Beyond Fierz-Pauli theory

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Fierz-Pauli Theory

- Fierz-Pauli theory (Fierz, Pauli, 1939)

$$S = M_{\rm Pl}^2 \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} m^2 (h_{\mu\nu}h^{\mu\nu} - h^2) \right]$$

Linearized Einstein-Hilbert term

Only allowed mass term which does not have ghost at linear order

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = -\frac{1}{2}(\Box h_{\mu\nu} - \partial_\mu\partial_\alpha h^\alpha_\nu - \partial_\nu\partial_\alpha h^\alpha_\mu + \partial_\mu\partial_\nu h^\alpha_\alpha - \eta_{\mu\nu}\Box h^\alpha_\alpha + \eta_{\mu\nu}\partial_\alpha\partial_\beta h^\alpha_\beta)$$

(1) Lorentz invariant theory

(2) No ghost (5 DOF = 2 tensor + 2 vector + 1 scalar)

(3) **Simple nonlinear extension (massive GR) contains ghost at nonlinear level**

(Boulware-Deser ghost, 6th DOF) (Boulware, Deser, 1971)

dRGT Massive Gravity

- dRGT massive gravity (de Rham, Gabadadze, Tolley 2011)

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_m[g_{\mu\nu}, \psi]$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{\delta^\mu{}_\nu - H^\mu{}_\nu} = \delta^\mu{}_\nu - \left(\sqrt{g^{-1}\eta}\right)^\mu{}_\nu$$

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

$$\sqrt{X^\mu{}_\alpha}\sqrt{X^\alpha{}_\nu} = X^\mu{}_\nu$$

$$\mathcal{U}_2 = 2\varepsilon_{\mu\alpha\rho\sigma}\varepsilon^{\nu\beta\rho\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta = 4\left([\mathcal{K}^2] - [\mathcal{K}]^2\right)$$

$$\mathcal{U}_3 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\rho}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta = -[\mathcal{K}]^3 + 3[\mathcal{K}][\mathcal{K}^2] - 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta\mathcal{K}^\rho{}_\sigma = -[\mathcal{K}]^4 + 6[\mathcal{K}]^2[\mathcal{K}^2] - 3[\mathcal{K}^2]^2 - 8[\mathcal{K}][\mathcal{K}^3] + 6[\mathcal{K}^4]$$

- Expanding the square root in the potential term

$$\mathcal{U}_2 = [H^2] - [H]^2 - \frac{1}{2}\left([H][H^2] - [H^3]\right) + \mathcal{O}(H^4)$$

Fierz-Pauli mass term **Infinite nonlinear corrections to eliminate BD ghost**

No BD ghost at full order (5 DOF) (Hassan, Rosen, 2011)

NON-CANONICAL KINETIC TERM

(**RK** & Yamauchi, 2013)

- dRGT mass term is uniquely determined
- Is dRGT theory a unique theory describing massive graviton without introducing other fields ?

$$S_{MG} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{m^2}{4} \left(\mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4 \right) \right] + S_{int} + S_m[g_{\mu\nu}, \psi],$$

New Kinetic terms???

- Candidates for derivative interactions

$$\mathcal{L}_{int} \supset M_{\rm Pl}^2 \sqrt{-g}\, g_{..}\, H_{..}\, R^{....}, M_{\rm Pl}^2 \sqrt{-g}\, H_{..}\, H_{..}\, R^{....}, \cdots$$

- **No-go theorem** - no derivative interaction cannot be introduced in dRGT theory (de Rham et al. 2013)

$$M_{\rm Pl}^2 \sqrt{-g}\, \nabla_. \nabla_.\, H^{\cdot}{}_. \, H^{\cdot\cdot}, M_{\rm Pl}^2 \sqrt{-g}\, \nabla_. \nabla_.\, H^{\cdot}{}_. \, H^{\cdot}{}_. \, H^{\cdot\cdot}, \cdots$$

General Spin-2 Theory

- Our Lagrangian (non-canonical kinetic term)

$$S = \int \mathrm{d}^4x \Big(-\mathcal{K}^{\alpha\beta|\mu\nu\rho\sigma} h_{\mu\nu,\alpha} h_{\rho\sigma,\beta} - \mathcal{M}^{\mu\nu\rho\sigma} h_{\mu\nu} h_{\rho\sigma} \Big)$$

$$\mathcal{K}^{\alpha\beta|\mu\nu\rho\sigma} = \kappa_1 \eta^{\alpha\beta}\eta^{\mu\rho}\eta^{\nu\sigma} + \kappa_2 \eta^{\mu\alpha}\eta^{\rho\beta}\eta^{\nu\sigma} + \kappa_3 \eta^{\alpha\mu}\eta^{\nu\beta}\eta^{\rho\sigma} + \kappa_4 \eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma}$$

$$\mathcal{M}^{\mu\nu\rho\sigma} = m_1 \eta^{\mu\rho}\eta^{\nu\sigma} + m_2 \eta^{\mu\nu}\eta^{\rho\sigma} \,,$$

$\kappa_i,\, m_i$: (constant) free parameters

General relativity

Linearized Einstein-Hilbert term $\quad \kappa_2 = -\kappa_3 = 2\kappa_4 = -2\kappa_1 \qquad m_1 = m_2 = 0$

Gauge symmetry $\quad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Fierz-Pauli theory

Kinetic term = Linearized Einstein-Hilbert term

Gauge invariance is broken by the mass term $\quad m_1 = -m_2$

Finding Theory

- SVT decomposition

$$h_{00} = h^{00} = -2\alpha\,, \qquad h_{0i} = -h^{0i} = \beta_{,i} + B_i \quad (B^i{}_{,i} = 0)$$

$$h_{ij} = h^{ij} = 2\mathcal{R}\delta_{ij} + 2\mathcal{E}_{,ij} + F_{i\,,j} + F_{j\,,i} + 2H_{ij} \quad (F^i{}_{,i} = 0\,, \quad H^i{}_i = H^{ij}{}_{,j} = 0)$$

- Tensor sector

$$S^T = 4\int \mathrm{d}t\,\mathrm{d}^3k\Big[\kappa_1 \dot{H}_{ij}^2 - (\kappa_1 k^2 + m_1)H_{ij}^2\Big]$$

  - Ghost-free condition $\kappa_1 > 0$
  - Degrees of freedom = 2

Vector Sector

- B_i and F_i has 4 d.o.f → one needs to eliminate one of them (ghost mode)

$$S^V = \int \mathrm{d}t\, \mathrm{d}^3k \Big[-(2\kappa_1 + \kappa_2)\dot{B}_i^2 + 2\kappa_1 (k\dot{F}_i)^2 + 2\kappa_2 k B_i (k\dot{F}_i)$$
$$+ 2\left(\kappa_1 k^2 + m_1\right) B_i^2 - \left(k^2(2\kappa_1 + \kappa_2) - 2m_1\right)(kF_i)^2 \Big]$$

↓ $2\kappa_1 + \kappa_2 = 0$

$$S^V = \int \mathrm{d}t\, \mathrm{d}^3k \Big[2\kappa_1 \dot{F}_i^2 - 4\kappa_1 k B_i \dot{F}_i + 2\left(\kappa_1 k^2 - m_1\right) B_i^2 - 2m_1 F_i^2 \Big]$$

- Canonical momenta

$$\pi_{B_i} = 0\,, \qquad \xrightarrow{\text{Primary constraints}} \qquad \mathcal{C}_1^{B_i} = \pi_{B_i} = 0$$

$$\pi_{F_i} = 4\kappa_1 (\dot{F}_i - kB_i)$$

- Secondary constraints

$$\mathcal{C}_2^{B_i} = \dot{\mathcal{C}}_1^{B_i} = \{\mathcal{C}_1^{B_i}, H_T\} = \{\mathcal{C}_1^{B_i}, H\} = k\pi_{F_i} + 4m_1 B_i \approx 0$$

- Time-evolution of the secondary constraints

$$\dot{\mathcal{C}}_2^{B_i} = \{\mathcal{C}_2^{B_i}, H_T\} = \{\mathcal{C}_2^{B_i}, H\} + \lambda_{B_j} \underline{\{\mathcal{C}_2^{B_i}, \mathcal{C}_1^{B_j}\}} \approx 0\,,$$

$$= 4m_1\,\delta_{ij}\,.$$

⬇

Case 1 : $m_1 \neq 0$

$$\text{vector DOFs} = \frac{4\times 2 - 4\,(2\text{ primary }\&\text{ 2 secondary})}{2} = 2$$

Case 2 : $m_1 = 0$

$$\text{vector DOFs} = \frac{4\times 2 - 4\,(2\text{ primary }\&\text{ 2 secondary})\times 2\,(\text{first-class})}{2} = 0$$

Scalar Sector

- Classification based on the Hamiltonian analysis

Case	DOF	Conditions	Comments
I	$3 = 2 + 0 + 1$	$m_1 = 0$ & $4\kappa_1^2 - 4\kappa_1\kappa_3 + 3\kappa_3^2 + 8\kappa_1\kappa_4 \neq 0$	Partially massless theory
II	$2 = 2 + 0 + 0$	$m_1 = 0$ & $4\kappa_1^2 - 4\kappa_1\kappa_3 + 3\kappa_3^2 + 8\kappa_1\kappa_4 = 0$	Only tensor modes
III	$5 = 2 + 2 + 1$	$m_1 \neq 0$ & $4\kappa_1^2 - 4\kappa_1\kappa_3 + 3\kappa_3^2 + 8\kappa_1\kappa_4 = 0$ & $m_1(4\kappa_1^2 - 6\kappa_1\kappa_3 + 3\kappa_3^2) + 4m_2\kappa_1^2 = 0$	New class of 5 DOF theory (Fierz-Pauli is included)
IV	$2 = 2 + 0 + 0$	$m_1 = m_2 = 0$ & $4\kappa_1^2 - 4\kappa_1\kappa_3 + 3\kappa_3^2 + 8\kappa_1\kappa_4 = 0$	Massless limit of case III (general relativity is included)

- All theories satisfies $2\kappa_1 + \kappa_2 = 0$
- I, II, & IV has gauge symmetries (containing first-class constraints)
- **Naive massless limit $m_1 \to 0$ of the case III reduces to the case IV,** whose gauge symmetry is given by

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + b\,\partial^\rho \xi_\rho \eta_{\mu\nu} \qquad b = -\frac{2\kappa_1 - \kappa_3}{2(\kappa_1 - \kappa_3)}$$

Decoupling Limit of Case III

(Massive, DOF=5 case)

- Stuckelberg decomposition

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \frac{1}{m}(\partial_\mu \hat{A}_\nu + \partial_\nu \hat{A}_\mu + b\,\partial^\rho \hat{A}_\rho \eta_{\mu\nu}) \qquad \hat{A}_\mu = A_\mu + \frac{1}{m}\partial_\mu \pi$$

- Matter coupling

$$\mathcal{L}_{\rm matter} = \frac{1}{M} h_{\mu\nu} T^{\mu\nu}$$

M : mass dimension parameter ($M_{\rm Pl}$ in GR)

m : graviton's mass ($m_1 := c_1 m^2$)

- Decoupling limit $\quad m \to 0, \quad M \to 0, \quad T \to \infty, \quad \Lambda_3$ and $\dfrac{T}{M}$ are fixed

$$\mathcal{L}^{(\rm DL)} = \mathcal{L}^{(\rm DL)}_{\rm tensor}[\tilde{h}] - \frac{6c_1}{\kappa_1}(\partial_\mu \pi)^2 + \frac{1}{M}\left[\tilde{h}_{\mu\nu}T^{\mu\nu} - \frac{c_1}{\kappa_1 - \kappa_3}\pi T\right] + \frac{b}{\Lambda_3^3}\Box \pi T$$

- **Matter is also coupled with the second derivatives of π !**

➡ Indicate the presence of higher derivatives ? (in progress)

SUMMARY

- **New** kinetic and mass interactions for spin-2 theories
  - 4 independent classes
    - Case I : DOF = 3 [2 tensor + 1 scalar] (partially-massless theory)
    - Case II : DOF = 2 [2 tensor] (3 first-class, 1 second-class constraints)
    - **Case III : DOF = 5 [2 tensor + 2 vector + 1 scalar]**
    - **Case IV : DOF = 2 [2 tensor]** (4 first-class constraints)
- Matter coupling might be a problem… (need to check !!)

$$\mathcal{L}^{(\mathrm{DL})} = \mathcal{L}^{(\mathrm{DL})}_{\mathrm{tensor}}[\tilde{h}] - \frac{6c_1}{\kappa_1}(\partial_\mu \pi)^2 + \frac{1}{M}\left[\tilde{h}_{\mu\nu}T^{\mu\nu} - \frac{c_1}{\kappa_1 - \kappa_3}\pi T\right] + \frac{b}{\Lambda_3^3}\Box \pi T$$