The 2νββ decay and a determination of the effective axial-vector coupling constant $g_A^{\text{eff}}$

Rastislav Dvornický and Fedor Šimkovic

JINR, Dubna, Russia & Comenius University, Bratislava, Slovakia
The $2\nu\beta\beta$ decay and a determination of the effective axial-vector coupling constant $g_A^{\text{eff}}$

Rastislav Dvornický and Fedor Šimkovic

Meant to be a supplementary talk to
So I will skip the introduction about the double beta decay
The weak form-factors

- Form factors introduced since proton/neutron are not elementary part.
- Depend on vector and axial weak charges of the proton and neutron.
- Two hypotheses:
  - Conservation of Vector Current (CVC):
  - Partial conservation of Axial Current (PCAC):

\[
F_V(q^2) = \frac{F_V(0)}{\left(1 - q^2 / 0.71\right)^2} \quad F_V(0) = 1 = g_V
\]

\[
G_A(q^2) = \frac{F_A(0)}{\left(1 - q^2 / 1.065\right)^2} \quad G_A(0) = g_A = -1.2573 \pm 0.028
\]

- For low energy neutrinos \(E_\nu \ll m_N\):

\[
\sigma(\nu_e n) = \sigma(\bar{\nu}_e p) = \frac{G_F \cos \theta_C}{\pi} E_\nu^2 \left[F_V(0)^2 + 3G_A(0)^2\right]
\]

\[
\approx 9.75 \times 10^{-42} \left(\frac{E_\nu}{10 \text{ MeV}}\right)^2 \text{cm}^2
\]
Axial-vector current in nuclei

- The axial current is not conserved!
- Thus, its extension to nuclei is not trivial.
- Nucleons interact in nuclei.

**Allowed Gamow-Teller transition**

\[ 0^+ \rightarrow 1^+ \]

\[ ft \sim \frac{1}{g_A^2 |M_A|^2} \]

\[ M_A^2 = \left| \langle \psi_i | GT | \psi_f \rangle \right|^2 \]

**Double Gamow-Teller transition**

\[ 0^+_{\text{g.s.}} \rightarrow 0^+_{\text{g.s.}} \]

\[ \frac{1}{T_{1/2}^{2\nu - \text{exp}}} = G^{2\nu}(E_0, Z) \ g_A^4 \ |M_{GT}^{2\nu}|^2 \]

\[ M_{GT}^{2\nu} = \sum_m \frac{\langle 0^+_f || \tau^+ \sigma || 1^+_m \rangle < 1^+_m || \tau^+ \sigma || 0^+_i \rangle}{E_m - E_i + \Delta} \]
Understanding of the $2\nu\beta\beta$-decay NMEs is of crucial importance for correct evaluation of the $0\nu\beta\beta$-decay NMEs.

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states. Both change two neutrons into two protons. Explaining $2\nu\beta\beta$-decay is necessary but not sufficient.

There is a need for reliable calculation of the $2\nu\beta\beta$-decay NMEs.

Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.) ISM (quenching, truncation of model space, spin-orbit partners)

Calculation via closure NME: IBM, PHFB

No calculation: EDF
Single State Dominance (\(^{100}\text{Mo}, \, ^{106}\text{Cd}, \, ^{116}\text{Cd}, \ldots\))

HSD, higher levels contribute to the decay

SSD, \(^1\) level dominates in the decay

(Abad et al., 1984, Ann. Fis. A 80, 9)

\[ E_i - E_f = -0.343 \text{ MeV} \]

\[ E_i - E_f = -0.041 \text{ MeV} \]

\[ E_i - E_f = 0.705 \text{ MeV} \]
SSD – theoretical studies


\[ M_{GT}^K = \sum_m \left( \frac{M^i_m(1^+)M^f_m(1^+)}{E_m - E_i + \varepsilon_{10} + \nu_{10}} + \frac{M^i_m(1^+)M^f_m(1^+)}{E_m - E_i + \varepsilon_{20} + \nu_{20}} \right) \]

SSD \Rightarrow \frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + \varepsilon_{10} + \nu_{10}} + \frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + \varepsilon_{20} + \nu_{20}} \Rightarrow 2\frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + \Delta}

HSD

Isotope f.s. \( T_{1/2}(\text{SSD})[y] \) \( T_{1/2}(\text{exp.})[y] \) \( 2\nu\beta^-\beta^- \)

<table>
<thead>
<tr>
<th>Isotope</th>
<th>f.s.</th>
<th>( T_{1/2}(\text{SSD})[y] )</th>
<th>( T_{1/2}(\text{exp.})[y] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{100}\text{Mo} )</td>
<td>0 g.s.</td>
<td>6.8 ( 10^{18} )</td>
<td>6.8 ( 10^{18} )</td>
</tr>
<tr>
<td></td>
<td>0 (^1)</td>
<td>4.2 ( 10^{20} )</td>
<td>6.1 ( 10^{18} )</td>
</tr>
<tr>
<td>(^{116}\text{Cd} )</td>
<td>0 g.s.</td>
<td>1.1 ( 10^{19} )</td>
<td>2.6 ( 10^{19} )</td>
</tr>
<tr>
<td>(^{128}\text{Te} )</td>
<td>0 g.s.</td>
<td>1.1 ( 10^{25} )</td>
<td>2.2 ( 10^{24} )</td>
</tr>
<tr>
<td></td>
<td>EC/EC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{106}\text{Cd} )</td>
<td>0 g.s.</td>
<td>&gt;4.4 ( 10^{21} )</td>
<td>&gt;5.8 ( 10^{17} )</td>
</tr>
<tr>
<td>(^{130}\text{Ba} )</td>
<td>0 g.s.</td>
<td>5.0 ( 10^{22} )</td>
<td>4.0 ( 10^{21} )</td>
</tr>
</tbody>
</table>

Domin, Kovalenko, Šimkovic, Semenov, NPA 753, 337 (2005)

The SSD prediction for the \( 2\nu\beta\beta \) half-life does not depend on quenching of \( g_A \)

\[
M^i_1(0^+) = \frac{1}{g_A} \sqrt{\frac{3D}{ft_{EC}}} \quad M^f_1(J^+) = \frac{1}{g_A} \sqrt{\frac{3D}{ft_{\beta^-}}}.
\]
SSD differential characteristics

$2\nu\beta^-\beta^-\text{decay}$

$0^+_{g.s.} \rightarrow 0^+_{g.s.}$

$0^+_{g.s.} \rightarrow 0^+_{1}$

$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$

$0^+_{g.s.} \rightarrow 2^+_{g.s.}$

Do not depend on $M^i M^f$

$2\nu EC/\beta^+\text{decay}$

$^{106}\text{Cd}$

$^{130}\text{Ba}$

$^{136}\text{Ce}$

$\varepsilon - m_e c^2 [\text{MeV}]$

$E - m_e c^2 (\text{MeV})$
Single electron spectrum different between SSD and HSD

Šimkovic, Šmotlák, Semenov

100Mo 2ν2β : Experimental Study of SSD Hypothesis

HSD: $T_{1/2} = 8.61 \pm 0.02$ (stat) $\pm$ 0.60 (syst) $\times 10^{18}$ y
SSD: $T_{1/2} = 7.72 \pm 0.02$ (stat) $\pm$ 0.54 (syst) $\times 10^{18}$ y

$\chi^2$/ndf = 139/36

Data
2β2ν HSD
Monte Carlo
Background subtracted

HSD higher levels $E_1 + E_2 > 2$ MeV

$\chi^2$/ndf = 40.7/36

Data
2β2ν SSD
Monte Carlo
Background subtracted

100Mo 2ν 2β single energy distribution in favour of Single State Dominant (SSD) decay
2νββ-decay rate

\[ T_{1/2}^{2\nu\beta\beta}(0^+) \]^{-1} = \frac{m_e}{8\pi^7 \ln 2} (G_\beta m_e^2)^4 I^{2\nu}(0^+), \]

\[ A^{2\nu} = g_V^4 \left[ \frac{1}{4} |M_F^K + M_F^L|^2 + \frac{3}{4} |M_F^K - M_F^L|^2 \right] \]
\[ - g_V g_A^2 \text{Re} \left\{ M_F^K \ast M_{GT}^L + M_F^K \ast M_{GT}^L \right\} \]
\[ + \frac{g_A^4}{3} \left[ \frac{3}{4} |M_{GT}^K + M_{GT}^L|^2 + \frac{1}{4} |M_{GT}^K - M_{GT}^L|^2 \right] \]

In the limit

\[ 2E_n - E_i - E_f = 0 \]

\[ A^{2\nu} = 0 \]
Quenching: 

\[ q = \frac{g_A}{g_{\text{free}}_A} \]

Free value of \( g_A \) (Particle Data Group 2016):

\[ g_{\text{free}}_A = 1.2723(23) \]

Effective value of \( g_A \):

\[ g_{\text{eff}}_A = q \cdot g_{\text{free}}_A \]
Quenching of $g_A$ (from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)

$(g_{eff}^A)^4 = 1.0$

$^{76}_{32}\text{Ge}_{44} \Rightarrow S^-_\beta - S^+_\beta = 3(N-Z) = 36$

Cross-section for charge exchange reaction:

$$\left[ \frac{d\sigma}{d\Omega} \right] = \left[ \frac{\mu}{\pi\hbar} \right]^2 \frac{k_f}{k_i} N_d \left| v_{\sigma\tau} \right|^2 \left| \langle f | \sigma \tau | i \rangle \right|^2$$

$q = 0!!$

largest at 100 - 200 MeV/A
Quenching of $g_A$ (from theory: $T_{1/2}^{0\nu}$ up 50 x larger)

$\left(g_{eff}^A\right)^4 \approx 0.66$ ($^{48}\text{Ca}$), 0.66 ($^{76}\text{Ge}$), 0.30 ($^{76}\text{Se}$), 0.20 ($^{130}\text{Te}$) and 0.11 ($^{136}\text{Xe}$).

The Interacting Shell Model (ISM), which describes qualitatively well energy spectra, does reproduce experimental values of $M^{2\nu}$ only by consideration of significant quenching of the Gamow-Teller operator, typically by 0.45 to 70%.

$\left(g_{eff}^A\right)^4 \approx (1.269 A^{-0.18})^4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

It has been determined by theoretical prediction for the 2$\nu$$\beta\beta$-decay half-lives, which were based on within closure approximation calculated corresponding NMEs, with the measured half-lives.

$(g_{\text{eff}}^A)^4 = 0.30$ and 0.50 for $^{100}\text{Mo}$ and $^{116}\text{Cd}$, respectively (The QRPA prediction).

$g_{\text{eff}}^A$ was treated as a completely free parameter alongside $g_{\text{pp}}$ (used to renormalize particle-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of $g_{\text{eff}}^A$ and $g_{\text{pp}}$, where possible, to the $\beta$-decay rate and $\beta^+$/EC rate of the $J = 1^+$ ground state in the intermediate nuclei involved in double-beta decay in addition to the $2\nu\beta\beta$ rates of the initial nuclei, leads to an effective $g_{\text{eff}}^A$ of about 0.7 or 0.8.

Extended calculation also for neighbor isotopes performed by F.F. Depisch and J. Suhonen, PRC 94, 055501 (2016)

Dependence of $g_{\text{eff}}^A$ on $A$ was not established.
Improved formalism of the $2\nu\beta\beta$-decay

Improved description of the $2\nu\beta\beta$-decay rate

$$
\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = \frac{m_e}{8\pi^7 \ln 2} (G_\beta m_e^2)^4 \left(g_A^\text{eff}\right)^4 I^{2\nu}
$$

Half-life without factorization of NMEs and phase space

$$
I^{2\nu} = \frac{1}{m_{e1}^{11}} \int_{m_e}^{E_i-E_f-m_e} F_0(Z_f, E_{e1}) p_{e1} E_{e1} dE_{e1}
\times \int_{m_e}^{E_i-E_f-E_{e1}} F_0(Z_f, E_{e2}) p_{e2} E_{e2} dE_{e2}
\times \int_0^{E_{\nu_1}^2 E_{\nu_2}^2} \mathcal{A}^{2\nu} dE_{\nu_1}
$$

$$
\mathcal{A}^{2\nu} = \left[ \frac{1}{4} |M_{GT}^K + M_{GT}^L|^2 + \frac{1}{12} |M_{GT}^K - M_{GT}^L|^2 \right]
$$

$$
M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{\left[E_n - (E_i + E_f)/2\right]^2 - \varepsilon_{K,L}^2}
$$

$$
M_n = \langle 0_f^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 1_n^+ \rangle \langle 1_n^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 0_i^+ \rangle
$$

$$
\varepsilon_K = (E_{e2} + E_{\nu_2} - E_{e1} - E_{\nu_1}) / 2
$$

$$
\varepsilon_L = (E_{e1} + E_{\nu_2} - E_{e2} - E_{\nu_1}) / 2
$$

The isospin conservation is assumed
\[ M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2} \]

Standard approximation which allows factorization of NME and phase space

\[ M_{GT}^{K,L} \simeq M_{GT}^{2\nu} = m_e \sum_n \frac{M_n}{E_n - (E_i + E_f)/2} \]

Let perform Taylor expansion

\[ \frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2} \]

\[ \varepsilon_{K,L} \in \left(-\frac{Q}{2}, \frac{Q}{2}\right) \]

\[ E_n - \frac{E_i + E_f}{2} = \frac{Q}{2} + m_e + (E_n - E_i) > |\varepsilon_{K,L}| \]
Improved description of the $0\nu\beta\beta$–decay rate

\[
\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \equiv \frac{\Gamma_{2\nu}}{\ln(2)} \simeq \frac{\Gamma_0^{2\nu} + \Gamma_2^{2\nu} + \Gamma_4^{2\nu}}{\ln(2)}
\]

\[
\frac{\Gamma_0^{2\nu}}{\ln(2)} = (g_A^{\text{eff}})^4 M_0 G_0^{2\nu}
\]

\[
\frac{\Gamma_2^{2\nu}}{\ln(2)} = (g_A^{\text{eff}})^4 M_2 G_2^{2\nu}
\]

\[
\frac{\Gamma_4^{2\nu}}{\ln(2)} = (g_A^{\text{eff}})^4 (M_4 G_4^{2\nu} + M_{22} G_{22}^{2\nu})
\]

Taylor expansion up to $\varepsilon^4$ order

\[
G_J^{2\nu} = \frac{c_{2\nu}}{m_e^{11/2}} \int_{m_e} E_i - E_f - m_e F_0(Z_f, E_{e1}) p_{e1} E_{e1} dE_{e1}
\]

\[
\times \int_{m_e} E_i - E_f - E_{e1} F_0(Z_f, E_{e2}) p_{e2} E_{e2} dE_{e2}
\]

\[
\times \int_{0} E_{\nu1}^2 E_{\nu2}^2 A_J^{2\nu} dE_{\nu1}, \ (J=0, 2, 4, 22)
\]

\[
A_0^{2\nu} = 1 \quad A_2^{2\nu} = \frac{\varepsilon_K^2 + \varepsilon_L^2}{(2m_e)^2},
\]

\[
A_4^{2\nu} = \frac{\varepsilon_K^2 \varepsilon_L^4}{(2m_e)^4} \quad A_{22}^{2\nu} = \frac{\varepsilon_K^4 + \varepsilon_L^4}{(2m_e)^4}
\]

<table>
<thead>
<tr>
<th>nucl.</th>
<th>$G_0^{2\nu}$ [yr$^{-1}$]</th>
<th>$G_2^{2\nu}$ [yr$^{-1}$]</th>
<th>$G_4^{2\nu}$ [yr$^{-1}$]</th>
<th>$G_{22}^{2\nu}$ [yr$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>$4.816 \times 10^{-20}$</td>
<td>$1.015 \times 10^{-20}$</td>
<td>$1.332 \times 10^{-21}$</td>
<td>$6.284 \times 10^{-22}$</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>$1.591 \times 10^{-18}$</td>
<td>$7.037 \times 10^{-19}$</td>
<td>$1.952 \times 10^{-19}$</td>
<td>$8.931 \times 10^{-20}$</td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>$3.303 \times 10^{-18}$</td>
<td>$1.509 \times 10^{-18}$</td>
<td>$4.320 \times 10^{-19}$</td>
<td>$1.986 \times 10^{-19}$</td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>$1.530 \times 10^{-18}$</td>
<td>$4.953 \times 10^{-19}$</td>
<td>$9.985 \times 10^{-20}$</td>
<td>$4.707 \times 10^{-20}$</td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td>$1.433 \times 10^{-18}$</td>
<td>$4.404 \times 10^{-19}$</td>
<td>$8.417 \times 10^{-20}$</td>
<td>$3.986 \times 10^{-20}$</td>
</tr>
</tbody>
</table>

Phase space factors
\[ \mathcal{M}_0 = \left| M_{GT-1}^{2\nu} \right|^2 \]
\[ \mathcal{M}_2 = \Re \{ M_{GT-1}^{2\nu} M_{GT-3}^{2\nu} \} \]
\[ \mathcal{M}_{22} = \frac{1}{3} \left| M_{GT-3}^{2\nu} \right|^2 \]
\[ \mathcal{M}_4 = \frac{1}{3} \left| M_{GT-3}^{2\nu} \right|^2 + \Re \{ M_{GT-1}^{2\nu} M_{GT-5}^{2\nu} \} \]

### QRPA

2νββ-decay NMEs and their ratios

<table>
<thead>
<tr>
<th>nucl.</th>
<th>( g_A^{\text{eff}} )</th>
<th>( M_{GT-1}^{2\nu} )</th>
<th>( M_{GT-3}^{2\nu} )</th>
<th>( M_{GT-5}^{2\nu} )</th>
<th>( \xi_{13}^{2\nu} )</th>
<th>( \xi_{15}^{2\nu} )</th>
<th>( P_0^{2\nu} )</th>
<th>( P_2^{2\nu} )</th>
<th>( P_4^{2\nu} )</th>
<th>( T_{1/2}^{2\nu-\exp} ) [yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>76Ge</td>
<td>0.800</td>
<td>0.175</td>
<td>0.0214</td>
<td>0.00445</td>
<td>0.1220</td>
<td>0.0254</td>
<td>0.9741</td>
<td>0.0250</td>
<td>0.0009</td>
<td>1.65 ( 10^{21} )</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.111</td>
<td>0.0133</td>
<td>0.00263</td>
<td>0.1204</td>
<td>0.0237</td>
<td>0.9745</td>
<td>0.0247</td>
<td>0.0008</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.269</td>
<td>0.689</td>
<td>0.00716</td>
<td>0.00716</td>
<td>0.1040</td>
<td>0.0170</td>
<td>0.9780</td>
<td>0.0214</td>
<td>0.0006</td>
<td></td>
</tr>
<tr>
<td>82Se</td>
<td>0.800</td>
<td>0.124</td>
<td>0.0216</td>
<td>0.00645</td>
<td>0.1745</td>
<td>0.0521</td>
<td>0.9213</td>
<td>0.0711</td>
<td>0.0076</td>
<td>0.92 ( 10^{20} )</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.0795</td>
<td>0.0129</td>
<td>0.00355</td>
<td>0.1620</td>
<td>0.0446</td>
<td>0.9271</td>
<td>0.0664</td>
<td>0.0065</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.269</td>
<td>0.0498</td>
<td>0.00643</td>
<td>0.00136</td>
<td>0.1290</td>
<td>0.0272</td>
<td>0.9421</td>
<td>0.0538</td>
<td>0.0041</td>
<td></td>
</tr>
<tr>
<td>100Mo</td>
<td>0.800</td>
<td>0.292</td>
<td>0.123</td>
<td>0.0453</td>
<td>0.4230</td>
<td>0.1553</td>
<td>0.8163</td>
<td>0.1578</td>
<td>0.0259</td>
<td>7.1 ( 10^{18} )</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.184</td>
<td>0.0876</td>
<td>0.0322</td>
<td>0.4752</td>
<td>0.1745</td>
<td>0.7972</td>
<td>0.1731</td>
<td>0.0297</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.269</td>
<td>0.112</td>
<td>0.0633</td>
<td>0.0233</td>
<td>0.5646</td>
<td>0.2075</td>
<td>0.7661</td>
<td>0.1976</td>
<td>0.0363</td>
<td></td>
</tr>
<tr>
<td>130Te</td>
<td>0.800</td>
<td>0.0466</td>
<td>0.00873</td>
<td>0.00239</td>
<td>0.1873</td>
<td>0.0512</td>
<td>0.9389</td>
<td>0.0569</td>
<td>0.0042</td>
<td>6.9 ( 10^{20} )</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.0298</td>
<td>0.00577</td>
<td>0.00144</td>
<td>0.1937</td>
<td>0.0482</td>
<td>0.9371</td>
<td>0.0588</td>
<td>0.0041</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.269</td>
<td>0.0185</td>
<td>0.00373</td>
<td>0.00078</td>
<td>0.2015</td>
<td>0.0420</td>
<td>0.9352</td>
<td>0.0610</td>
<td>0.0038</td>
<td></td>
</tr>
<tr>
<td>136Xe</td>
<td>0.800</td>
<td>0.0268</td>
<td>0.00706</td>
<td>0.00232</td>
<td>0.2637</td>
<td>0.0866</td>
<td>0.9190</td>
<td>0.0745</td>
<td>0.0065</td>
<td>2.19 ( 10^{21} )</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.0170</td>
<td>0.00526</td>
<td>0.00169</td>
<td>0.3098</td>
<td>0.0995</td>
<td>0.9059</td>
<td>0.0863</td>
<td>0.0078</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.269</td>
<td>0.0104</td>
<td>0.00403</td>
<td>0.00126</td>
<td>0.3867</td>
<td>0.1207</td>
<td>0.8848</td>
<td>0.1051</td>
<td>0.0101</td>
<td></td>
</tr>
</tbody>
</table>
Normalized to unity different partial energy distributions
The single electron energy distribution

\[ \frac{1}{\Gamma} \frac{d\Gamma}{dT_e} \]

For isotopes of $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{96}\text{Zr}$, and $^{100}\text{Mo}$, showing different curves for $N=0$, $N=2$, and $N=4$. The graph includes a legend indicating full, dashed, and dotted lines for the respective nuclear spins.
The sum electron energy distribution

![Graph showing the sum electron energy distribution for different isotopes and N values.]

- $^{48}\text{Ca}$
- $^{116}\text{Cd}$
- $^{130}\text{Te}$
- $^{136}\text{Xe}$

The graph illustrates the distribution of electron energy for various isotopes, with different N values as indicated by the legend: full contr., N=0, N=2, and N=4.
The endpoint of the spectrum of differential decay rate vs. the sum of kinetic energy of emitted electrons.
A new method to determine effective $g_A$

Improved description of the $0
\bar{\nu}\beta\beta$–decay rate

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

Let us perform Taylor expansion

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2}, \quad \varepsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

We get

$$[T_{1/2}^{2\nu\beta\beta}]^{-1} \approx (g_A^{\text{eff}})^4 |M_{GT-3}^{2\nu}|^2 \frac{1}{|\xi_{13}^{2\nu}|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3} \quad \xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

The $g_A^{\text{eff}}$ can be determined with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)
The running sum of the $2\nu\beta\beta$–decay NMEs

\[ M^{2\nu}_{GT-I} \]

\[ E_{ex} \text{ [MeV]} \]

\[ 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \]

\[ 0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \]

\[ \text{I=1} \]

\[ \text{I=3} \]
The running sum of the $2\nu\beta\beta$–decay NMEs
\( \xi_{13} \) tells us about importance of higher lying states of int. nucl.

\( \xi_{13} \) can be determined phenomenologically from the shape of energy distributions of emitted electrons.

HSD: \( \xi_{13} = 0 \)

Šimkovic, Šmotlák, Semenov

\( ^{100}\text{Mo} \)
The change of the $2\nu\beta\beta$–decay energy distributions

HSD: $\xi_{13}=0$
Solution: NEMO3/SuperNemo measurement of $\xi$ and calculation of $M_{GT-3}$

\[
\left( g_A^{\text{eff}} \right)^2 = \frac{1}{M_{GT-3}^{2\nu}} \cdot \frac{|\xi^{2\nu}|}{\sqrt{T_{1/2}^{2\nu-exp}} \left( G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu} \right)}
\]

\[
g_A^{\text{eff}}(^{100}\text{Mo}) = \frac{0.251}{\sqrt{M_{GT-3}^{2\nu}}}
\]

\[
g_A^{\text{eff}}(^{100}\text{Cd}) = \frac{0.214}{\sqrt{M_{GT-3}^{2\nu}}}
\]

$M_{GT-3}$ have to be calculated by nuclear theory - ISM
Conclusions

• We presented an improved formalism of the $2\nu\beta\beta$-decay, which takes into account the effect of lepton energies in energy denominators.
• There is one additional parameter $\xi_{13}$, which needs to be fitted for the determination of the $2\nu\beta\beta$-decay half-life.
• The phenomenological determination of the $\xi_{13}$ and calculation of $M_{GT-3}$ (within the ISM) might allow to determine $g_{A}^{\text{eff}}$. The NEMO3 and KamlandZEN analysis are under way.