Positivity bounds on vector boson scattering in the Standard Model EFT

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Cen Zhang & **SYZ**, arXiv:1808.00010 [hep-ph]

Effective field theory (EFT)

- EFTs are widely used in modern physics

GR, inflation, dark energy, BSM physics,…

- Separation of physics at different scales

write down all local operators consistent
with symmetries suppressed by cut-off scale

$$\mathscr{L} = \sum_i \Lambda^4 f_i \mathscr{O}_i \left(\frac{\text{boson}}{\Lambda}, \frac{\text{fermion}}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right)$$

Are all EFTs allowed?

Answer: No!

UV completion:

$$e^{\frac{i}{\hbar}S_W[\text{light}]} = \int D[\text{heavy}]\ e^{\frac{i}{\hbar}S_{\text{UV}}[\text{light,heavy}]}$$

Lorentz invariance, unitarity, locality,
causality, analyticity, crossing symmetry,…

↓

Recent recognition: Positivity bounds

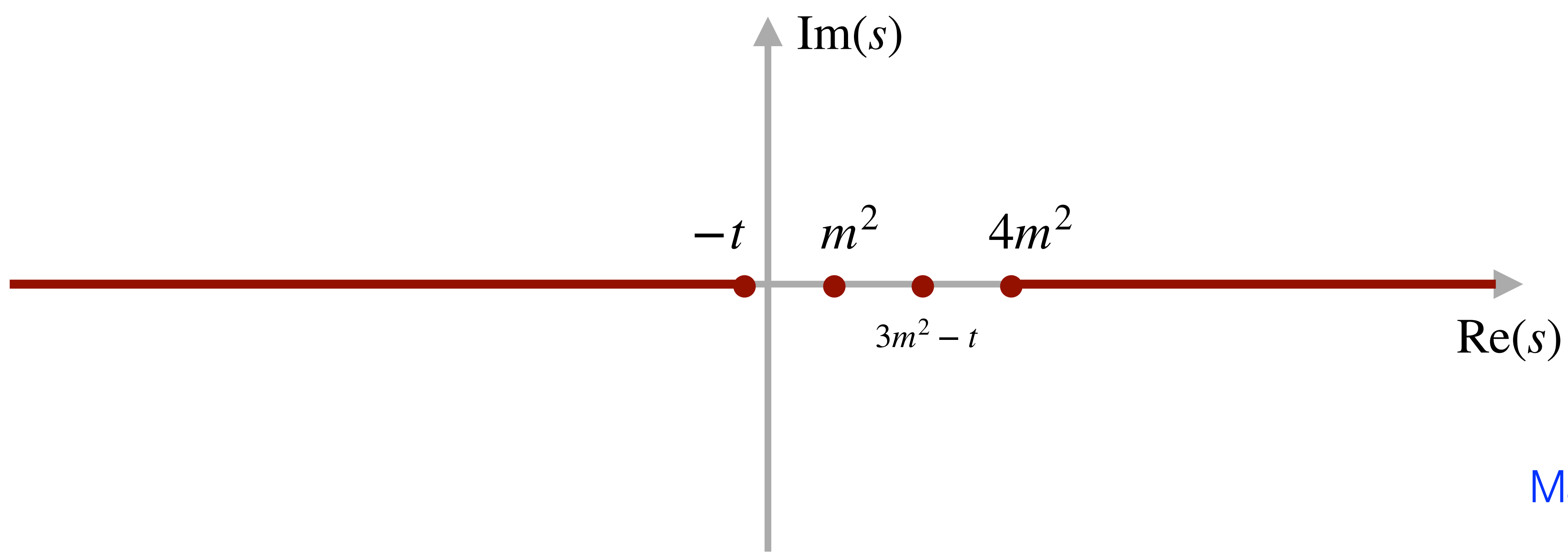
How does it work?

Analyticity

2 to 2 scattering:

$$s = -(p_1 + p_2)^2 = E_{CM}^2$$
$$t = -(p_1 + p_1')^2 = -(s - 4m^2)(1 - \cos\theta)/2$$
$$u = -(p_1 + p_2')^2 = 4m^2 - s - t$$

Complex amplitude $A(s,t)$

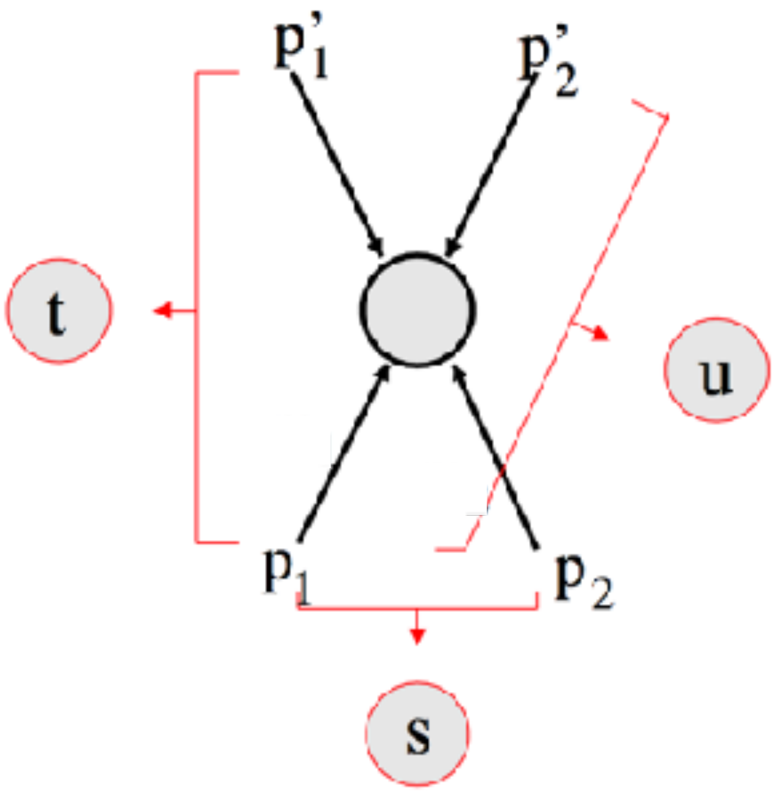


Martin, 1965

Crossing symmetry

Spin 0:

$$A(s,t) = A(u,t)$$



Spin >0:

restrict to forward limit $t = \theta = 0$

choose linear polarization vectors

$$\epsilon_\mu^{(3)} = (\epsilon_\mu^{(1)})^*, \quad \epsilon_\mu^{(4)} = (\epsilon_\mu^{(2)})^*$$

$$A(s,0) = A(u,0)$$

Unitarity

Unitarity:

$$S^\dagger S = 1, \qquad S = 1 + iT \quad \longrightarrow \quad (T - T^\dagger) = iT^\dagger T$$

Acting initial and final states:

$$\langle F|T|I\rangle - \langle I|T|F\rangle^* = \sum_f \int d\Pi_f \langle f|T|F\rangle^* \langle f|T|I\rangle$$

Optical theorem: $\mathrm{Im}[A(s,0)] = \sqrt{s(s-4m^2)}\sigma(s) > 0$

$$\frac{\partial^n}{\partial t^n}\mathrm{Im}[A(s,t)] > 0 \quad \forall \quad s \geq 4m^2\,,\, 0 \leq t < 4m^2.$$

see Scott's talk

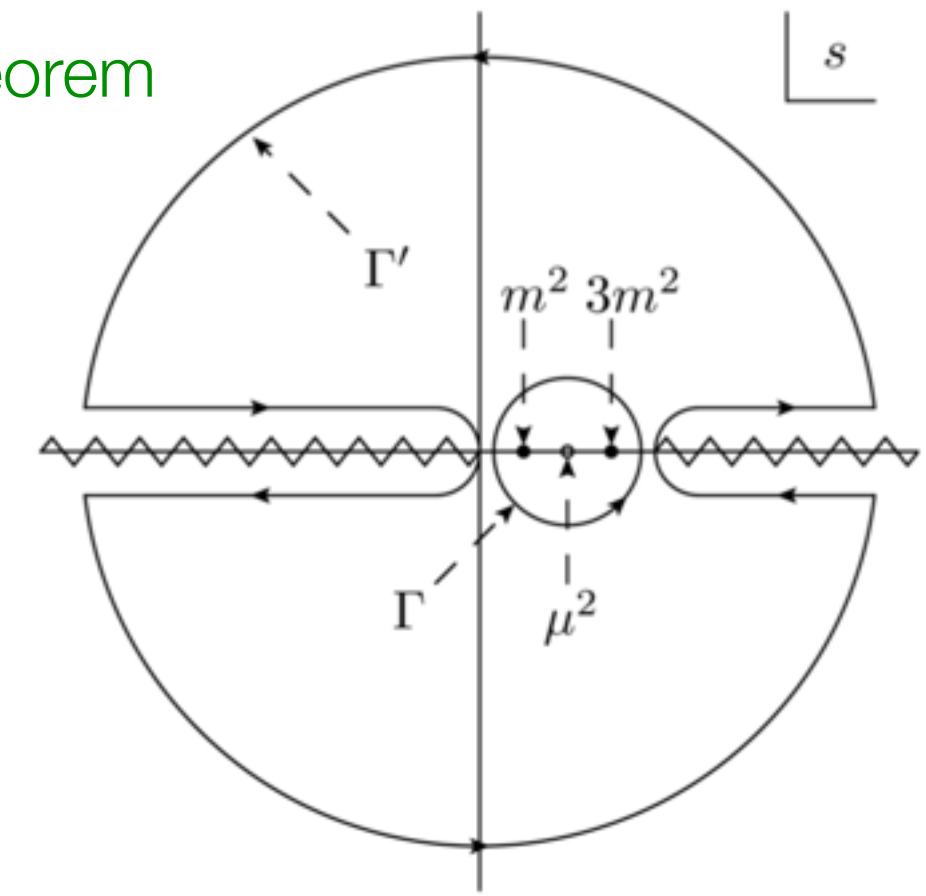
Frossart bound: as $s \to \infty, |A(s,0)| < Cs\ln^2 s$

Dispersion relation

$$f = \frac{1}{2\pi i} \oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$$

Cauchy's theorem

Froissart, 1961



Frossart bound: as $s \to \infty, |A(s,0)| < Cs\ln^2 s$

$$f = \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{+\infty}\right) \mathrm{d}s \frac{\mathrm{Disc}\, A(s,0)}{(s-\mu^2)^3}$$

Crossing symmetry

$$f = \frac{1}{\pi}\int_{4m^2}^{\infty} \mathrm{d}s \left[\frac{1}{(s-\mu^2)^3} + \frac{1}{(s+\mu^2-4m^2)^3}\right] \mathrm{Im} A(s,0)$$

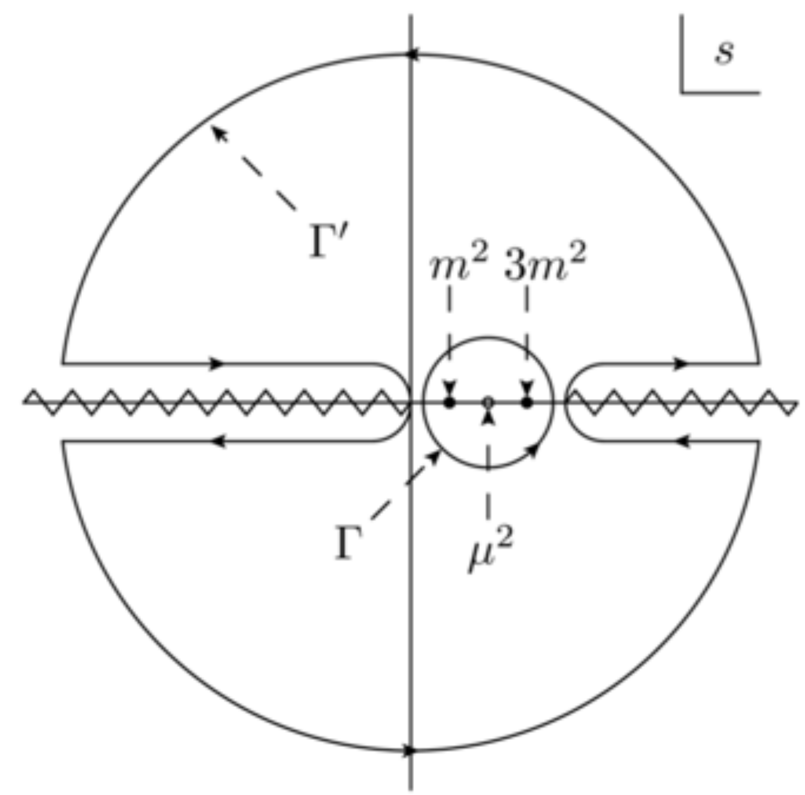
Positivity bound

Optical theorem: $\text{Im}[A(s,0)] = \sqrt{s(s-4m^2)}\sigma(s) > 0$

$$f = \frac{1}{\pi}\int_{4m^2}^{\infty} \mathrm{d}s \left[\frac{\sqrt{s(s-4m^2)}}{(s-\mu^2)^3} + \frac{\sqrt{s(s-4m^2)}}{(s+\mu^2-4m^2)^3}\right]\sigma(s)$$

For $s > 4m^2,\ 0 < \mu^2 < 4m^2$

$$f > 0$$



Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006

$$f = \sum_{\Gamma} \text{Res}\left[\frac{A(s,0)}{(s-\mu^2)^3}\right]$$

Calculable within low energy EFT!

Simple example: P(X)

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \cdots$$

$$A_{\text{tree}}(s,0) = \frac{\lambda}{\Lambda^4}\left[s^2 + (4m^2 - s)^2 - 4m^2\right]$$

Positivity bound: $f > 0$ ➡ $\lambda > 0$

Theories with $\lambda < 0$ do not have a local and Lorentz invariant UV completion

Application to the SMEFT

Standard Model Effective Field Theory

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \sum_i \frac{f_i O_i}{\Lambda^4}$$

- SM particle contents
- SM gauge group structure $SU(3)_c \times SU(2)_L \times U(1)$

Vector boson scattering (VBS)

VBS: a sensitive probe to new physics

$$V_1 + V_2 \to V_3 + V_4, \quad V_i \in \{Z, W^+, W^-, \gamma\}$$

$$\begin{aligned}
O_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] \\
O_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] \\
O_{S,2} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi] \\
O_{M,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,1} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,2} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,3} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,4} &= \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\mu\Phi\right] \times \hat{B}^{\beta\nu} \\
O_{M,5} &= \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\nu\Phi\right] \times \hat{B}^{\beta\mu} \\
O_{M,7} &= \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right]
\end{aligned}$$

$$\begin{aligned}
O_{T,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right] \\
O_{T,1} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right] \\
O_{T,2} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right] \\
O_{T,5} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,6} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu} \\
O_{T,7} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \\
O_{T,8} &= \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,9} &= \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},
\end{aligned}$$

We are interested in tree level amplitudes.

6D operators

$$\epsilon_1^\mu = \left(a_3 p_1/m_i, a_1, a_2, a_3 E_1/m_i\right)$$
$$\epsilon_2^\mu = \left(b_3 p_2/m_i, b_1, b_2, b_3 E_2/m_i\right)$$

If 6D ops alone, all positivity bounds violated

$$\mathcal{O}(\Lambda^{-4}): \qquad \sum_i (-c_i)\left(\sum_j d_j f_j^{(6)}\right)^2 \geq 0, \quad c_i > 0$$

Positivity bounds require the existence of higher-D ops!

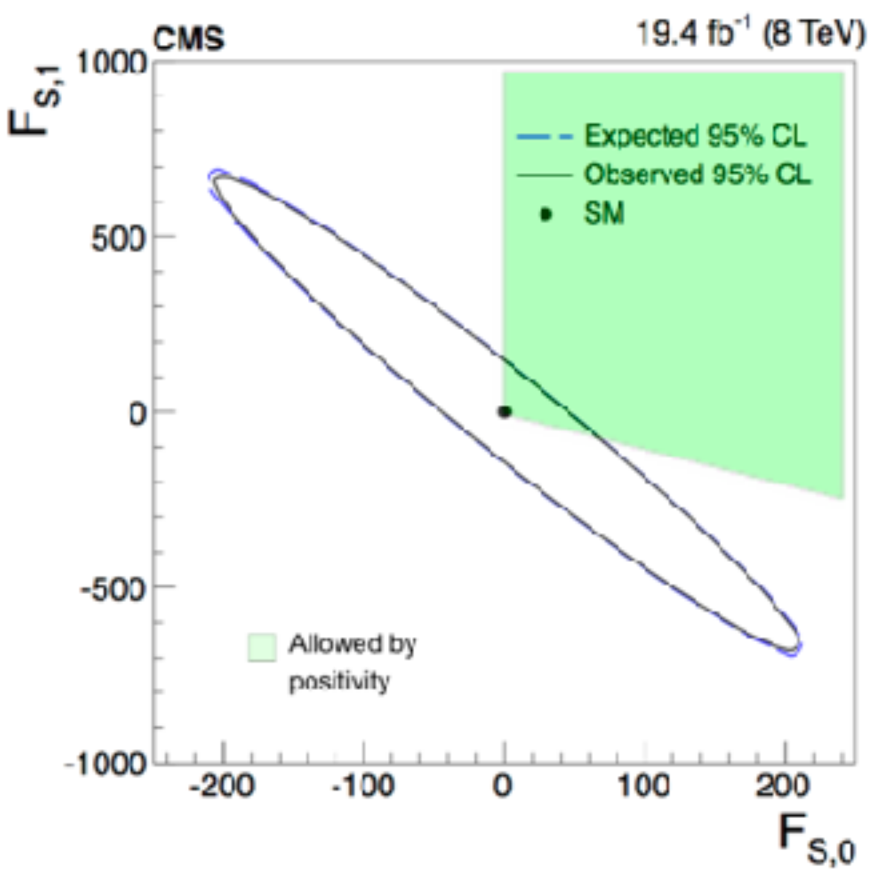
Cen Zhang & **SYZ**, arXiv:1808.00010

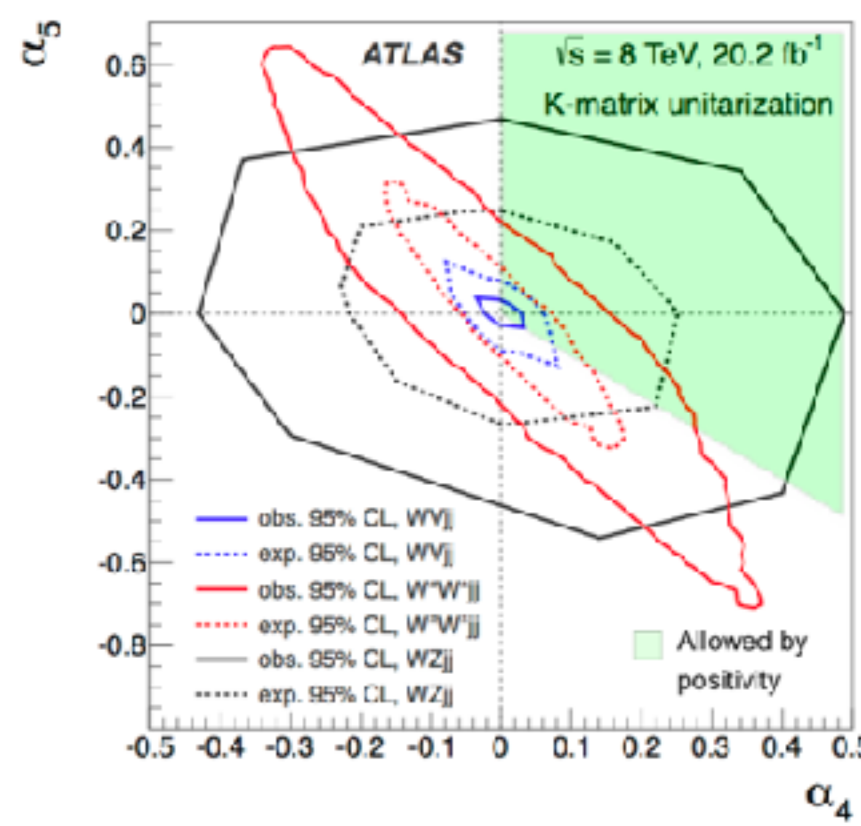
8D operators (1)

Positivity bounds (neglect 6D ops):

Cen Zhang & **SYZ**, arXiv:1808.00010

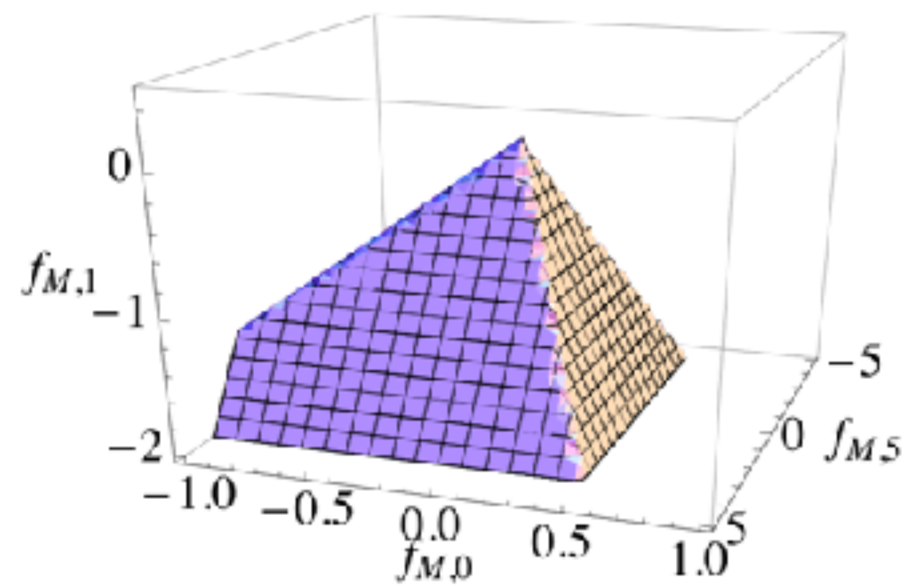
$$\mathcal{O}(\Lambda^{-4}) : \qquad \sum_i C_i f_i \geq 0$$





8D operators (2)

More parameters:



All 18 parameters:

Only ~3% of the total parameter space admit a local/Lorentz invariant UV completion!

Cen Zhang & **SYZ**, arXiv:1808.00010

Summary

Not all EFTs have a UV completion!

Positivity bounds: new constraints on coupling constants

Most of the parameter space of the
SMEFT do not admit a UV completion.