Unification and “Invisible” Axion

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Konkuk Univ., 7 Aug 2018

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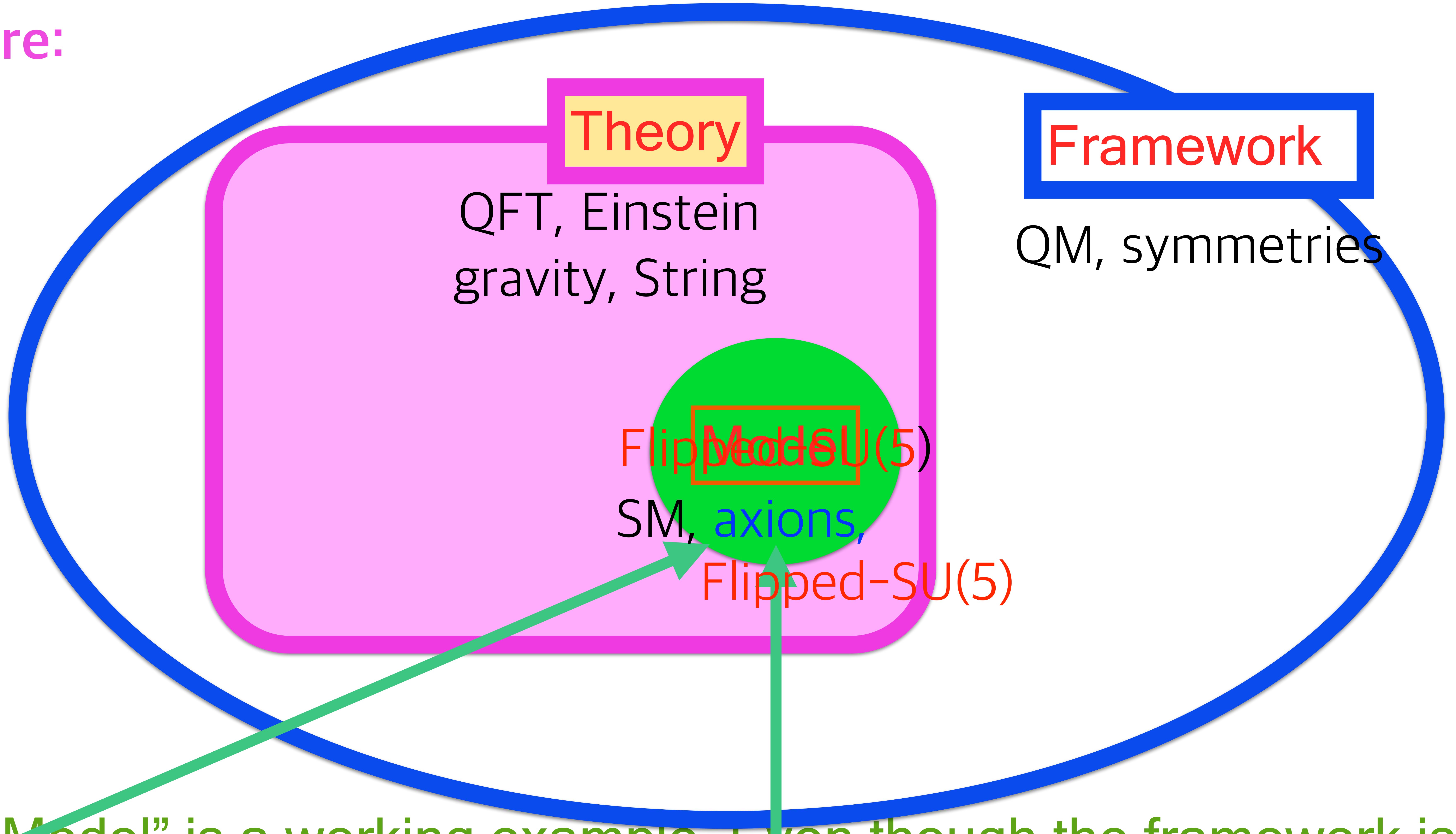
1. Introduction

Gross's picture:



"Model" is a working example. Even though the framework is fantastic, without a model example some will say that it is a religion.

Efforts to find a working model is our job toward MODEL/THEORY/FRAMEWORK paradigm.

Let us work in string compactification.

1st BSM

Weak-Interaction Singlet and Strong *CP* Invariance

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(Received 16 February 1979)

Strong *CP* invariance is *automatically* preserved by a spontaneously broken chiral $U(1)_A$ symmetry. A weak-interaction singlet heavy quark Q, a new scalar meson σ^0, and *a very light axion* are predicted. Phenomenological implications are also included.

attempts[1-4] to incorporate the observed made the Lagrangian *CP* invariant. In gen

mplicated and the
ed in the present

he axion properties,
menological impli-
a new scalar σ^0, and

principle, the col-
can be arbitrary.
e the same as light
is $\frac{2}{3}$ or $-\frac{1}{3}$, the
served in high-en-
PEP and PETRA,
arge is 0, there
lor-singlet hadrons
. Hence, the ob-

The new scalar σ^0.—By the spontaneous symmetry breaking of $U(1)_A$, σ will be split into a scalar boson σ^0 of mass $(2\mu_0)^{1/2}$ and an axion a. This σ^0 is *not* a Higgs meson, because it does not break the gauge symmetry, but the phenomenology of it is similar to the Higgs because of its coupling to quark as m_Q/v'. If this scalar mass is $\gtrsim 2m_Q$, we will see spectacular final state of stable particles such as $(Q\bar{u})$ and $(\bar{Q}u)$. If its mass is $\lesssim 2m_Q$, the effective interaction through loops $(c/v')F_{\mu\nu}{}^a F^{a\mu\nu}\sigma^0$, with numerical constant c, will describe the decay $\sigma^0 \to$ ordinary hadrons. The order of magnitude of its lifetime is $\tau(\sigma^0) \approx \tau(\pi^0)(v'/f_\pi)^2(m_{\pi^0}/m_{\sigma^0})^3 \approx 2\times10^{-10}$ sec for $v' \approx 10^5$ GeV and $m_{\sigma^0} \approx 10$ GeV. This kind of particle can be identified as a jet in pp high-energy collisions,

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Neutrino magnetic moment*

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(Received 1 June 1976)

The neutrino magnetic moment $f^{\nu\nu'}$ is calculated in the SU(2)⊗U(1) gauge model with the $a(\nu, L^-)_L$, $b(\nu', L^-)_R$. The order of magnitude of $f^{\nu\nu'}$ is barely within the upper bound for f^{ν_μ} ob $\bar{\nu}_\mu$-e elastic scattering data.

A neutrino, which is massless and electrically neutral, can have electromagnetic properties through its weak interactions with charged particles. In the past, an estimate for these properties was obtained indirectly from astrophysical data.[1] Recent neutral-current experiments, however, give valuable information[2] on the upper bounds of muonic-neutrino charge radius (r) and magnetic moment (f), viz. $r \leq 10^{-15}$ cm and $f \leq 10^{-8}$.

the neutrino will give valuable inf heavy-lepton mass in a specified

For a specific calculational pur two weak doublets with neutrinos identical or distinct neutrinos) in

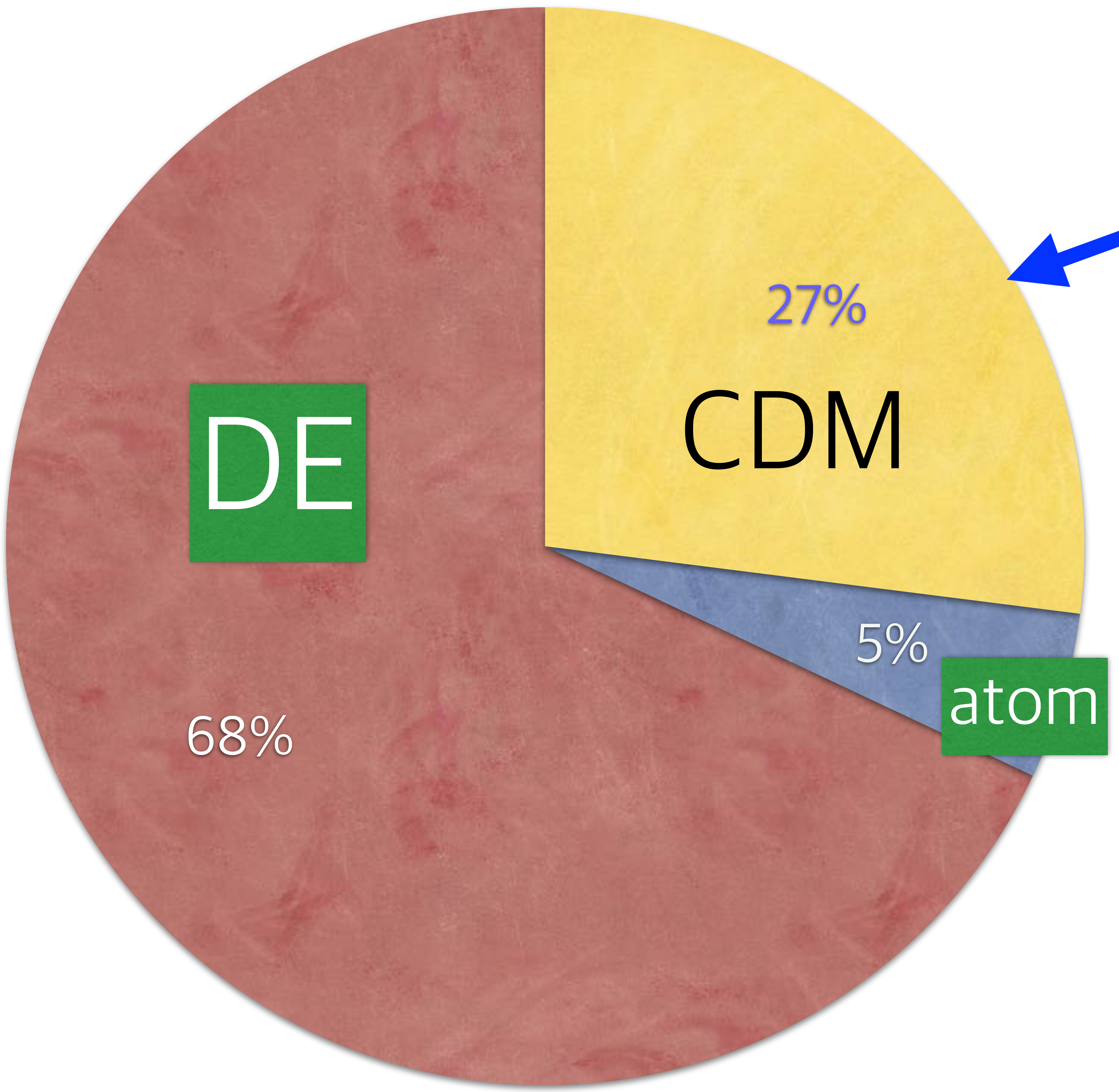
$$a\begin{pmatrix}\nu \\ L^-\end{pmatrix}_L, \quad b\begin{pmatrix}\nu' \\ L^-\end{pmatrix}_R,$$

which can be a substructure of

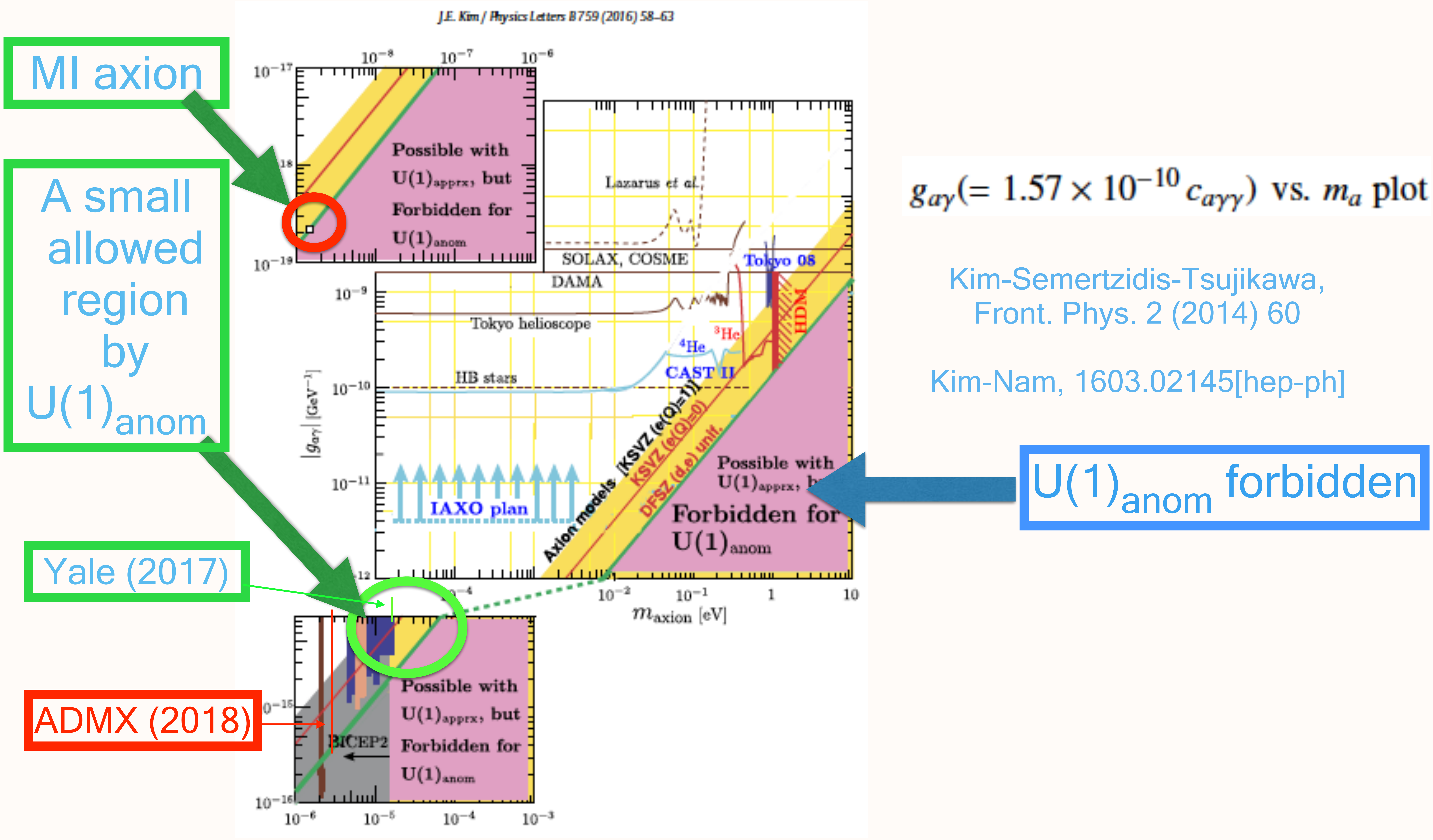
ronic currents.

(ii) Some have conjectured a large electron-neutrino magnetic moment to explain the solar-neutrino nondetection,[7] but the gauge-theory calculation does not give such a large moment as order of 10^{-4}.

In this paper, I have shown that the neutrino magnetic moment arises even for the massless neutrinos if one introduces two neutrino helicity states coupled to the same heavy lepton, and it is very close to the presently available upper bound. Of course, if one assumes a small mass of the neutrino, one can always obtain the magnetic moment proportional to the neutrino mass without the assumption of two neutrino helicities.



In addition, “Invisible” axion can be a part of

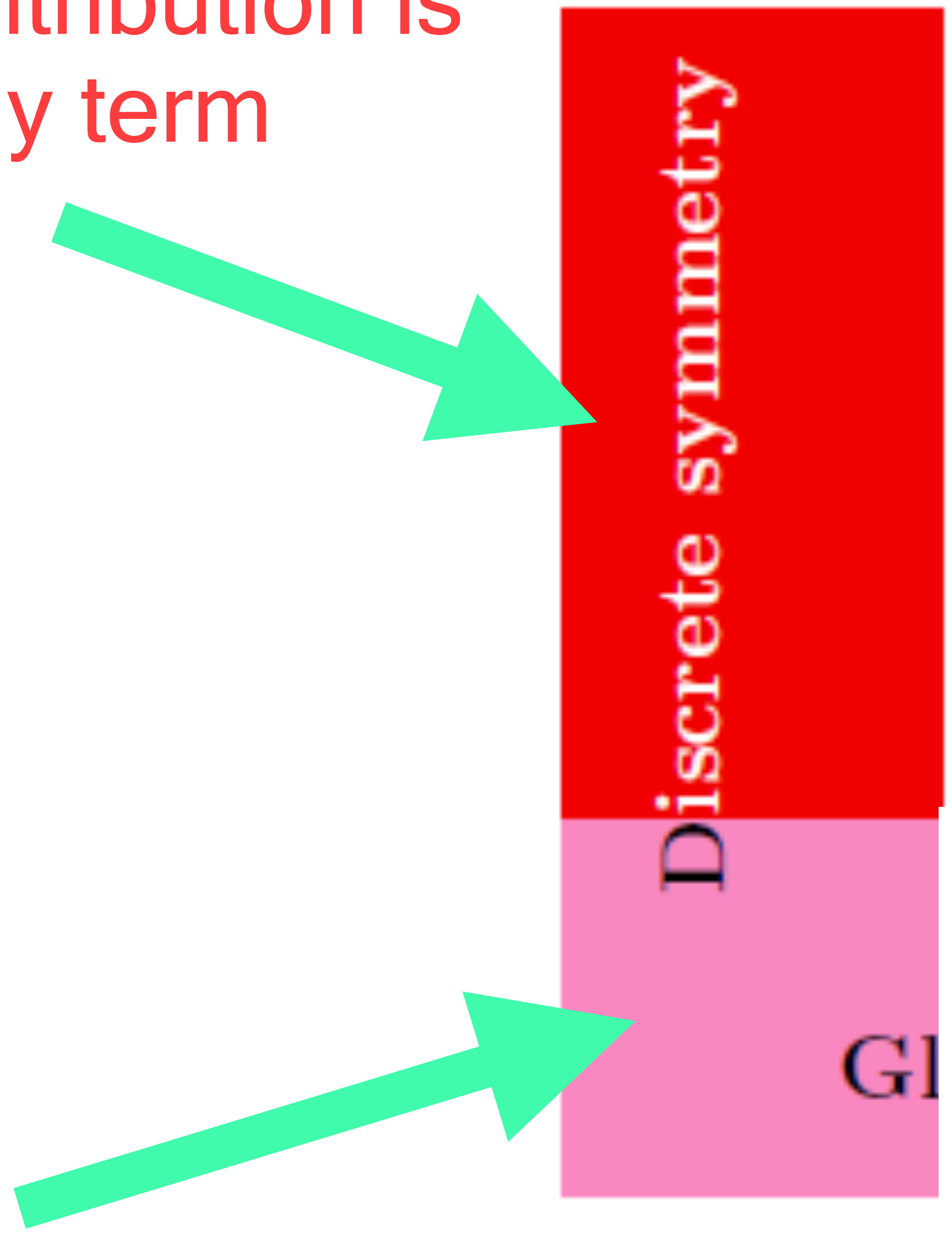


$g_{a\gamma}(= 1.57 \times 10^{-10} c_{a\gamma\gamma})$ vs. m_a plot

Kim-Semertzidis-Tsujikawa, Front. Phys. 2 (2014) 60

Kim-Nam, 1603.02145[hep-ph]

The dominant contribution is QCD anomaly term



JEK, Nam, Semertzidis, arXiv:1712.08648 [hep-ph] [Int. J. Mod. Phys. A 33 (2018) 183002]

2. $U(1)_{anom}$ as the symmetry for the “invisible” axion

JEK, Kyae, Nam, 1703.05345 [Eur. Phys. J. C77 (2017) 847]

't Hooft mechanism:

If a gauge symmetry and a global symmetry are broken by one complex scalar by the BEHGHK mechanism, then the gauge symmetry is broken and a global symmetry remains unbroken.

Q_{gauge}	Q_{global}
1	1

Unbroken $X = Q_{global} - Q_{gauge}$

$$\phi \to e^{i\alpha(x)Q_{\text{gauge}}} e^{i\beta Q_{\text{global}}} \phi$$

the α direction becomes the longitudinal mode of heavy gauge boson. The above transformation can be rewritten as

$$\phi \to e^{i(\alpha(x)+\beta)Q_{\text{gauge}}} e^{i\beta(Q_{\text{global}}-Q_{\text{gauge}})} \phi$$

Redefining the local direction as $\alpha'(x) = \alpha(x) + \beta$, we obtain the transformation

$$\phi \to e^{i\alpha'(x)Q_{\text{gauge}}} e^{i\beta(Q_{\text{global}}-Q_{\text{gauge}})} \phi.$$

$$|D_\mu \phi|^2 = |(\partial_\mu - igQ_a A_\mu)\phi|^2_{\rho=0} = \frac{1}{2}(\partial_\mu a_\phi)^2 - gQ_a A_\mu \partial^\mu a_\phi + \frac{g^2}{2} Q_a^2 v^2 A_\mu^2$$

$$= \frac{g^2}{2} Q_a^2 v^2 (A_\mu - \frac{1}{gQ_a v}\partial^\mu a_\phi)^2$$

So, the gauge boson becomes heavy and there remains the x-independent transformation parameter beta. The corresponding charge is a combination:

$X = Q_{\text{global}} - Q_{\text{gauge}}$

The MI axion

$$H_{\mu\nu\rho} = M_{MI}\epsilon_{\mu\nu\rho\sigma}\,\partial^\sigma a_{MI}.$$

$$\frac{1}{2}\partial^\mu a_{MI}\partial_\mu a_{MI} + M_{MI}A_\mu\partial^\mu a_{MI}$$

This is the Higgs mechanism, i.e. a_{MI} becomes the longitudinal mode of the gauge boson. [JEK, Kyae, Nam, 1703.05345]

$$\frac{1}{2}(\partial_\mu a_{MI})^2 + M_{MI}A_\mu\partial^\mu a_{MI} + \frac{1}{2\cdot 3!}A_\mu A^\mu \to \frac{1}{2}M_{MI}^2(A_\mu + \frac{1}{M_{MI}}\partial_\mu a_{MI})^2.$$

It is the 't Hooft mechanism working in the string theory. So, the continuous direction $a_{MI} \to a_{MI}$ + (constant) survives as a global symmetry at low energy:

"Invisible" axion!! appearing at $10^{10\text{-}11}$ GeV scale when the global symmetry is broken.

3. QCD phase transition

JEK, 1805.08153

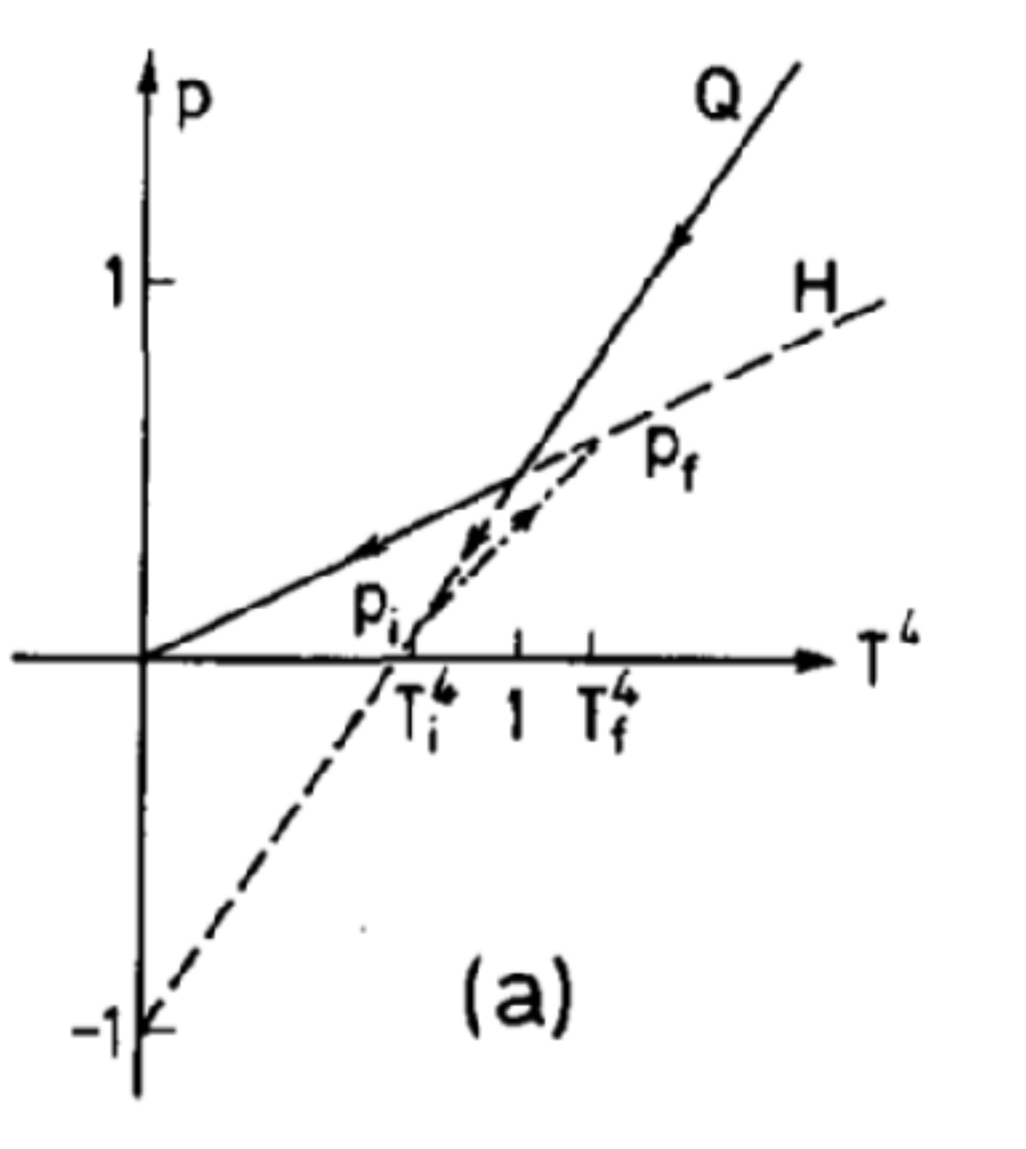
$$\text{Before}\begin{cases}\rho = \frac{\pi^2}{30}\, g_*^i T^4 \\ s = \frac{2\pi^2}{45}\, g_*^i T^3\end{cases}$$

$$\text{After}\begin{cases}\rho = \frac{\pi^2}{30}\, g_*^f T^4 \\ s = \frac{2\pi^2}{45}\, g_*^f T^3\end{cases}$$

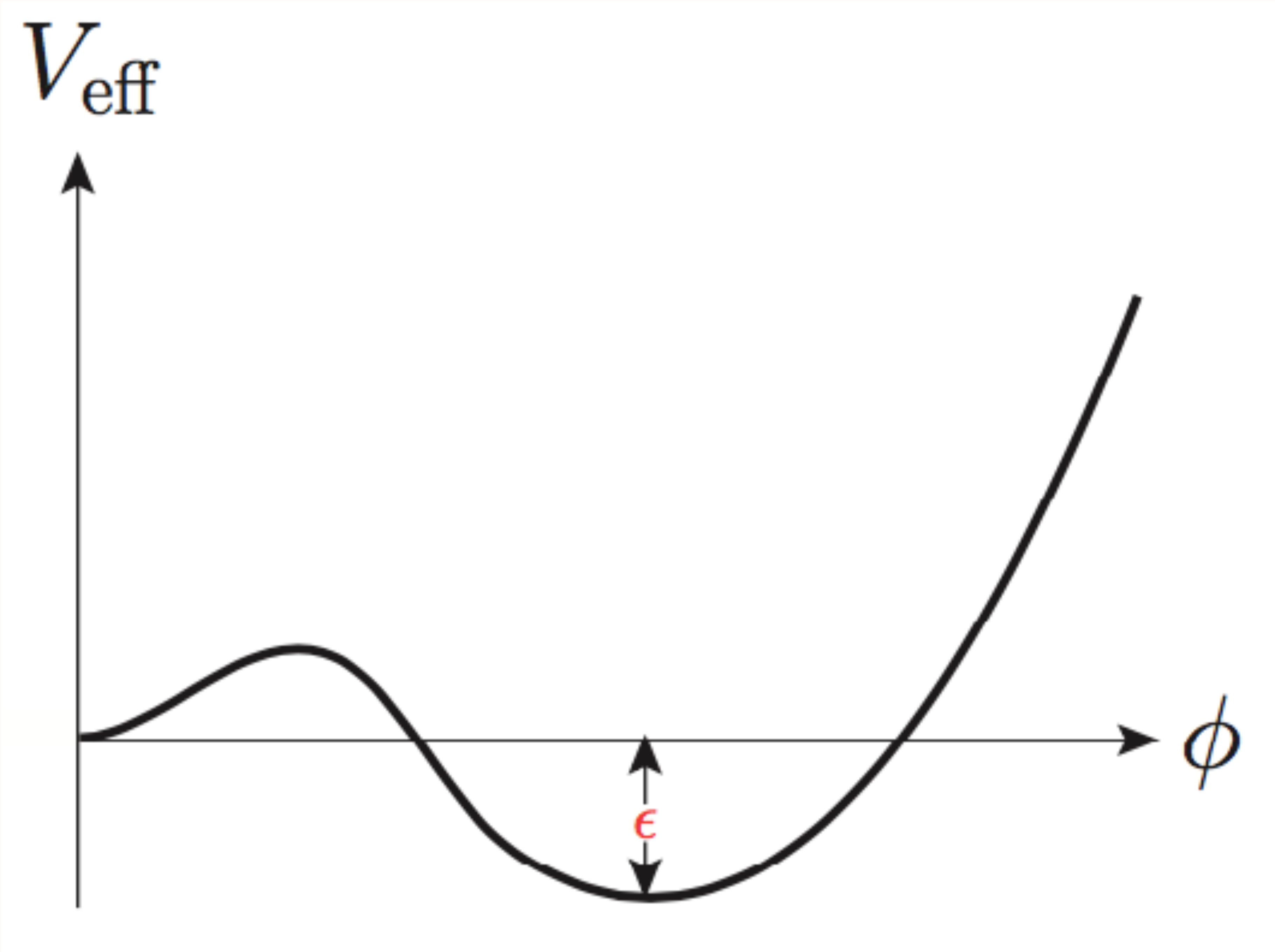
$$g_*^i = 51.25, \;\; g_*^f = 17.25.$$

37, 3, for hadrons only

DeGrand and callaborators studied QCD phase transition and axion since 84 with MIT bag model. This calculation has a complicated behavior.



Kolb and Turner studied with a phenomenological Lagrangian with ϵ parameter.



We calculate the phase transition from the first principles.

Here, we study the following two parameter differential equation on the fraction of h-phase in the evolving universe.

$$\frac{df_h}{dt} = \alpha(1 - f_h) + \frac{3}{(1 + Cf_h(1 - f_h))(t + R_i)} f_h$$

It is possible to calculate it because the critical temperature is pinned down now at 165+-5 MeV in the lattice community.

- Knowledge on the axion mass is important.
- Susceptibility χ is important.

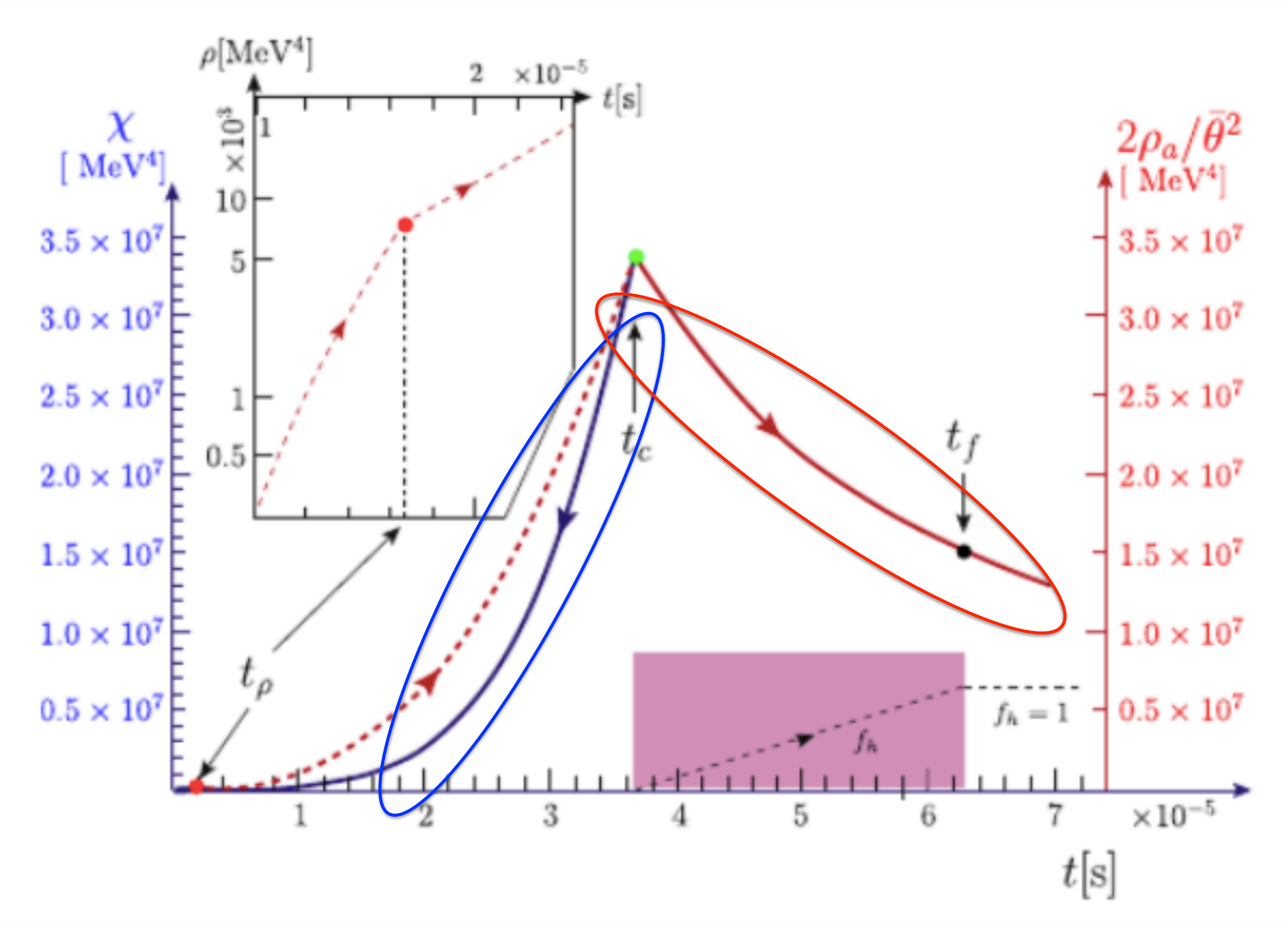
Quark and gluon phase with Λ_{QCD} :

$$f_a^2 m_a^2 = \frac{(\sin^2\bar{\theta}/\bar{\theta}^2)}{2Z\cos\bar{\theta}+1+Z^2}\, m_u^2 \Lambda_{QCD}^2 \left(\frac{1}{2}\bar{\theta}^2\right),$$

Hadronic phase in terms of $f_{\pi^0}^2 m_{\pi^0}^2$:

$$f_a^2 m_a^2 = \frac{Z(\sin^2\bar{\theta}/\bar{\theta}^2)}{2Z\cos\bar{\theta}+1+Z^2}\, f_{\pi^0}^2 m_{\pi^0}^2 \left(\frac{1}{2}\bar{\theta}^2\right),$$

Lattice susceptibility χ : $f_a^2 m_a^2 = \chi\left(\frac{1}{2}\bar{\theta}^2\right)$,



χ
[MeV⁴]
3.5 × 10⁷
3.0 × 10⁷
2.5 × 10⁷
2.0 × 10⁷
1.5 × 10⁷
1.0 × 10⁷
0.5 × 10⁷
ρ[MeV⁴]
2
×10⁻⁵
t[s]
×10³
1
10
5
1
0.5
tρ
tc
tf
fh = 1
fh
2ρa/θ̄²
[MeV⁴]
1
2
3
4
5
6
7
×10⁻⁵
t[s]

There are two aspects in this study: (1) the strong interaction, (2) axion energy density evolution in the evolving Universe.

At and below Tc, the quark-gluon phase and the hadronic phase co-exist. So, at Tc we know what is the energy density in the q&g-phase. And pressure is just 1/3 of it. So, we know the pressure of h-phase since the pressures are the same during the 1st order phase transition.

Now, at Tc the pion pressure is known. So, it is known below Tc also.

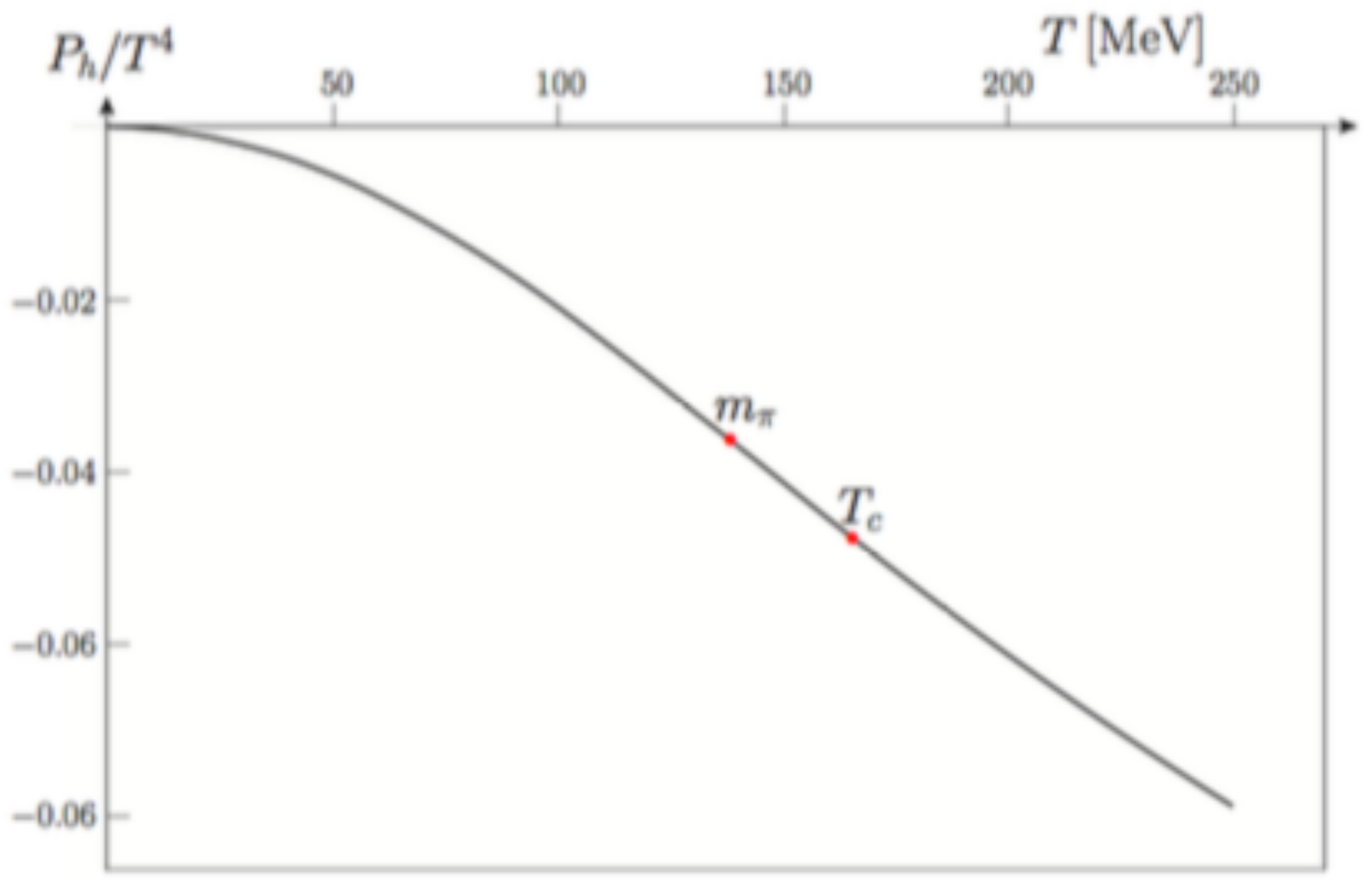


FIG. 3: P_h versus T.

We used Eq. (8.55) of Huang's book, "Statistical Mechanics", in realtivistic form.

- In the expanding Universe, the free energy is conserved,

$$(-SdT - PdV + \mu dN)_q + (-SdT - PdV + \mu dN)_h = 0.$$

Using $dV_q = -dV_h$,

$$(P_h - P_q)dV_h = (S_q - S_h)dT + \mu_h dN_h - \mu_q dN_q = (S_q - S_h)dT.$$

$$\frac{1}{V}\frac{dV_h}{dt} = \frac{(S_q - S_h)}{(P_h - P_q)}\frac{dT}{dt}.$$

$$\alpha(T) = \frac{(S_q - S_h)}{(P_h - P_q)}\frac{dT}{dt} \approx \frac{-37\pi^2}{45(P_h - P_q)}\frac{T^6}{\text{MeV}}, \text{ with } T^2 t_{[s]} \simeq \text{MeV}.$$

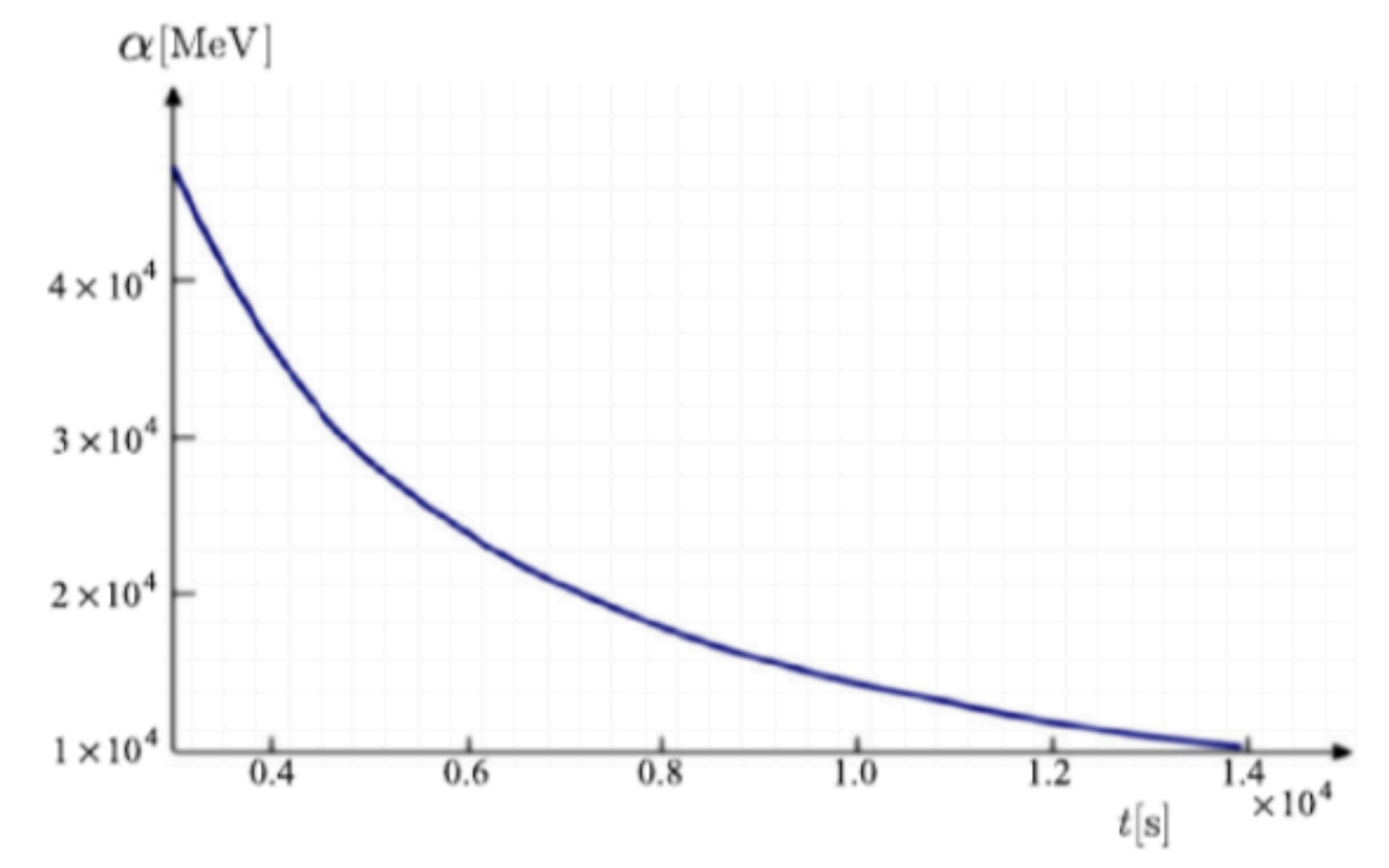
$$dU = dQ - PdV + \mu dN,$$ Used in the1st law

$$dA = -SdT - PdV + \mu dN,$$ Used in the evolveing Univ.

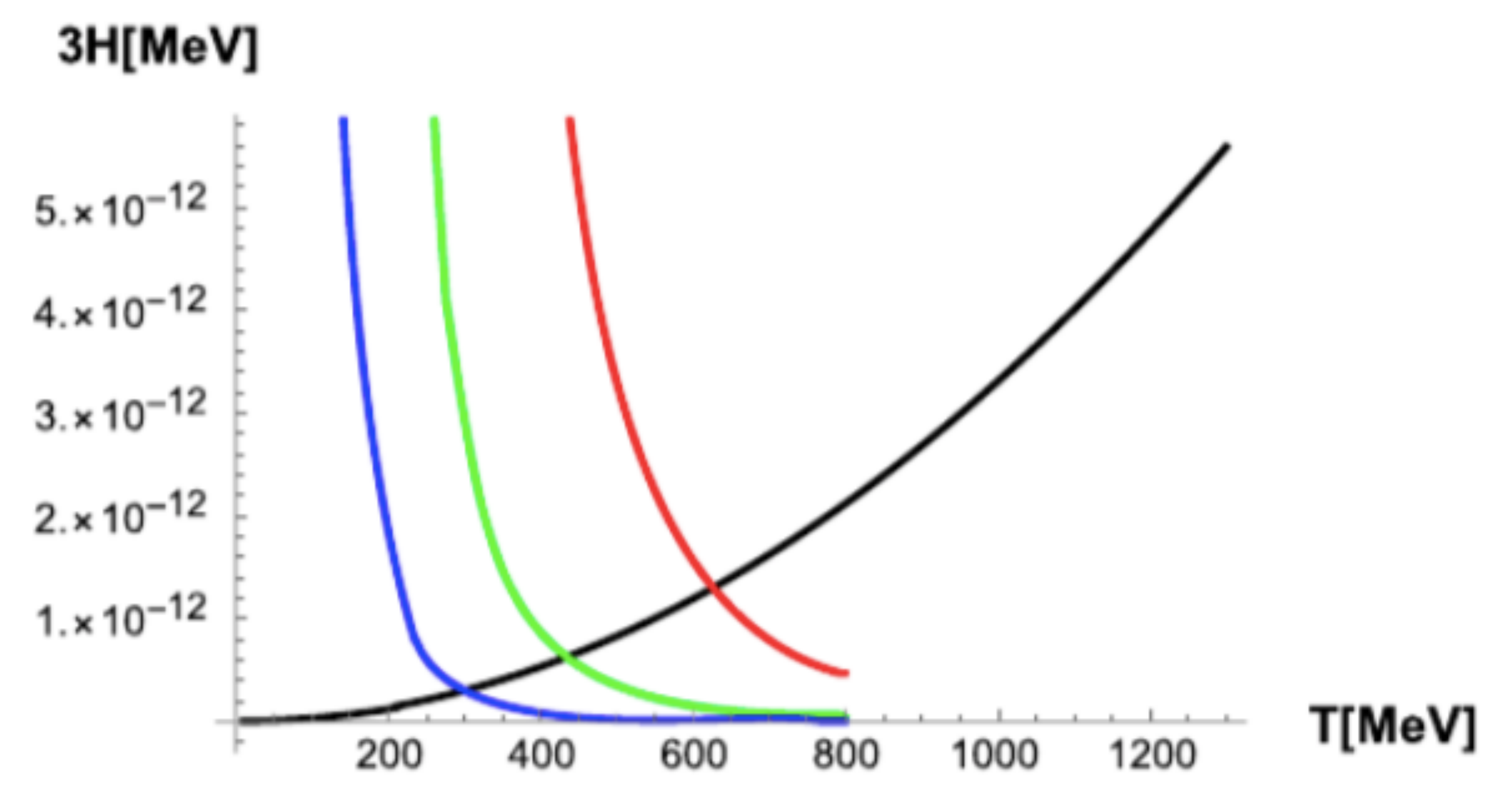
$$dG = -SdT + VdP + \mu dN,$$

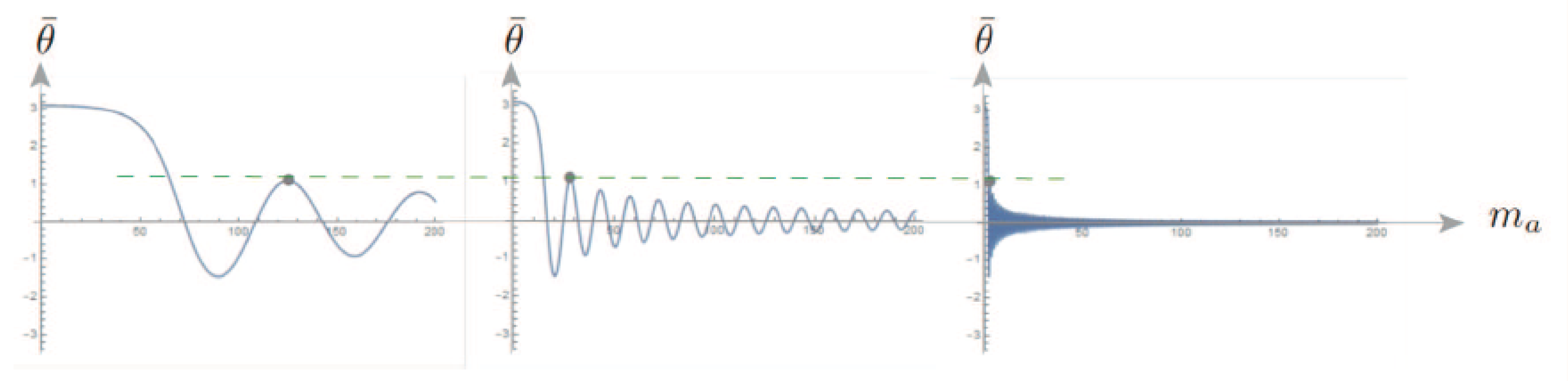
During the 1st order(cross-over) phase transition, the Gibbs free energy is conserved. At the same temperature and pressure. We know P of massless quarks and gluons at temperatures T, 1/3 of energy density.

Now, we have to know P of massive pions at and below Tc.

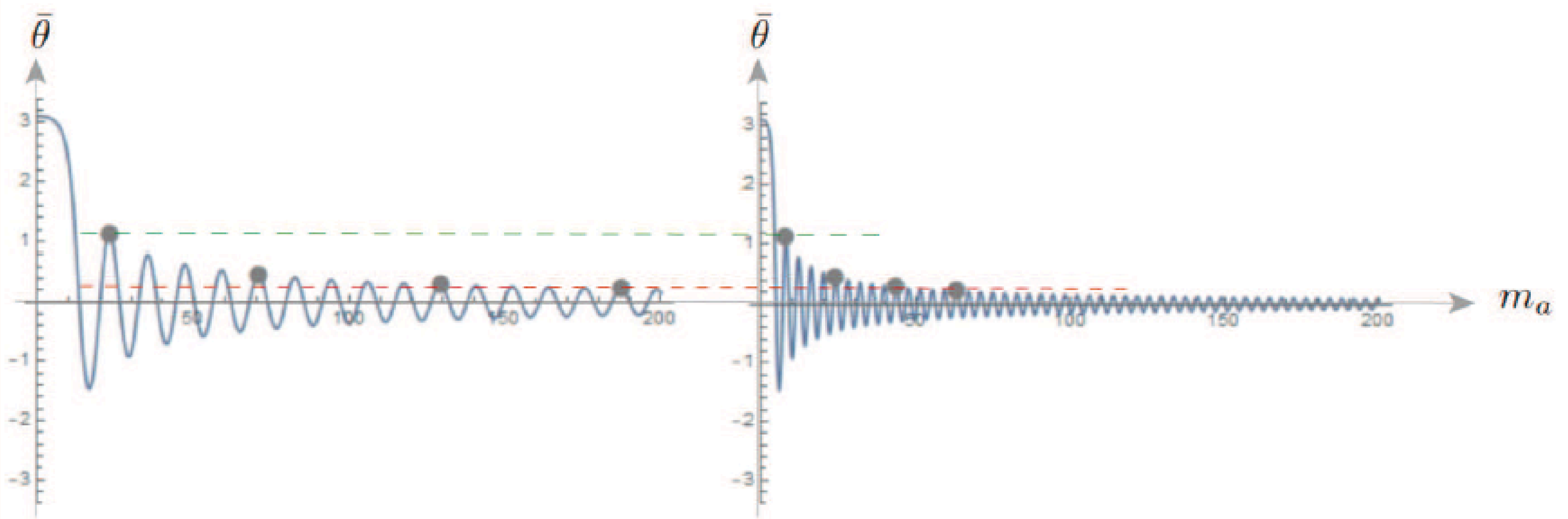


α[MeV]
4×10^4
3×10^4
2×10^4
1×10^4
0.4
0.6
0.8
1.0
1.2
1.4
$\times10^4$
t[s]

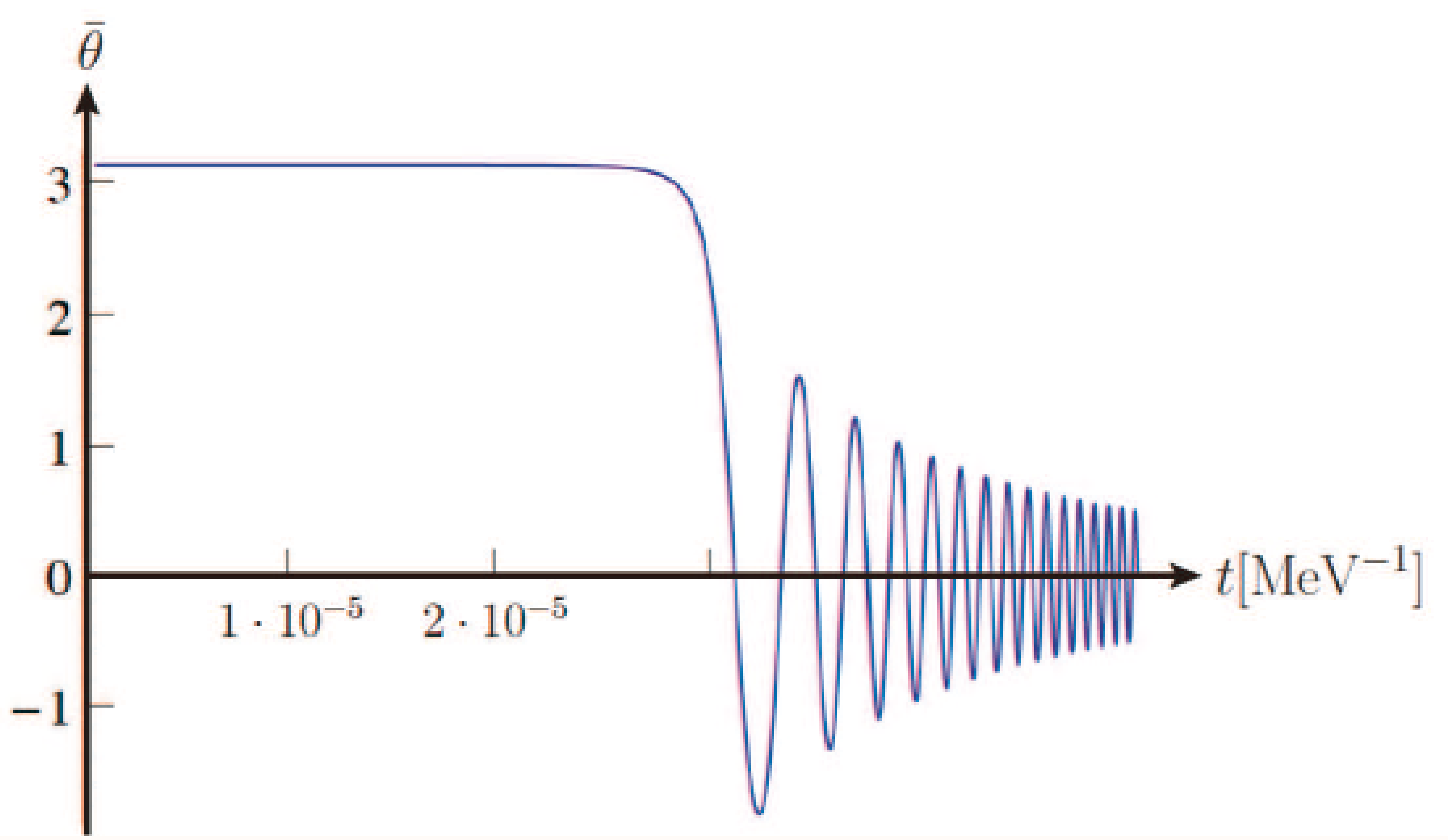

3H[MeV]
5.×10^-12
4.×10^-12
3.×10^-12
2.×10^-12
1.×10^-12
200
400
600
800
1000
1200
T[MeV]



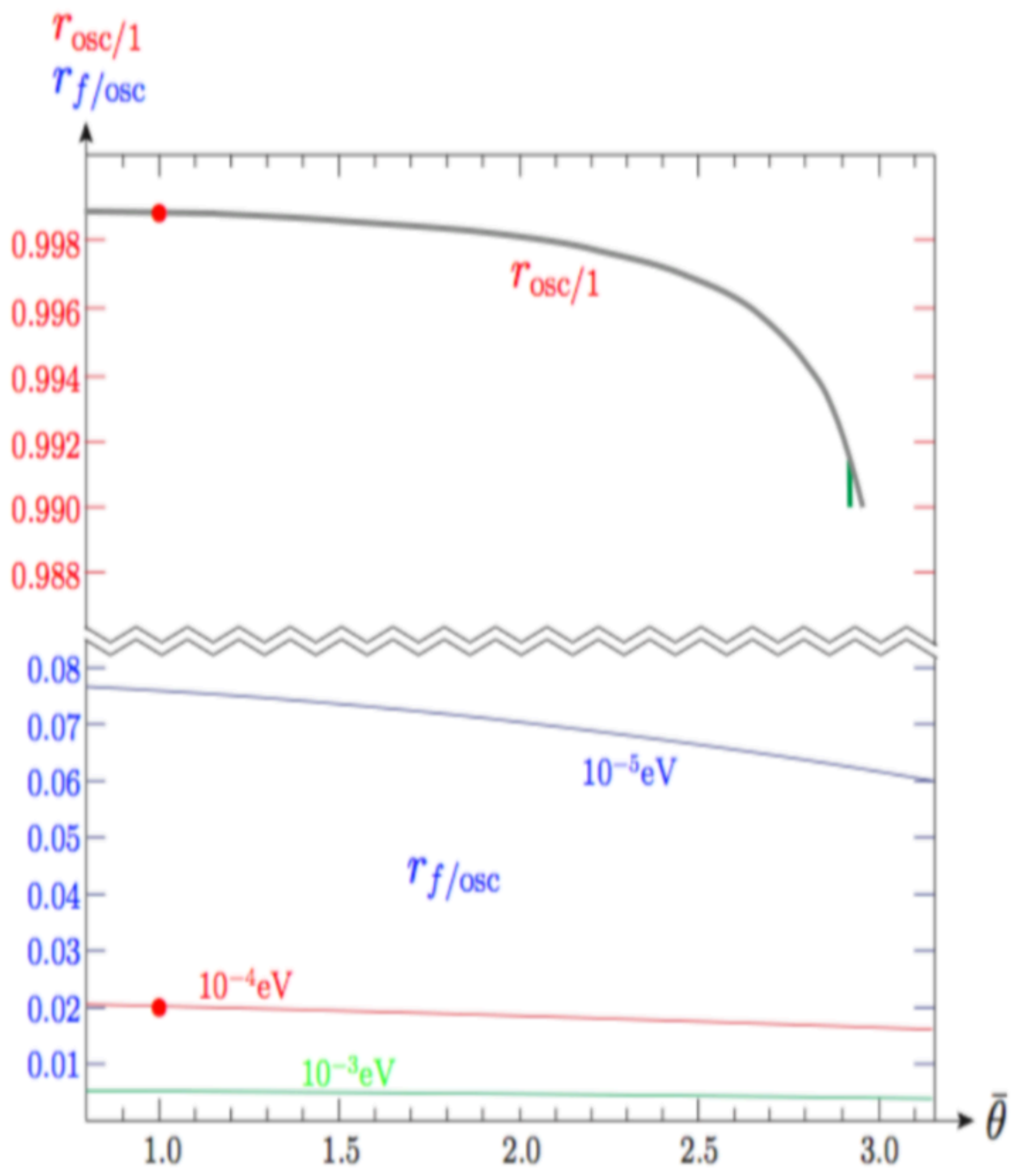
$\bar{\theta}$
$\bar{\theta}$
$\bar{\theta}$
m_a
50
100
150
200



$\bar{\theta}$
$\bar{\theta}$
3
2
1
-1
-2
-3
50
100
150
200
m_a



Show movie for m_a independence



$$\bar\theta_{\rm now} \simeq \bar\theta_1 \cdot r_{f/1} \cdot \left(\frac{\bar\theta_{\rm now}}{\bar\theta_f}\right)$$

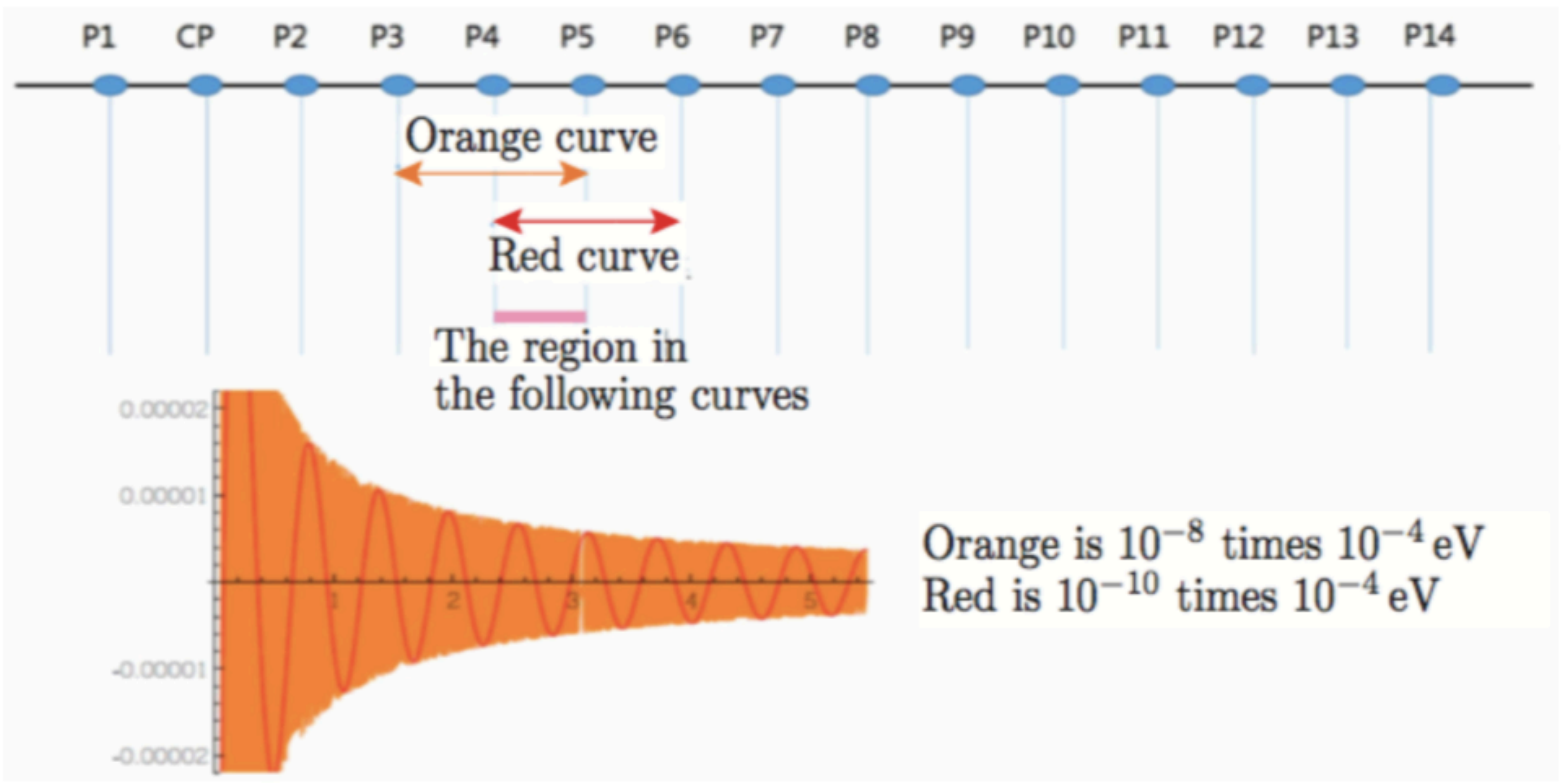
$$r_{f/1} \simeq 0.02 \left(\frac{m_a}{10^{-4}\,{\rm eV}}\right)^{-0.591\pm0.008}$$

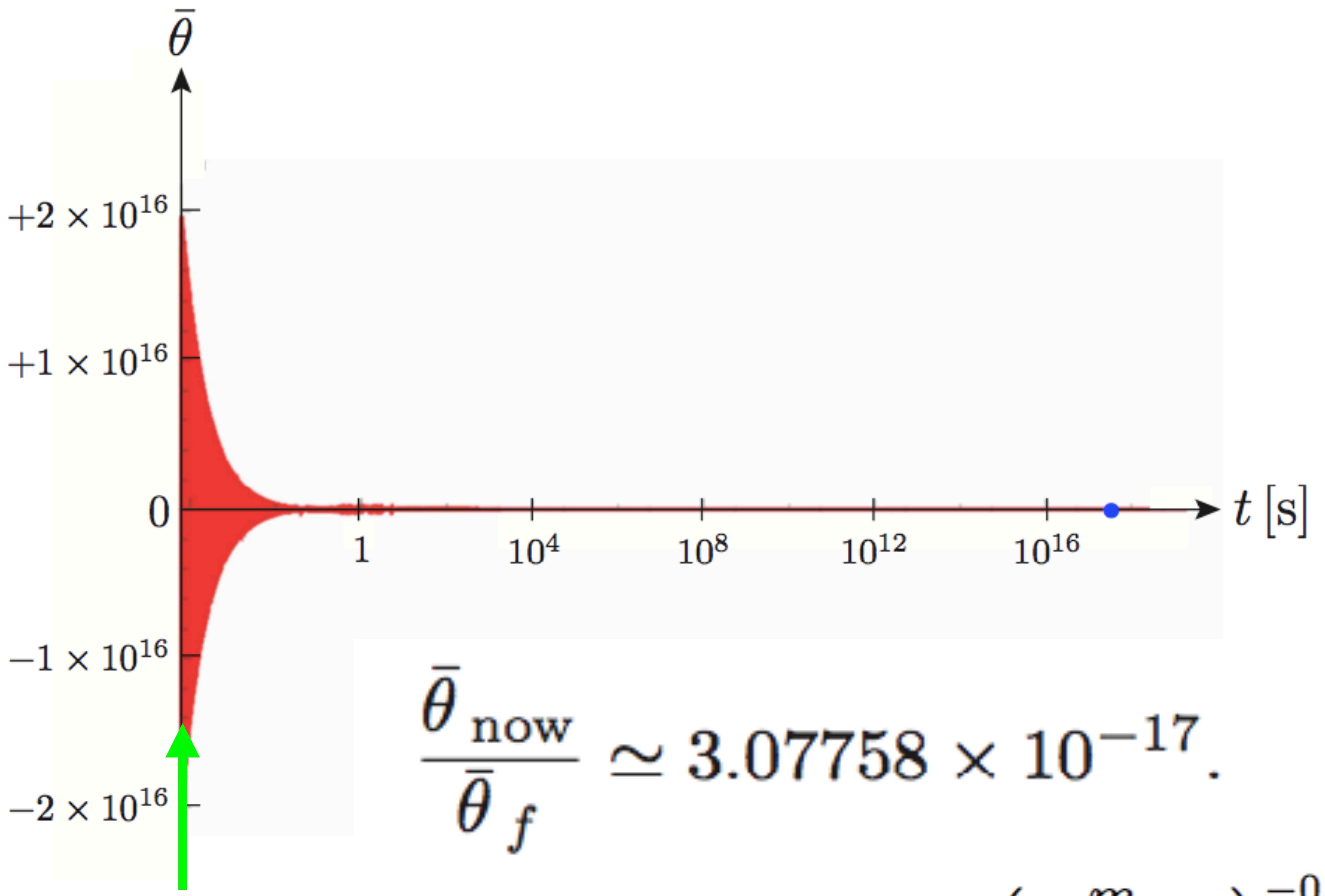
FIG. 7: The ratios $r_{\rm osc/1} \equiv \bar\theta_{\rm osc}/\bar\theta_1$ and $r_{f/\rm osc} \equiv \bar\theta_f/\bar\theta_{\rm osc}$ as functions of $\bar\theta_1$ for three $m_a(0)(=10^{-3}\,{\rm eV(green)}, 10^{-4}\,{\rm eV(red)}, 10^{-5}\,{\rm eV(blue)}$. In the upper figure, these curves are almost overlapping (shown as gray) except the green for a large $\bar\theta_1$. [See also Supplement.] $t_{\rm osc}$ is the time of the 1st oscillation after which the harmonic motion is a good description. Different T_1's are used for different $m_a(0)$, as presented in Fig. 4.

After t_f, we solve

$$\ddot{\bar{\theta}} + 3H\dot{\bar{\theta}} + \frac{m_0^2}{2}\bar{\theta} \simeq 0.$$

$$m_0 \to 10^{-n} m_0$$
$$\bar{\theta} \to 10^n \bar{\theta}\,.$$





$$\frac{\bar{\theta}_{\text{now}}}{\bar{\theta}_f} \simeq 3.07758 \times 10^{-17}.$$

$$r_{f/1} \simeq 0.02 \left(\frac{m_a}{10^{-4}\,\text{eV}} \right)^{-0.591 \pm 0.008}$$

From t_f to t_{now}: (JEK, S. Kim, Nam, 1803.03517)

3.07758×10^{-17}

We calculated a new number F_{now}.

The final factor is

$$\bar\theta_{\rm now} \simeq \bar\theta_1 \cdot r_{f/1} \cdot \left(\frac{\bar\theta_{\rm now}}{\bar\theta_f}\right) = 0.62\times10^{-18}\,\bar\theta_1$$

4. Flipped SU(5) from string

SO(10) —> $SU(5) \times U(1)_X$

e^c d^c

GG SU(5) with X=0: 10^*_0, 5_0, (1_0)

(Higgs) 5_0, 5^*_0

d^c u^c e^c

Flipped SU(5) with X: (matter) 10^*_{-1}, 5_{+3}, 1_{-5}

(Higgs) 5_{-2}, 5^*_{+2}

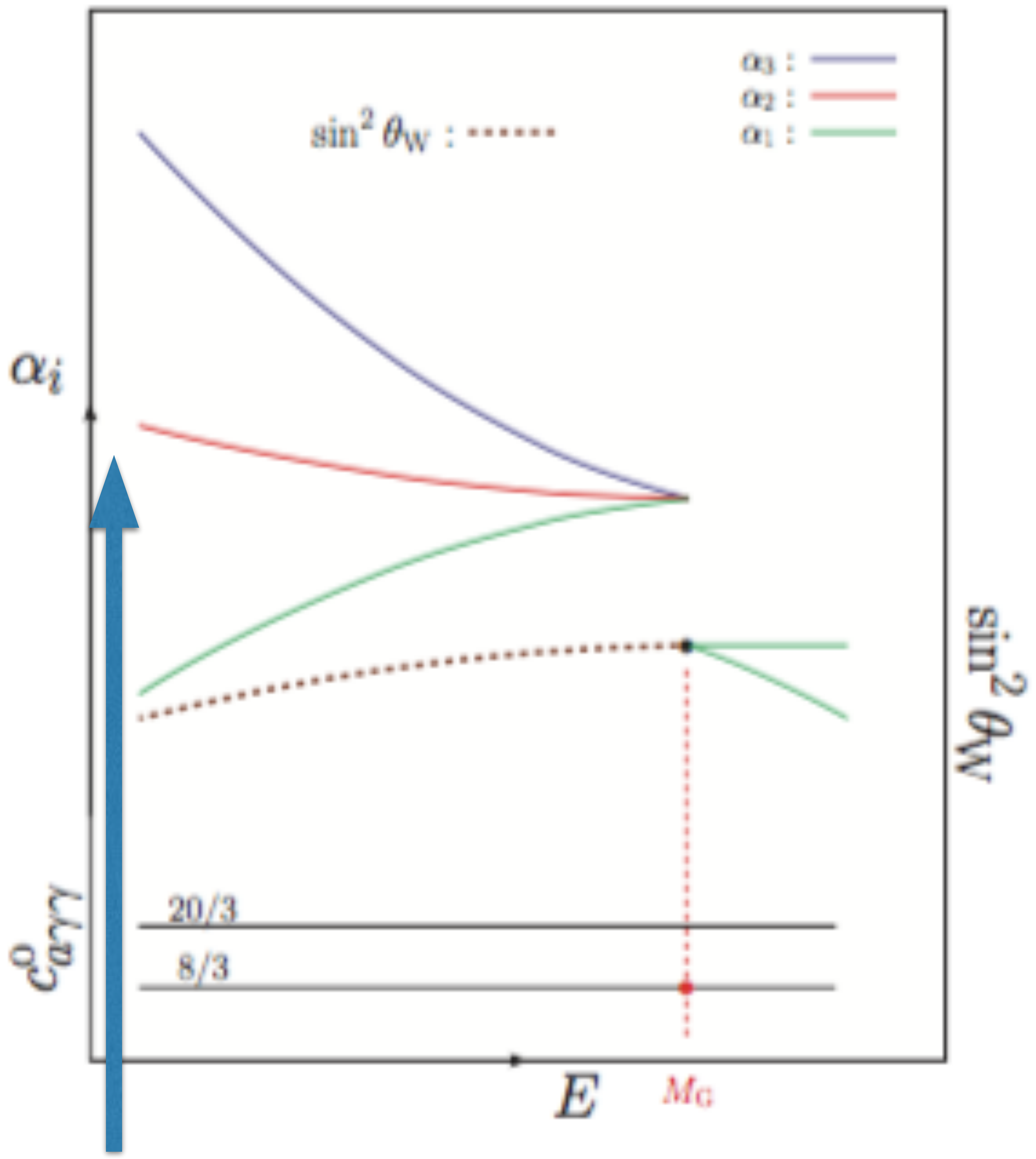
The flipped SU(5) arises from compactification of heterotic string, easily

Georgi-Quinn-Weinberg expression is

$$\sin^2\theta_W = \frac{\mathrm{Tr}\, T_3^2}{\mathrm{Tr}\, Q_{\mathrm{em}}^2}$$

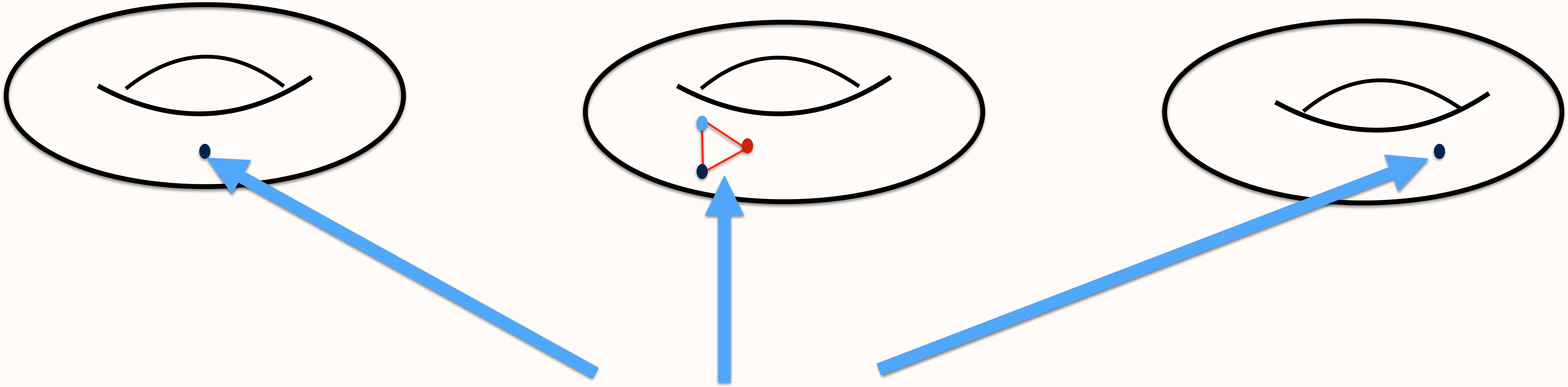
It depends on symmetry breaking.
If there is no more funny particles
beyond 16 of SO(10),

$$\sin^2\theta_W = \frac{3}{8}$$



Which is renormalized to 0.233 at EW scale
[Kim et al, RMP 51 (1981) 211] ;
LHC confirmed, i.e. loop corrections in the SM work.

$Z_{12\text{-}I}$



Fixed points on two tori.
Simplest in number of fixed points.
Dixon-Kaplunovsky-Louise, 1990

arXiv:1703.05345 [hep-ph]

Z(12-I) orbifold compactification:

a flipped SU(5) model x SU(5)’ x SU(2)’ x U(1)s [Huh-Kim-Kyae:0904.1108]

7 U(1)s: $U(1)_Y$, $U(1)_1$, $U(1)_2$, $U(1)_3$, $U(1)_4$, $U(1)_5$, $U(1)_6$.

$$Q_1 = (0^5; 12, 0, 0)(0^8)',$$
$$Q_2 = (0^5; 0, 12, 0)(0^8)',$$
$$Q_3 = (0^5; 0, 0, 12)(0^8)',$$
$$Q_4 = (0^8)(0^4, 0; 12, -12, 0)',$$
$$Q_5 = (0^8)(0^4, 0; -6, -6, 12)',$$
$$Q_6 = (0^8)(-6, -6, -6, -6, 18; 0, 0, 6)'.$$

Flipped SU(5) is the simplest GUT from heterotic string compactification: Adjoint representation is not needed to break the GUT.

$$X = (-2, -2, -2, -2, -2\,;\, 0^3)(0^8)',$$
$$Q_{\rm anom} = 84Q_1 + 147Q_2 - 42Q_3 - 63Q_5 - 9Q_6,$$

$U(1)_{\rm anom}$, is enough. It is working for the invisible axion.

These are family q.n.

Table 1 The $SU(5)\times U(1)_X$ states. Here, + represents helicity $+\frac{1}{2}$ and – represents helicity $-\frac{1}{2}$. Sum of $Q_{\rm anom}$ is multiplied by the index of the fundamental representation of $\mathrm{SU(3)}_c$, $\frac{1}{2}$. The PQ symmetry, being chiral, counts quark and antiquark in the same way. The right-handed states in T_3 and T_5 are converted to the left-handed ones of T_9 and T_7, respectively. The bold entries are $Q_{\rm anom}/126$

Sect.	Colored states	$\mathrm{SU(5)}_X$	Mult.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label	$Q_a^{\gamma\gamma}$
U	$(\underline{+++--};--+)(0^8)'$	$\overline{\mathbf{10}}_{-1}$		-6	-6	$+6$	0	0	0	$-1638(\mathbf{-13})$	C_2	-3276
U	$(\underline{+----};+--)(0^8)'$	$\mathbf{5}_{+3}$		$+6$	-6	-6	0	0	0	$-126(\mathbf{-1})$	C_1	-294
T_4^0	$(\underline{+----};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^8)'$	$\mathbf{5}_{+3}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_3$	-882
T_4^0	$(\underline{+++--};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^8)'$	$\overline{\mathbf{10}}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_4$	-756
T_4^0	$(\underline{10000};\frac{1}{3}\frac{1}{3}\frac{1}{3})(0^8)'$	$\mathbf{5}_{-2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_5$	$+1008$
T_4^0	$(\underline{-10000};\frac{1}{3}\frac{1}{3}\frac{1}{3})(0^8)'$	$\overline{\mathbf{5}}_{+2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_6$	$+1008$
T_6^0	$(\underline{10000};000)(0^5;\frac{-1}{2}\frac{+1}{2}0)'$	$\mathbf{5}_{-2}$	3	0	0	0	-12	0	0	0	$3C_7$	0
T_6^0	$(\underline{-10000};000)(0^5;\frac{+1}{2}\frac{-1}{2}0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
T_7^0	$(\underline{-10000};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_9	-1296
T_7^0	$(\underline{+10000};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	$\mathbf{5}_{-2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_{10}	-1296
T_3^0	$(\underline{+++--};000)(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	$\overline{\mathbf{10}}_{-1}$	1	0	0	0	0	$+9$	$+3$	$-594(-\frac{33}{7})$	C_{11}	-1188
T_9^0	$(\underline{++---};000)(0^5;\frac{+1}{4}\frac{+1}{4}\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	-9	-3	$+594(+\frac{33}{7})$	C_{12}	$+1188$
				-16	-28	$+8$	0	$+18$	$+6$	-3492		-5406

Two families from T4 and one family from U

$$c_{a\gamma\gamma} \simeq \frac{-9312}{-3492} - 2 = \frac{2}{3}$$

The unification value

Adding contributions from other tables, -9312

3rd family members

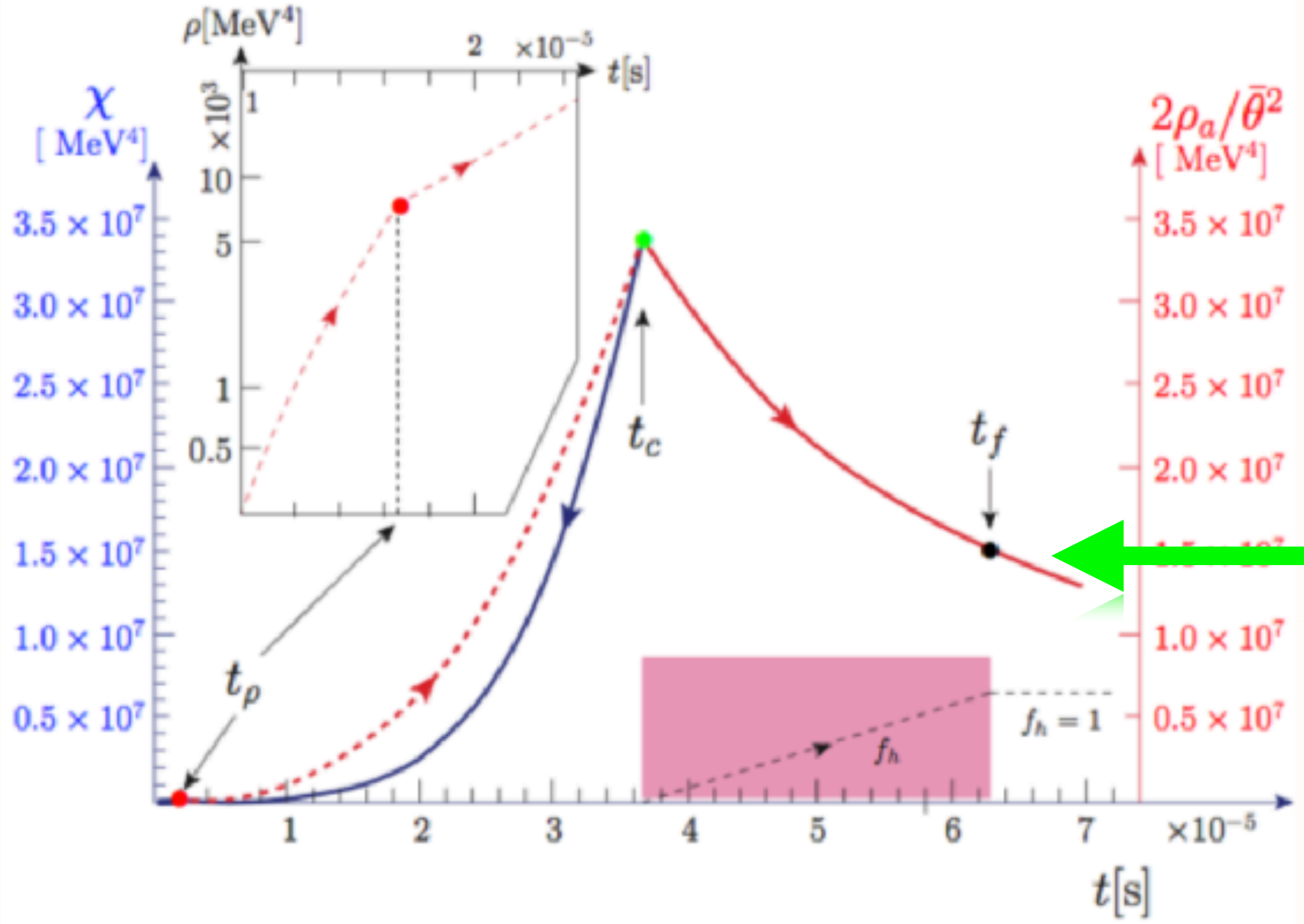
$SU(5)_{\rm flip}$ (Symbol)	Sect.	$U(1)_{\rm anom}$ (Charge)	$SU(5)_{\rm flip}$ (Symbol)	Sect.	$U(1)_{\rm anom}$ (Charge)
$\mathbf{1}_{-5}(S_1)$	U	$+5$	$\mathbf{1}_0(\sigma_1)$	T_4^0	-12
$\mathbf{1}_{-5}(S_{24}^a)$	T_4^0	-3	$\mathbf{1}_0(\sigma_2)$	T_4^0	-2
$\mathbf{1}_{-5}(S_{24}^b)$	T_4^0	-3	$\mathbf{1}_0(\sigma_3)$	T_4^0	-8
$\overline{\mathbf{10}}_{-1}(C_2)$	U	-13	$\mathbf{1}_0(\sigma_4)$	T_4^0	$+10$
$\mathbf{5}_{+3}(C_1)$	U	-1	$\mathbf{1}_0(\sigma_5)$	T_6	$+14$
$\mathbf{5}_{+3}(C_{3a})$	T_4^0	-3	$\mathbf{1}_0(\sigma_6)$	T_6	-4
$\mathbf{5}_{+3}(C_{3b})$	T_4^0	-3	$\mathbf{1}_0(\sigma_9)$	T_2^0	-6
$\overline{\mathbf{10}}_{-1}(C_{4a})$	T_4^0	-3	$\mathbf{1}_0(\sigma_{10})$	T_2^0	-6
$\overline{\mathbf{10}}_{-1}(C_{4b})$	T_4^0	-3	$\mathbf{1}_0(\sigma_{13})$	T_3	$+\frac{124}{7}$
$\overline{\mathbf{10}}_{-1}(C_{11})$	T_3	$-\frac{33}{7}$	$\mathbf{10}_{+1}(C_{12})$	T_9	$+\frac{33}{7}$
$\mathbf{5}_{-2}(H_u)$	T_6	0	$\mathbf{1}_0(\sigma_{15})$	T_9	$+\frac{30}{7}$
$\overline{\mathbf{5}}_{+2}(H_d)$	T_6	0	$\mathbf{1}_0(\sigma_{21})$	T_1^0	$+\frac{12}{7}$

There are only three 5 s. So, there must be one anti-symmetric combination from T4.

Matter under $SU(5)\times U(1)_X$

The SM singlet VEVs for the FN mechanism

$$\frac{\rho_a}{[\mathrm{eV}^4]} \simeq 5.68 \cdot 10^{-6}\, \bar{\theta}_1^{\,2} \left(\frac{m_a}{10^{-4}\ \mathrm{eV}}\right)^{-1.591\pm 0.008} \simeq 2.1 \cdot 10^{-6}\, \bar{\theta}_1^{\,2} \left(\frac{f_a}{10^{11}\ \mathrm{GeV}}\right)^{1.591\pm 0.008}$$



If $x = \frac{1}{10}$, we need $\frac{\bar{\theta}_{\mathrm{now}}}{\bar{\theta}_f} \approx 10^{-20}$ for the axion CDM for $\bar{\theta}_1 = 1$ and $f_a = 10^{11}$ GeV.