Shell-crossing structure of cold dark matter with Lagrangian perturbation theory

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arXiv:1805.08787 (S.S, Atsushi Taruya, Stéphane Colombi)
COSMO18, Institute for Basic Science, Daejeon, 08/27-08/31

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Contents

1. Introduction

Λ**CDM** model

Cold Dark Matter:

- Initially **negligible velocity dispersion**
  - ➢ single-stream flow = fluid approximation is valid.
- Breakdown of single-stream flow at **shell-crossing**
  - ➢ singular density field
  - ➢ after the shell-crossing, cold dark matters undergo multi-stream flow

$\delta \lesssim 1$: Quasi non-linear regime ↔ single-stream flow
$\delta \gtrsim 1$: Non-linear regime ↔ multi-stream flow

On small scales, **multi-stream flow** is important.

1.1 How much do we know multi-stream flow?

Simulation:

ColDICE: 6D Vlasov-Poisson solver

T.Sousbie and S.Colombi [1509.07720]

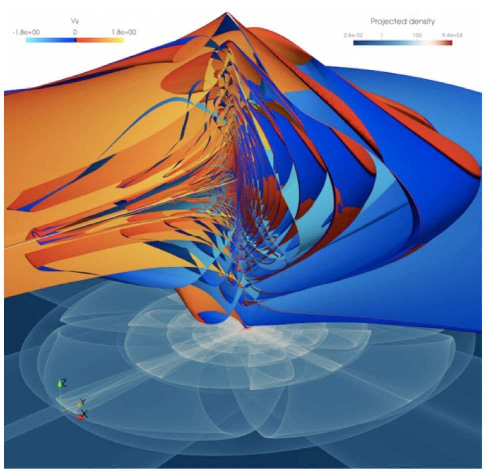
ScM: 4D Schrödinger-Poisson solver

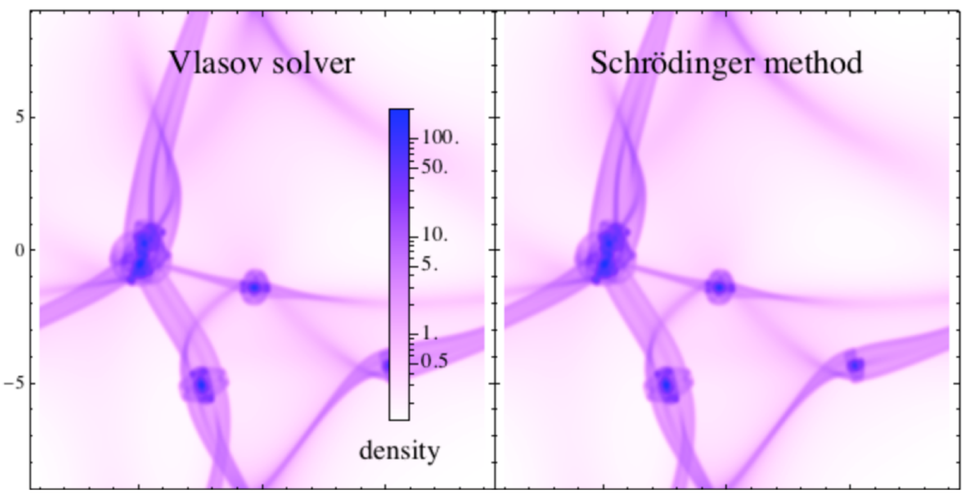
Solve the Schrödinger-Poisson equation: wave function ψ

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV(\boldsymbol{r})\psi \qquad \nabla^2 V = 4\pi G\psi\psi^*$$

$$f \sim |\psi|^2 + \boxed{\hbar \text{ dependent term}}$$

M.Kopp, K.Vattis, and C.Skordis [1711.00140]





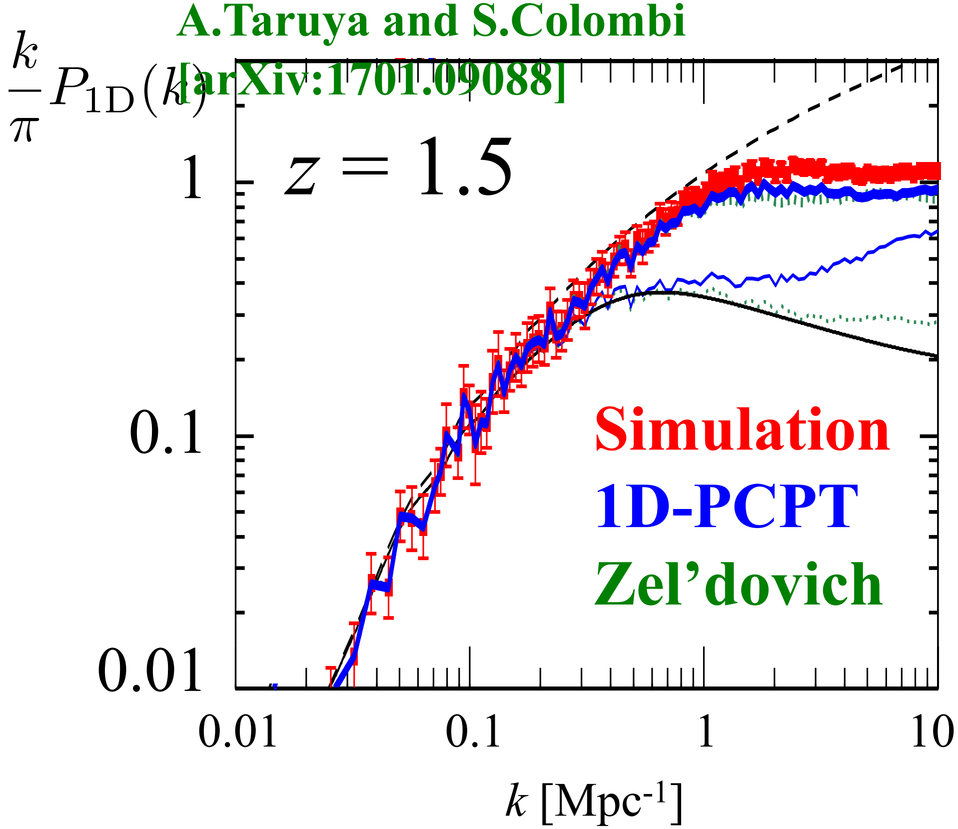
1.2 Why do we need Vlasov-Poisson solver?

https://vlasix.org/

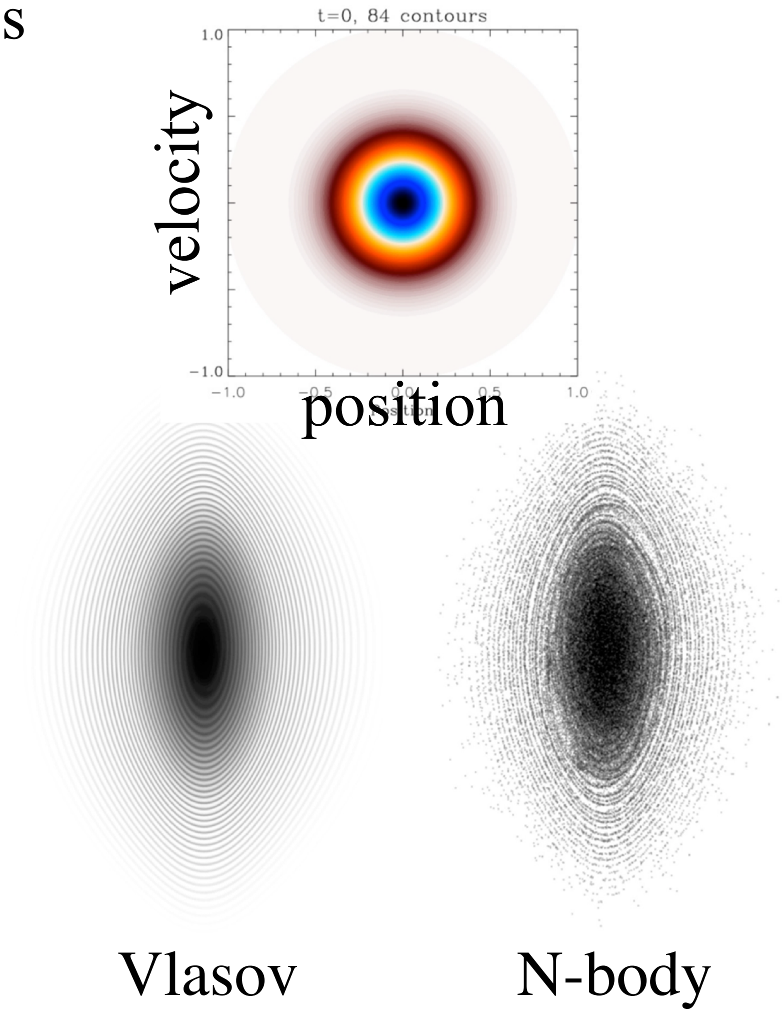
Solving in phase-space:

- ✓ including shell-crossing process
- ✓ without shot noise
- ✓ available in the sparse region

A.Taruya and S.Colombi [arXiv:1701.09088]



1D collapse



S.Colombi et al. [1504.07337]

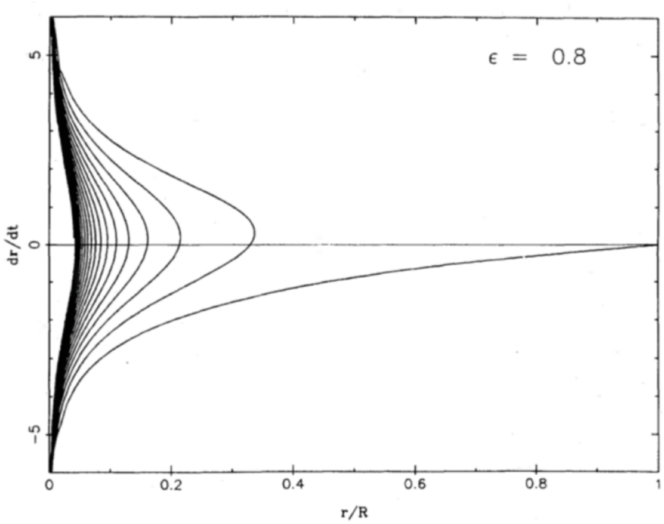
1.3 How much do we know multi-stream flow?

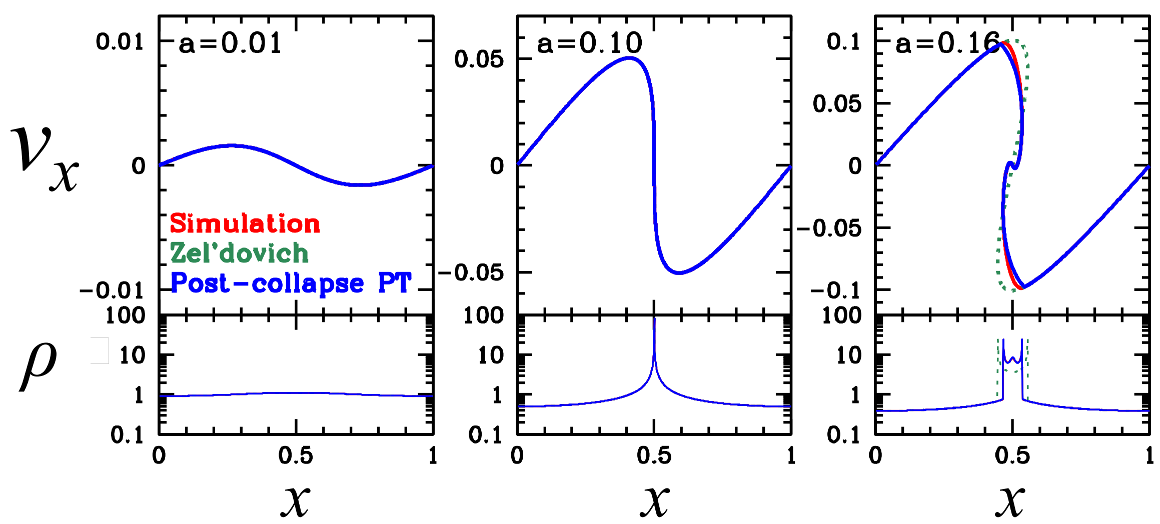
Theory:

- self-similar solution with a certain symmetry
  e.g., Planar (1D), Cylindrical (2D), Spherical (3D)

  J.A.Fillmore and P.Goldreich [ApJ,281(1984)]

- Post-collapse perturbation theory (1D)





1.4 Our work

(1D) Post-collapse perturbation theory (1D-PCPT)

Pre-collapse

Zel'dovich solution

$$x(q,t) = q + \psi(q) D_+(t)$$

Post-collapse

1D-PCPT

based on **Zel'dovich solution**

(= post-collapse dynamics)

A.Taruya and S.Colombi [arXiv:1701.09088]

(2D/3D) Post-collapse perturbation theory (2D/3D-PCPT)

Pre-collapse

Perturbation

$x(q,t) =$ **This work**

Post-collapse

2D/3D-PCPT

based on **?**

(= post-collapse dynamics)

2 Lagrangian Perturbation Theory

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t)$$

Pressureless, Newtonian gravity:

Longitudinal part: $\nabla_x \cdot \left[\ddot{\boldsymbol{\Psi}} + 2H\dot{\boldsymbol{\Psi}}\right] = -4\pi G\bar{\rho}_{\mathrm{m}}\delta$

Transverse part: $\nabla_x \times \left[\ddot{\boldsymbol{\Psi}} + 2H\dot{\boldsymbol{\Psi}}\right] = 0$

Nonlinear equation for the displacement field $\boldsymbol{\Psi}$

T.Matsubara [arXiv:1505.01481]

Longitudinal part: $\left(\hat{\mathcal{T}} - 4\pi G\bar{\rho}_{\mathrm{m}}\right)\Psi_{i,i} = -\epsilon_{ijk}\epsilon_{ipq}\Psi_{j,p}\left(\hat{\mathcal{T}} - 2\pi G\bar{\rho}\right)\Psi_{k,q}$ **2D**

$\hat{\mathcal{T}} \equiv \frac{\partial^2}{\partial t^2} + 2H\frac{\partial}{\partial t}$ $\qquad - \frac{1}{2}\epsilon_{ijk}\epsilon_{pqr}\Psi_{i,p}\Psi_{j,q}\left(\hat{\mathcal{T}} - \frac{4\pi G}{3}\bar{\rho}\right)\Psi_{k,r}$ **3D**

Transverse part: $\hat{\mathcal{T}}\epsilon_{ijk}\Psi_{j,k} = -\epsilon_{ijk}\Psi_{a,j}\hat{\mathcal{T}}\Psi_{a,k}$ **2D, 3D**

$$\Psi_i = \nabla^{-2}\left[\nabla_i\Psi_{a,a} + \epsilon_{iab}\nabla_a\left(\epsilon_{bcd}\Psi_{c,d}\right)\right]$$

2.1 How to solve the recursion relations?

We need to perform time- and spatial-integration.

- **Time-part**

  Fastest-growing mode

$$\boldsymbol{\Psi}(\boldsymbol{q},\eta) = \sum_{n=1}^{\infty} D_+^n \boldsymbol{\Psi}^{(n)}(\boldsymbol{q}) = \sum_{n=1}^{\infty} e^{n\eta} \boldsymbol{\Psi}^{(n)}(\boldsymbol{q})$$

- **Spatial-part**

  Specific initial condition (three crossed initial sine waves)

$$\boldsymbol{\Psi}^{(1)}(\boldsymbol{q},t) = D_+(t) \begin{pmatrix} \epsilon_x & \sin q_x \\ \epsilon_y & \sin q_y \\ \epsilon_z & \sin q_z \end{pmatrix}$$

We finally obtain LPT solutions

up to 10th order.

3 Results

<u>ColDICE</u>: Vlasov-Poisson solver

T.Sousbie and S.Colombi [1509.07720]

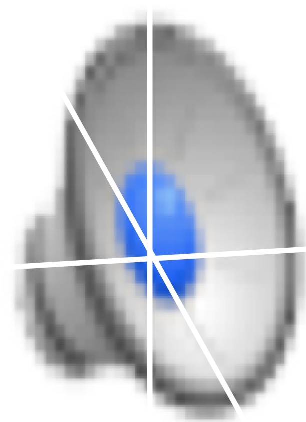
https://vlasix.org/

$$\frac{\partial f(t,\boldsymbol{x},\boldsymbol{u})}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla} f(t,\boldsymbol{x},\boldsymbol{u}) - \boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla} f(t,\boldsymbol{x},\boldsymbol{u}) = 0$$

$$\nabla^2\phi = 4\pi G\int \mathrm{d}^3\boldsymbol{u} f(t,\boldsymbol{x},\boldsymbol{u})$$

Initial condition:

$$\boldsymbol{\Psi}^{(1)}(\boldsymbol{q},t) = D_+(t)\begin{pmatrix} \epsilon_x & \sin q_x \\ \epsilon_y & \sin q_y \\ \epsilon_z & \sin q_z \end{pmatrix}$$



Q1D

$(\varepsilon_x, \varepsilon_y, \varepsilon_z) = (-24, -4, -3)$

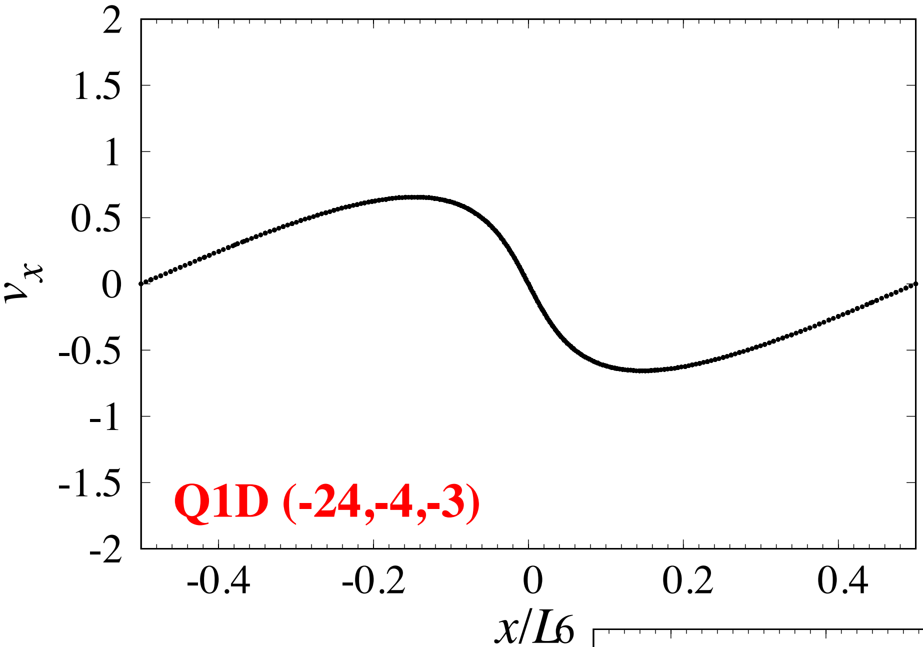
ASY

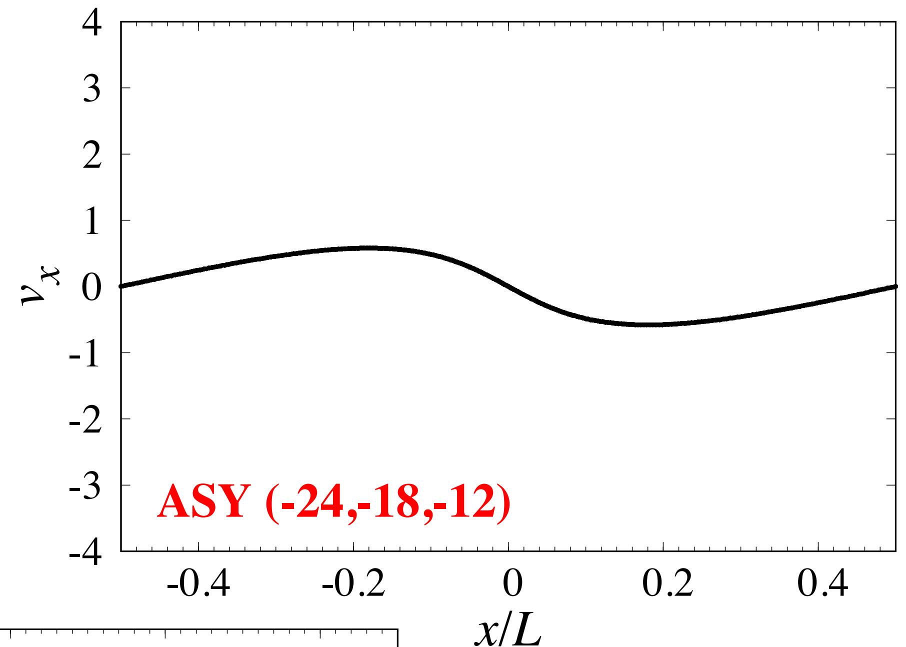
$(\varepsilon_x, \varepsilon_y, \varepsilon_z) = (-24, -18, -12)$

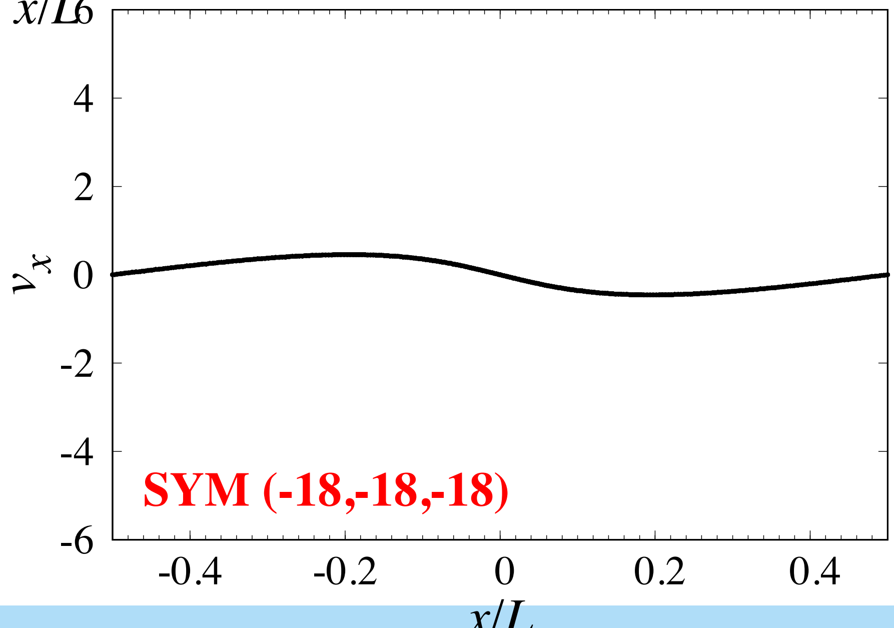
SYM

$(\varepsilon_x, \varepsilon_y, \varepsilon_z) = (-18, -18, -18)$

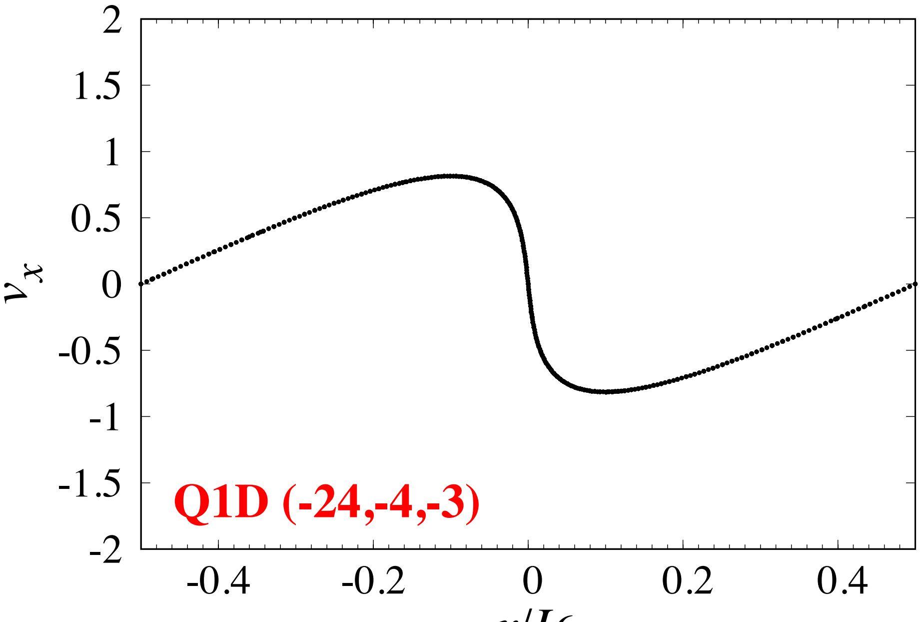
3.1 Vlasov-Poisson simulation (Pre-collapse)

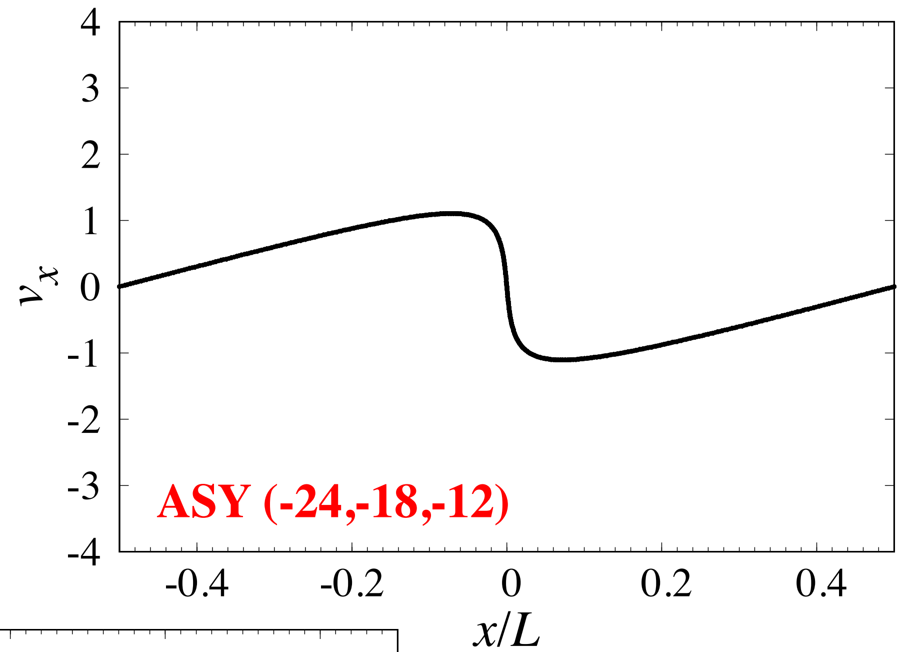


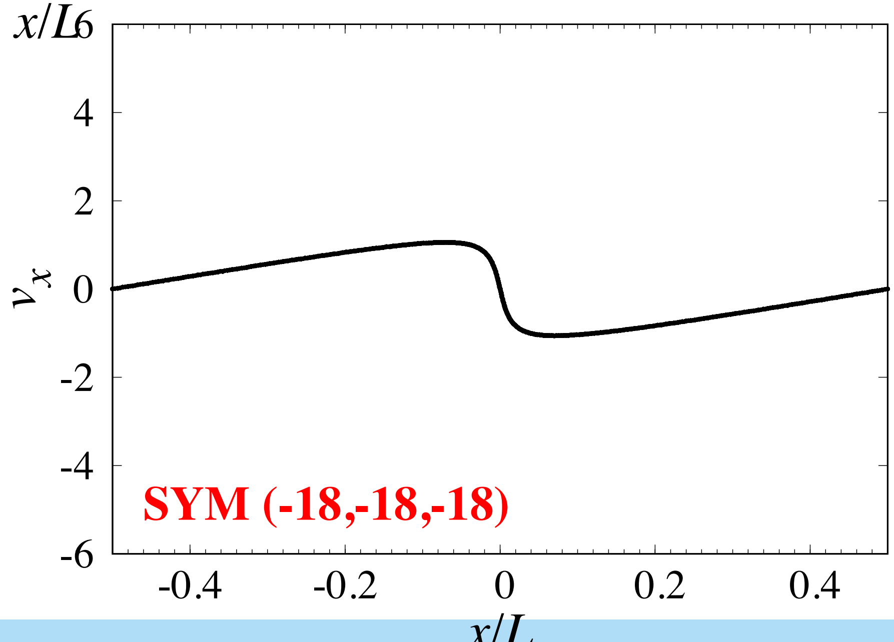




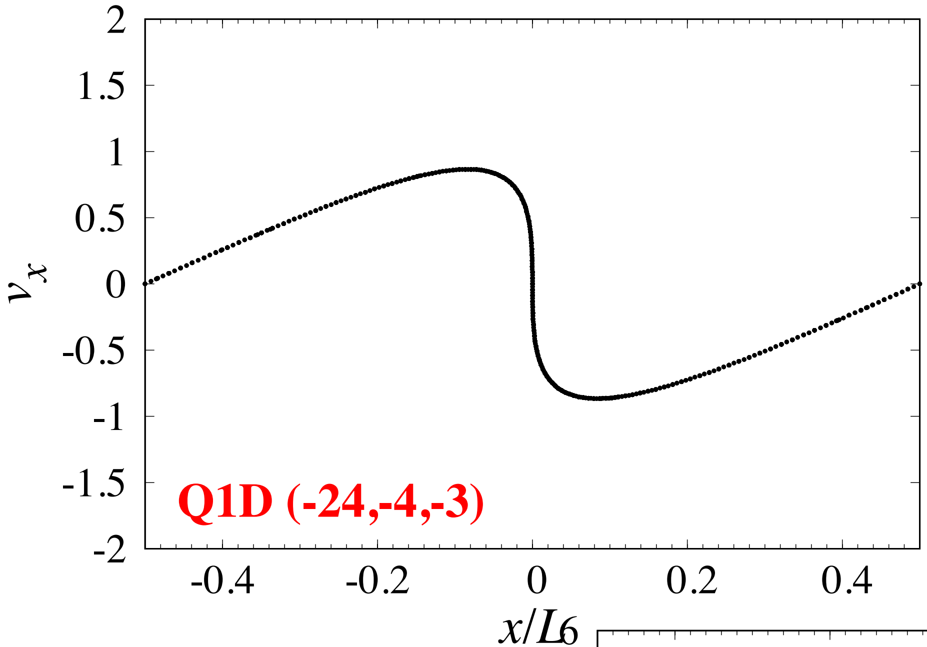
3.1 Vlasov-Poisson simulation (Pre-collapse)

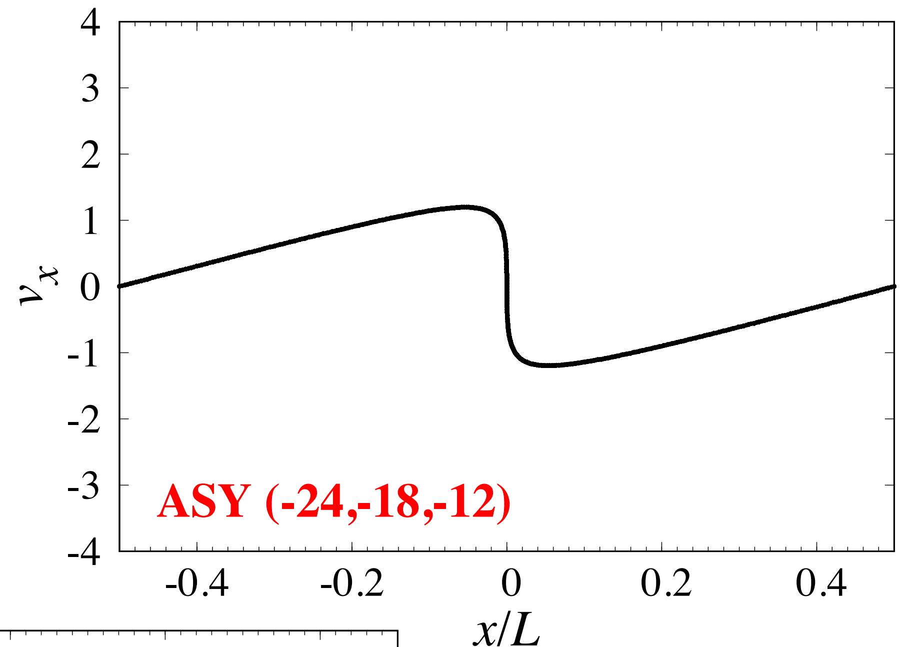


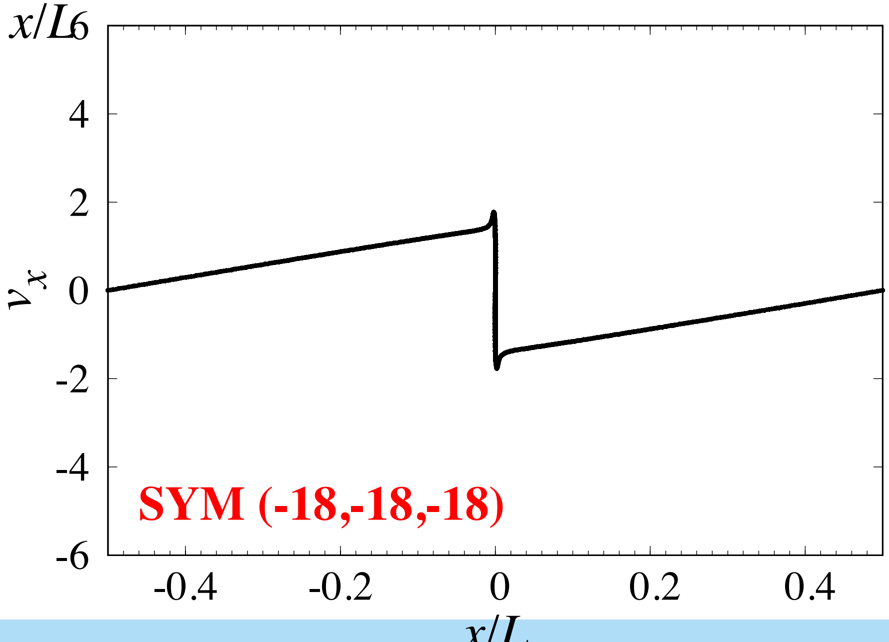




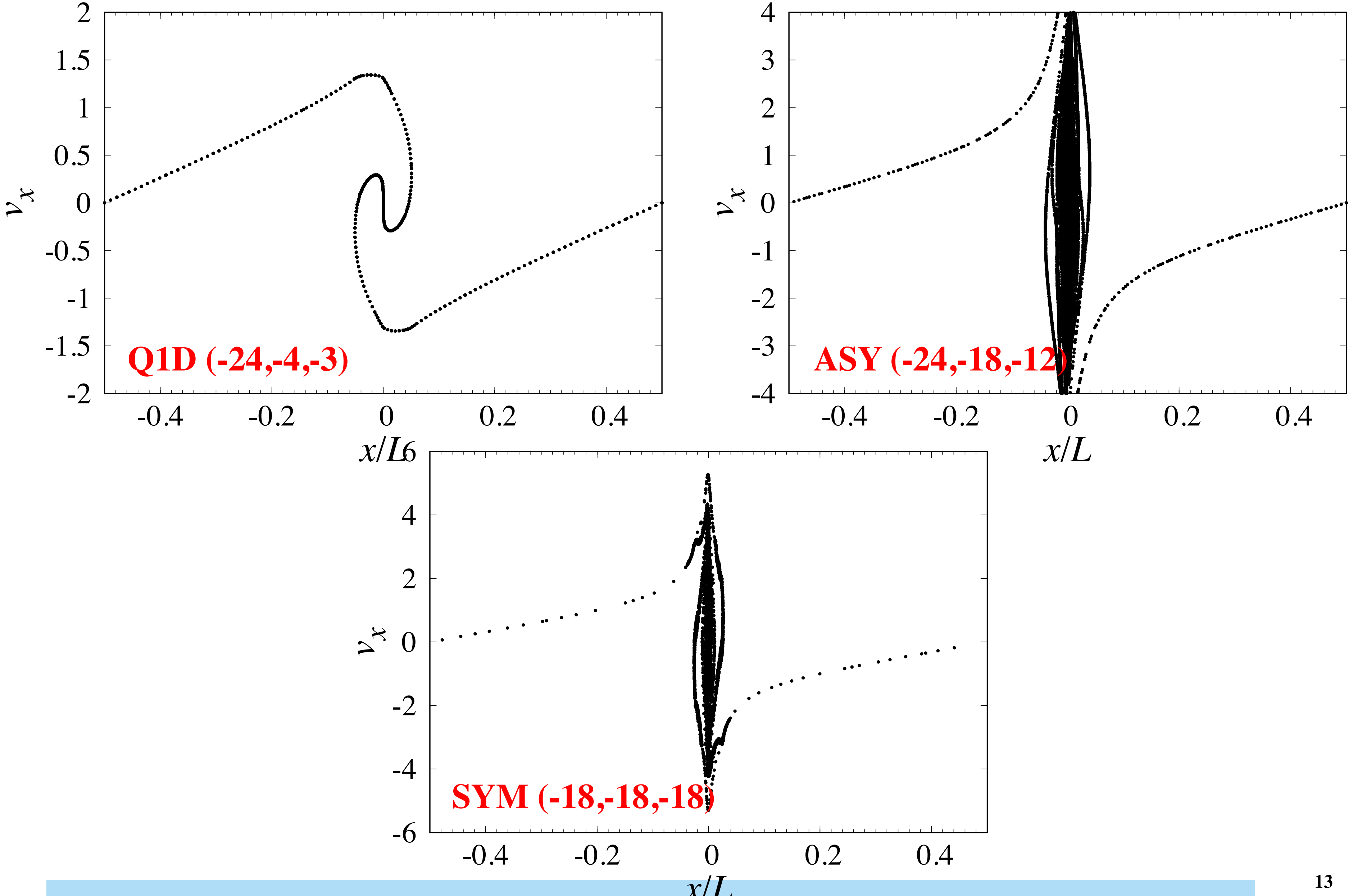
3.1 Vlasov-Poisson simulation (Shell-crossing)



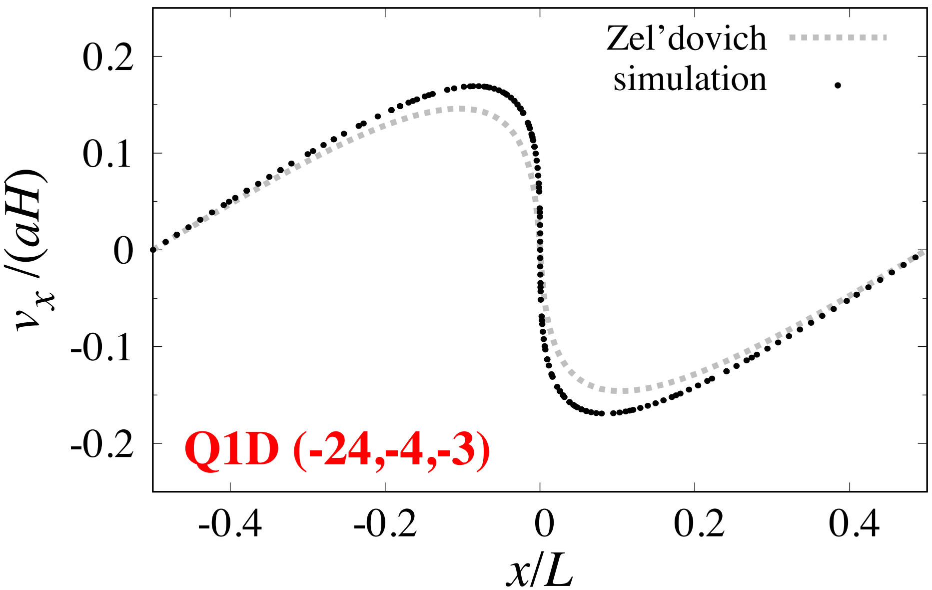


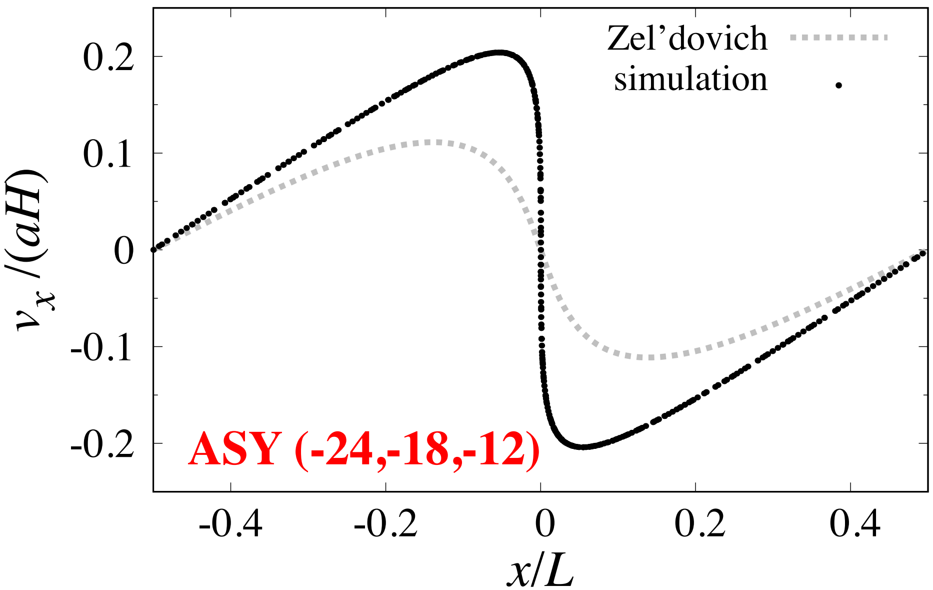


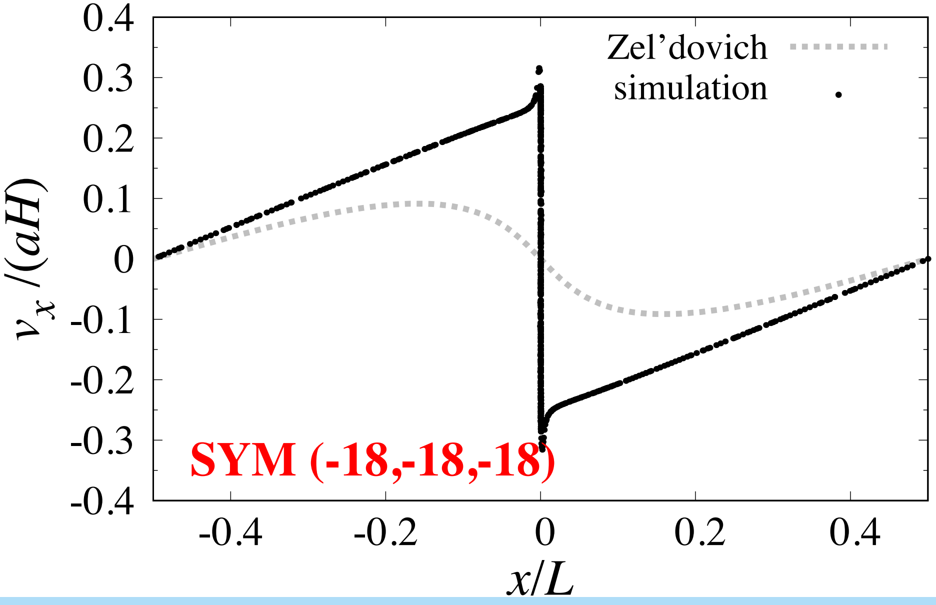
3.1 Vlasov-Poisson simulation (Post-collapse)



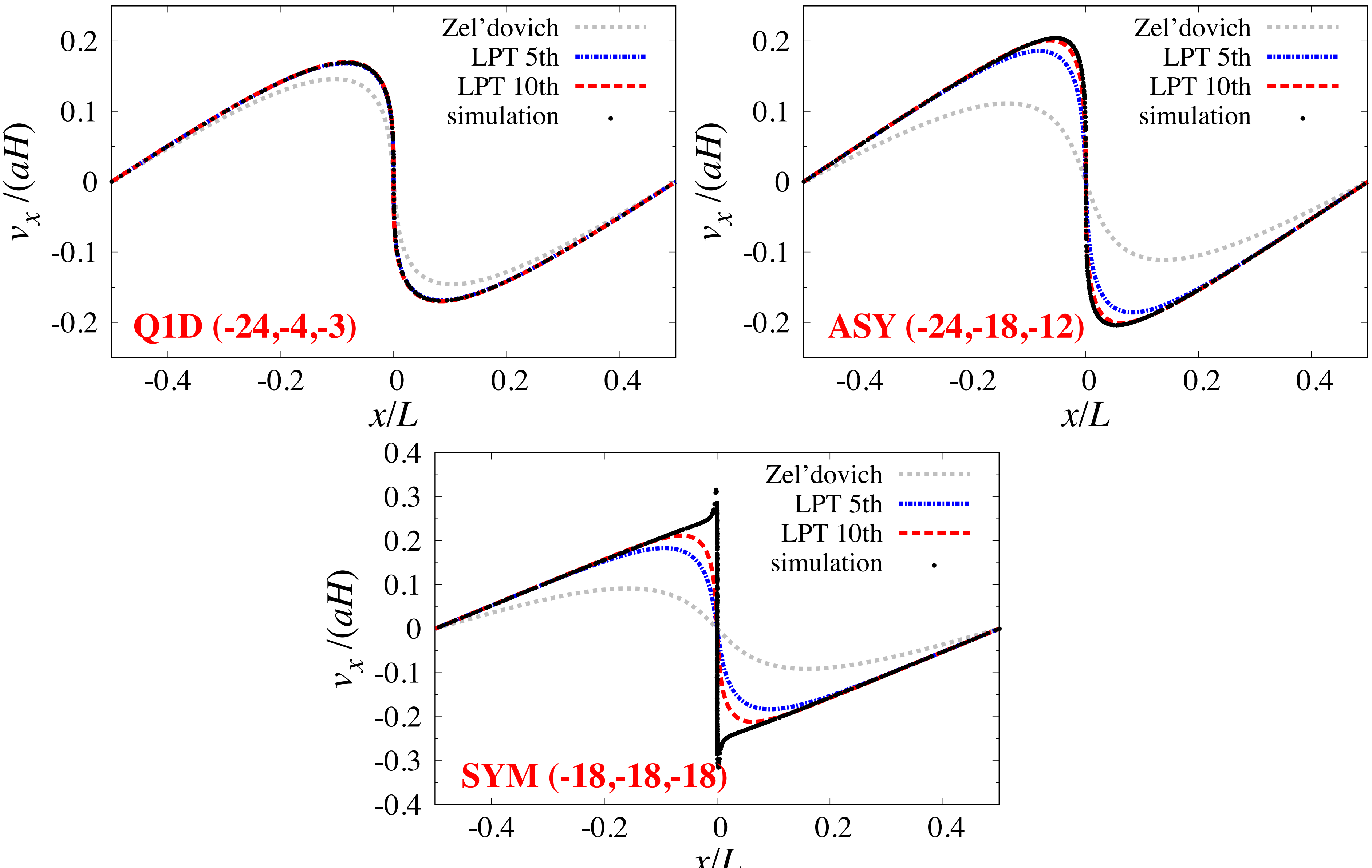
3.2 Zel'dovich approximation at shell-crossing







3.3 10th LPT at shell-crossing



3.4 Fitting function and Extrapolation

We found the fitting formula $x(\boldsymbol{q}, n)$ as a function of order n.

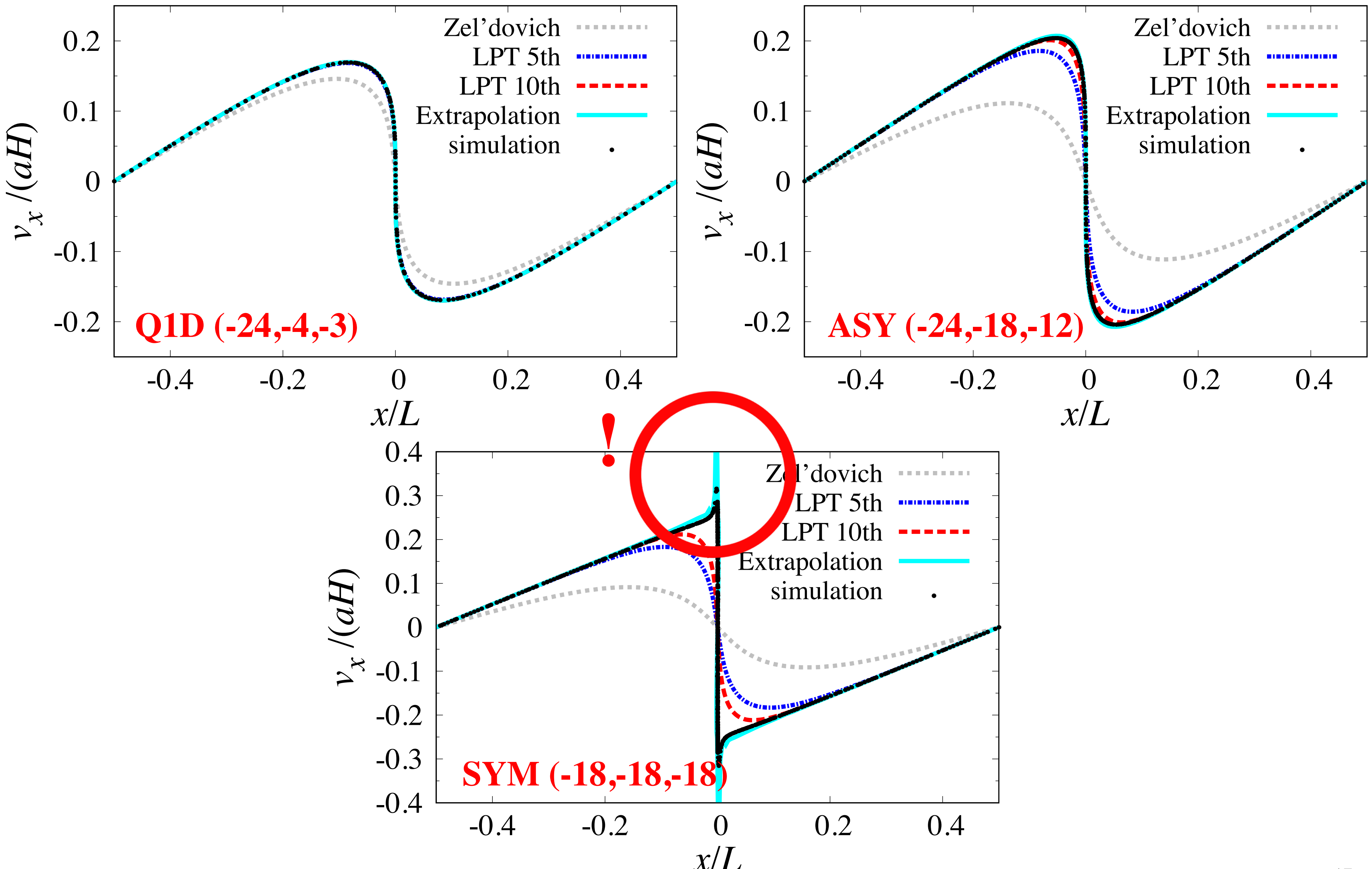
$$x_i(\boldsymbol{q}, t, n) = q_i + \sum_{a=1}^{n} \Psi_i^{(a)}(\boldsymbol{q}, t)$$

$$= a_i(\boldsymbol{q}, t) + \frac{1}{b_i(\boldsymbol{q}, t) + c_i(\boldsymbol{q}, t) \exp\left[d_i(\boldsymbol{q}, t) n^{e_i(\boldsymbol{q}, t)}\right]}$$

(in an empirical way)

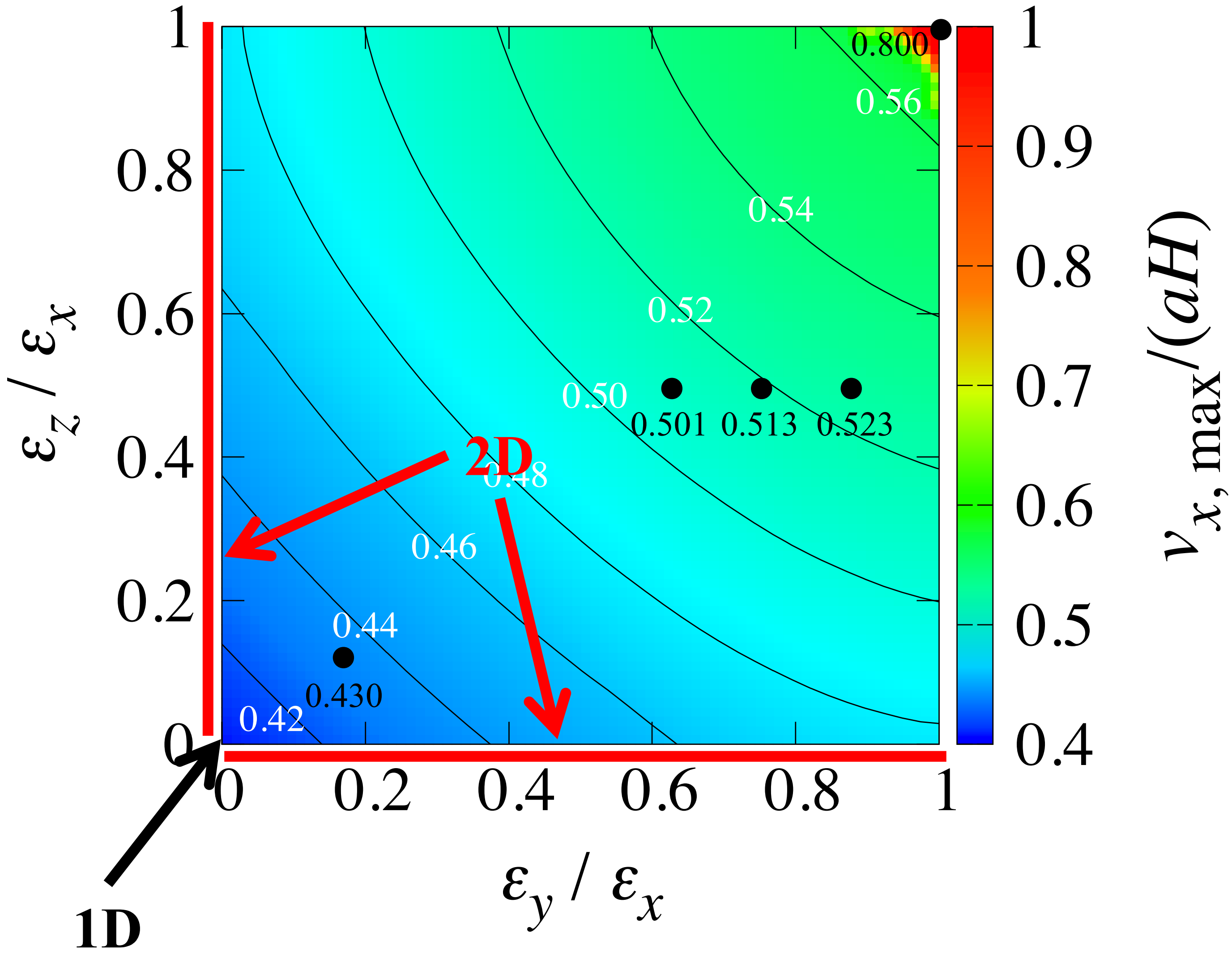
$n \longrightarrow \infty$: “Effective” infinite-order LPT

3.5 Extrapolation at Shell-crossing



3.6 Exploring parameter space@Shell-crossing

Focusing on maximum velocity as a function of $(\varepsilon_y/\varepsilon_x, \varepsilon_z/\varepsilon_x)$.



4. Summary

- We consider primordial dark matter halos seeded by three crossed initial sine waves.

- We perform **10th order LPT and beyond**.

- Except for rare cases such as the triaxial symmetric configuration, **collapse is generally expected to produce a planar singularity.**

1D → 2D: No transitions
2D → 3D: Spiky feature appears!