COSMO-18 30 Aug 2018

Positivity Constraints for Gravity and Cosmology

Scott Melville

Imperial College London

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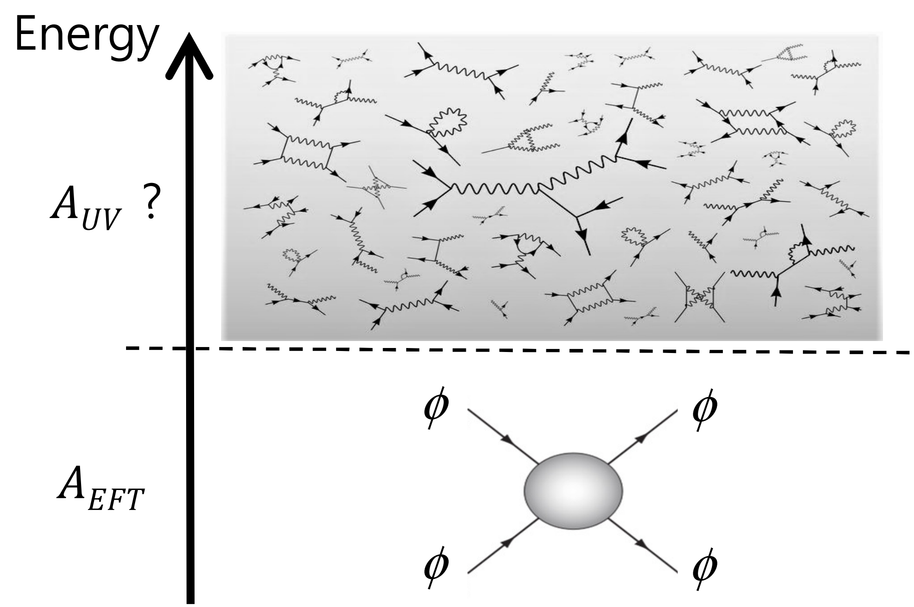

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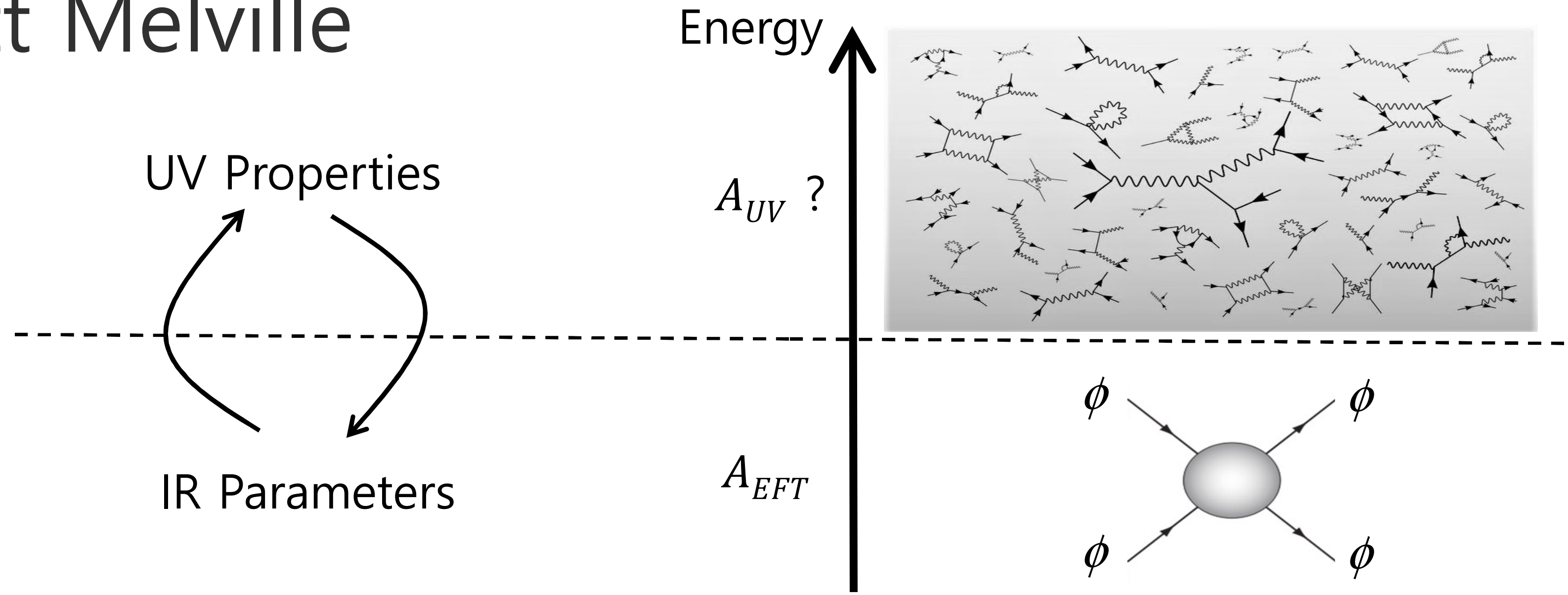

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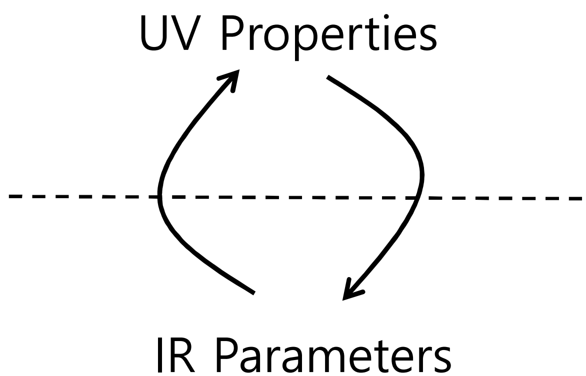

All EFTs

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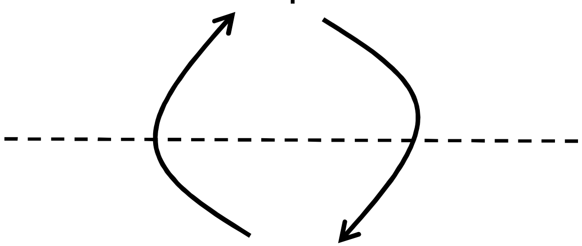

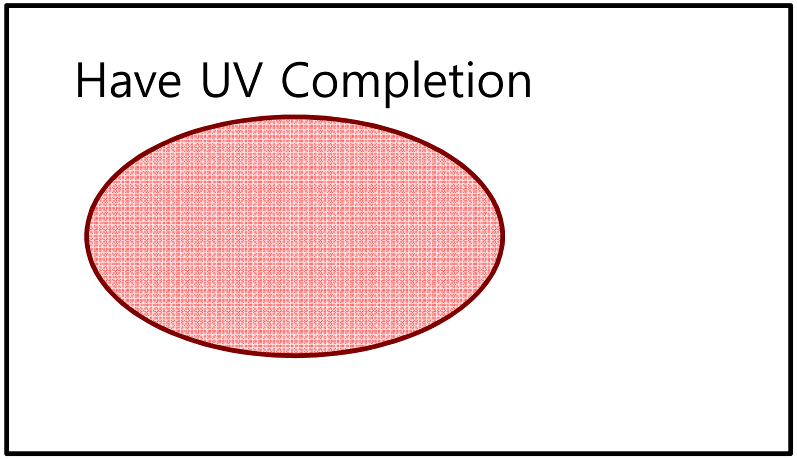

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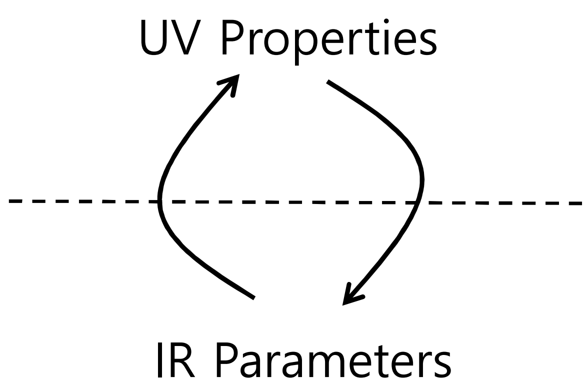

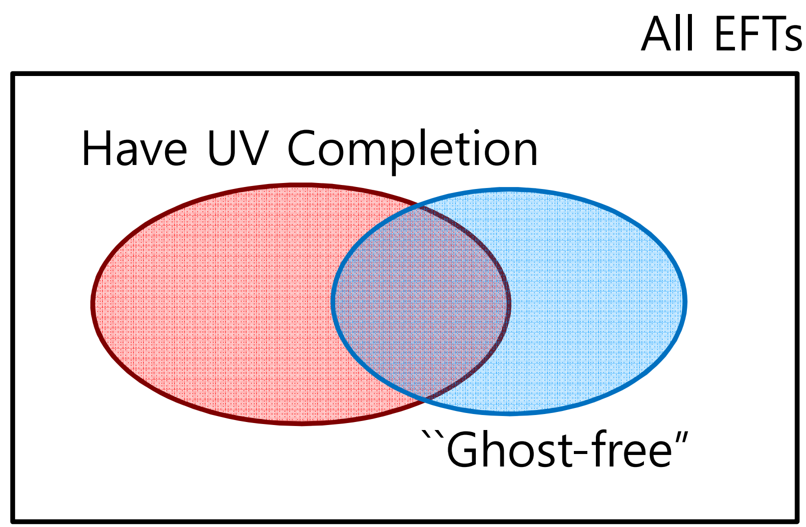

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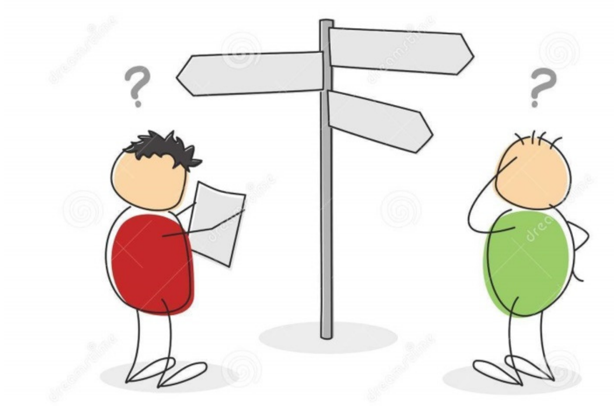

Where do they come from?

hep-ph/9607351 Donoghue
hep-th/0602178 Adams et al
hep-th/0609159 Jenkins et al
1605.06111 Bellazzini
1702.06134 SM et al
1706.02712 SM et al

What good are they?

1204.6388 Dvali
1502.07304 Baumann et al
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1601.04068 Cheung et al
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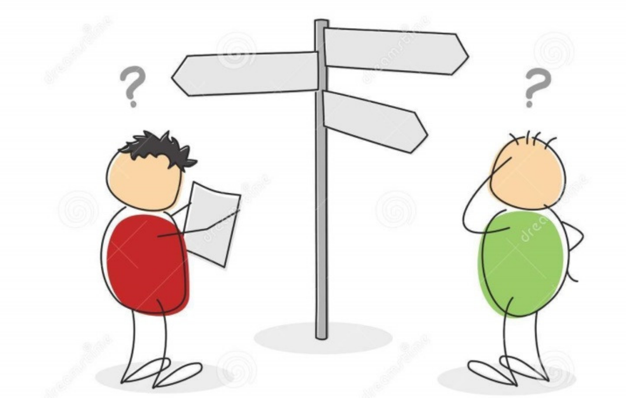
1710.02539 Bellazzini
1710.09611 SM et al
1804.10624 SM et al

- Big Picture

Only some EFTs have sensible UV completions…



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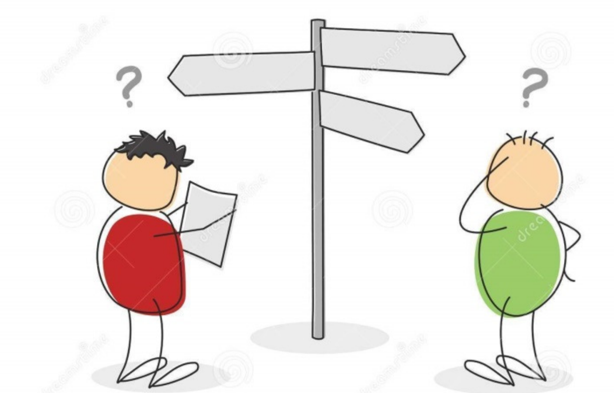
- Big Picture
- Positivity Constraints

Only some EFTs have sensible UV completions…

… and this gives us constraining power!



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- Big Picture — Only some EFTs have sensible UV completions…
- Positivity Constraints — … and this gives us constraining power!
- Applications — $P(X)$, Proca, Massive Gravity

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- Big Picture — Only some EFTs have sensible UV completions…
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What good are they?

Scalars

1204.6388	Dvali
1502.07304	Baumann et al
1702.08577	SM et al

Fermions

0802.4081	Adams et al

Vectors

1405.2960	Bellazzini et al
1808.00010	Zhang, Zhou

Gravity

1509.00851	Bellazzini et al
1601.04068	Cheung et al
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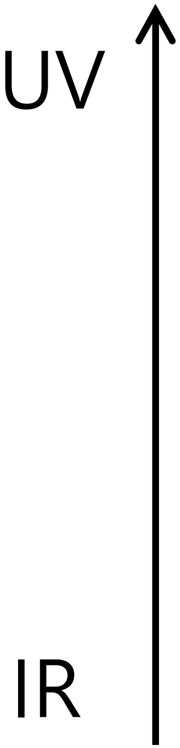
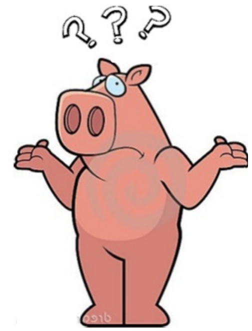
Big Picture

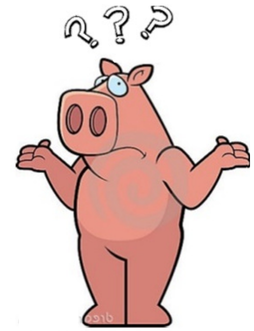

UV
IR

UV

$$\mathcal{L}_{UV} = \quad ?$$



IR



UV

$$\mathcal{L}_{UV} = \quad ?$$

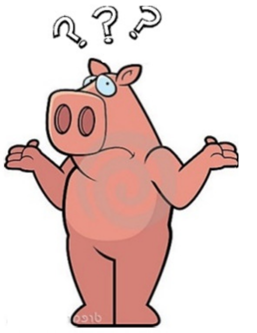
Λ

$$\mathcal{L}_{EFT} = \sum \frac{c_n}{\Lambda^n} \mathcal{O}_n$$

IR

UV

$$\mathcal{L}_{UV} = \quad ?$$



Λ

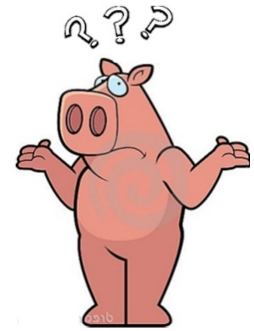
$$\mathcal{L}_{EFT} = \sum \frac{c_n}{\Lambda^n} \mathcal{O}_n$$

IR

$$\mathcal{L} = (\partial\phi)^2 + \frac{c_4}{\Lambda^4}(\partial\phi)^4 + \frac{c_6}{\Lambda^6}(\partial\phi)^6 + \cdots$$

UV

$$\mathcal{L}_{UV} = \quad ?$$



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$$\mathcal{L}_{EFT} = \sum^{N} \frac{c_n}{\Lambda^n} \mathcal{O}_n$$

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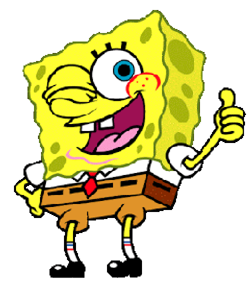
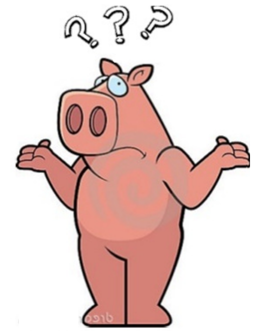


Finite Number of Local Operators

UV

$$\mathcal{L}_{UV} = \quad ?$$



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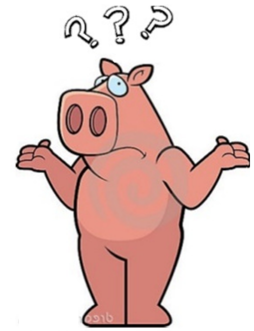
Finite Number of Local Operators

with undetermined coefficients

UV

$$\mathcal{L}_{UV} = \quad ?$$



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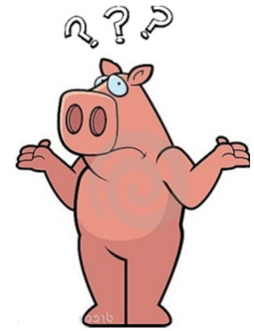

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Finite Number of Local Operators

with undetermined coefficients

(1) Many operators

=> Need many measurements

UV

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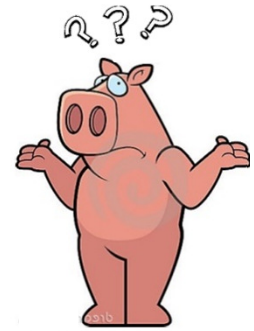

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UV

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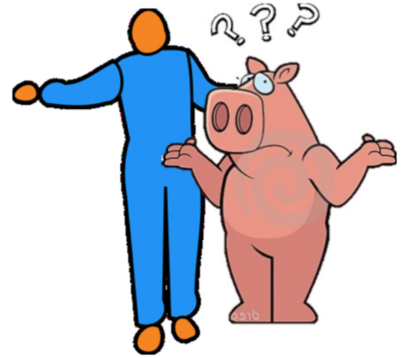

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Finite Number of Local Operators

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UV

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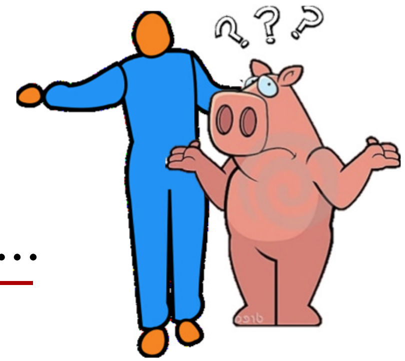
Unitary, Causal, Local, …



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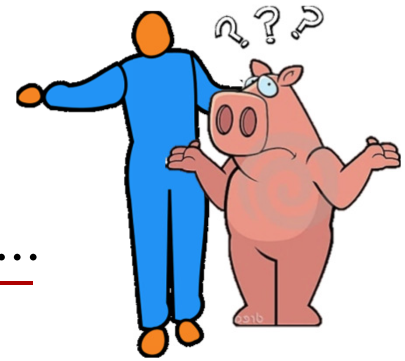

Unitary, Causal, Local, …

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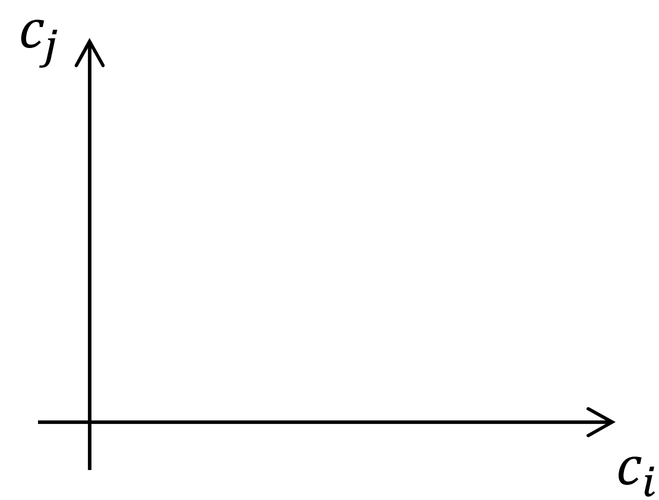

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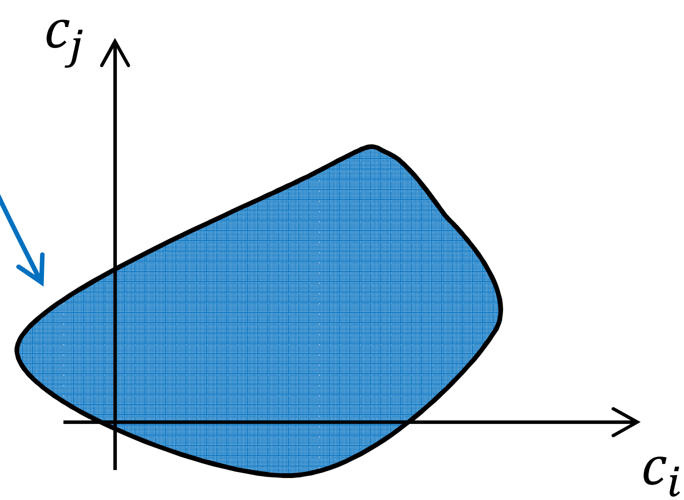

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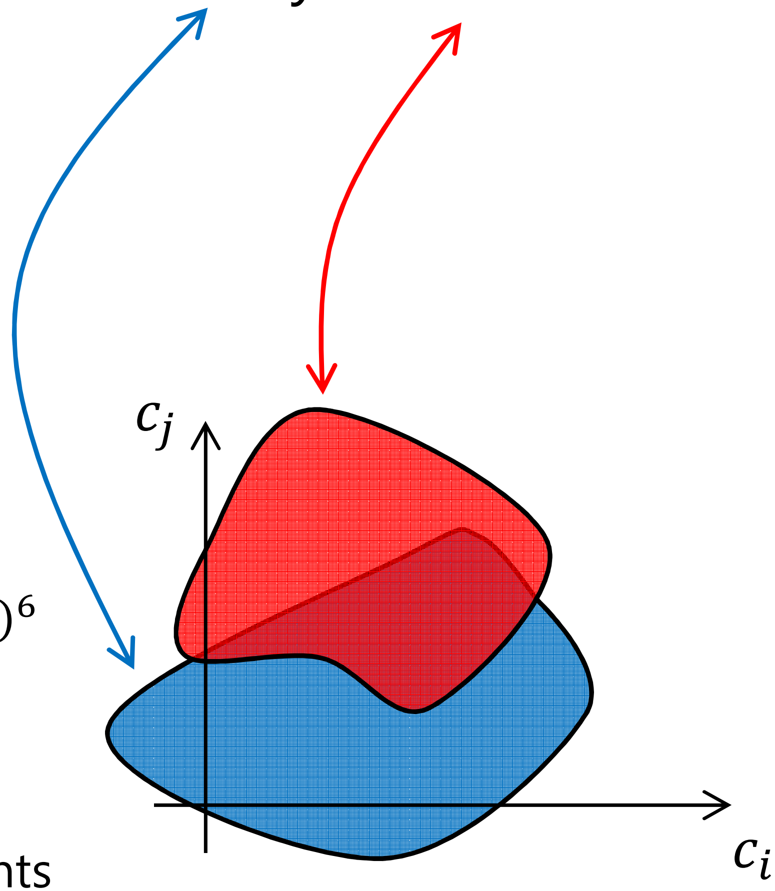

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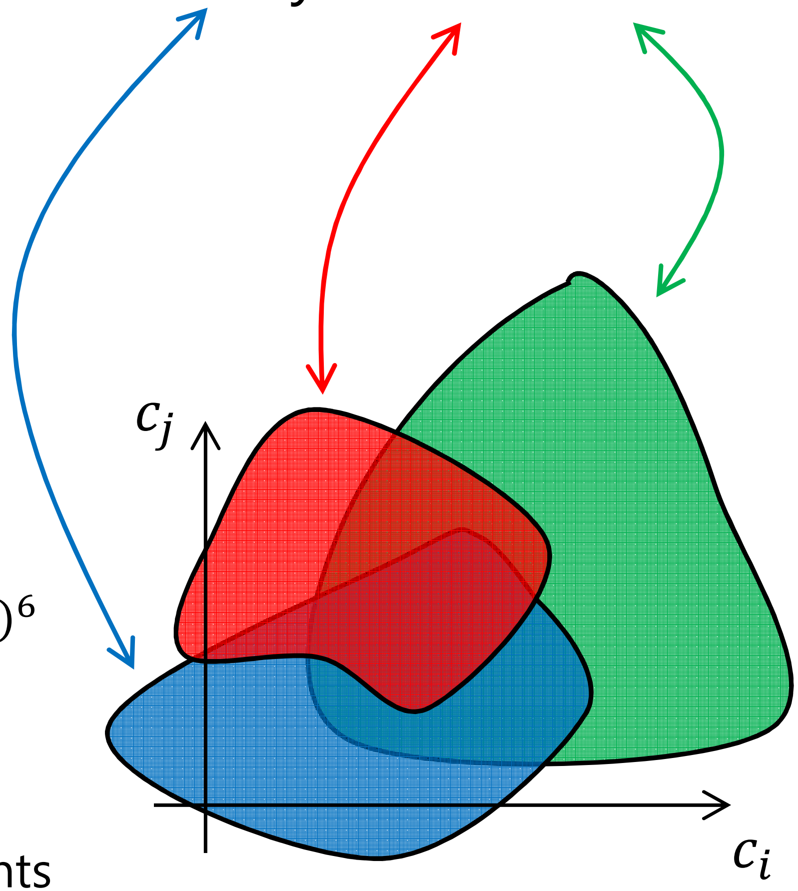

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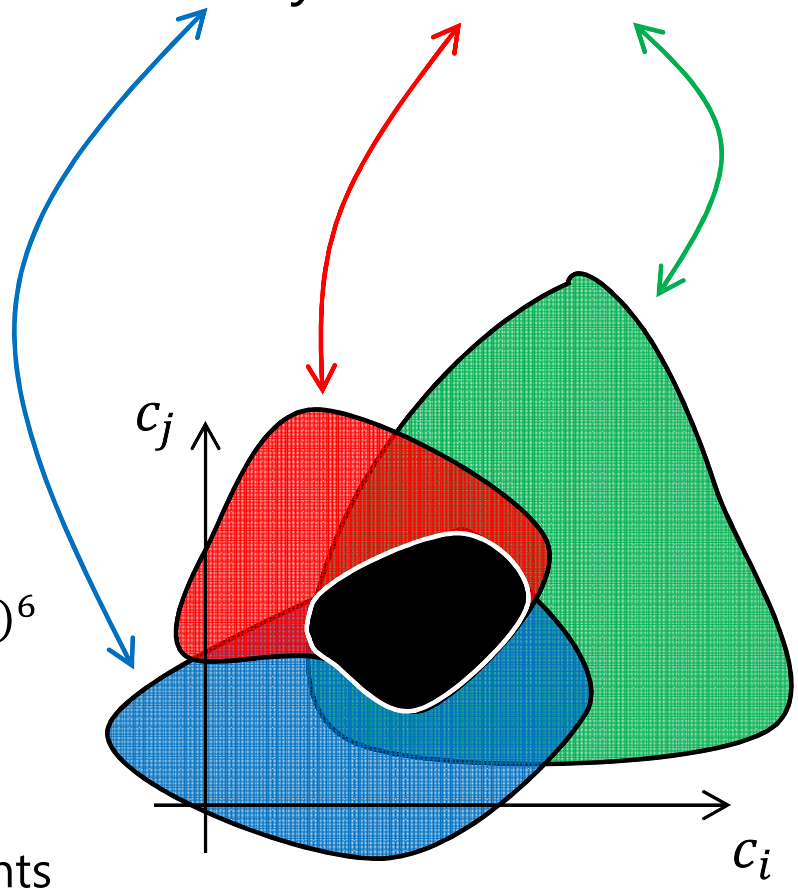

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Unitary, Causal, Local, ...

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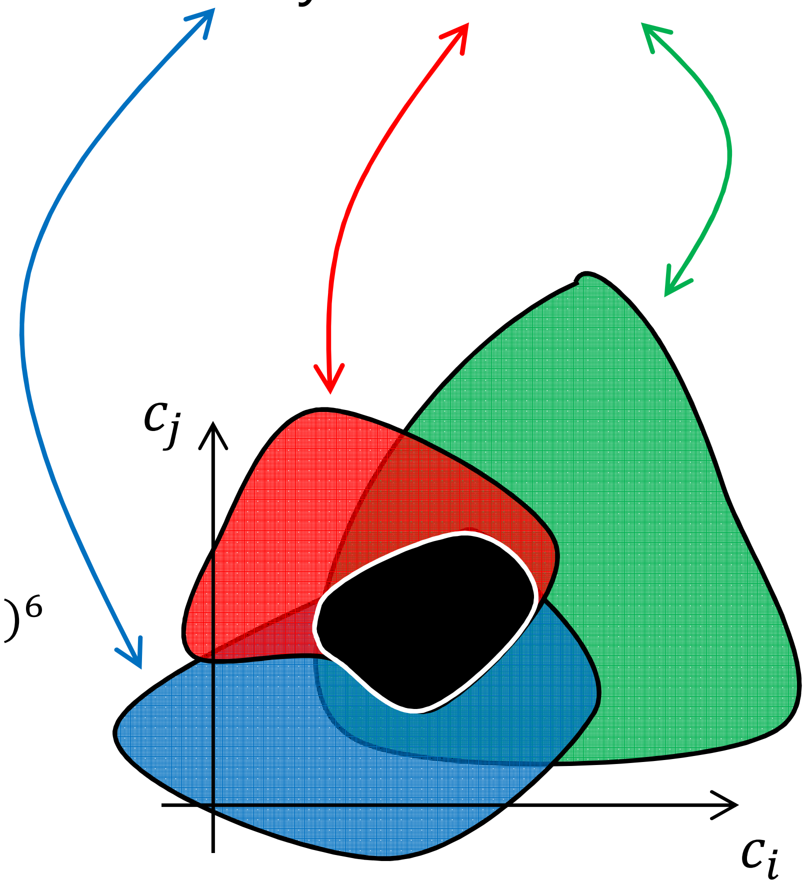

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✓ (1) Many operators
=> Many constraints

(2) No deeper understanding of UV

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$$\mathcal{L}_{UV} = \quad ?$$

Unitary, Causal, Local, ...

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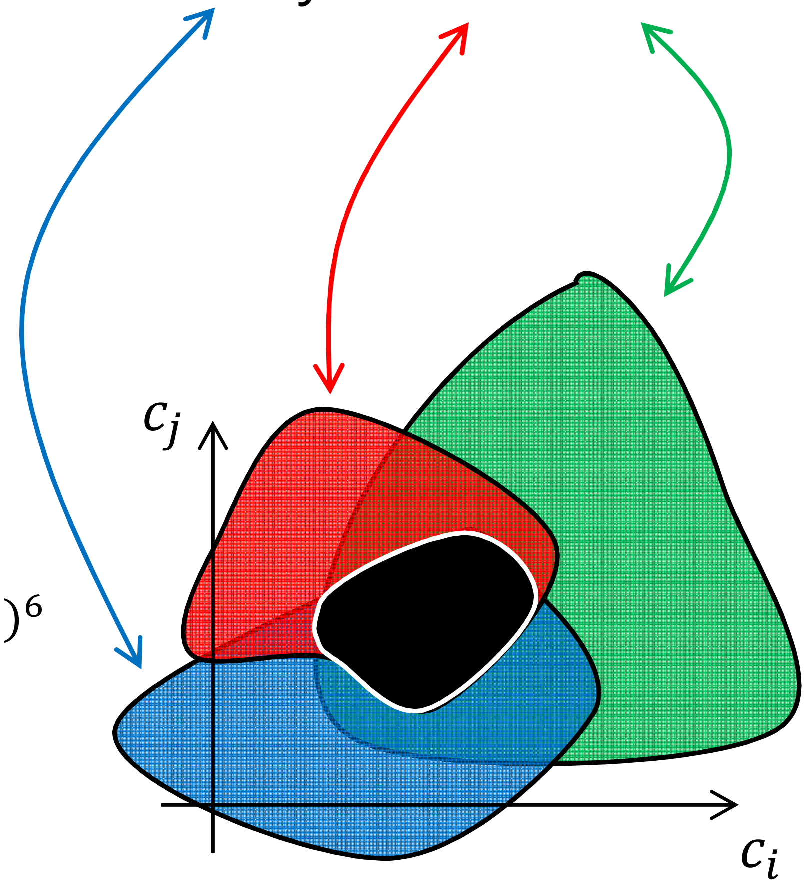

(1) Many operators
=> Many constraints

(2) Can infer UV properties from IR measurements

UV

$$\mathcal{L}_{UV} = \quad ?$$

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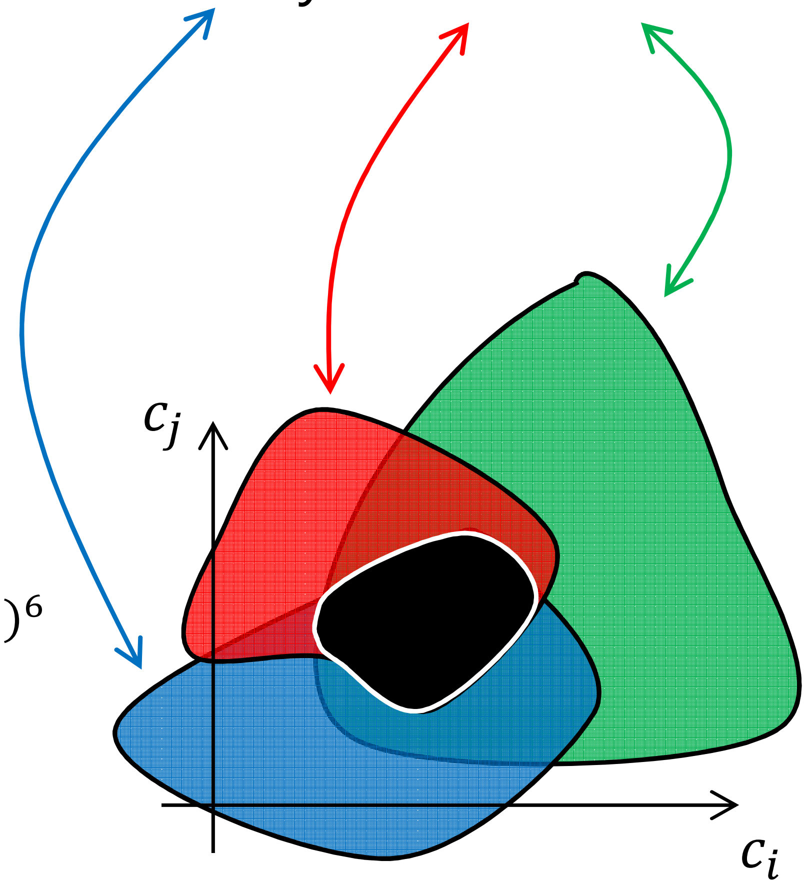
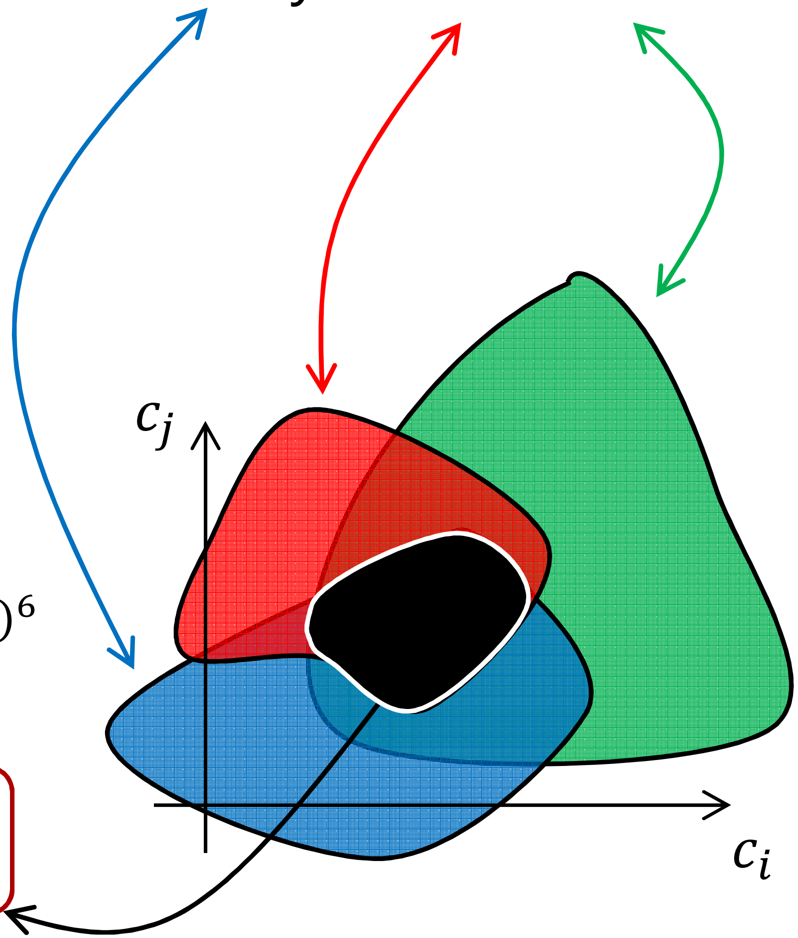
Unitary, Causal, Local, …



UV

$$\mathcal{L}_{UV} = \quad ?$$

Unitary, Causal, Local, ...



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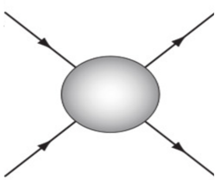
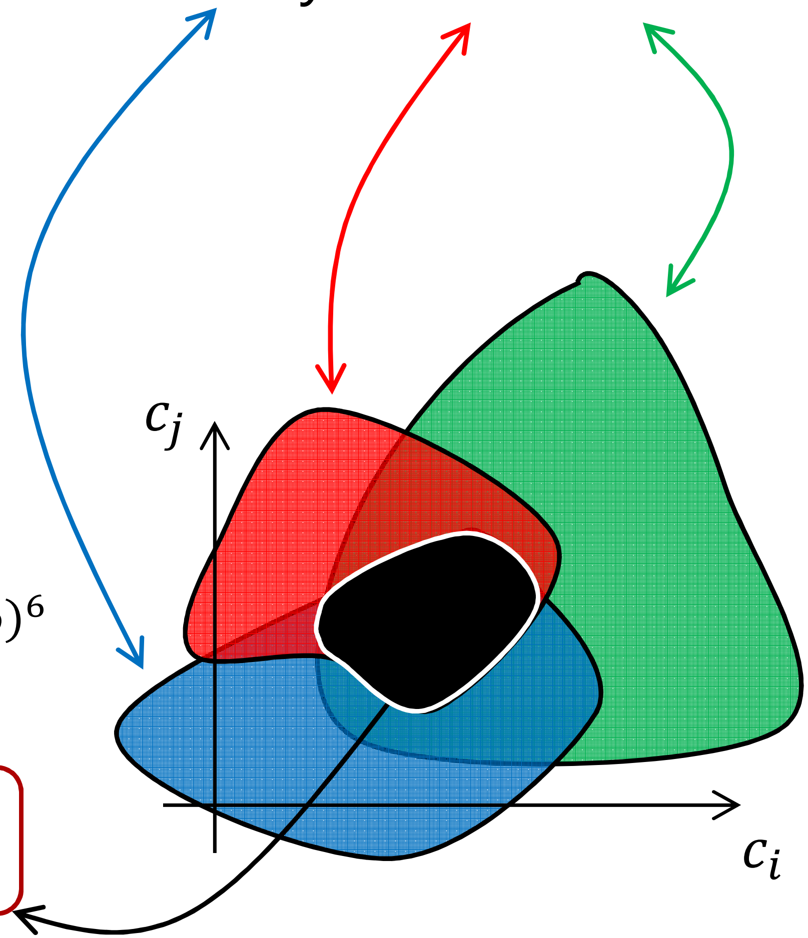


A_4

UV

$$\mathcal{L}_{UV} = \quad ?$$

Unitary, Causal, Local, …



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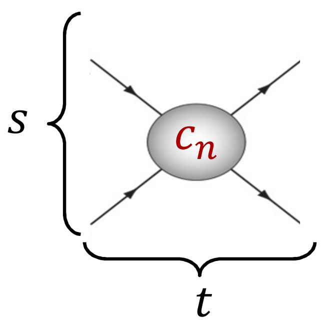

$$\boxed{A_4}$$

UV

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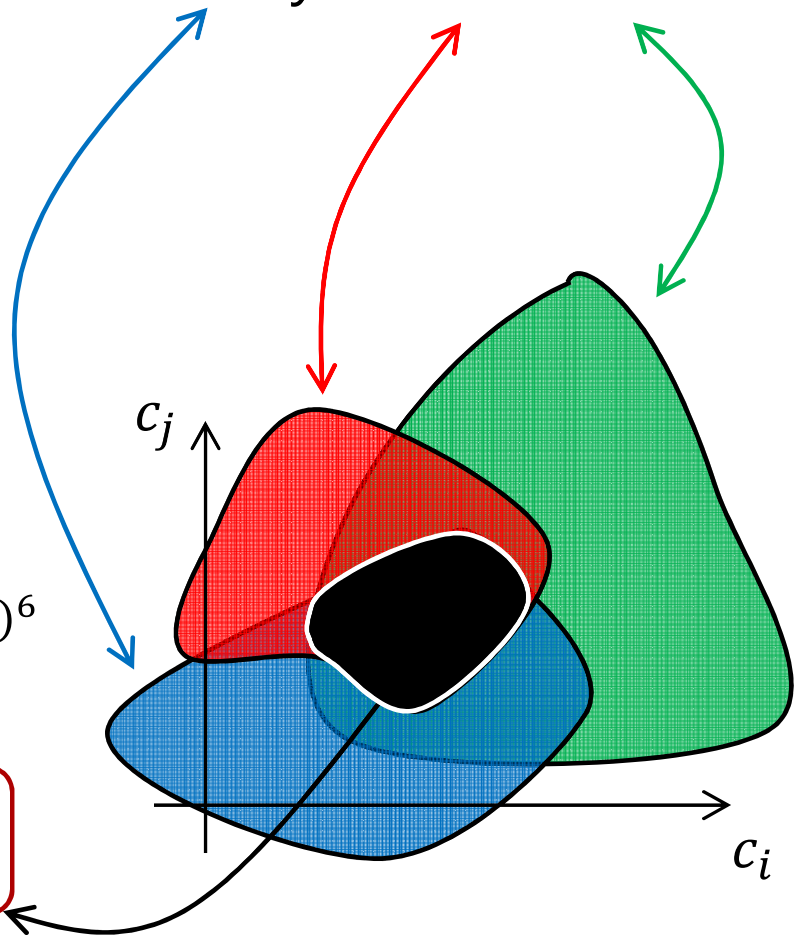
Unitary, Causal, Local, ...



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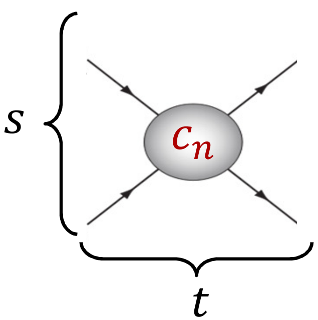

$$\boxed{A_4(s,t;c_n)}$$

UV

$$\mathcal{L}_{UV} = \quad ?$$

Unitary, Causal, Local, …

Im A **positive**

$$\mathcal{L}_{EFT} = \sum \frac{c_n}{\Lambda^n} \mathcal{O}_n$$

IR

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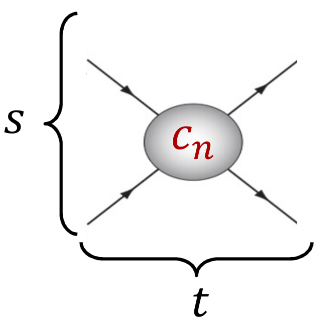
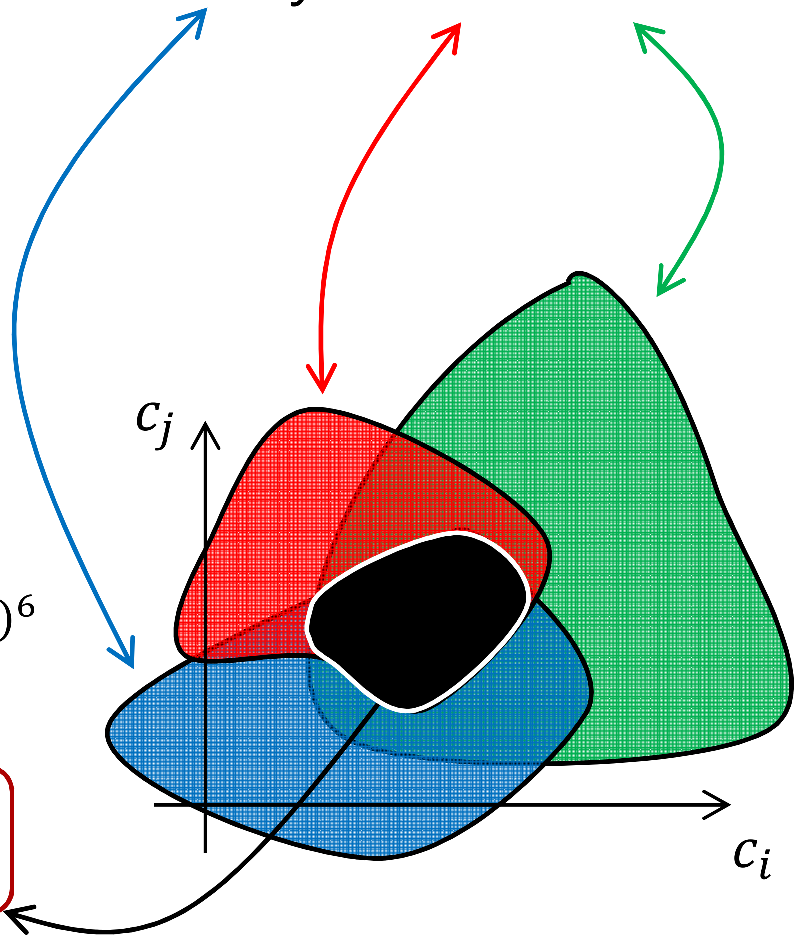


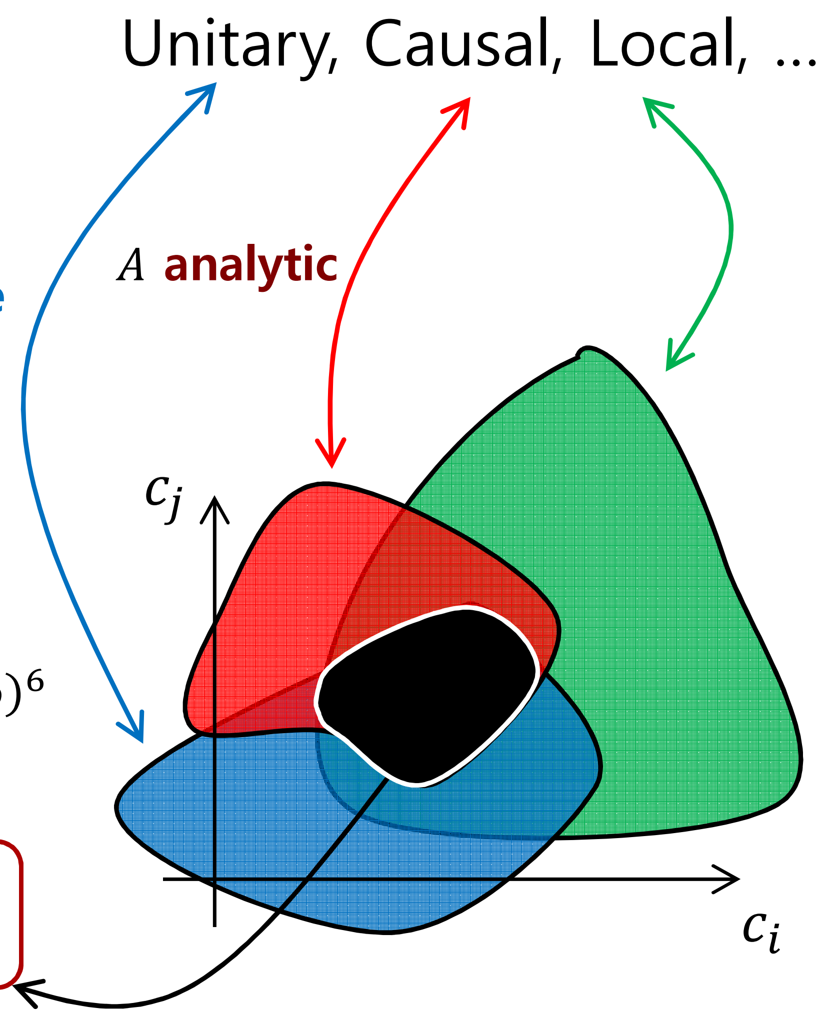
$$A_4(s,t;c_n)$$



UV

$$\mathcal{L}_{UV} = \quad ?$$

Im A **positive**



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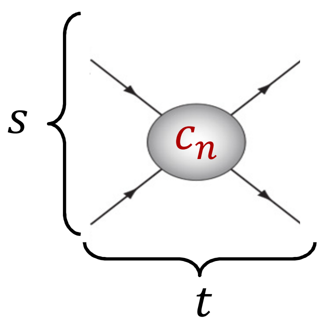

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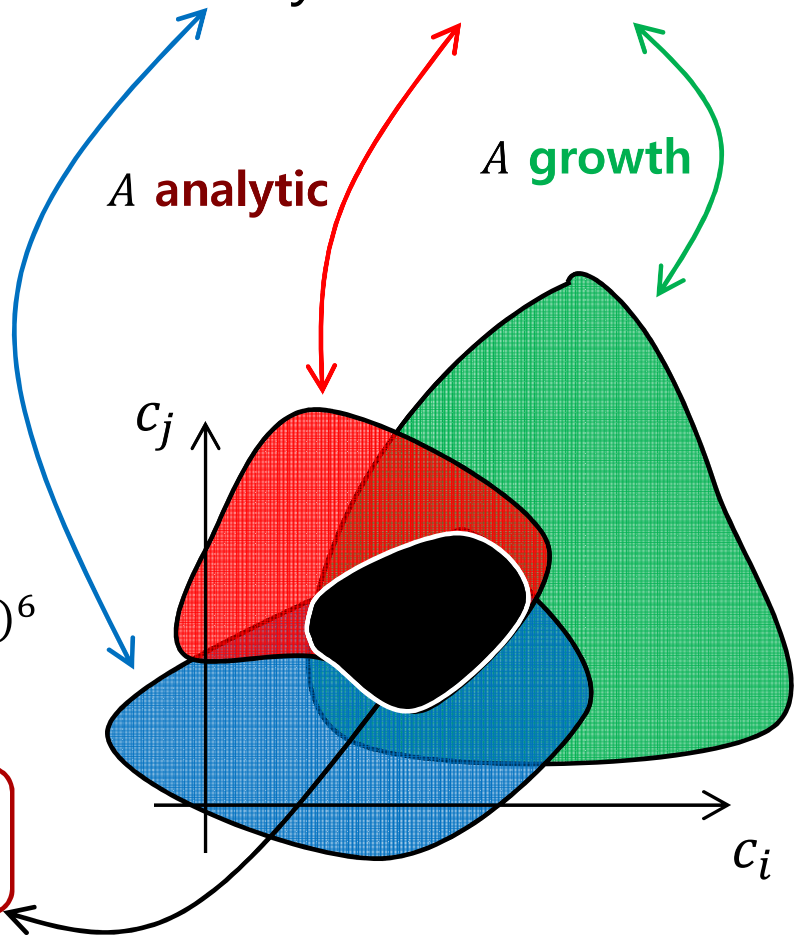

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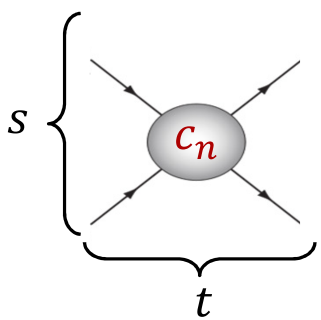

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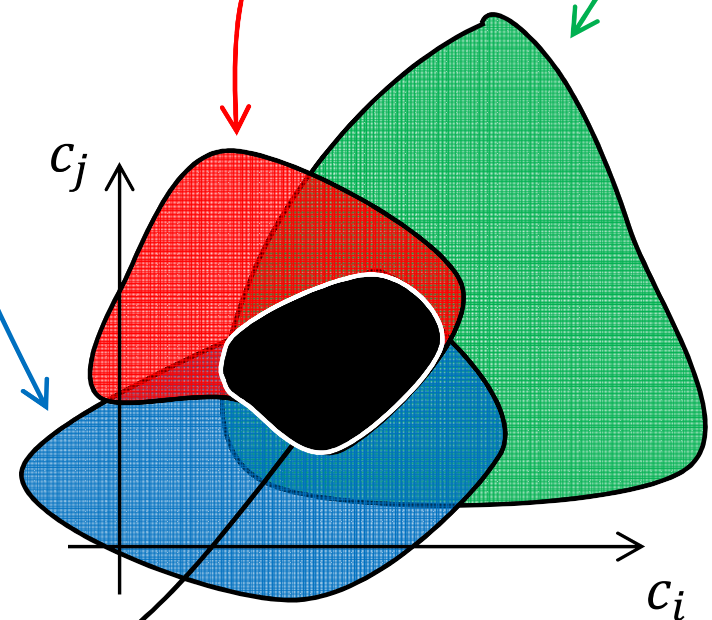

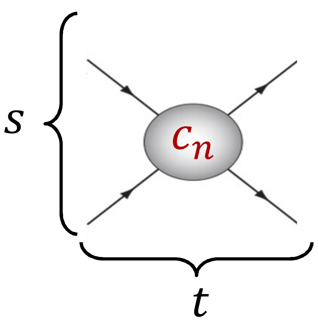

$$\boxed{\partial_t^{\#}\partial_s^{\#} A_4(s,t;c_n) > 0}$$

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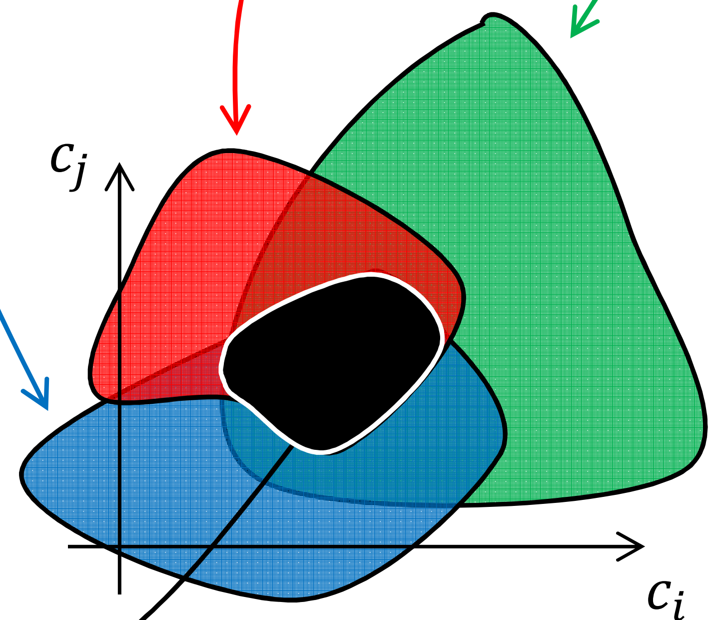

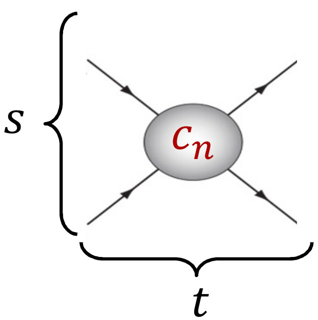

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Positivity Bound

Positivity Bounds

Analyticity

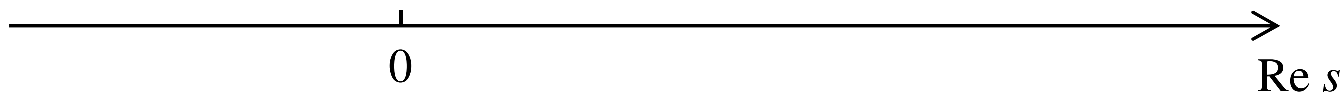
[hep-th/0602178, 1702.06134, 1706.02712]

Analyticity connects IR EFT with UV unknowns

Analyticity

[hep-th/0602178, 1702.06134, 1706.02712]

Analyticity connects IR EFT with UV unknowns

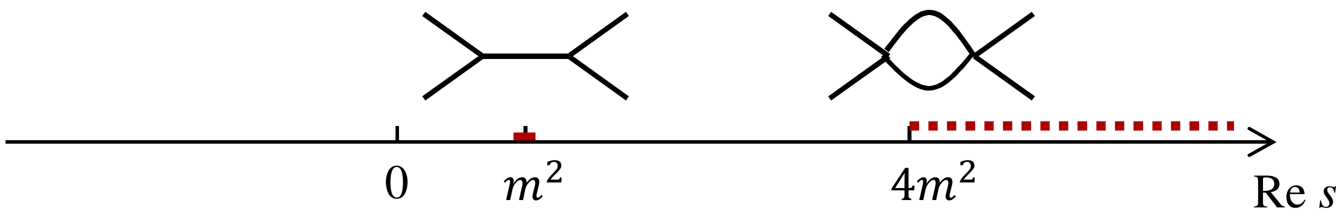


$$A(s,t) = \sum_{p,q} s^p\, t^q + \text{poles} + \text{branch cuts}$$

Analyticity

[hep-th/0602178, 1702.06134, 1706.02712]

Analyticity connects IR EFT with UV unknowns



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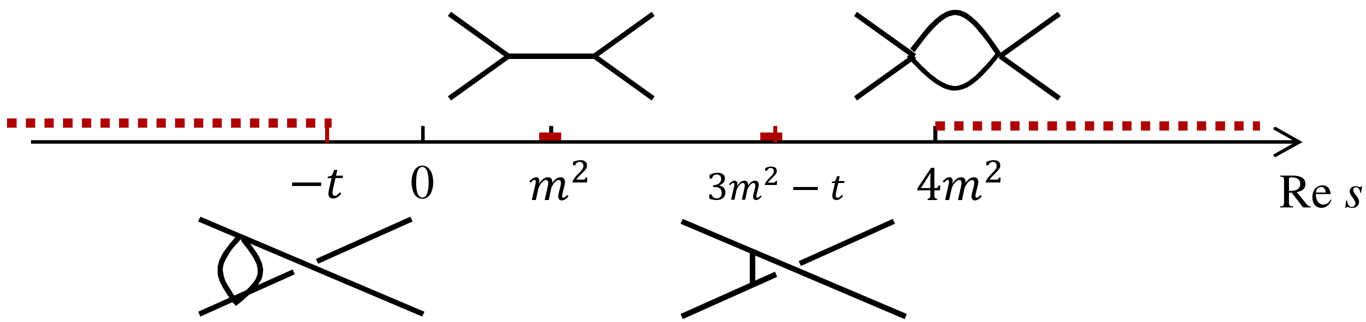

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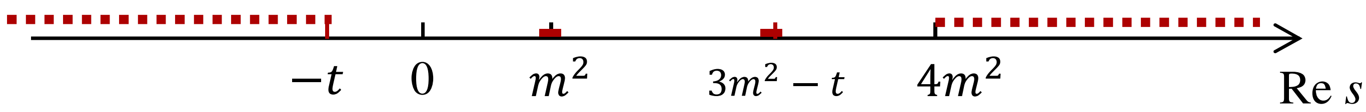

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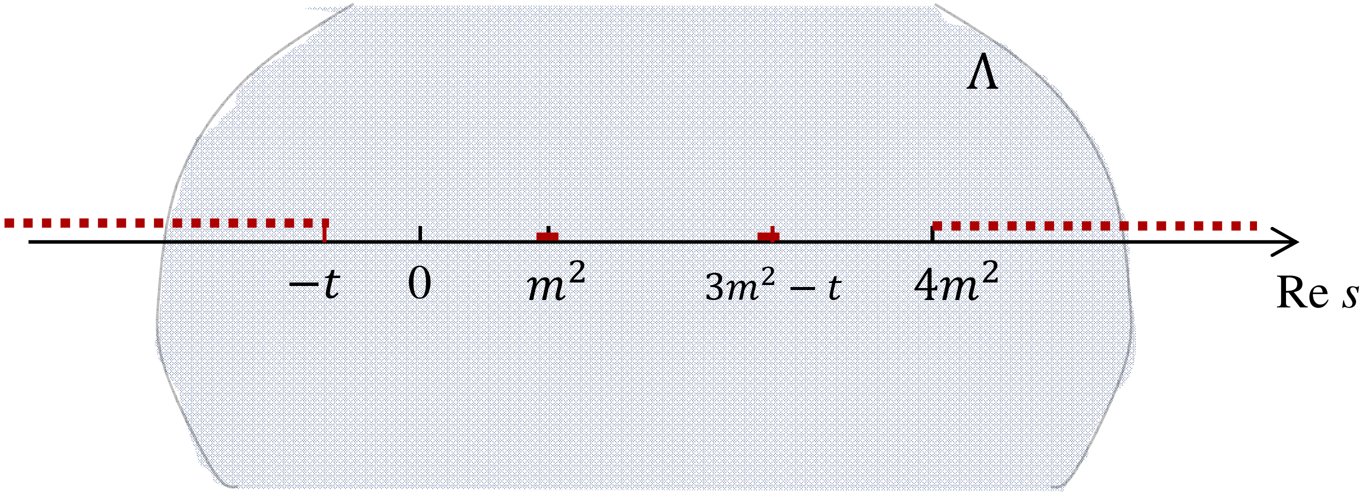

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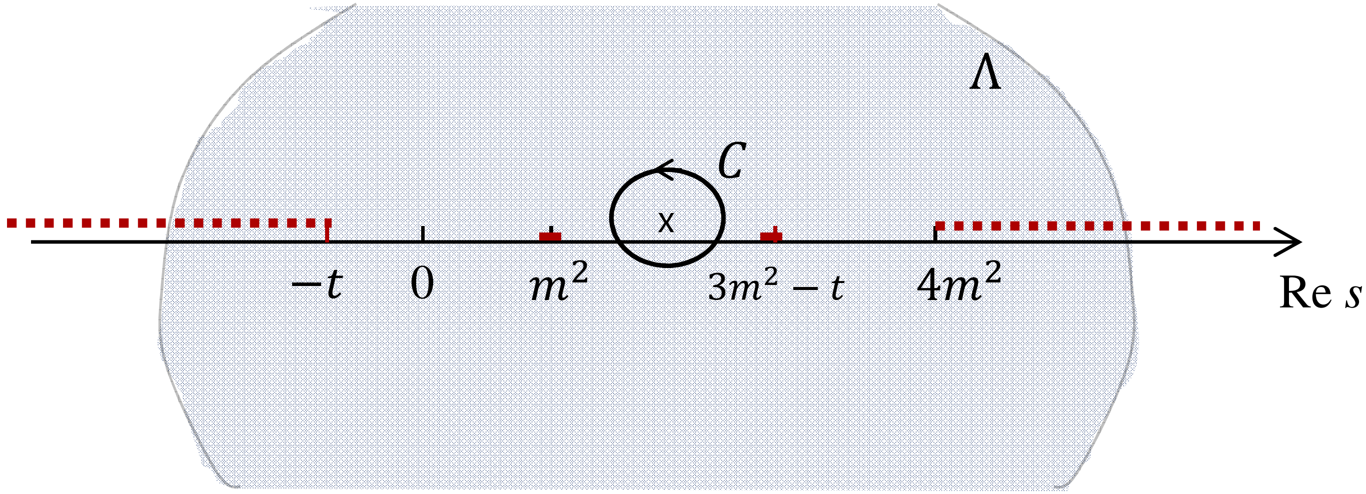

Analyticity

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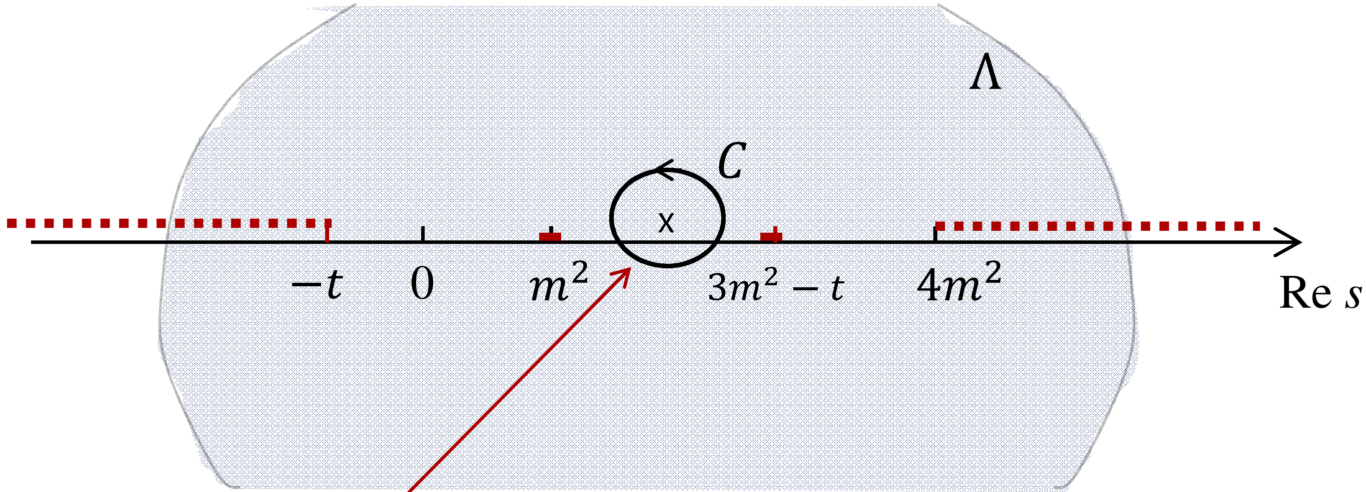

$$\partial_s^N A_{EFT}(s,t) = \oint_C \frac{d\mu}{2\pi i} \frac{A(\mu,t)}{(\mu - s)^{N+1}}$$

Analyticity

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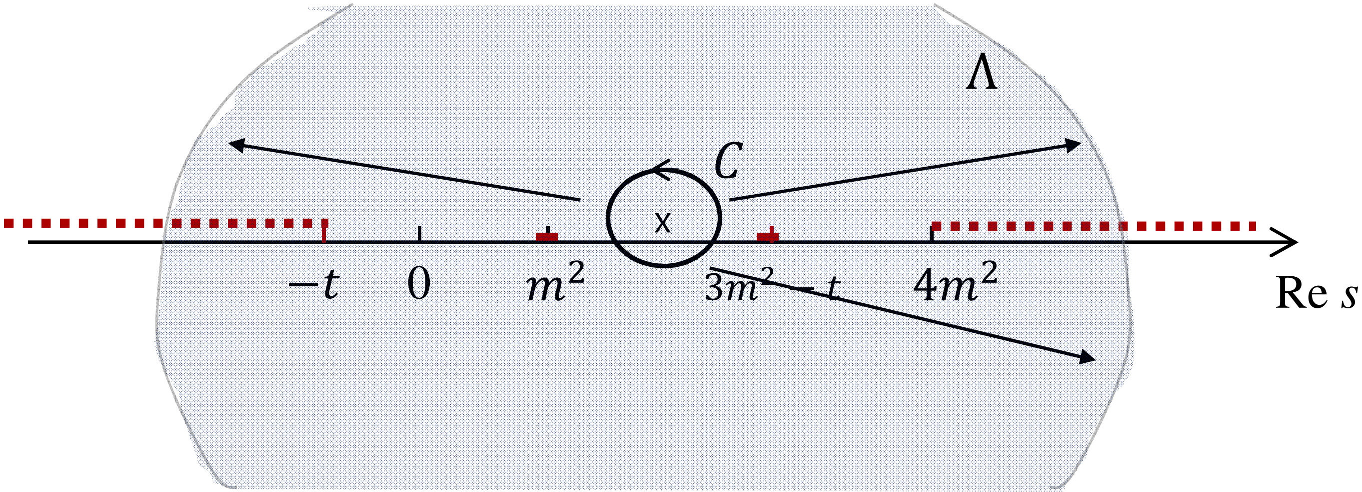

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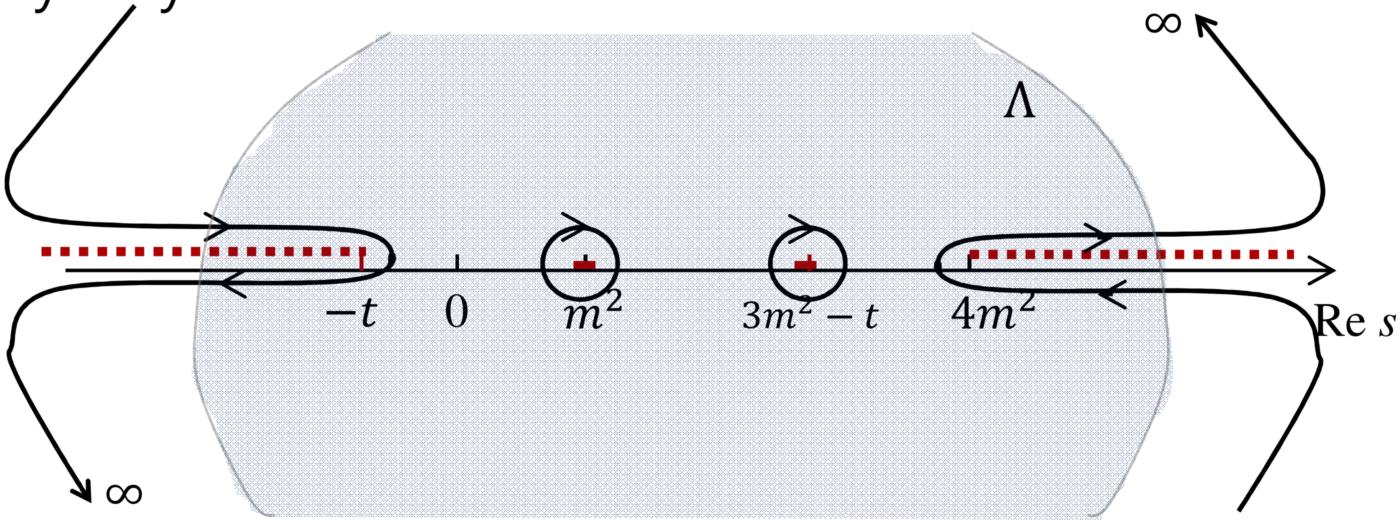

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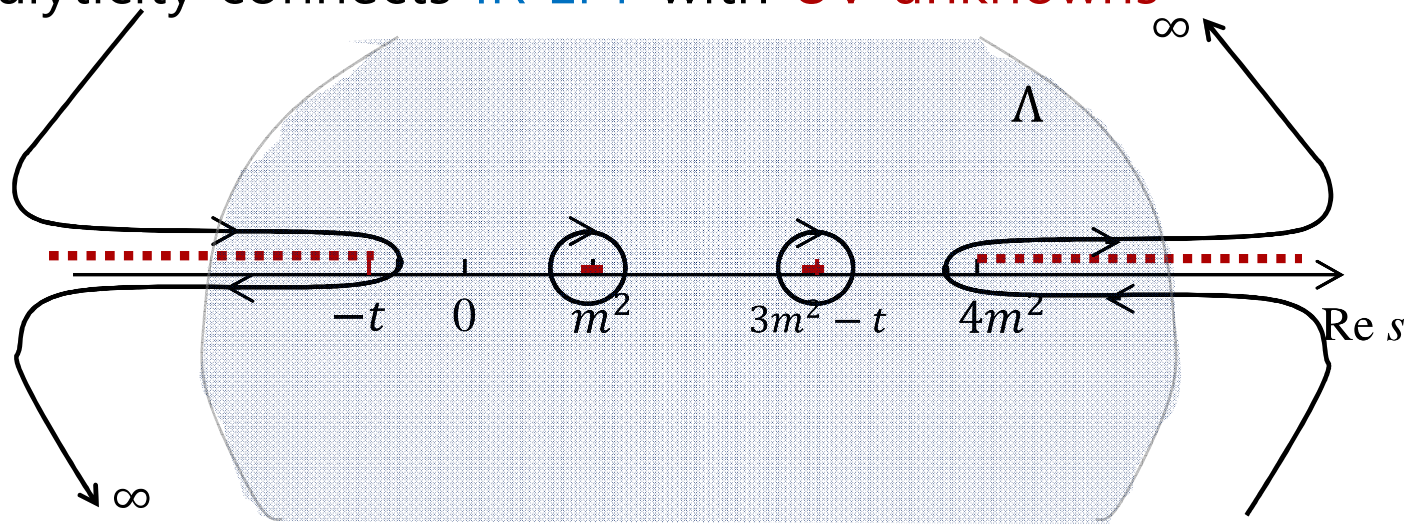

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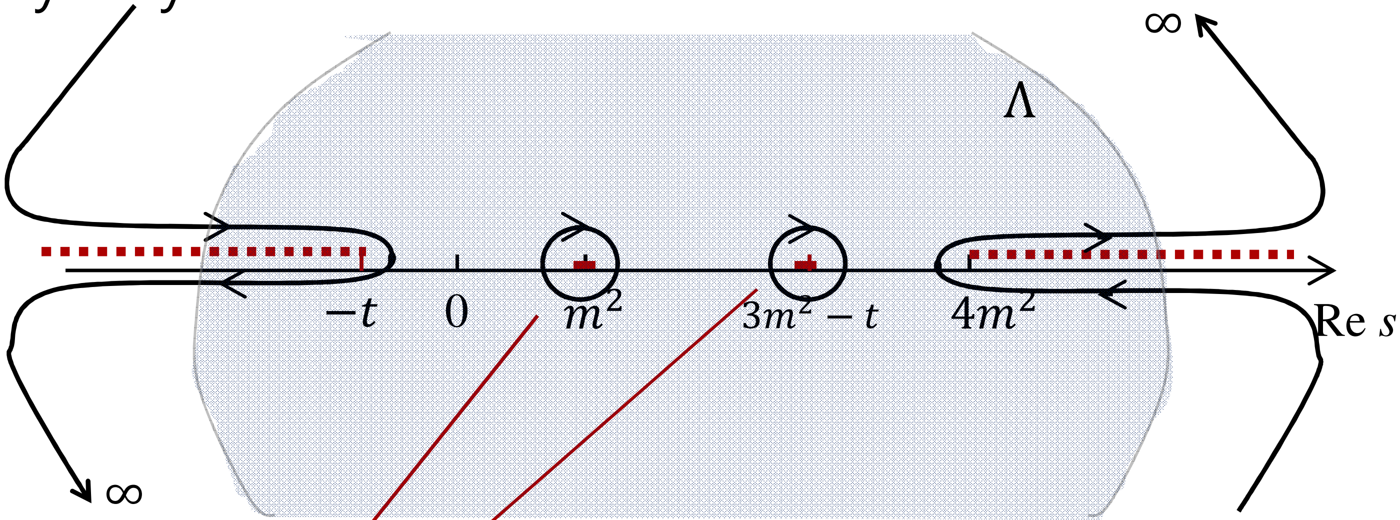

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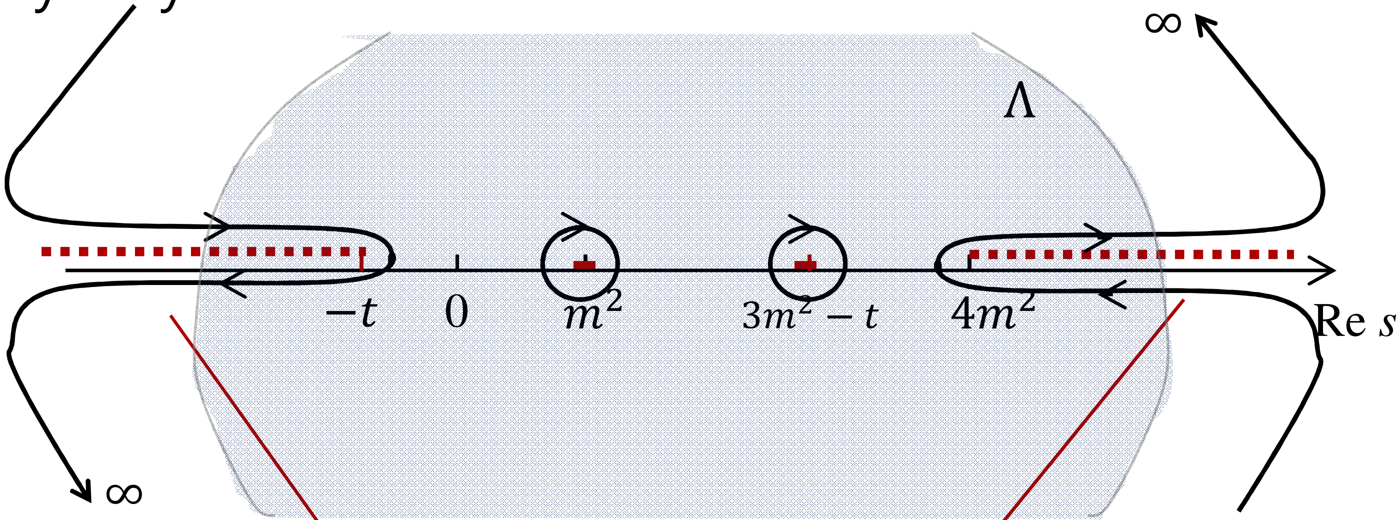

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Known Function

Analyticity

[hep-th/0602178, 1702.06134, 1706.02712]

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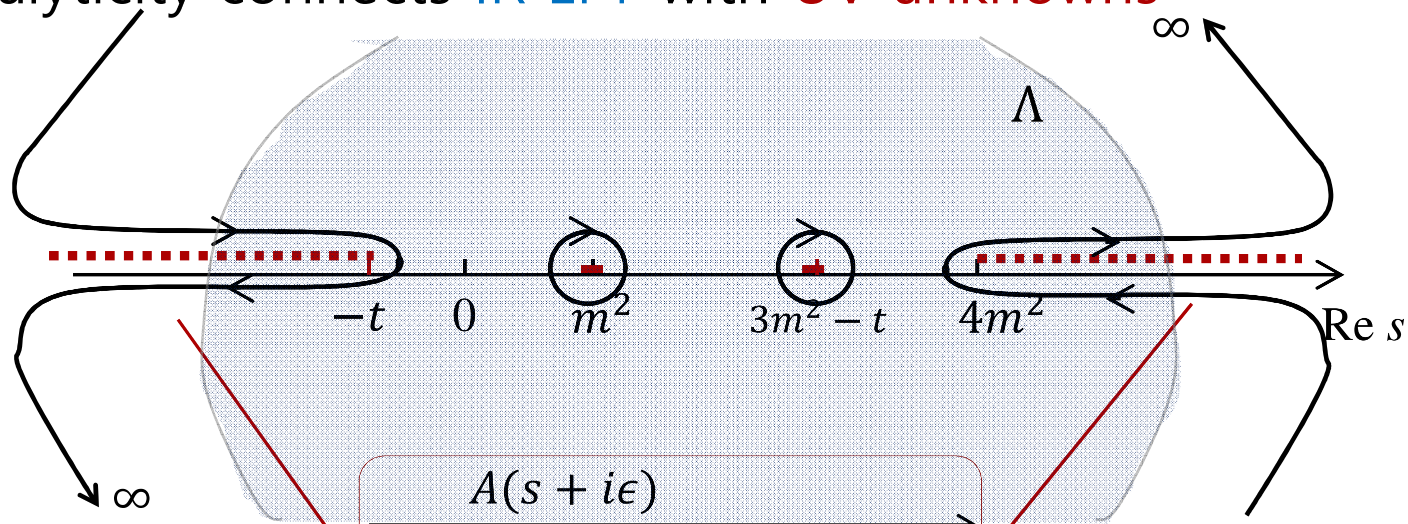

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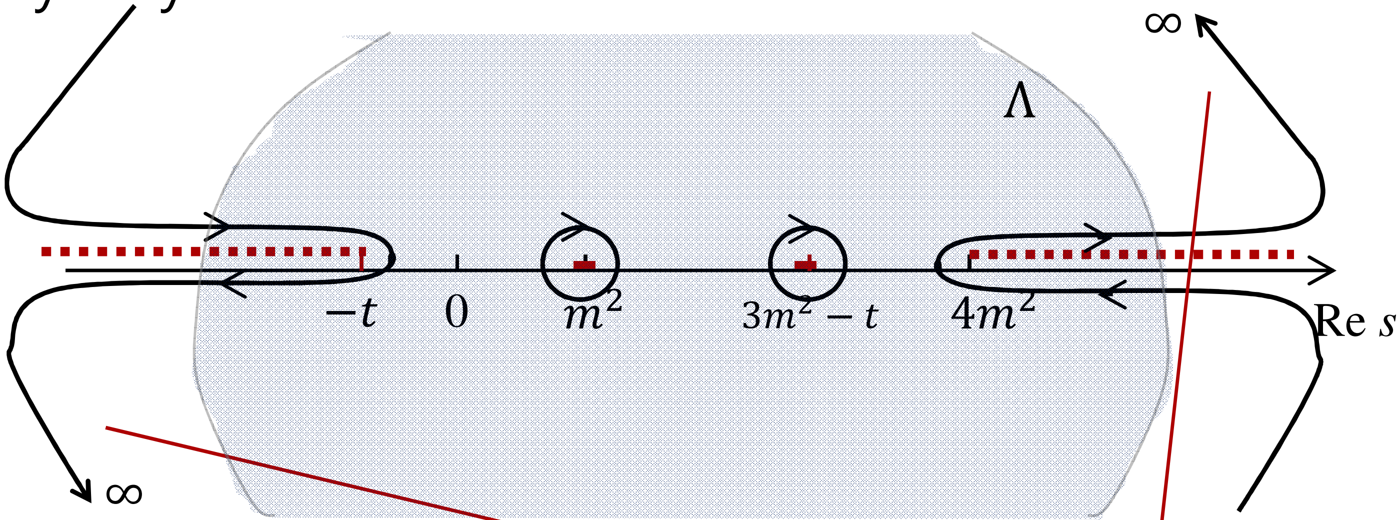

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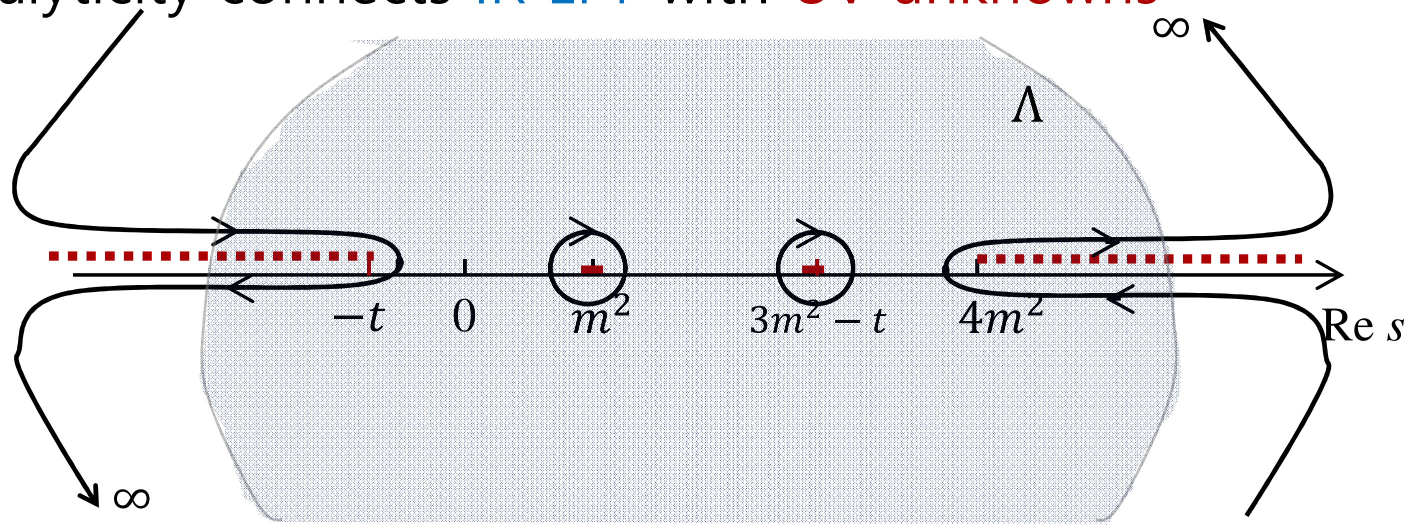
Dispersion Relation

[hep-th/0602178, 1702.06134, 1706.02712]

Analyticity connects IR EFT with UV unknowns



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[hep-th/0602178, 1702.06134, 1706.02712]

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(Locality => $N = 2$)

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Applications

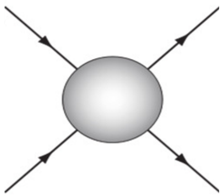
Higher derivative scalar

[hep-th/0602178, 1204.6388]

$$\mathcal{L} = -\tfrac{1}{2}(\partial\varphi)^2 + \frac{c}{\Lambda^8}(\partial\partial\varphi)^4 + \ldots$$

Higher derivative scalar

[hep-th/0602178, 1204.6388]



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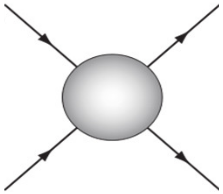

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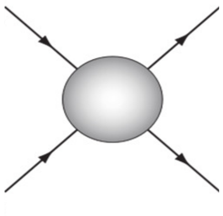
$$\boxed{c > 0}$$

Higher derivative scalar

[hep-th/0602178, 1204.6388]



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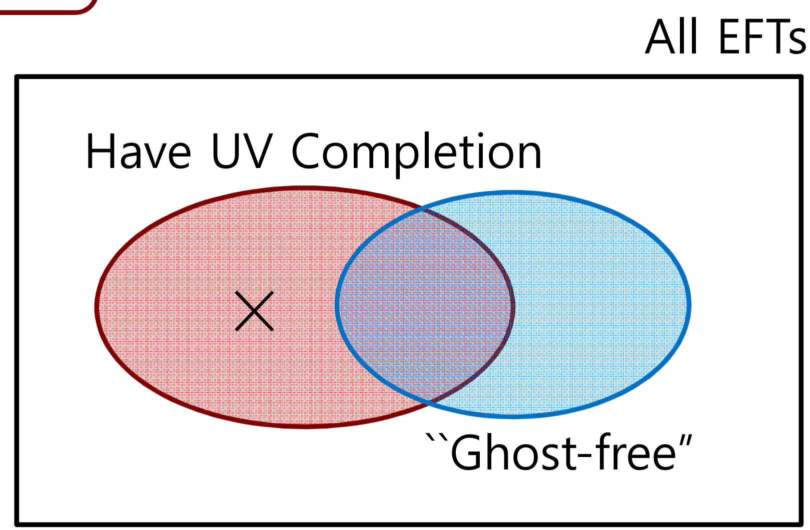

Ghost, but may
have UV completion

Proca

[1804.10624]

$$\mathcal{L} = -\frac{1}{4}F^2 - \frac{1}{2}m^2A^2$$

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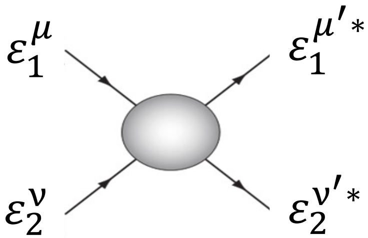

A has spin

$\Rightarrow$ Bounds for each polarization

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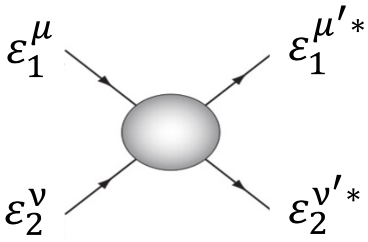
Let’s focus on ghost-free:

$$c_2 A^2\left[\left(\partial_\mu A_\nu\right)^2 - \left(\partial_\mu A^\mu\right)^2\right]$$



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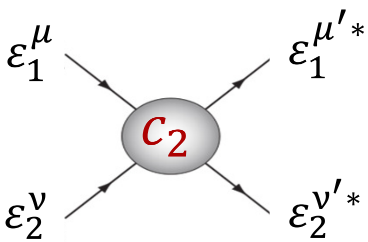

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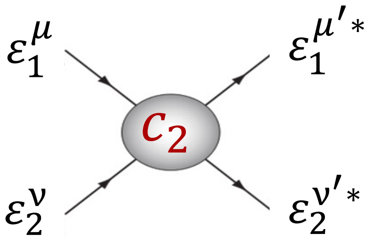

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No ghost, but no UV completion!

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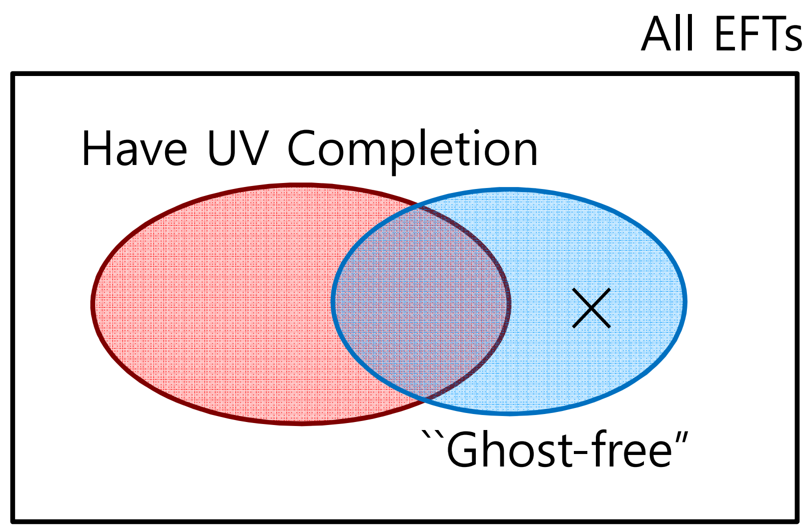

Massive Gravity

[1804.10624]

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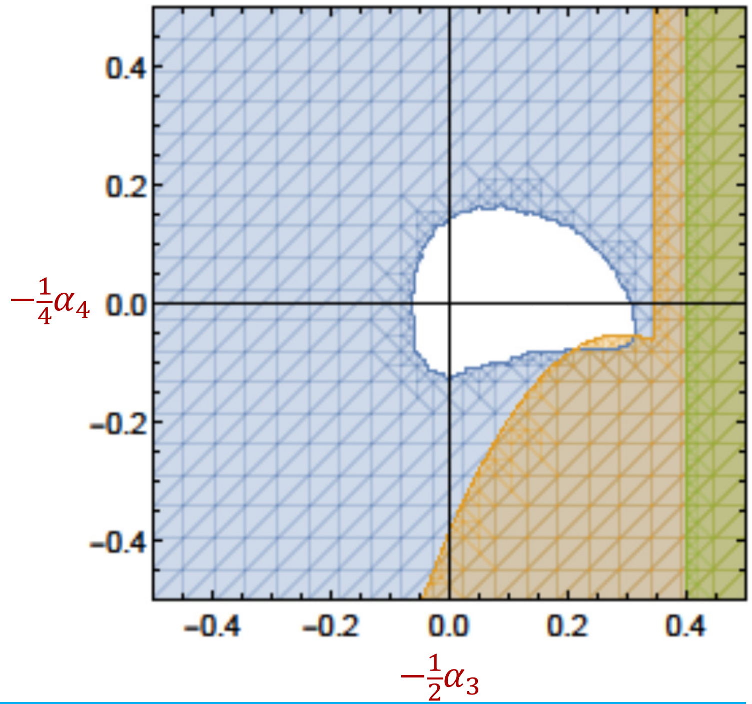

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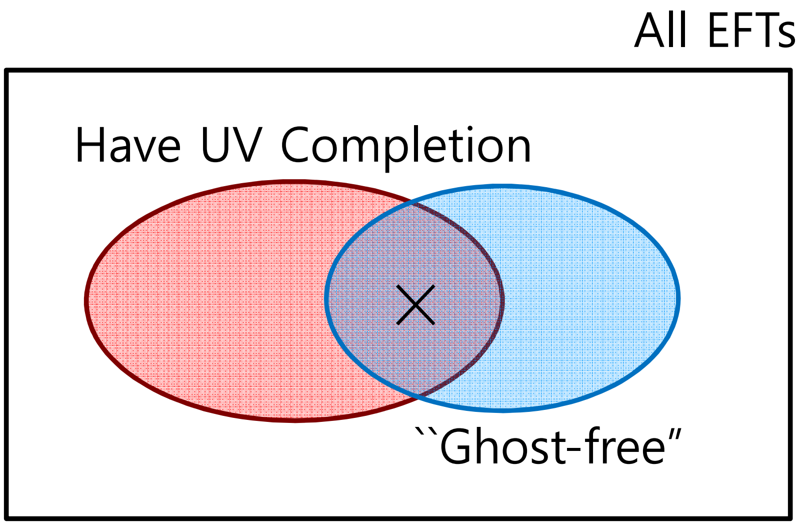
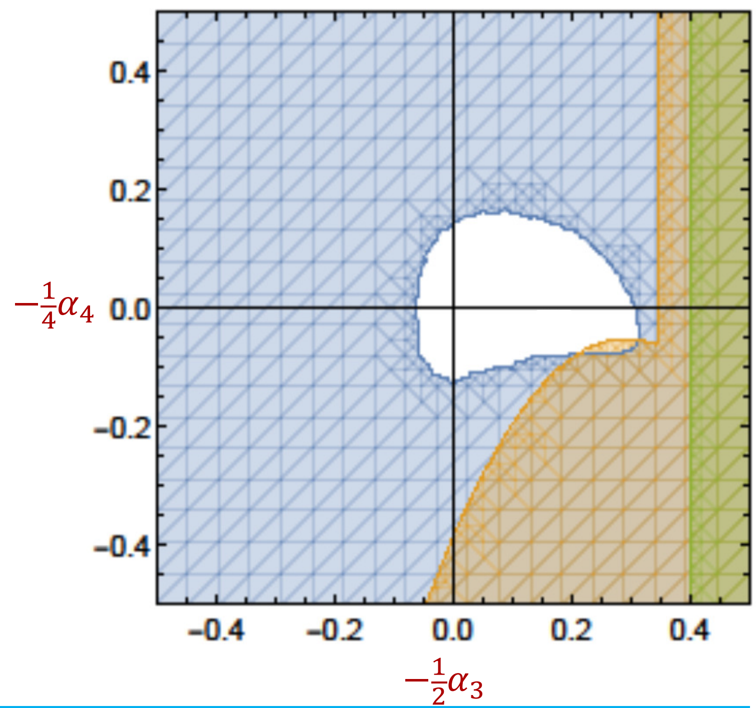
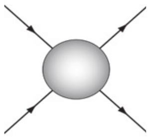




Summary

Only some Effective Fields Theories have UV Completions



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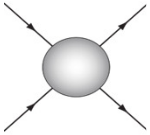

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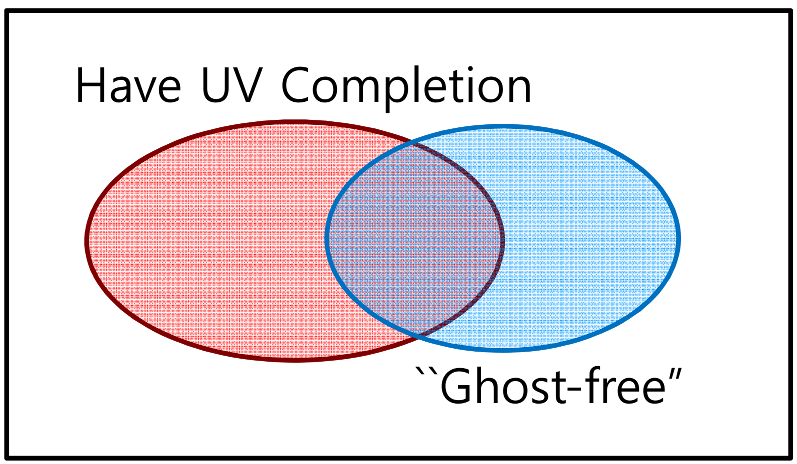

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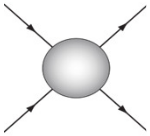

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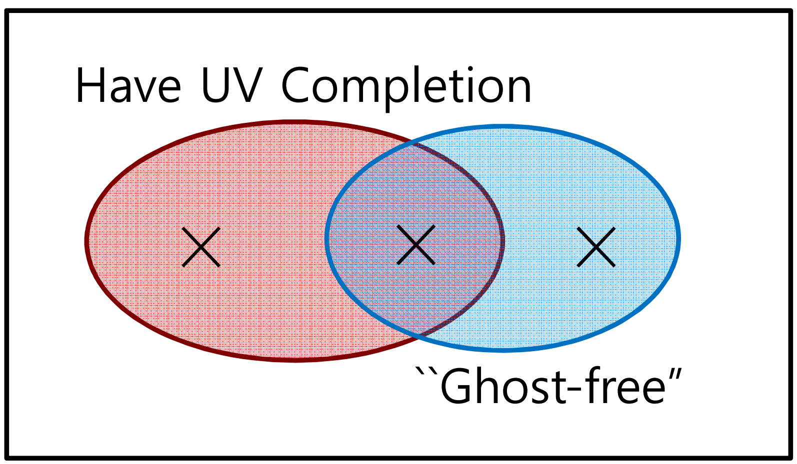

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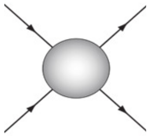

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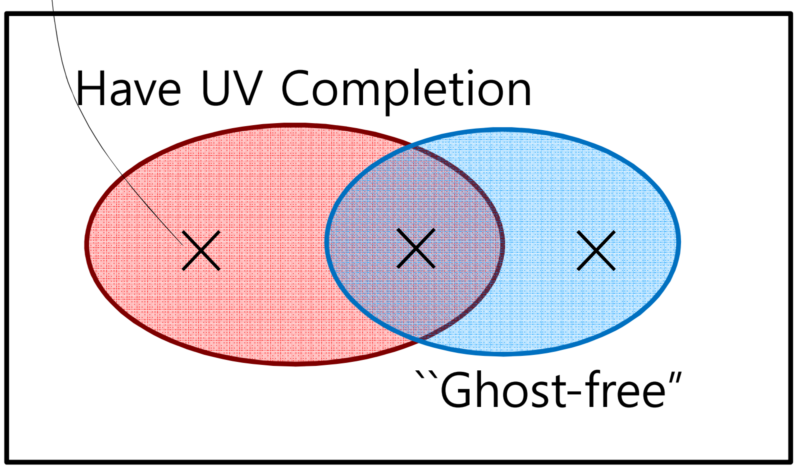

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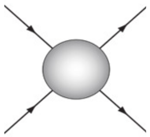

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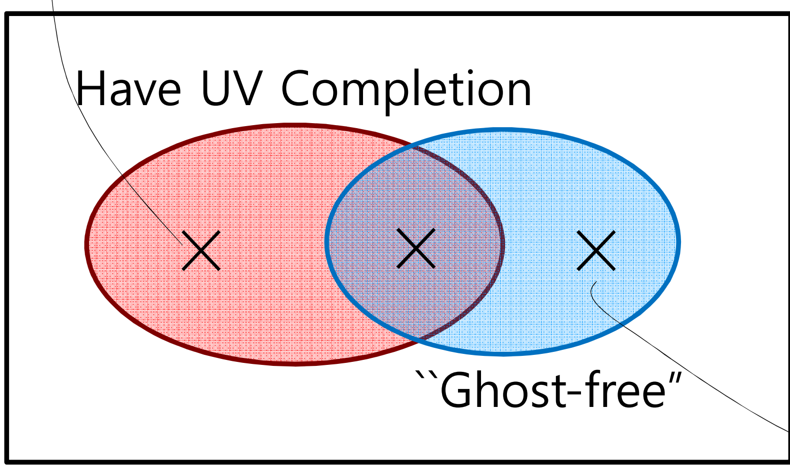

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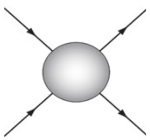

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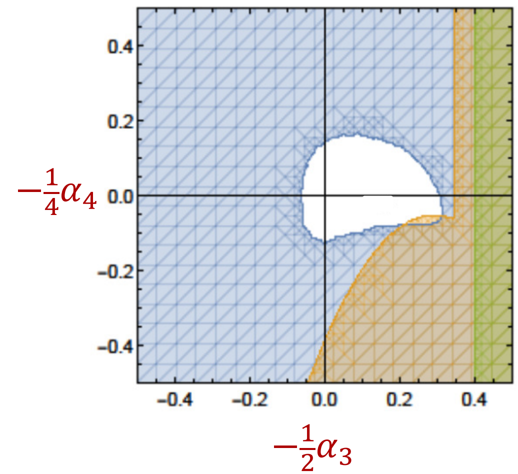

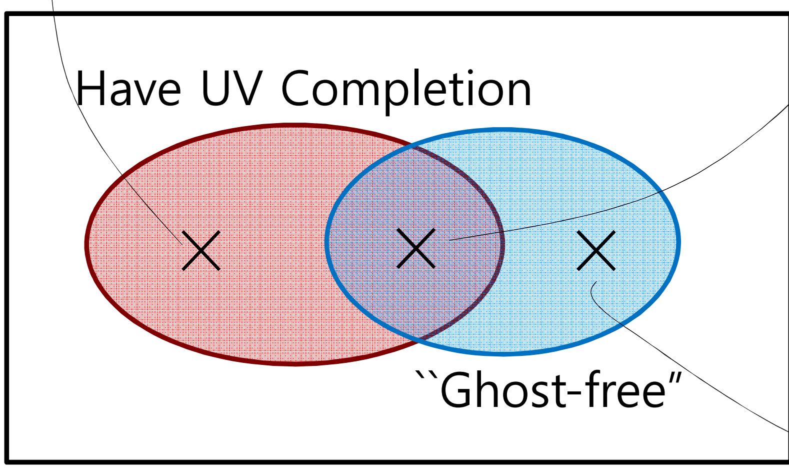

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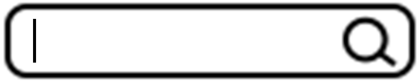
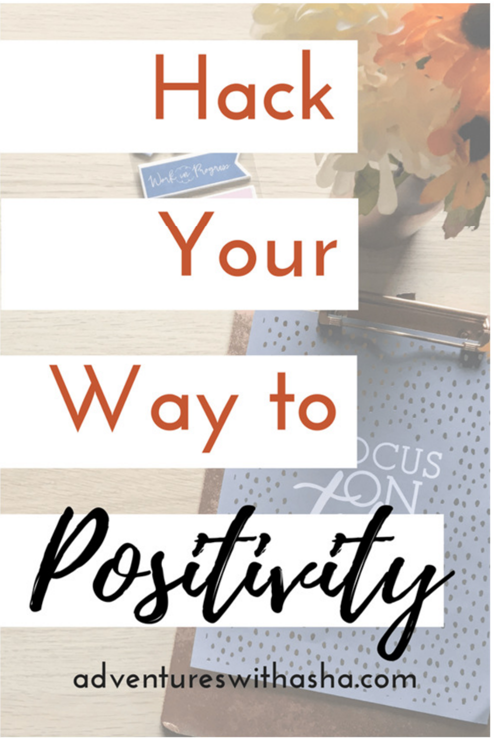


positivity|

positivity|



1. Find your happy place,
   and go there often

2. Remember your why

3. Give yourself reminders

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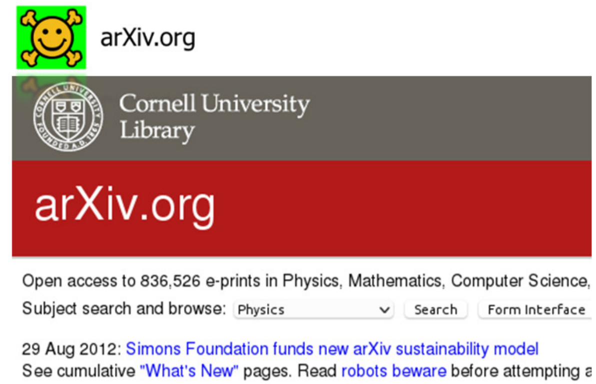

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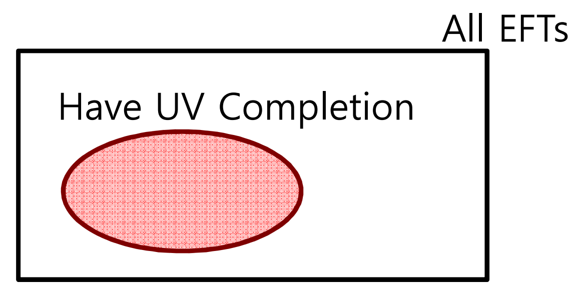

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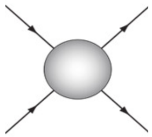

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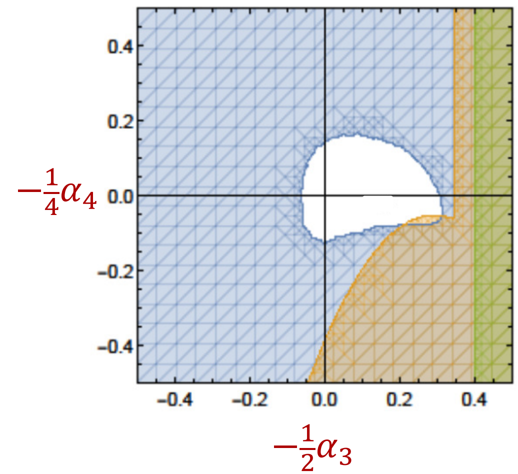

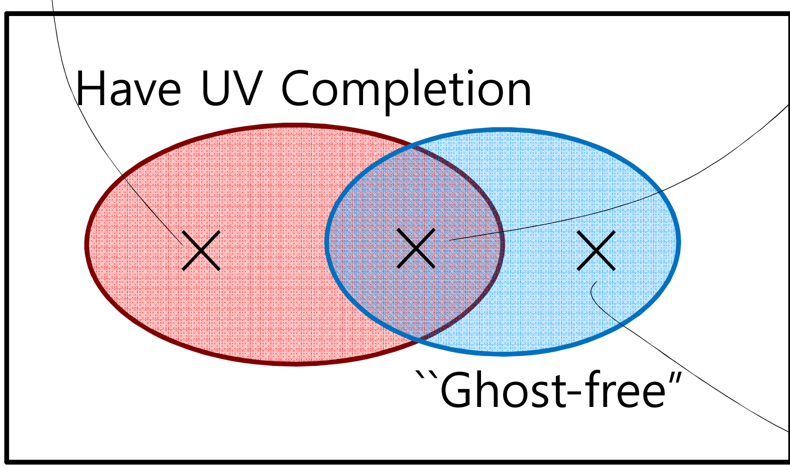

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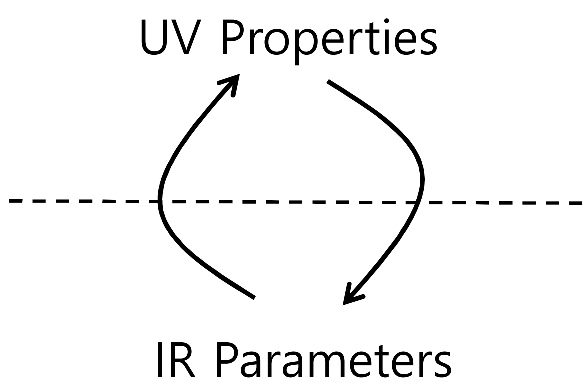
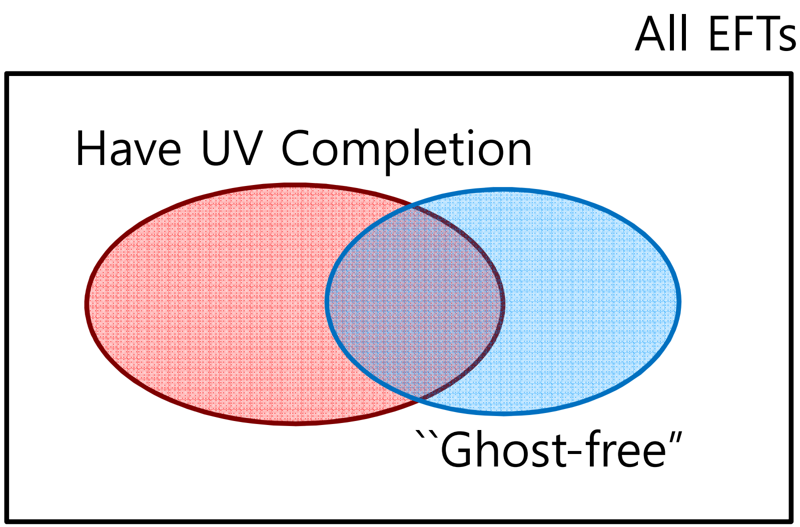
COSMO-18 30 Aug 2018

Positivity Constraints for Gravity and Cosmology

Scott Melville





Imperial College London

Additional Slides

Massive Galileon

[1701.08577]

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Shift: $\varphi \to \varphi + a$

$$\mathcal{L} = -\tfrac{1}{2}(\partial\varphi)^2 + c(\partial\varphi)^4 + \cdots$$

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Galileon: $\varphi \to \varphi + a + v_\mu x^\mu$

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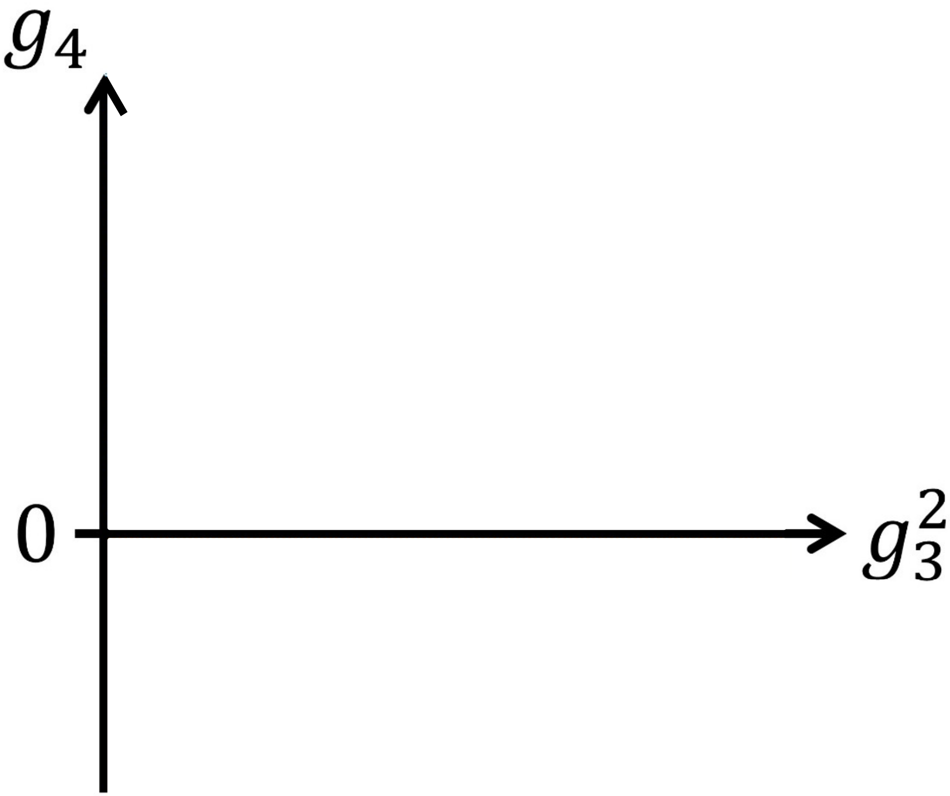

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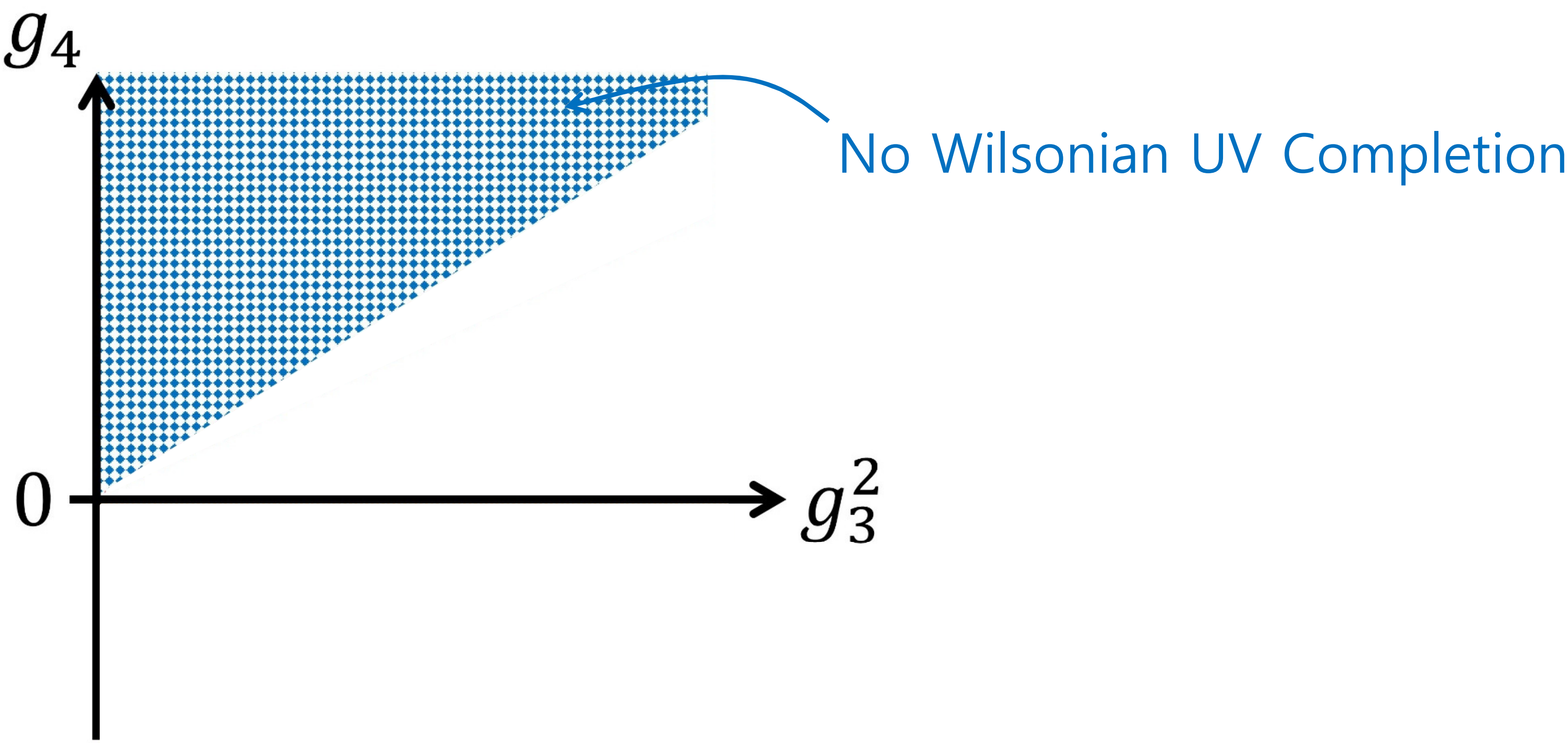

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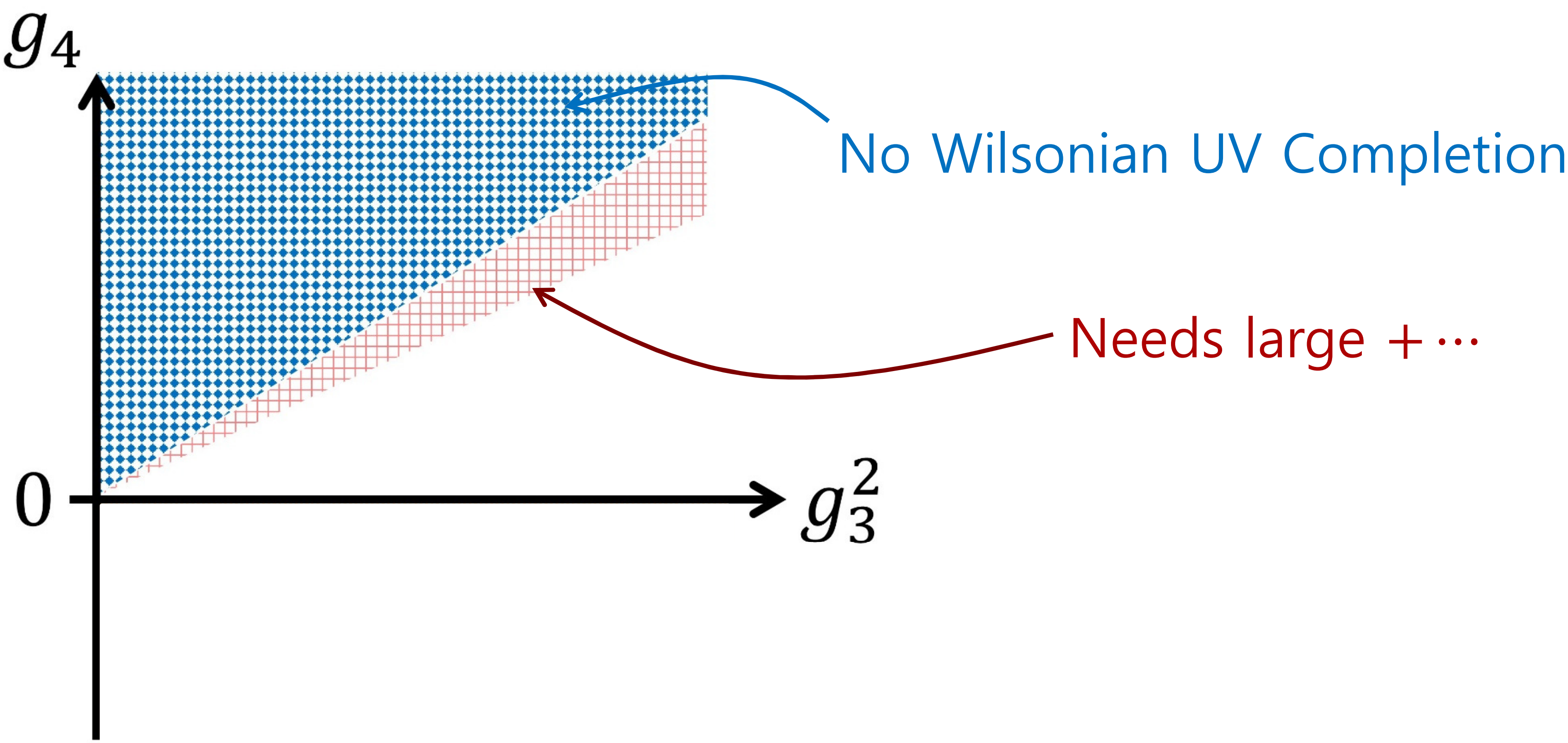

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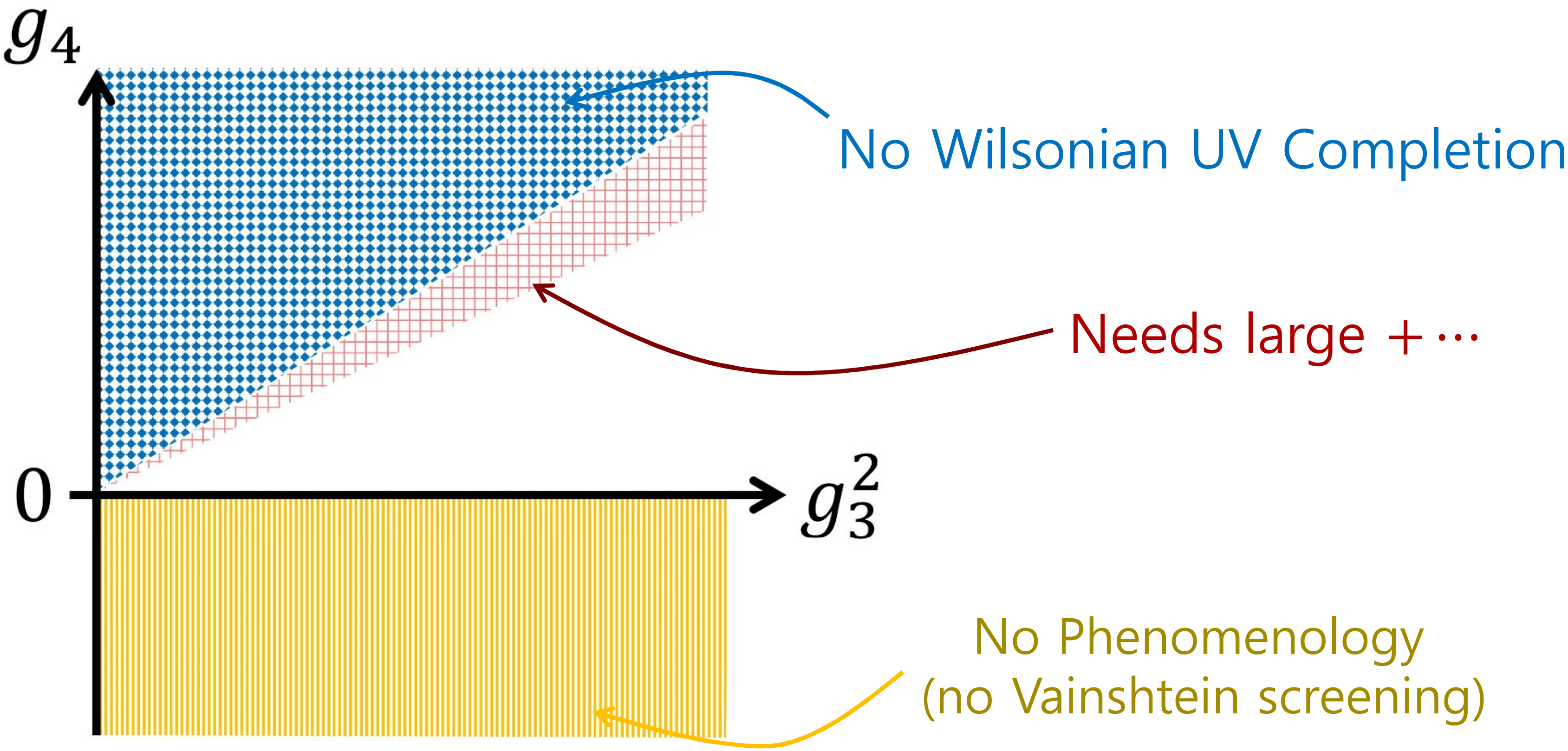

Tree Vs Loop Bounds [1702.08577, 1710.09611]

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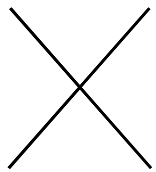
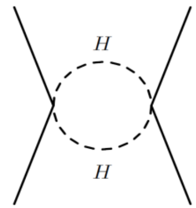
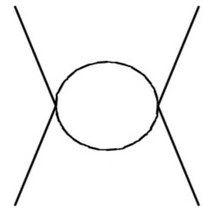
Tree Heavy loops Light loops

$A_{\mathrm{UV}} =$  $+$ 

$+$ 

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Tree Heavy loops Light loops

$A_{\mathrm{UV}} =$

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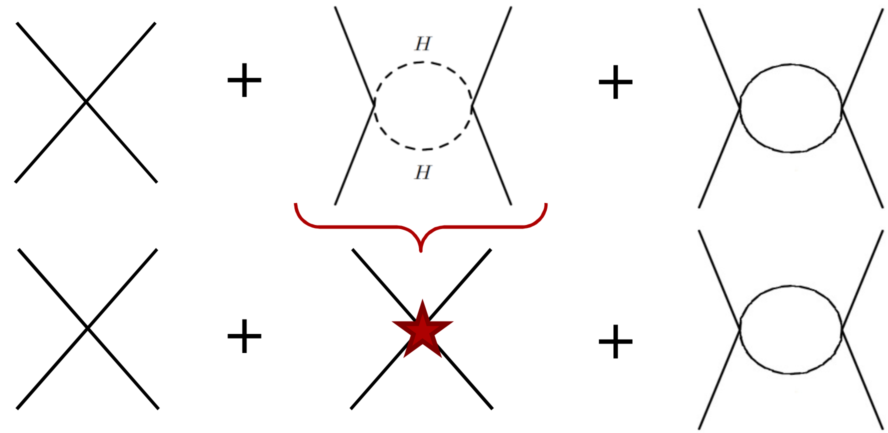

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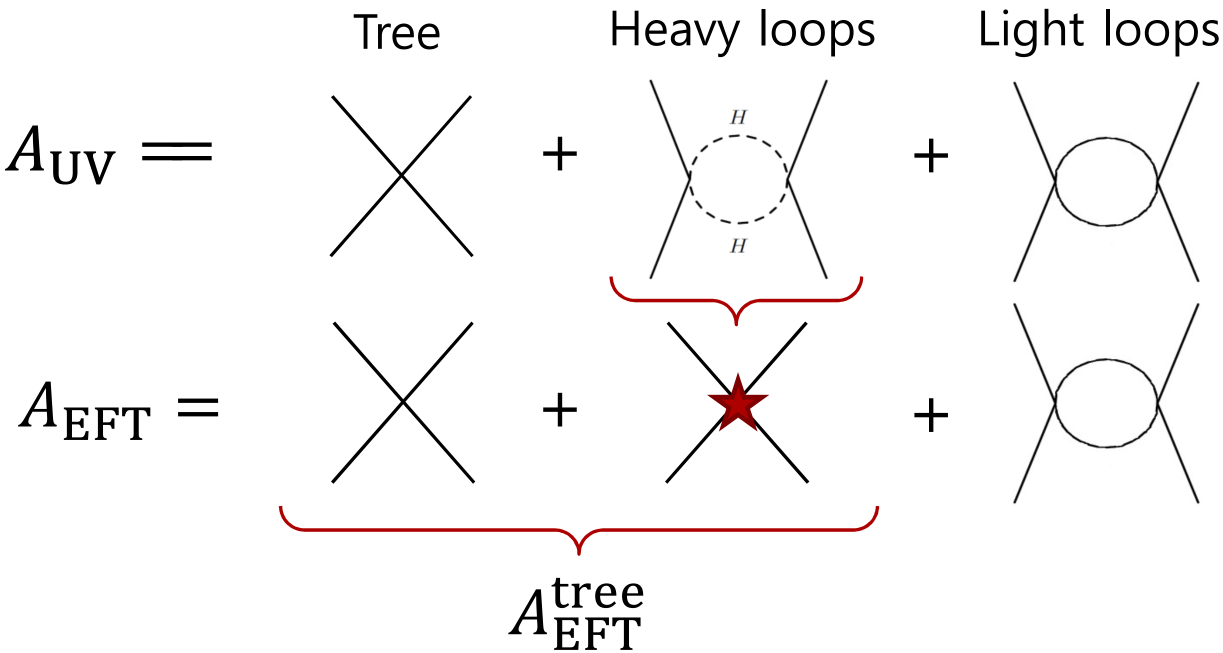

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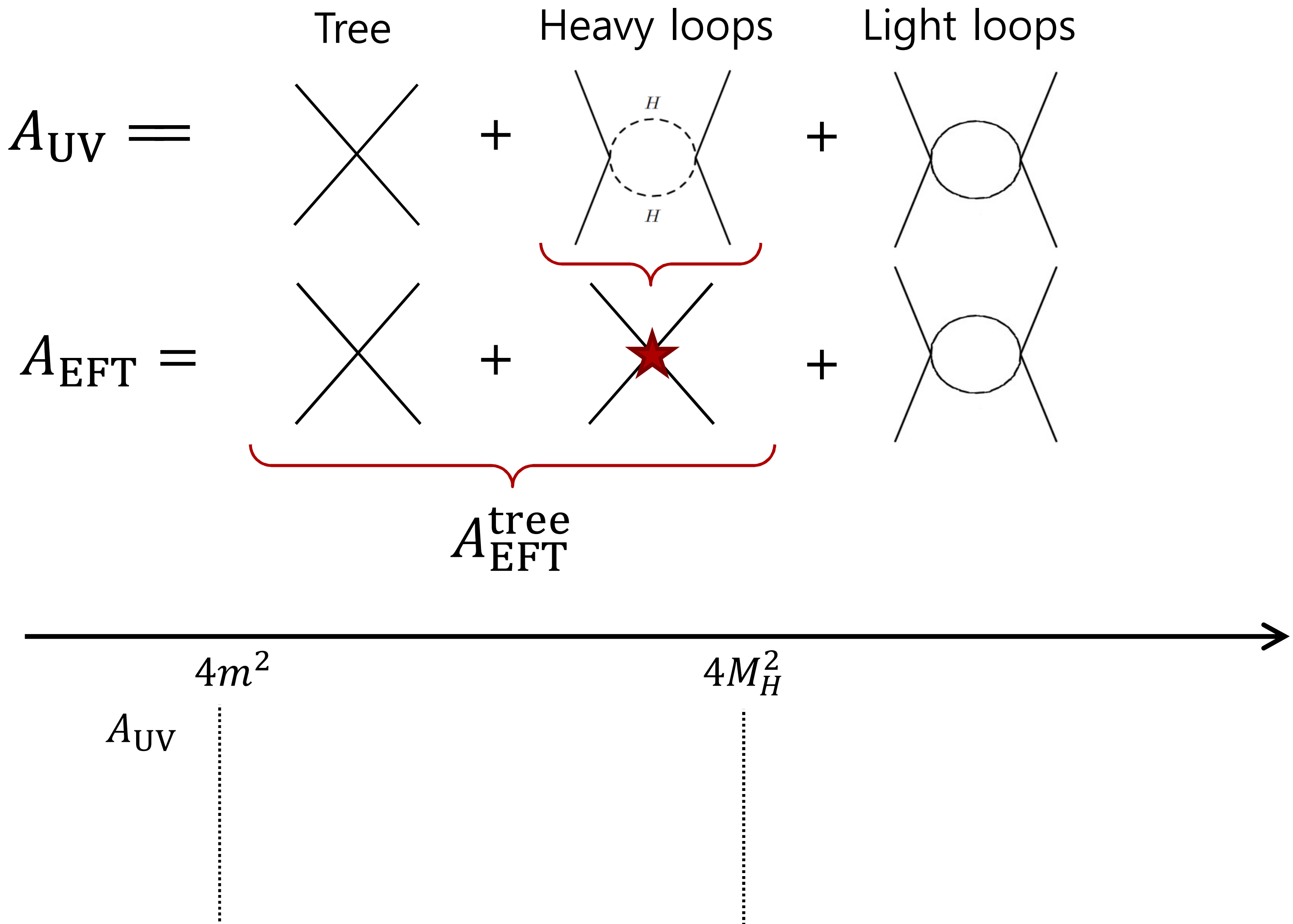

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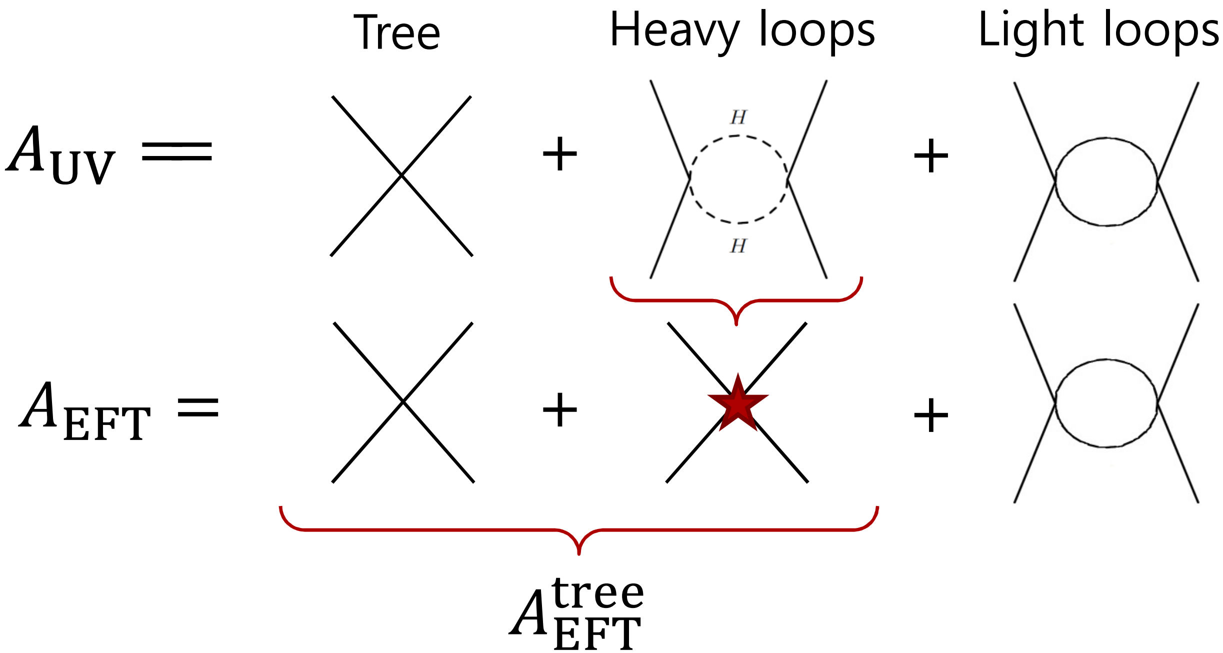

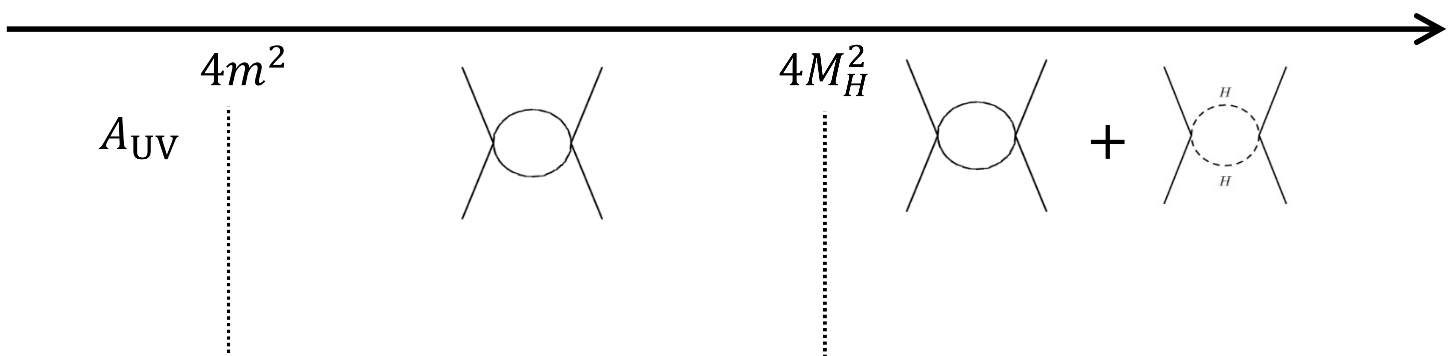

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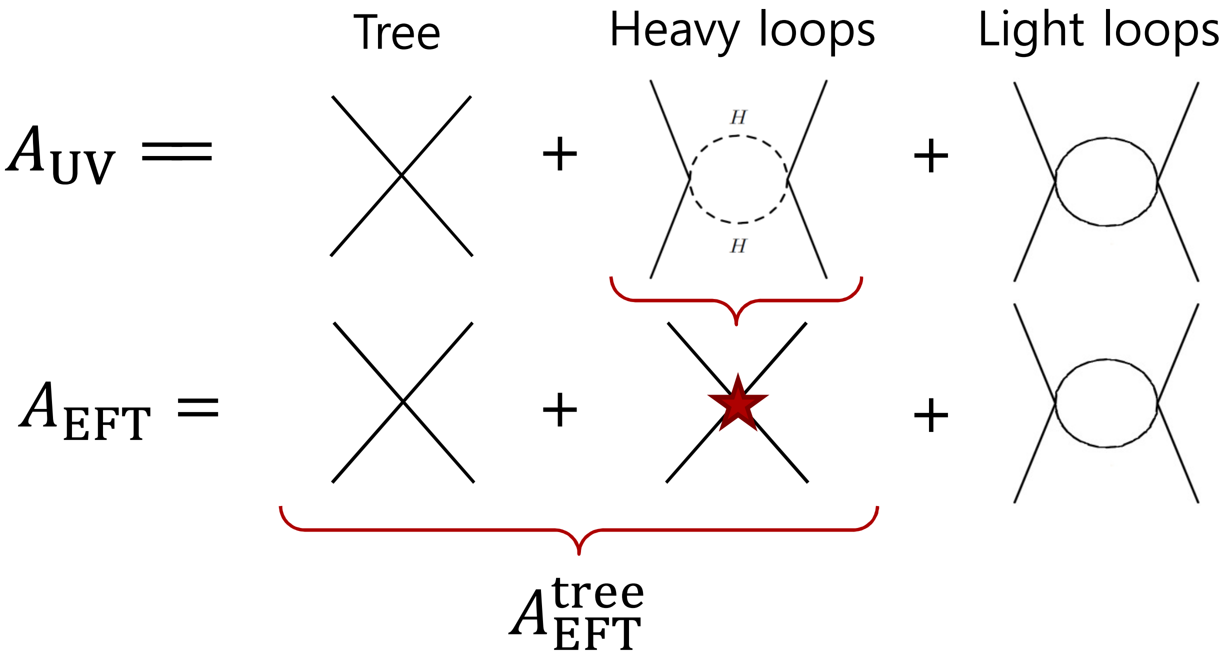

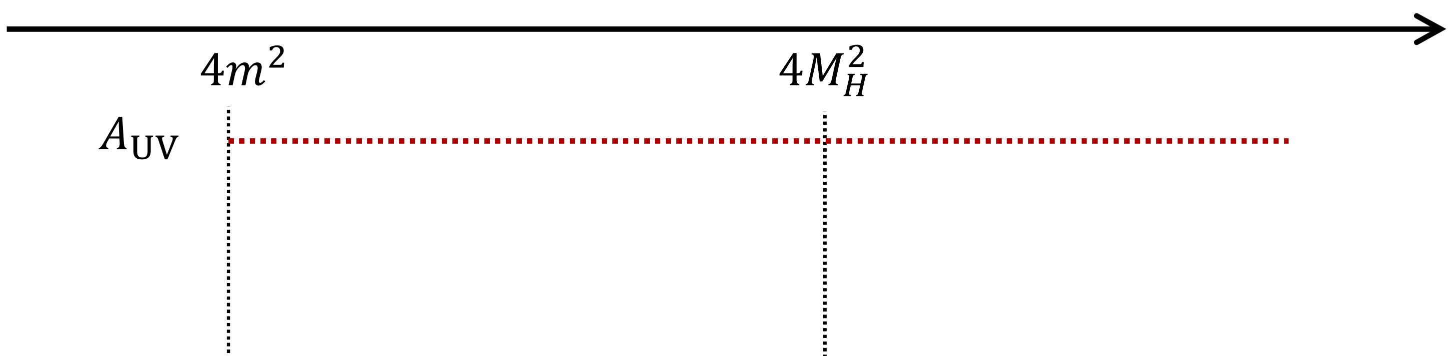

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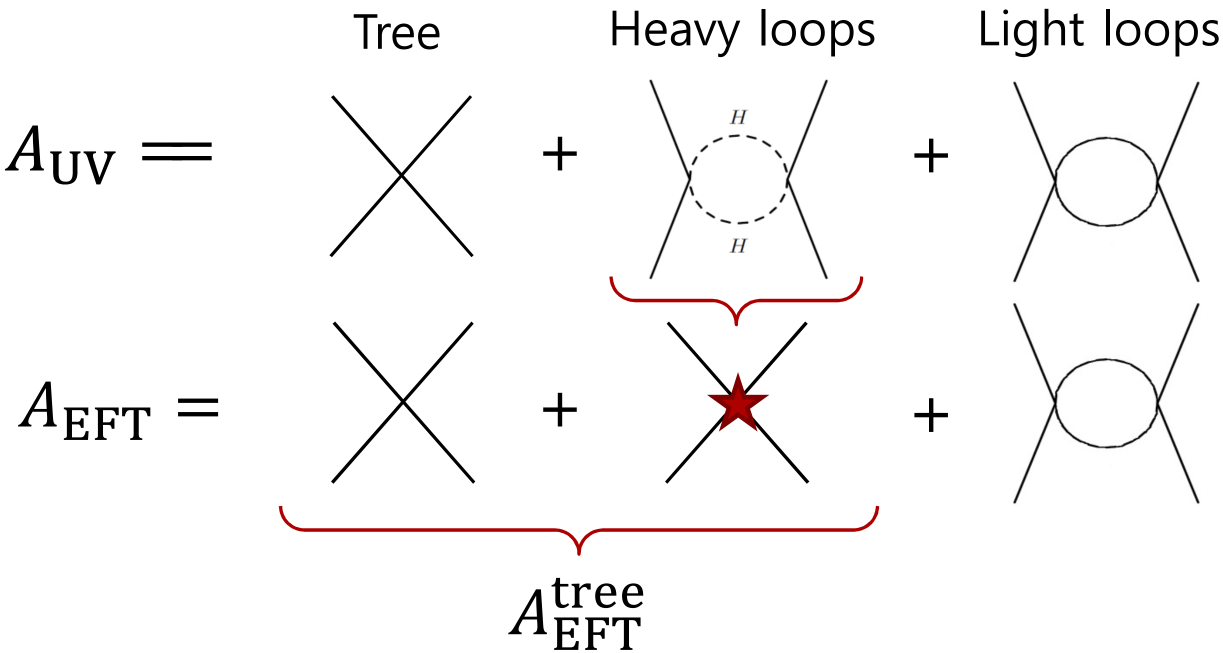
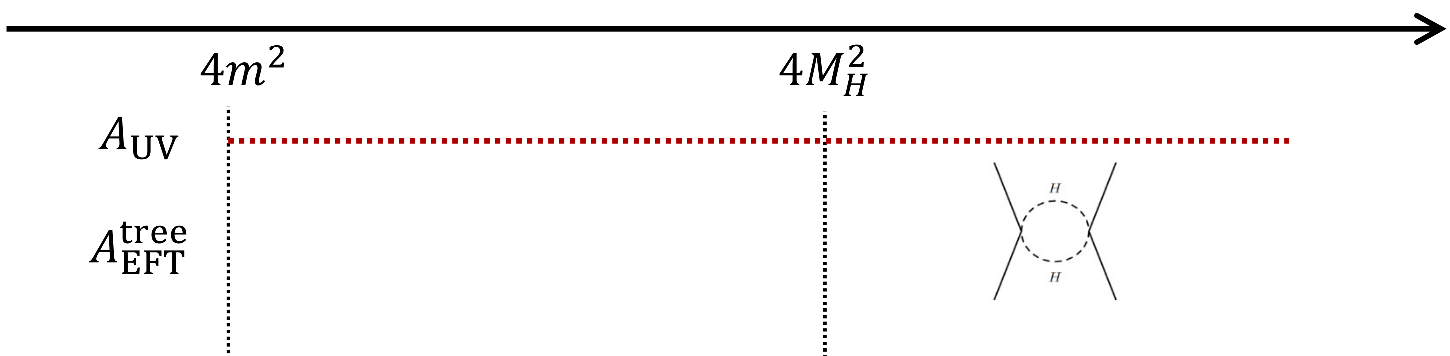
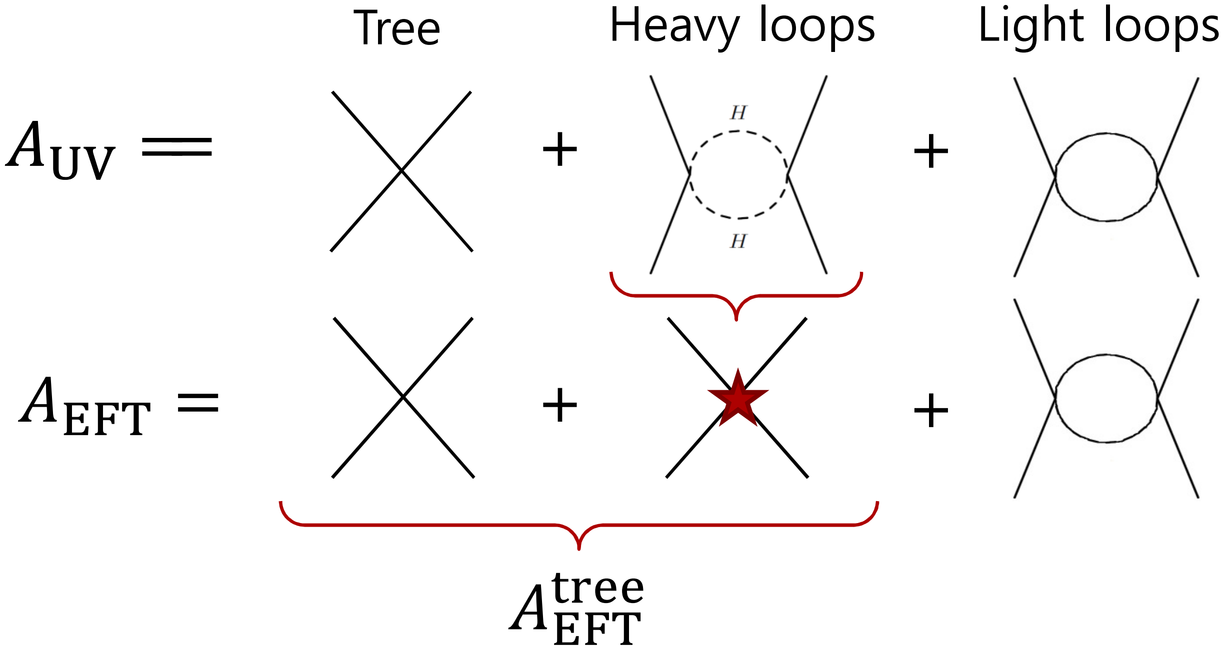
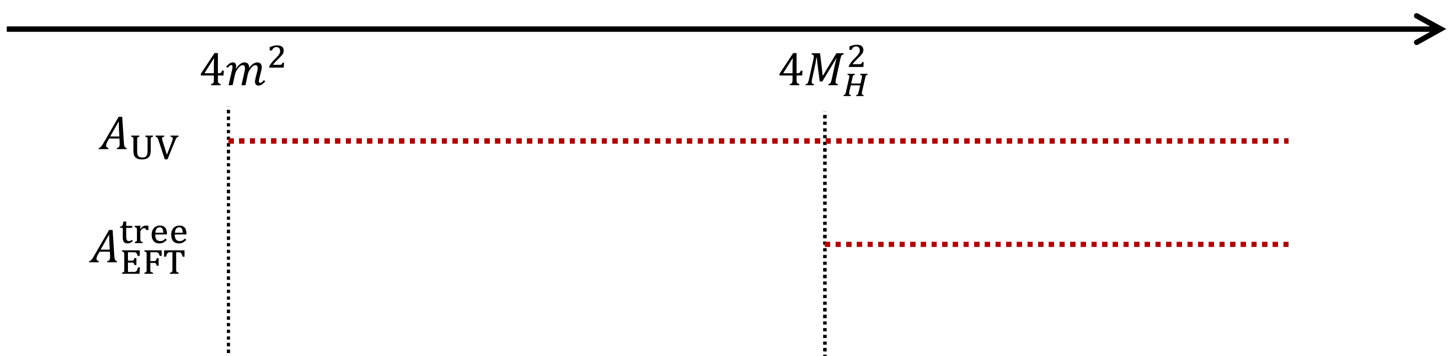




Tree Vs Loop Bounds [1702.08577, 1710.09611]





Tree Vs Loop Bounds [1702.08577, 1710.09611]

Light Loops

$$\partial_s^2 \tilde{A}(s,t) = \int_{4m^2}^{\infty} d\mu \;\; \mathrm{Im}\, A(\mu,t)\, D(\mu;s,t)$$

$$Y^{(N,M)}(s,t) \;= \sum \frac{c_{i,j}}{(4m^2)^{\#}}\, \partial_t^j \partial_s^i\, \tilde{A}(s,t)$$

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Light Trees

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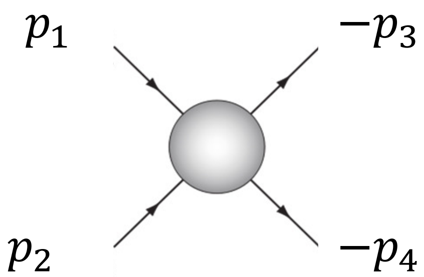

$$\langle \Phi\, \chi\, \Phi\, \chi \rangle = A(s,t)$$

$$s = 4\omega^2 = -(p_1+p_2)^2$$
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=>

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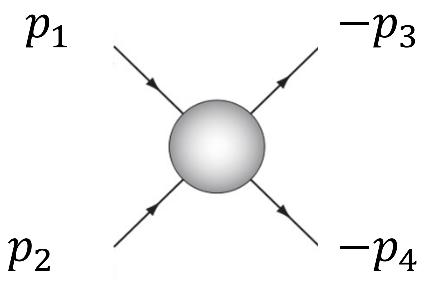

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Key Assumptions

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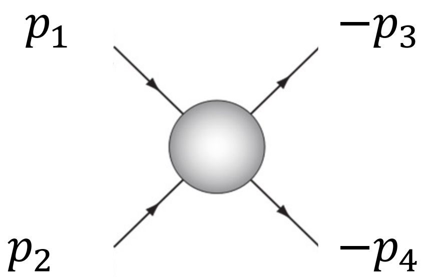
$(0 \le t < 4m^2)$



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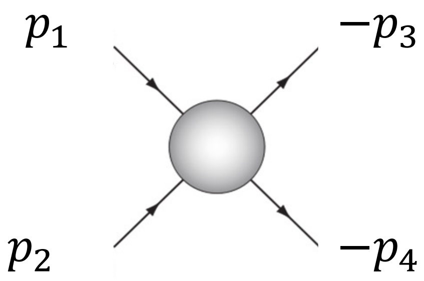

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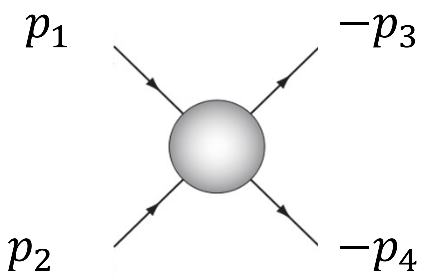

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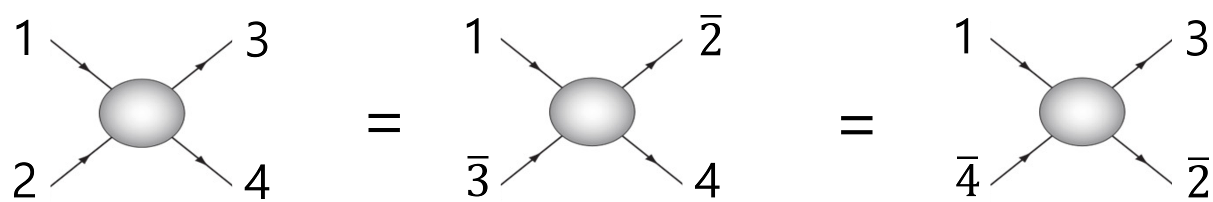
4. Crossing



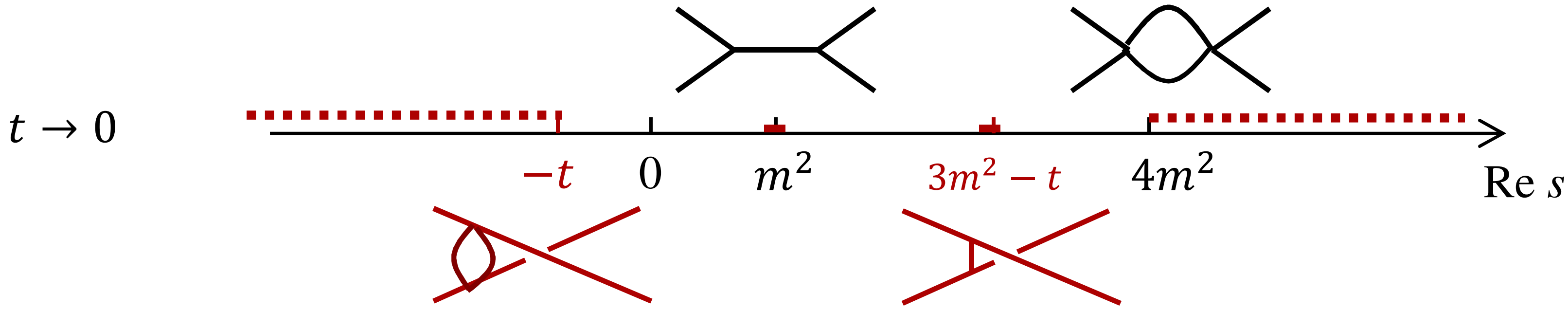
Bounds for

MASSLESS

Particles

Problem #1: Analyticity

$$A(s,t) = \sum_{p,q} s^p \, t^q + \text{poles} + \text{branch cuts}$$



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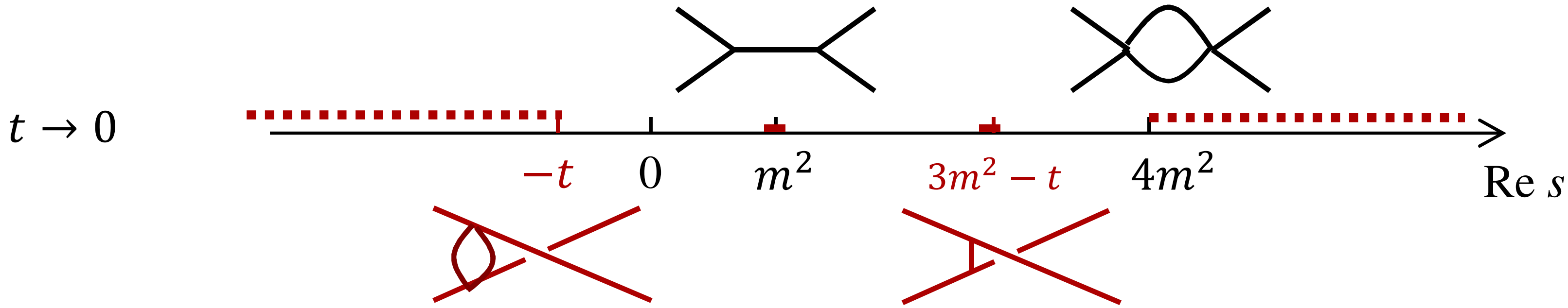

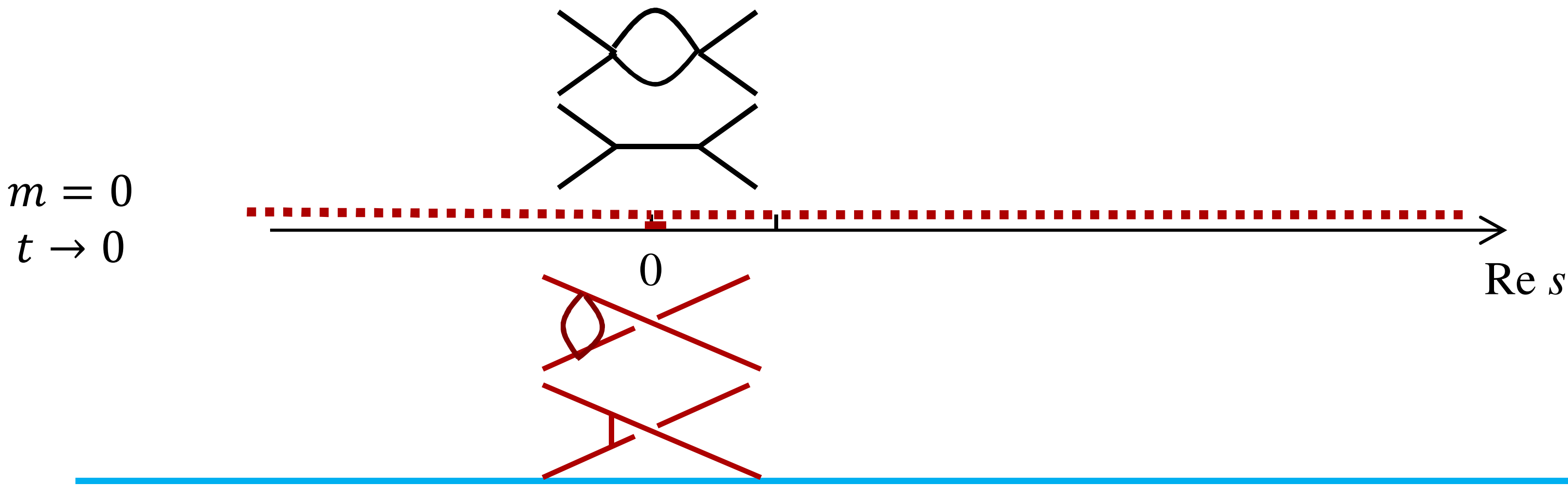

Problem #1: Analyticity => Use Tree Level Amplitudes

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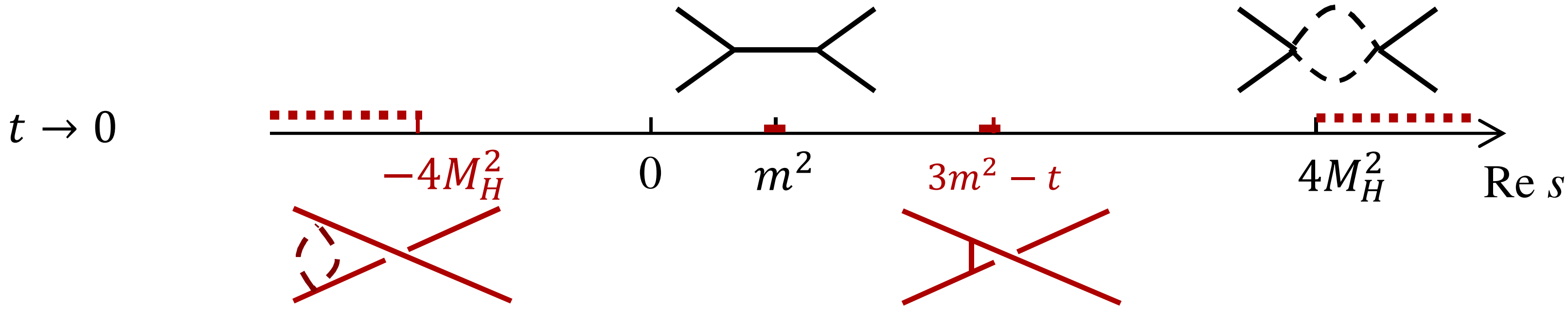

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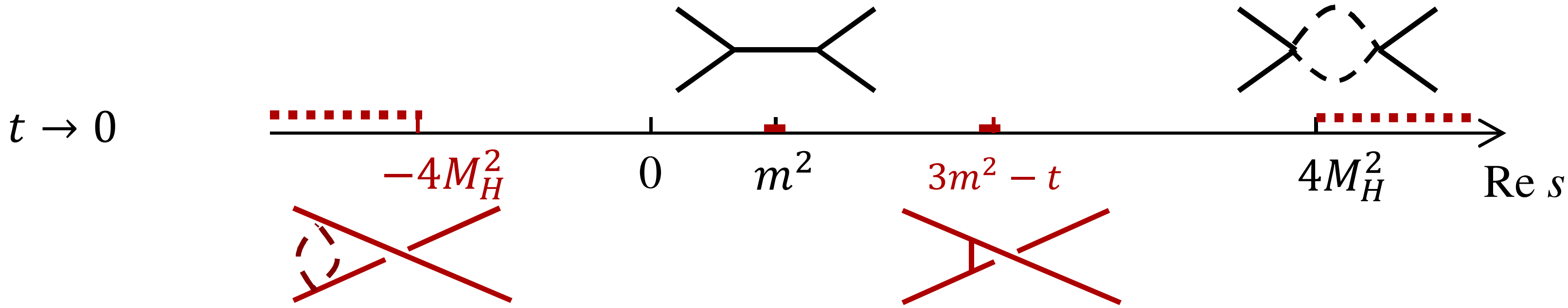
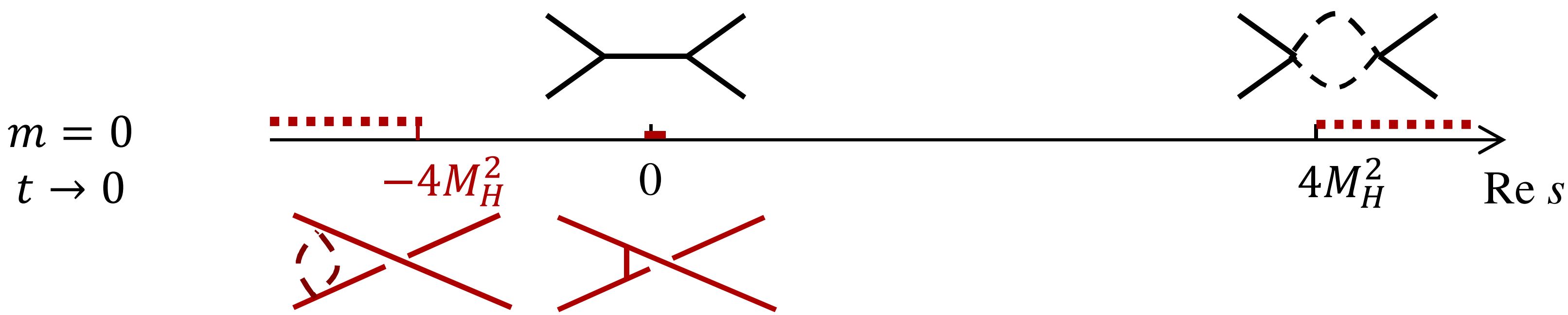
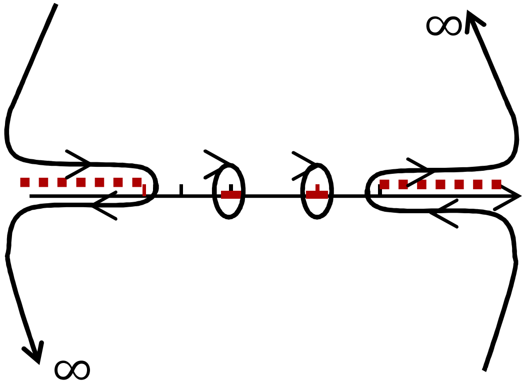




Problem #2: Convergence => More Subtractions



$$\lim_{s\to\infty} A(s) \sim s \log^2 s$$

$\partial_s^2 \tilde{A}$ Converges

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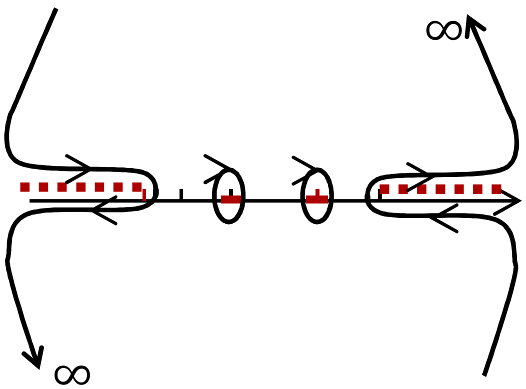

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$\partial_s^2 \tilde{A}$ Converges

$m = 0$ => No Froissart Bound

$$\lim_{s\to\infty} A(s) < s^N$$

$\partial_s^N \tilde{A}$ Converges

Problem #2: Convergence => More Subtractions

Regge Behaviour [0802.4081]

$\mathcal{L}_{UV}$: Spin-J Bound State

$\mathcal{L}_{EFT}$: $A(s,t) \sim s^J$

$$\boxed{\partial_s^n \tilde{A} > 0 \;\; \forall \; n > N \;\; \text{only} \;\; \Rightarrow \;\; \text{Spin-}N \text{ bound states}}$$