Universiteit Leiden

# New conservation law in low scale leptogenesis

Shintaro Eijima (Leiden Univ.)

Based on: SE and Mikhail Shaposhnikov, PLB 771 (2017) 288-296

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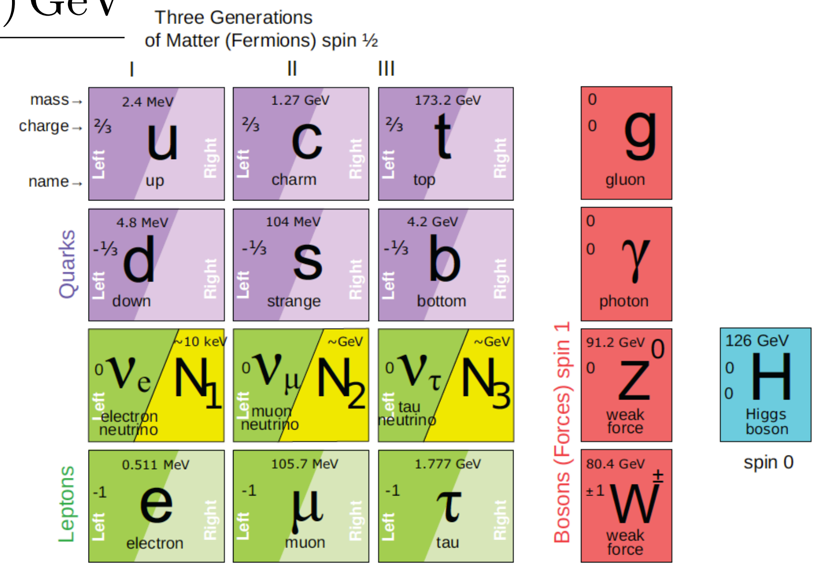
# Neutrino Minimal Standard Model (νMSM)

[Asaka, Blanchet, Shaposhnikov ('05)] [Asaka, Shaposhnikov ('05)]

SM + 3 right-handed neutrinos with $M \lesssim \mathcal{O}(100)\,\mathrm{GeV}$




Heavy Neutral Leptons (HNLs)

$N_1$ ▪ **Dark matter**

mass $\sim \mathcal{O}(10)\mathrm{keV}$

Yukawa couplings $\sim \mathcal{O}(10^{-13})$

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$N_2\,, N_3$ ▪ **Non-zero neutrino masses**

by Seesaw mechanism [Minkowski ('77)] [Yanagida ('79)] [Gell-Mann, Ramond, Slansky ('79)] [Glashow ('79)] [Mohapatra, Senjanovic ('79)]

▪ **Baryon asymmetry of the universe (BAU)** [Akhemedov, Rubakov, Smirnov ('98)]

by Baryogenesis via neutrino oscillations [Asaka, Shaposhnikov ('05)]

Masses are in GeV range and quasi-degenerate $M \sim \mathcal{O}(1)\,\mathrm{GeV}$

Yukawa couplings $\sim \mathcal{O}(10^{-7})$ $\Delta M/M \ll 1$

# Baryogenesis via neutrino oscillations

[Akhemedov, Rubakov, Smirnov ('98)] [Asaka, Shaposhnikov ('05)]

Interactions of HNLs are out-of-equilibrium due to the small Yukawa couplings

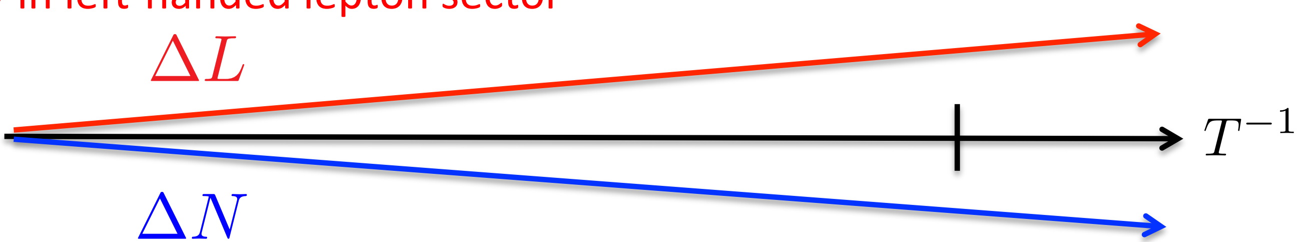
HNL oscillations : Flavor transitions of left-handed leptons :


Lepton asymmetries are generated by CP violation in neutrino Yukawa couplings

$$\Gamma(L_\alpha \to L_\beta) \neq \Gamma(\bar{L}_\alpha \to \bar{L}_\beta)$$

Effective lepton number conservation due to $M \ll T$ leads $\Delta L \approx -\Delta N$

Asymmetry in left-handed lepton sector




Asymmetry in right-handed neutrino sector

$T_{\rm SF}^{-1} \approx (130\ \mathrm{GeV})^{-1}$

[D'Onofrio, Rummukainen, Tranberg ('14)]

# Baryogenesis via neutrino oscillations

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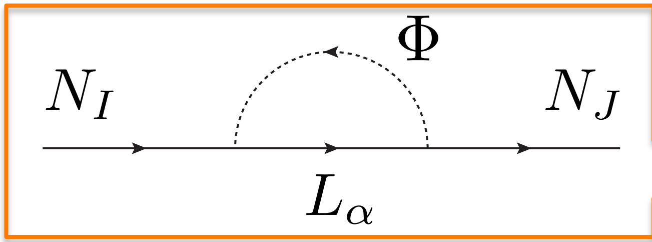
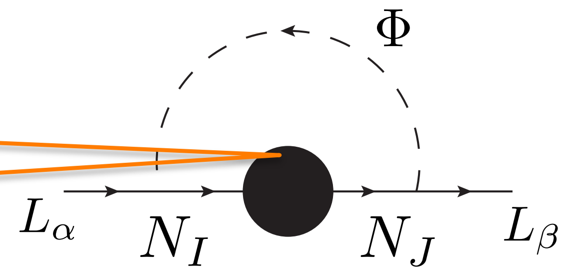



Flavor transitions of left-handed leptons :

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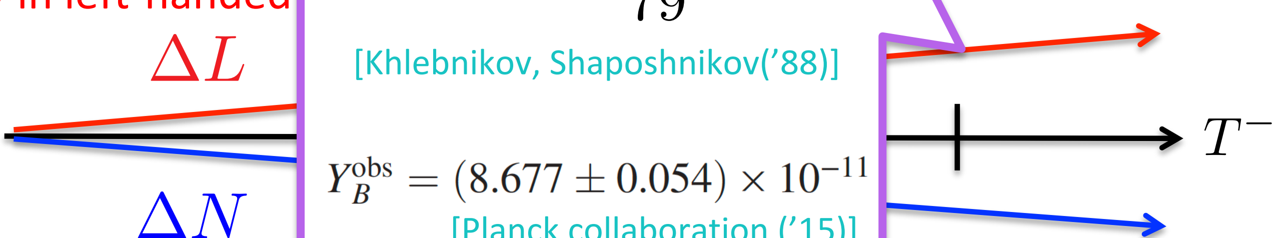
Asymmetry in left-handed




Asymmetry in right-hande

$\approx (130 \text{ GeV})^{-1}$

ukainen, Tranberg ('14)]

Sphaleron freeze-out

$$\Delta B = -\frac{28}{79}\Delta L$$

[Khlebnikov, Shaposhnikov('88)]

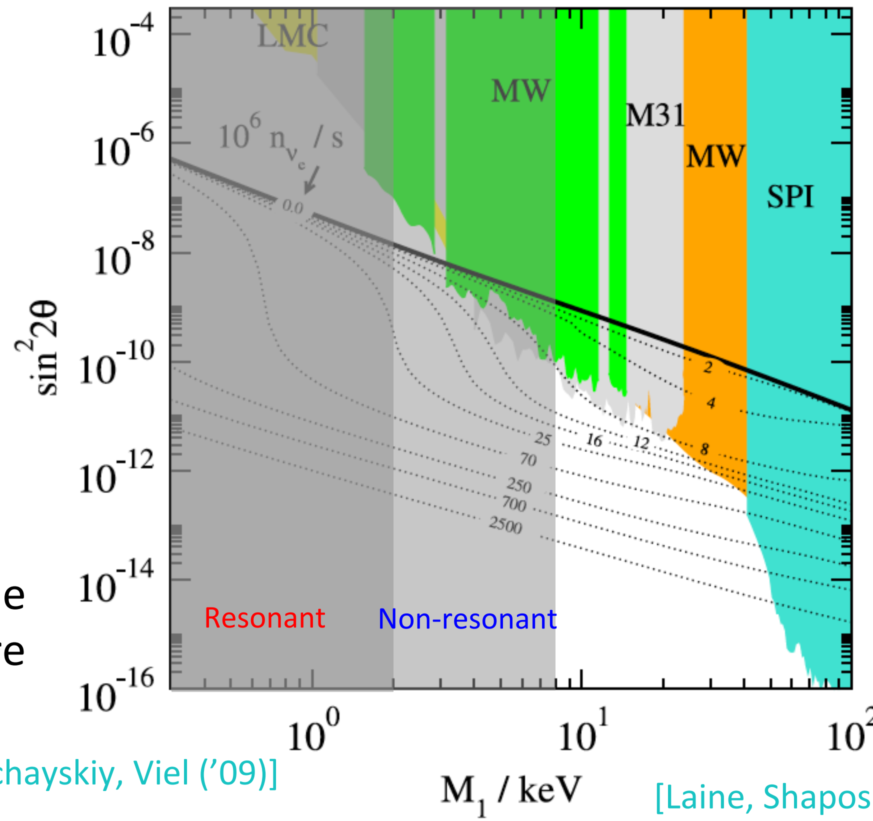
$$Y_B^{\text{obs}} = (8.677 \pm 0.054) \times 10^{-11}$$

[Planck collaboration ('15)]

$$\to \sum_\alpha n_{\nu_\alpha}/s \simeq 10^{-10}$$

# Problem : DM production

Allowed parameter space of the lightest HNL




Upper bound on the mixing from X-ray observation

Abundance of DM

Produced by scattering with SM particles through neutrino mixing [Dodelson, Widrow ('94)]

Lepton asymmetry induces the resonant production [Shi, Fuller ('99)]

Lower bound on the mass from structure formation

[Boyarsky, Lesgourgues, Ruchayskiy, Viel ('09)]

[Laine, Shaposhnikov ('08)]

**The resonant production requires large lepton asymmetries,** $n_{\nu_\alpha}/s \gtrsim 10^{-5}$**, at** $T_{\mathrm{DM}} \sim \mathcal{O}(100)\mathrm{MeV}$**, and only** $N_{2,3}$ **can be their source.**

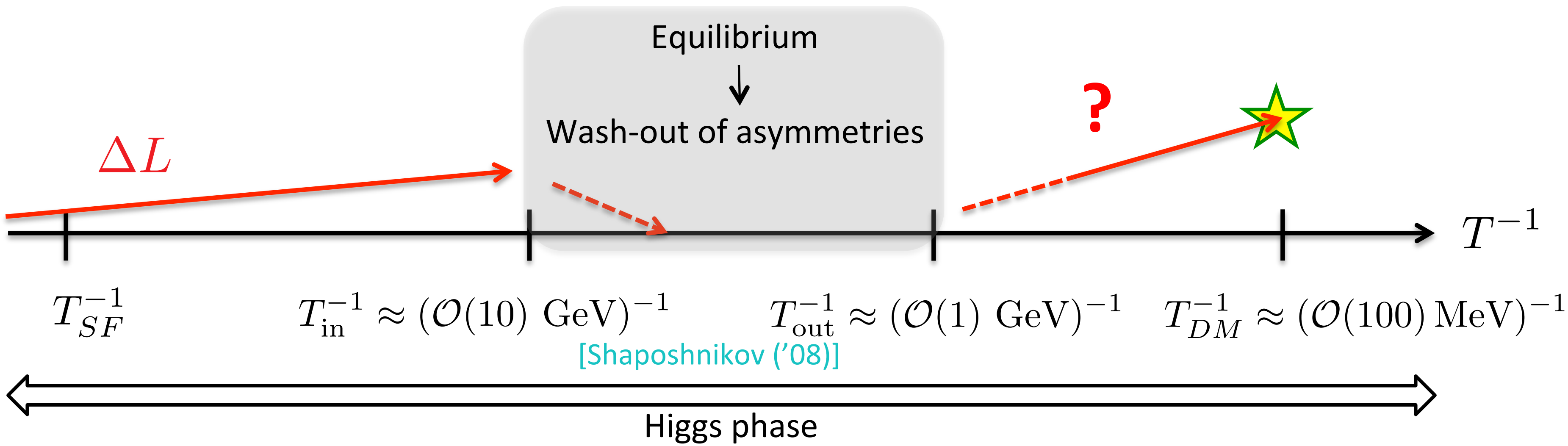
# Problem : Late-time leptogenesis

Late-time leptogenesis for the resonant dark matter production

Even after the sphaleron freeze-out the lepton asymmetry generation can continue.

The generated asymmetries are washed out when HNLs come into equilibrium.

How can we tackle this problem?




One idea :
HNL oscillations and decays at $T < T_{\rm out}$ [Canetti, Drewes, Frossard, Shaposhnikov ('13)]

**New idea :**
**Protection of asymmetries generated at $T > T_{\rm in}$ due to a conservation law!**

# Kinetic equations in the Higgs phase

To treat coherent and incoherent processes at the same time

Matrix of density : $\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$ — Diagonal elements : occupation numbers; Off-diagonal elements : correlations

For HNLs and anti-HNLs (2x2 matrix)

$$\frac{d\rho_N}{dt} = -i\left[H_N, \rho_N\right] - \frac{1}{2}\{\Gamma_N, \rho_N - \rho_N^{\rm eq}\} - \frac{1}{2}\sum_\alpha \left[\tilde{\Gamma}_N^\alpha \Delta\rho_{\nu_\alpha}\right]$$

Oscillation — Production and Destruction — Communication (back-reaction)

$$\frac{d\rho_{\bar N}}{dt} = -i\left[H_N^*, \rho_{\bar N}\right] - \frac{1}{2}\{\Gamma_N^*, \rho_{\bar N} - \rho_N^{\rm eq}\} + \frac{1}{2}\sum_\alpha \left[\left(\tilde{\Gamma}_N^\alpha\right)^* \Delta\rho_{\nu_\alpha}\right]$$

For lepton asymmetries $\Delta\rho_{\nu_\alpha}$ $(\alpha = e, \nu, \tau)$

Damping — Communication (back-reaction)

$$\frac{d\Delta\rho_{\nu_\alpha}}{dt} = -\Gamma_{\nu_\alpha}\Delta\rho_{\nu_\alpha} + \mathrm{Tr}[\tilde{\Gamma}_{\nu_\alpha}\rho_{\bar N}] - \mathrm{Tr}[\tilde{\Gamma}^*_{\nu_\alpha}\rho_N]$$

**All terms in the equations are described only by parameters in the theory**

# Effective Hamiltonian and production rates

## Effective Hamiltonian

$$H_N = H_0 + H_I,$$

$$H_0 = -\frac{\Delta M M}{E_N}\sigma_1$$

$$H_I = h_+ \sum_\alpha Y^N_{+,\alpha} + h_- \sum_\alpha Y^N_{-,\alpha},$$

## Production and back-reaction rates

$$\Gamma_N = \gamma_+ \sum_\alpha Y^N_{+,\alpha} + \gamma_- \sum_\alpha Y^N_{-,\alpha}$$

$$\tilde{\Gamma}^\alpha_N = -\gamma_+ Y^N_{+,\alpha} + \gamma_- Y^N_{-,\alpha}$$

$$\Gamma_{\nu_\alpha} = (\gamma_+ + \gamma_-) \sum_I h_{\alpha I} h^*_{\alpha I}$$

$$\tilde{\Gamma}_{\nu_\alpha} = -\gamma_+ Y^\nu_{+,\alpha} + \gamma_- Y^\nu_{-,\alpha}$$

## Coefficients

$$h_+ = \frac{2\langle\Phi\rangle^2 E_\nu (E_N + k)(E_N + E_\nu)}{k E_N (4(E_N + E_\nu)^2 + \gamma_\nu^2)},$$

$$h_- = \frac{2\langle\Phi\rangle^2 E_\nu (E_N - k)(E_N - E_\nu)}{k E_N (4(E_N - E_\nu)^2 + \gamma_\nu^2)},$$

$$\gamma_+ = \frac{2\langle\Phi\rangle^2 E_\nu (E_N + k)\gamma_\nu}{k E_N (4(E_N + E_\nu)^2 + \gamma_\nu^2)},$$

$$\gamma_- = \frac{2\langle\Phi\rangle^2 E_\nu (E_N - k)\gamma_\nu}{k E_N (4(E_N - E_\nu)^2 + \gamma_\nu^2)},$$

$$E_N = \sqrt{k^2 + M^2} \qquad E_\nu = k - b_L$$

## Matrices of yukawa couplings

$$Y^N_{+,\alpha} = \begin{pmatrix} h_{\alpha 3} h^*_{\alpha 3} & -h_{\alpha 3} h^*_{\alpha 2} \\ -h_{\alpha 2} h^*_{\alpha 3} & h_{\alpha 2} h^*_{\alpha 2} \end{pmatrix},$$

$$Y^N_{-,\alpha} = \begin{pmatrix} h_{\alpha 2} h^*_{\alpha 2} & -h_{\alpha 3} h^*_{\alpha 2} \\ -h_{\alpha 2} h^*_{\alpha 3} & h_{\alpha 3} h^*_{\alpha 3} \end{pmatrix},$$

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**Subscripts "+" and "-" correspond to fermion number conserving and violating contributions, respectively.**

# New conservation law

Combinations of leptonic asymmetries for HNLs and active flavors [Shaposhnikov ('08)]

$$L_\pm \equiv \Delta N \mp \Delta L \quad \text{where} \quad \begin{cases} \Delta N = \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}(\rho_N - \rho_{\bar{N}}) \\ \Delta L = \sum_\alpha \left[ \int \frac{d^3k}{(2\pi)^3} \Delta\rho_{\nu_\alpha} \right] \end{cases}$$

[ When total lepton number is conserved $\Delta L \approx -\Delta N$
$\rightarrow L_+ \neq 0, L_- \approx 0$ ]

From the derived kinetic equations

$$\frac{dL_+}{dt} \propto \gamma_+ \quad \text{and} \quad \frac{dL_-}{dt} \propto \gamma_-$$

$$\left[ \Gamma_N = \Gamma_+ + \Gamma_- \qquad \begin{aligned} \Gamma_+ &= \gamma_+ \sum_\alpha Y^N_{+,\alpha} \\ \Gamma_- &= \gamma_- \sum_\alpha Y^N_{-,\alpha} \end{aligned} \right]$$

If $\Gamma_+$ $(\Gamma_-)$ doesn't come into equilibrium, $L_+$ $(L_-)$ is a conserved number!

**In such a case generated asymmetries can potentially survive from the wash-out up to smaller temperatures even if HNLs are equilibrated.**

# Thermalization of HNLs

Interaction rate : $\Gamma_N = \Gamma_+ + \Gamma_-,$

$$\Gamma_+ = \gamma_+ \sum_\alpha Y^N_{+,\alpha},$$

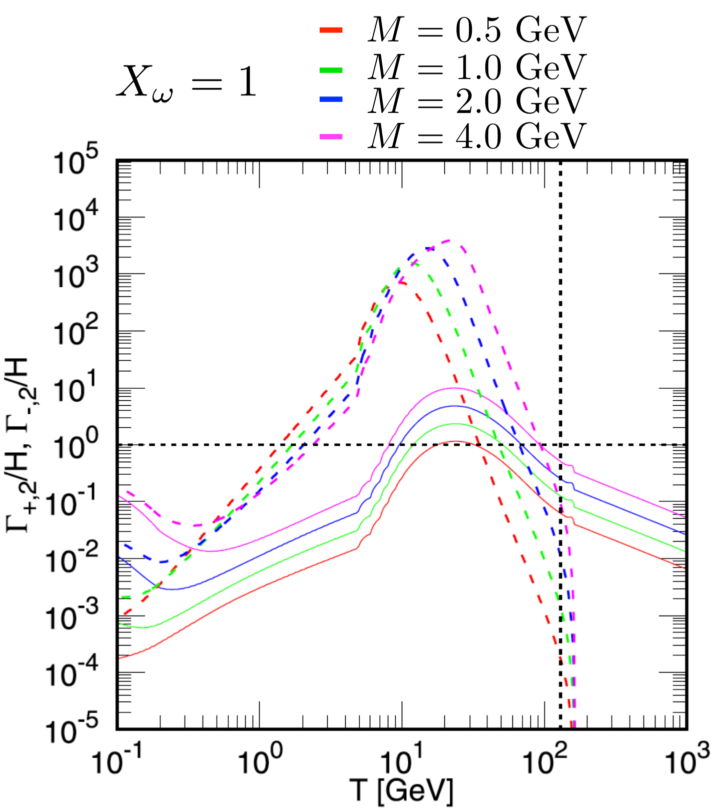
$$\Gamma_- = \gamma_- \sum_\alpha Y^N_{-,\alpha},$$

$\gamma_\pm$ depend on $M$

$\sum_\alpha Y^N_{\pm,\alpha}$ depend on $M$, active neutrino masses and a complex angle $\omega$

Largest eigenvalues : $\Gamma_{\pm,2} \propto X_\omega^2 \quad X_\omega \equiv e^{\mathrm{Im}\,\omega}$

Largest eigenvalues of $\Gamma_+$(solid lines) and $\Gamma_-$(dashed lines) divided by $H$

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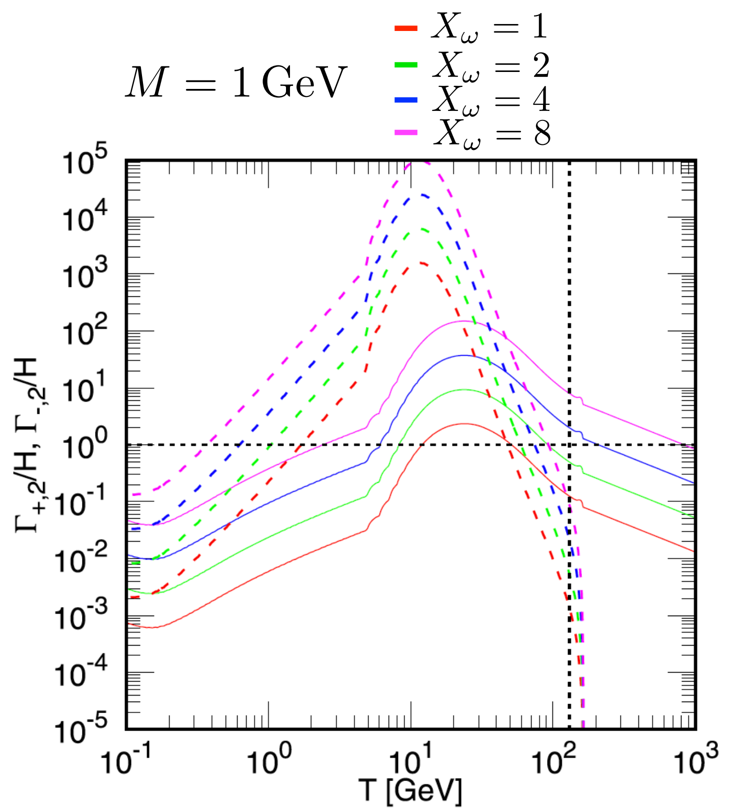


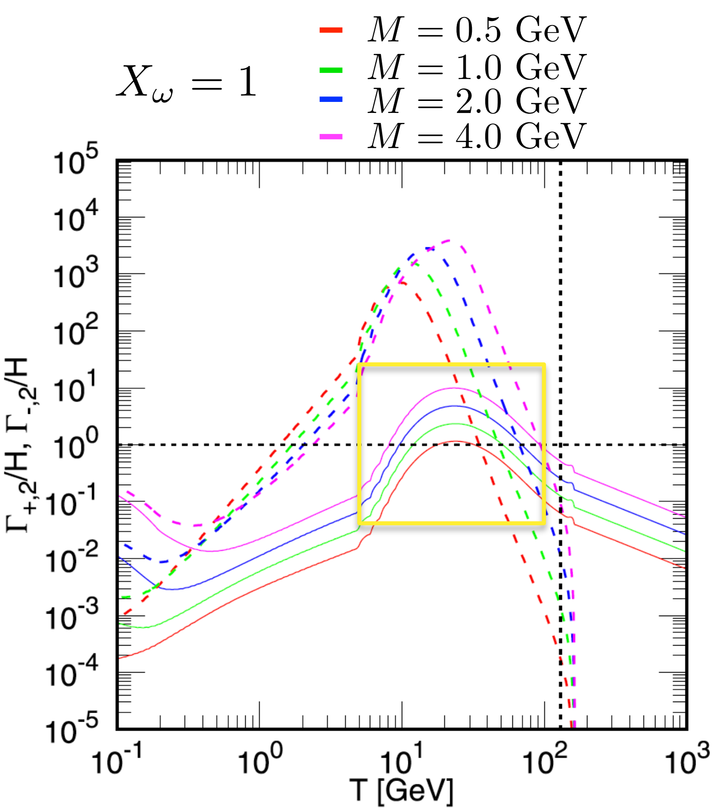
# Impact on the dark matter production

$\Gamma_-/H \gg 1$

HNLs must be thermalized.

No conservation of $L_-$

Largest eigenvalues of $\Gamma_+$(solid lines) and




$\Gamma_+/H \lesssim \mathcal{O}(1)$

for $M \lesssim 2$ GeV and $X_\omega \sim 1$

Effective conservation of $L_+$

**Protection of lepton asymmetries from wash-out.**

Further before the thermalization of HNLs $\Gamma_+/H$ gets close to unity.

**Good condition to generate a large $L_+$.**

**This conservation law can be new solution of problem in the resonant DM production!**

# Summary

In the νMSM the heavier heavy neutral leptons have to generate lepton asymmetries not only for the baryon asymmetry of the universe but also for the resonant production of the lightest HNL as dark matter.

We found **HNLs can potentially generate the late-time lepton asymmetries due to the protection from wash-out induced by new conservation law.**

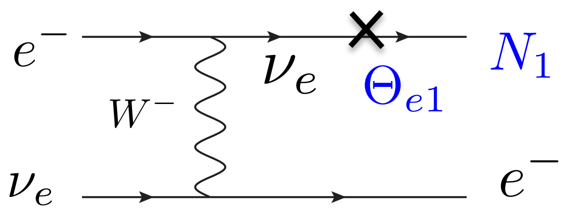
Quantitative analysis for both baryogenesis and late-time leptogenesis requires numerical parameter scanning, which will be performed in future work.

# Backup

# DM production

**Produced by scattering with SM particles through neutrino mixing**

[Dodelson, Widrow ('94)]




**Resonant production** [Shi, Fuller ('99)]

$$\Gamma_N \sim \Theta_M^2(T)\Gamma_\nu \sim \Theta_M^2(T) G_F^2 T^5$$

$$\Theta_M^2(T) \simeq \frac{\Theta_1^2}{\left(1 + \frac{2p}{M_1^2}(b(p,T) \pm c(T))\right)^2 + \Theta_1^2}$$

$$\begin{cases} b(p,T) = \dfrac{16 G_F^2}{\pi \alpha_W} p (2 + \cos^2\theta_W) \dfrac{7\pi^2 T^4}{360} \\ c(T) = 3\sqrt{2} G_F (1 + 4\sin^2\theta_W)(n_{\nu_e} - n_{\bar{\nu}_e}) \end{cases}$$

[Boyarsky, Ruchayskiy, Shaposhnikov ('09)]

When $\left[1 + \frac{2p}{M_1^2}(b(p,T) \pm c(T))\right] \simeq 0$ the mixing is enhanced

→ The level crossing between ν and $N_1$ leads the rapid transition

# Derivation of kinetic equations

Lagrangian in Pseudo-Dirac basis : $\Psi = N_2 + N_3^c$

$$\mathcal{L} = \mathcal{L}_{SM} + \overline{\Psi} i \partial_\mu \gamma^\mu \Psi - M \overline{\Psi} \Psi + \mathcal{L}_{int},$$

$$\mathcal{L}_{int} = -\frac{\Delta M}{2}(\overline{\Psi}\Psi^c + \overline{\Psi^c}\Psi) - (h_{\alpha 2}\langle\Phi\rangle\overline{\nu_{L\alpha}}\Psi + h_{\alpha 3}\langle\Phi\rangle\overline{\nu_{L\alpha}}\Psi^c + h.c.),$$

$\longrightarrow$ **Interaction with left-handed $\nu$ , HNL oscillation and lepton number violation**

HNL field

$$\Psi = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2k_0}}\sum_\sigma \left[a_\sigma(k) u_\sigma(k) e^{-ik\cdot x} + b_\sigma^\dagger(k) v_\sigma(k) e^{ik\cdot x}\right],$$

Fermion numbers of creation operators.

| plus, particles | minus, anti-particles |
|---|---|
| $a_+^\dagger(k)$, $b_-^\dagger(k)$ | $a_-^\dagger(k)$, $b_+^\dagger(k)$ |

---

The density matrix can treat coherent and incoherent processes simultaneously.

$$\rho_N = \begin{pmatrix} \mathrm{Tr}[a_+^\dagger(k) a_+(k)\rho] & \mathrm{Tr}[a_+^\dagger(k) b_-(k)\rho] \\ \mathrm{Tr}[b_-^\dagger(k) a_+(k)\rho] & \mathrm{Tr}[b_-^\dagger(k) b_-(k)\rho] \end{pmatrix}$$

$\rho$ : density matrix operator of complete system

# Derivation of kinetic equations

Time derivative of number density

$$i\dot{q}_i^0 = \mathrm{Tr}\left([\mathbf{H}, Q_i^0]\rho\right) \ ,$$

Number density : $q_i^0 = \mathrm{Tr}[Q_i^0 \rho]$

Number density operator : $Q_i^0$

Total Hamiltonian : $\mathbf{H} = H_2 + H_{int} + H_{int}^{SM}$

$H_2$ : quadratic part including HNLs and active neutrinos

$H_{int}$ : corresponding to $\mathcal{L}_{int}$

$H_{int}^{SM}$ : SM interactions

$H_{int}^{SM}$ can be accounted for by modification of neutrino energy with the thermal potential $b_L$ and interaction rate $\gamma_\nu$.

Computing commutators with $H_2 + H_{int}$ perturbatively in the second order in yukawa couplings and first order in $\Delta M$ we derived the kinetic equations.

# Fermion number violation effects

**Coefficients**

$$h_+ = \frac{2\langle\Phi\rangle^2 E_\nu (E_N + k)(E_N + E_\nu)}{k E_N (4(E_N + E_\nu)^2 + \gamma_\nu^2)},$$

$$h_- = \frac{2\langle\Phi\rangle^2 E_\nu (E_N - k)(E_N - E_\nu)}{k E_N (4(E_N - E_\nu)^2 + \gamma_\nu^2)},$$

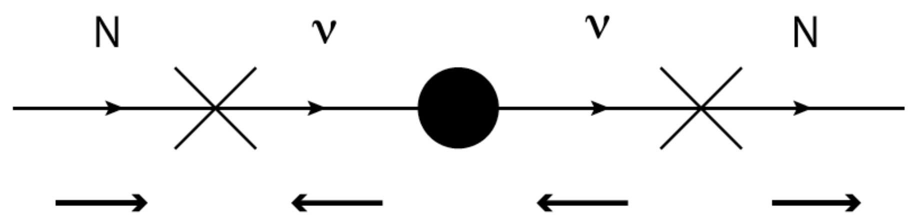
$$\gamma_+ = \frac{2\langle\Phi\rangle^2 E_\nu (E_N + k)\gamma_\nu}{k E_N (4(E_N + E_\nu)^2 + \gamma_\nu^2)},$$

$$\gamma_- = \frac{2\langle\Phi\rangle^2 E_\nu (E_N - k)\gamma_\nu}{k E_N (4(E_N - E_\nu)^2 + \gamma_\nu^2)},$$

"+" processes

$$h_{\alpha 2}\, a_+(k)\, b_\nu(-k)\, e^{-i(E_N+E_\nu)t},$$

$$h^*_{\alpha 2}\, b^\dagger_\nu(-k)\, a^\dagger_+(k)\, e^{i(E_N+E_\nu)t},$$

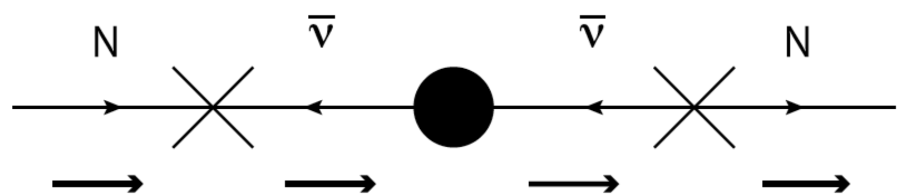



(a) Fermion number conserving process

"-" processes

$$h^*_{\alpha 3}\, a_+(k)\, b_\nu(k)^\dagger\, e^{-i(E_N-E_\nu)t},$$

$$h_{\alpha 3}\, b_\nu(k)\, a^\dagger_+(k)\, e^{i(E_N-E_\nu)t}.$$




(b) Fermion number violating process

**Fermion number violating effects are obtained automatically!**

# Improved kinetic equations

To make more realistically the coefficients are modified by adding contributions from "direct" processes (non-vanishing processes for $\langle\Phi\rangle \to 0$ )

Higgs phase

$$h_+ = \mathcal{K}(m_h)\frac{T^2}{8k} + \frac{2\langle\Phi\rangle^2 E_\nu(E_N+k)(E_N+E_\nu)}{kE_N(4(E_N+E_\nu)^2+\gamma_\nu^2)},$$

$$\gamma_+ = \gamma_+^{\text{direct}} + \frac{2\langle\Phi\rangle^2 E_\nu(E_N+k)\gamma_\nu}{kE_N(4(E_N+E_\nu)^2+\gamma_\nu^2)},$$

$$\gamma_+^{\text{direct}} = \mathcal{K}(m_h)\frac{1}{E_N}\mathrm{Im}\,\Pi_R + \gamma_{ph},$$

$$\mathcal{K}(m_h) = \frac{3}{\pi^2 T^3}\int_0^\infty dp\, p^2 f_b(E_h)(1+f_b(E_h)),$$

$$\gamma_{ph} = \frac{1}{E_N}\frac{m_h^2 T}{32\pi k}\ln\left\{\frac{1+e^{-\frac{m_h^2}{4kT}}}{1-e^{-\frac{1}{T}\left(k+\frac{m_h^2}{4k}\right)}}\right\},$$ [Ghiglieri, Laine ('16)]

$$h_- = \frac{2\langle\Phi\rangle^2 E_\nu(E_N-k)(E_N-E_\nu)}{kE_N(4(E_N-E_\nu)^2+\gamma_\nu^2)},$$

$$\gamma_- = \frac{2\langle\Phi\rangle^2 E_\nu(E_N-k)\gamma_\nu}{kE_N(4(E_N-E_\nu)^2+\gamma_\nu^2)},$$

Ultra-relativistic limit

$\langle\Phi\rangle \to 0$

Symmetric phase

$$h_+ = \frac{T^2}{8k},$$

$$\gamma_+ = \frac{1}{E_N}\mathrm{Im}\,\Pi_R,$$

$$h_- = 0,$$

$$\gamma_- = 0,$$

# Conserved quantum number

$$\frac{d}{dt}L_+ = -2\int \frac{d^3k}{(2\pi)^3}\gamma_+ \sum_\alpha \Big[ h_{\alpha 2}h^*_{\alpha 2}(\rho_{N,11} - \rho_{\bar{N},11}) + h_{\alpha 3}h^*_{\alpha 3}(\rho_{N,22} - \rho_{\bar{N},22}) - 2\mathrm{Re}(h_{\alpha 2}h^*_{\alpha 3})(\mathrm{Re}\rho_{N,12} - \mathrm{Re}\rho_{\bar{N},12}) + 2\mathrm{Im}(h_{\alpha 2}h^*_{\alpha 3})(\mathrm{Im}\rho_{N,12} + \mathrm{Im}\rho_{\bar{N},12}) - (h_{\alpha 2}h^*_{\alpha 2} + h_{\alpha 3}h^*_{\alpha 3})\,\omega_{\alpha\beta}\left[\frac{12\Delta_\beta}{T^3} f_f(1-f_f)\right]\Big].$$

$$\frac{d}{dt}L_- = -2\int \frac{d^3k}{(2\pi)^3}\gamma_- \sum_\alpha \Big[ h_{\alpha 2}h^*_{\alpha 2}(\rho_{N,11} - \rho_{\bar{N},11}) + h_{\alpha 3}h^*_{\alpha 3}(\rho_{N,22} - \rho_{\bar{N},22}) - 2\mathrm{Re}(h_{\alpha 2}h^*_{\alpha 3})(\mathrm{Re}\rho_{N,12} - \mathrm{Re}\rho_{\bar{N},12}) + 2\mathrm{Im}(h_{\alpha 2}h^*_{\alpha 3})(\mathrm{Im}\rho_{N,12} + \mathrm{Im}\rho_{\bar{N},12}) + (h_{\alpha 2}h^*_{\alpha 2} + h_{\alpha 3}h^*_{\alpha 3})\,\omega_{\alpha\beta}\left[\frac{12\Delta_\beta}{T^3} f_f(1-f_f)\right]\Big],$$

# Yukawa coupling constants from seesaw

From the seesaw mass matrix $M_\nu = -\langle\Phi\rangle^2 F\, M_M^{-1}\, F^T$, the yukawa coupling constants in mass basis are

$$F = (i/\langle\Phi\rangle) U\, D_\nu^{\frac{1}{2}}\, \Omega\, D_N^{\frac{1}{2}}$$

(3x2 matrix) [Casas, Ibarra ('01)]

- $D_\nu^{\frac{1}{2}} = \mathrm{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$
- $D_N^{\frac{1}{2}} = \mathrm{diag}(\sqrt{M_1}, \sqrt{M_2}) = \mathrm{diag}(\sqrt{M_N - \Delta M}, \sqrt{M_N + \Delta M})$
- $U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\eta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- $\Omega = \begin{pmatrix} 0 & 0 \\ \cos\omega & \sin\omega \\ -\xi\sin\omega & \xi\cos\omega \end{pmatrix}$ (*e.g.* NH)

$\omega$ :complex parameter $\quad \xi = \pm 1$

$$F \propto \exp(\mathrm{Im}\,\omega) \equiv X_\omega \quad (\mathrm{Im}\,\omega > 0)$$
$$F \propto \exp(-\mathrm{Im}\,\omega) \equiv X_\omega^{-1} \quad (\mathrm{Im}\,\omega < 0)$$

$$F_{\alpha I} = h_{\alpha J}[U_N^*]_{JI}, \quad U_N = \frac{1}{\sqrt{2}}\begin{pmatrix} -i & 1 \\ i & 1 \end{pmatrix}$$

ν osc. is guaranteed as long as this parameterization is relevant.