Quantum Diffusion During Inflation and Primordial Black Holes

Chris Pattison

Based on arXiv:1707.00537 (JCAP10 2017 046)
Collaboration with David Wands, Vincent Vennin and Hooshyar Assadullahi

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Outline

- Primordial black holes (brief introduction!)
- Introduction to stochastic-δN inflation
- Characteristic function formalism
- Summary

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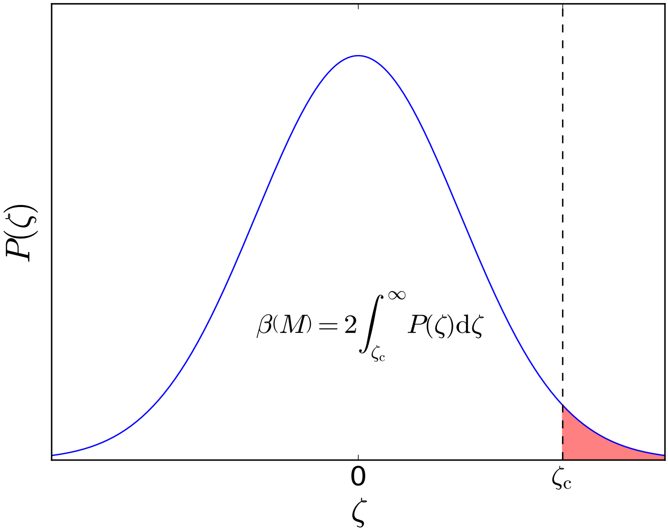
$$\beta\left[M\left(\phi\right)\right] = 2\int_{\zeta_{\rm c}}^{\infty} P\left(\zeta, \phi\right) {\rm d}\zeta\,.$$

This gives the mass fraction of the universe contained in PBHs.

Gaussian Example

It is typically assumed ζ has a Gaussian distribution.



Inflation

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This gives simplified equation of motion

$$\dot{\phi}_{\rm SR} \simeq -\frac{V'(\phi)}{3H}\,.$$

Stochastic Formalism

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Then ϕ is described by a Langevin equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi\left(N\right) ,$$

where $\langle\xi\left(N\right)\rangle = 0$ and $\langle\xi\left(N\right)\xi\left(N'\right)\rangle = \delta\left(N - N'\right)$, $k < aH$ and $N = \int H\mathrm{d}t$.

Inflaton evolves under Langevin equation until ϕ reaches ϕ_{end} where inflation ends.

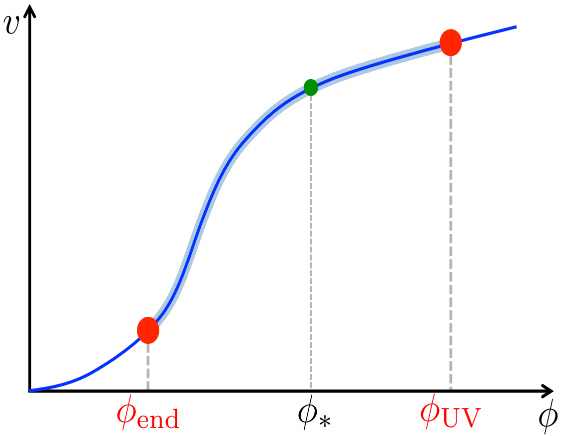

Figure 1: A reflective wall is added at ϕ_{uv} to prevent the field from exploring arbitrarily large values.

Separate Universe (Wands et al, 2000)

The primordial curvature perturbation ζ is

$$\zeta(t, \mathbf{x}) = N(t, \mathbf{x}) - N_0(t) \equiv \delta N \,,$$

where N is the local number of e-folds of inflation, and N_0 is the amount of expansion in an unperturbed universe.

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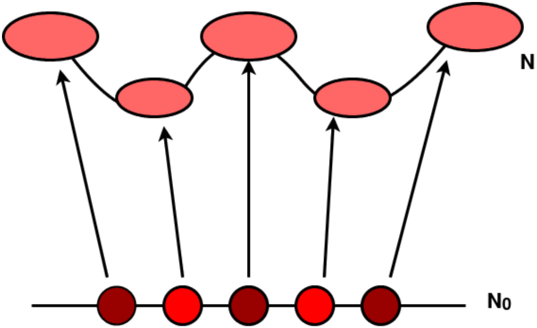

Figure 2: N_0 is at a zero curvature surface, final slice is constant density.

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The stochastic formalism treats the number of e-folds N as a random variable, denoted $\mathcal{N}$.

This reduces calculating curvature perturbations to calculating statistics of $\mathcal{N}$ realised under Langevin equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi\left(N\right) .$$

Characteristic Function Formalism (Pattison et al, 2017)

We want to know about **all** the moments of $\mathcal{N}$, and set $f_n(\phi) = \langle \mathcal{N}^n(\phi) \rangle$. Characteristic function $\chi_{\mathcal{N}}(t, \phi)$ is

$$\begin{aligned} \chi_{\mathcal{N}}(t, \phi) &= \left\langle e^{it\mathcal{N}(\phi)} \right\rangle \\ &= \sum_{n=0}^{\infty} \frac{(it)^n}{n!} f_n(\phi) \, . \end{aligned}$$

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$$= \sum_{n=0}^{\infty} \frac{(it)^n}{n!} f_n(\phi) \, .$$

We derive an ODE to solve for $\chi_\mathcal{N}$ and then find the PDF $P(\mathcal{N},\phi)$ by an inverse Fourier transform.

Simple Example

As an example, we take the potential

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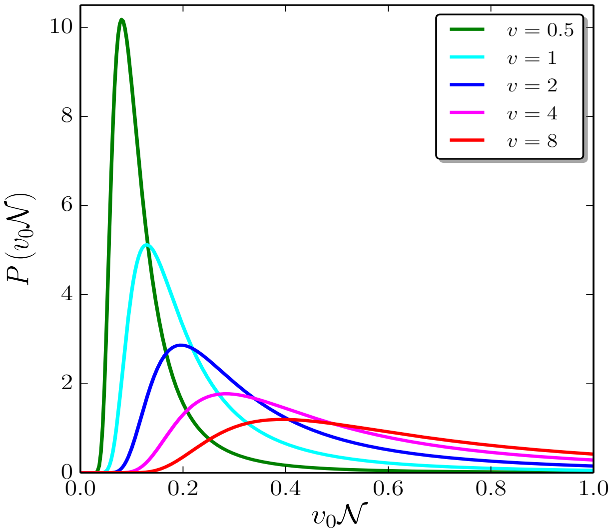

Figure 3: Plot of the PDF of $\mathcal{N}$ against $\mathcal{N}$, for the potential $v(\phi) = v_0 \left(\frac{\phi}{M_{\rm Pl}}\right)^2$.

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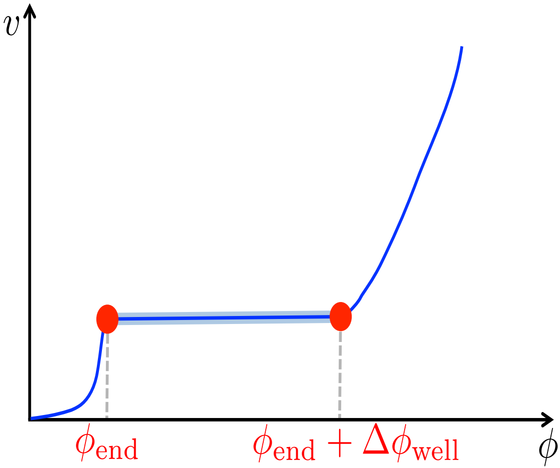
Inflationary models that produce $\zeta > \zeta_c$ can be approximated by a flat potential at the end of inflation, so $v \simeq v_0$. For $v = v_0$, we can solve for $\chi_{\mathcal{N}}$ exactly.



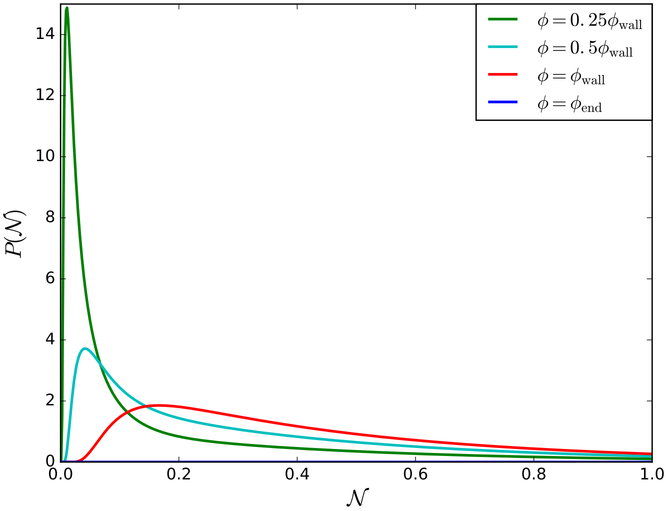


Figure 4: The PDF we obtain for a flat potential.

Summary

- The stochastic-$\delta\mathcal{N}$ formalism is needed to analyse curvature perturbations and PBH formation.
- We developed a characteristic function formalism to calculate the PDF of large fluctuations.
- Large non-Gaussianities are possible and can impact PBH production.

Future Work

- Stochastic inflation needs to be extended beyond slow-roll (Biagetti et al 1804.07124, Ezquiaga et al 1805.06731)
  - We have started a USR background analysis (Pattison et al 1806.09553)
- Study higher-order corrections to the tail of the Gaussian, even in the classical case
- Extend the formalism to include multi-field inflation.