Can deep infrared modes beyond current horizon scale affect observable primordial fluctuations?

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in collaboration with Takahiro Tanaka（Kyoto Univ. ＆YITP）

Refs. : JT and T.Tanaka JCAP02(2018)014, arXiv: 1708.01734

JT and T.Tanaka arXiv:1806.03262

Working in progress.

2018.8.30 Cosmo18@IBS

plan

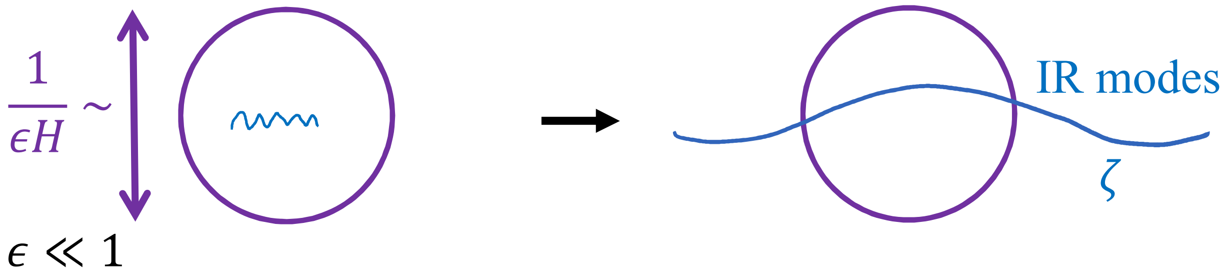
- 1. Introduction and Motivation (8-10mins.)
- 2. Our works (5-3 mins.)
- 3. Summary (1 min.)

Inflation-perturbations

- Microscopic fluctuations → Macroscopic. (IR modes)

(super-horizon modes)

$$\lambda_{\mathrm{ph}} = \frac{2\pi a(t)}{k} \approx \frac{2\pi e^{Ht}}{k}$$



- After inflation, super-horizon modes enter into the horizon.

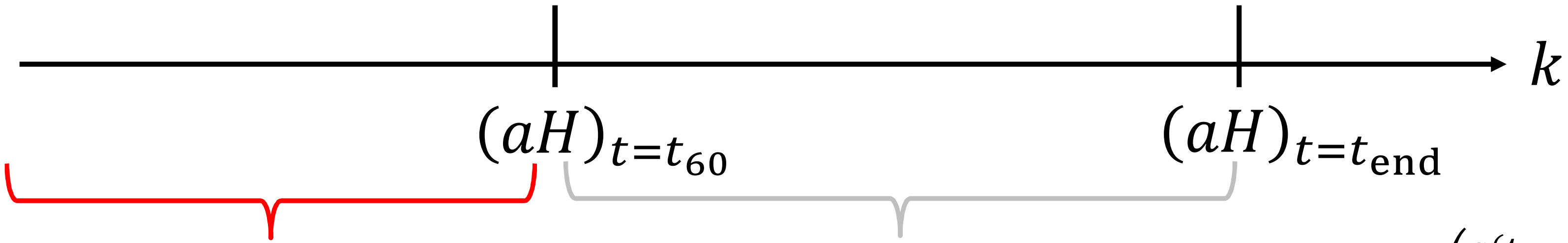
``horizon re-enter''

$$k_{\mathrm{ph}} = \frac{k}{a} \quad \text{vs} \quad H$$

$$\partial_t \left(\frac{H}{k_{\mathrm{ph}}} \right) = \frac{1}{k} \partial_t (aH) = \frac{\ddot{a}}{k} < 0$$

Observables = Quantum fluctuations.

- These fluctuations are the seeds of various objects, *e.g.*, galaxies.

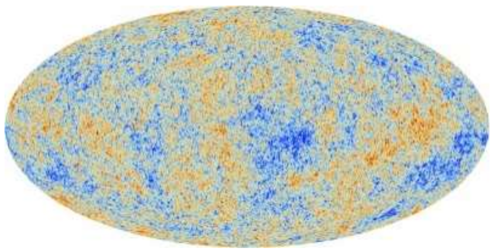


Deep IR modes
(beyond current horizon scale)

with $\ln\left(\frac{a(t_{\rm end})}{a(t_{60})}\right) = 60$

Observed density perturbations

e.g. Cosmic Microwave Background (CMB)



ESA, Planck Collaboration (2013)

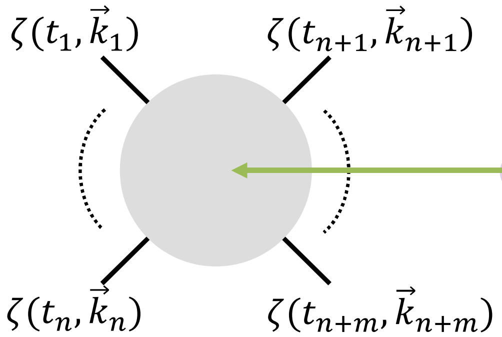
Naively,

Observables = QFT expectation values of ζ: $< \zeta \cdots \zeta >$

IR secular growth

- $< \zeta \cdots \zeta >$ contains very large loop corrections:

ζ, h_{ij} : metric perturbations
ϕ : other light scalar fields



loops of $\zeta, h_{\rm ij}$ and loops of ϕ

$\Downarrow$

divergent or significantly large in deep IR.

- If IR loops affect observables, it would be interesting because

1. IR loops may **modify the current predictions,**
2. the dynamics **before** 60 e-folds may be imprinted on observables!

Be suspicious.

IR loops affect observables for local observers (us)?

- Large IR loops may signal the inappropriate definition of observables (*e.g.*, QED: one should consider charged particle state dressed with soft photon) .

- We should reconsider which quantities are really observables for us.

Large IR loops can affect observables?

- IR loops are classified into two cases:

1. loops of ζ, h_{ij} (metric perturbations) T. Tanaka and Y. Urakawa (2009,…)

Redefined observables, which are genuinely *gauge invariant,* do not contain IR loops:

IR loops of ζ, h_{ij} = *gauge* artifact

2. loops of ϕ (isocurvature modes)

→ The above discussion **does not** apply.

Hint: Stochastic approach.

A. A. Starobinsky (1986) A. A. Starobinsky and J. Yokoyama(1994)

Stochastic Formalism

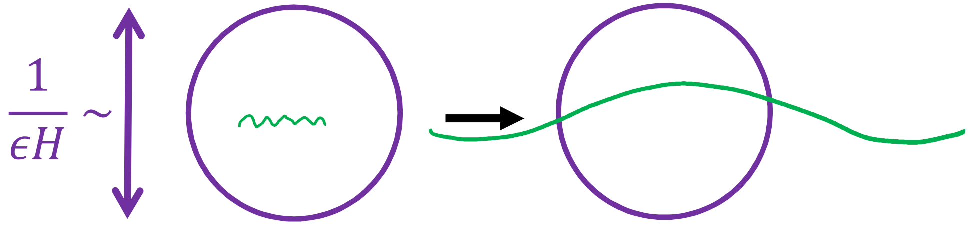
IR dynamics of $\phi_{\rm IR}$=Brownian motion with an external force.

$$\dot{\phi}_{\rm IR} = \underbrace{-\frac{1}{3H}V'(\phi_{\rm IR})}_{\text{deterministic}} + \underbrace{\xi}_{\text{stochastic}} \qquad < \xi(x_1)\xi(x_2) > = \frac{H^3}{4\pi^2}\delta(t_1 - t_2)\frac{\sin(\epsilon a(t_1)H|\vec{x_1} - \vec{x_2}|)}{\epsilon a(t_1)H|\vec{x_1} - \vec{x_2}|}$$

coarse-graining scale

※Approximation: UV modes = harmonic oscillators

Physical origin of ξ



Short-wavelength (UV) modes

are transferred to

IR modes (phase is random)

Stochastic force

✓ This eq. can be solved non-perturbatively.

✓ This eq. can correctly recover part of IR secular effects.

N. C. Tsamis and R. P. Woodard (2005)

Stochastic Formalism

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coarse-graining scale

※Approximation: UV modes = harmonic oscillators

e.g. $\lambda\phi^4$ theory in de Sitter

$$\frac{<\phi_{\rm IR}^2(x)>}{H^2} \sim \ln\frac{a}{a_0}+1 +\lambda\left[\left(\ln\frac{a}{a_0}\right)^3+\left(\ln\frac{a}{a_0}\right)^2+\left(\ln\frac{a}{a_0}\right)+1\right] +\lambda^2\left[\left(\ln\frac{a}{a_0}\right)^5+\left(\ln\frac{a}{a_0}\right)^4+\left(\ln\frac{a}{a_0}\right)^3+\cdots\right] +\cdots$$

$k \geq a_o H$: an IR cutoff

- ✓ This eq. can be solved non-perturbatively.
- ✓ This eq. can correctly recover part of IR secular effects.

N. C. Tsamis and R. P. Woodard (2005)

Classical *Stochastic Picture of the inflationary universe*

A. Linde(1986) A. A. Starobinsky (1986) Y. Nambu and M. Sasaki (1989)

The time evolution of the inflationary universe = **classical stochastic process**

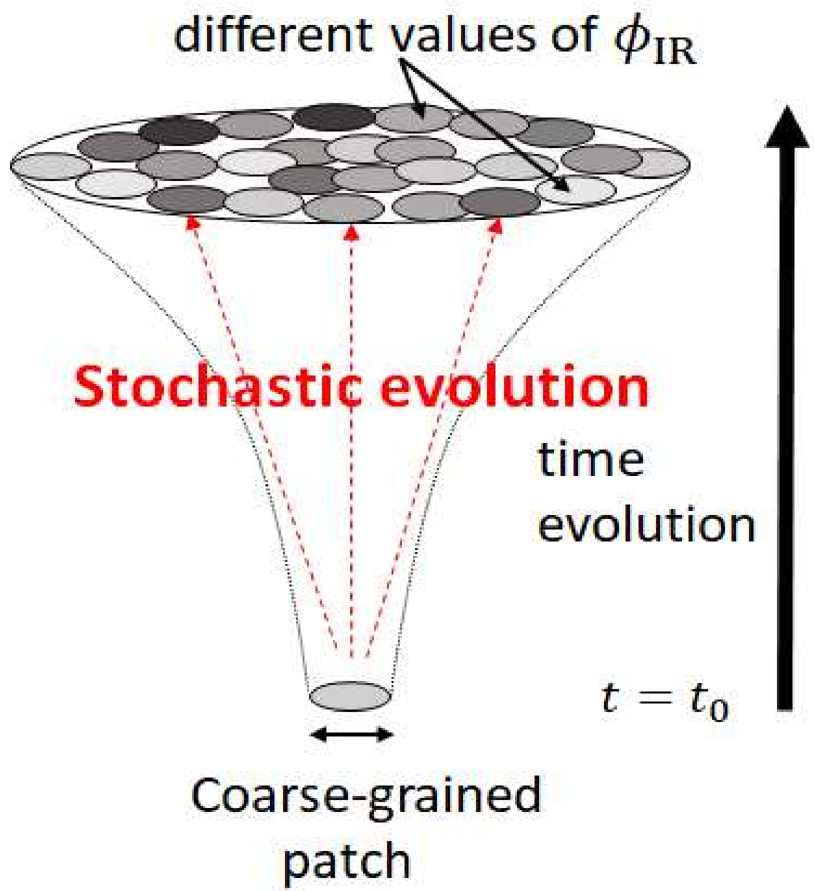
Brownian Motion

In this picture, one assumes that

a certain value of ϕ_{IR} is **classically realized** at each patch.

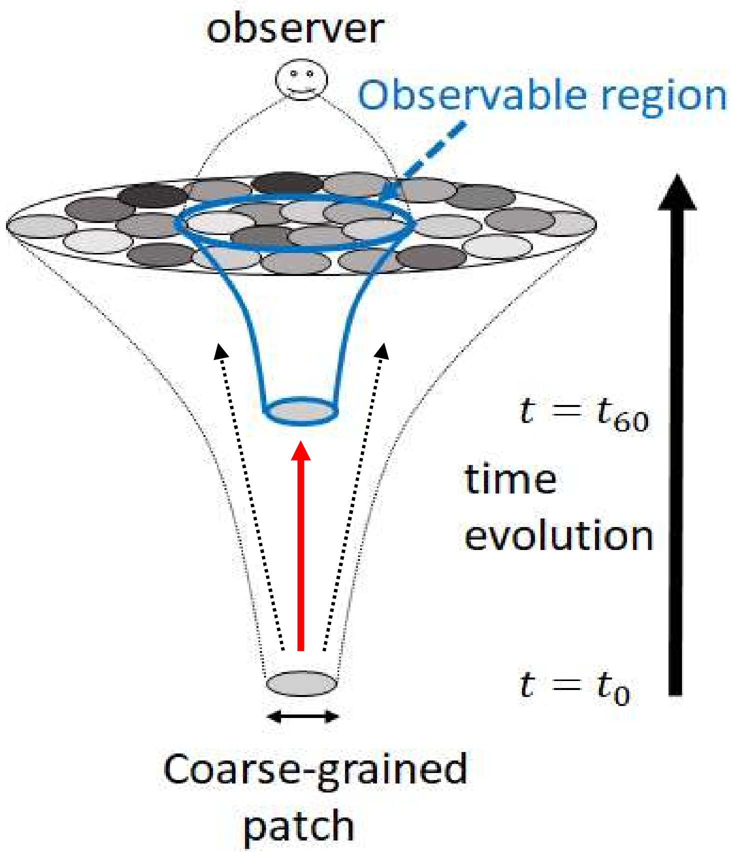
$$\dot{\phi}_{\mathrm{IR}} = -\frac{1}{3H} V'(\phi_{IR}) + \xi$$

※ transition probability is non-negative.



Observables in the classical stochastic picture

In this picture, fluctuations of deep infrared modes do not affect observed primordial fluctuations.

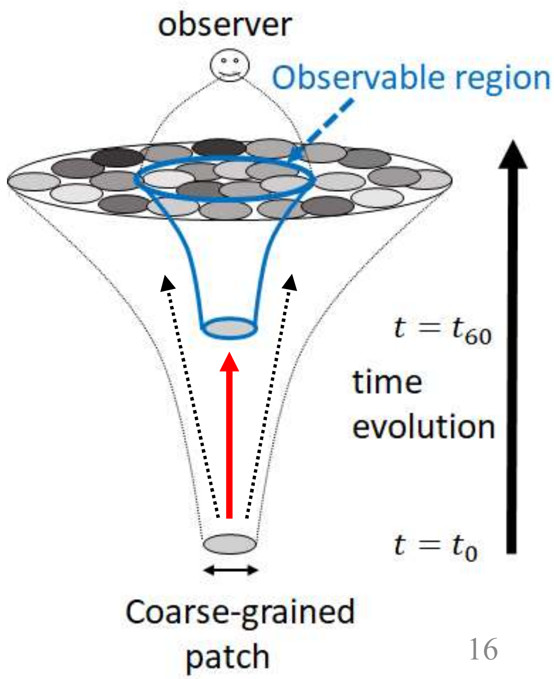


However, …

- Remember that the origin of this classical stochastic picture is $\dot{\boldsymbol{\phi}}_{\mathbf{IR}} = -\frac{\mathbf{1}}{\mathbf{3}\boldsymbol{H}}\boldsymbol{V}'(\boldsymbol{\phi}_{\boldsymbol{IR}}) + \boldsymbol{\xi}$.
- Non-linearities of UV modes are neglected in this equation.

These terms are not correctly recovered.

$$\frac{<\phi_{\mathrm{IR}}^2(x)>}{H^2} \sim \ln\frac{a}{a_0} + 1$$
$$+\lambda\left[\left(\ln\frac{a}{a_0}\right)^3 + \left(\ln\frac{a}{a_0}\right)^2 + \left(\ln\frac{a}{a_0}\right) + 1\right]$$
$$+\lambda^2\left[\left(\ln\frac{a}{a_0}\right)^5 + \left(\ln\frac{a}{a_0}\right)^4 + \left(\ln\frac{a}{a_0}\right)^3 + \cdots\right]$$
$$+ \cdots$$



Classical stochastic picture can capture these IR secular effects?

What is ``classicality''?

- The following conditions are often thought to be *necessary* conditions for the validity of the classical stochastic interpretations:

(1) Correlation functions are correctly reproduced by the classical stochastic process in a good approximation.

(2) Quantum decoherence occurs sufficiently.

Off-diagonal elements of reduced density matrix ρ_{IR} decay:

$$\hat{\rho}_{\mathrm{IR}}(t) = \int \mathrm{d}\phi_{\mathrm{IR}} \int \mathrm{d}\phi'_{\mathrm{IR}}\, \rho(\phi_{\mathrm{IR}}, \phi'_{\mathrm{IR}})\, |\phi_{\mathrm{IR}}\rangle\langle\phi'_{\mathrm{IR}}| \approx \int \mathrm{d}\phi_{\mathrm{IR}}\, p(\phi_{\mathrm{IR}})|\phi_{\mathrm{IR}}\rangle\langle\phi_{\mathrm{IR}}|$$

D. Polarski, C. Kiefer, A. A. Starobinsky *et al.*('96,98,06,…)
C.P. Burgess *et al.*('14,16) E. Nelson *et al.*('16)

Our work.

1. How to derive the effective IR dynamics which can correctly recover all the IR secular effects.

   JT and T. Tanaka JCAP02(2018)14

2. We show that all the IR secular effects can correctly reproduced by the classical stochastic process in a good approximation at a late time.

   JT and T. Tanaka arXiv:1806.03262

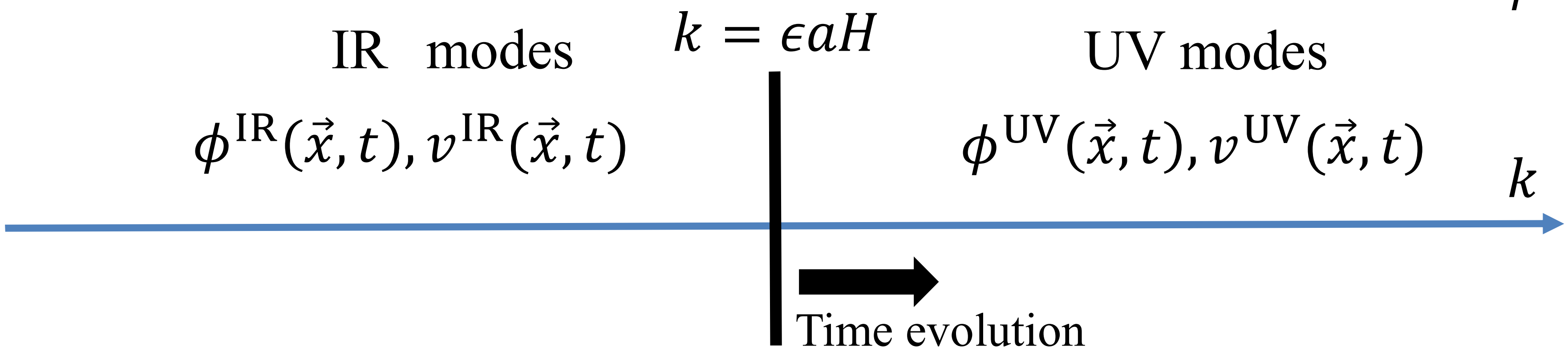
Our work: Setup

JT and T. Tanaka (2017)

- Model : $\mathcal{H} = \frac{1}{2}v^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi, v)$ on de Sitter background.

- Defs. of UV modes and IR modes

$v \equiv \dot{\phi}$



- Assumptions

1. No IR mode initially (at $t = t_0$).
2. $\mathrm{V}(\phi, v)$ is turned on at $t = t_0$, and the initial state is set to the Bunch-Davies vacuum states for a free field.

Integrate out UV modes

JT and T. Tanaka (2017)

Derive an effective IR dynamics by integrating out UV modes

Decompose the path integral

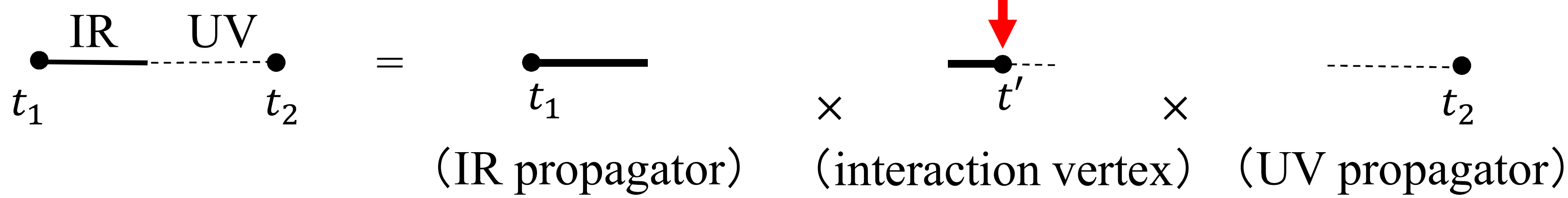
$$\int \mathcal{D}\phi_+\mathcal{D}v_+\mathcal{D}\phi_-\mathcal{D}v_-\; e^{i(S^+-S^-)} \qquad S=\int \mathrm{d}^4x\, \mathrm{a}^3\left(v\dot{\phi}-\mathcal{H}(v,\phi)\right)$$

$$\to \int \mathcal{D}\phi_+^{\mathrm{IR}}\mathcal{D}v_+^{\mathrm{IR}}\mathcal{D}\phi_-^{\mathrm{IR}}\mathcal{D}v_-^{\mathrm{IR}}\; e^{iS_{\mathrm{IR}}} \int \mathcal{D}\phi_+^{\mathrm{UV}}\mathcal{D}v_+^{\mathrm{UV}}\mathcal{D}\phi_-^{\mathrm{UV}}\mathcal{D}v_-^{\mathrm{UV}}\; e^{iS_{\mathrm{UV}}}\boxed{e^{iS_{\mathrm{int}}^{\mathrm{UV-IR}}}}$$

Interactions between UV modes and IR modes.

non-linear interactions

Bilinear interactions which describes UV $\to$ IR transition



JT and T. Tanaka (2017)

Derive the effective EoM

$$\int \mathcal{D}\phi_+^{\mathrm{IR}}\mathcal{D}v_+^{\mathrm{IR}}\mathcal{D}\phi_-^{\mathrm{IR}}\mathcal{D}v_-^{\mathrm{IR}}\, e^{iS_{\mathrm{IR}}} \boxed{\int \mathcal{D}\phi_+^{\mathrm{UV}}\mathcal{D}v_+^{\mathrm{UV}}\mathcal{D}\phi_-^{\mathrm{UV}}\mathcal{D}v_-^{\mathrm{UV}}\, e^{iS_{\mathrm{UV}}} e^{iS_{\mathrm{int}}^{\mathrm{UV-IR}}}}$$

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$$\equiv e^{i\left(\Gamma_{(\rm s)}+\Gamma_{(\rm d)}\right)}$$

Linear in $\phi_\Delta^{\rm IR}$ or $v_\Delta^{\rm IR}$.

$$\phi_c \equiv \frac{\phi_+ + \phi_-}{2},\ \phi_\Delta \equiv \phi_+ - \phi_-$$

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$$\int \mathcal{D}\phi_+^{\rm IR}\mathcal{D}v_+^{\rm IR}\mathcal{D}\phi_-^{\rm IR}\mathcal{D}v_-^{\rm IR}\, e^{iS_{\rm IR}} \boxed{\int \mathcal{D}\phi_+^{\rm UV}\mathcal{D}v_+^{\rm UV}\mathcal{D}\phi_-^{\rm UV}\mathcal{D}v_-^{\rm UV}\, e^{iS_{\rm UV}} e^{iS_{\rm int}^{\rm UV-}}}$$

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$$\equiv e^{i\left(\Gamma_{(\rm s)}+\Gamma_{(\rm d)}\right)}$$

Linear in $\phi_\Delta^{\rm IR}$ or $v_\Delta^{\rm IR}$.

$$e^{i\Gamma_{(\rm s)}} = \int \mathcal{D}\xi_\phi \mathcal{D}\xi_v\, P\left[\xi_\phi, \xi_v; \phi_c^{\rm IR}, v_c^{\rm IR}\right] e^{\underline{i\xi_\phi v_\Delta^{\rm IR} - i\xi_v \phi_\Delta^{\rm IR}}}$$

linear in $\phi_\Delta^{\rm IR}$ or $v_\Delta^{\rm IR}$

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$$\int \mathcal{D}\phi_+^{\rm IR}\mathcal{D}v_+^{\rm IR}\mathcal{D}\phi_-^{\rm IR}\mathcal{D}v_-^{\rm IR}\, e^{iS_{\rm IR}} \boxed{\int \mathcal{D}\phi_+^{\rm UV}\mathcal{D}v_+^{\rm UV}\mathcal{D}\phi_-^{\rm UV}\mathcal{D}v_-^{\rm UV}\, e^{iS_{\rm UV}} e^{iS_{\rm int}^{\rm UV-I}}}$$

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linear in $\phi_\Delta^{\rm IR}$ or $v_\Delta^{\rm IR}$

$$e^{i\left(\Gamma_{(\rm s)}+\Gamma_{(\rm d)}\right)} \longrightarrow e^{i\left[\phi_\Delta^{\rm IR}(\cdots)+v_\Delta^{\rm IR}(\cdots)\right]}$$

JT and T. Tanaka (2017)

Derive the effective EoM

$$\int \mathcal{D}\phi_+^{\rm IR}\mathcal{D}v_+^{\rm IR}\mathcal{D}\phi_-^{\rm IR}\mathcal{D}v_-^{\rm IR}\, e^{iS_{\rm IR}} \boxed{\int \mathcal{D}\phi_+^{\rm UV}\mathcal{D}v_+^{\rm UV}\mathcal{D}\phi_-^{\rm UV}\mathcal{D}v_-^{\rm UV}\, e^{iS_{\rm UV}} e^{iS_{\rm int}^{\rm UV-I}}}$$

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linear in $\phi_\Delta^{\rm IR}$ or $v_\Delta^{\rm IR}$

$$\int \mathcal{D}\phi_\Delta^{\rm IR} \mathcal{D}\, v_\Delta^{\rm IR}$$

$$e^{i\left(\Gamma_{(\rm s)}+\Gamma_{(\rm d)}\right)} \longrightarrow e^{i\left[\phi_\Delta^{\rm IR}(\cdots)+v_\Delta^{\rm IR}(\cdots)\right]}$$

An effective IR dynamics

$$\dot\phi_c^{\rm IR} = v_c^{\rm IR} + \mu_1\left(\phi_c^{\rm IR}, v_c^{\rm IR}\right) + \xi_\phi$$

$$\dot v_c^{\rm IR} = -3H v_c^{\rm IR} - \mu_2\left(\phi_c^{\rm IR}, v_c^{\rm IR}\right) + \xi_v$$

Stochastic noises

Derive the effective EoM

$$\int \mathcal{D}\phi_+^{\mathrm{IR}}\mathcal{D}v_+^{\mathrm{IR}}\mathcal{D}\phi_-^{\mathrm{IR}}\mathcal{D}v_-^{\mathrm{IR}}\, e^{iS_{\mathrm{IR}}} \boxed{\int \mathcal{D}\phi_+^{\mathrm{UV}}\mathcal{D}v_+^{\mathrm{UV}}\mathcal{D}\phi_-^{\mathrm{UV}}\mathcal{D}v_-^{\mathrm{UV}}\, e^{iS_{\mathrm{UV}}} e^{iS_{\mathrm{int}}^{\mathrm{UV-IR}}}}$$

$$\phi_c \equiv \frac{\phi_+ + \phi_-}{2},\ \phi_\Delta \equiv \phi_+ - \phi_-$$

$$\equiv e^{i\left(\Gamma_{(\mathrm{s})}+\Gamma_{(\mathrm{d})}\right)}$$

Linear in $\phi_\Delta^{\mathrm{IR}}$ or v_Δ^{IR}.

$$e^{i\Gamma_{(\mathrm{s})}} = \int \mathcal{D}\xi_\phi \mathcal{D}\xi_v\, P\left[\xi_\phi, \xi_v; \phi_c^{\mathrm{IR}}, v_c^{\mathrm{IR}}\right] e^{\underline{i\xi_\phi v_\Delta^{\mathrm{IR}} - i\xi_v \phi_\Delta^{\mathrm{IR}}}}$$

linear in $\phi_\Delta^{\mathrm{IR}}$ or v_Δ^{IR}

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Stochastic noises

Probability distribution of noises = P

JT and T.Tanaka (2018)

Path integral for IR modes can be written as

$$\int \mathcal{D}\phi_c^{\rm IR}\mathcal{D}v_c^{\rm IR}\mathcal{D}\phi_\Delta^{\rm IR}\mathcal{D}v_\Delta^{\rm IR} e^{i\int d^4x\, a^3 v_\Delta^{\rm IR}(\dot\phi_c^{\rm IR}-v_c^{\rm IR}-\mu_1)} e^{i\Gamma_{(s)}\left[v_\Delta^{\rm IR},\phi_\Delta^{\rm IR},v_c^{\rm IR},\phi_c^{\rm IR}\right]} e^{i\int d^4x\, a^3\phi_\Delta^{\rm IR}(-\dot v_c^{\rm IR}-3Hv_c^{\rm IR}-\mu_2)}$$

Integration by parts over $v_c^{\rm IR}$. $\phi_\Delta^{\rm IR}(t)\to -\hat F \equiv i\int_t^{t_f}\frac{dt'}{a^3(t')}\frac{\delta}{\delta v_c^{\rm IR}(t')}(1+\cdots)$

$$\int \mathcal{D}\phi_c^{\rm IR}\mathcal{D}v_\Delta^{\rm IR}\mathcal{D}v_c^{\rm IR}\, \underline{e^{i\Gamma_{(s)}}\Big|_{\phi_\Delta^{\rm IR}\to\hat F}\left[e^{i\int d^4x\, a^3 v_\Delta^{\rm IR}(\dot\phi_c^{\rm IR}-v_c^{\rm IR}-\mu_1)}\right]} \int \mathcal{D}\phi_\Delta^{\rm IR} e^{i\int d^4x\, a^3\phi_\Delta^{\rm IR}(-\dot v_c^{\rm IR}-3Hv_c^{\rm IR}-\mu_2)}$$

$$\simeq e^{i\int d^4x\, a^3 v_\Delta^{\rm IR}(\dot\phi_c^{\rm IR}-v_c^{\rm IR}-\mu)} e^{-\frac{1}{2}\int d^4x_1\, a^3(t_1)\int d^4x_2\, a^3(t_2) v_\Delta^{\rm IR}(x_1)v_\Delta^{\rm IR}(x_2)A(x_1,x_2)}$$

$$A(x_1,x_2)=\frac{H^3}{4\pi^2}\delta(t_1-t_2)\frac{\sin(\epsilon a(t_1)H|\vec{x_1}-\vec{x_2}|)}{\epsilon a(t_1)H|\vec{x_1}-\vec{x_2}|}$$

$$\dot\phi_c^{\rm IR}=v_c^{\rm IR}+\mu_1+\xi$$ Approximately Gaussian noise

$$\dot v_c^{\rm IR}=-3Hv_c^{\rm IR}-\mu_2$$

→Positive probability !

※ $v_c^{\rm IR}$ no longer corresponds to conjugate momentum.

Dynamics of $\phi_c^{\rm IR}$ = a classical stochastic process.

Summary

Motivation

IR loops of light scalar ϕ_{IR}: very large→ What are observables for us?

If the classical stochastic picture holds, one would be allowed to propose observables which do not suffer from IR secular effects.

Conclusions and Discussions

1. Correlation functions can be correctly recovered by the classical stochastic process in a good approximation.

2. This suggests that one can consistently construct the IR finite observables based on the stochastic picture.

3. Decoherence ($\langle\phi_{\mathrm{IR}}|\hat{\rho}_{\mathrm{IR}}|\phi'_{\mathrm{IR}}\rangle \approx 0$) will be also necessary to justify the classical stochastic picture of IR dynamics.

backup

Positioning of our work

Inflationary paradigm

Various models (actions)

Standard single field inflation
Multi-field inflation
Gauge fields
String-motivated … etc.

Framework

Predictions

Power spectrum, tilts,
Bispectrum…

Our work

Definitions of observables are really correct?

Key issues: IR divergence problem

An effective IR dynamics=Classical process?

$$P = \int \mathcal{D}v_\Delta^{\rm IR}\mathcal{D}\phi_\Delta^{\rm IR}\left(e^{-A_1 {v_\Delta^{\rm IR}}^2 - iA_2 {v_\Delta^{\rm IR}}^3 - \cdots}\right)\left(e^{-B_1 {\phi_\Delta^{\rm IR}}^2 - \cdots}\right)\left(e^{-C_1 v_\Delta^{\rm IR}\phi_\Delta^{\rm IR} - \cdots}\right) e^{i\xi_\phi v_\Delta^{\rm IR} - i\xi_v \phi_\Delta^{\rm IR}}$$

$= \exp[i\Gamma_{(s)}]$

$\frac{A_1}{H^4} \sim O(1)$, $\frac{B_1}{H^2} \sim O(\lambda^2)$, $\frac{C_1}{H^3} \sim O(\lambda)$ λ: coupling constant

Gaussian part of $\phi_\Delta^{\rm IR}$: suppressed by λ.

Regarding $\phi_\Delta^{\rm IR}$, non-Gaussian parts contribute at the same order.

It seems impossible to ensure the non-negativity of $P[\xi_\phi, \xi_v; \phi_c^{\rm IR}, v_c^{\rm IR}]$ within the validity of the perturbation theory.

JT and T.Tanaka (2018)

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$$\int \mathcal{D}\phi_c^{\rm IR}\mathcal{D}v_c^{\rm IR}\mathcal{D}\phi_\Delta^{\rm IR}\mathcal{D}v_\Delta^{\rm IR} e^{i\int d^4x\, a^3 v_\Delta^{\rm IR}(\dot\phi_c^{\rm IR}-v_c^{\rm IR}-\mu_1)} e^{i\Gamma_{(s)}\left[v_\Delta^{\rm IR},\phi_\Delta^{\rm IR},v_c^{\rm IR},\phi_c^{\rm IR}\right]} e^{i\int d^4x\, a^3\phi_\Delta^{\rm IR}(-\dot v_c^{\rm IR}-3Hv_c^{\rm IR}-\mu_2)}$$

Integration by parts over $v_c^{\rm IR}$. $\phi_\Delta^{\rm IR}(t)\to -\hat F \equiv i\int_t^{t_f}\frac{dt'}{a^3(t')}\frac{\delta}{\delta v_c^{\rm IR}(t')}(1+\cdots)$

$$\int \mathcal{D}\phi_c^{\rm IR}\mathcal{D}v_\Delta^{\rm IR}\mathcal{D}v_c^{\rm IR}\; \underline{e^{i\Gamma_{(s)}}\Big|_{\phi_\Delta^{\rm IR}\to\hat F}\left[e^{i\int d^4x\, a^3 v_\Delta^{\rm IR}(\dot\phi_c^{\rm IR}-v_c^{\rm IR}-\mu_1)}\right]}\int \mathcal{D}\phi_\Delta^{\rm IR} e^{i\int d^4x\, a^3\phi_\Delta^{\rm IR}(-\dot v_c^{\rm IR}-3Hv_c^{\rm IR}-\mu_2)}$$

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→Positive probability！

※ $v_c^{\rm IR}$ no longer corresponds to conjugate momentum.

Dynamics of $\phi_c^{\rm IR}$ = a classical stochastic process.

What is ``classicality”?

In our previous work, we showed that

- All correlation functions are correctly recovered in a good approximation by the classical stochastic process.

Is this sufficient for justifying the classical stochastic interpretation of the IR secular effects?

No. Decoherence is needed. (necessary condition)

Off-diagonal elements of reduced density matrix $\rho_{\rm IR}$ decay:

$$\hat{\rho}_{\rm IR}(t) = \int \mathrm{d}\phi_{\rm IR} \int \mathrm{d}\phi'_{\rm IR}\, \rho(\phi_{\rm IR}, \phi'_{\rm IR})\, |\phi_{\rm IR}\rangle\langle\phi'_{\rm IR}\,| \approx \int \mathrm{d}\phi_{\rm IR}\, p(\phi_{\rm IR}) |\phi_{\rm IR}\rangle\langle\phi_{\rm IR}\,|$$

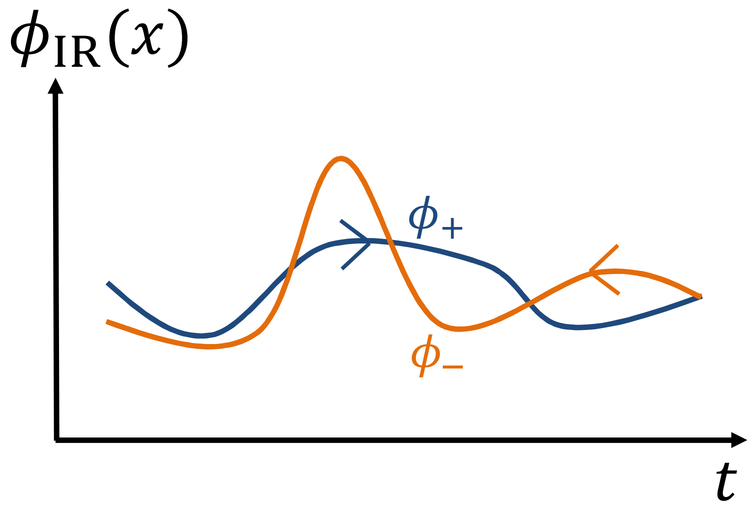
✓ Decoherence itself is well investigated in justifying the emergence of classical properties of primordial perturbations during inflation.

D. Polarski, C. Kiefer, A. A. Starobinsky *et al.*(’96,98,06,…)
C.P Burgess *et al.*(’14,16) E. Nelson *et al.*(’16)

Role of decoherence in classical stochastic interpretation

JT and T. Tanaka *work in progress*

- But its importance on the problem of IR secular effects is not discussed in detail so far (as far as I know).
- Decoherence can be also understood in the Schwinger-Keldysh formalism.



$\alpha \neq \alpha'$

$D(\alpha, \alpha') \neq 0 \Leftrightarrow$ Coherence exists.

$$D(\alpha, \alpha') \equiv \int \mathcal{D}\phi_+ \delta(\{\phi_+\} - \alpha) \mathcal{D}\phi_- \, \delta(\{\phi_-\} - \alpha') e^{i\Gamma}$$

The contribution from two field trajectories α and α' to the path integral.

Classical stochastic picture and field trajectories

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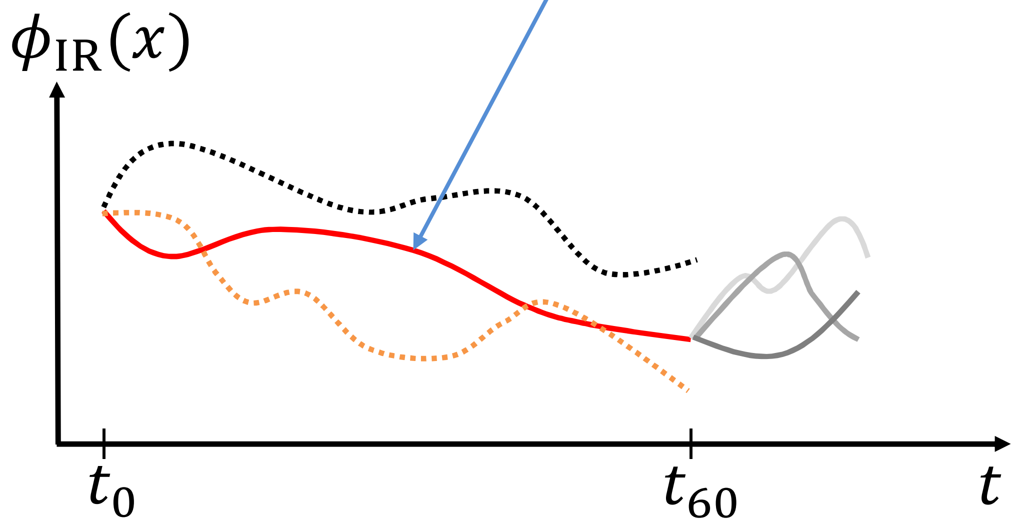
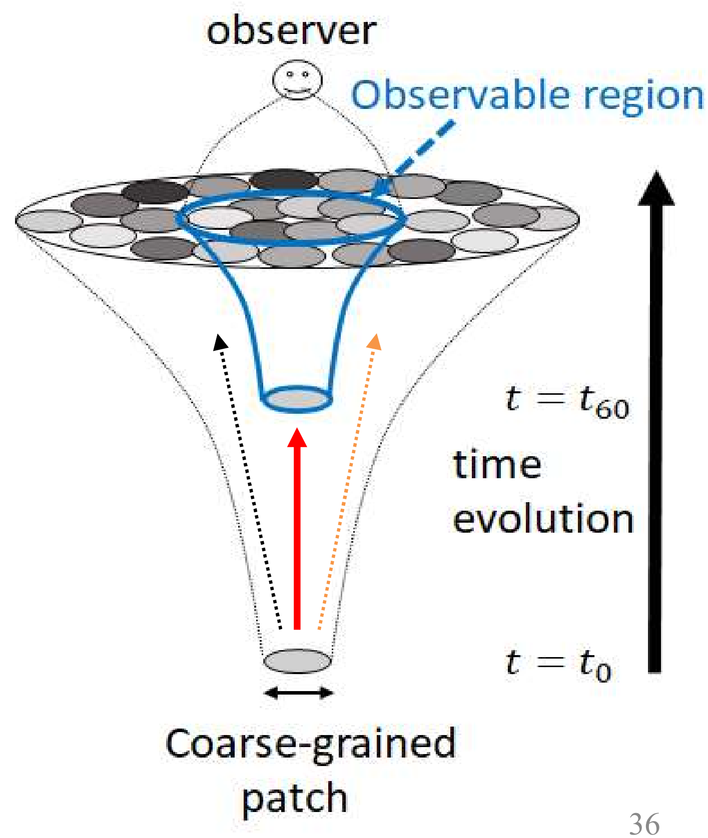
- In this picture, for deep IR modes, local observer can observe only one particular realization history of $\phi_{\vec{k}}$.





Probability assigned to each trajectory α:
$$P(\alpha) = D(\alpha, \alpha).$$

Role of decoherence in classical stochastic interpretation

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- For the validity of this classical stochastic picture,

sum rule $$P(\bar{\alpha}) = \sum_{\alpha\in\bar{\alpha}} P(\alpha)$$ $\bar{\alpha}$: coarse-grained trajectory

should be satisfied.

- However, when decoherence does not occur, the above sum rule is violated, because

$$P(\bar{\alpha}) \equiv D(\bar{\alpha},\bar{\alpha}) = \sum_{\alpha\in\bar{\alpha}} P(\alpha) + \sum_{\alpha,\alpha'\in\bar{\alpha},\alpha\neq\alpha'} D(\alpha,\alpha') \neq \sum_{\alpha\in\bar{\alpha}} P(\alpha)$$

Role of decoherence in classical stochastic interpretation

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$$P(\bar{\alpha}) \equiv D(\bar{\alpha},\bar{\alpha}) = \sum_{\alpha\in\bar{\alpha}} P(\alpha) + \sum_{\alpha,\alpha'\in\bar{\alpha},\alpha\neq\alpha'} D(\alpha,\alpha') \neq \sum_{\alpha\in\bar{\alpha}} P(\alpha)$$

- It is known that decoherence during inflation would not be exact.
- This violation of the sum rule will quantify the degree of the ``error'' of the classical stochastic interpretation of IR secular effects.