

The Effects of an Extended Neutrino Sphere on Supernova Neutrino Oscillations

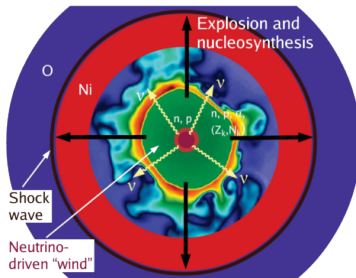
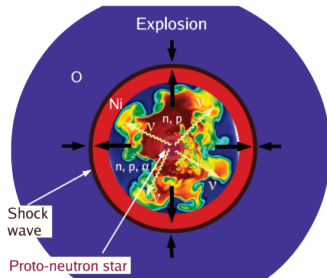
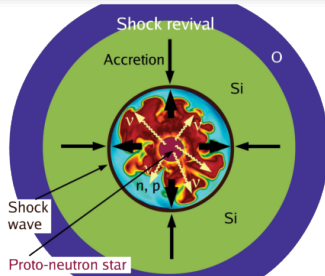
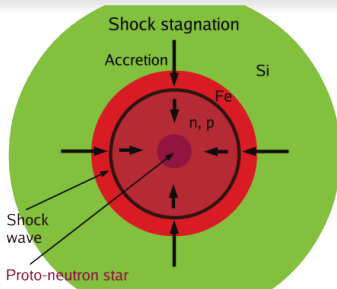


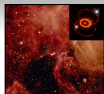
Rasmus S. L. Hansen



NDM 2018, IBS, Korea

July 2, 2018





Impact of neutrino oscillations

- Supernova explosion mechanism:

$$E(\nu_{\mu/\tau}) > E(\nu_e)$$

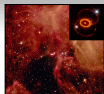
- Modify the neutrino signal:

$$\nu_e \rightarrow \nu_{\mu/\tau} \quad \nu_{\mu/\tau} \rightarrow \nu_e$$

- Nucleosynthesis depends on the neutron fraction:

$$p + \bar{\nu}_e \rightarrow n + e^+ \quad n + \nu_e \rightarrow p + e^-$$

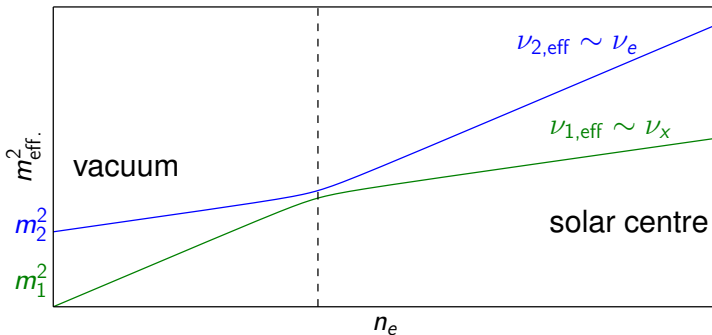
(See also talk by Baha Balantekin on Friday)

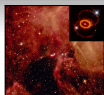


Matter effect

Electron background shifts the energy eigenvalues.

$$H = \frac{\Delta m^2}{2E} \begin{pmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{pmatrix} + \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$





Density matrix formalism

In the mean field approximation:

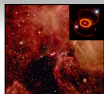
$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix} = \frac{1}{2}(P_0 + \vec{P} \cdot \vec{\sigma}).$$

Similarly for the Hamiltonian:

$$H = \frac{1}{2} \begin{pmatrix} V_z & V_x - iV_y \\ V_x + iV_y & -V_z \end{pmatrix} = \frac{1}{2}\vec{V} \cdot \vec{\sigma}.$$

Equation of motion (in absence of collisions, [see also poster by Hirokazu Sasaki](#)):

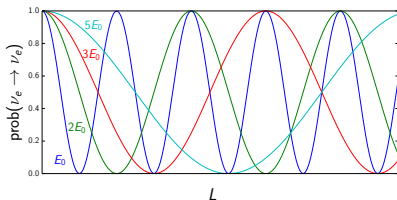
$$i\dot{\rho} = [H, \rho] \quad \Leftrightarrow \quad \dot{\vec{P}} = \vec{V} \times \vec{P}$$



Collective oscillations

Normal oscillations:

$$\text{prob}(\nu_e \rightarrow \nu_e) \propto \cos^2(\Delta m^2 L / 4E).$$

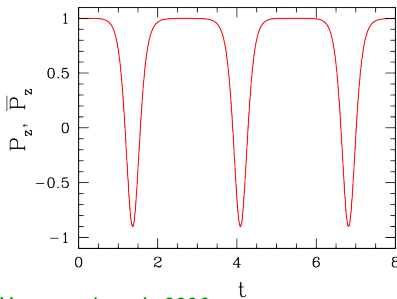


Neutrino background:

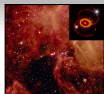
$$H_{\nu\nu} = \sqrt{2}G_F \int dp(\rho - \bar{\rho}).$$

Conversion independent of E .

Non-linear problem $\rightarrow$ hard to solve in a realistic setting.



Hannestad et al. 2006



Linear equations, RSLH and Smirnov, 2018

Observation

Considering the individual neutrino, its oscillations in a SN is a linear problem.

Idea

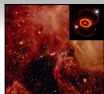
How much of the neutrino-neutrino refraction can we describe using linear equations.

Ultimate goal

General conclusions about the behaviour of neutrino oscillations in presence of neutrino-neutrino refraction.

Methods

- Solve equations from first principles, analytic and numeric.
- Describe complicated systems using effective potentials.



General equations

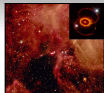
Probe neutrino in arbitrary neutrino and matter background.

$$H^{(p)} = \frac{1}{2} \begin{pmatrix} -c_{2\theta}\omega_p + V_e + V_\nu & s_{2\theta}\omega_p + 2\bar{V}_\nu e^{i\phi_B} \\ s_{2\theta}\omega_p + 2\bar{V}_\nu e^{-i\phi_B} & c_{2\theta}\omega_p - V_e - V_\nu \end{pmatrix}, \quad (1)$$

$$V_\nu = \int d\mathbf{k} V_\nu^0(\mathbf{k}) [\rho_{ee}(\mathbf{k}) - \rho_{\tau\tau}(\mathbf{k})],$$

$$\bar{V}_\nu e^{i\phi_B} = \int d\mathbf{k} V_\nu^0(\mathbf{k}) \rho_{e\tau}(\mathbf{k})$$

$$V_\nu^0(\mathbf{k}) = \sqrt{2} G_F n(\mathbf{k}) (1 - \mathbf{v}_{bg} \cdot \mathbf{v}_p)$$



General equations - rotated

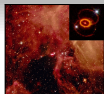
Rewrite the off-diagonal as $V'e^{i\phi'} = s_{2\theta}\omega_p + 2\bar{V}_\nu e^{i\phi_B}$. Apply the transformation $U = \text{diag}\left(e^{i\phi'/2}, e^{-i\phi'/2}\right)$.

$$H^{(p)} = \frac{1}{2} \begin{pmatrix} V^r & V' \\ V' & -V^r \end{pmatrix},$$

where

$$V' = \sqrt{4\bar{V}_\nu^2 + 4s_{2\theta}\omega_p \cos \phi_B \bar{V}_\nu + s_{2\theta}^2 \omega_p^2},$$

$$V^r = V_e + V_\nu + \dot{\phi}' - c_{2\theta}\omega_p.$$



Conditions for a large conversion

Our Ansatz is that every case where a large conversion happens can be described in terms of at least one of these frameworks:

1 Resonance

Vanishing diagonal:

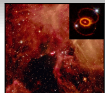
$$V_e + V_\nu + \dot{\phi}' - c_{2\theta}\omega_p = 0.$$

2 Adiabatic conversion

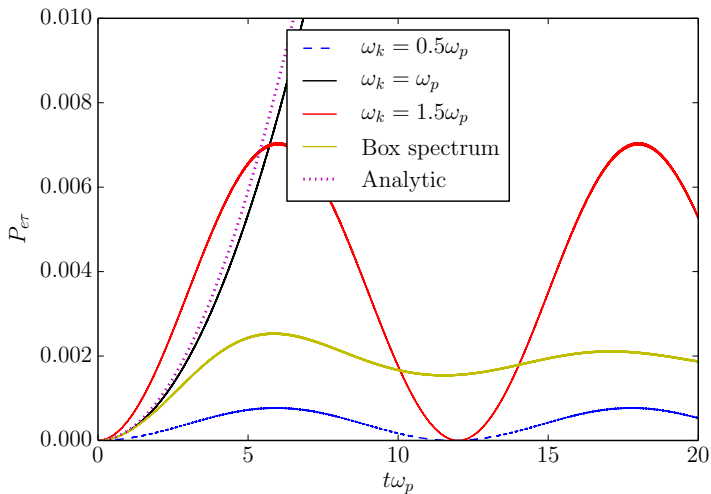
Fast oscillations in V^r and V' can be removed by going to a rotating frame. This can result in a Hamiltonian describing adiabatic evolution.

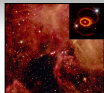
3 Parametric enhancement

Present if the period of oscillation equals the period of change of mixing angle.



Energy spectrum





Neutrino emission

$\nu_{\mu/\tau}$ decouple first, then $\bar{\nu}_e$ and last ν_e .

$\nu_{\mu/\tau}$ have:

Number sphere:

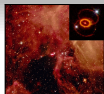
$$e^+ e^- \rightarrow \nu_{\mu/\tau} \bar{\nu}_{\mu/\tau}$$

Energy sphere:

$$e \nu_{\mu/\tau} \rightarrow e \nu_{\mu/\tau}$$

Transport sphere:

$$N \nu_{\mu/\tau} \rightarrow N \nu_{\mu/\tau}$$



Neutrino emission

$\nu_{\mu/\tau}$ decouple first, then $\bar{\nu}_e$ and last ν_e .

$\nu_{\mu/\tau}$ have:

Number sphere:

$$e^+ e^- \rightarrow \nu_{\mu/\tau} \bar{\nu}_{\mu/\tau}$$

Energy sphere:

$$e \nu_{\mu/\tau} \rightarrow e \nu_{\mu/\tau}$$

Transport sphere:

$$N \nu_{\mu/\tau} \rightarrow N \nu_{\mu/\tau}$$

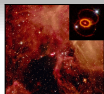
$\bar{\nu}_e$ and ν_e are dominated by absorption and emission from nucleons. ($n_n > n_p$)

Mean free path:

$$\frac{1}{\lambda} \approx \frac{G_F^2}{\pi} (g_V^2 + 3g_A^2) E^2 n_N.$$

Emissivity:

$$j = \frac{1}{\lambda} \exp(-E/T)$$



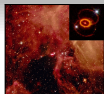
Extended source

Rough estimates:

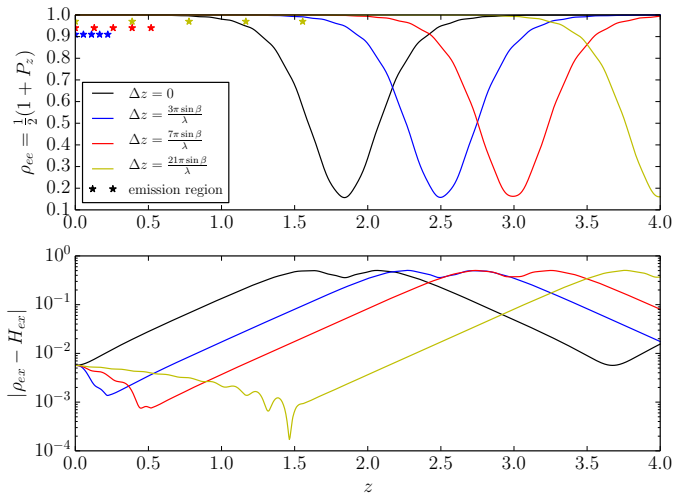
- Neutrino sphere: $\sim 10\text{km}$.
- Width of neutrino sphere: $\sim 1\text{km}$.
- Oscillation length: $\sim \frac{1}{G_F n_e} \sim 10^{-8} - 10^{-7}\text{km}$.

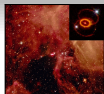
Average over emission region suppresses oscillatory terms by $10^7 - 10^8$.

Parametric resonance is not removed as such.

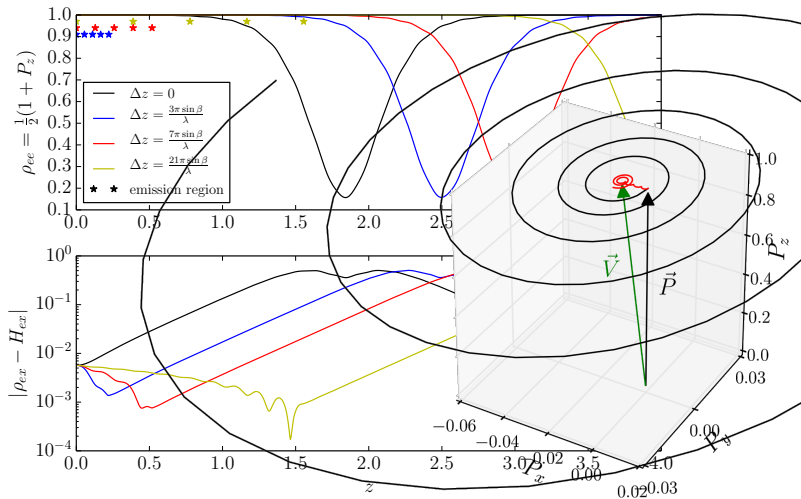


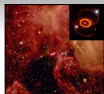
Extended source - non-linear





Extended source - non-linear





Changing background - only matter

Coordinate system with $\vec{V}$ along z-axis:

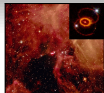
$$\rho_{12}(r) = \rho_{12,\text{initial}} \exp \left(i \int_{r_e}^r \omega_m(r') dr' \right).$$

Average over emission point:

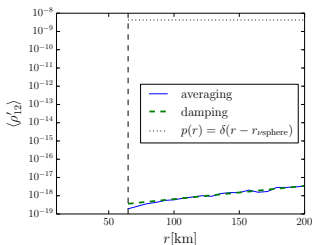
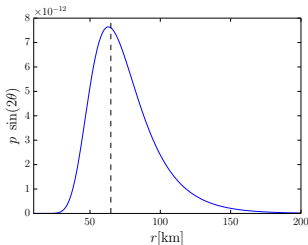
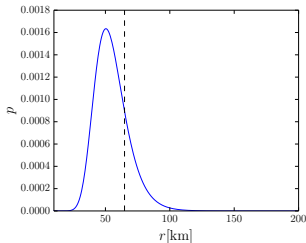
$$\langle \rho_{12}(r) \rangle = \int_0^r p(r_e) \frac{1}{2} \sin 2\theta_m(r_e) \exp \left(i \int_{r_e}^r \omega_m(r') dr' \right) dr_e,$$

where

$$p(r_e) = \frac{1}{\lambda(r_e)} \exp(-E/T) \exp \left(- \int_{r_e}^{\infty} \frac{1}{\lambda(r')} dr' \right).$$



Changing background - only matter



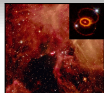
Include effect of damping, D :

$$\dot{\vec{P}} = \vec{V} \times \vec{P} - D\vec{P}_T.$$

For V_z and D large and $P_z \approx 1$:

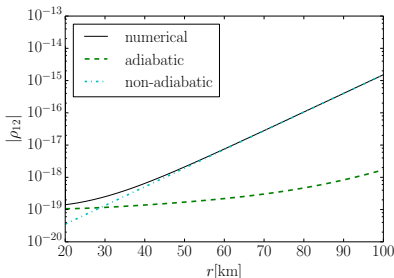
$$P_x \approx \frac{V_x V_z}{V_z^2 + D^2}, P_y \approx \frac{-V_x D}{V_z^2 + D^2}.$$

Bell et al. 1998, Hannestad et al. 2012



Effect of non-adiabaticity

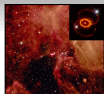
Solve $\dot{\vec{P}} = \vec{V} \times \vec{P} - D\vec{P}_T$
numerically:



Non-adiabatic effects for $D=0$:

$$\vec{P} = \begin{pmatrix} \frac{V_x}{V_z} + \frac{V_x \partial_r V_z}{V_z^3} \sin \left(\int V_z(r') dr' \right) \\ \frac{V_x \partial_r V_z}{V_z^3} (1 - \cos \left(\int V_z(r') dr' \right)) \\ 1 + \frac{V_x^2 \partial_r V_z}{V_z^4} \sin \left(\int V_z(r') dr' \right) \end{pmatrix}$$

$$|\rho_{12}| \approx \left| \frac{V_x \partial_r V_z}{2V_z^3} \right| \approx \left| \frac{V_x}{2V_z^2 r_0} \right|$$



Linear stability analysis Banerjee, Dighe, Raffelt 2011

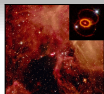
Linear analysis demonstrate stability or instability.

$$\rho = \frac{f_{\nu_e} + f_{\nu_x}}{2} + \frac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s & S \\ S^* & -s \end{pmatrix}$$

$$|S| = |P_x + iP_y| \ll 1, \quad s^2 + S^2 = 1 \Rightarrow s = P_z \approx 1.$$

Linearised equation:

$$i\dot{S} = (\omega + \lambda + \mu)S - \mu \int d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') S',$$



Linear stability analysis

In Fourier space: $S = e^{-i\Omega t} Q$

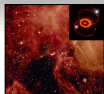
$$\Omega Q = (\omega + \lambda + \mu)Q - \mu \int d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') Q'$$

Unstable if $\text{Im}(\Omega) \neq 0$. (See also Capozzi et al. 2017)

Discrete modes: solve matrix equation.

(Continuous modes: Decompose in independent functions.)

Can also be formulated as a dispersion relation. (Izaguirre, Raffelt and Tamborra, 2016)



Simple model

Linearised equation: ($\omega = -1$, $\lambda = 30$, $\mu = 3$)

$$i\dot{\vec{S}} = \begin{pmatrix} -\omega + \lambda - \mu & \mu \\ -\mu & \omega + \lambda + \mu \end{pmatrix} \vec{S}$$

Eigenvalues:

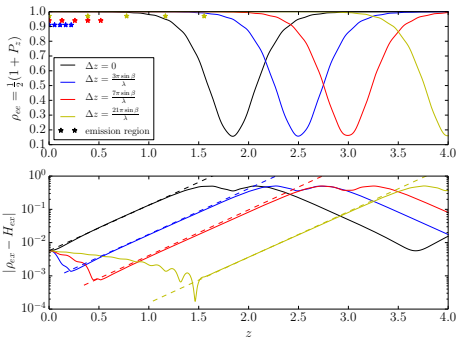
$$\Omega = \lambda \pm \sqrt{\omega(2\mu + \omega)}$$

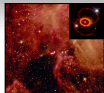
Growth rate = $\text{Im}(\Omega)$.

Emission point = $\frac{2}{3}\Delta z$.

Start value = $\frac{1}{\Delta z \lambda} \sin 2\theta_m$.

Does not work for large Δz .



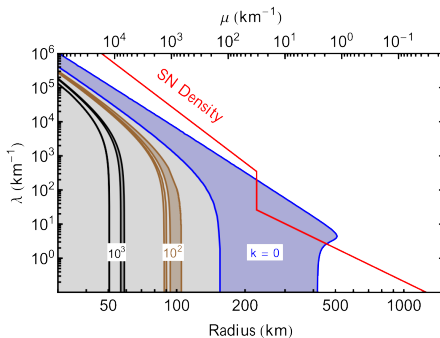
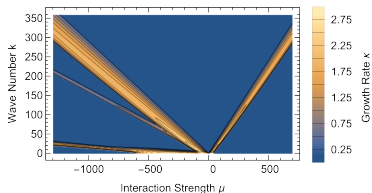


Multiple angles, in-homogeneous

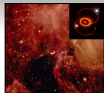
Linear stability analysis of a more realistic model:

$$(\Omega + \mathbf{v} \cdot \mathbf{k})Q = (\omega + \lambda + \mu(\epsilon - \mathbf{v} \cdot \phi))Q - \mu \int d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') g' Q'$$

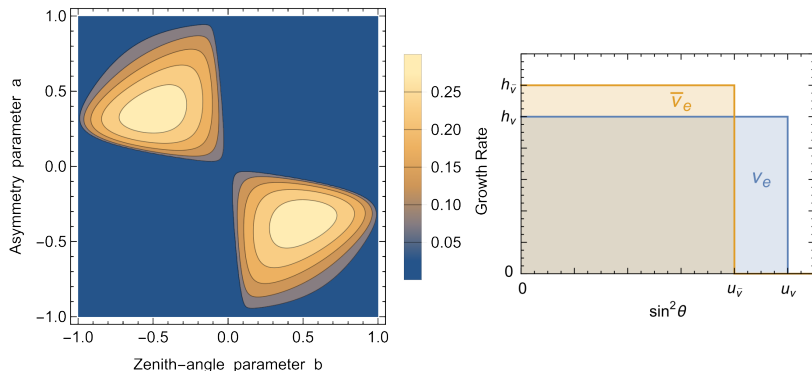
- μ and λ functions of r .
- Multi angle matter effect.
- Homogeneous mode: $k = 0$



Chakraborty, RSLH, Izaguirre and Raffelt, 2015



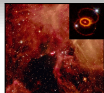
Very fast flavour conversion R. F. Sawyer



Chakraborty, RSLH, Izaguirre and Raffelt, 2016

Conversion on meter-scale.

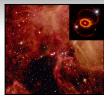
Can also occur in a supernova. Dasgupta, Mirizzi and Sen, 2016



Summary

- The non-negligible width of the neutrino sphere affects the neutrino state due to averaging over different emission points.
- The angle between the neutrino state and the Hamiltonian in polarization space is reduced by a factor 10^8 at the neutrino sphere by the averaging.
- A small adiabaticity violation increases the angle significantly as the neutrino propagates out through the supernova.
- The onset of neutrino conversion can be analysed using linear stability analysis, and for a given model, it can be calculated if conversion has the potentially to occur.

Thanks for your attention!

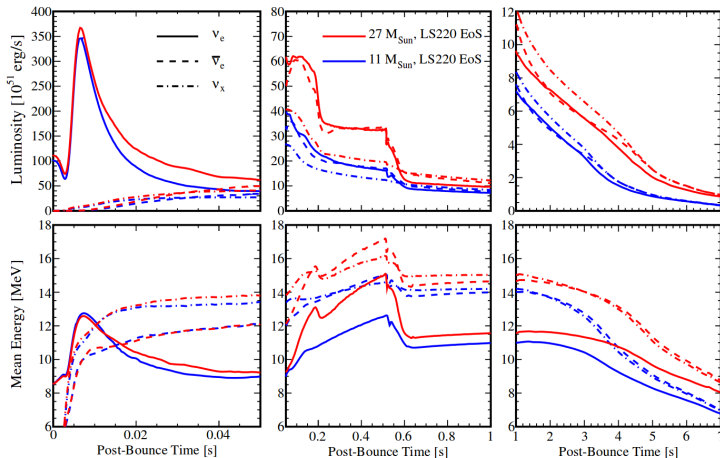


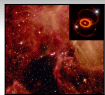
Neutrino emission

Deleptonisation

Accretion

Cooling



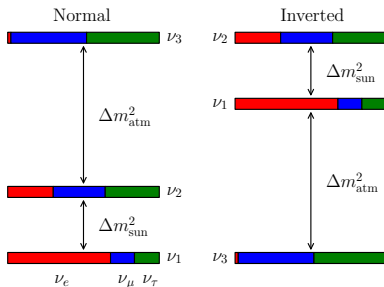


Neutrino mixing

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}} W_{\mu}^{+} \bar{\nu}_L^i \gamma^{\mu} l_L + h.c.$$

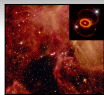
Interaction states and mass states are different:

$$\nu^I = U \nu^i.$$



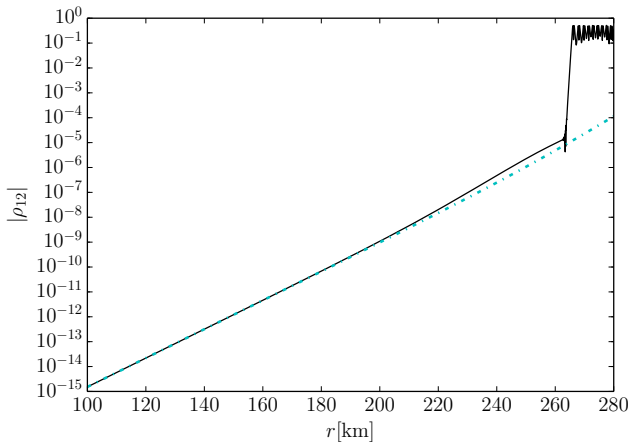
Mixing matrix:

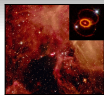
$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$



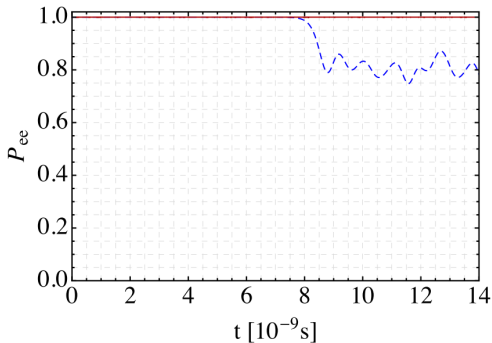
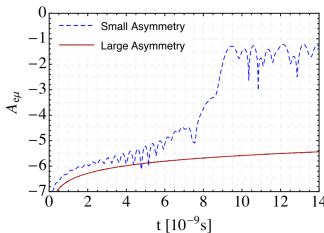
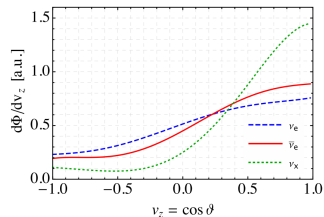
Collective oscillations

Can collective oscillations still occur? YES!





Very fast flavour conversion



Dasgupta, Mirizzi and Sen, 2016