UNIVERSITÉ
DE GENÈVE

COSMO-18
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IBS Science and Culture Center
Daejeon, Korea

Topics on primordial black holes from inflation

Gabriele Franciolini

Aug – 2018

Outline

- ***Primordial Black holes:*** *(short review)*

- ***Non-Gaussianities:***

based on: «*Primordial Black Holes from Inflation and non-Gaussianity*»

[arXiv:1801.09415] ***GF***, A. Kehagias, S. Matarrese, A. Riotto

Primordial Black Holes

- ***Hawking & Carr, 1974***
  "Black holes in the early universe"

- ***Chapline, 1975***
  "Cosmological effects of primordial black holes"

- ***Meszaros, 1975***

- ***...***

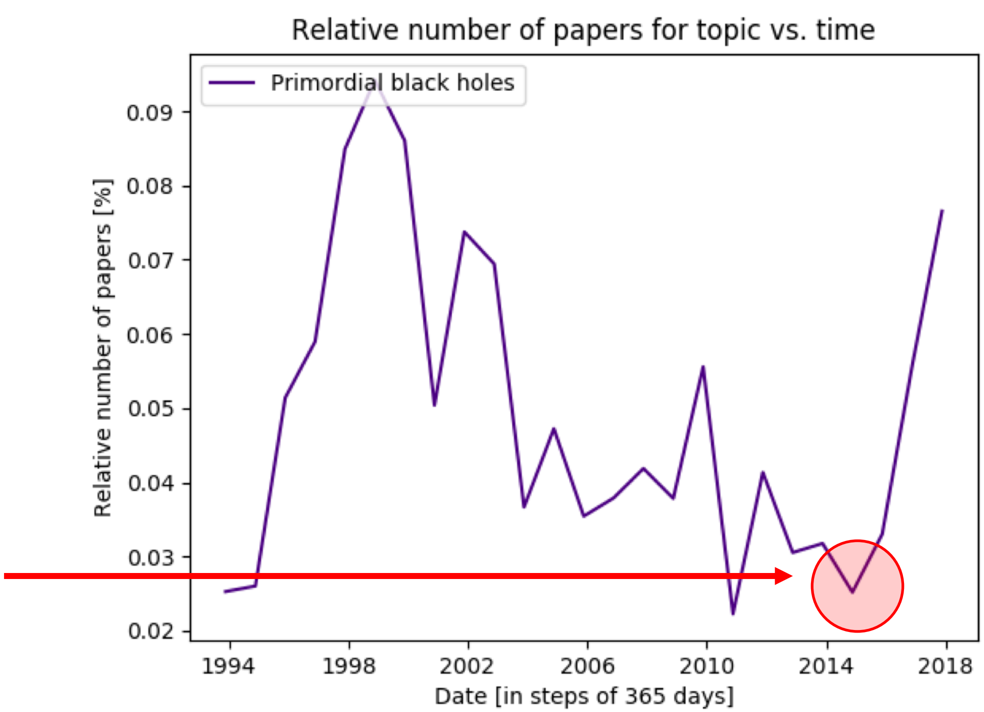
2015

Did LIGO detect dark matter?

Simeon Bird,* Ilias Cholis, Julian B. Muñoz, Yacine Ali-Haïmoud, Marc Kamionkowski, Ely D. Kovetz, Alvise Raccanelli, and Adam G. Riess[1]

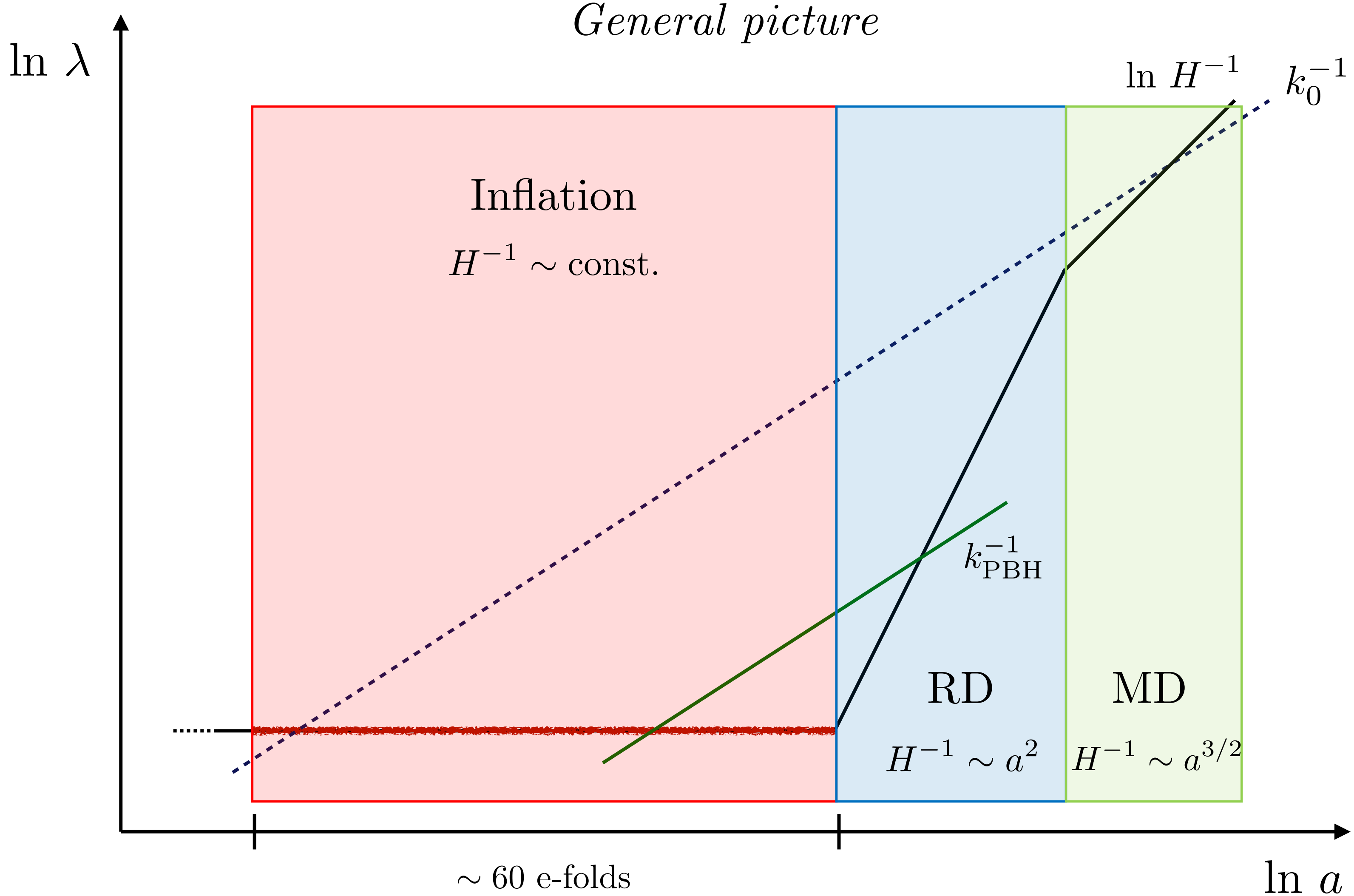
[1] *Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles St., Baltimore, MD 21218, USA*

[Source: www.benty-fields.com/trending]



Primordial Black Holes

General picture

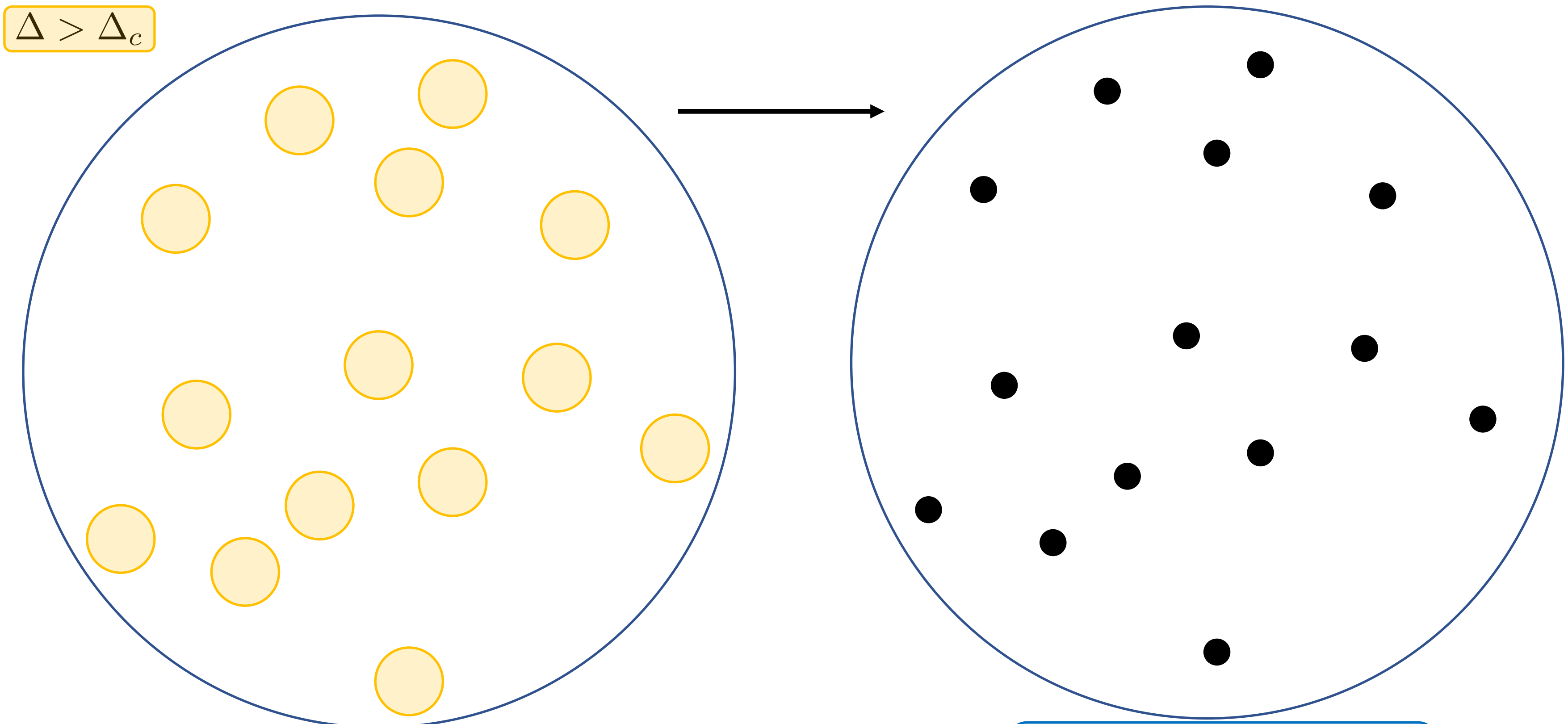


Primordial Black Holes

Density contrast $= \Delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$

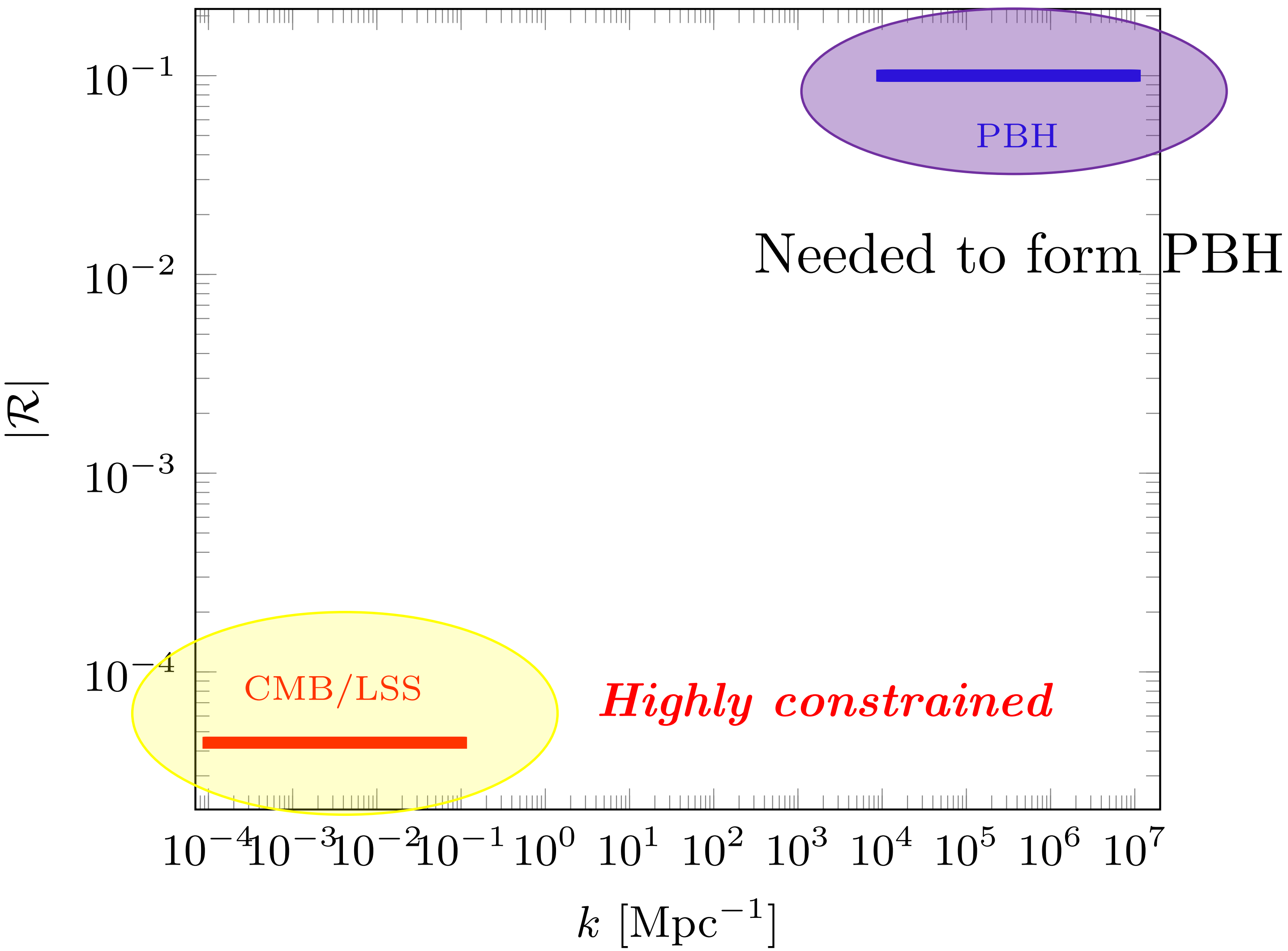
After horizon re-entry

$\boxed{\Delta > \Delta_c}$



$$\boxed{M_{\rm PBH} \sim \gamma \frac{4\pi}{3} \rho_{\rm form} H_{\rm form}^{-3}}$$

Primordial Black Holes



Primordial Black Holes

- ***Beta fraction:*** mass fraction of PBH ***at formation***

$$\beta = \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}}\Big|_{\text{formation}} = \left(\frac{H_0}{H_{\text{form}}}\right)^2 \left(\frac{a_{\text{form}}}{a_0}\right)^{-3} \Omega_{\text{CDM}} f_{\text{PBH}}$$

Fraction of DM in PBH

Density parameter

- ***Press–Schechter*** formalism for beta:

$$\beta = \int_{\Delta_c}^{\infty} \text{P}(\Delta)\text{d}\Delta \longleftarrow \text{P}(\Delta) = \frac{1}{\sqrt{2\pi}\sigma_\Delta} \exp\left[-\frac{\Delta^2}{2\sigma_\Delta^2}\right] \quad (*)$$

Probability distribution for the density contrast perturbations

(under the assumption of gaussian probability)

- Beta fraction can be regarded as the probability that the density contrast is larger than the threshold for the PBH

$$\sigma_\Delta^2(R_H) = \frac{16}{81}\int_0^\infty \text{d}\ln q (qR_H)^4 W^2(qR_H)\mathcal{P}_\mathcal{R}(q) \qquad \Delta(\vec{x}) = \frac{4}{9a^2H^2}\nabla^2\mathcal{R}(\vec{x})$$

Volume normalized window function: select the modes on scales of horizon re-entry

Impact of Non-Gaussianities

Impact of Non-Gaussianities

With the assumption of gaussian statistics:

$$\boxed{P(\zeta_{R_H} > \zeta_c) = \beta_{\rm prim}(M) = \int_{\zeta_c} \frac{\mathrm{d}\zeta_{R_H}}{\sqrt{2\pi}\,\sigma} e^{-\zeta_{R_H}^2/2\sigma^2}}$$

if we define $\nu(M) = \dfrac{\zeta_c}{\sigma(M)}$ $\xrightarrow{\nu \gg 1}$ $\beta_{\rm prim}(M) \simeq \sqrt{\dfrac{1}{2\pi\nu^2}} e^{-\nu^2/2}$

(complementary error fraction)

How can we relax the ***assumption of gaussian statistic*** and include the effects on ***non-gaussianity*** in the calculation of ***beta***?

$$\boxed{P(\zeta_R > \zeta_c) = \frac{1}{\sqrt{2\pi\nu^2}} \exp\left\{-\nu^2/2 + \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \xi_R^{(n)}(0)\,(\zeta_c/\sigma_R^2)^n\right\}} \qquad \nu \gg 1$$

Increase / decrease the impact of tails of the PDF

Criterion for the impact of ***non-gaussianities***

Cumulants:
$$S_n = \frac{\xi^{(n)}_{\nu,R_H}(0)}{\left(\xi^{(2)}_{\nu,R_H}(0)\right)^{n-1}} = \frac{\langle \overbrace{\zeta_{R_H}(\vec{x}) \cdots \zeta_{R_H}(\vec{x})}^{n-\text{times}} \rangle}{\sigma^{2(n-1)}_{R_H}}$$

Define ***"fine tuning"*** parameter:
$$\Delta_n = \frac{\mathrm{d} \ln \beta_{\mathrm{prim}}(M)}{\mathrm{d} \ln S_n}$$

so that
$$\boxed{\frac{\beta^{\mathrm{NG}}_{\mathrm{prim}}(M)}{\beta^{\mathrm{G}}_{\mathrm{prim}}(M)} = e^{\Delta_n}}$$

In order ***not*** to have a significant impact of ***NG***:

$$\boxed{|\Delta_n| \leq 1} \longrightarrow \boxed{|S_n| \leq \left(\frac{\sigma_{R_H}}{\zeta_c}\right)^2 \frac{n!}{\zeta_c^{n-2}}}$$

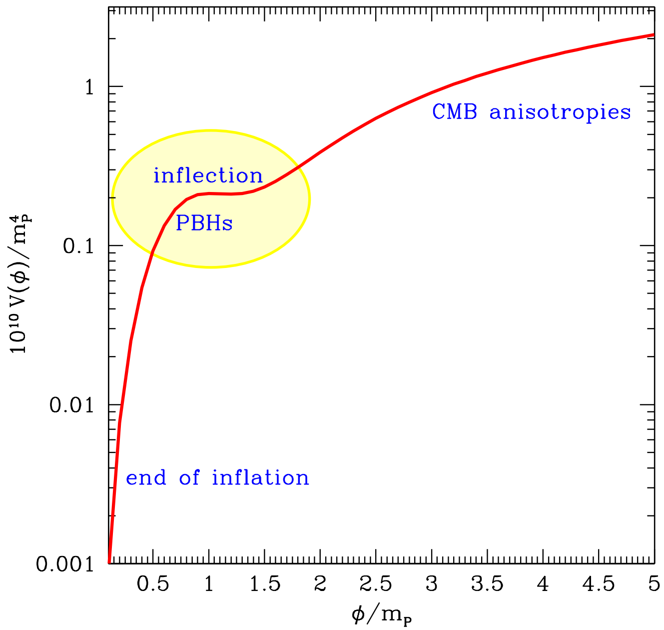
Impact of Non-Gaussianities

Single field models

No significant effect of the skewness if:

$$S_3 \leq 0.06 \left(\frac{\sigma^2_{R_H}}{0.02}\right)\left(\frac{1.3}{\zeta_c}\right)^3$$

Normalized to the approx. value of the variance in order to have the right abundance of ***DM*** ad PBH



In USR, using shift symmetry and dilation symmetry fixes the three-pt. function

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle = 3\cdot(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\left[P_\zeta(k_1)P_\zeta(k_2) + \text{two terms}\right]$$

[Finelli et al arXiv:1711.03737]

$$S_3 = \frac{1}{\sigma^4_{R_H}}\int\frac{\mathrm{d}^3k_1}{(2\pi)^3}\int\frac{\mathrm{d}^3k_2}{(2\pi)^3}\int\frac{\mathrm{d}^3k_3}{(2\pi)^3}W(k_1,R_H)W(k_2,R_H)W(k_3,R_H)\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle \simeq 9$$

Rough estimate of window function effect. With gaussian window function and spiky PS we checked that the result gets correction of a factor ~ 1

Multi-field models: axion-curvaton model

Inflaton Curvaton [Kawasaki et al, arXiv:1207.2550]

$$V(\phi_0) = \frac{1}{2}\lambda H^2 \phi_0^2 \quad V_\chi = \Lambda^4[1-\cos(\chi/f)] \sim \frac{1}{2}m_\chi^2\chi^2$$

$$\mathcal{P}_\zeta(k) = \mathcal{P}_{\zeta,\mathrm{infl}}(k) + \boxed{\mathcal{P}_{\zeta,\mathrm{curv}}(k)}$$

enhancement at small scales

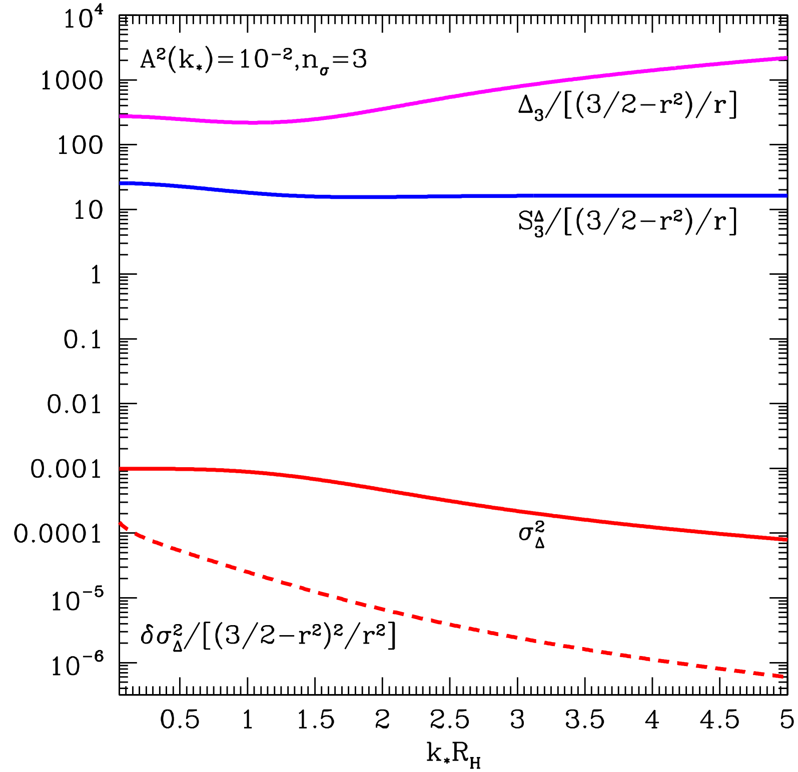
NG are induced at second order

$$\zeta_2(\vec{x}) = \zeta_1(\vec{x}) + \frac{1}{r}\left(\frac{3}{2} - r^2\right)(\zeta_1(\vec{x}))^2$$

$$r = \frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma}.$$

[Bartolo, Matarrese, Riotto, arXiv:0309033]

$$\boxed{S_3 \leq 0.06\left(\frac{\sigma^2_{R_H}}{0.02}\right)\left(\frac{1.3}{\zeta_c}\right)^3}$$



(parameter $k_* \equiv$ scale at which ϕ reaches the minimum)

Clustering properties

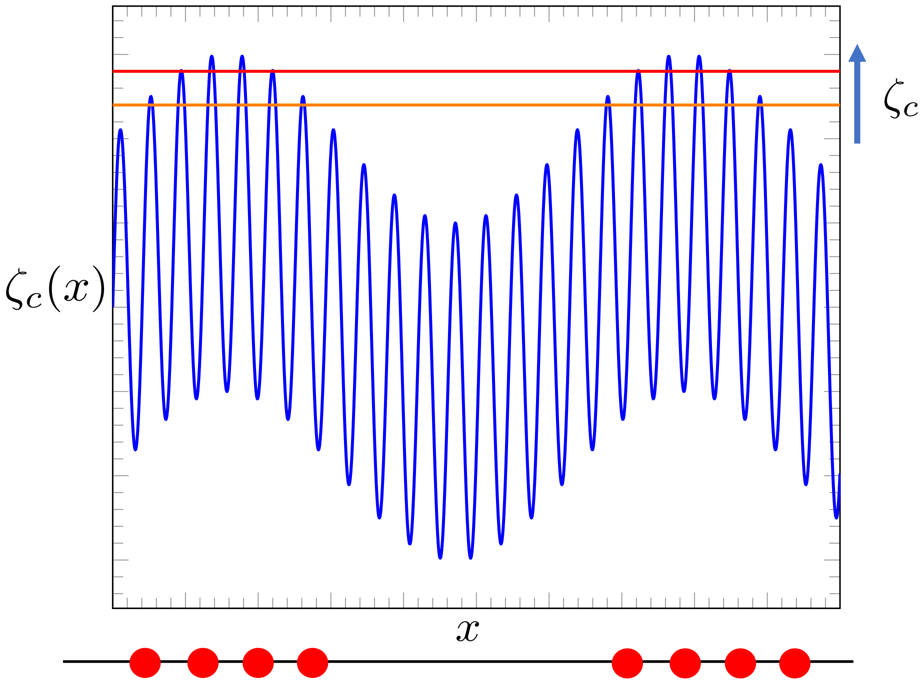
PBH two point function (bias model) at large separations:

$$\boxed{\xi^{(2)}_{\nu,R_H}(\vec{x}_1,\vec{x}_2) = -1 + \exp\left(\nu^2 w^{(2)}_{R_H}(\vec{x}_1,\vec{x}_2)\right)}$$

$$w^{(2)}_R(\vec{x}_1,\vec{x}_2) = \sigma_R^{-2}\,\xi^{(2)}_R(\vec{x}_1,\vec{x}_2)$$

H. D. Politzer and M. B. Wise, Astrophys. J. 285, L1 (1984)

$$\nu\ \omega^{(2)}_{R_H}(x_1,x_2) \ll 1$$



The two point function measures the ***excess probability*** (over ***random***) of finding pairs with a separation r

NG change clustering properties

For $\nu \gg 1$ the two point function (two point probability of over-density)

$$\boxed{\xi^{(2)}_{\nu,R}(\vec{x}_1,\vec{x}_2) = -1 + \exp\left\{\sum_{n=2}^{\infty}\sum_{j=1}^{n-1}\frac{\nu^n\sigma_R^{-n}}{j!(n-1)!}\xi_R^{(n)}\Bigg(\underbrace{\vec{x}_1,\cdots,\vec{x}_1}_{j-\text{times}},\underbrace{\vec{x}_2,\cdots,\vec{x}_2}_{(n-j)-\text{times}}\Bigg)\right\}}$$

$$\nu\ \omega^{(2)}_{R_H}(x_1,x_2) \ll 1$$

If we focus only to the three point function contribution

$$\xi^{(2)}_{\nu,R_H}(\vec{x}_1,\vec{x}_2) = -1 + \exp\left(\nu^2/\sigma^2_{R_H}\xi^{(2)}_{R_H}(\vec{x}_1,\vec{x}_2)\right)\exp\left(\nu^3/\sigma^3_{R_H}\xi^{(3)}_{R_H}(\vec{x}_1,\vec{x}_2,\vec{x}_2)+\cdots\right)$$

E.g. in the single field model of inflation (USR) we get:

$$\boxed{\frac{\nu^3}{\sigma^3_{R_H}}\xi^{(3)}_{R_H}(k) \sim 6\zeta_c\left[\frac{\nu^2}{\sigma^2_{R_H}}\xi^{(2)}_{R_H}(k)\right]}$$

$$\zeta_c \sim \mathcal{O}(1)$$

(in Fourier transform)

Result: *NG* must be taken into account!

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Thanks!

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Backup Slides

Impact of Non-Gaussianities

> A path-integral approach to large-scale matter distribution originated by non-Gaussian fluctuations
> S. Matarrese, F. Lucchin and S. A. Bonometto, Astrophys. J. 310, L21 (1986)

- Using threshold statistics, define peak over-density $\rho_{\nu,R}(\vec{x}) = \Theta(\zeta_R(\vec{x}) - \nu\sigma_R)$

- The joint probability of over-density in N points $(\vec{x}_1, \cdots, \vec{x}_N)$

$$\Pi^{(N)}_{\nu,R}(\vec{x}_1, \cdots, \vec{x}_N) = \left\langle \prod_{i=1}^{N} \rho_{\nu,R}(\vec{x}_i) \right\rangle = \int [D\zeta(\vec{x})] P[\zeta(\vec{x})] \prod_{i=1}^{N} \Theta\Big(\zeta_R(\vec{x}_i) - \nu\sigma_R\Big).$$

- The Heaviside step function can be written as $\Theta(\vec{x}) = \int_{-x}^{\infty} \mathrm{d}a \int_{-\infty}^{\infty} \frac{\mathrm{d}\phi}{2\pi}\, e^{i\phi a}$

- Identify the partition function $Z[J] = \exp[W[J]] = \int [D\zeta(\vec{x})] P[\zeta(\vec{x})] e^{i\int \mathrm{d}^3x J(\vec{x})\zeta(\vec{x})}$

  which is an expansion in the (smoothed) ***connected correlation function***

$$\xi^{(N)}_R(\vec{x}_1, \cdots, \vec{x}_N) = \int \xi^{(N)}_R(\vec{y}_1, \cdots, \vec{y}_n) \prod_{i=1}^{N} \mathrm{d}^3 y_i W(|\vec{x}_i - \vec{y}_i|, R) \qquad (\xi^{(2)}_R(0) = \sigma^2_R)$$

Impact of Non-Gaussianities

A path-integral approach to large-scale matter distribution originated by non-Gaussian fluctuations
S. Matarrese, F. Lucchin and S. A. Bonometto, Astrophys. J. 310, L21 (1986)

Using path-integral techniques... exact formula for the joint probability

$$\Pi^{(N)}_{\nu,R}(\vec{x}_1, \cdots, \vec{x}_N)$$

- We are interested in the single-point probability

$$P(\zeta_R > \zeta_c) = \langle \rho_{\nu,R}(\vec{x}) \rangle$$

which determines the mass fraction of formation $\beta_{\rm prim}(M)$

$$\boxed{P(\zeta_R > \zeta_c) = \frac{1}{\sqrt{2\pi\nu^2}} \exp\left\{ -\nu^2/2 + \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \xi^{(n)}_R(0) \left(\zeta_c/\sigma^2_R\right)^n \right\}}$$

Valid under the only assumption that $\nu \gg 1$

Impact of Non-Gaussianities

An estimate of the skewness impact

- Using δN formalism ($\zeta(t,\mathbf{x}) = N(t,\mathbf{x}) - N(t) \equiv \delta N$, with $\tilde{a}(t,\mathbf{x}) = a(t)e^{\zeta(t,\mathbf{x})}$)

$$\zeta(\vec{x}) = \sum_{n\geq 1} \mathcal{N}_n (\zeta_1(\vec{x}))^n \qquad \mathcal{N}_n = \frac{3^{n-1}}{n}$$

[Sasaki et al, arXiv:1712.09998]

linear curvature perturbation

- Estimating only the connected piece of the form

$$S_n \supset s_n = \frac{n\mathcal{N}_{n-1}}{\sigma_{R_H}^{2(n-1)}} \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} W(k_1, R_H) \cdots \int \frac{\mathrm{d}^3 k_n}{(2\pi)^3} W(k_n, R_H) \left\langle \zeta_{1,\vec{k}_1} \cdots \zeta_{1,\vec{k}_{n-1}} (\zeta_1)^{n-1}_{\vec{k}_n} \right\rangle \simeq n!\,\mathcal{N}_{n-1}$$

One is then able to re-sum the series to get

$$\boxed{P_1(\zeta_R > \zeta_c) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi\nu^2}} \xrightarrow{\zeta_c = 1/3} P_{\mathrm{NG}}(\zeta_R > \zeta_c) \simeq \frac{e^{-1.6\,\nu^2/2}}{\sqrt{2\pi\nu^2}}}$$

Not an exact result, enough to signal a ***potentially large impact of NG***

Primordial Black Holes

- ***Beta fraction:*** mass fraction of PBH ***at formation***

Fraction of DM in PBH

$$\boxed{\beta = \frac{\rho_{\rm PBH}}{\rho_{\rm tot}}\bigg|_{\rm formation} = \left(\frac{H_0}{H_{\rm form}}\right)^2 \left(\frac{a_{\rm form}}{a_0}\right)^{-3} \Omega_{\rm CDM} f_{\rm PBH}}$$

Density parameter

- Related to today's abundance of PBH (neglecting accretion, merging, etc.)

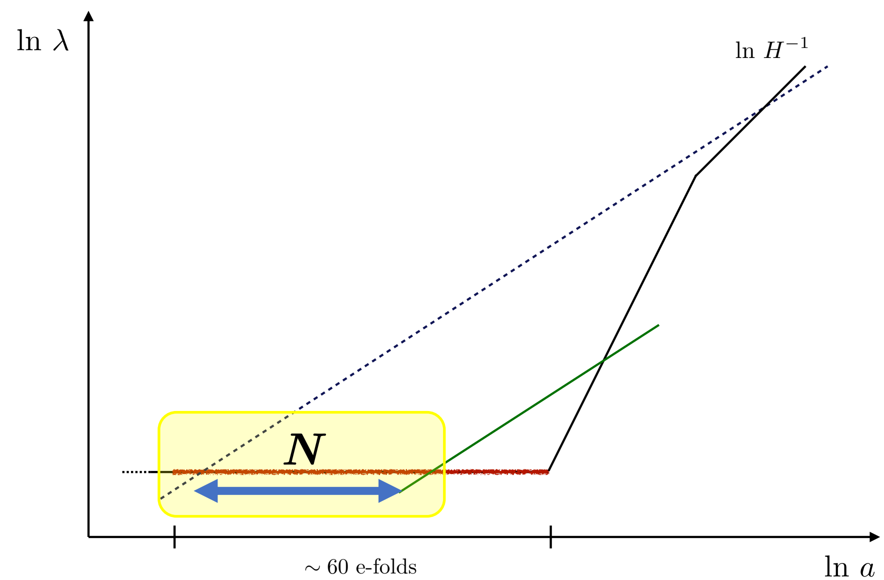
$$\left(\frac{\Omega_{\rm DM}(M)h^2}{0.12}\right) \simeq \left(\frac{\beta_{\rm prim}(M)}{7\cdot 10^{-9}}\right)\left(\frac{\gamma}{0.2}\right)^{1/2}\left(\frac{106.75}{g_*}\right)^{1/4}\left(\frac{M_\odot}{M}\right)^{1/2}$$

Mass

- ***Mass*** related to

$$N = 18.4 - \frac{1}{12}\ln\left(\frac{g^*}{g^*_0}\right) + \frac{1}{2}\ln\gamma - \frac{1}{2}\ln\left(\frac{M}{M_\odot}\right)$$

Effective number of d.o.f.



Models

- ***Single field models:*** J. Garcia-Bellido, E. Ruiz Morales [arXiv:1702.03901]
  K. Kannike, L. Marzola, M. Raidal and H. Veerme [arXiv:1705.06225]
  G. Ballesteros and M. Taoso [arXiv:1709.05565]
  M. Cicoli, V. A. Diaz and F. G. Pedro [arXiv:1803.02837]
  O.Özsoy, S.Parameswaran, G.Tasinato, I.Zavala [arXiv:1803.07626]

- ***Spectator fields:*** M. Kawasaki, N. Kitajima, T. T. Yanagida [arXiv:1207.2550]
  B. Carr, F. Kuhnel and M. Sandstad [arXiv:1607.06077]
  J. Garcia-Bellido, M. Peloso and C. Unal [arXiv:1610.03763]
  B. Carr, T. Tenkanen and V. Vaskonen [arXiv:1706.03746]
  + [Ando et al, arXiv:1805.07757] (yesterday)

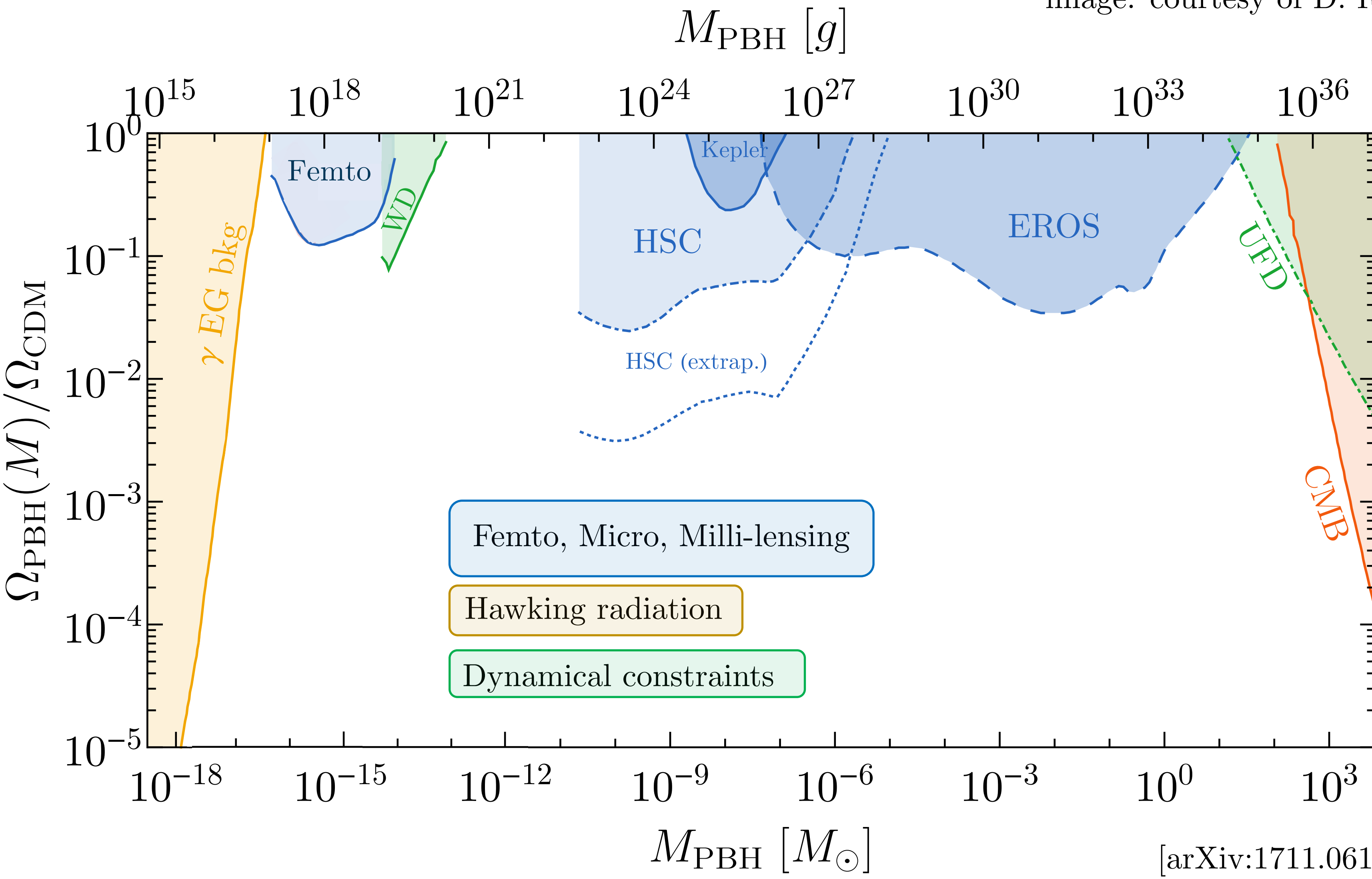
- ***Higgs Instability:*** J. R. Espinosa, D. Racco and A. Riotto [arXiv:1710.11196]

- ...

(if you don't mind fine-tuning)

PBH - Constraints

image: courtesy of D. Racco



[arXiv:1711.06129]

PBH - Constraints

[arXiv:1711.06129]

Femto, Micro, Milli-lensing

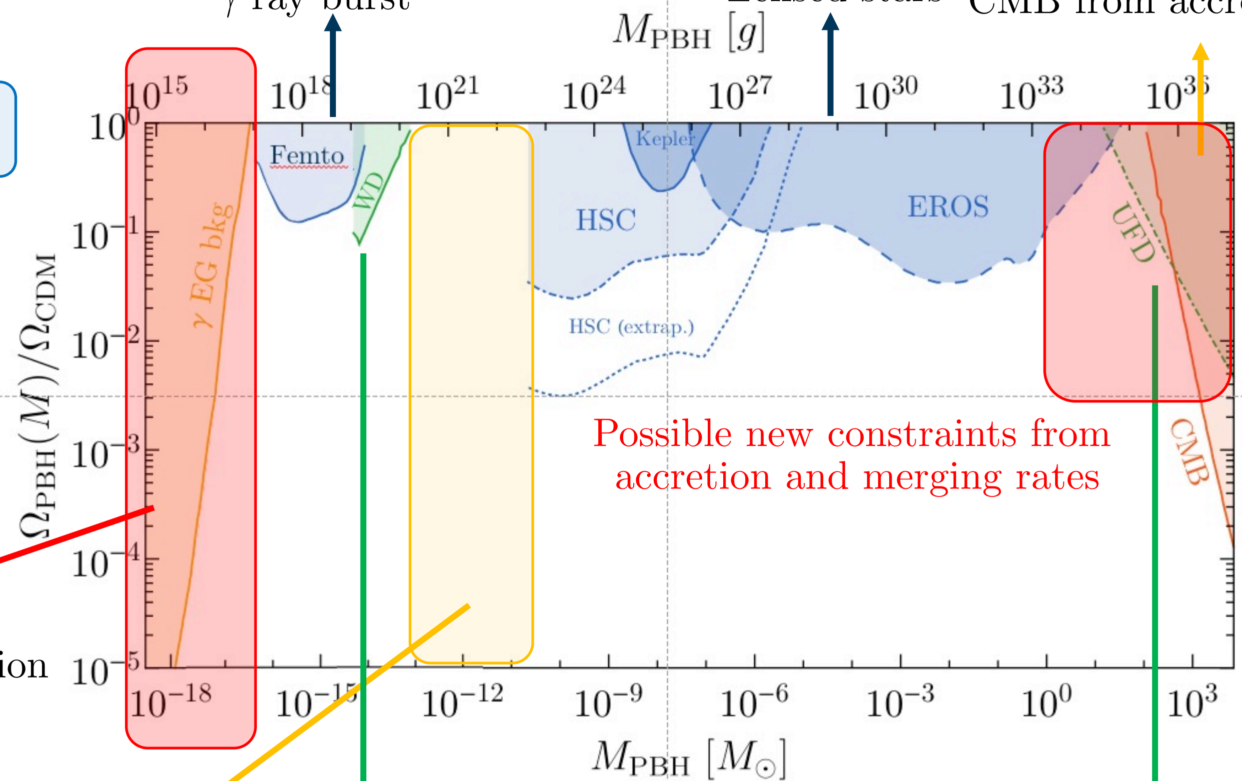
Hawking radiation

Dynamical constraints

γ ray burst

Lensed stars

CMB from accretion



Uncertainties on Hawking radiation

Gap due to "wave effects"

$\lambda \sim r_H(\text{lens})$

WD = disruption of white dwarfs (pbh passage ignite nuclear reaction)

UFD = ultra faint dwarfs galaxies

Grav. interaction equalise kinetic energies $\rightarrow$ stars spread

Caveats on formation mass fraction

- ***Critical spherical collapse:*** $M \sim k M_H (\Delta - \Delta_c)^\gamma$

Non spherical collapse effects?

- ***Threshold:*** dependence on the shape of the over-density peaks

[Germani, Musco, arXiv:1805.04087]
[Yoo et al, arXiv:1805.03946]

"Jeans length" argument for critical density for PBH formation

Compaction function criterion: mass excess identifying the Horizon formation fully taking into account peak shape

- ***Window function*** [arXiv:1802.06393]

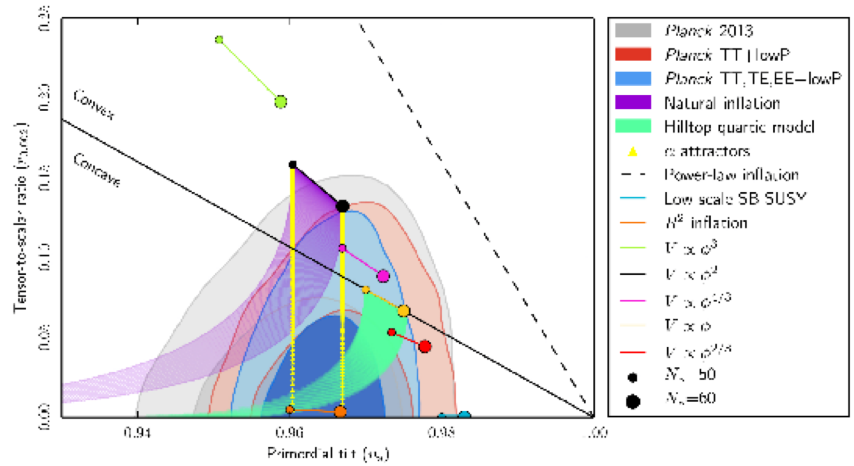
real-space top-hat window function
Gaussian window function
k-space top-hat window function

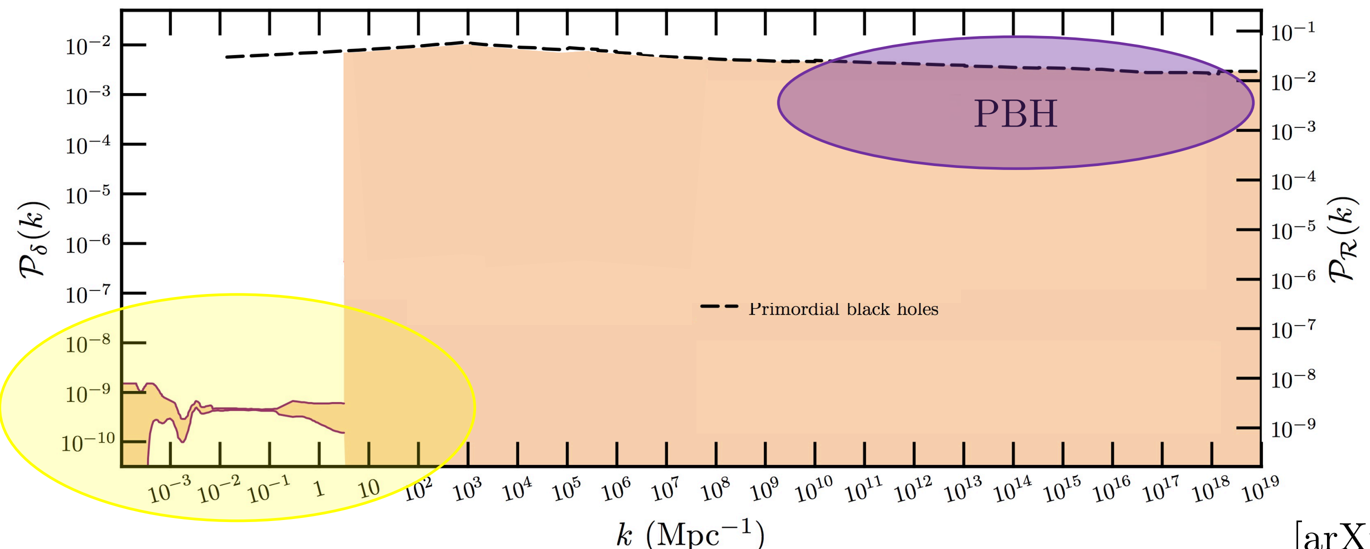
Constraints on inflation

- Planck 2015

$$\ln(10^{10}A_s) = 3.093 \pm 0.034,$$
$$n_s = 0.9673 \pm 0.0043,$$
$$\alpha_s = -0.0125 \pm 0.0091,$$
$$r < 0.166$$

[arXiv:1502.02114]





[arXiv:1110.2484]

PBH from single field model of inflation

PBH in Single Field Inflation

- ***Inflection point***

$$\mathcal{P}_{\mathcal{R}}^{1/2} = \frac{H^2}{2\pi|\dot{\phi}|}\bigg|_{k=aH}$$

(computed at Hubble crossing!)

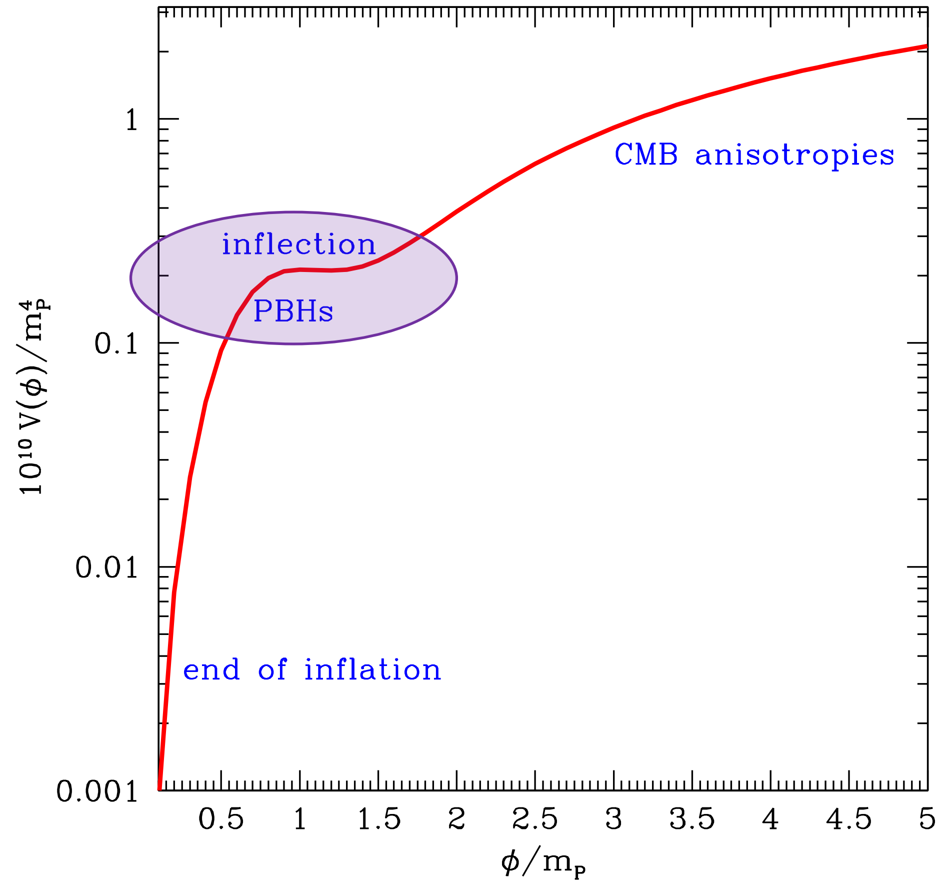
↓

in order to get an enhancement

Slow-roll violation:

$\boxed{\epsilon \ll 1}$ $\boxed{\eta \sim \mathcal{O}(1)}$

$\boxed{|\Delta \ln \epsilon / \Delta N| \geq \mathcal{O}(1)}$



In ***few*** N-efolds ϵ jumps of many orders of magnitude,

where the Hubble SR parameters are: $\epsilon = -\frac{\dot{H}}{H^2}$ $\eta = -\frac{\ddot{\phi}}{H\dot{\phi}}$

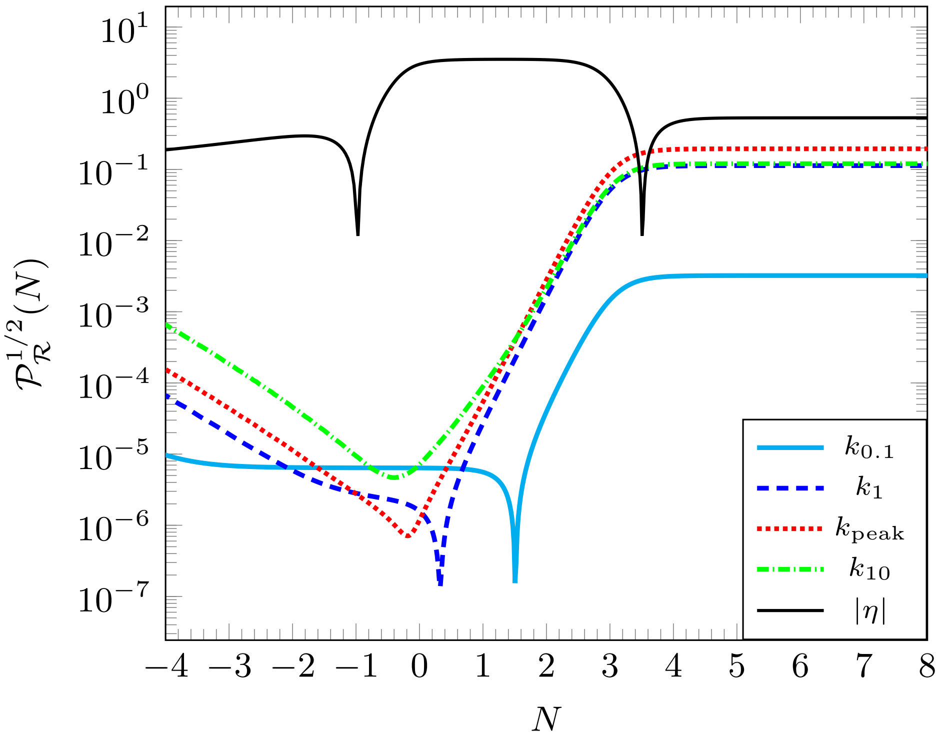
PBH in Single Field Inflation

- ***Enhancement*** during the non-attractor phase

$$\mathcal{R}_{,\tau\tau} + 2\frac{z_{,\tau}}{z}\mathcal{R}_{,\tau} + k^2\mathcal{R} = 0$$

$$z = a\dot{\phi}/H$$

$$\frac{z_{,\tau}}{z} = aH\left(1 + \epsilon - \eta\right)$$



$$u_k(\tau) = \mathcal{R}z$$

Bunch-Davies initial condition

$$\lim_{k\eta\to-\infty} u_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}$$

$k_{0.1}$

Super-Hubble before non-attractor phase

Modes on super-Hubble scales grow!