# The 2νββ decay and a determination of the effective axial-vector coupling constant $g_A^{eff}$


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Meant to be a supplementary talk to

So I will skip the introduction about the double beta decay

# The weak form-factors

- Form factors introduced since proton/neutron are not elementary part.
- Depend on vector and axial weak charges of the proton and neutron.
- Two hypotheses:
  - Conservation of Vector Current (CVC):
  - Partial conservation of Axial Current (PCAC):

$$F_V(q^2) = \frac{F_V(0)}{\left(1 - q^2/0.71\right)^2} \qquad F_V(0) = 1 = \mathbf{g_V}$$

$$\boldsymbol{G_A}(q^2) = \frac{F_A(0)}{\left(1 - q^2/1.065\right)^2} \qquad \boldsymbol{G_A}(0) = g_A = -1.2573 \pm 0.028$$

- For low energy neutrinos ($E_\nu << m_N$):

$$\sigma(\nu_e n) = \sigma(\bar{\nu}_e p) = \frac{\left(G_F \cos\theta_C\right)^2 E_\nu^{\;2}}{\pi}\left[F_V(0)^2 + \boldsymbol{3G_A}(0)^2\right]$$

$$\approx 9.75 \times 10^{-42} \left(\frac{E_\nu}{10\ MeV}\right)^2 cm^2$$

# Axial-vector current in nuclei

- The axial current is not conserved!
- Thus, its extension to nuclei is not trivial.
- Nucleons interact in nuclei.

**Allowed Gamow-Teller transition**

$0^+ \rightarrow 1^+$

$$ft \sim \frac{1}{g_A^2 |M_A|^2}$$

$$M_A^2 = \left| \langle \psi_i | GT | \psi_f \rangle \right|^2$$

**Double Gamow-Teller transition**

$0^+_{g.s.} \rightarrow 0^+_{g.s.}$

$$\frac{1}{T_{1/2}^{2\nu-exp}} = G^{2\nu}(E_0, Z)\ g_A^4\ |M_{GT}^{2\nu}|^2$$

$$M_{GT}^{2\nu} = \sum_m \frac{< 0_f^+ || \tau^+ \sigma || 1_m^+ >< 1_m^+ || \tau^+ \sigma || 0_i^+ >}{E_m - E_i + \Delta}$$

## *Understanding of the 2$\nu\beta\beta$-decay NMEs is of crucial importance for correct evaluation of the 0$\nu\beta\beta$-decay NMEs*

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

*Both 2$\nu\beta\beta$ and 0$\nu\beta\beta$ operators connect the same states.*
*Both change two neutrons into two protons.*

*Explaining 2$\nu\beta\beta$-decay is necessary but not sufficient*

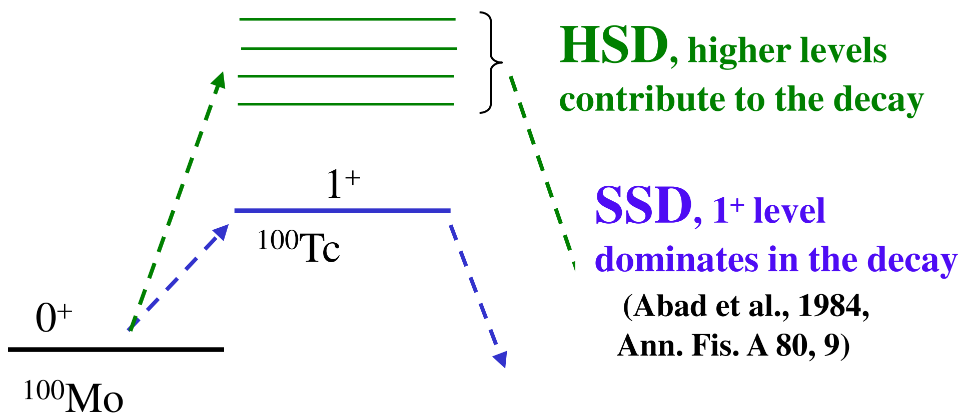
**There is a need for reliable calculation of the *2$\nu\beta\beta$-decay NMEs***

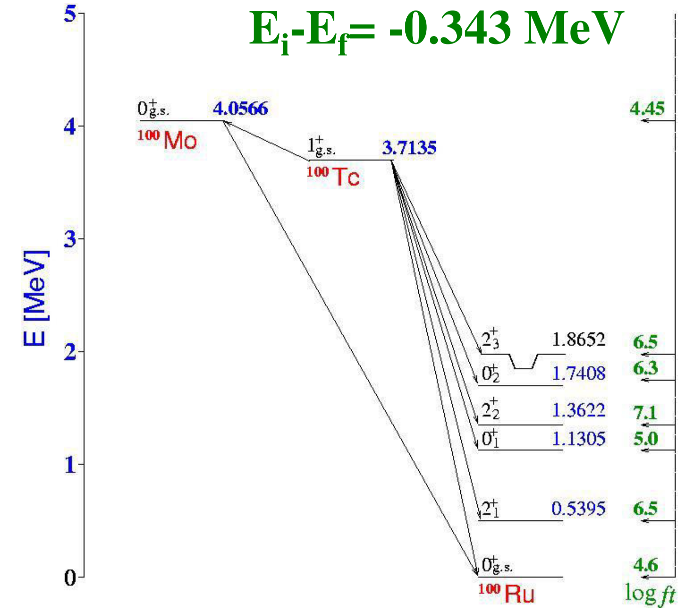
**Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.)**
**ISM (quenching, truncation of model space, spin-orbit partners)**

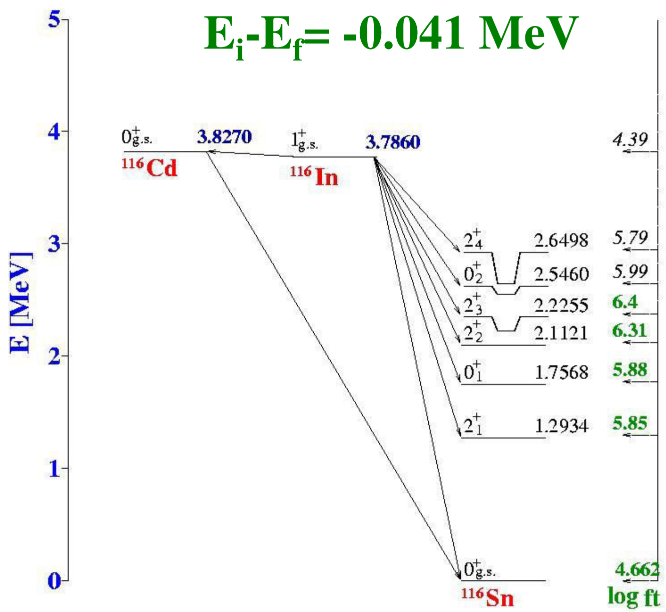
**Calculation via closure NME: IBM, PHFB**

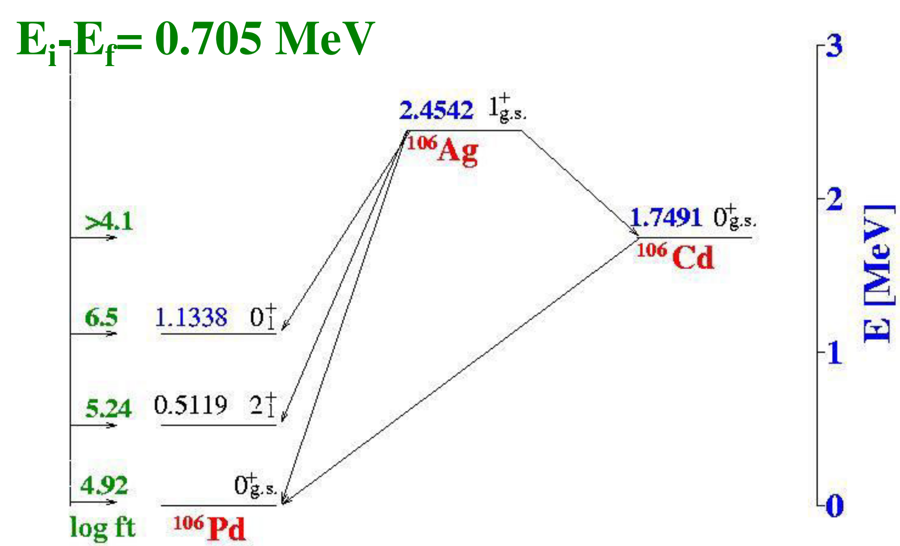
**No calculation: EDF**

# Single State Dominance ( $^{100}$Mo, $^{106}$Cd, $^{116}$Cd, …)



# SSD – theoretical studies

Šimkovic, Šmotlák, Semenov, J. Phys. G, 27, 2233, 2001

$$M_{GT}^{K} = \sum_m \left( \frac{M_m^i(1^+) M_m^f(1^+)}{E_m - E_i + e_{10} + \nu_{10}} + \frac{M_m^i(1^+) M_m^f(1^+)}{E_m - E_i + e_{20} + \nu_{20}} \right) \qquad M_{GT}^{K} = M_{GT}^{L}(\nu_{10} \leftrightarrow \nu_{20})$$

SSD

$$\Rightarrow \frac{M_1^i(1^+) M_1^f(1^+)}{E_1 - E_i + e_{10} + \nu_{10}} + \frac{M_1^i(1^+) M_1^f(1^+)}{E_1 - E_i + e_{20} + \nu_{20}} \Rightarrow 2\frac{M_1^i(1^+) M_1^f(1^+)}{E_1 - E_i + \Delta}$$

HSD

common approx.

$$e_{10} + \nu_{10} \approx e_{20} + \nu_{20} \approx (E_i - E_f)/2 \equiv \Delta$$

| Isotope | f.s. | $T_{1/2}$(SSD)[y] | $T_{1/2}$(exp.)[y] |
|---|---|---|---|
| | | $2\nu\beta^-\beta^-$ | |
| $^{100}$Mo | $0_{g.s.}$ | $6.8\ 10^{18}$ | $6.8\ 10^{18}$ |
| | $0_1$ | $4.2\ 10^{20}$ | $6.1\ 10^{18}$ |
| $^{116}$Cd | $0_{g.s.}$ | $1.1\ 10^{19}$ | $2.6\ 10^{19}$ |
| $^{128}$Te | $0_{g.s.}$ | $1.1\ 10^{25}$ | $2.2\ 10^{24}$ |
| | | EC/EC | |
| $^{106}$Cd | $0_{g.s.}$ | $>4.4\ 10^{21}$ | $>5.8\ 10^{17}$ |
| $^{130}$Ba | $0_{g.s.}$ | $5.0\ 10^{22}$ | $4.0\ 10^{21}$ |

**$E_1$-$E_i$ ≈ 0 or neg. ⇒ sensitivity to lepton energies in energy denominators**
**⇒ SSD and HSD offer different differential characteristics**

**The SSD prediction for the 2νββ half-life does not depend on quenching of $g_A$**

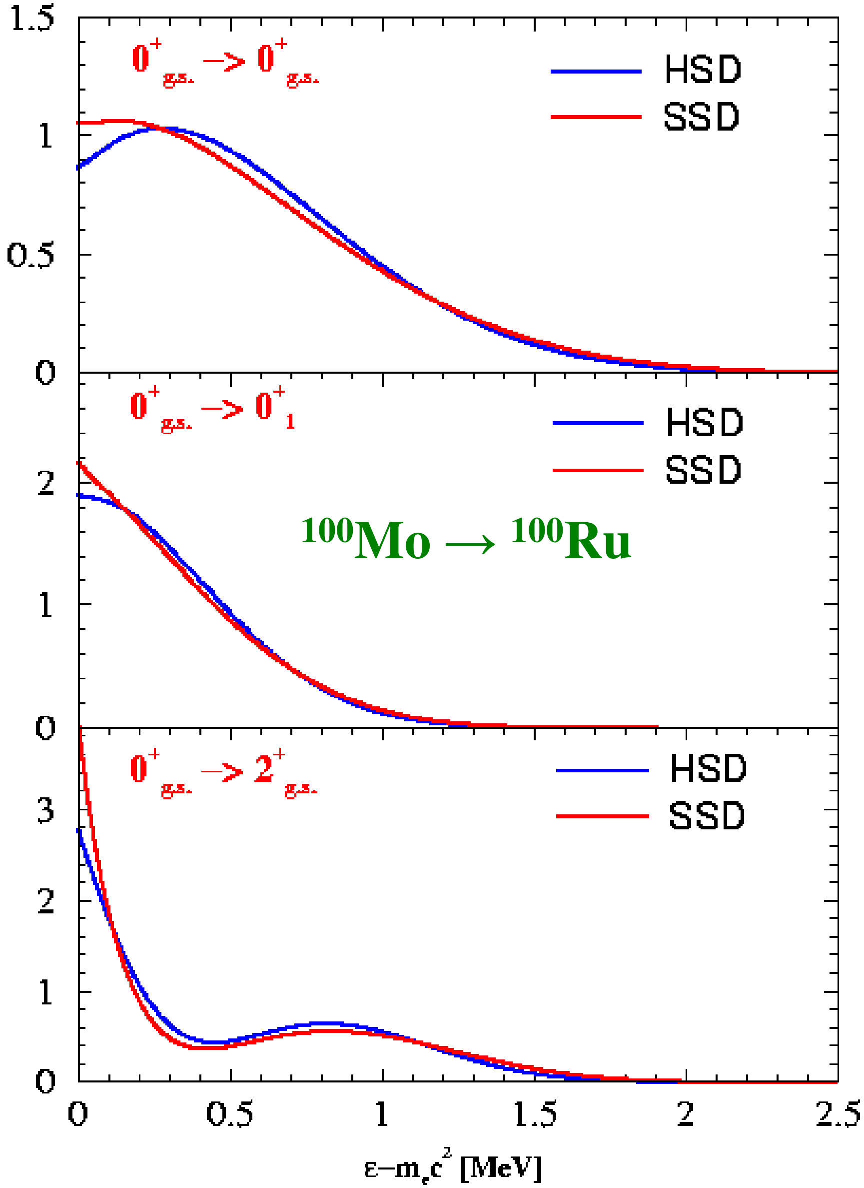
Domin, Kovalenko, Šimkovic, Semenov, NPA 753, 337 (2005)

$$M_1^i(0^+) = \frac{1}{g_A}\sqrt{\frac{3D}{ft_{EC}}} \qquad M_1^f(J^+) = \frac{1}{g_A}\sqrt{\frac{3D}{ft_{\beta^-}}}$$

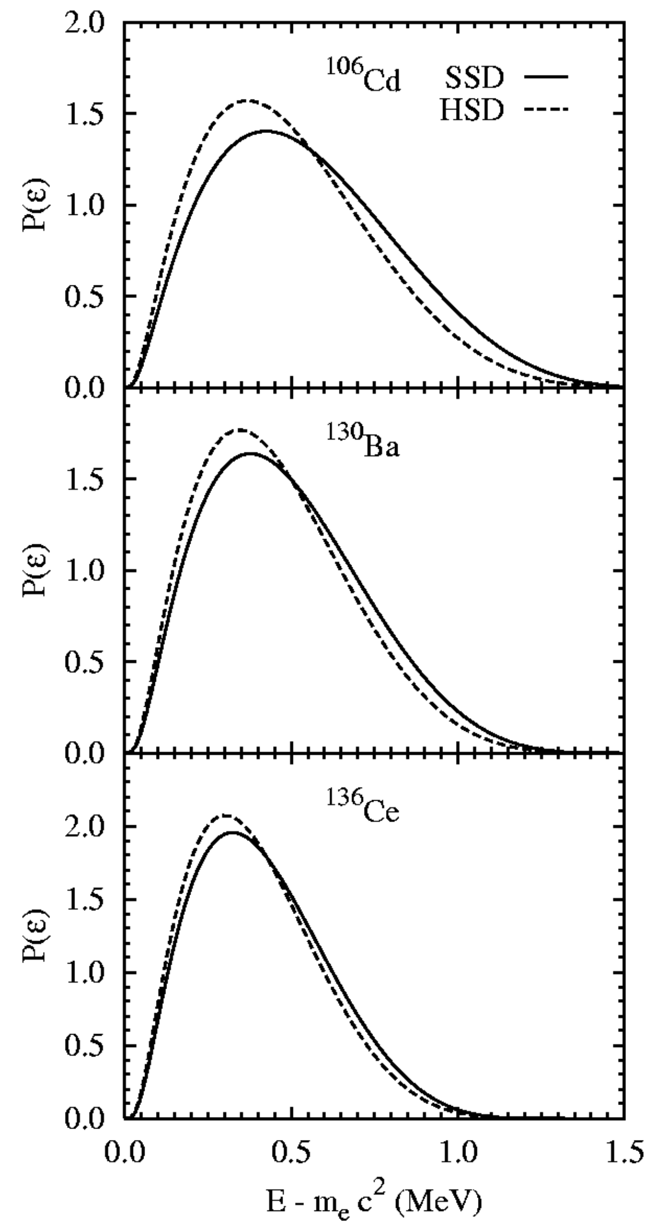
$2\nu\beta^-\beta^-$–decay

# SSD differential characteristics

$2\nu EC/\beta^+$–decay

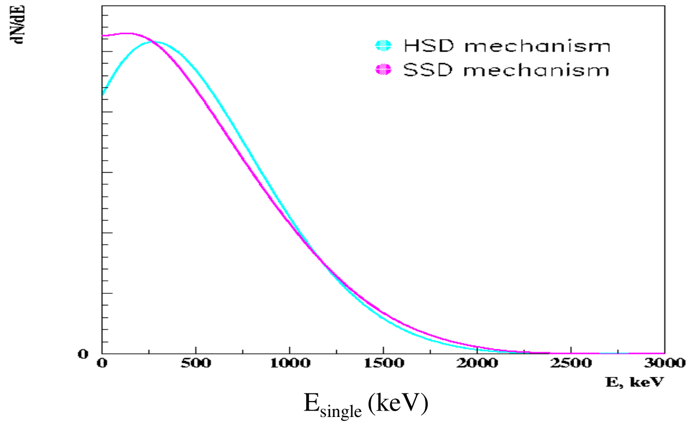



**Do not depend on $M^iM^f$**



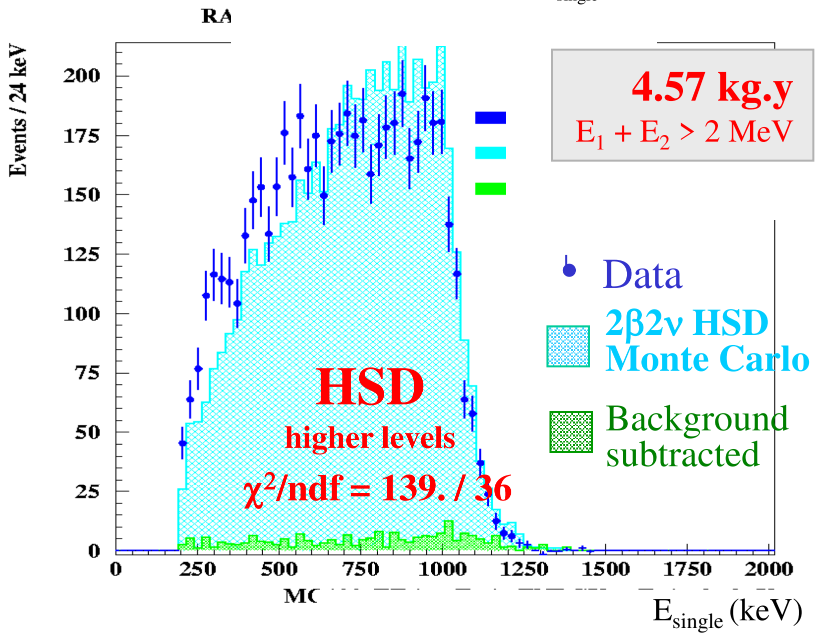
# $^{100}$Mo 2ν2β : Experimental Study of SSD Hypothesis

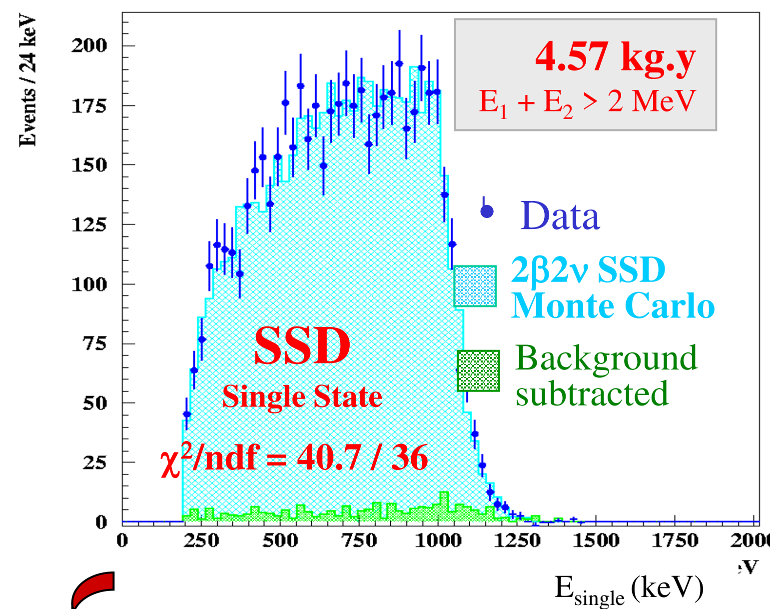
**NEMO 3 exp.**




**Single electron spectrum different between SSD and HSD**

**Šimkovic, Šmotlák, Semenov**
**J. Phys. G, 27, 2233, 2001**




**HSD**: $T_{1/2} = 8.61 \pm 0.02$ (stat) $\pm 0.60$ (syst) $\times 10^{18}$ y

**SSD**: $T_{1/2} = 7.72 \pm 0.02$ (stat) $\pm 0.54$ (syst) $\times 10^{18}$ y

**$^{100}$Mo 2ν 2β single energy distribution in favour of Single State Dominant (SSD) decay**

## 2νββ-decay rate

**Double Fermi + DGT transitions, only $s_{1/2}$ lepton states and no recoil.**

$$\left[T^{2\nu\beta\beta}_{1/2}(0^+)\right]^{-1} = \frac{m_e}{8\pi^7 \ln 2}(G_\beta m_e^2)^4 I^{2\nu}(0^+),$$

$$I^{2\nu}(0^+) = \frac{1}{m_e^9}\int_{m_e}^{E_i-E_f-m_e} F_0(Z_f,E_{e_1})p_{e_1}E_{e_1}dE_{e_1} \times \int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f,E_{e_2})p_{e_2}E_{e_2}dE_{e_2} \times \int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}^{2\nu} dE_{\nu_1}.$$

$$\mathcal{A}^{2\nu} = g_V^4\left[\frac{1}{4}|M_F^K + M_F^L|^2 + \frac{3}{4}|M_F^K - M_F^L|^2\right] - g_V^2 g_A^2 \mathrm{Re}\left\{M_F^{K*}M_{GT}^L + M_{GT}^{K*}M_F^L\right\} + \frac{g_A^4}{3}\left[\frac{3}{4}|M_{GT}^K + M_{GT}^L|^2 + \frac{1}{4}|M_{GT}^K - M_{GT}^L|^2\right]$$

$$M_F^K = \sum_n \frac{K(0_n^+)}{2}F_n, \quad M_F^L = \sum_n \frac{L(0_n^+)}{2}F_n,$$

$$M_{GT}^K = \sum_n \frac{K(1_n^+)}{2}G_n, \quad M_{GT}^L = \sum_n \frac{L(1_n^+)}{2}G_n.$$

$$F_n = \langle 0_f^+ \| \sum_m \tau_m^- \| 0_n^+\rangle\langle 0_n^+ \| \sum_m \tau_m^- \| 0_i^+\rangle,$$

$$G_n = \langle 0_f^+ \| \sum_m \tau_m^-\sigma_m \| 1_n^+\rangle\langle 1_n^+ \| \sum_m \tau_m^-\sigma_m \| 0_i^+\rangle$$

$$K_n(J^+) = \frac{2}{(2E_n(J^+)-E_i-E_f)+\epsilon_K} + \frac{2}{(2E_n(J^+)-E_i-E_f)-\epsilon_K}$$

$$L_n(J^+) = \frac{2}{(2E_n(J^+)-E_i-E_f)+\epsilon_L} + \frac{2}{(2E_n(J^+)-E_i-E_f)-\epsilon_L}$$

$$\epsilon_K = E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1}$$
$$\epsilon_L = E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1}$$

**In the limit** $2E_n - E_i - E_f = 0$ ⇒ $\mathcal{A}^{2\nu} = 0$

# *Quenching of $g_A$*

**Quenching:**

$q = g_A / g^{free}_A$

**Free value of $g_A$ (Particle Data Group 2016):**

$g^{free}_A = 1.2723(23)$

**Effective value of $g_A$:**

$g^{eff}_A = q\ g^{freeA}$

Jouni

$g_A^4 = (1.269)^4 = 2.6$

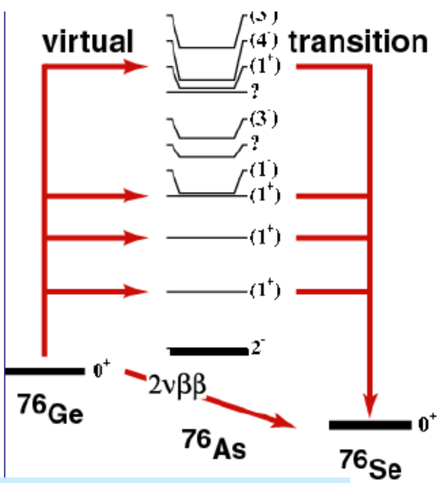
# *Quenching of $g_A$ (from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)*

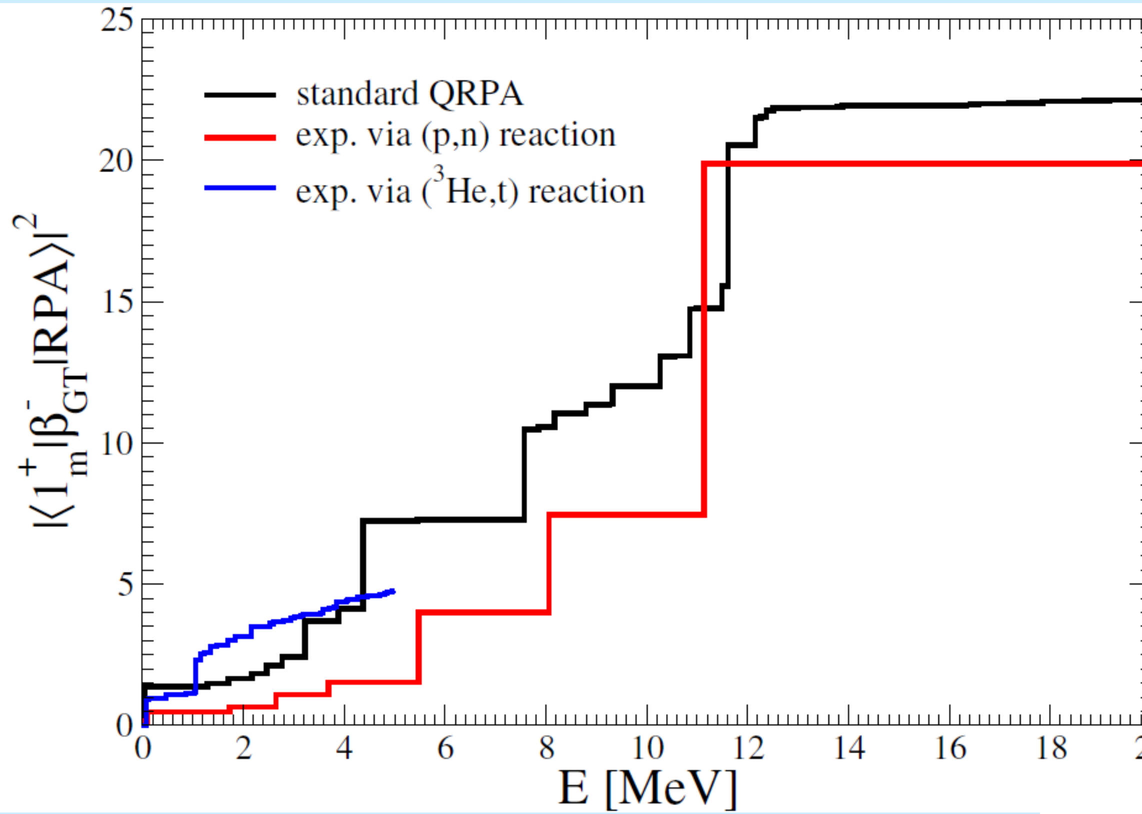
$(g^{eff}_A)^4 = 1.0$

**Strength of GT trans. (approx. given by Ikeda sum rule =3(N-Z)) has to be quenched to reproduce experiment**

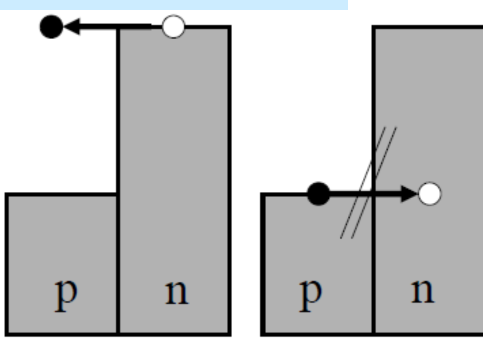
$^{76}_{32}Ge_{44} \Rightarrow$

$S_{\beta^-} - S_{\beta^+} = 3(N-Z) = 36$




**Pauli blocking**




**Cross-section for charge exchange reaction:**

$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} N_d \, |V_{\sigma\tau}|^2 \, |\langle f \,|\, \sigma\tau \,|\, i\rangle|^2$$

$q = 0!!$

largest at 100 - 200 MeV/A

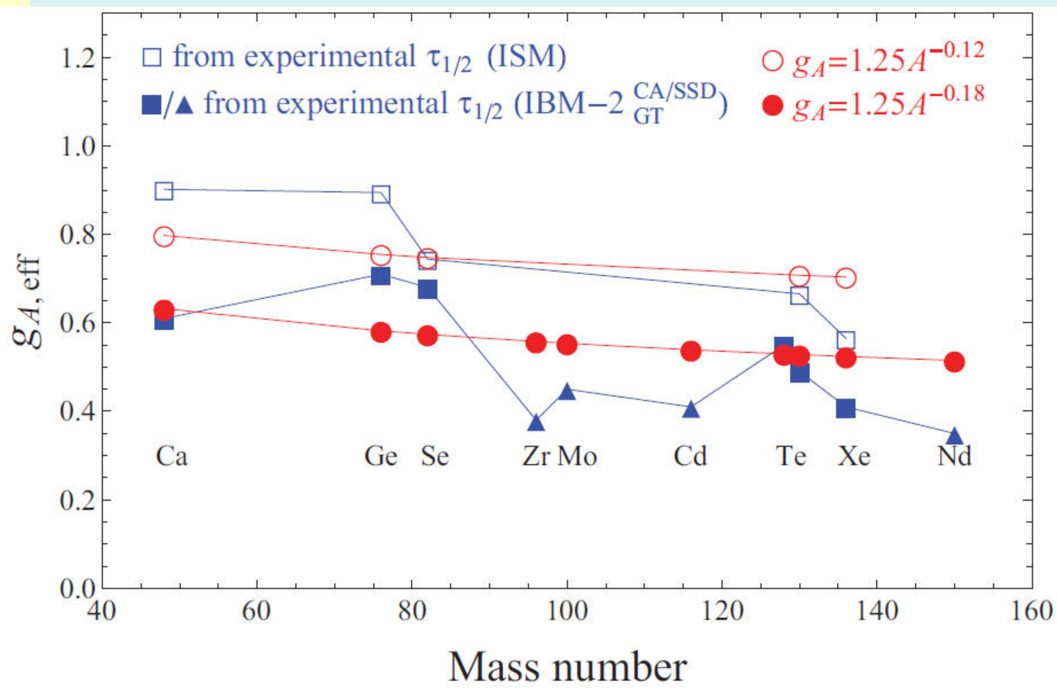
# *Quenching of $g_A$ (from theory: $T_{1/2}^{0\nu}$ up 50 x larger)*

**$(g^{eff}_A)^4 \simeq 0.66$ ($^{48}$Ca), 0.66 ($^{76}$Ge), 0.30 ($^{76}$Se), 0.20 ($^{130}$Te) and 0.11 ($^{136}$Xe)**
**The Interacting Shell Model (ISM), which describes qualitatively well energy spectra, does reproduce experimental values of $M^{2\nu}$ only by consideration of significant quenching of the Gamow-Teller operator, typically by 0.45 to 70%.**

**$(g^{eff}_A)^4 \simeq (1.269\ A^{-0.18})4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.**

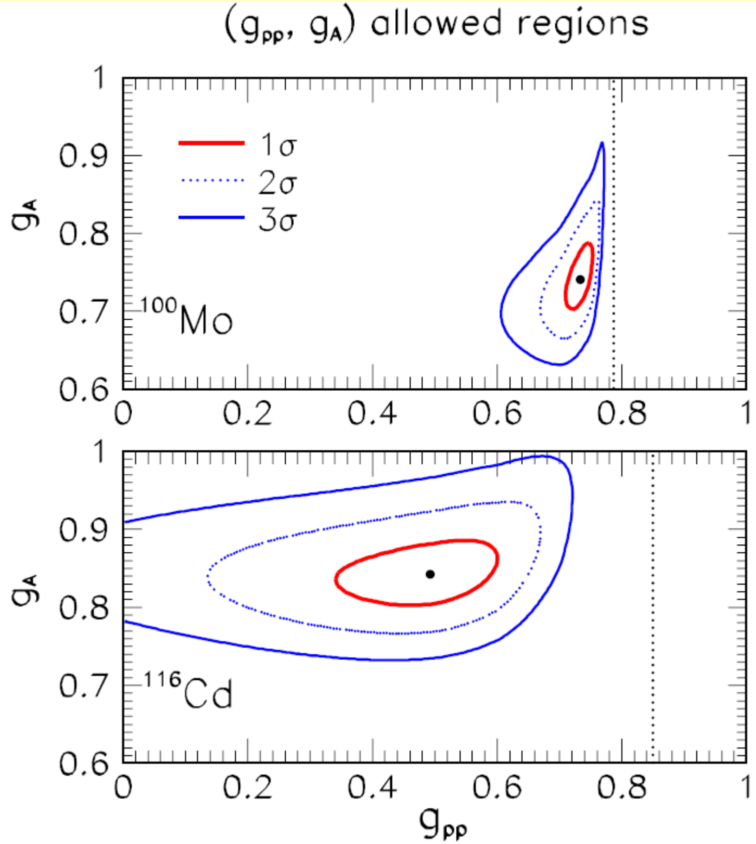
**J. Barea, J. Kotila, F. Iachello, PRC 87, 014315 (2013).**

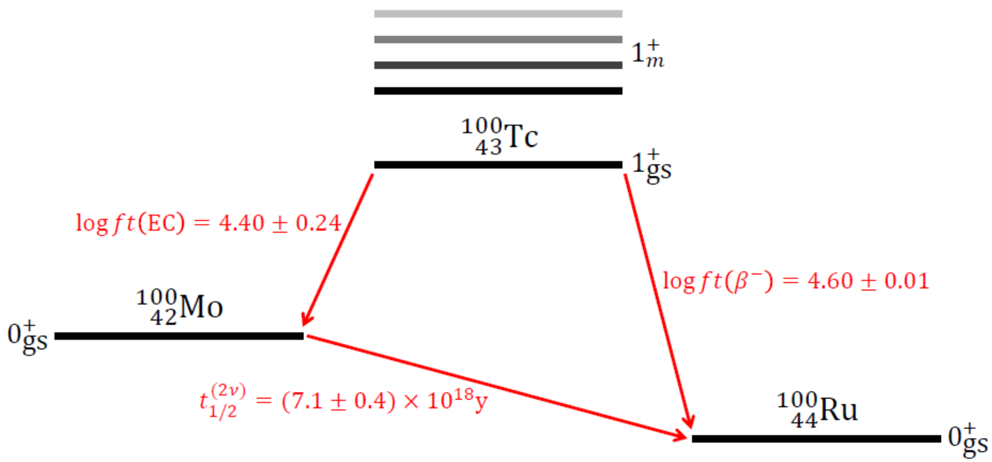
**It has been determined by theoretical prediction for the 2νββ-decay half-lives, which were based on within closure approximation calculated corresponding NMEs, with the measured half-lives.**



**Faessler, Fogli, Lisi, Rodin, Rotunno, Šimkovic, J. Phys. G 35, 075104 (2008).**

**$(g^{eff}_A)^4$ = 0.30 and 0.50 for $^{100}$Mo and $^{116}$Cd, respectively (The QRPA prediction).**
**$g^{eff}_A$ was treated as a completely free parameter alongside $g_{pp}$ (used to renormalize particle-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of $g^{eff}_A$ and $g_{pp}$, where possible, to the β-decay rate and β+/EC rate of the J = 1$^+$ ground state in the intermediate nuclei involved in double-beta decay in addition to the 2νββ rates of the initial nuclei, leads to an effective $g^{eff}_A$ of about 0.7 or 0.8.**




**Extended calculation also for neighbor isotopes performed by**

**F.F. Depisch and J. Suhonen, PRC 94, 055501 (2016)**

**Dependence of $g^{eff}_A$ on A was not established.**

# *Improved formalism of the $2\nu\beta\beta$-decay*

**F. Šimkovic, R. Dvornický, D. Štefánik, A. Faessler, PRC 97 (2018) 034315**

# *Improved description of the 2νββ-decay rate*

**The isospin conservation is assumed**

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = \frac{m_e}{8\pi^7 \ln 2}(G_\beta m_e^2)^4 \left(g_A^{\rm eff}\right)^4 I^{2\nu}$$

**Half-life without factorization of NMEs and phase space**

$$\begin{aligned} I^{2\nu} &= \frac{1}{m_e^{11}} \int_{m_e}^{E_i-E_f-m_e} F_0(Z_f, E_{e_1}) p_{e_1} E_{e_1} dE_{e_1} \\ &\times \int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f, E_{e_2}) p_{e_2} E_{e_2} dE_{e_2} \\ &\times \int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}^{2\nu} dE_{\nu_1} \end{aligned}$$

$$\mathcal{A}^{2\nu} = \left[\frac{1}{4}|M_{GT}^K + M_{GT}^L|^2 + \frac{1}{12}|M_{GT}^K - M_{GT}^L|^2\right]$$

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i+E_f)/2}{[E_n - (E_i+E_f)/2]^2 - \varepsilon_{K,L}^2}$$

$$M_n = \langle 0_f^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 1_n^+ \rangle \langle 1_n^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 0_i^+ \rangle$$

$$\begin{aligned} \epsilon_K &= (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2 \\ \epsilon_L &= (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2 \end{aligned}$$

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

**Standard approximation which allows factorization of NME and phase space**

$$M_{GT}^{K,L} \simeq M_{GT}^{2\nu} = m_e \sum_n \frac{M_n}{E_n - (E_i + E_f)/2}$$

## Let perform Taylor expansion

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2} \qquad \epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

$$E_n - \frac{E_i + E_f}{2} = \frac{Q}{2} + m_e + (E_n - E_i) > |\epsilon_{K,L}|$$

# Improved description of the 0νββ–decay rate

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \equiv \frac{\Gamma^{2\nu}}{\ln(2)} \simeq \frac{\Gamma_0^{2\nu}+\Gamma_2^{2\nu}+\Gamma_4^{2\nu}}{\ln(2)}$$

$$\frac{\Gamma_0^{2\nu}}{\ln(2)} = \left(g_A^{\rm eff}\right)^4 \mathcal{M}_0 G_0^{2\nu}$$
$$\frac{\Gamma_2^{2\nu}}{\ln(2)} = \left(g_A^{\rm eff}\right)^4 \mathcal{M}_2 G_2^{2\nu}$$
$$\frac{\Gamma_4^{2\nu}}{\ln(2)} = \left(g_A^{\rm eff}\right)^4 \left(\mathcal{M}_4 G_4^{2\nu} + \mathcal{M}_{22} G_{22}^{2\nu}\right)$$

**Taylor expansion up to ε⁴ order**

$$G_J^{2\nu} = \frac{c_{2\nu}}{m_e^{11}} \int_{m_e}^{E_i-E_f-m_e} F_0(Z_f,E_{e_1}) p_{e_1} E_{e_1} dE_{e_1}$$
$$\times \int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f,E_{e_2}) p_{e_2} E_{e_2} dE_{e_2}$$
$$\times \int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}_J^{2\nu} dE_{\nu_1}, \quad (\mathrm{J}{=}0,\ 2,\ 4,\ 22)$$

$$\mathcal{A}_0^{2\nu} = 1 \quad \mathcal{A}_2^{2\nu} = \frac{\varepsilon_K^2+\varepsilon_L^2}{(2m_e)^2},$$
$$\mathcal{A}_{22}^{2\nu} = \frac{\varepsilon_K^2\varepsilon_L^2}{(2m_e)^4} \quad \mathcal{A}_4^{2\nu} = \frac{\varepsilon_K^4+\varepsilon_L^4}{(2m_e)^4}$$

**Phase space factors**

| | 2νββ-decay | | | |
|---|---|---|---|---|
| nucl. | $G_0^{2\nu}$ [yr$^{-1}$] | $G_2^{2\nu}$ [yr$^{-1}$] | $G_4^{2\nu}$ [yr$^{-1}$] | $G_{22}^{2\nu}$ [yr$^{-1}$] |
| $^{76}$Ge | $4.816\ 10^{-20}$ | $1.015\ 10^{-20}$ | $1.332\ 10^{-21}$ | $6.284\ 10^{-22}$ |
| $^{82}$Se | $1.591\ 10^{-18}$ | $7.037\ 10^{-19}$ | $1.952\ 10^{-19}$ | $8.931\ 10^{-20}$ |
| $^{100}$Mo | $3.303\ 10^{-18}$ | $1.509\ 10^{-18}$ | $4.320\ 10^{-19}$ | $1.986\ 10^{-19}$ |
| $^{130}$Te | $1.530\ 10^{-18}$ | $4.953\ 10^{-19}$ | $9.985\ 10^{-20}$ | $4.707\ 10^{-20}$ |
| $^{136}$Xe | $1.433\ 10^{-18}$ | $4.404\ 10^{-19}$ | $8.417\ 10^{-20}$ | $3.986\ 10^{-20}$ |

**3 different NMEs**

$$\mathcal{M}_0 = \left| M^{2\nu}_{GT-1} \right|^2$$

$$\mathcal{M}_2 = \Re\{ M^{2\nu}_{GT-1} M^{2\nu}_{GT-3} \}$$

$$\mathcal{M}_{22} = \frac{1}{3} \left| M^{2\nu}_{GT-3} \right|^2$$

$$\mathcal{M}_4 = \frac{1}{3} \left| M^{2\nu}_{GT-3} \right|^2 + \Re\{ M^{2\nu}_{GT-1} M^{2\nu}_{GT-5} \}$$

$$M^{2\nu}_{GT-1} \equiv M^{2\nu}_{GT}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\, m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

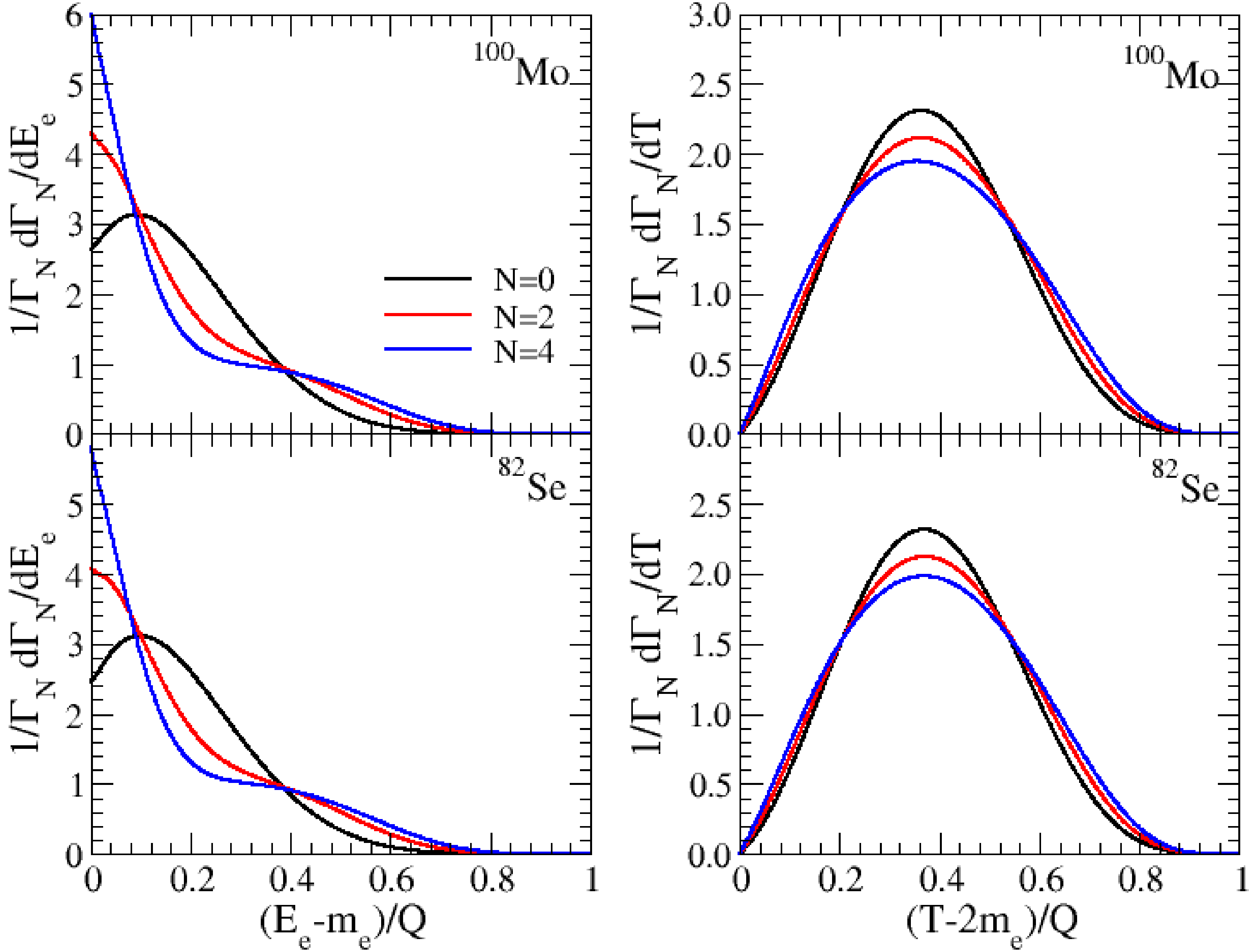
$$M^{2\nu}_{GT-5} = \sum_n M_n \frac{16\, m_e^5}{(E_n - (E_i + E_f)/2)^5}$$

**QRPA**

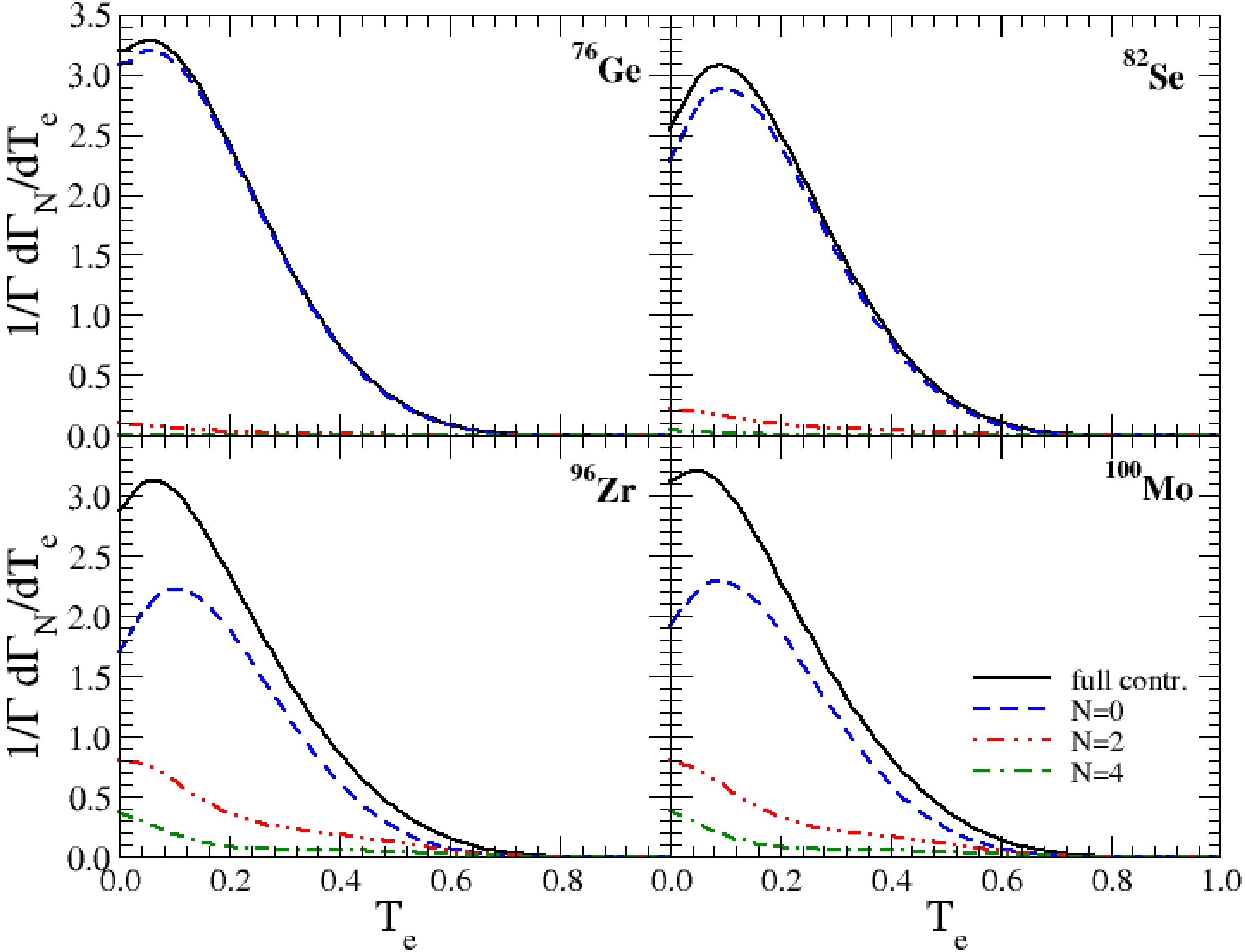
$2\nu\beta\beta$-decay NMEs and their ratios

| nucl. | $g_A^{\rm eff}$ | $M^{2\nu}_{GT-1}$ | $M^{2\nu}_{GT-3}$ | $M^{2\nu}_{GT-5}$ | $\xi^{2\nu}_{13}$ | $\xi^{2\nu}_{15}$ | $P^{2\nu}_0$ | $P^{2\nu}_2$ | $P^{2\nu}_4$ | $T^{2\nu-exp}_{1/2}$ [yr] |
|---|---|---|---|---|---|---|---|---|---|---|
| $^{76}$Ge | 0.800 | 0.175 | 0.0214 | 0.00445 | 0.1220 | 0.0254 | 0.9741 | 0.0250 | 0.0009 | $1.65\ 10^{21}$ |
| | 1.000 | 0.111 | 0.0133 | 0.00263 | 0.1204 | 0.0237 | 0.9745 | 0.0247 | 0.0008 | |
| | 1.269 | 0.689 | 0.00716 | 0.00716 | 0.1040 | 0.0170 | 0.9780 | 0.0214 | 0.0006 | |
| $^{82}$Se | 0.800 | 0.124 | 0.0216 | 0.00645 | 0.1745 | 0.0521 | 0.9213 | 0.0711 | 0.0076 | $0.92\ 10^{20}$ |
| | 1.000 | 0.0795 | 0.0129 | 0.00355 | 0.1620 | 0.0446 | 0.9271 | 0.0664 | 0.0065 | |
| | 1.269 | 0.0498 | 0.00643 | 0.00136 | 0.1290 | 0.0272 | 0.9421 | 0.0538 | 0.0041 | |
| $^{100}$Mo | 0.800 | 0.292 | 0.123 | 0.0453 | 0.4230 | 0.1553 | 0.8163 | 0.1578 | 0.0259 | $7.1\ 10^{18}$ |
| | 1.000 | 0.184 | 0.0876 | 0.0322 | 0.4752 | 0.1745 | 0.7972 | 0.1731 | 0.0297 | |
| | 1.269 | 0.112 | 0.0633 | 0.0233 | 0.5646 | 0.2075 | 0.7661 | 0.1976 | 0.0363 | |
| $^{130}$Te | 0.800 | 0.0466 | 0.00873 | 0.00239 | 0.1873 | 0.0512 | 0.9389 | 0.0569 | 0.0042 | $6.9\ 10^{20}$ |
| | 1.000 | 0.0298 | 0.00577 | 0.00144 | 0.1937 | 0.0482 | 0.9371 | 0.0588 | 0.0041 | |
| | 1.269 | 0.0185 | 0.00373 | 0.00078 | 0.2015 | 0.0420 | 0.9352 | 0.0610 | 0.0038 | |
| $^{136}$Xe | 0.800 | 0.0268 | 0.00706 | 0.00232 | 0.2637 | 0.0866 | 0.9190 | 0.0745 | 0.0065 | $2.19\ 10^{21}$ |
| | 1.000 | 0.0170 | 0.00526 | 0.00169 | 0.3098 | 0.0995 | 0.9059 | 0.0863 | 0.0078 | |
| | 1.269 | 0.0104 | 0.00403 | 0.00126 | 0.3867 | 0.1207 | 0.8848 | 0.1051 | 0.0101 | |

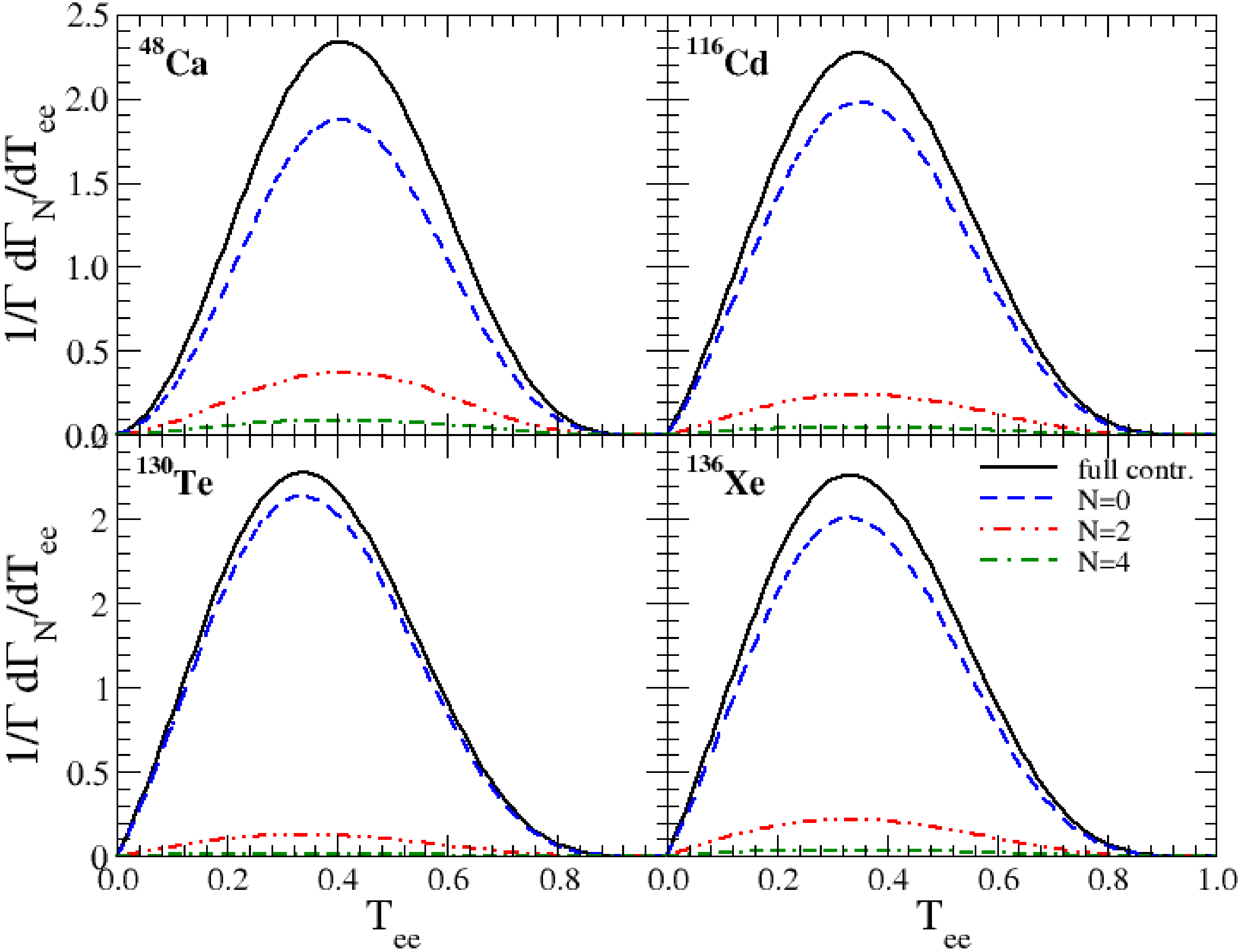
# Normalized to unity different partial energy distributions



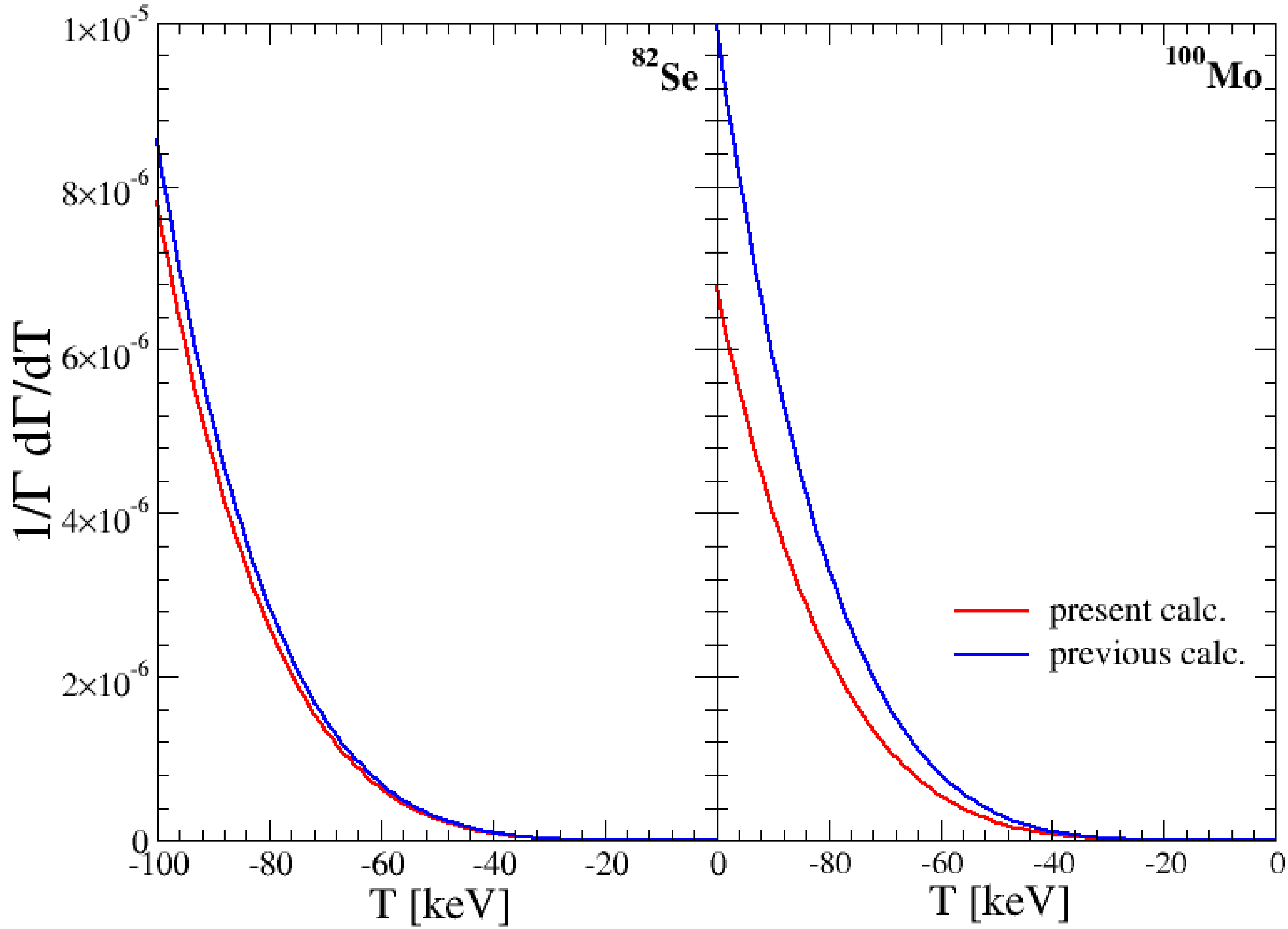
# The single electron energy distribution



# The sum electron energy distribution



# The endpoint of the spectrum of differential decay rate vs. the sum of kinetic energy of emitted electrons



# *A new method to determine effective $g_A$*

F. Šimkovic, R. Dvornický, D. Štefánik, A. Faessler, PRC 97 (2018) 034315

# Improved description of the 0νββ–decay rate

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

**Let us perform Taylor expansion**

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2} \qquad \epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

**We get**

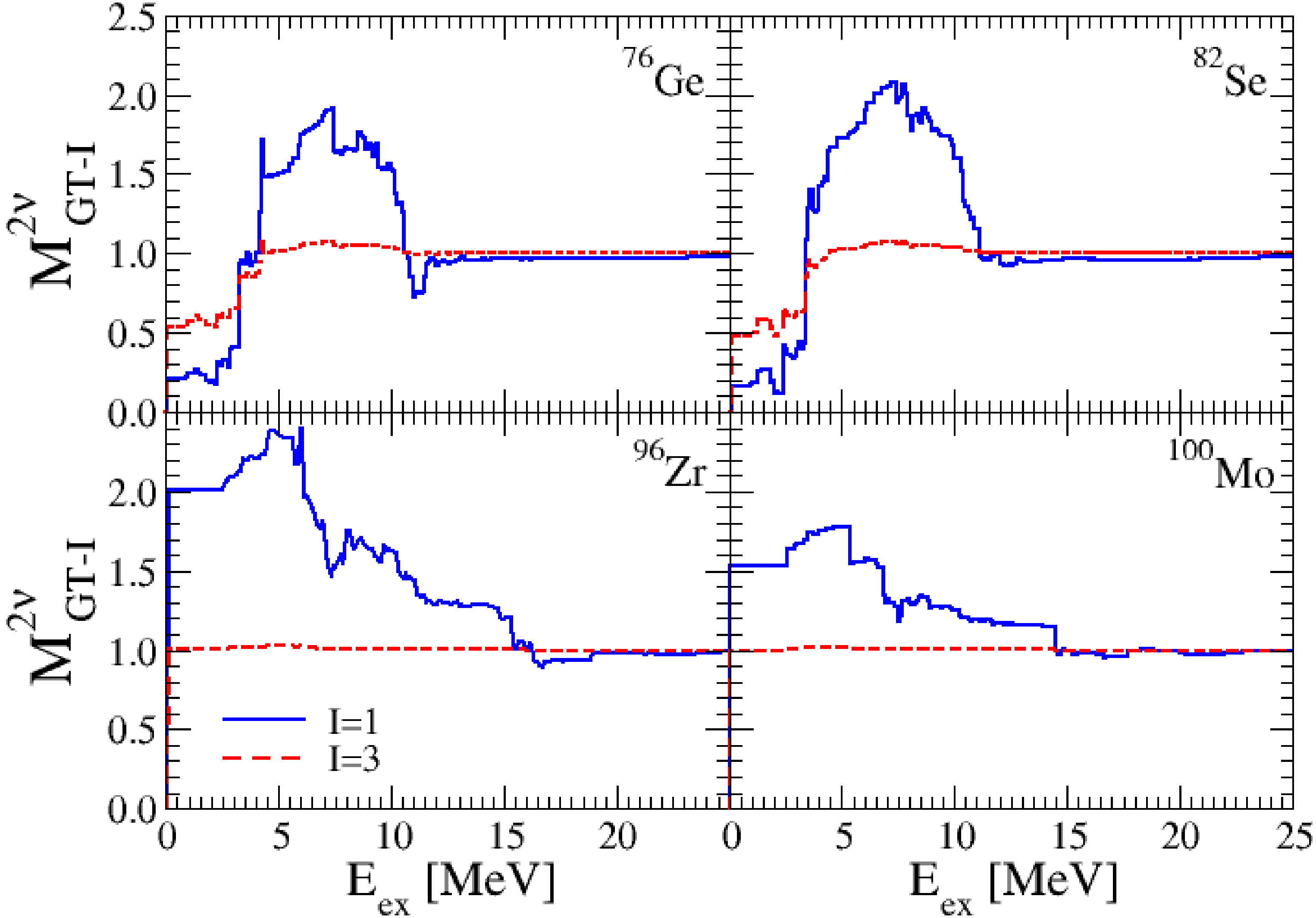
$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\rm eff}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{\left|\xi_{13}^{2\nu}\right|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

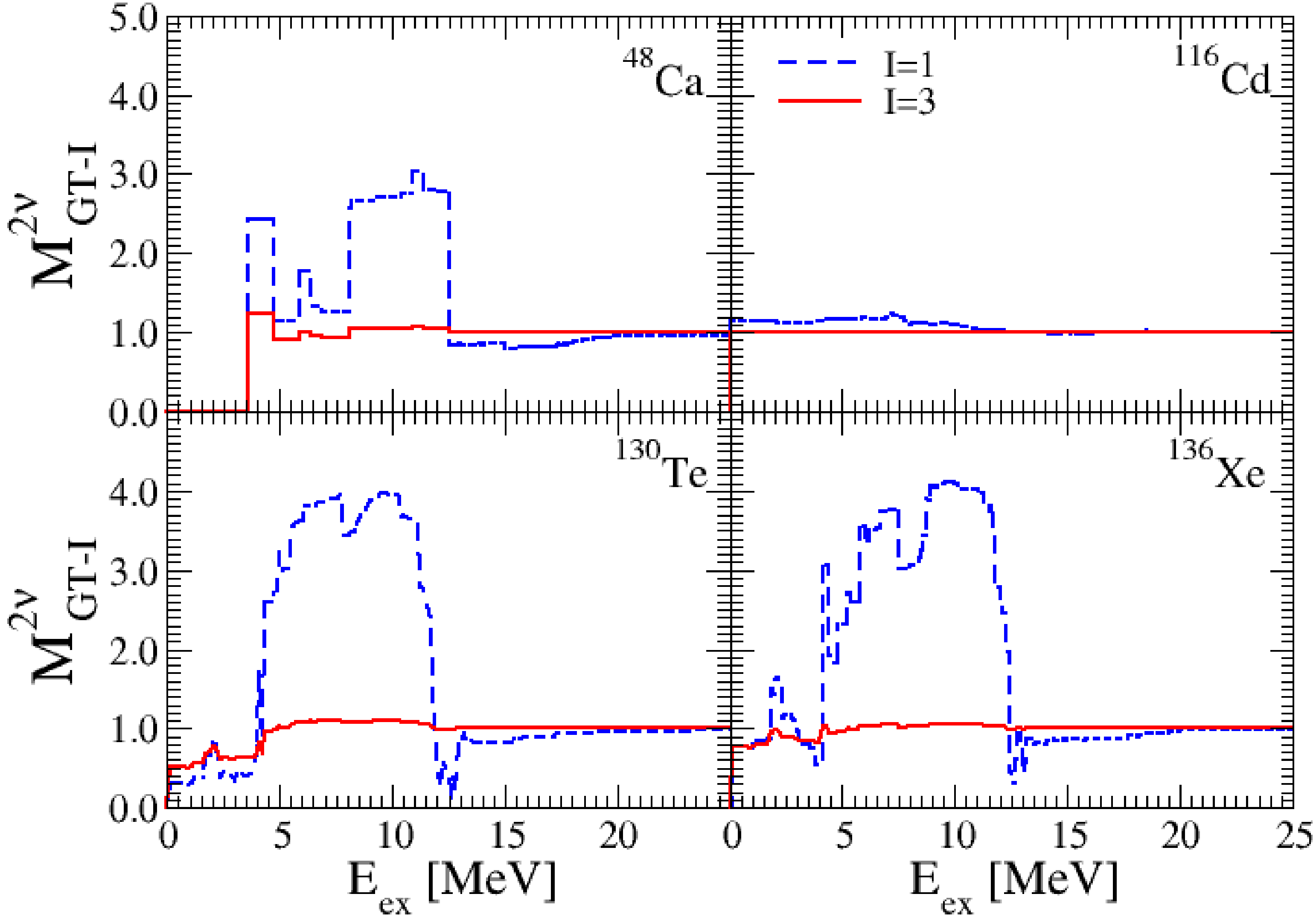
$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3} \qquad \xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

**The $g_A^{eff}$ can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)**

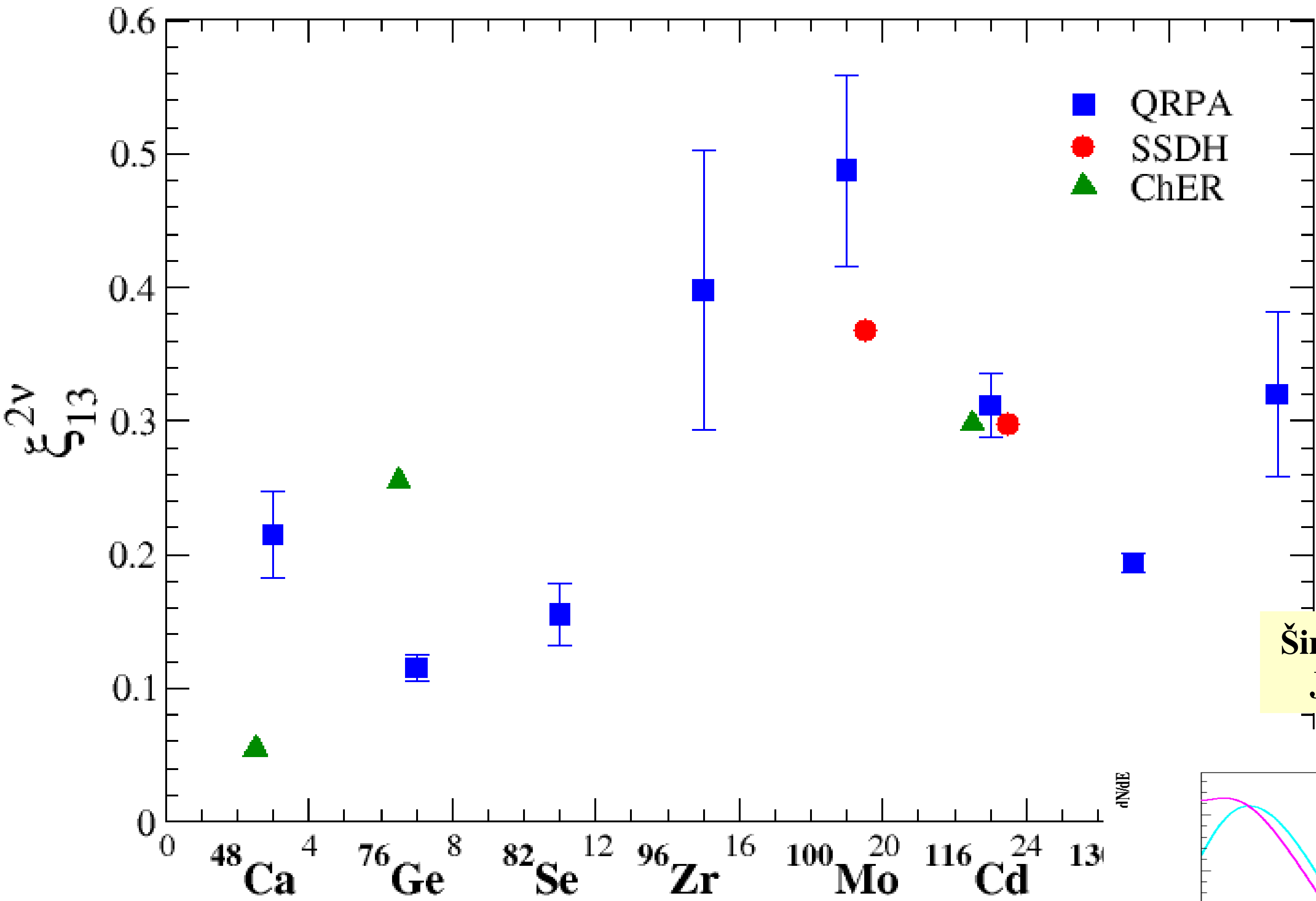
# The running sum of the 2νββ–decay NMEs



# The running sum of the 2νββ–decay NMEs

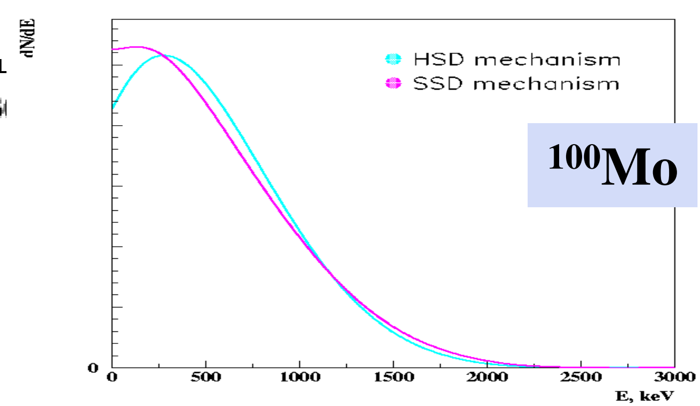


# $\xi_{13}$ tells us about importance of higher lying states of int. nucl.




**HSD: $\xi_{13}$=0**

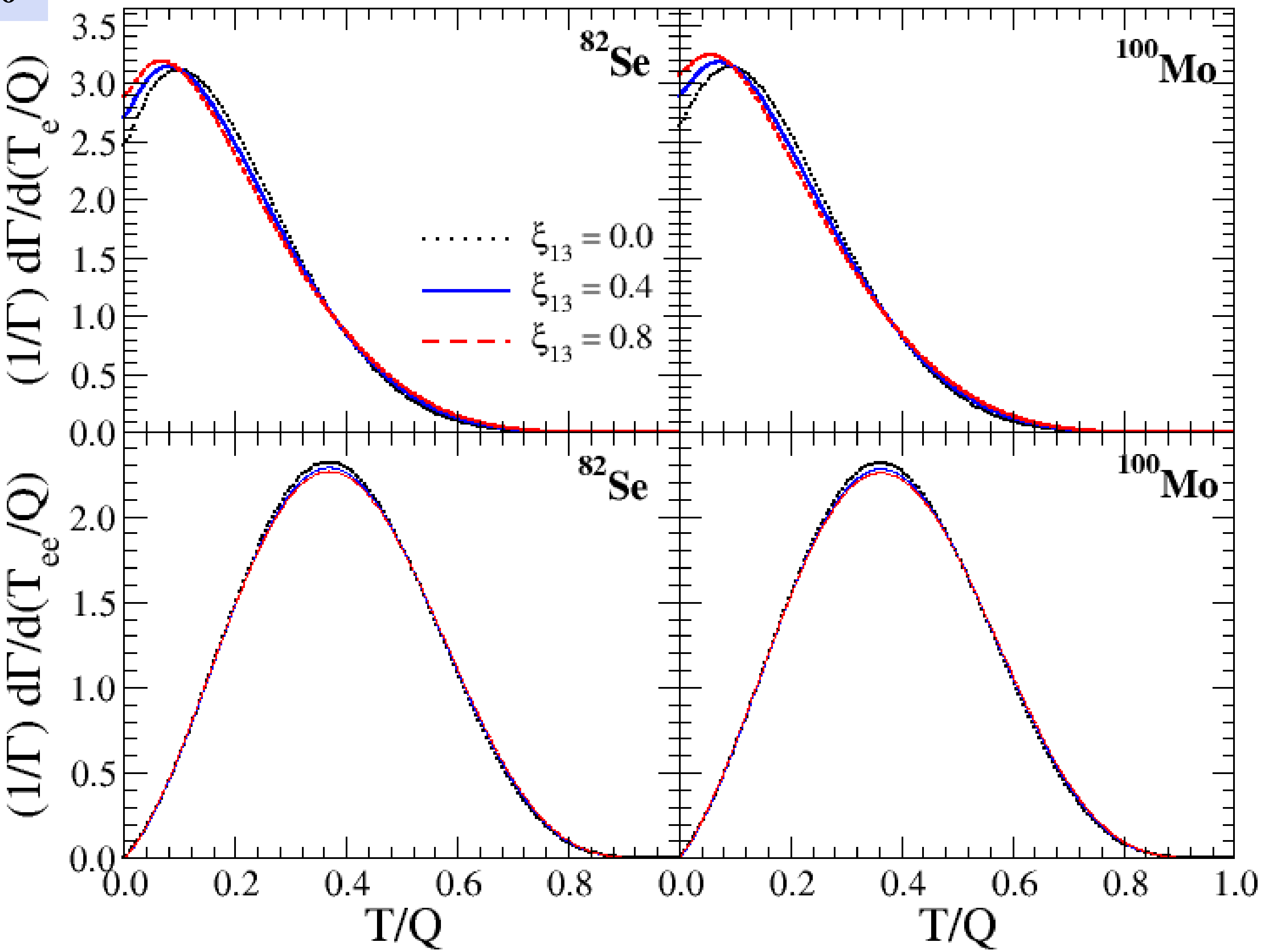
**Šimkovic, Šmotlák, Semenov**
**J. Phys. G, 27, 2233, 2001**




**$\xi_{13}$ can be determined phenomenologically from the shape of energy distributions of emitted electrons**

# The change of the 2νββ–decay energy distributions

**HSD: $\xi_{13}$=0**

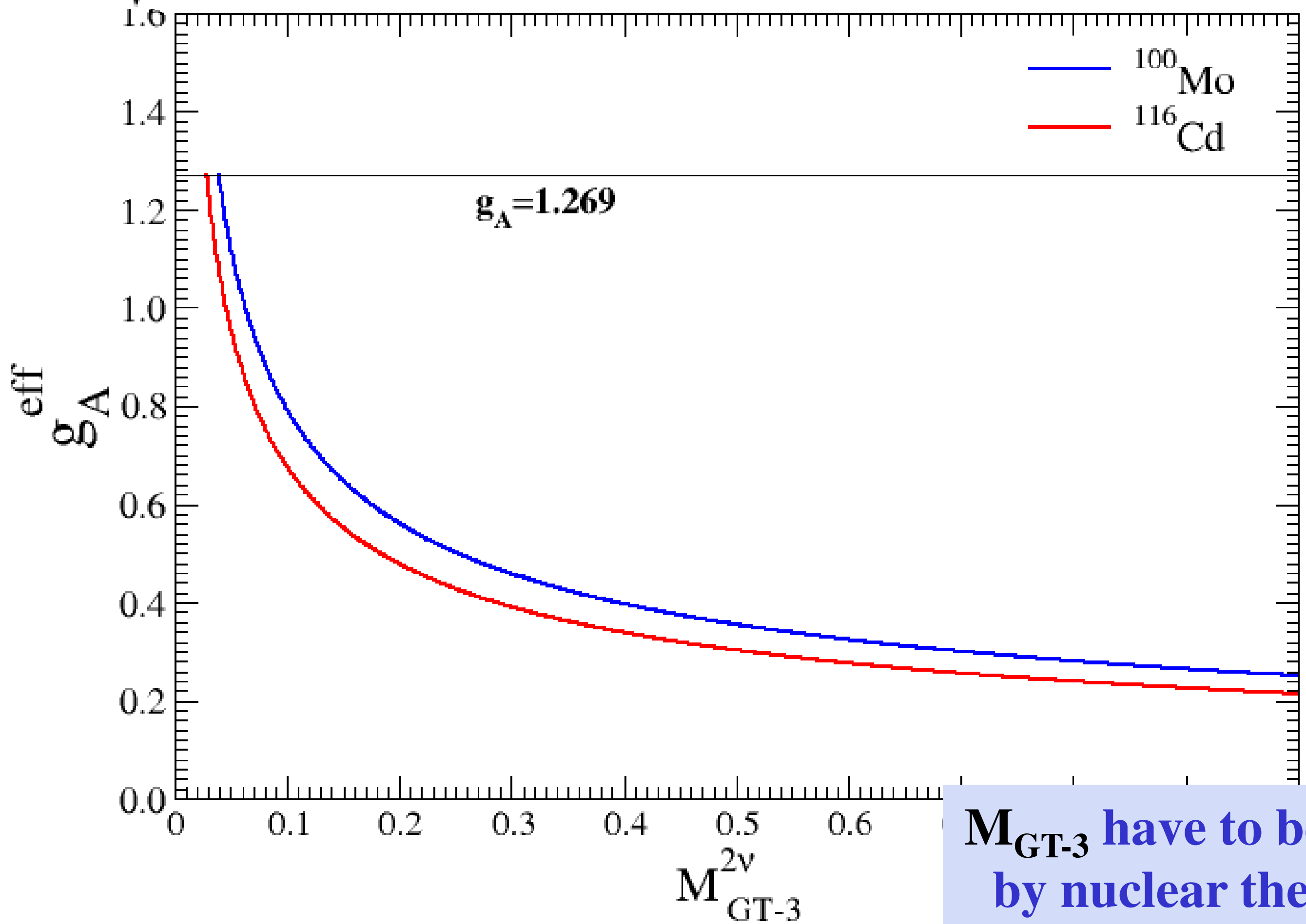


**Solution: NEMO3/SuperNemo measurement of ξ and calculation of $M_{GT-3}$**

$$\left(g_A^{\rm eff}\right)^2 = \frac{1}{\left|M^{2\nu}_{GT-3}\right|}\frac{\left|\xi^{2\nu}_{13}\right|}{\sqrt{T^{2\nu-exp}_{1/2}\left(G^{2\nu}_0 + \xi^{2\nu}_{13} G^{2\nu}_2\right)}}$$

$$g_A^{\rm eff}(^{100}{\rm Mo}) = \frac{0.251}{\sqrt{M^{2\nu}_{GT-3}}}$$

$$g_A^{\rm eff}(^{100}{\rm Cd}) = \frac{0.214}{\sqrt{M^{2\nu}_{GT-3}}}$$




**$M_{GT-3}$ have to be calculated by nuclear theory - ISM**

# Conclusions

- **We presented an improved formalism of the 2νββ-decay, which takes into account the effect of lepton energies in energy denominators**
- **There is one additional parameter $\xi_{13}$, which needs to be fitted for the determination of the 2νββ-decay half-life**
- **The phenomenological determination of the $\xi_{13}$ and calculation of $M_{GT-3}$ (within the ISM) might allow to determine $g_A^{eff}$. The NEMO3 and KamlandZEN analysis are under way.**