Ab-Initio Theory and $\beta \beta$ Decay

July 2, 2018
Review of $0\nu\beta\beta$ Decay

Forbidden in Standard Model.
New physics inside blobs.
Review of $0\nu\beta\beta$ Decay

Standard operator

I’ll focus on this one.

Forbidden in Standard Model.
New physics inside blobs.
Nuclear Matrix Element (Simplified)

\[ M^{0\nu} = g_A^2 M_{GT}^{0\nu} - g_V^2 M_{F}^{0\nu} + \ldots \]

with

\[ M_{GT}^{0\nu} = \langle f | \sum_{a,b} H_{GT}(r_{ab}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle \]

\[ M_{F}^{0\nu} = \langle f | \sum_{a,b} H_{F}(r_{ab}) \tau_a^+ \tau_b^+ | i \rangle \]

\[ H_{GT}(r) \approx H_{F}(r) \approx \frac{R_{\text{nucl.}}}{r} \]
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Also:

\[ M_{2\nu} = g_A^2 \sum_m \frac{\langle f | \sum_a \vec{\sigma}_a \tau_a^+ | m \rangle \cdot \langle m | \sum_b \vec{\sigma}_b \tau_b^+ | i \rangle}{E_m - \frac{E_f + E_i}{2}} \]
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\[ \mathcal{M}_{GT}^{0\nu} = \langle f | \sum_{a,b} H_{GT}(r^{ab}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau^+_a + \tau^+_b | i \rangle \]

But the idea that there is a single “\( g_A \) in medium” is too much of a simplification.

\[ H_{GT}(r) \approx H_F(r) \approx \frac{R_{nucl.}}{r} \]

Also:

\[ \mathcal{M}_{2\nu} = g_A^2 \sum_m \frac{\langle f | \sum_a \vec{\sigma}_a \tau^+_a | m \rangle \cdot \langle m | \sum_b \vec{\sigma}_b \tau^+_b | i \rangle}{E_m - \frac{E_f + E_i}{2}} \]
Ab Initio Nuclear Structure

Often starts with chiral effective-field theory

Nucleons, pions sufficient below chiral-symmetry breaking scale. Expansion of operators in powers of \( Q/\Lambda \chi \).

\( Q = m_\pi \) or typical nucleon momentum.

<table>
<thead>
<tr>
<th>Order</th>
<th>2N Force</th>
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<tbody>
<tr>
<td>LO</td>
<td>((Q/\Lambda \chi)^0)</td>
<td>(\chi)</td>
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</tr>
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<td>((Q/\Lambda \chi)^2)</td>
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<td>(N^3)LO</td>
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Figure 1: Hierarchy of nuclear forces in ChPT. Solid lines represent nucleons and dashed lines pions. Small dots, large solid dots, solid squares, and solid diamonds denote vertices of index 0, 1, 2, and 4, respectively. Further explanations are given in the text.

The reason why we talk of a hierarchy of nuclear forces is that two- and many-nucleon forces are created on an equal footing and emerge in increasing number as we go to higher and higher orders. At NNLO, the first set of nonvanishing three-nucleon forces (3NF) occur \([70, 71]\), cf. column '3N Force' of Fig. 1. In fact, at the previous order, NLO, irreducible 3N graphs appear already, however, it has been shown by Weinberg \([52]\) and others \([70, 127, 128]\) that these diagrams all cancel. Since nonvanishing 3NF contributions happen first at order \((Q/\Lambda \chi)^3\), they are very weak as compared to 2NF which start at \((Q/\Lambda \chi)^0\).

More 2PE is produced at \(\Lambda_\pi = 4\), next-to-next-to-next-to-leading order (N3LO), of which we show only a few symbolic diagrams in Fig. 1. Two-loop 2PE graphs show up for the first time and so does three-pion exchange (3PE) which necessarily involves two loops. 3PE was found to be negligible at this order \([57, 58]\).

Most importantly, 15 new contact terms \(\sim Q^4\) arise and are represented by the four-nucleon-leg graph with a solid diamond. They include a quadratic spin-orbit term and contribute up to \(D\)-waves. Mainly due to the increased number of contact terms, a quantitative description of the two-nucleon interaction up to about 300 MeV lab. energy is possible, at N3LO (for details, see below). Besides further 3NF, four-nucleon forces (4NF) start at this order. Since the leading 4NF come into existence one order higher than the leading 3NF, 4NF are weaker than 3NF. Thus, ChPT provides a straightforward explanation for the empirically known fact that 2NF < 3NF < 4NF . . . .

4. Two-nucleon interactions

The last section was just an overview. In this section, we will fill in all the details involved in the ChPT development of the \(NN\) interaction; and 3NF and 4NF will be discussed in Section 5. We start by talking...
Many-Body Methods

All require many CPU-hours.

- Quantum Monte Carlo in light nuclei: More or less exact solution of many-body Schrödinger equation.
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- Coupled-clusters ansatz:

\[ |\Psi\rangle = \exp \left( t_{ij} a_i^\dagger a_j + t_{ijkl} a_i^\dagger a_j^\dagger a_k a_l + \ldots \right) |\text{Slater det.}\rangle \]
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- In-medium similarity-renormalization group: Flow equations that gradually decouple low-lying states (will explain this).
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Ab Initio Shell Model

Partition of Full Hilbert Space

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$P = \text{valence space}$
$Q = \text{the rest}$

**Task:** Find unitary transformation to make $H$ block-diagonal in $P$ and $Q$, with $H_{\text{eff}}$ in $P$ reproducing $d$ most important eigenvalues.

Shell model done here.
Ab Initio Shell Model

Partition of Full Hilbert Space

\[ P \quad Q \]

\[ P \quad H_{\text{eff}} \quad Q \]

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Q = the rest

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For transition operator \( ^\text{M} \), must apply same transformation to get \( ^\text{M}_{\text{eff}} \).

As difficult as solving full problem. But idea is that N-body effective operators beyond \( N > 2 \) or 3 can be treated approximately.

Shell model done here.
Ab Initio Shell Model

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\[ P \quad Q \]

\[ P \]

\[ H_{\text{eff}} \]

\[ Q \]

\[ H_{\text{eff-Q}} \]

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As difficult as solving full problem. But idea is that N-body effective operators beyond $N > 2$ or 3 can be treated approximately.
In-Medium Similarity Renormalization Group

One way to determine the transformation

Flow equation for effective Hamiltonian. Gradually decouples shell-model space.

Trick is to keep all 1- and 2-body terms in $H$ at each step after normal ordering (approximate treatment of 3-, 4- ... terms).

If shell-model space contains just a single state, approach yields ground-state energy. If it is a typical valence space, result is effective interaction and operators.
One way to determine the transformation

Flow equation for effective Hamiltonian. Gradually decouples shell-model space.

Transformation can also be done via the coupled-cluster expansion.

Trick is to keep all 1- and 2-body terms in $H$ at each step after normal ordering (approximate treatment of 3-, 4- ... terms).

If shell-model space contains just a single state, approach yields ground-state energy. If it is a typical valence space, result is effective interaction and operators.
Ab Initio Calculations of Spectra

Deformed nuclei

Neutron-rich oxygen isotopes

Ab initio
Phenom.
Expt.
Ab Initio $^{76}$Ge and $^{76}$Se

Stroberg, Holt, et al.

Ab Initio $0^\nu\beta\beta$-Decay Predictions from Valence-Space IMSRG

Conventional SM: phenomenological wavefunctions

Ab initio SM: wavefunctions from chiral NN+3N forces

1) Ab initio energies in medium/heavy-mass region

Valence-space IM-SRG for all medium-mass nuclei

Deformation challenging for large-space methods

First ab initio calculation of $^{76}$Ge/$^{76}$Se

$$M_0 = h f_x \mathbf{a} \cdot \mathbf{b} \nu + a \nu + b \nu$$

$$E(0^+) \text{ (MeV)}$$

$^{76}$Ge

-666.61

Exp.

-661.60

IMSRG

$^{76}$Se

-668.38

Exp.

-662.07

IMSRG
Gamow-Teller $\beta$ Decay

Leading order decay operator is $\vec{\sigma}\tau_+$. 

50-Year-Old Problem: Effective $g_A$ needed in all calculations of shell-model (or related) type.

Many suggestions about the cause but, until recently, no consensus.
Other Tests of $\bar{\nu}\tau$ Strength Also Show Suppression

Only about 2/3 of theoretically expected strength observed.
And $2\nu\beta\beta$ Decay...

From F. Iachello

![Graph showing $g_{A,\text{eff}}$ vs. mass number for different nuclei with data points and curves indicating trends.](image-url)

- From experimental $\tau_{1/2}$ (ISM)
- $g_{A,\text{eff}}^{\text{ISM}} = 1.269A^{-0.12}$
- From experimental $\tau_{1/2}$ (IBM–2 CA/SSD)
- $g_{A,\text{eff}}^{\text{IBM–2}} = 1.269A^{-0.18}$

Mass number

Nuclei: Ca, Ge, Se, ZrMo, Cd, Te, Xe, Nd
And $2\nu\beta\beta$ Decay...

What explains all the over-prediction of matrix elements?

In ab initio calculation with chiral EFT, the answer must be a combination of many-body approximations and truncation of chiral expansion of current operator.
Axial Weak Current in Chiral EFT

$\beta$ Decay (simplified) with electron lines omitted

**Leading order:**

Usual $\beta$-decay current.

Finite-momentum corrections at next order.
Axial Weak Current in Chiral EFT

β Decay (simplified) with electron lines omitted

Leading order:

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Higher order:
Axial Weak Current in Chiral EFT

β Decay (simplified) with electron lines omitted

Leading order:

Usual β-decay current.
Finite-momentum corrections at next order.

Higher order:

Coefficients same as in three-body interaction:
Quenching in the sd and pf Shells

IMSRG calculation, Holt et al, preliminary

Shell model seems to include most correlations.
Bulk of quenching comes from two-body current.
...And in $^{100}\text{Sn}$

Coupled-Cluster Calculation of $\beta$ Decay

Hagen et al, unpublished

Again, good part of the quenching accounted for by two-body current.

Quenching increases with mass, at least up to Sn.

Spectator nucleons contribute coherently to two-body current.
Gamow-Teller Strength in $^{132}\text{Sn}$

Strength vs. energy

Running sum

Almost 20% of strength above 30 MeV and 10% above 50 MeV.
And $0\nu\beta\beta$ Decay?

Preliminary results in $^{48}$Ca

Two-body currents not yet included, but preliminary indications are that their effects are not as large as in $\beta$ decay.

Coupled -clusters result is red bar at bottom.

But $^{48}$Ca is not typical. $^{76}$Ge coming soon.
Small Fly in the Ointment

Usual light neutrino exchange:

must be supplemented, at same order in chiral EFT, by short-range operator (representing high-energy $\nu$ exchange):

Coefficient of this term is unknown.

Looking for ways to fit to, e.g., pion double-charge exchange
So, to Sum Up…

1. Chiral EFT + new many-body methods are the tools required to compute matrix elements with controlled uncertainty. Already doing preliminary calculations of $0\nu\beta\beta$ matrix elements.

2. Quenching of single $\beta$ decay mostly understood in this framework as due to combination of previously un-captured correlations and two-body current.

3. Application of chiral EFT to $0\nu\beta\beta$ decay implies short-range contribution to neutrino exchange with unknown coefficient. We're investigating...

4. A similar issue hampers our ability to fully examine effects of the two-body current in $0\nu\beta\beta$ decay, though the part for which we do know coefficients seems to quench very little.

5. Coordinated effort on this stuff by U.S. DOE Topical Theory Collaboration. Should make more progress.

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