

Institute for Basic Science HQ, Daejeon, Korea

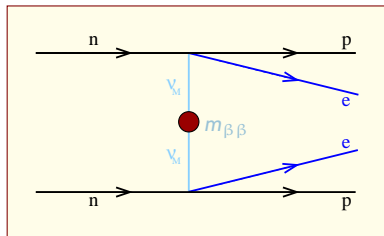
Ab-Initio Theory and $\beta\beta$ Decay

조나단 엔겔

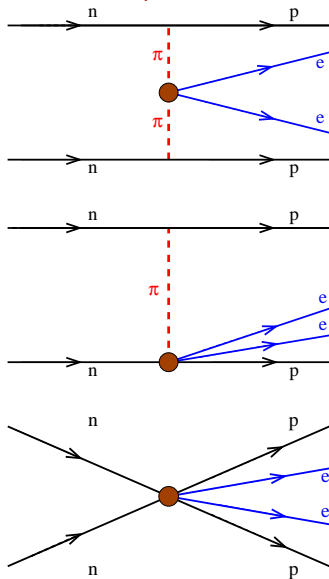
July 2, 2018

Review of $0\nu\beta\beta$ Decay

Standard operator



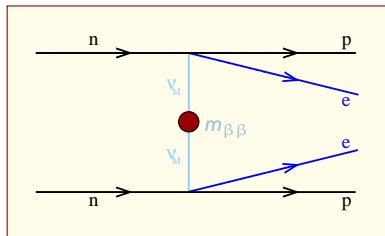
Other possibilities



Forbidden in Standard Model.
New physics inside blobs.

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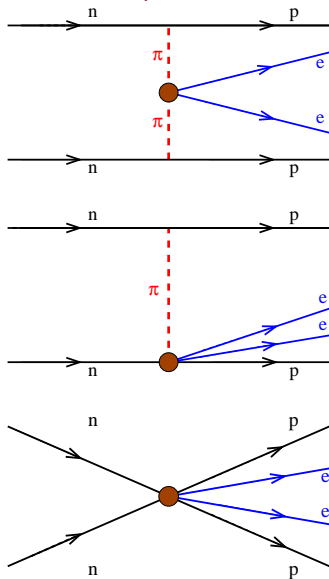
Standard operator



I'll focus on this one.

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New physics inside blobs.

Other possibilities



Nuclear Matrix Element (Simplified)

$$M^{0\nu} = g_A^2 M_{GT}^{0\nu} - g_V^2 M_F^{0\nu} + \dots$$

Dominant
piece

with

$$M_{GT}^{0\nu} = \langle f | \sum_{a,b} H_{GT}(r_{ab}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle$$

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Also:

$$M_{2\nu} = g_A^2 \sum_m \frac{\langle f | \sum_a \vec{\sigma}_a \tau_a^+ | m \rangle \cdot \langle m | \sum_b \vec{\sigma}_b \tau_b^+ | i \rangle}{E_m - \frac{E_f + E_i}{2}}$$

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But the idea that there is a single “ g_A in medium” is too much of a simplification.

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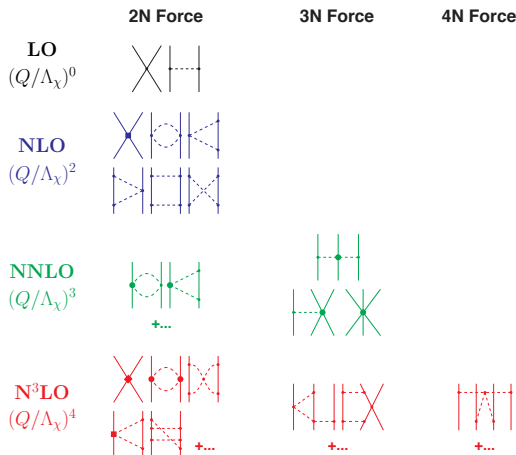
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Ab Initio Nuclear Structure

Often starts with chiral effective-field theory

Nucleons, pions sufficient below chiral-symmetry breaking scale.
Expansion of operators in powers of Q/Λ_χ .

$Q = m_\pi$ or typical nucleon momentum.



Many-Body Methods

All require many CPU-hours.

- ▶ Quantum Monte Carlo in light nuclei: More or less exact solution of many-body Schrödinger equation.

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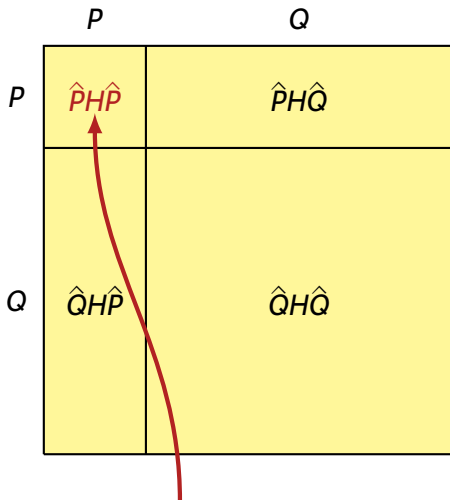
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- ▶ \vdots

Ab Initio Shell Model

Partition of Full Hilbert Space



Shell model done here.

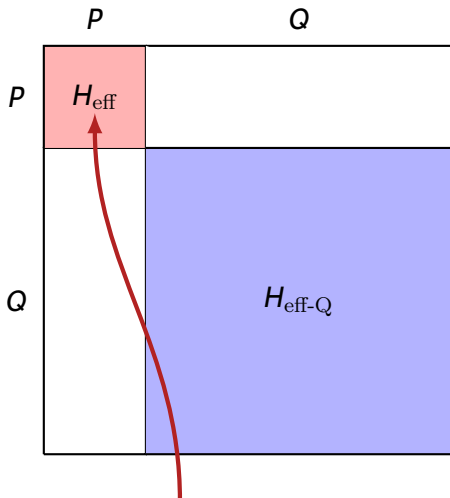
P = valence space

Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q , with H_{eff} in P reproducing d most important eigenvalues.

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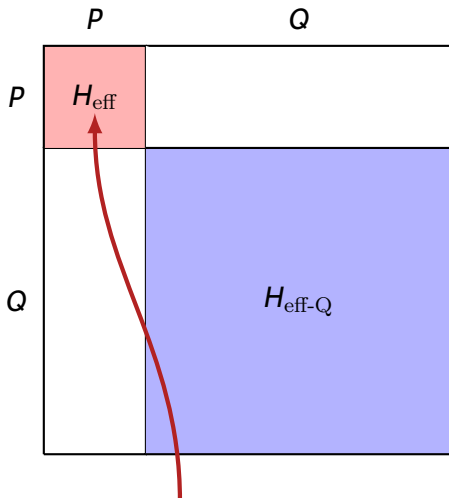
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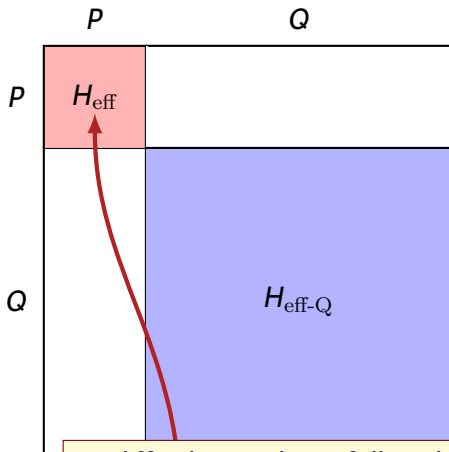
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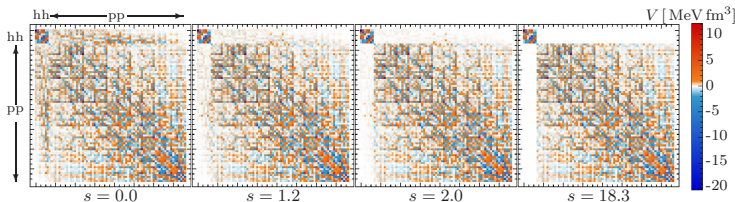
As difficult as solving full problem. But idea is that N-body effective operators beyond $N > 2$ or 3 can be treated approximately.

Shell model done here.

In-Medium Similarity Renormalization Group

One way to determine the transformation

Flow equation for effective Hamiltonian.
Gradually decouples shell-model space.



Hergert et al.

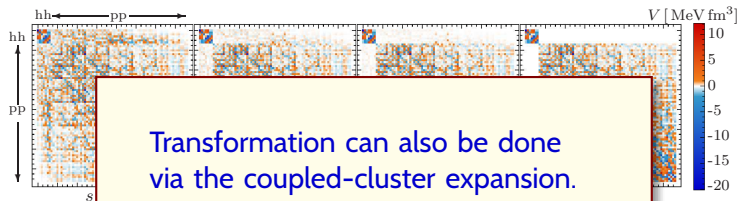
Trick is to keep all 1- and 2-body terms in H at each step *after normal ordering* (approximate treatment of 3-, 4- ... terms).

If shell-model space contains just a single state, approach yields ground-state energy. If it is a typical valence space, result is effective interaction and operators.

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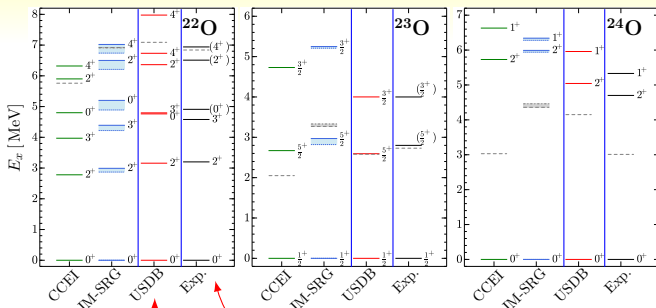
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Ab Initio Calculations of Spectra



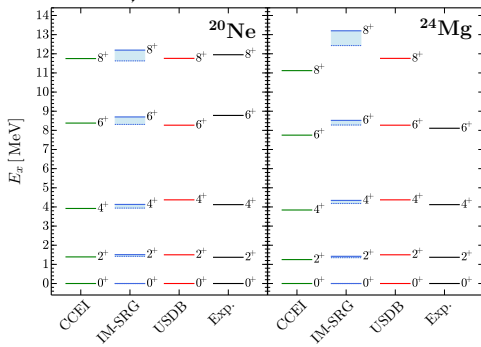
Neutron-rich
oxygen isotopes

Ab
initio

Phenom.

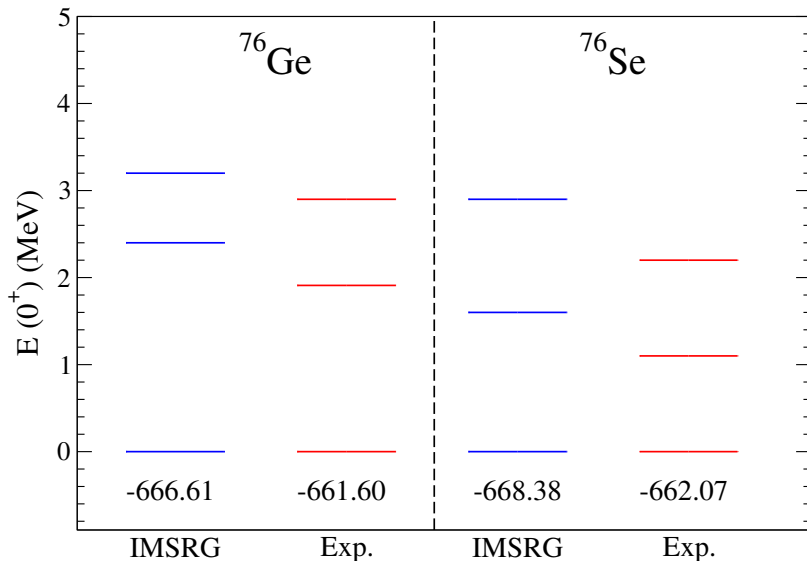
Expt.

Deformed nuclei



Ab Initio ^{76}Ge and ^{76}Se

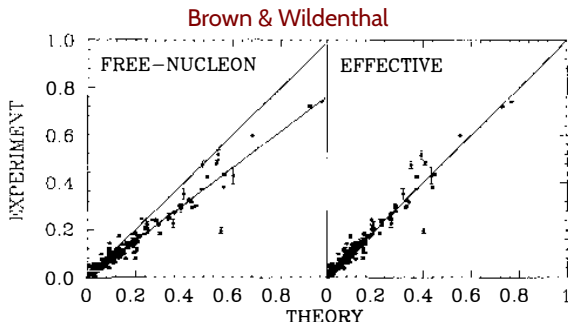
Stroberg, Holt, et al.



Gamow-Teller β Decay

Leading order decay operator is $\vec{\sigma}\tau_+$.

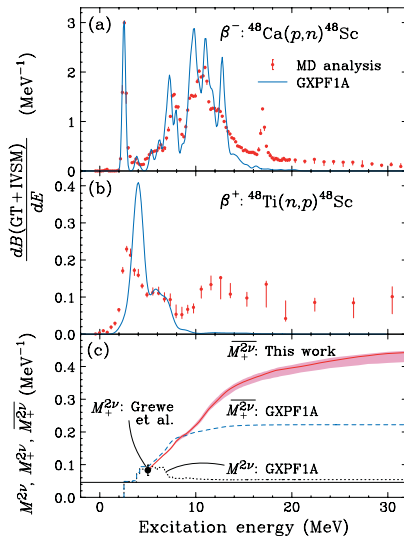
50-Year-Old Problem: Effective g_A needed in all calculations of shell-model (or related) type.



Many suggestions about the cause but, until recently, no consensus.

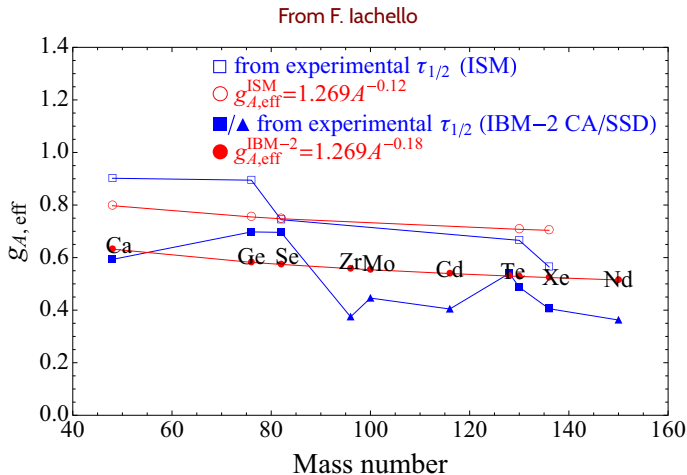
Other Tests of $\vec{\sigma}\tau$ Strength Also Show Suppression

From Yako et al., PRL 103, 012503 (2009)



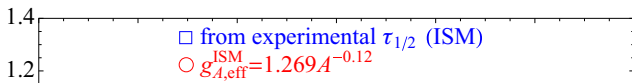
Only about 2/3 of theoretically expected strength observed.

And $2\nu\beta\beta$ Decay...



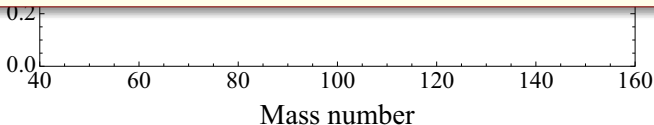
And $2\nu\beta\beta$ Decay...

From F. Iachello



What explains all the over-prediction of matrix elements?

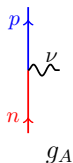
In ab initio calculation with chiral EFT, the answer must be a combination of many-body approximations and truncation of chiral expansion of current operator.



Axial Weak Current in Chiral EFT

β Decay (simplified) with electron lines omitted

Leading order:



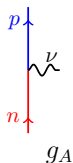
Usual β -decay current.

Finite-momentum corrections at next order.

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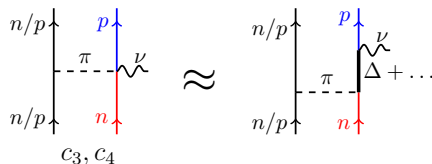
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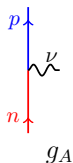
Higher order:



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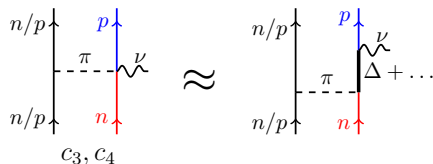
Leading order:



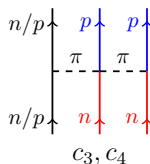
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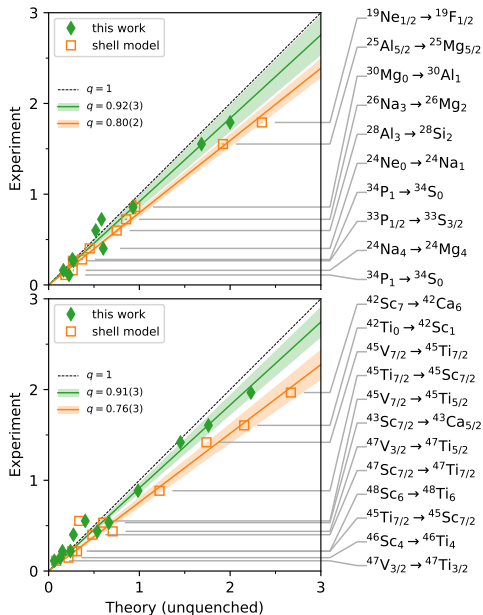
Higher order:



Coefficients same as in three-body interaction:



Quenching in the *sd* and *pf* Shells



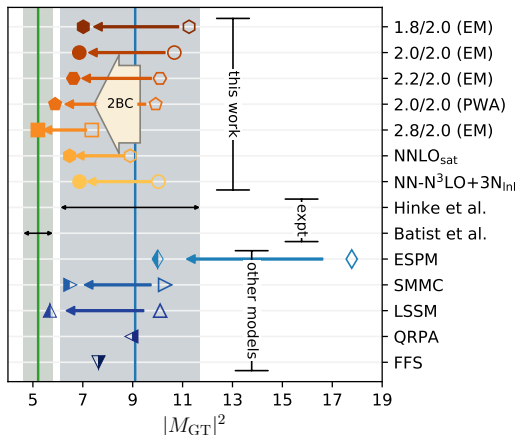
IMSRG calculation, Holt et al,
preliminary

Shell model seems to
include most correlations.
Bulk of quenching comes
from two-body current.

...And in ^{100}Sn

Coupled-Cluster Calculation of β Decay

Hagen et al, unpublished

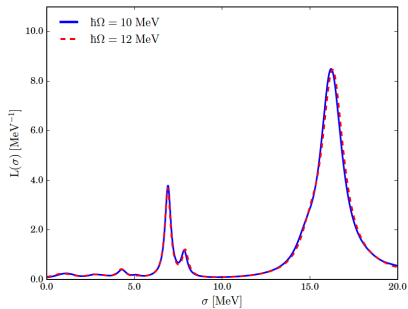


Again, good part of the quenching accounted for by two-body current.

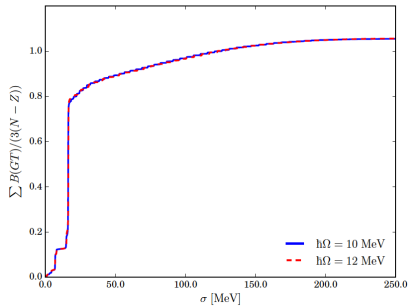
Quenching increases with mass, at least up to Sn.

Spectator nucleons contribute coherently to two-body current.

Gamow-Teller Strength in ^{132}Sn



Strength vs. energy



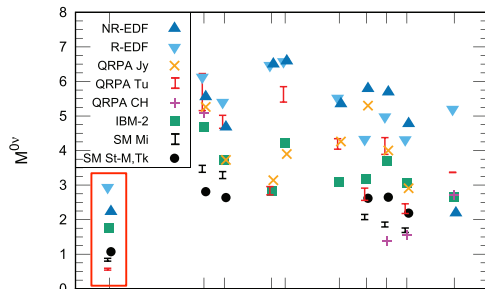
Running sum

Almost 20% of strength above 30 MeV and 10% above 50 MeV.

And $0\nu\beta\beta$ Decay?

Preliminary results in ^{48}Ca

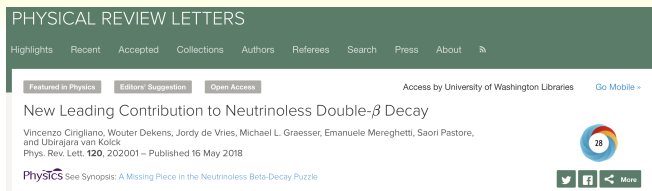
Two-body currents not yet included, but preliminary indications are that their effects are not as large as in β decay.



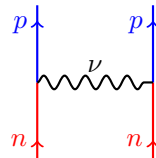
Coupled -clusters result is red bar at bottom.

But ^{48}Ca is not typical. ^{76}Ge coming soon.

Small Fly in the Ointment



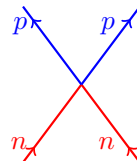
Usual light neutrino exchange:



must be supplemented, **at same order in chiral EFT**, by short-range operator (representing high-energy ν exchange):

Coefficient of this term is unknown.

Looking for ways to fit to, e.g., pion double-charge exchange



So, to Sum Up...

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That's all; thanks.
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