

# Effective Value of Weak Axial Coupling: A Review

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NDM2018 (6th Symposium on Neutrinos and Dark Matter in  
Nuclear Physics 2018)

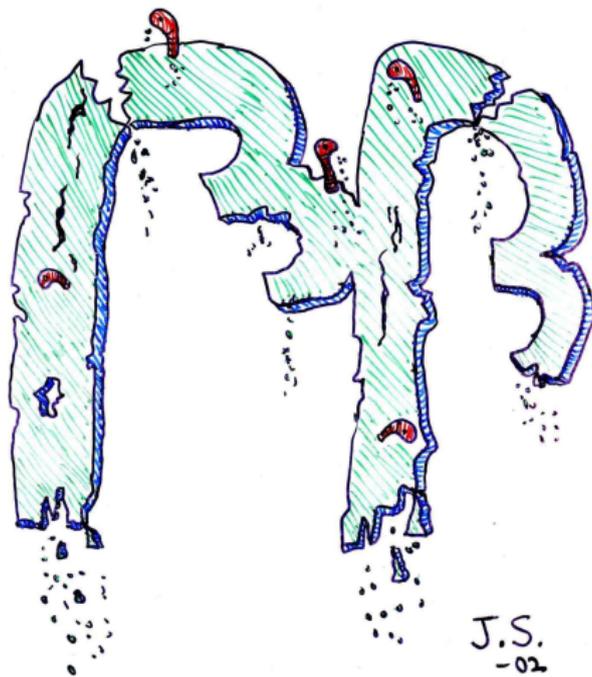
Daejeon, South Korea, June 29 - July 4, 2018



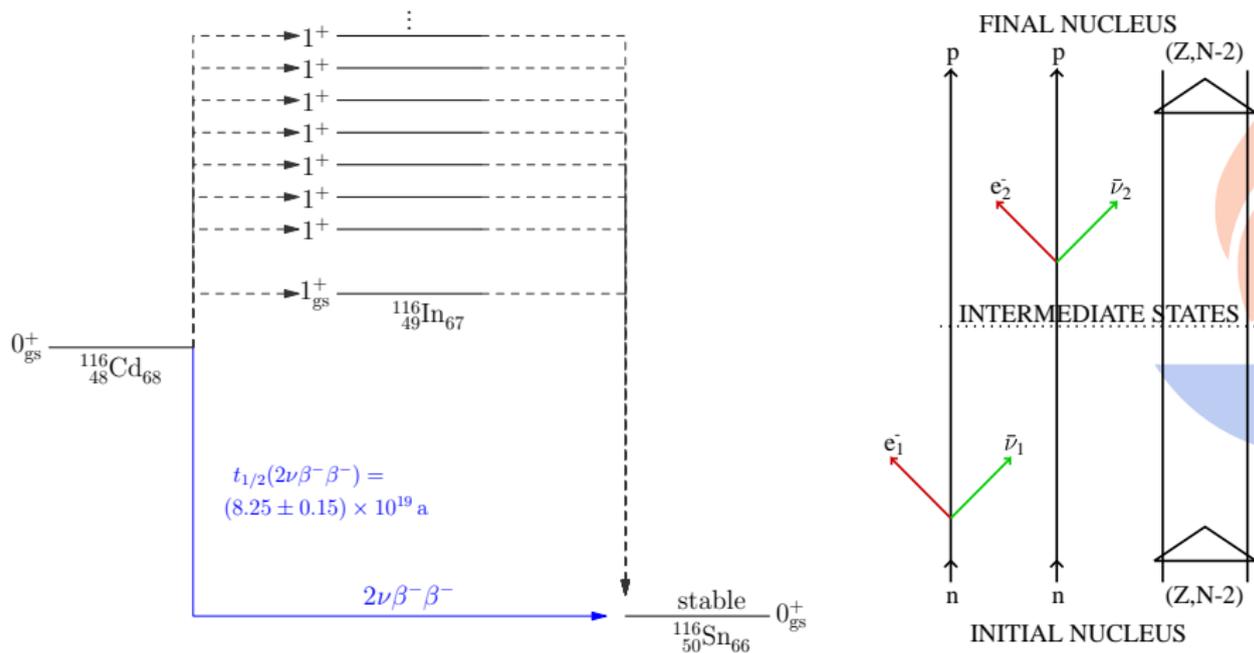
## Contents:

- Double-beta-decay rates
- Effective value of  $g_A$ :
- from allowed  $\beta$  Decays
- from forbidden  $\beta$  Decays
- Effects on  $0\nu\beta\beta$  NMEs
- Is there any reactor- $\bar{\nu}$  anomaly?

# Motivation for the Work: Double Beta Decay

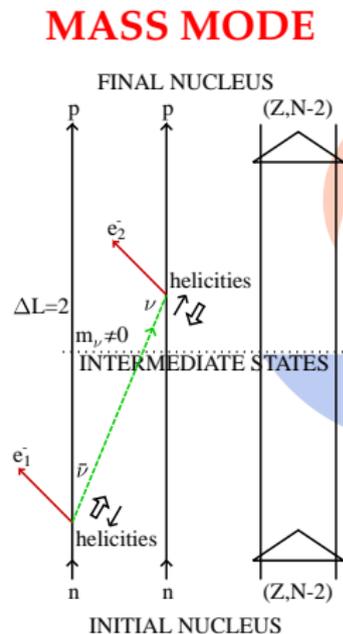
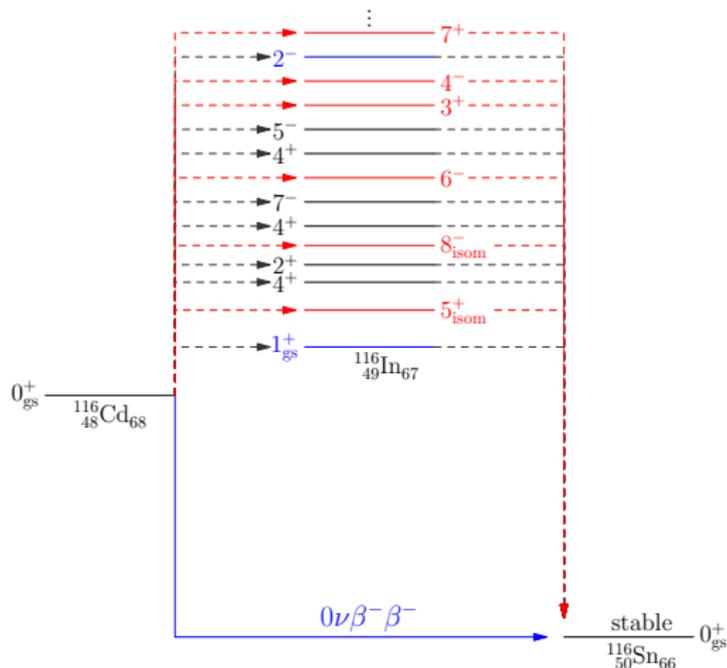


# Two-Neutrino Double Beta Decay of $^{116}\text{Cd}$



$$2\nu\beta\beta - \text{rate} \sim \left| M_{\text{GTGT}}^{(2\nu)} \right|^2 = (g_A)^4 \left| \sum_{m,n} \frac{M_L(1_m^+) M_R(1_n^+)}{D_m} \right|^2$$

# Neutrinoless Double Beta Decay of $^{116}\text{Cd}$



$$0\nu\beta\beta - \text{rate} \sim \left| M_{\text{GTGT}}^{(0\nu)} \right|^2 = (g_{A,0\nu})^4 \left| \sum_{J^\pi} \langle 0^+_{\text{f}} || \mathcal{O}_{\text{GTGT}}^{(0\nu)}(J^\pi) || 0^+_{\text{i}} \rangle \right|^2$$

# Definitions

See also: “Value of the axial-vector coupling strength in  $\beta$  and  $\beta\beta$  decays: A review” published in **Frontiers in Physics** 5 (2017) 55.

Nucleon weak current in a nucleus:

$$j_N^\mu = g_V \gamma^\mu - g_A \gamma^\mu \gamma^5$$

Quenching:

$$q = g_A / g_A^{\text{free}}$$

Free value of  $g_A$  (Particle Data Group 2016) from the decay of free neutron:

$$g_A^{\text{free}} = 1.2723(23)$$

Effective value of  $g_A$ :

$$g_A^{\text{eff}} = q g_A^{\text{free}}$$

# Gamow-Teller $\beta$ decays

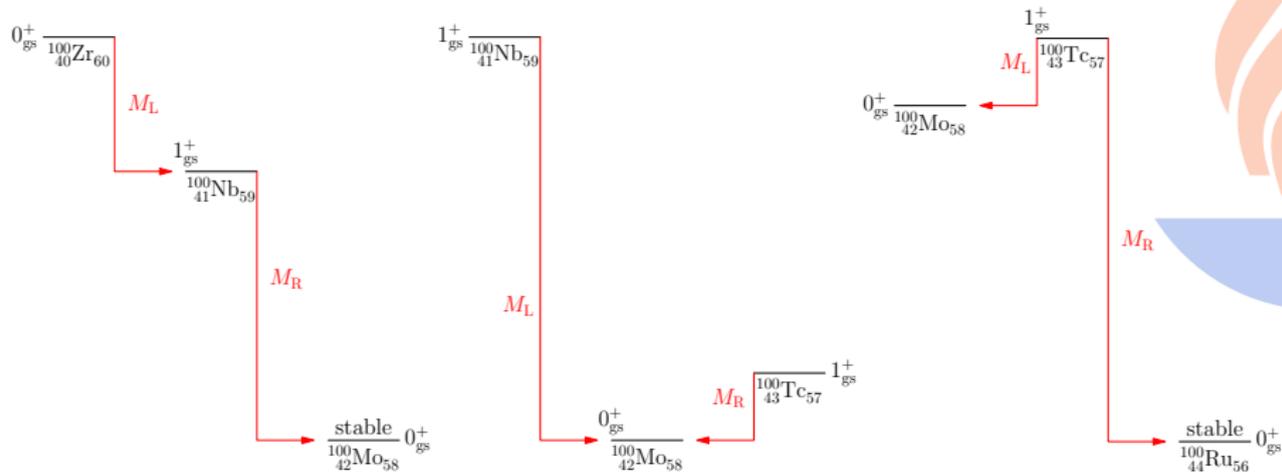
There are data on:

## Gamow-Teller $\beta$ TRANSITIONS

Theoretical approaches:

ISM (Interacting Shell Model)  
pnQRPA (proton-neutron QRPA)

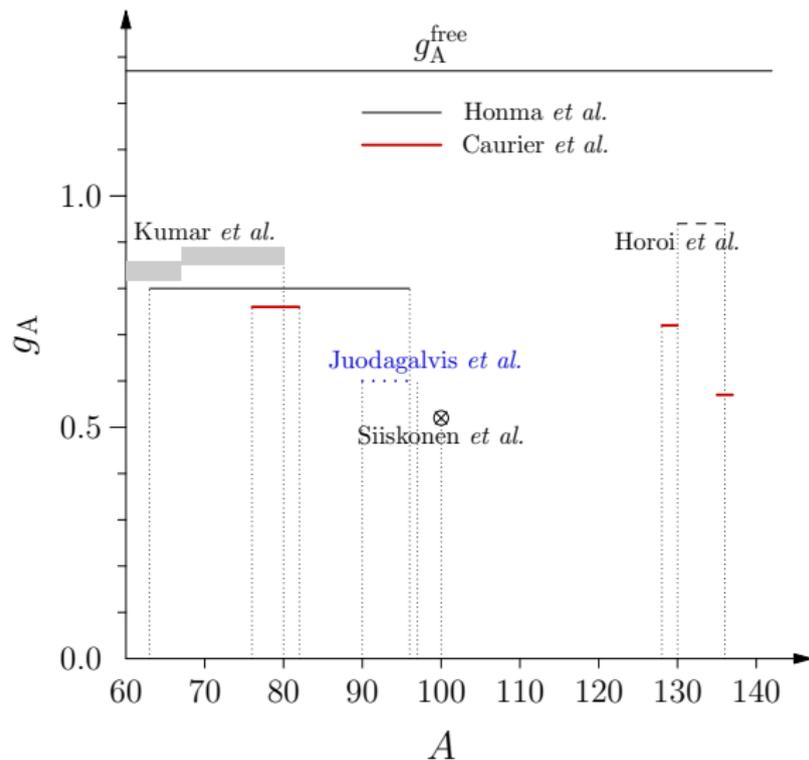
# Typical Gamow-Teller $\beta$ transitions



Results from:

## Quenching of $g_A$ in the ISM calculations

# Results from the ISM



- Kumar *et al.*: J. Phys. G 43 (2016) 105104
- Honma *et al.*: J. Phys. Conf. Ser. 49 (2006) 45
- Caurier *et al.*: Phys. Lett. B 711 (2012) 62
- Horoi *et al.*: Phys. Rev. C 93 (2016) 024308
- Juodagalvis *et al.*: Phys. Rev. C 72 (2005) 024306
- Siiskonen *et al.*: Phys. Rev. C 63 (2001) 055501

# Proton-neutron Quasiparticle Random-Phase Approximation (pnQRPA)

Results from:

## Quenching of $g_A$ in the pnQRPA calculations

# Results from the pnQRPA analyses

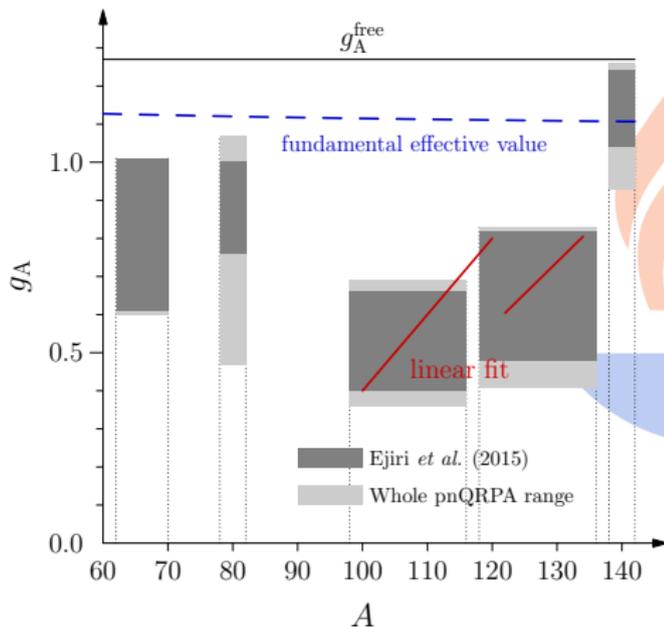
$A$	$pn$ Conf.	$\bar{g}_A^{\text{eff}}$ [1]
62 – 70	$1p_{3/2} - 1p_{1/2}$	$0.81 \pm 0.20$
78 – 82	$0g_{9/2} - 0g_{9/2}$	$0.88 \pm 0.12$
98 – 116	$0g_{9/2} - 0g_{7/2}$	$0.53 \pm 0.13$
118 – 136	$1d_{5/2} - 1d_{5/2}$	$0.65 \pm 0.17$
138 – 142	$1d_{5/2} - 1d_{3/2}$	$1.14 \pm 0.10$

[1] H. Ejiri, J. S., J. Phys. G 42 (2015)  
055201

Other analyses in the whole range:

[2] P. Pirinen, J. S., Phys. Rev. C 91  
(2015) 054309

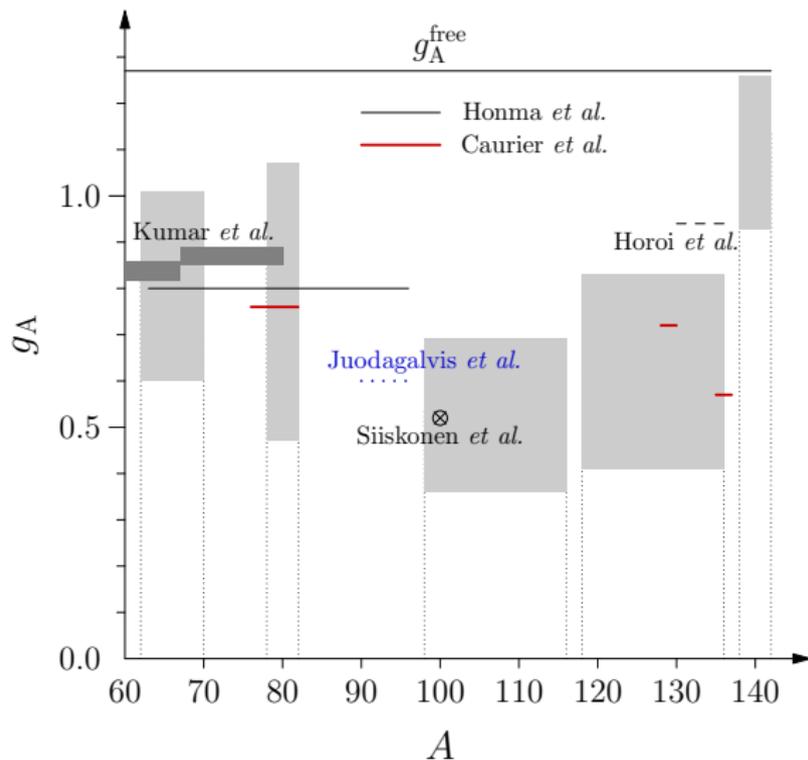
[3] F. Deppisch, J. S., Phys. Rev. C 94  
(2016) 055501



**Fundamental quenching:** M. Ericson (1971); M. Ericson *et al.* (1973);  
M. Rho (1974); D. H. Wilkinson (1974)

(Meson-exchange currents  $\rightarrow$  effective two-body operators)

# Results from the ISM on top of the pnQRPA ranges

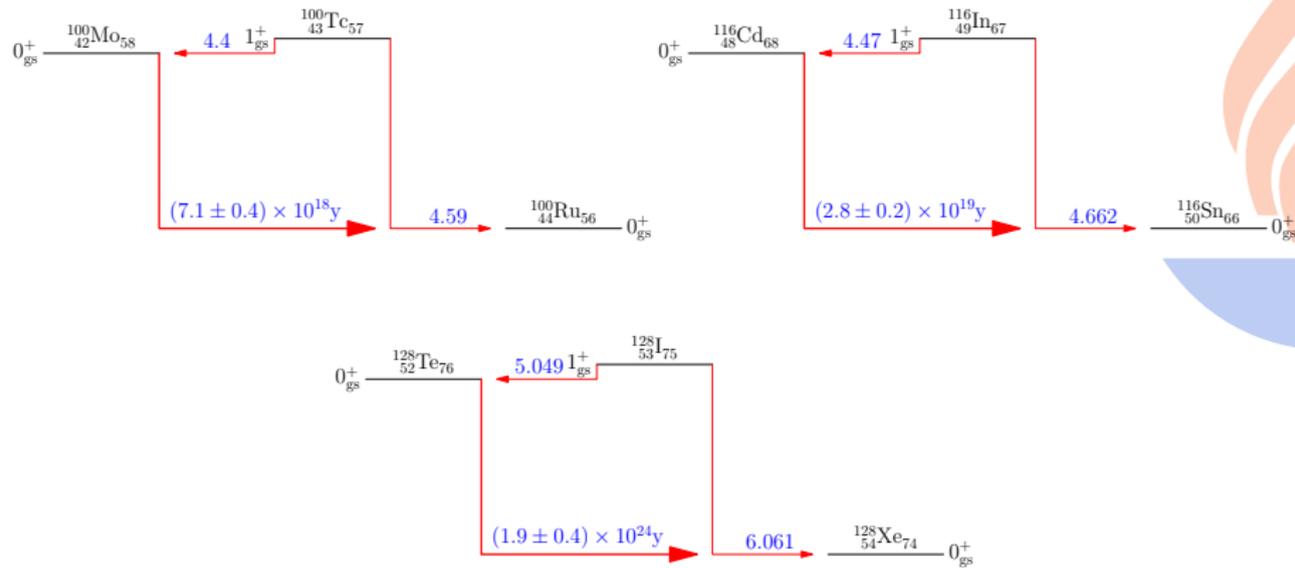


- Kumar *et al.*: J. Phys. G 43 (2016) 105104
- Honma *et al.*: J. Phys. Conf. Ser. 49 (2006) 45
- Caurier *et al.*: Phys. Lett. B 711 (2012) 62
- Horoi *et al.*: Phys. Rev. C 93 (2016) 024308
- Juodagalvis *et al.*: Phys. Rev. C 72 (2005) 024306
- Siiskonen *et al.*: Phys. Rev. C 63 (2001) 055501

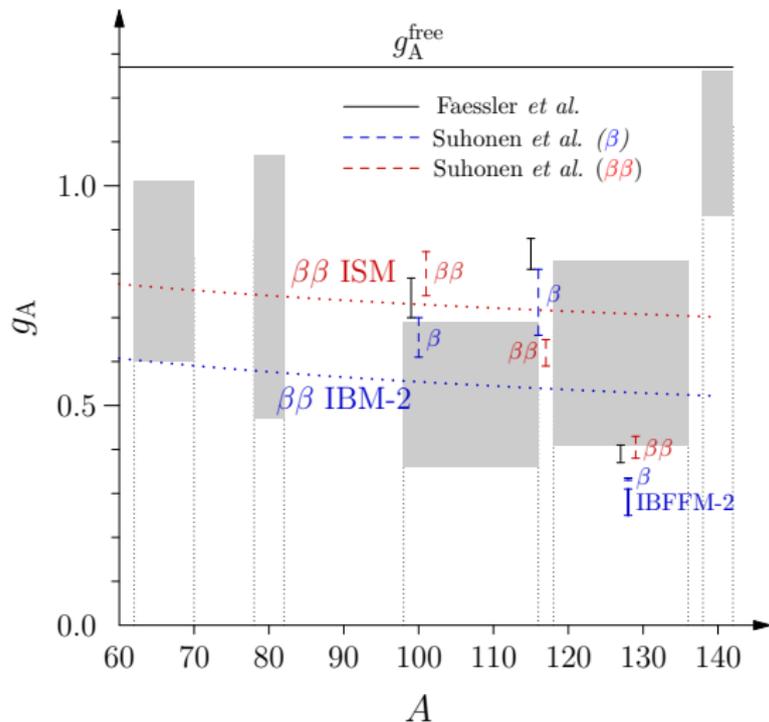
Results from:

Quenching of  $g_A$   
in the pnQRPA-based,  
ISM-based and  
IBM-based calculations  
of  $\beta$  decays and  $\beta\beta$  decays

# The studied cases



# Results from the $\beta+\beta\beta$ calculations against the pnQRPA ranges from Gamow-Teller $\beta$ decays

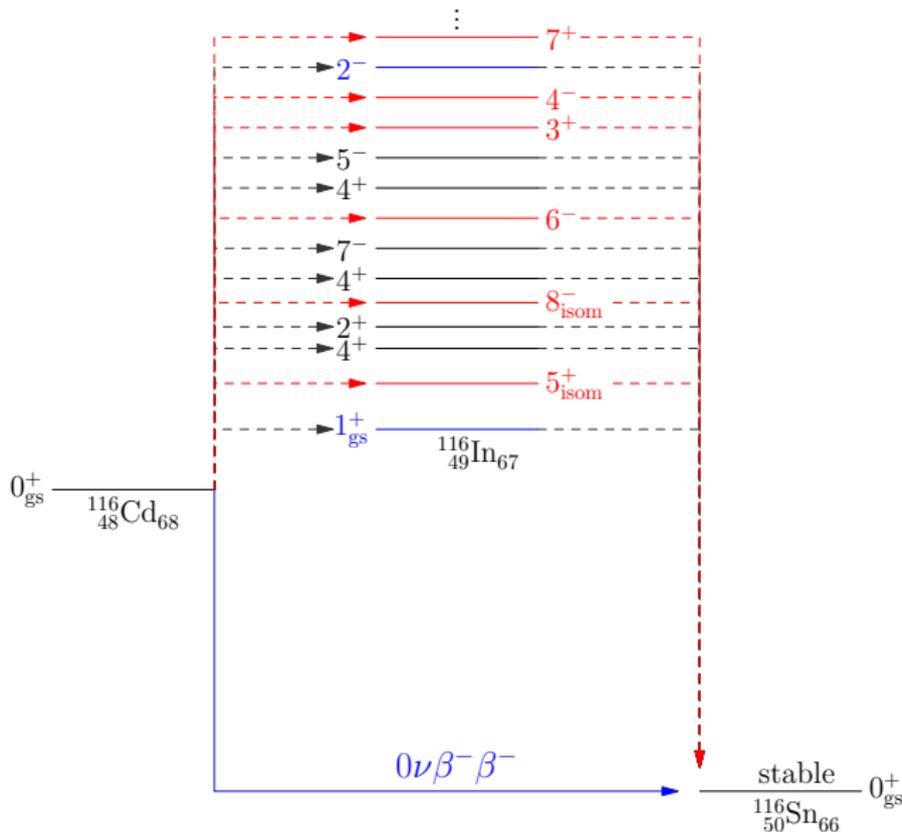


- **pnQRPA: Faessler *et al.***, A. Faessler, G. L. Fogli, E. Lisi, V. Rodin, A. M. Rotunno, F. Šimkovic, arXiv 0711.3996v1 [Nucl-th]
- **pnQRPA: Suhonen *et al.***, J. Suhonen, O. Civitarese, Nucl. Phys. A 924 (2014) 1
- **$\beta\beta$  ISM and IBM-2:** J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 87 (2013) 014315
- **IBFFM-2:** N. Yoshida, F. Iachello, Prog. Theor. Exp. Phys. 2013 (2013) 043D01

Results from:

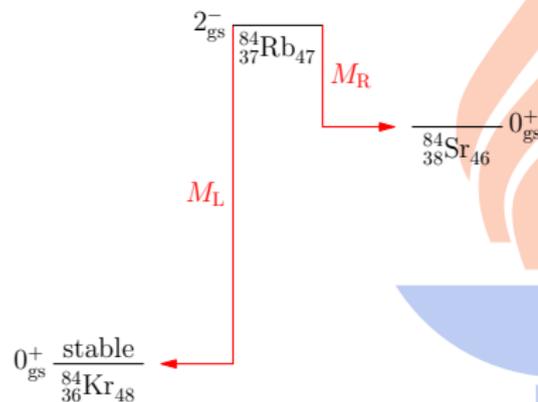
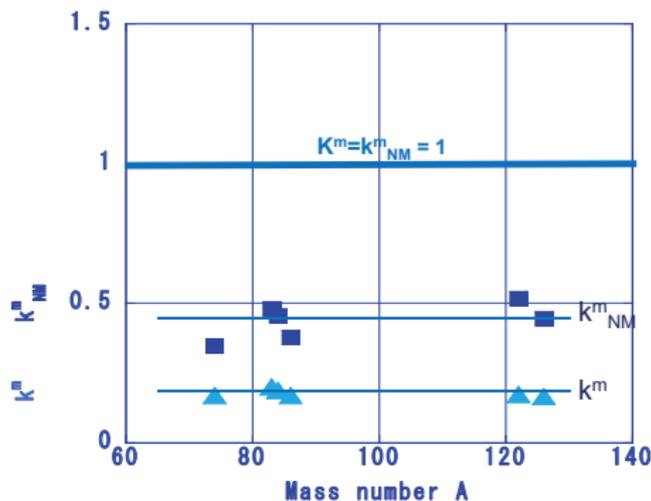
Quenching of  $g_A$   
as derived from  
 $\beta$  decays  
of forbiddenness  $F$

# INCENTIVE: $0\nu\beta\beta$ decay through the higher angular-momentum states



# Global study for the first-forbidden ( $F = 1$ ) $2_{gs}^- \rightarrow 0_{gs}^+$ decays

H. Ejiri, N. Soukouti and J. S., Spin-dipole nuclear matrix elements for double beta decays and astro-neutrinos, Phys. Lett. B 729 (2014) 27



$$\bar{M}(\text{SD}2^-) = \sqrt{M_L M_R}$$

$$\langle k \rangle = \left\langle \frac{\bar{M}_{\text{exp}}(\text{SD}2^-)}{M_{\text{qp}}(\text{SD}2^-)} \right\rangle \approx 0.18$$

$$\langle k_{\text{NM}} \rangle = \left\langle \frac{\bar{M}_{\text{exp}}(\text{SD}2^-)}{M_{\text{pnQRPA}}(\text{SD}2^-)} \right\rangle \approx 0.45$$

$$\Rightarrow \bar{g}_A^{\text{eff}} \approx 0.57$$

## Extrapolation to $\beta$ decays of higher forbiddenness ( $F \geq 2$ )

Based on the global studies in

H. Ejiri, J. S., J. Phys. G: Nucl. Part. Phys. 42 (2015) 055201

H. Ejiri, N. Soukouti, J. S., Phys. Lett. B 729 (2014) 27

J. Kostensalo, J.S., Phys. Rev. C 95 (2017) 014322 conclude that all unique-forbidden  $\beta$  transitions roughly evenly quenched)

**Then:** Low-energy quenching of  $g_A$  derivable from the hatched regions of the Gamow-Teller studies in the pnQRPA framework:

Mass range	$A = 76 - 82$	$A = 100 - 116$	$A = 122 - 136$
$g_{A,0\nu}^{\text{eff}}$	0.5 – 1.0	0.4 – 0.7	0.5 – 0.8

**Assumption:** Also the forbidden non-unique virtual transitions behave like the forbidden unique virtual transitions. **BUT:** How to study the forbidden non-unique decays?

**Caveat:**  $0\nu\beta\beta$  decay is a high-momentum transfer process ( $q \sim 100$  MeV)  $\Rightarrow$  less quenching (J. Menéndez, D. Gazit, A. Schwenk, PRL 107 (2011) 062501)

Results from:

Effective value of  $g_A$   
as derived from  
electron spectra of  
forbidden non-unique  $\beta$  decays

# Spectrum shape of higher-forbidden non-unique $\beta$ decays

Half-life:

$$t_{1/2} = \kappa / \tilde{C}.$$

Dimensionless integrated shape function:

$$\tilde{C} = \int_1^{w_0} C(w_e) p w_e (w_0 - w_e)^2 F_0(Z_f, w_e) dw_e.$$

Shape factor:

$$C(w_e) = \sum_{k_e, k_\nu, K} \lambda_{k_e} \left[ M_K(k_e, k_\nu)^2 + m_K(k_e, k_\nu)^2 - \frac{2\gamma_{k_e}}{k_e w_e} M_K(k_e, k_\nu) m_K(k_e, k_\nu) \right],$$

where

$$\lambda_{k_e} = \frac{F_{k_e-1}(Z, w_e)}{F_0(Z, w_e)}; \quad \gamma_{k_e} = \sqrt{k_e^2 - (\alpha Z_f)^2},$$

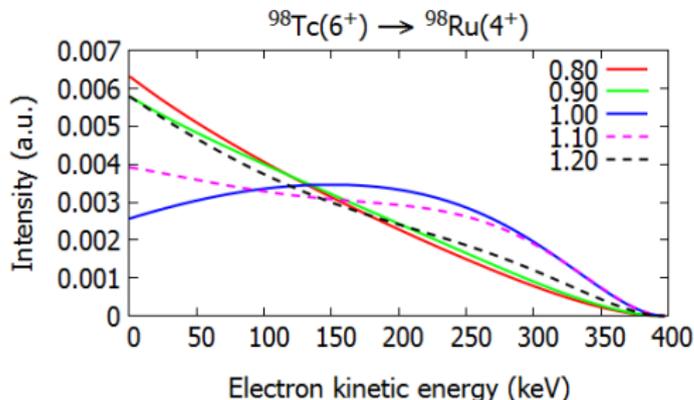
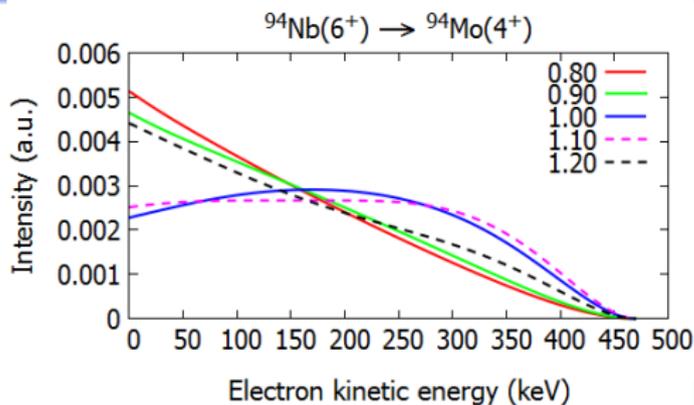
$F_{k-1}(Z, w_e)$  being the generalized Fermi function.

Decomposition of the shape factor:

$$C(w_e) = g_V^2 C_V(w_e) + g_A^2 C_A(w_e) + g_V g_A C_{VA}(w_e).$$

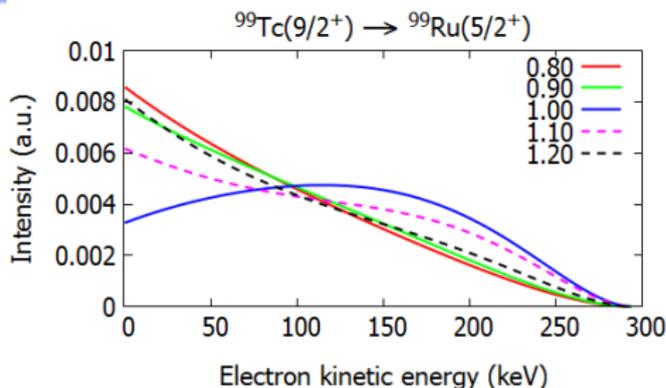
# ISM-computed $\beta$ spectra for different values of $g_A$

Normalized  
ISM-computed  
electron spectra for  
the  $2nd$ -forbidden  
nonunique  $\beta^-$   
decays of  $^{94}\text{Nb}$  and  
 $^{98}\text{Tc}$  ( $g_V = 1.0$ ).

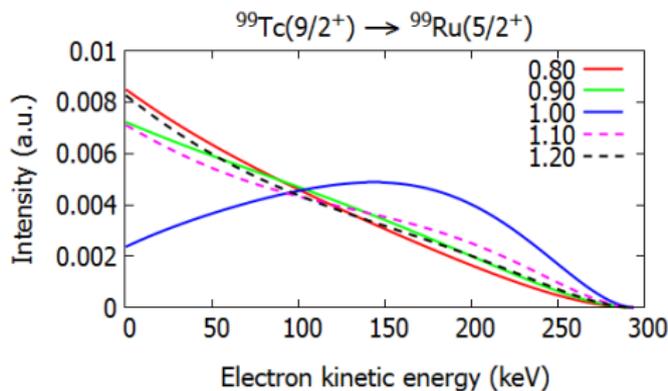


# Example: ISM- and MQPM-computed electron spectra

Normalized ISM-  
and  
MQPM-computed  
electron spectra for  
the 2nd-forbidden  
nonunique  $\beta^-$  decay  
of  $^{99}\text{Tc}$  ( $g_V = 1.0$ )  
using different  
values of  $g_A$ .



(ISM)



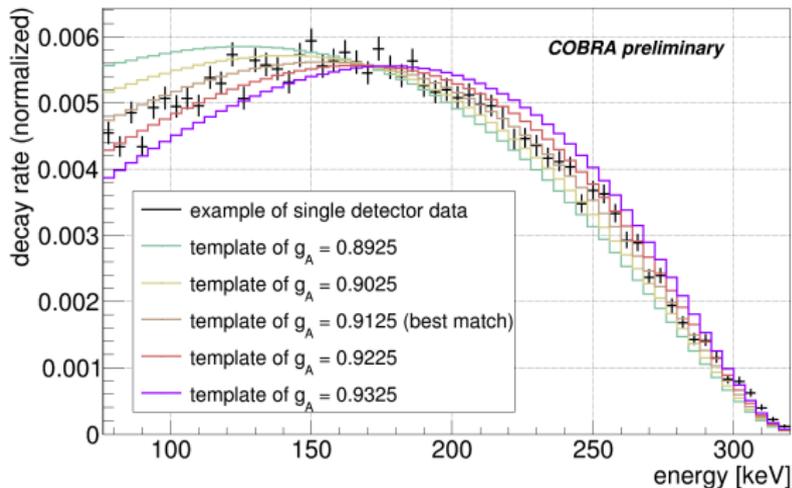
(MQPM)

# Example: Decay of $^{113}\text{Cd}$ – Comparison with data

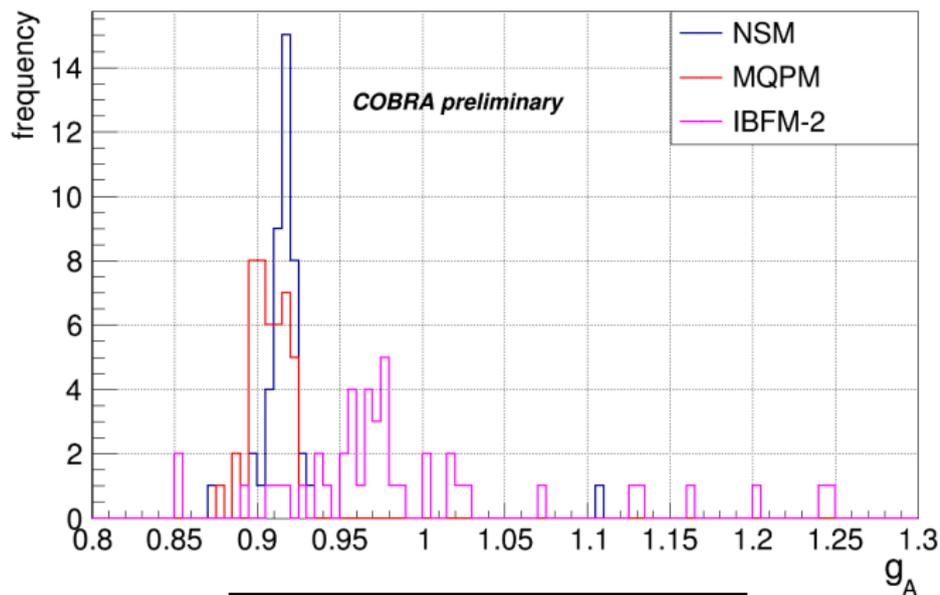
Normalized electron spectra  
for the 4th-forbidden  
nonunique  $\beta^-$  decay  
 $^{113}\text{Cd}(1/2^+) \rightarrow ^{113}\text{In}(9/2^+)$   
( $g_V = 1.0$ ).

Experimental data from:

The **COBRA** collaboration,  
L. Bodenstein-Dresler *et al.*, arXiv:1806.02254  
[nucl-ex] 6 Jun 2018



# Distribution of the best-match $g_A$ values from 44 detector units



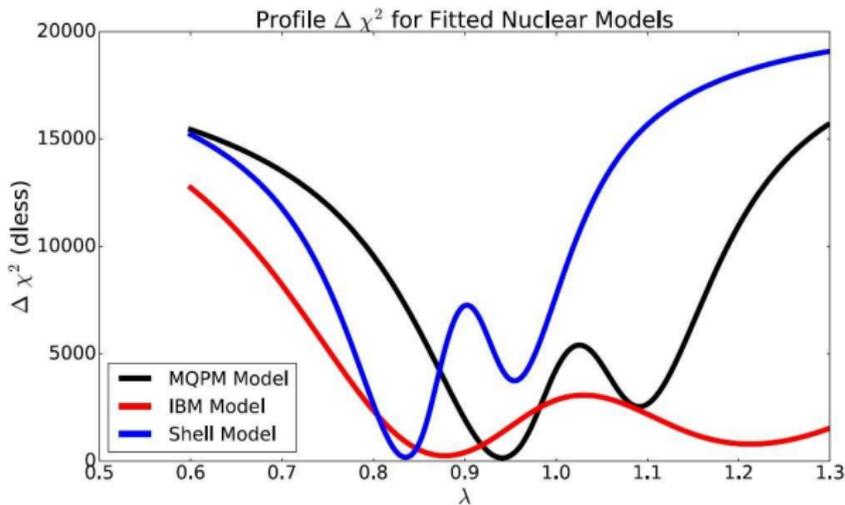
$$\begin{aligned}\bar{g}_A(\text{ISM}) &= 0.92 \pm 0.02 \\ \bar{g}_A(\text{MQPM}) &= 0.91 \pm 0.01 \\ \bar{g}_A(\text{IBFM-2}) &= 0.94 \pm 0.09\end{aligned}$$

# Example: Decay of $^{115}\text{In}$ – Comparison with data

Normalized electron spectra  
for the 4th-forbidden  
nonunique  $\beta^-$  decay  
 $^{115}\text{In}(9/2^+) \rightarrow ^{115}\text{Sn}(1/2^+)$   
( $g_V = 1.0$ ).

Result from:

The MIT-CSNSM-Jyväskylä  
collaboration, A. Leder *et al.*, to be submitted.



$$\begin{aligned}\bar{g}_A(\text{ISM}) &= 0.83 \pm 0.03 \\ \bar{g}_A(\text{IBFM-2}) &= 0.88 \pm 0.06 \\ \bar{g}_A(\text{MQPM}) &= 0.94^{+0.03}_{-0.04}\end{aligned}$$

# Summary of the exploratory work on $\beta$ spectra

Transition	$J_i^{\pi_i}$ (gs)	$J_f^{\pi_f}$ ( $n_f$ )	Branching	K	Sensitivity	Nucl. model
$^{36}\text{Cl} \rightarrow ^{36}\text{Ar}$	2+	0+ (gs)	<b>98%</b>	2	None	ISM
$^{48}\text{Ca} \rightarrow ^{48}\text{Sc}$	0+	4+ (2)	$\sim 0\%$	4	None	ISM
$^{48}\text{Ca} \rightarrow ^{48}\text{Sc}$	0+	6+ (gs)	$\sim 0\%$	6	None	ISM
$^{50}\text{V} \rightarrow ^{50}\text{Cr}$	6+	2+ (1)	$\sim 0\%$	4	Weak	ISM
$^{60}\text{Fe} \rightarrow ^{60}\text{Co}$	0+	2+ (1)	<b>100%</b>	2	None	ISM
$^{85}\text{Br} \rightarrow ^{85}\text{Kr}$	3/2-	9/2+ (gs)	$\sim 0\%$	3	Moderate	MQPM
$^{87}\text{Rb} \rightarrow ^{87}\text{Sr}$	3/2-	9/2+ (gs)	<b>100%</b>	3	<b>Moderate</b>	MQPM, ISM
$^{92}\text{Rb} \rightarrow ^{92}\text{Sr}$	0-	0+ (gs)	<b>95%</b>	1	Weak	ISM
$^{93}\text{Zr} \rightarrow ^{93}\text{Nb}$	5/2+	9/2+ (gs)	$5 \leq \%$	2	Weak	MQPM
$^{93}\text{Y} \rightarrow ^{93}\text{Zr}$	1/2-	1/2+ (1)	<b>2%</b>	1	Moderate	ISM
$^{94}\text{Nb} \rightarrow ^{94}\text{Mo}$	6+	4+ (2)	<b>100%</b>	<b>2</b>	<b>Strong</b>	NSM
$^{95}\text{Sr} \rightarrow ^{95}\text{Y}$	1/2+	1/2- (gs)	<b>56%</b>	1	Weak	ISM
$^{96}\text{Zr} \rightarrow ^{96}\text{Nb}$	0+	4+ (2)	$\sim 0\%$	4	None	ISM
$^{96}\text{Zr} \rightarrow ^{96}\text{Nb}$	0+	6+ (gs)	$\sim 0\%$	6	<b>Strong</b>	ISM
$^{96}\text{Y} \rightarrow ^{96}\text{Zr}$	0-	0+ (gs)	<b>96%</b>	1	weak	ISM
$^{97}\text{Zr} \rightarrow ^{97}\text{Nb}$	1/2+	9/2+ (gs)	$\sim 0\%$	4	<b>Strong</b>	MQPM
$^{97}\text{Y} \rightarrow ^{97}\text{Zr}$	1/2+	1/2- (gs)	<b>40%</b>	1	Weak	ISM
$^{98}\text{Tc} \rightarrow ^{98}\text{Ru}$	6+	4+ (3)	<b>100%</b>	<b>2</b>	<b>Strong</b>	ISM
$^{99}\text{Tc} \rightarrow ^{99}\text{Ru}$	9/2+	5/2+ (gs)	<b>100%</b>	<b>2</b>	<b>Strong</b>	MQPM, ISM

# Summary on $\beta$ spectra continues . . .

Transition	$J_i^{\pi_i}$ (gs)	$J_f^{\pi_f}$ ( $n_f$ )	Branching	K	Sensitivity	Nucl. model
$^{101}\text{Mo} \rightarrow ^{101}\text{Tc}$	1/2 <sup>+</sup>	9/2 <sup>+</sup> (gs)	~0%	4	<b>Strong</b>	MQPM
$^{113}\text{Cd} \rightarrow ^{113}\text{In}$	1/2 <sup>+</sup>	9/2 <sup>+</sup> (gs)	<b>100%</b>	<b>4</b>	<b>Strong</b>	MQPM, ISM, IBFM-2
$^{115}\text{Cd} \rightarrow ^{115}\text{In}$	1/2 <sup>+</sup>	9/2 <sup>+</sup> (gs)	~0%	4	<b>Strong</b>	MQPM
$^{115}\text{In} \rightarrow ^{115}\text{Sn}$	9/2 <sup>+</sup>	1/2 <sup>+</sup> (gs)	<b>100%</b>	<b>4</b>	<b>Strong</b>	MQPM, ISM, IBFM-2
$^{117}\text{Cd} \rightarrow ^{117}\text{In}$	1/2 <sup>+</sup>	9/2 <sup>+</sup> (gs)	~0%	4	<b>Strong</b>	MQPM
$^{119}\text{In} \rightarrow ^{119}\text{Sn}$	9/2 <sup>+</sup>	1/2 <sup>+</sup> (gs)	~0%	4	<b>Strong</b>	MQPM
$^{123}\text{Sn} \rightarrow ^{123}\text{Sb}$	11/2 <sup>-</sup>	1/2 <sup>+</sup> (4)	~0%	5	Weak	MQPM
$^{125}\text{Sb} \rightarrow ^{125}\text{Te}$	7/2 <sup>+</sup>	9/2 <sup>-</sup> (3)	7.2%	1	None	MQPM
$^{126}\text{Sn} \rightarrow ^{126}\text{Sb}$	0 <sup>+</sup>	2 <sup>+</sup> (5)	<b>100%</b>	2	None	ISM
$^{133}\text{Sn} \rightarrow ^{133}\text{Sb}$	7/2 <sup>-</sup>	7/2 <sup>+</sup> (gs)	<b>85%</b>	1	Weak	ISM
$^{134}\text{Sb} \rightarrow ^{134}\text{Te}$	0 <sup>-</sup>	0 <sup>+</sup> (gs)	<b>98%</b>	1	Weak	ISM
$^{135}\text{Cs} \rightarrow ^{135}\text{Ba}$	7/2 <sup>+</sup>	3/2 <sup>+</sup> (gs)	<b>100%</b>	2	None	MQPM
$^{135}\text{Te} \rightarrow ^{135}\text{I}$	7/2 <sup>-</sup>	7/2 <sup>+</sup> (gs)	<b>62%</b>	1	Weak	ISM
$^{137}\text{Cs} \rightarrow ^{137}\text{Ba}$	7/2 <sup>+</sup>	3/2 <sup>+</sup> (gs)	5.4%	2	None	MQPM, ISM
$^{137}\text{Xe} \rightarrow ^{137}\text{Cs}$	7/2 <sup>-</sup>	7/2 <sup>+</sup> (gs)	<b>67%</b>	1	Weak	ISM
$^{138}\text{Cs} \rightarrow ^{138}\text{Ba}$	3 <sup>-</sup>	3 <sup>+</sup> (1)	<b>44%</b>	1	<b>Strong</b>	ISM
$^{139}\text{Ba} \rightarrow ^{139}\text{La}$	7/2 <sup>-</sup>	7/2 <sup>+</sup> (gs)	<b>70%</b>	1	Weak	ISM
$^{139}\text{Cs} \rightarrow ^{139}\text{Ba}$	7/2 <sup>+</sup>	7/2 <sup>-</sup> (gs)	<b>85%</b>	1	Weak	ISM

# Summary on $\beta$ spectra continues . . .

Transition	$J_i^{\pi_i}$ (gs)	$J_f^{\pi_f}$ ( $n_f$ )	Branching	K	Sensitivity	Nucl. model
$^{141}\text{Ce} \rightarrow ^{141}\text{Pr}$	$7/2^-$	$5/2^+$ (gs)	31%	1	Weak	MQPM
$^{142}\text{Pr} \rightarrow ^{142}\text{Nb}$	$2^-$	$2^+$ (1)	3.7%	1	Weak	ISM
$^{143}\text{Pr} \rightarrow ^{143}\text{Nb}$	$7/2^+$	$7/2^-$ (gs)	<b>100%</b>	1	Weak	ISM
$^{159}\text{Gd} \rightarrow ^{159}\text{Tb}$	$3/2^-$	$5/2^+$ (1)	26%	1	None	MQPM
$^{161}\text{Tb} \rightarrow ^{161}\text{Dy}$	$3/2^+$	$5/2^-$ (1)	$\sim 0\%$	1	None	MQPM
$^{169}\text{Er} \rightarrow ^{169}\text{Tm}$	$1/2^-$	$3/2^+$ (1)	<b>45%</b>	1	None	MQPM
$^{210}\text{Bi} \rightarrow ^{210}\text{Po}$	$1^-$	$0^+$ (gs)	<b>100%</b>	1	<b>Strong</b>	ISM
$^{211}\text{Pb} \rightarrow ^{211}\text{Bi}$	$9/2^+$	$9/2^-$ (gs)	<b>91%</b>	1	Weak	ISM
$^{213}\text{Bi} \rightarrow ^{213}\text{Po}$	$9/2^-$	$9/2^+$ (gs)	<b>66%</b>	1	Weak	ISM

# Effects of quenched values of $g_A$

Results from:

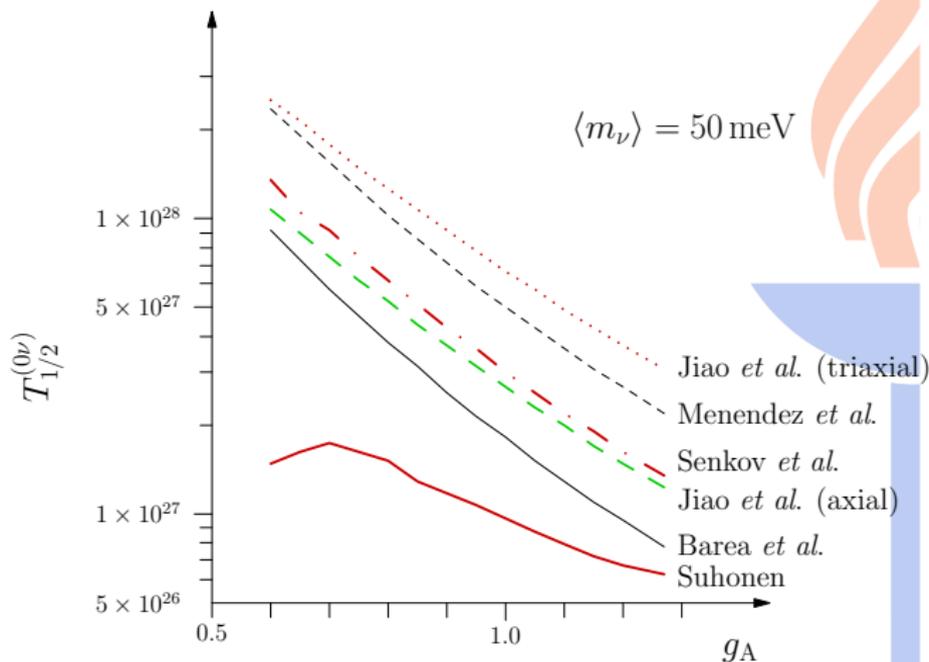
Effects of a quenched  $g_A$   
on NMEs of  $0\nu\beta\beta$  decays:

$$\left[ T_{1/2}^{(0\nu)} \right]^{-1} = (g_{A,0\nu})^4 G^{(0\nu)} |M^{(0\nu)}|^2 \left( \frac{\langle m_\nu \rangle}{m_e} \right)^2$$

$$M^{(0\nu)} = M_{\text{GT}}^{(0\nu)} - \left( \frac{g_V}{g_{A,0\nu}} \right)^2 M_{\text{F}}^{(0\nu)} + M_{\text{T}}^{(0\nu)}$$

# Example: $0\nu\beta\beta$ NMEs of $^{76}\text{Ge}$ , effect on the half-life

- **Jiao *et al.*:** Phys. Rev. C 96 (2017) 054310 (GCM+ISM)
- **Menendez *et al.*:** Nucl. Phys. A 818 (2009) 139 (ISM)
- **Senkov *et al.*:** Phys. Rev. C 93 (2016) 044334 (ISM)
- **Barea *et al.*:** Phys. Rev. C 91 (2015) 034304 (IBM-2)
- **Suhonen:** Phys. Rev. C 96 (2017) 055501 (pnQRPA + isospin restoration + data on  $2\nu\beta\beta$ )



# Novel application of electron spectra of forbidden decays

Try to investigate:

## Reactor- $\bar{\nu}$ anomaly and the spectral shoulder

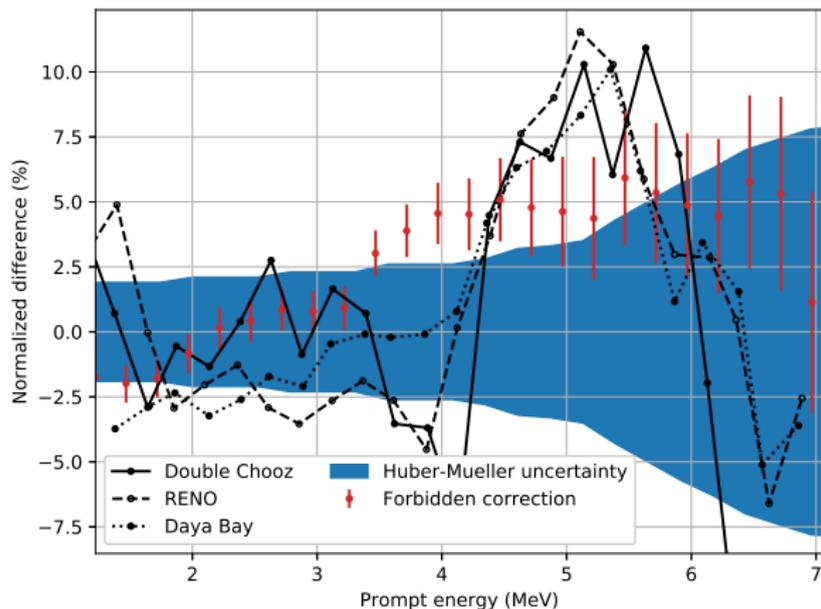
See: L. Heyen, J. Kostensalo, N. Severijns, J.S., First forbidden transitions in the reactor anomaly, arXiv:1805.12259 [nucl-th] 30 May 2018

# Results from the analyses (see the parallel talk of Joel Kostensalo on Tuesday evening!)

Taking into account the  
(first-forbidden)  
decays of

$^{86}\text{Br}(0^+)$ ,  $^{86}\text{Br}(2^+)$ ,  $^{87}\text{Se}$ ,  $^{88}\text{Rb}$ ,  
 $^{89}\text{Br}(3/2^+)$ ,  $^{89}\text{Br}(5/2^+)$ ,  $^{90}\text{Rb}$ ,  
 $^{91}\text{Kr}(5/2^-)$ ,  $^{91}\text{Kr}(3/2^-)$ ,  $^{92}\text{Rb}$ ,  
 $^{92}\text{Y}$ ,  $^{93}\text{Rb}$ ,  $^{94}\text{Y}(0^+)$ ,  $^{94}\text{Y}(0^+)$ ,  
 $^{95}\text{Rb}(7/2^+)$ ,  $^{95}\text{Rb}(3/2^+)$ ,  $^{95}\text{Sr}$ ,  
 $^{96}\text{Y}$ ,  $^{97}\text{Y}$ ,  $^{98}\text{Y}$ ,  $^{133}\text{Sn}$ ,  $^{134m}\text{Sb}(6^+)$ ,  
 $^{134m}\text{Sb}(6^+?)$ ,  $^{135}\text{Te}$ ,  $^{136m}\text{I}$ ,  $^{137}\text{I}$ ,  
 $^{138}\text{I}$ ,  $^{139}\text{Xe}$ ,  $^{140}\text{Cs}$ ,  $^{142}\text{Cs}$

decreases the  $\bar{\nu}$  flux by  
5% !



The spectral sholder appears due to forbidden  
spectral corrections !

# Conclusions and Outlook

## Conclusions:

- The long chain of ISM calculations and the recent pnQRPA and IBM-2 calculations of Gamow-Teller  $\beta$  decays and  $2\nu\beta\beta$  decays are (surprisingly!) consistent with each other and clearly point to a **A-dependent quenched  $g_A$**
- Studies on GT  $1^+$  and SD  $2^-$   $\beta$  decays shed light on the **suppression chain: quasiparticle NME  $\rightarrow$  pnQRPA NME  $\rightarrow$  experimental NME**
- Studies of **high-forbidden** unique  $\beta$  decays ( $F \geq 2$ )  $\rightarrow$  uniform quenching  $\rightarrow$  speculations about modifications in the  **$0\nu\beta\beta$ -decay half-lives**
- The **spectrum-shape method (SSM)** for forbidden non-unique  $\beta$  decays is a **robust tool** (largely independent of the nuclear model, the assumed Hamiltonian and mean field) to search for the **effective value of  $g_A$**  and to try to solve other problems, like those related to the **reactor- $\bar{\nu}$  spectra**.

## Outlook:

- Urge **measurements of the  $\beta$  spectra** for the interesting decays amenable to the SSM
- The **effective value of  $g_A$**  is involved in all weak processes, and thus has impact on **studies of rare  $\beta$  decays, neutrino physics and astrophysics**