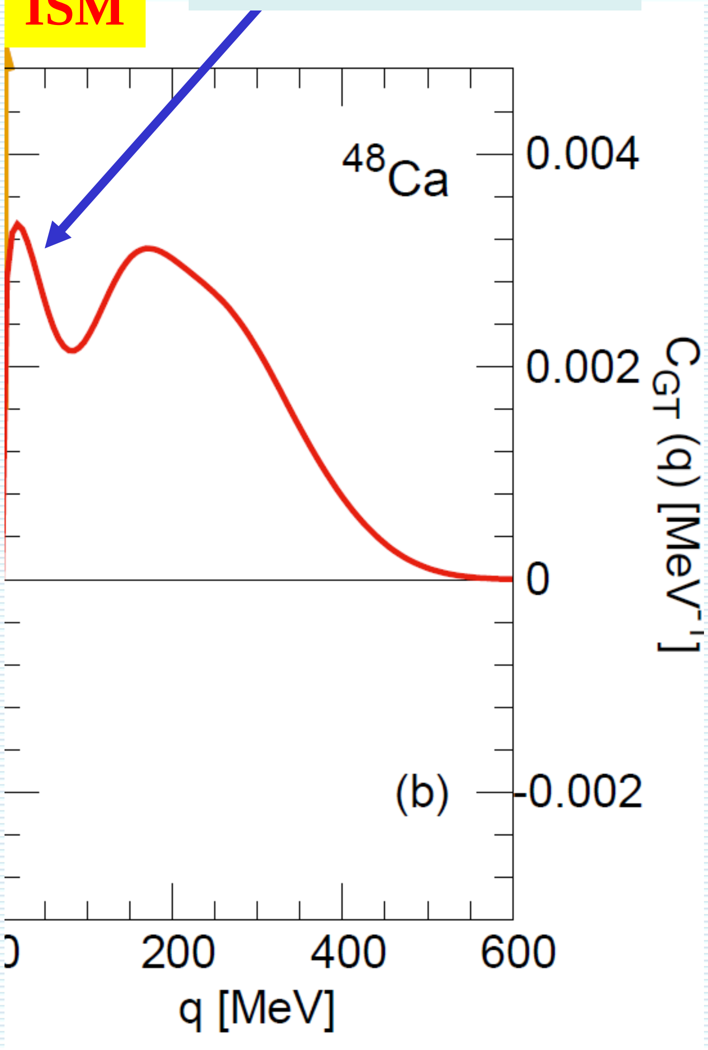
## Size of the model space



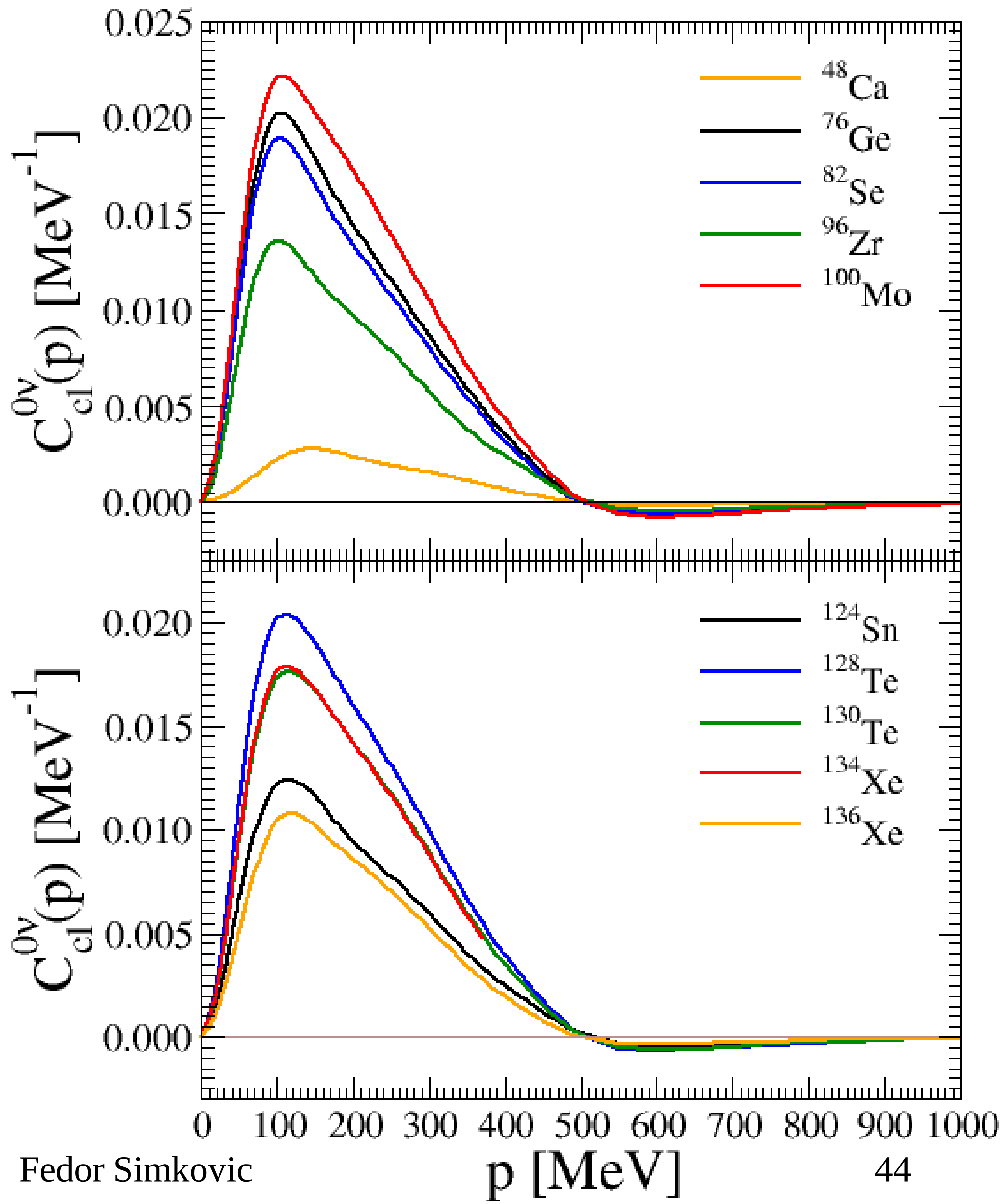
What is the origin of this peak?

ISM




N. Shimizu, J. Menendez, K. Yako, Phys. Rev. Lett. 120, 14502 (2018)

QRPA



$M^{2\nu}_{F\text{-}cl} = 0$

# *The DBD Nuclear Matrix Elements and the SU(4) symmetry*

**D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)**

$M^{2\nu}_{GT\text{-}cl} = 0$

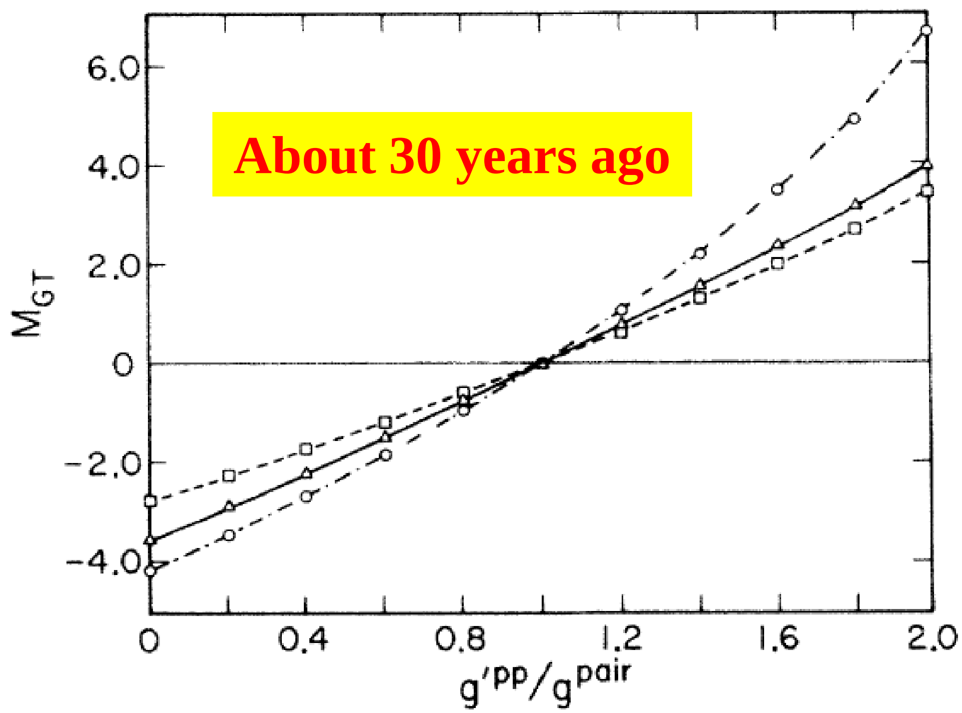
**Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects**

**P. Vogel, M.R. Zirnbauer, PRL (1986) 3148**

**O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11**

**E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53**

…




**The isospin is known to be a good approximation in nuclei**

**In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.**



**What is beyond this behavior? Is it an artifact of the QRPA?**

**s.p. mean-field**

**Conserves SU(4) symmetry**

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^\dagger_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^\dagger_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+ (g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g_{pp}^{T=1})A^\dagger_{0,1}(0,0)A_{0,1}(0,0)\,.}_{H_I}$$

**$H_I$ violates SU(4) symmetry**

**$g_{pair}$- strength of isovector like nucleon pairing (L=0, S=0, T=1, $M_T$=±1)**
**$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, $M_T$=0**
**$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)**
**$g_{ph}$- strength of particle-hole force**

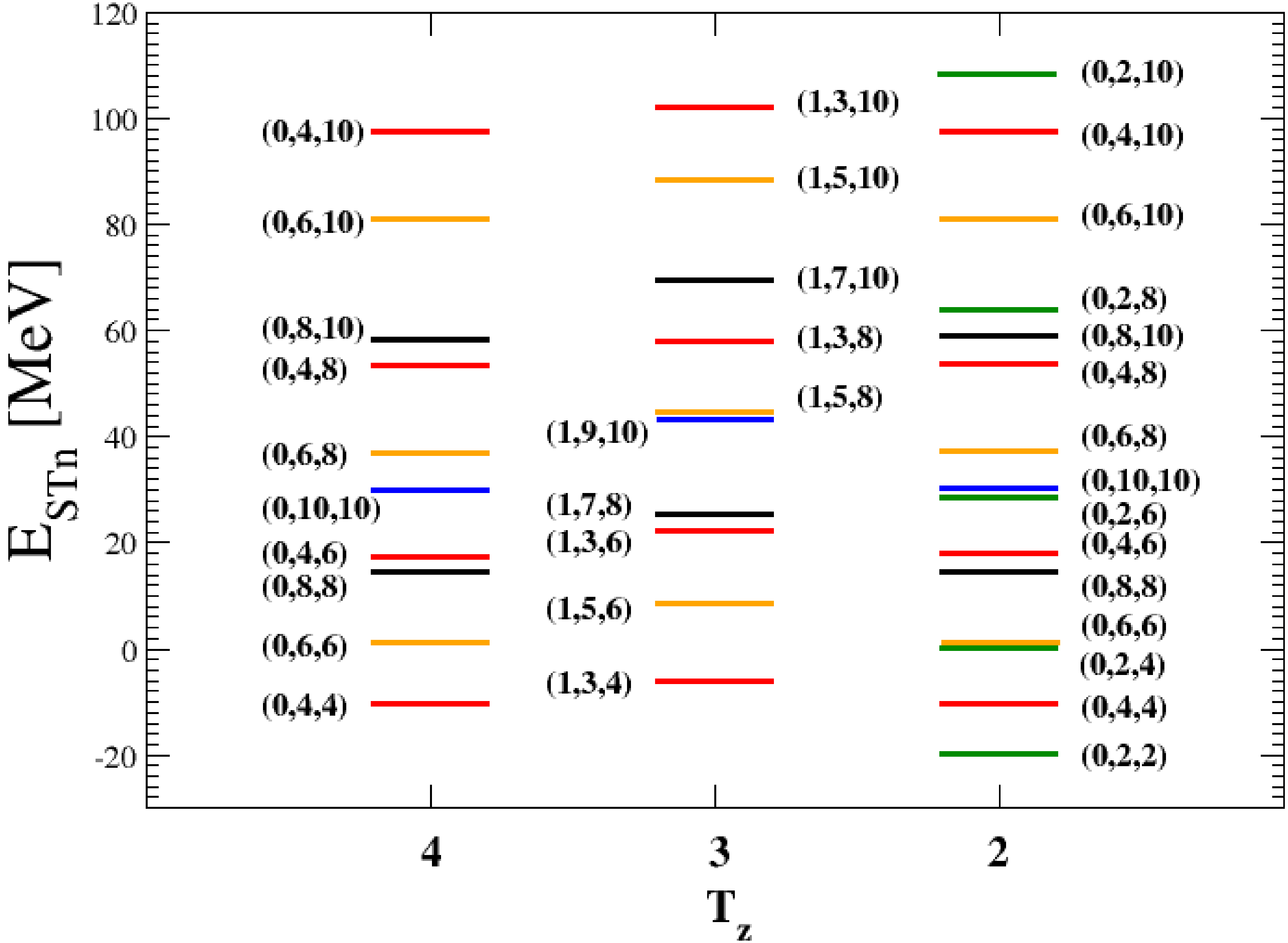
**$M_F$ and $M_{GT}$** do not depend on the mean-field part of **H** and are governed by a weak violation of the **SU(4)** symmetry by the particle-particle interaction of **H**

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}\,(g_{pair} - g_{pp}^{T=1})}{(5g_{pair}+3g_{ph})(10g_{pair}+6g_{ph})}$$

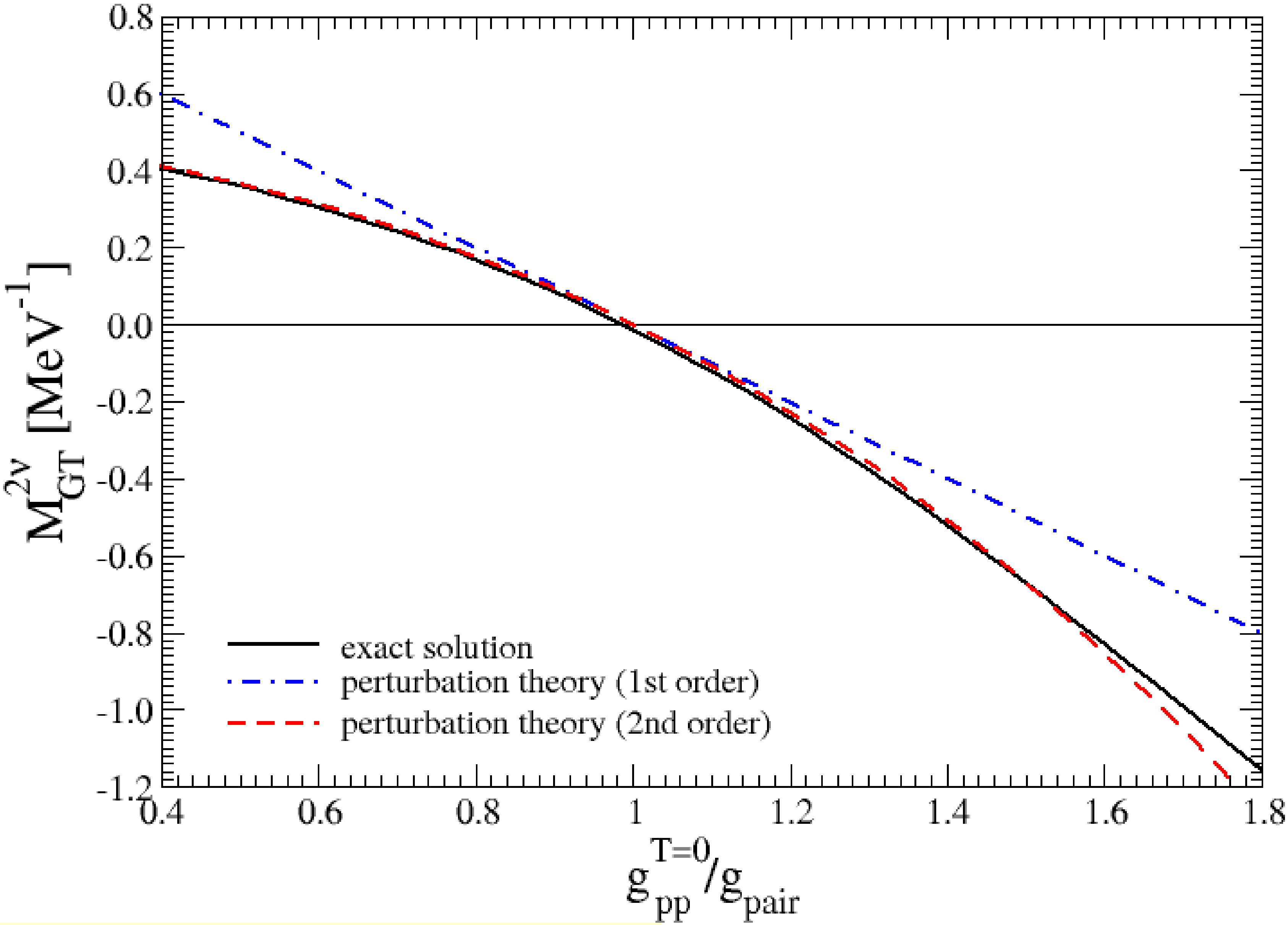
$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair}+9g_{ph}}\left\{\frac{(g_{pair} - g_{pp}^{T=0})}{(10g_{pair}+20g_{ph})} + \frac{2g_{ph}(g_{pair} - g_{pp}^{T=1})}{(10g_{pair}+20g_{ph})(10g_{pair}+6g_{ph})}\right\}$$

## Energies of excited states for the case of conserved SU(4) symmetry

**$M_F$=0, $M_{GT}$=0 (see SU(4) multiplets)**



## $M_{GT}$ up to the second order of perturbation theory due to violation of the SU(4) symmetry by the particle-particle interaction of H




D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

## *Quenching of $g_A$*

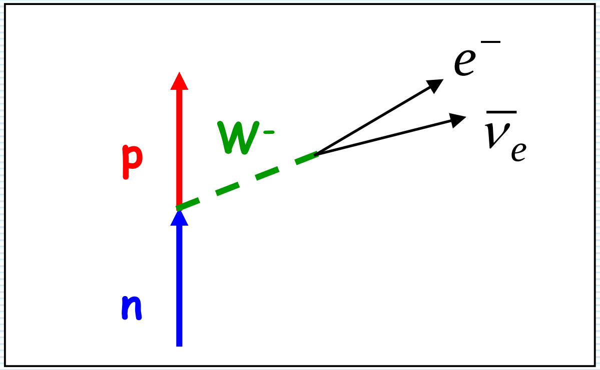
***(A novel method to determine $g_A^{eff}$,***

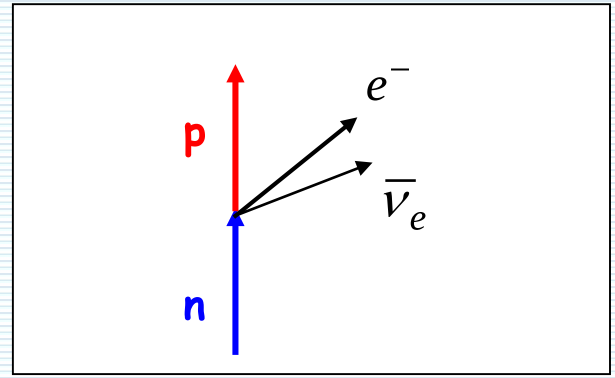
***in details in presentation of R. Dvornický)***

**F.Š., R. Dvornický, D. Štefánik, PRC 97, 034315 (2018)**

# Quenching in nuclear matter: $g^{eff}_A = q\, g^{free}_A$

*(from theory: $T_{1/2}^{0\nu}$ up 50 x larger)*




$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}} \left[\overline{u}\gamma^\alpha(1-\gamma^5)d\right] \left[\overline{e}\gamma^\alpha(1-\gamma^5)\nu_e\right]$$

$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}} \left[\overline{p}\gamma^\alpha(g_V - g_A\gamma^5)n\right] \left[\overline{e}\gamma^\alpha(1-\gamma^5)\nu_e\right]$$

## *CVC hypothesis*

**$g_V$ =1 at the quark level**

**$g_V$ =1 at the nucleon level**

**$g_V$ =1 inside nuclei**

## *Quenching of $g_A$*

**$g_A$ =1 at the quark level**

**$g^{free}_A$ =1.27 at the nucleon level**

**$g^{eff}_A$ = ? inside nuclei**

**ISM:** $(g^{eff}_A)^4 \simeq 0.66$ ($^{48}$Ca), 0.66 ($^{76}$Ge), 0.30 ($^{76}$Se), 0.20 ($^{130}$Te) and 0.11 ($^{136}$Xe)

**IBM:** $(g^{eff}_A)^4 \simeq (1.269\, A^{-0.18})^4 = 0.063$

**QRPA:** $(g^{eff}_A)^4 = 0.30$ and 0.50 for $^{100}$Mo and $^{116}$Cd

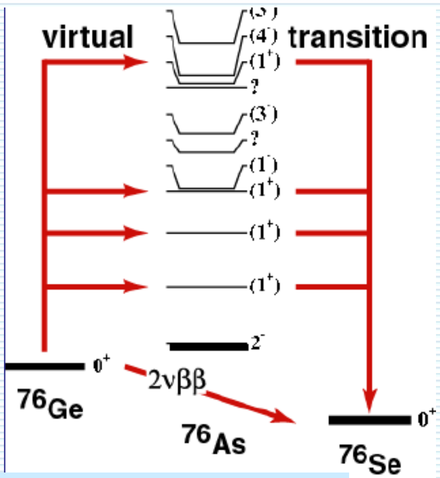
$g_A^4 = (1.269)^4 = 2.6$

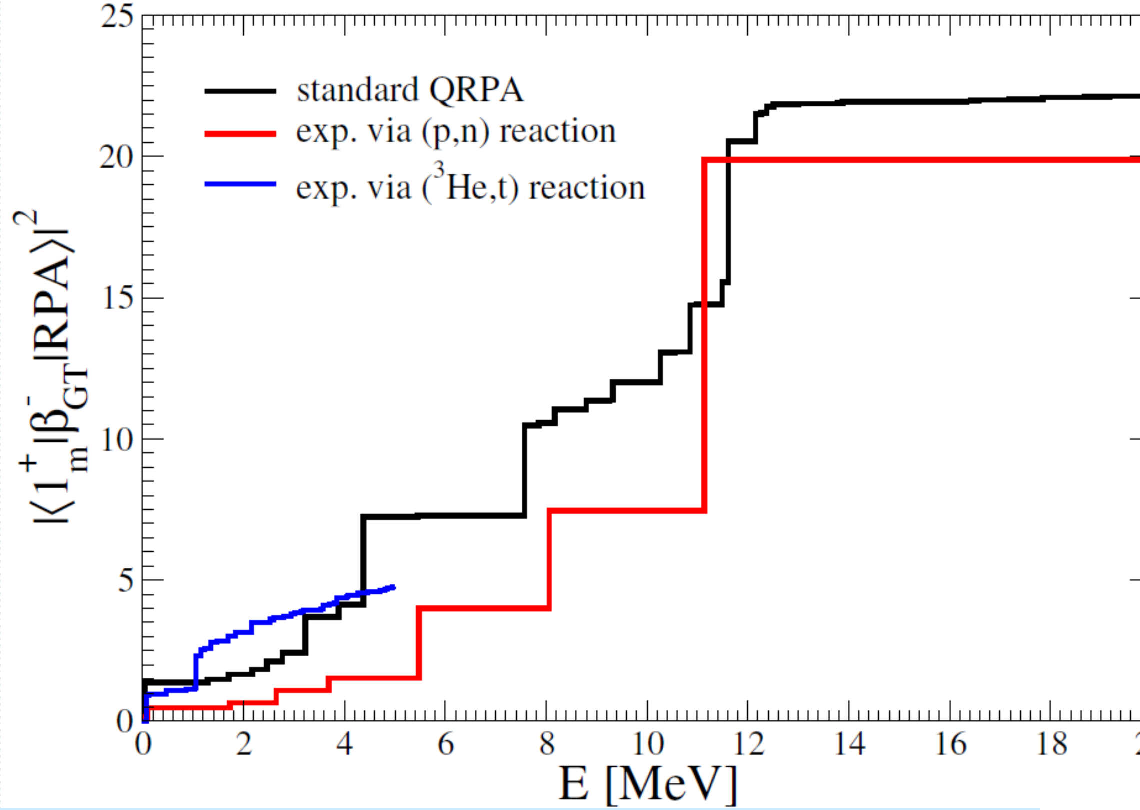
# *Quenching of $g_A$* *(from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)*

$(g^{eff}_A)^4 = 1.0$

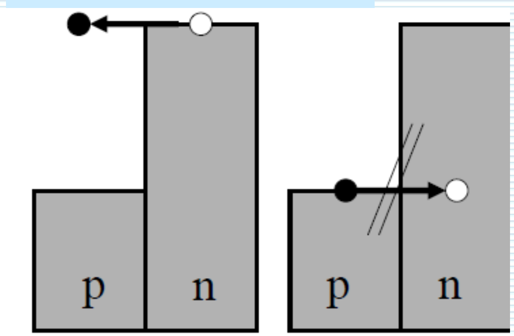
**Strength of GT trans. (approx. given by Ikeda sum rule =3(N-Z)) has to be quenched to reproduce experiment**

$${}^{76}_{32}\mathrm{Ge}_{44} \Rightarrow$$
$$S_{\beta^-} - S_{\beta^+} = 3(N-Z) = 36$$




**Pauli blocking**




**Cross-section for charge exchange reaction:**

$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} N_d \, |V_{\sigma\tau}|^2 \, |\langle f \,|\, \sigma\tau \,|\, i \rangle|^2$$

$$q = 0!!$$

largest at 100 - 200 MeV/A

# Improved description of the 0νββ–decay rate

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

**Let perform Taylor expansion**

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2} \qquad \epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

**We get**

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\rm eff}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{\left|\xi_{13}^{2\nu}\right|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

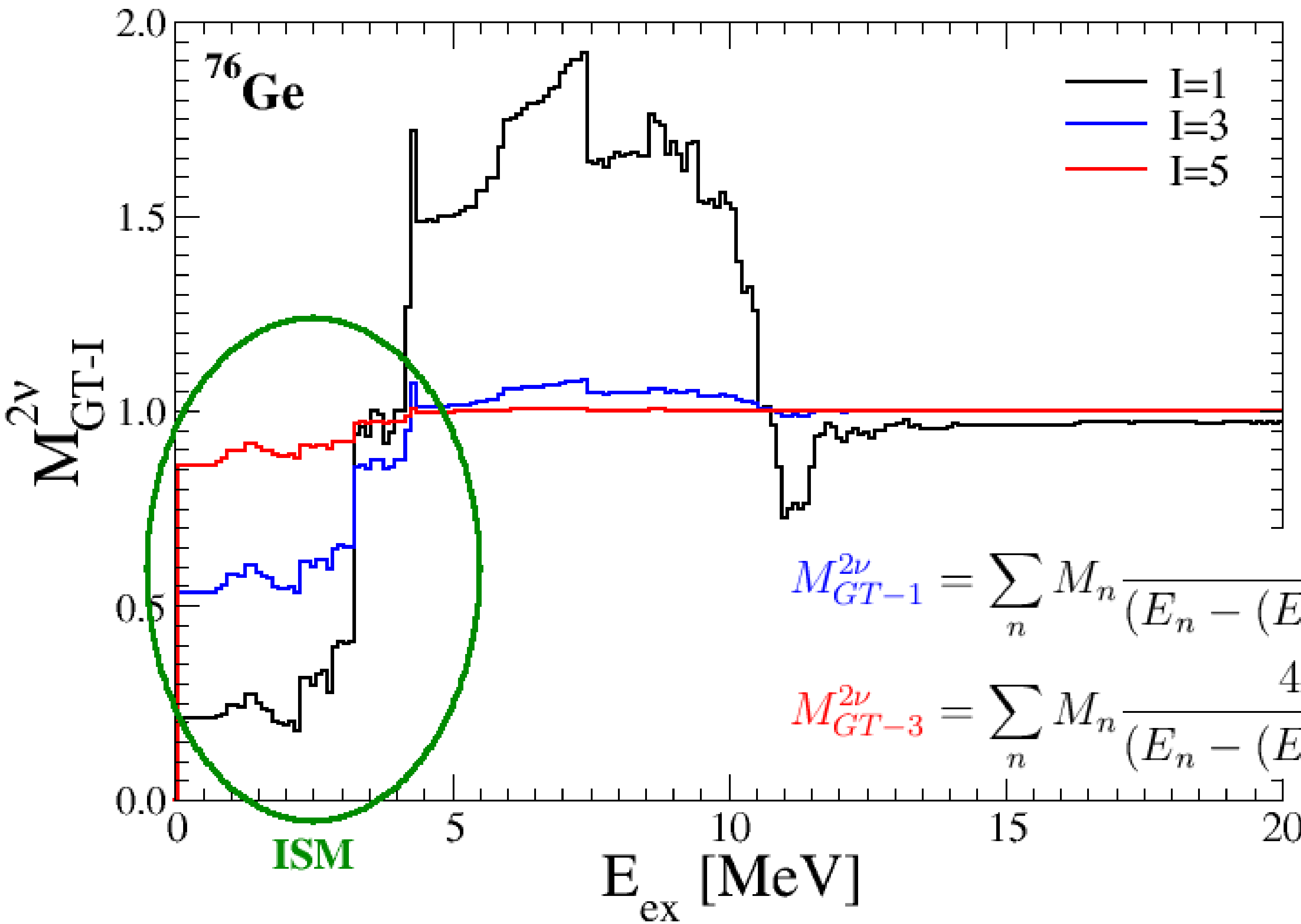
$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4\, m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

$$\xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

**The $g_A^{eff}$ can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)**

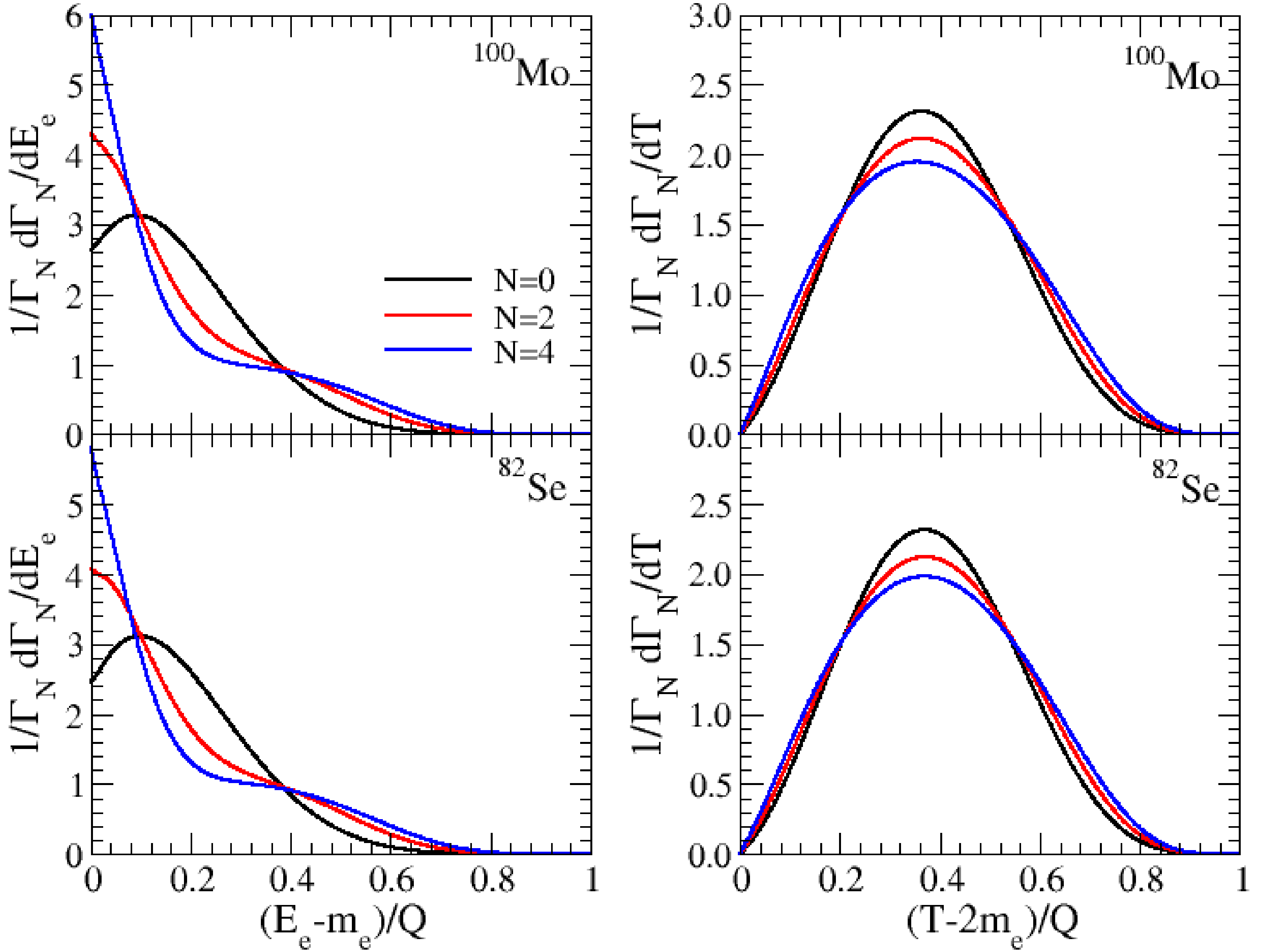
# The running sum of the 2νββ–decay NMEs



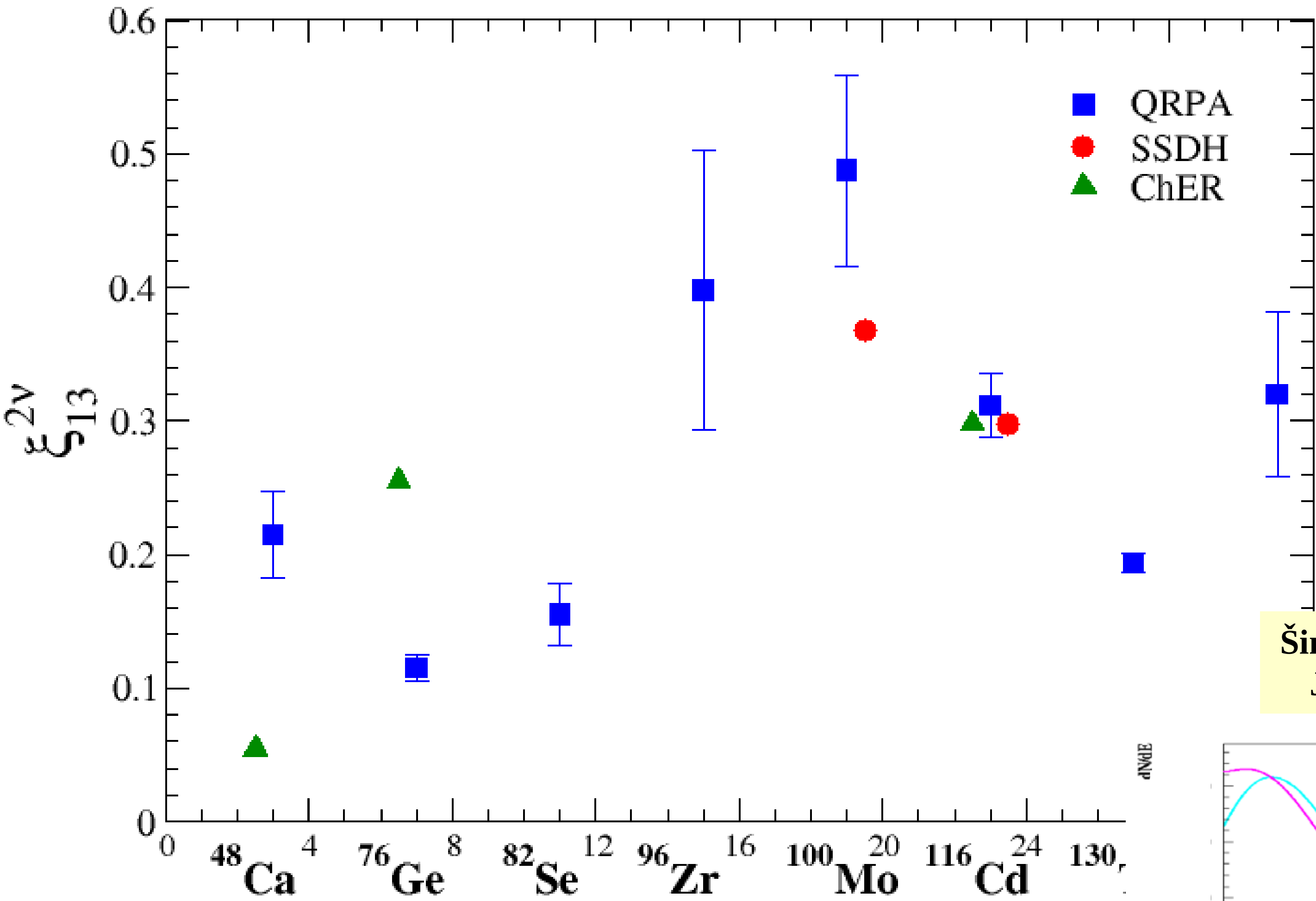

$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

# Normalized to unity different partial energy distributions

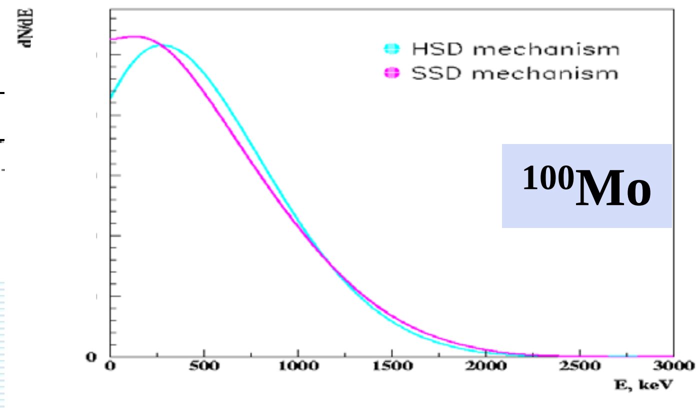


# $\xi_{13}$ tell us about importance of higher lying states of int. nucl.




HSD: $\xi_{13}$=0

**Šimkovic, Šmotlák, Semenov**
**J. Phys. G, 27, 2233, 2001**




$^{100}$Mo

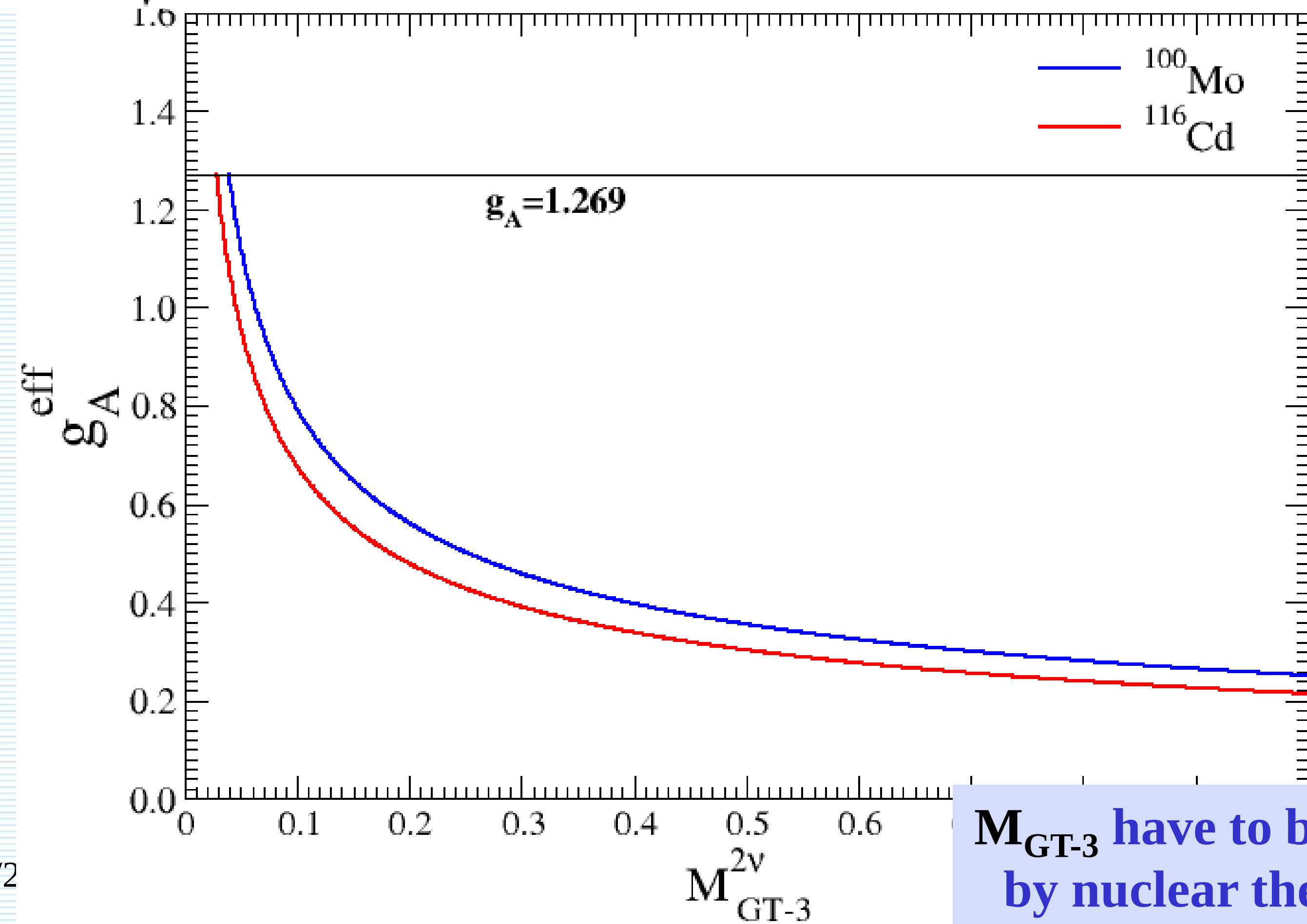
$\xi_{13}$ can be determined phenomenologically from the shape of energy distributions of emitted electrons

## Solution: NEMO3/Supernemo measurement of $\xi$ and calculation of $M_{GT-3}$

$$\left(g_A^{\mathrm{eff}}\right)^2 = \frac{1}{\left|M_{GT-3}^{2\nu}\right|}\frac{\left|\xi_{13}^{2\nu}\right|}{\sqrt{T_{1/2}^{2\nu-exp}\left(G_0^{2\nu}+\xi_{13}^{2\nu}G_2^{2\nu}\right)}}$$

$$g_A^{\mathrm{eff}}({}^{100}\mathrm{Mo}) = \frac{0.251}{\sqrt{M_{GT-3}^{2\nu}}}$$

$$g_A^{\mathrm{eff}}({}^{100}\mathrm{Cd}) = \frac{0.214}{\sqrt{M_{GT-3}^{2\nu}}}$$




$M_{GT-3}$ **have to by calculated by nuclear theory - ISM**

## ***New modes of the double beta decay***

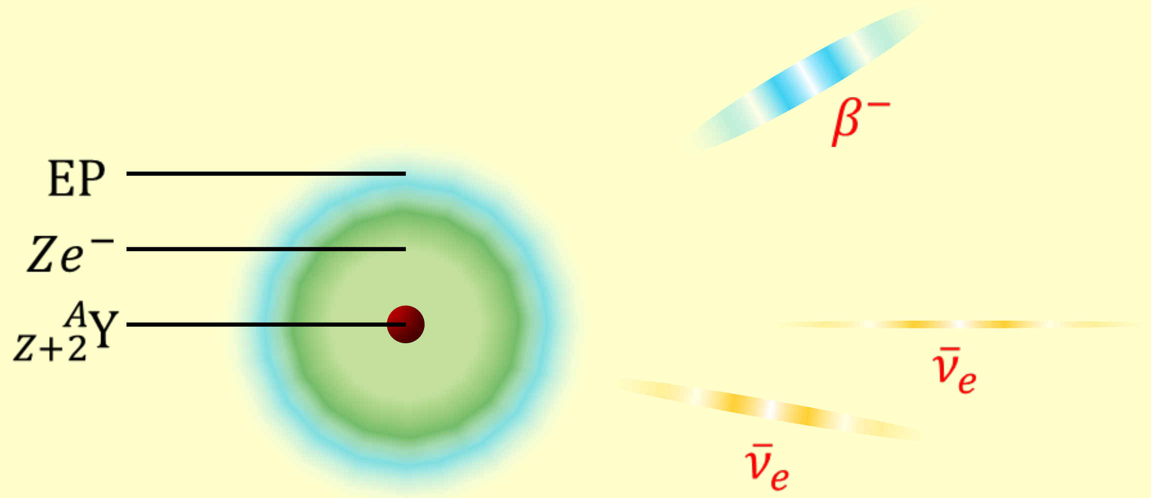
# Double Beta Decay with emission of a single electron

A. Babič, M.I. Krivoruchenko, F.Š., arXiv:1805.07815 [hep-ph]

**[Jung *et al.* (GSI), 1992] observed beta decay of $^{163}_{66}\mathrm{Dy}^{66+}$ ions with Electron Production (EP) in K or L shells:** $T^{\mathrm{EP}}_{1/2} = 47$ d

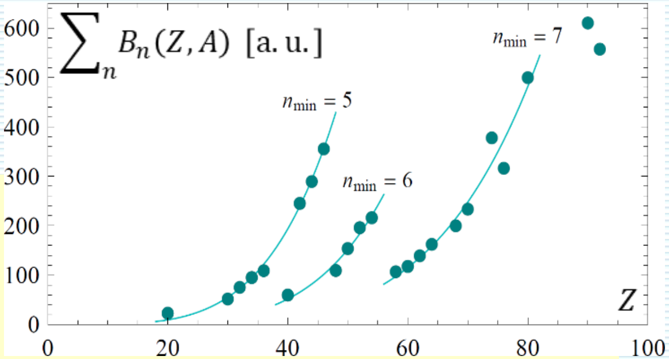
**Bound-state double-beta decay** $0\nu\mathrm{EP}\beta^-$ $(2\nu\mathrm{EP}\beta^-)$ **with EP in available** $s_{1/2}$ **or** $p_{1/2}$ **subshell of daughter 2+ ion:**

$$^{A}_{Z}\mathrm{X} \longrightarrow {}^{\;\;A}_{Z+2}\mathrm{Y} + e^-_{\mathrm{b}} + e^- + (\bar{\nu}_e + \bar{\nu}_e)$$




**Search for possible manifestation in single-electron spectra…**

# Phase space factors




## $0\nu EP\beta^-$ and $2\nu EP\beta^-$ phase-space factors:

$$G^{0\nu\mathrm{EP}\beta}(Z,Q) = \frac{G_\beta^4\, m_e^2}{32\pi^4 R^2 \ln 2} \sum_{n=n_{\min}}^{\infty} B_n(Z,A)\, F(Z+2,E)\, E\, p$$

$$G^{2\nu\mathrm{EP}\beta}(Z,Q) = \frac{G_\beta^4}{8\pi^6 m_e^2 \ln 2} \sum_{n=n_{\min}}^{\infty} B_n(Z,A) \int_{m_e}^{m_e+Q} \mathrm{d}E\, F(Z+2,E)\, E\, p \int_0^{m_e+Q-E} \mathrm{d}\omega_1\, \omega_1^2\, \omega_2^2$$

## Single-electron spectra for $^{82}Se$ ($Q = 2.998\ MeV$):

**Bound- and free-electron Fermi functions:**

$$B_n(Z,A) = f_{n,-1}^2(R) + g_{n,+1}^2(R)$$
$$F(Z,E) = f_{-1}^2(R,E) + g_{+1}^2(R,E)$$

**Relativistic electron wave functions in central field:**

$$\psi_{\kappa\mu}(\vec{r}) = \begin{pmatrix} f_\kappa(r)\,\Omega_{\kappa\mu}(\hat{r}) \\ i g_\kappa(r)\,\Omega_{-\kappa\mu}(\hat{r}) \end{pmatrix}$$

$$\kappa = (l-j)(2j+1) = \pm 1, \pm 2, \ldots$$
$$j = |l \pm 1/2| \qquad \mu = -j, \ldots, +j$$

# CALCULATION: GRASP2K

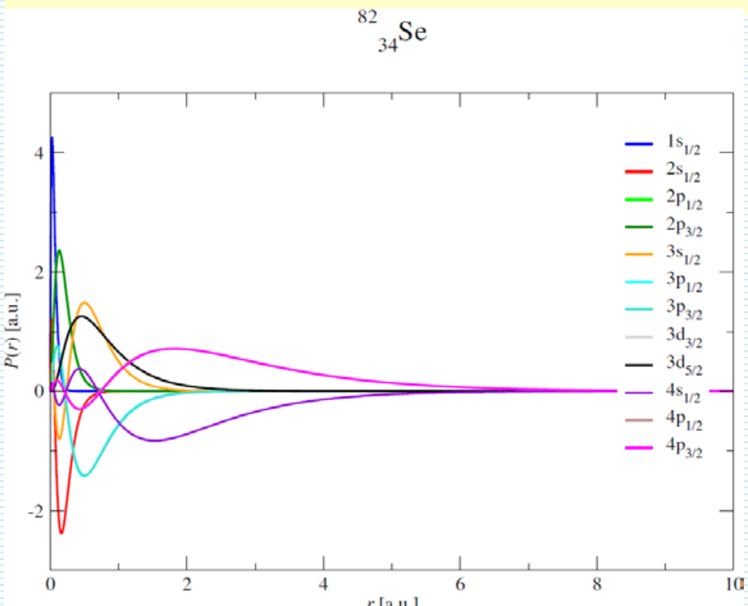
**Stationary $N$-particle Dirac eq. with separable central atomic Hamiltonian $[a.u.]$:**

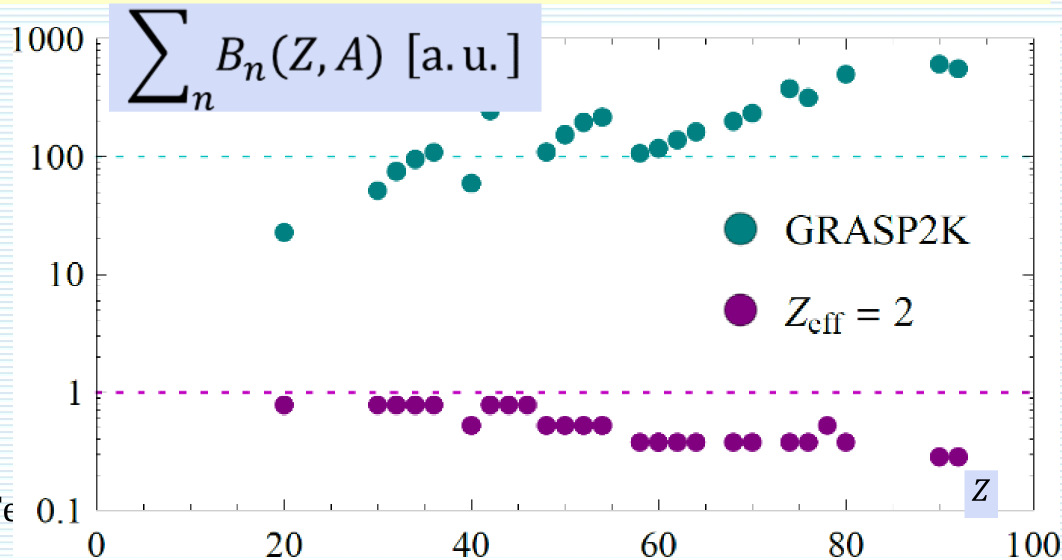
$$\left[\sum_{i=1}^{N} -i\nabla_i \cdot \vec{\alpha} c + \beta c^2 - \frac{Z}{r_i} + V(r_i)\right]\Psi = E\,\Psi$$

$$\Psi = \frac{1}{\sqrt{N!}}\begin{vmatrix} \psi_1(\vec{r}_1) & \cdots & \psi_1(\vec{r}_N) \\ \vdots & \ddots & \vdots \\ \psi_N(\vec{r}_1) & \cdots & \psi_N(\vec{r}_N) \end{vmatrix}$$

**Multiconfiguration Dirac–Hartree–Fock package GRASP2K:**

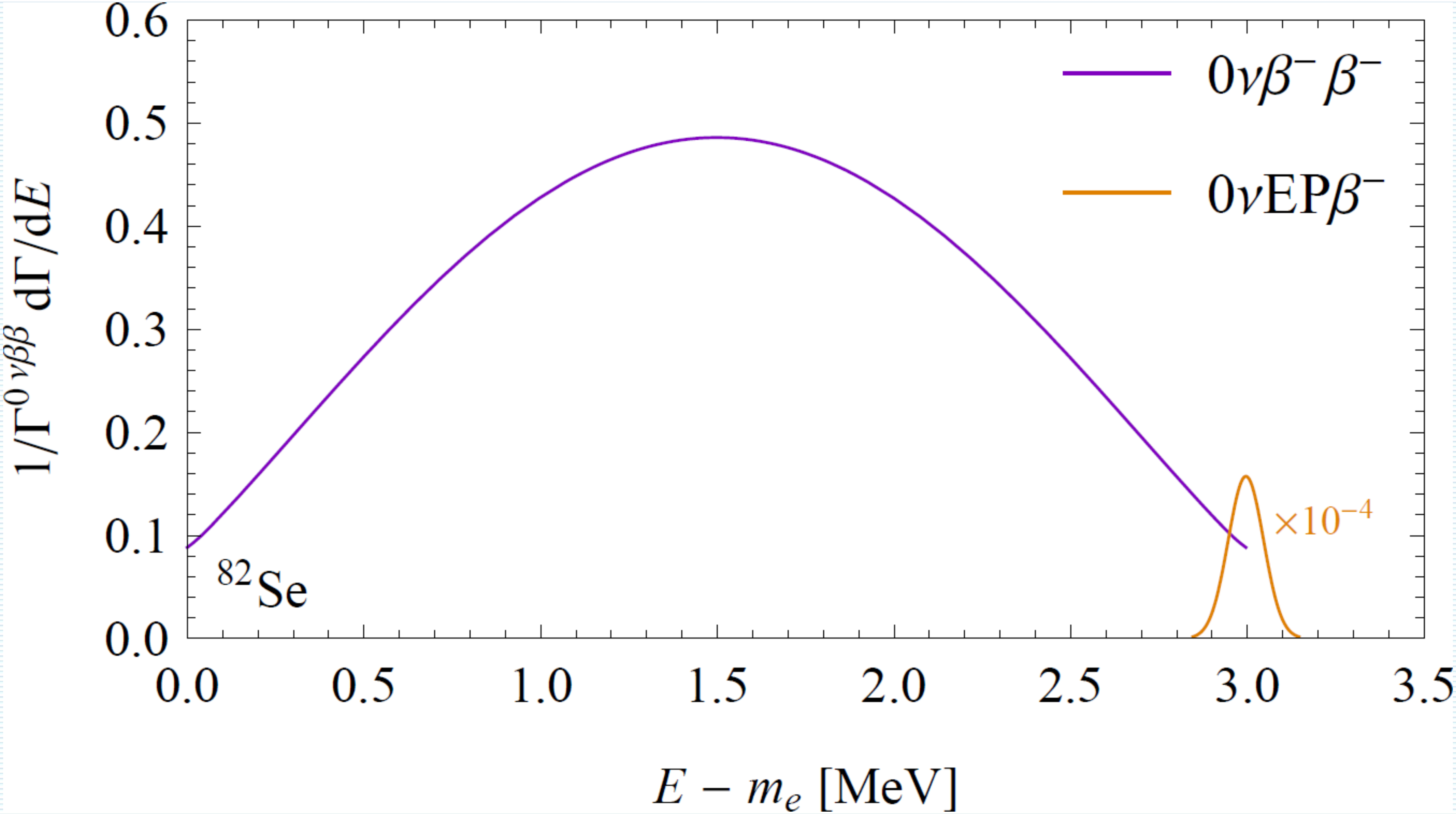
- **Fit of non-convergent orbitals:** $f_{n,-1}^2, g_{n,+1}^2(R) \approx aZ^b$
- **Fit of orbitals beyond $n = 9$:** $f_{n,-1}^2, g_{n,+1}^2(R) \approx cn^d$



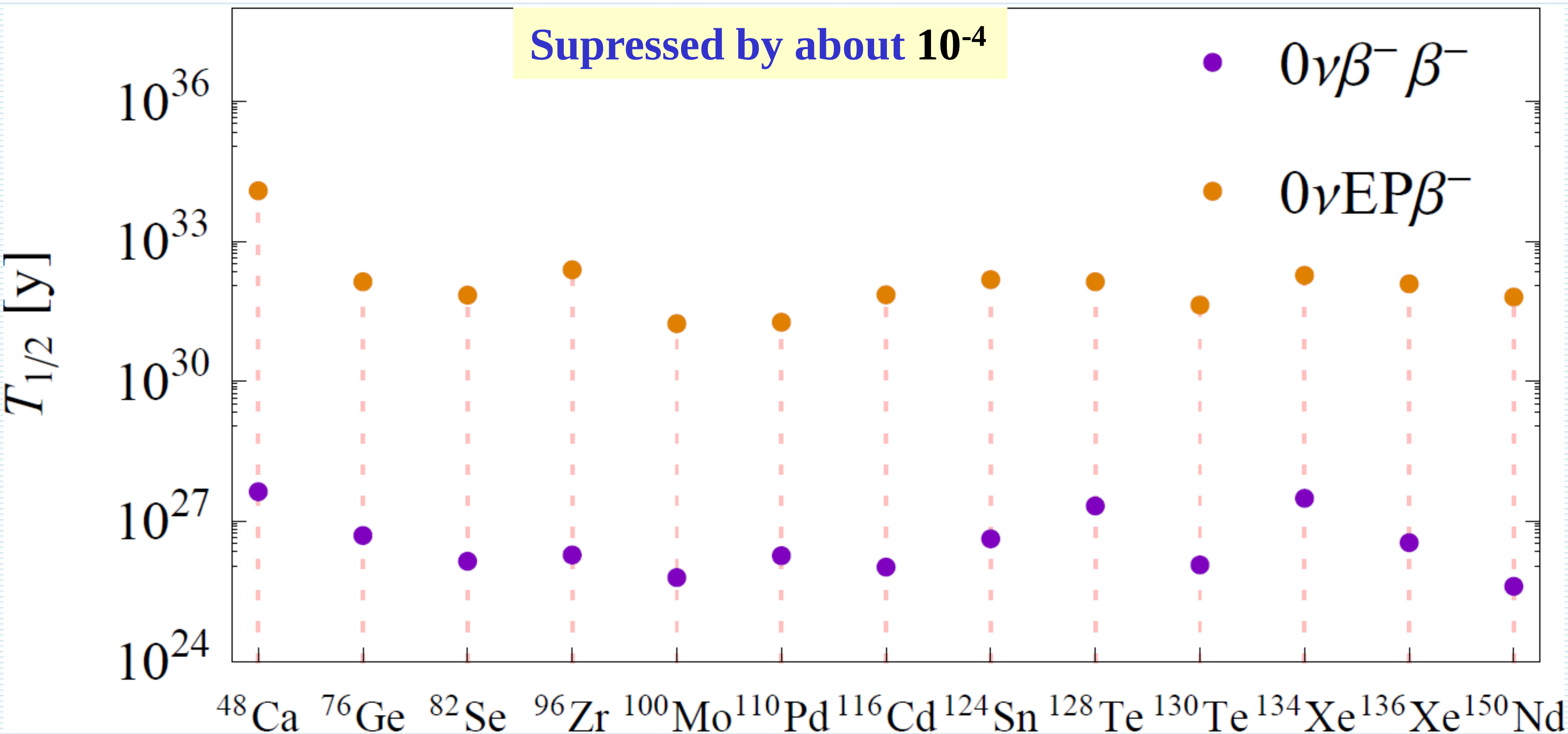
# $0\nu EP\beta^-$ Single-Electron Spectrum ($^{82}$Se)

$0\nu\beta^-\beta^-$ **and** $0\nu EP\beta^-$ **single-electron spectra** $1/\Gamma^{0\nu\beta\beta}\, d\Gamma/dE$ ***vs.*** **electron kinetic energy** $E - m_e$ **for** $^{82}$Se ($Q = 2.996$ MeV)



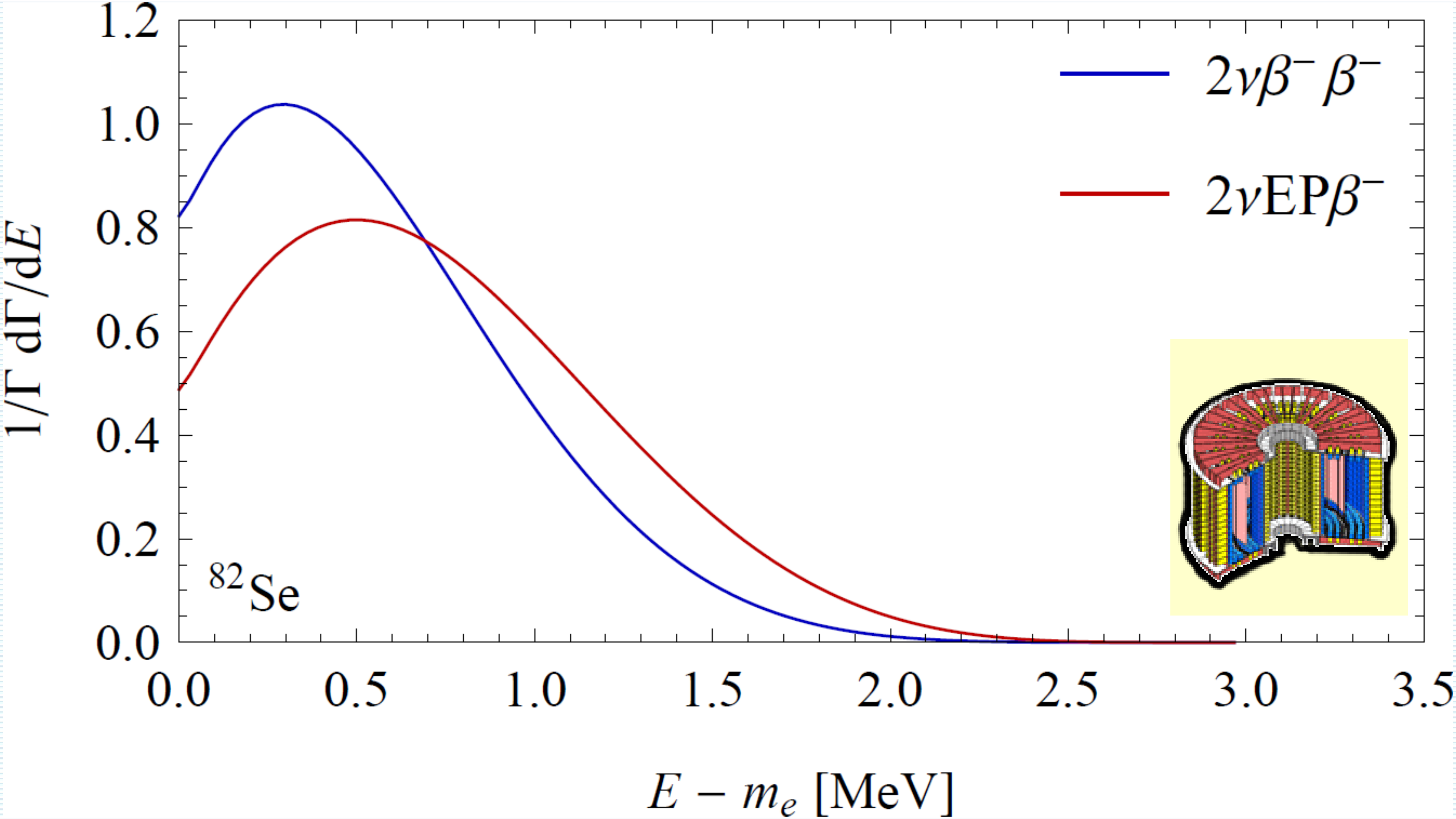
# $0\nu\text{EP}\beta^-$ Half-Lives

$0\nu\beta^-\beta^-$ and $0\nu\text{EP}\beta^-$ half-lives $T_{1/2}^{0\nu\beta\beta}$ and $T_{1/2}^{0\nu\text{EP}\beta}$ estimated for $\beta^-\beta^-$ isotopes with known NME $|M^{0\nu\beta\beta}|$, assuming unquenched $g_A = 1.269$ and $|m_{\beta\beta}| = 50\ \text{meV}$



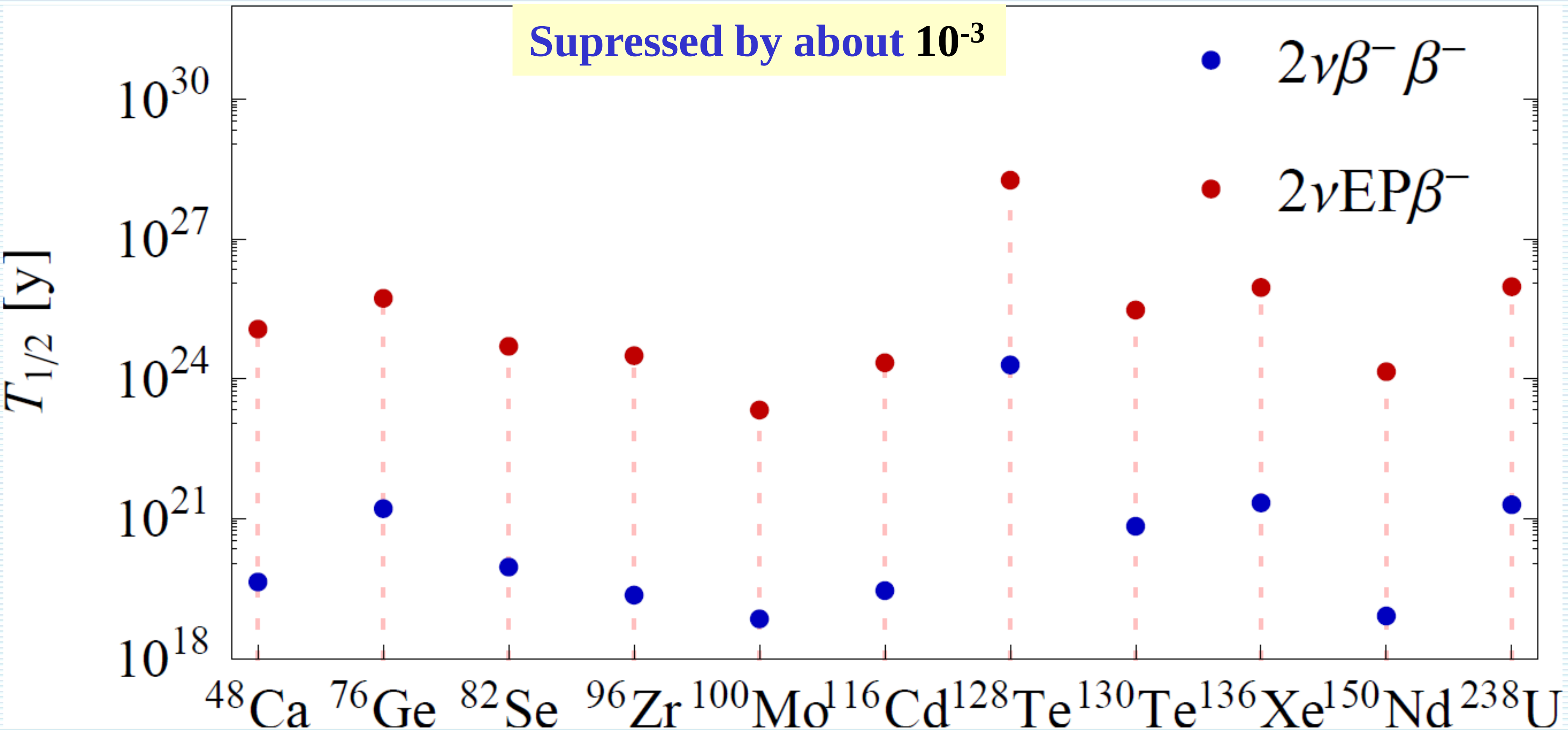
# $2\nu\mathrm{EP}\beta^-$ Single-Electron Spectrum ($^{82}$Se)

$2\nu\beta^-\beta^-$ **and** $2\nu\mathrm{EP}\beta^-$ **single-electron spectra** $1/\Gamma\,\mathrm{d}\Gamma/\mathrm{d}E$ ***vs.*** **electron kinetic energy** $E - m_e$ **for** **$^{82}$Se** ($Q = 2.996$ MeV)



# $2\nu EP\beta^-$ Half-Lives predictions
**(independent on $g_A$ and value of NME)**

$2\nu\beta^-\beta^-$ **and** $2\nu EP\beta^-$ **half-lives** $T_{1/2}^{2\nu\beta\beta}$ **and** $T_{1/2}^{2\nu EP\beta}$ **calculated for** $\beta^-\beta^-$ **isotopes observed experimentally, assuming unquenched** $g_A = 1.269$



# Instead of Conclusions

**Neutrino physics**

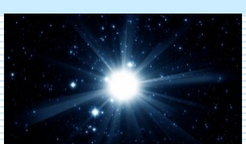
$$\frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)}$$

**LHC physics**

$$\frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$


**Progress in nuclear structure calculations is highly required**

**We are at the beginning of the Beyond Standard Model Road…**



**The future of neutrino physics is bright**